Introduction to theoretical uncertainties

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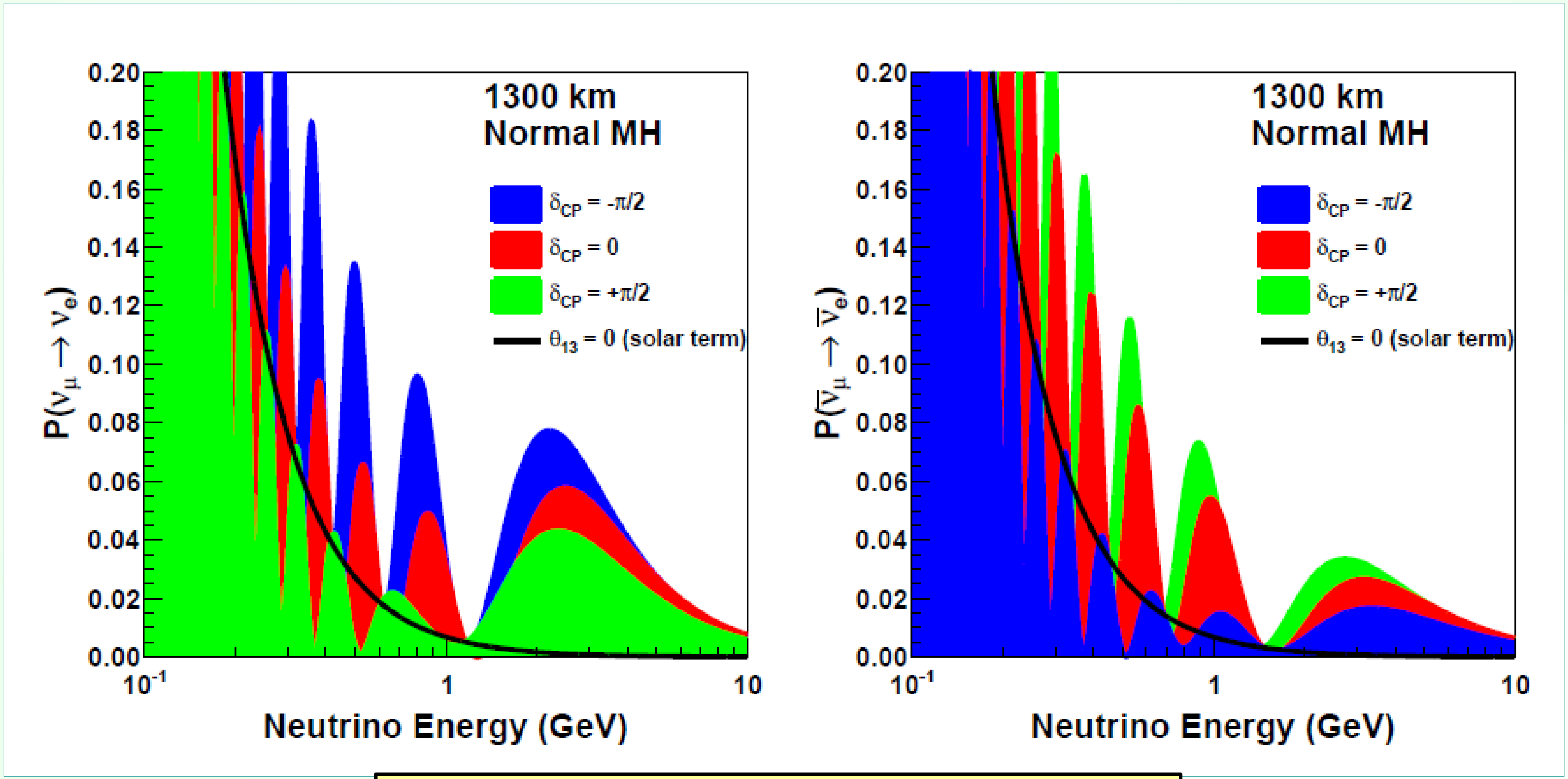
δ_{CP} from $\overset{(-)}{\nu}_e$ event distributions

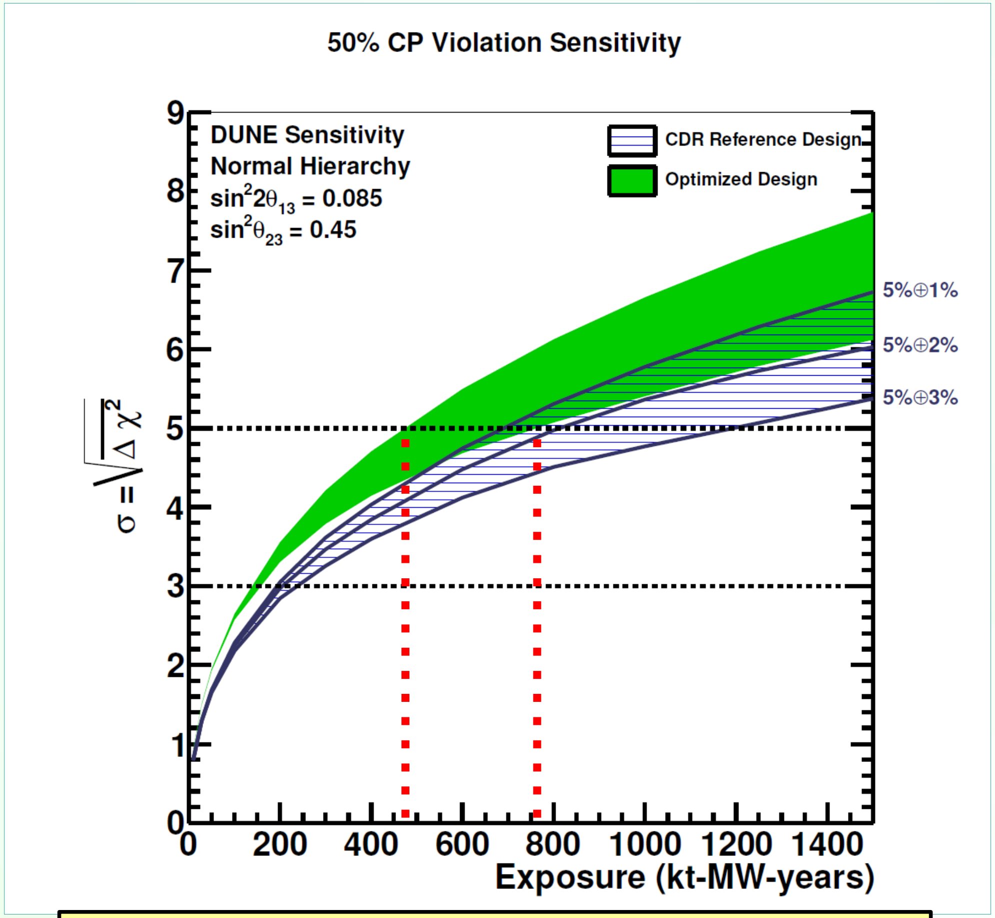
To find P($\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_e$) from event distributions, precise $\overset{(-)}{\nu}_e$ cross sections are necessary.



Acciari *et al.* (DUNE), arXiv:1512.06148

How relevant is the precision?

Dependence of DUNE's ~~CP~~ sensitivity on exposure for $\sigma(\nu_e)/\sigma(\nu_\mu)$ uncertainty between 1% and 3%.



Acciari *et al.* (DUNE), arXiv:1512.06148

ν_e to ν_μ cross sections' ratio

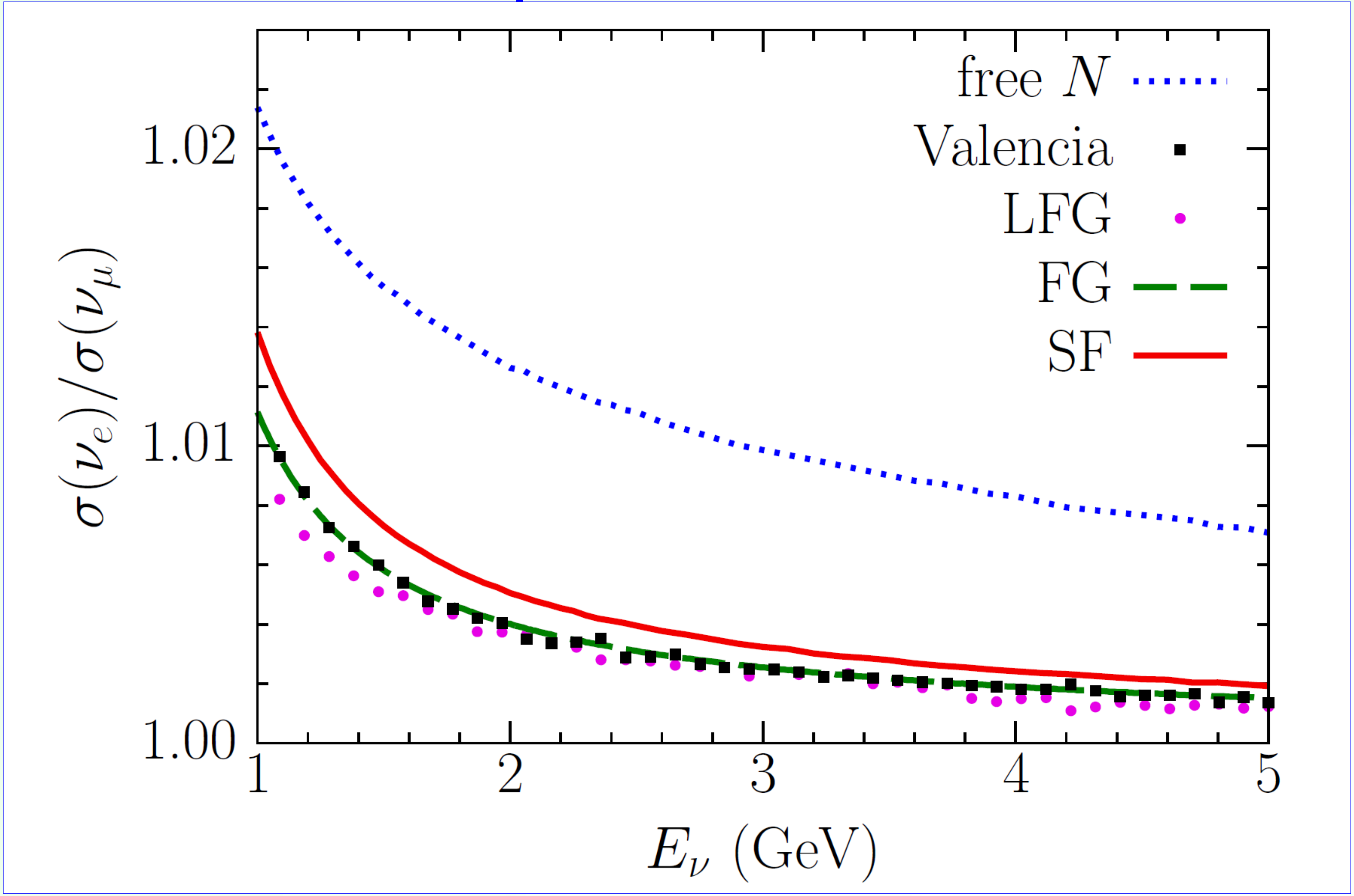
- In near detectors, event statistics lower ~100x for ν_e's then for ν_μ's. Higher flux and detector-response uncertainties.

- New concept of tagging $K^+ \rightarrow e^+\nu_e\pi^0$ events should allow the ν_e cross section determination with 1% uncertainty. [Longhin *et al.*, EPJ C 75, 155 (2015)]

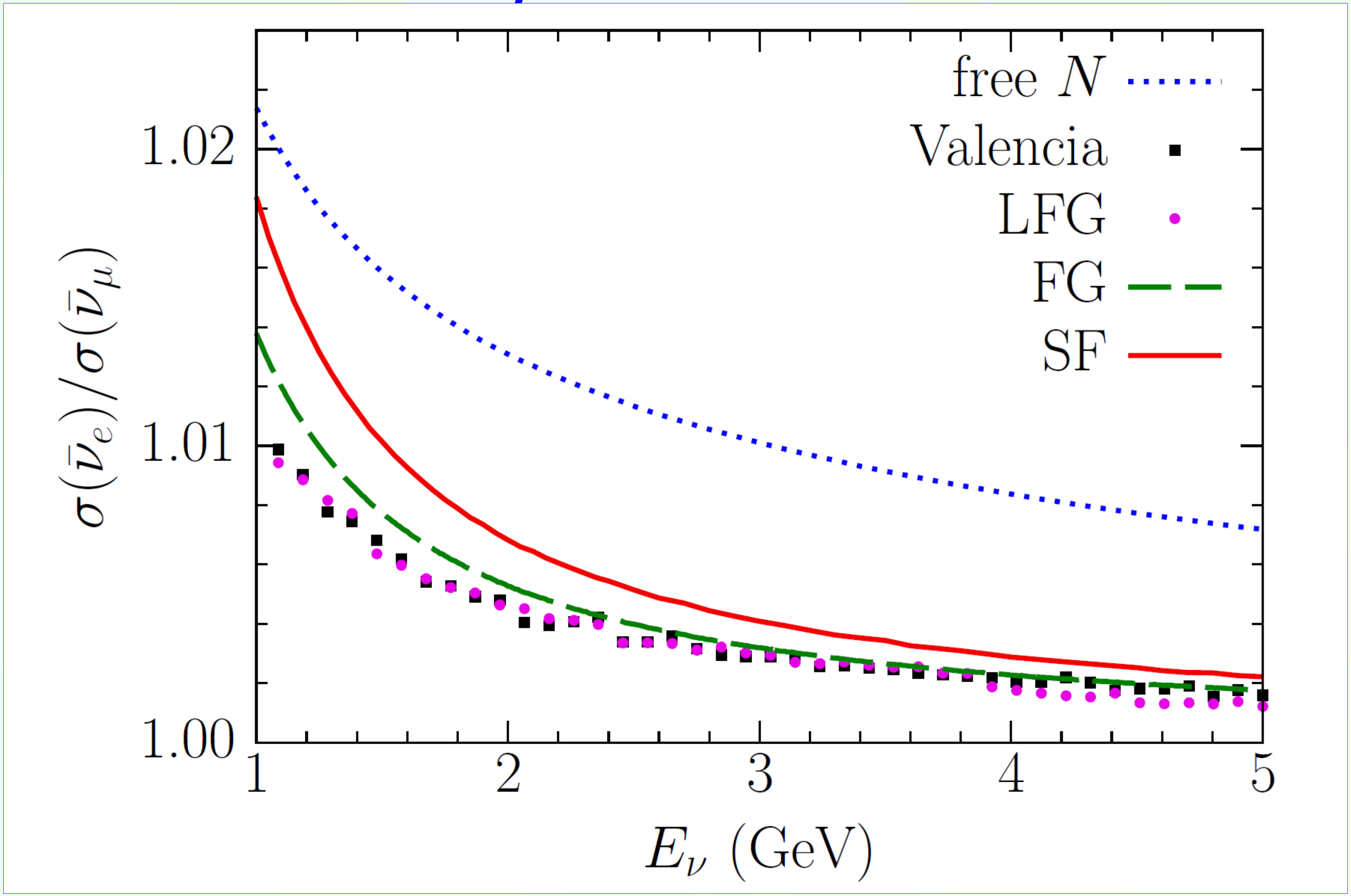
- Cross-section's dependence on the charged-lepton's mass is well known, knowing accurately nuclear $\sigma(\nu_\mu)$ we can obtain accurate $\sigma(\nu_e)$.

- Radiative corrections may be relevant [Day & McFarland, PRD 86, 053003 (2012)] but can be calculated with required precision.

CCQE ν_e to ν_μ cross sections' ratio



CCQE $\bar{\nu}_e$ to $\bar{\nu}_\mu$ cross sections' ratio



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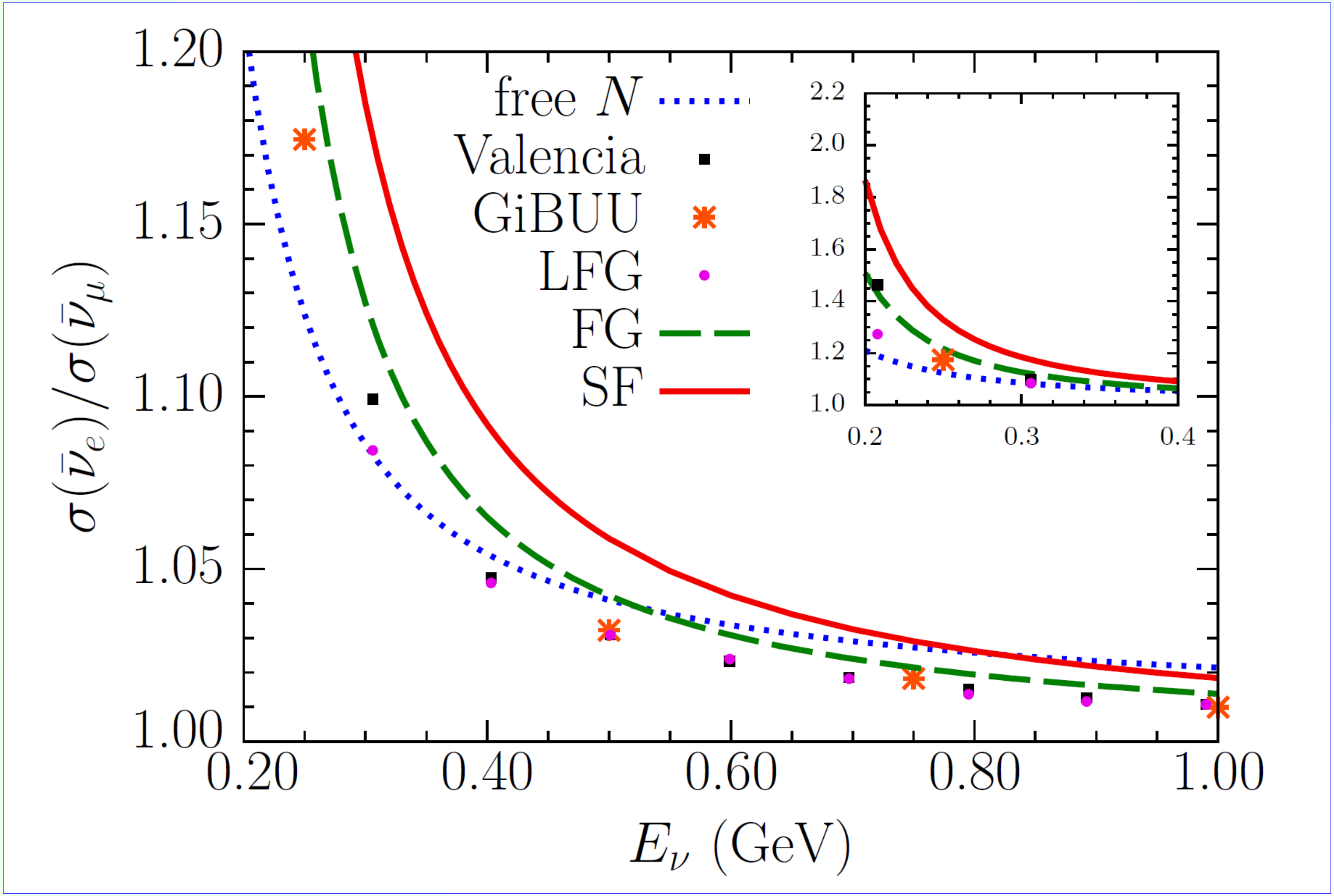

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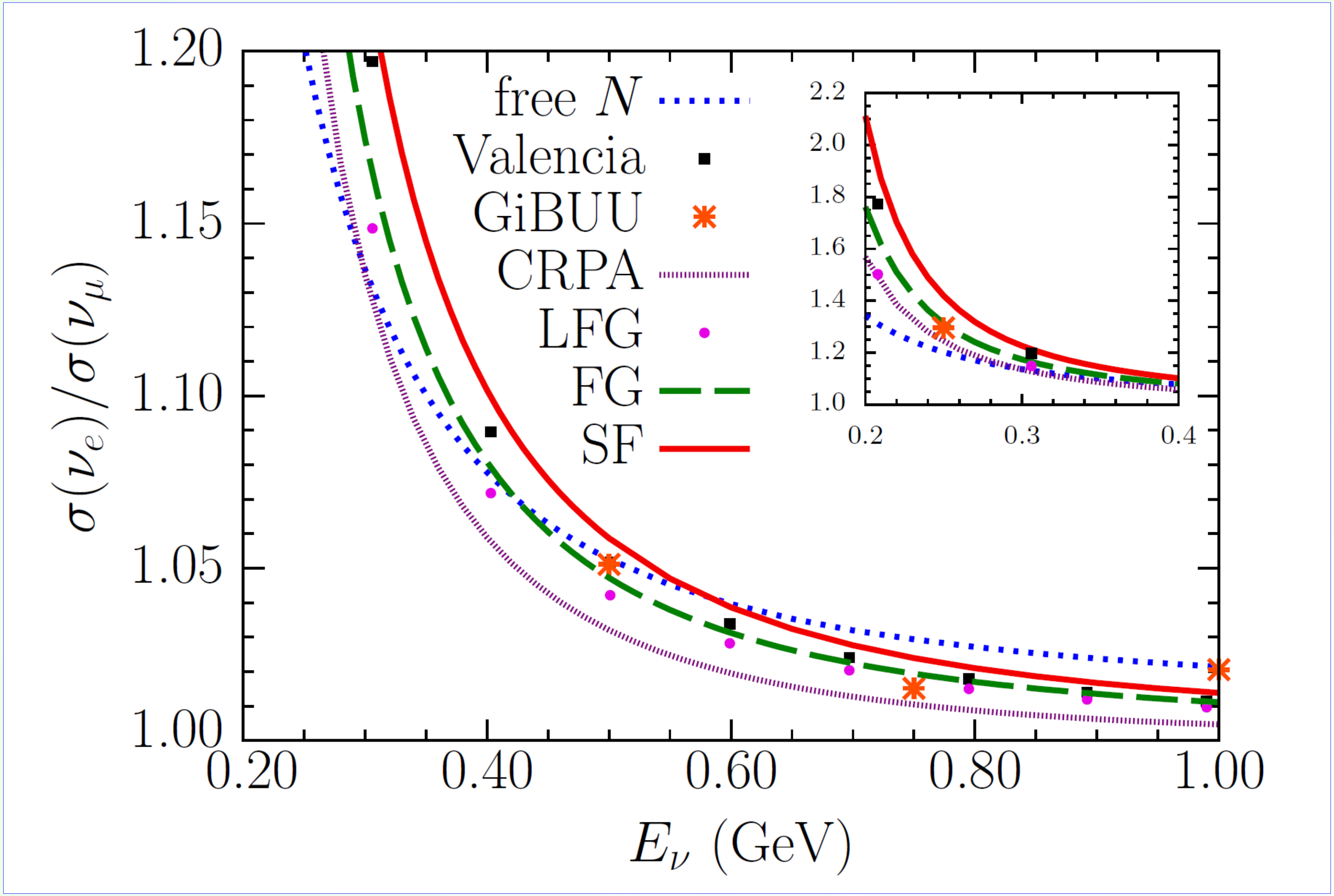
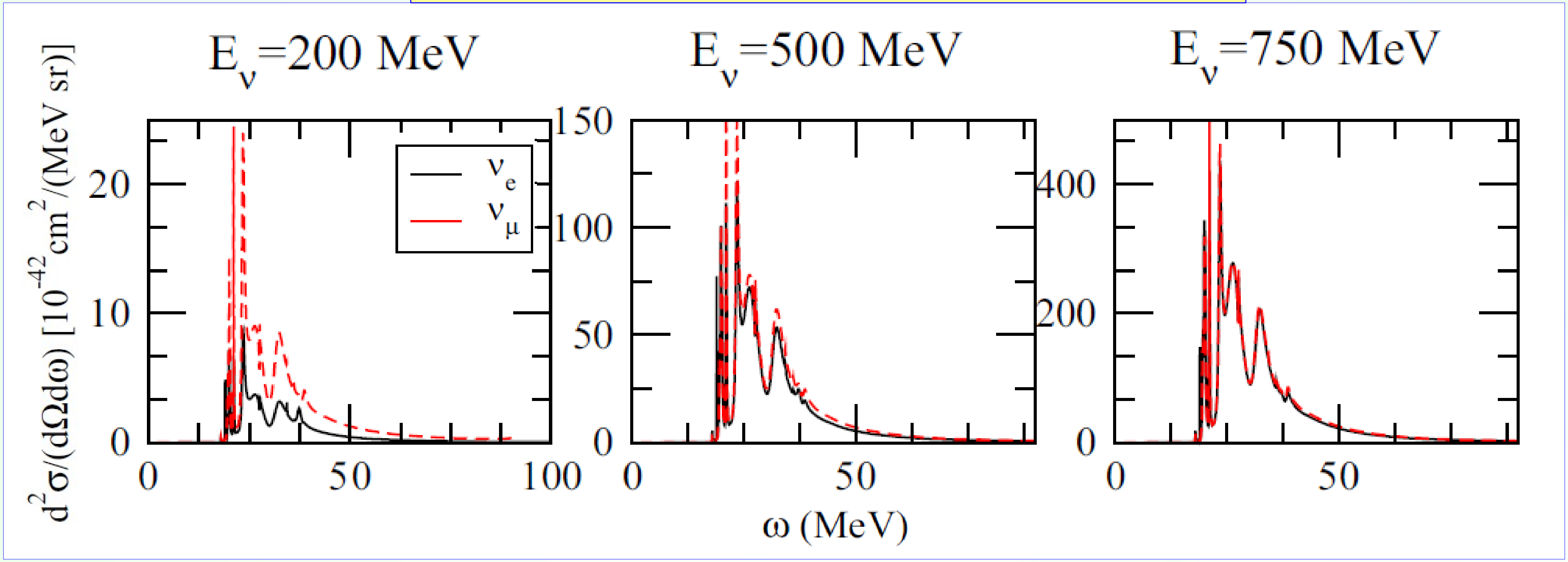


CCQE ν_μ and ν_e cross sections at 5°

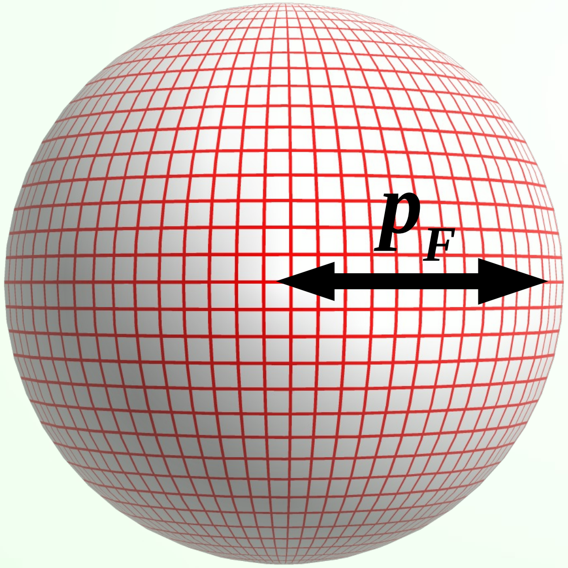
M. Martini *el al.*, PRC **94**, 015501 (2016)

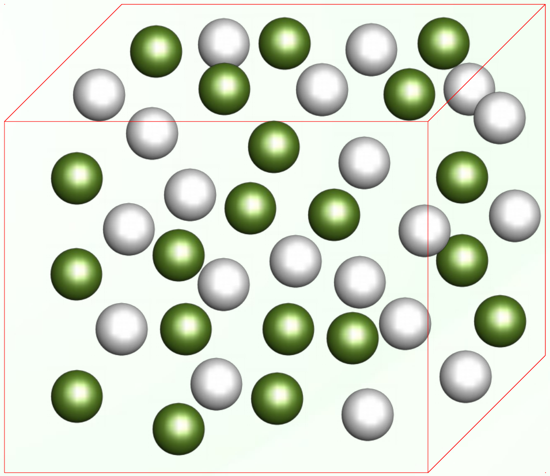


Conclusion: While at higher energies the ν_μ and ν_e cross sections practically coincide, at low energies and small scattering angles the ν_μ cross section is **higher** than the ν_e one, due to different |q|'s.

Fermi gas model

Nucleus treated as a fragment of non-interacting infinite nuclear matter of constant density. Eigenstates have definite momenta and energies $E_p = \sqrt{M^2 + \mathbf{p}^2} - \epsilon$.





Momentum space

Coordinate space

Relativistic Fermi gas

Lepton kinematics

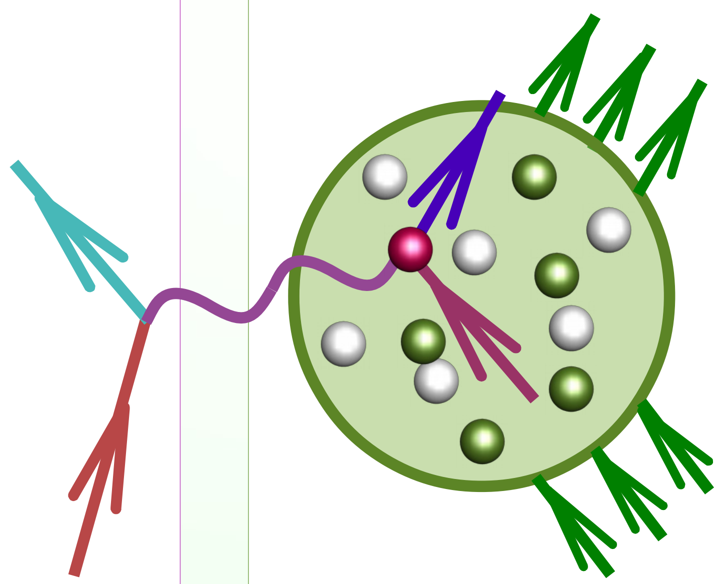
$$|\mathbf{q}| = \sqrt{E_\nu^2 - 2E_\nu|\mathbf{k}'|\cos\theta + |\mathbf{k}'|^2},$$

$$|\mathbf{k}'| = \sqrt{(E_\nu - \omega)^2 - m^2}.$$

Nucleon kinematics

$$|h - p_F| \le |\mathbf{q}| \le h + p_F,$$

$$h = \sqrt{(\omega - \epsilon + E_F)^2 - M^2} \text{ and } E_F = \sqrt{M^2 + p_F^2}$$



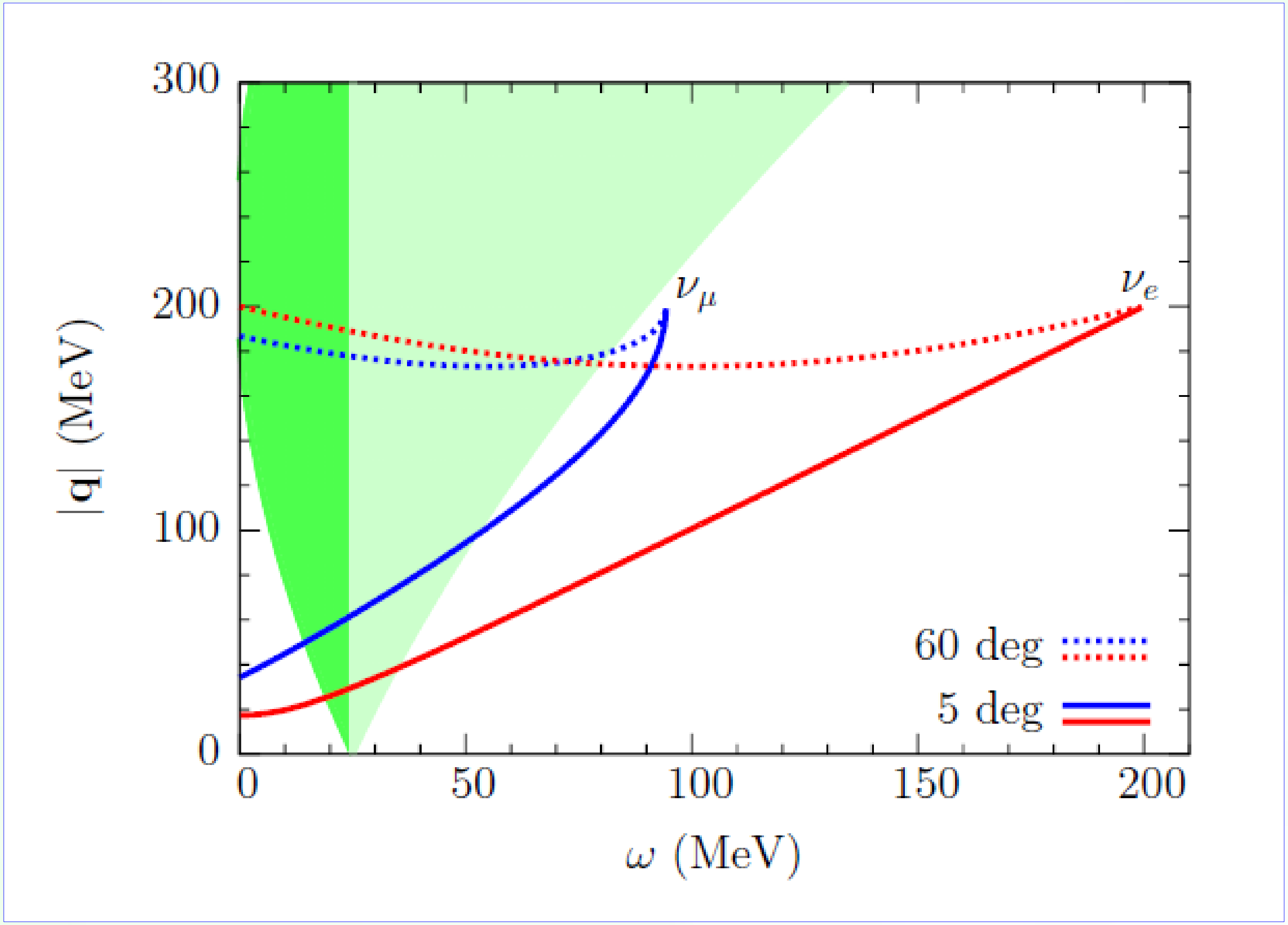
w/o Pauli blocking

$$\omega_{\min} = M - E_F + \epsilon$$

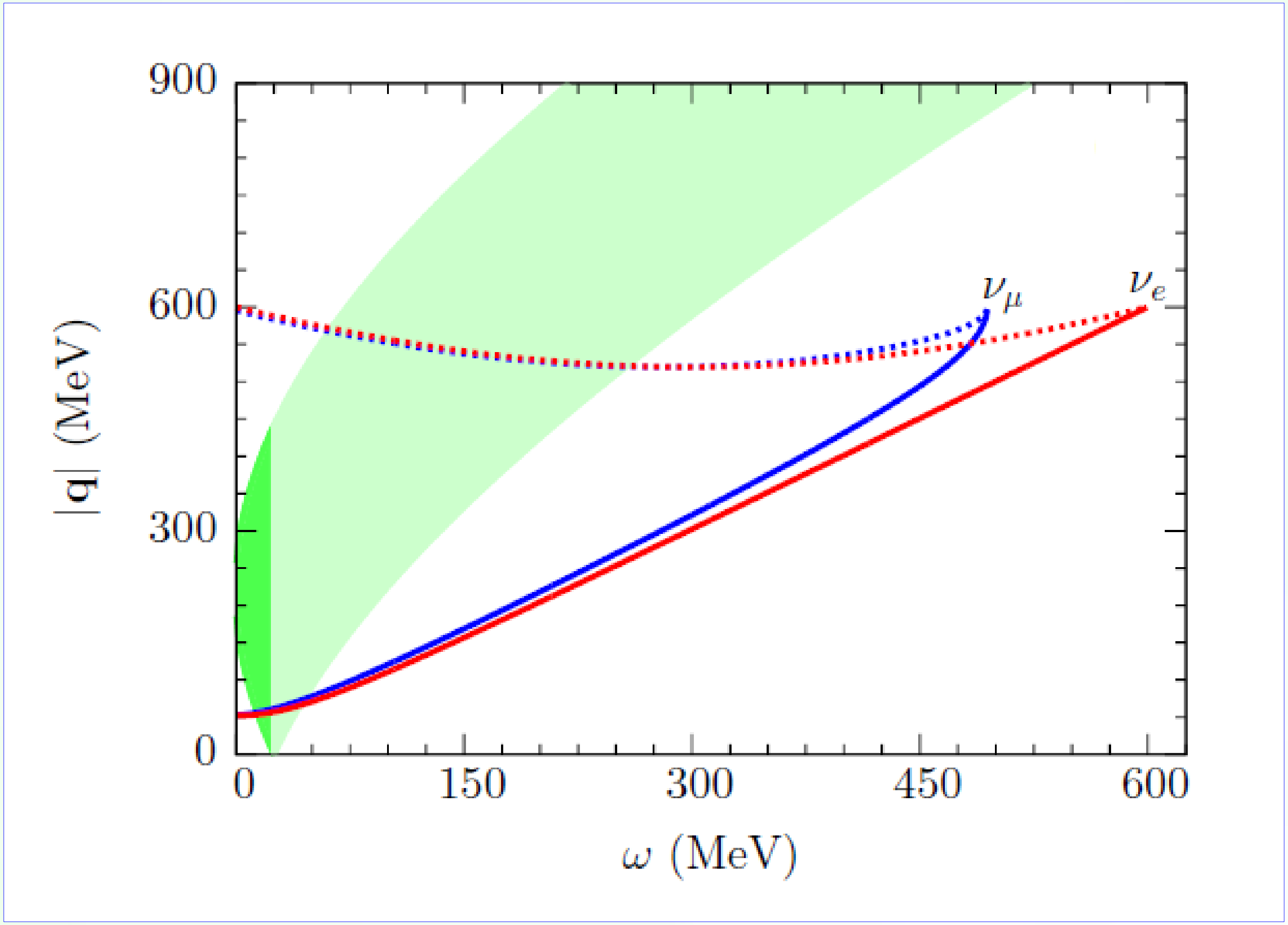
w/ Pauli blocking

$$\omega_{\min} = \epsilon$$

RFG, CCQE scattering at 200 MeV



RFG, CCQE scattering at 600 MeV



Cross-sections' ratio $\frac{d\sigma(\nu_\mu)}{d\cos\theta} \Big/ \frac{d\sigma(\nu_e)}{d\cos\theta}$

	$E_\nu = 200$ MeV		$E_\nu = 600$ MeV	
	5°	60°	5°	60°
RFG w/ PB	1.57	0.62	1.03	0.97
RFG w/o PB	0.73	0.71	0.96	0.97

Details of nuclear model can qualitatively change the mass dependence of the cross section.

The behavior is driven by the phase-space availability, rather than the kinematics.

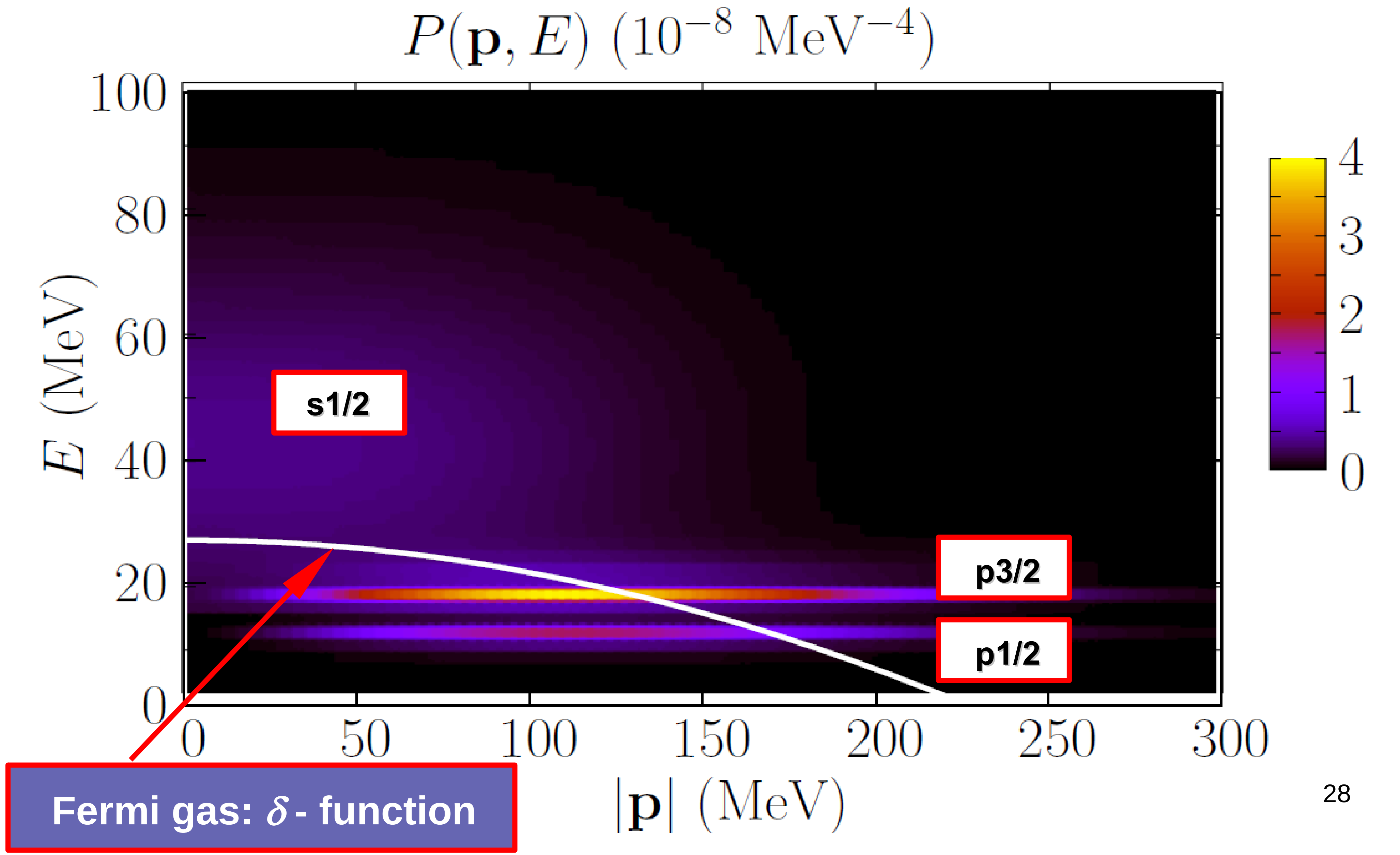
Relativistic Fermi gas

Reducing the available phase space, Pauli blocking changes the behavior of the cross section.

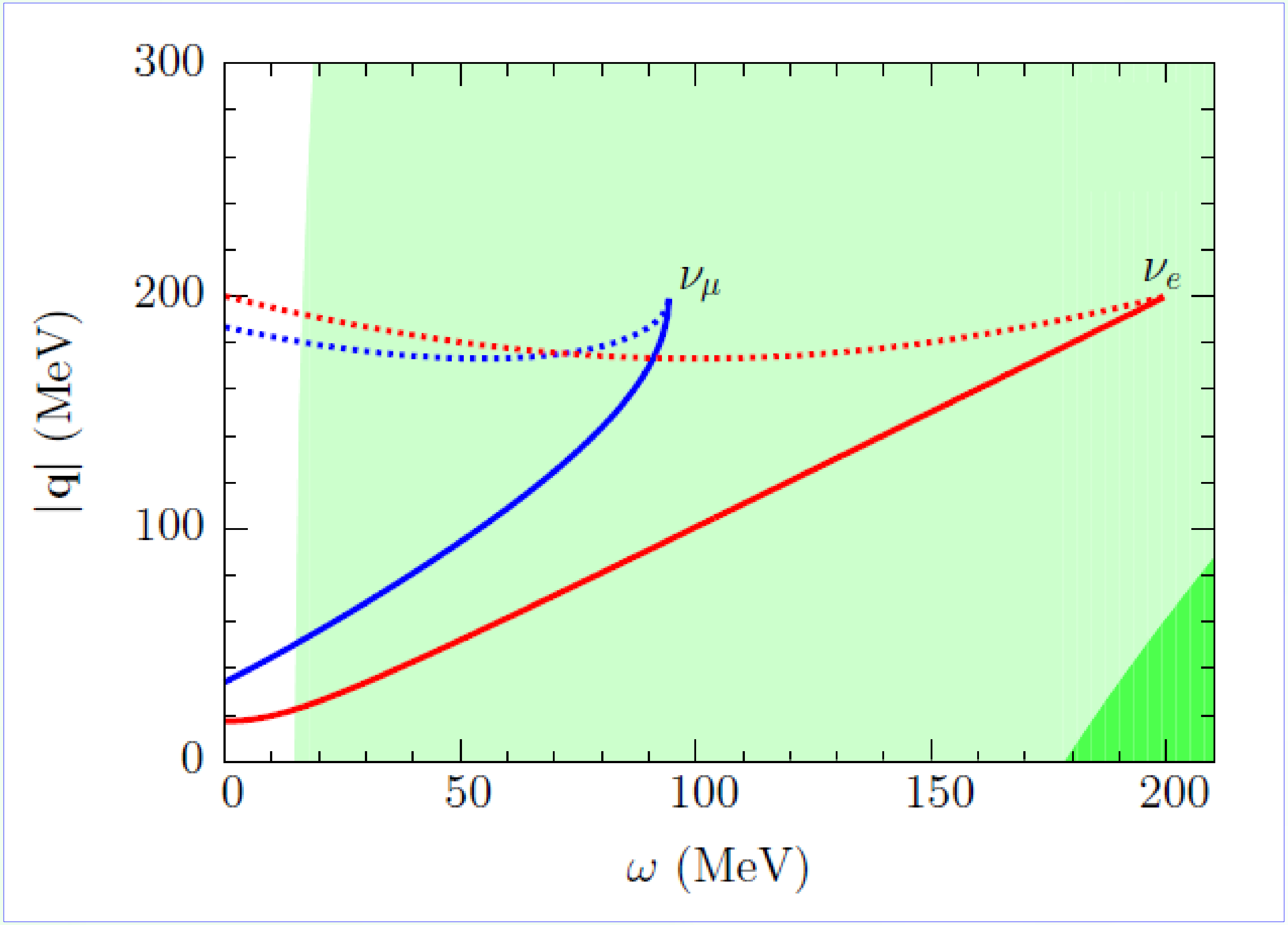
- Without Pauli blocking, the ν_e cross section converges to the ν_μ one from below, when energy increases.
- With Pauli blocking, close to the threshold the ν_μ cross section is lower than the ν_e one for any angle. At higher energies, there is a range of angles where

$$d\sigma(\nu_\mu)/d\cos\theta > d\sigma(\nu_e)/d\cos\theta.$$

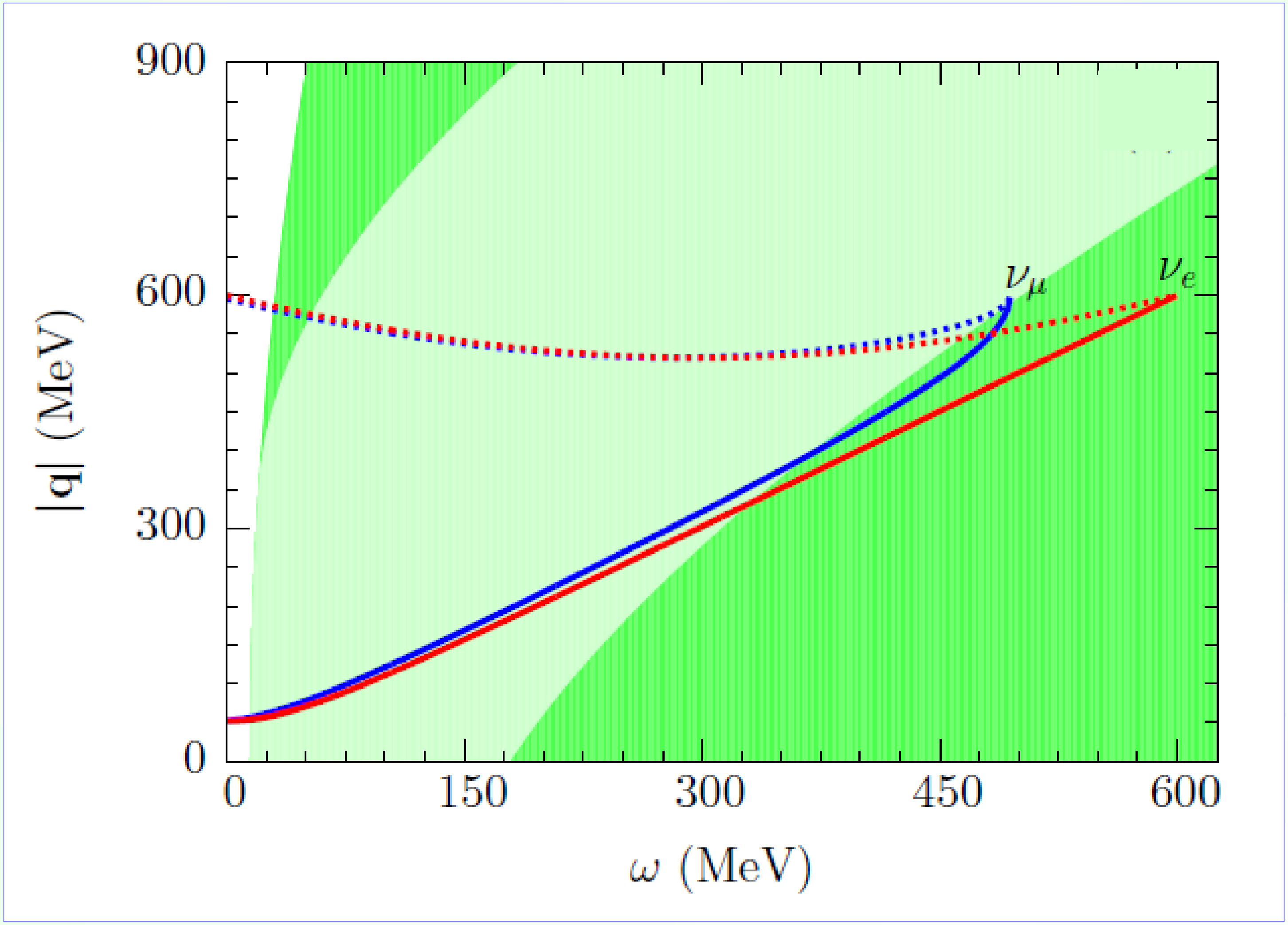
Shell model and spectral function



SF, CCQE scattering at 200 MeV



SF, CCQE scattering at 600 MeV



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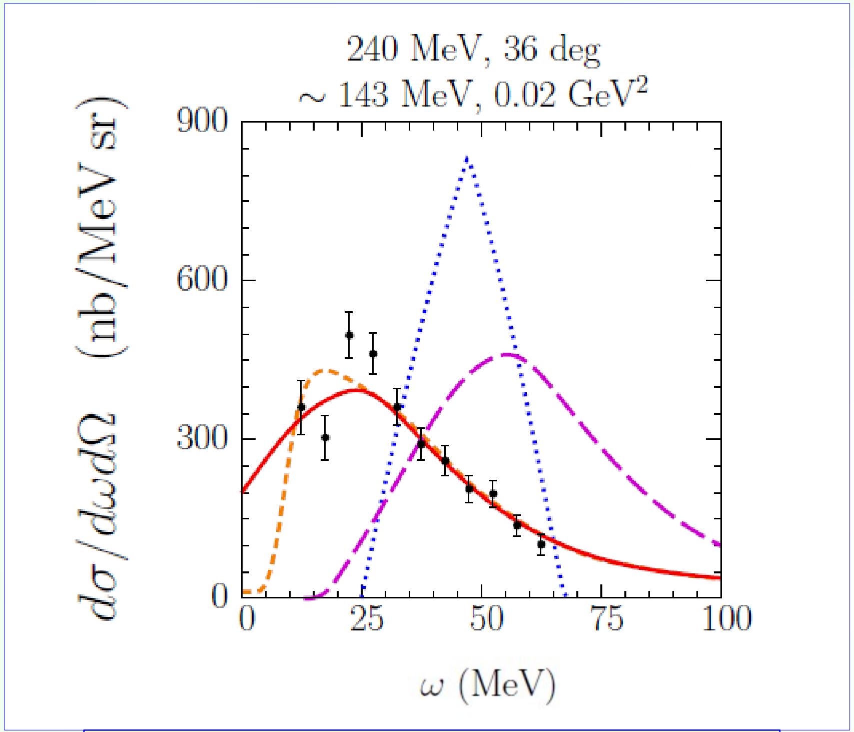
RFG w/ PB	1.57	0.62	1.03	0.97
RFG w/o PB	0.73	0.71	0.96	0.97
Mean-field SF	0.72	0.53	0.96	0.97
Full SF	0.71	0.52	0.96	0.97

The ratio is governed mostly by the shell contribution, extracted from (*e*,*e'p*) data.

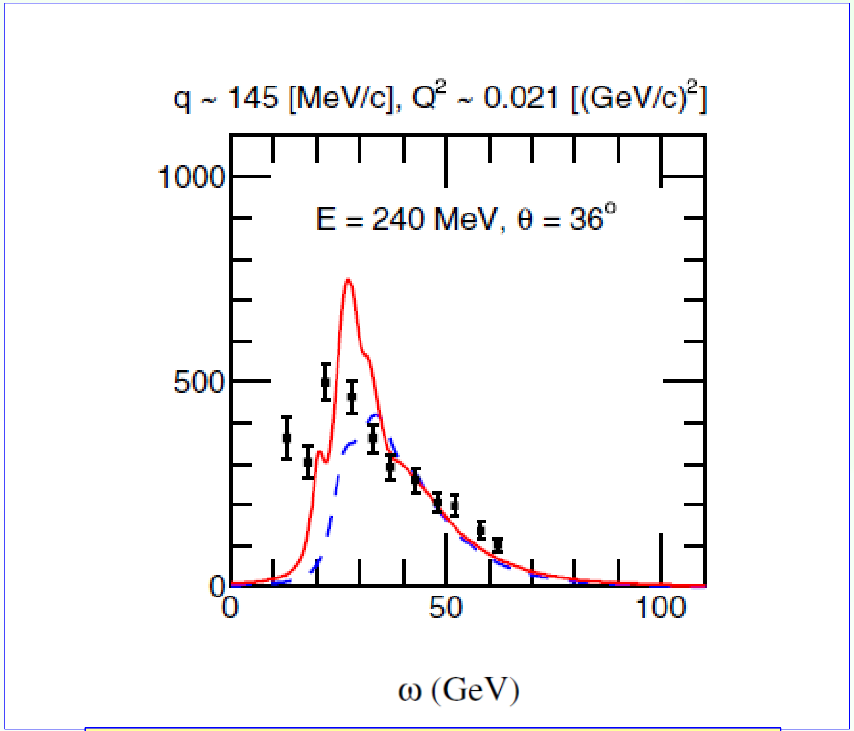
Shell model/Spectral function

- The available phase space is broad due to energy and momentum distributions of the shell states.
- The ν_μ cross section is lower than the ν_e one **for any angle**, but converges to it as energy increases.

Nuclear models are converging

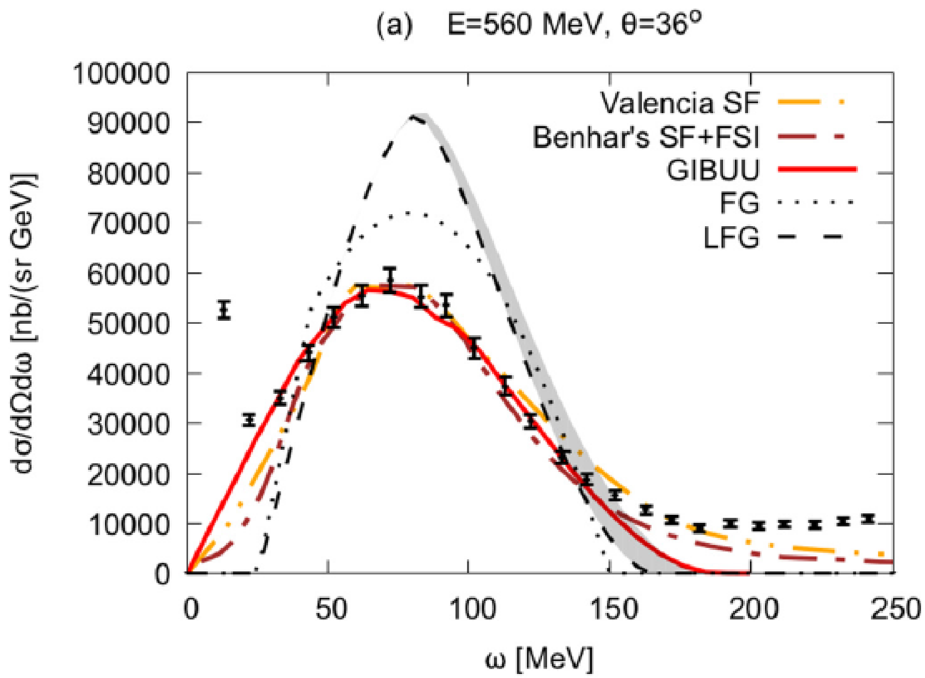


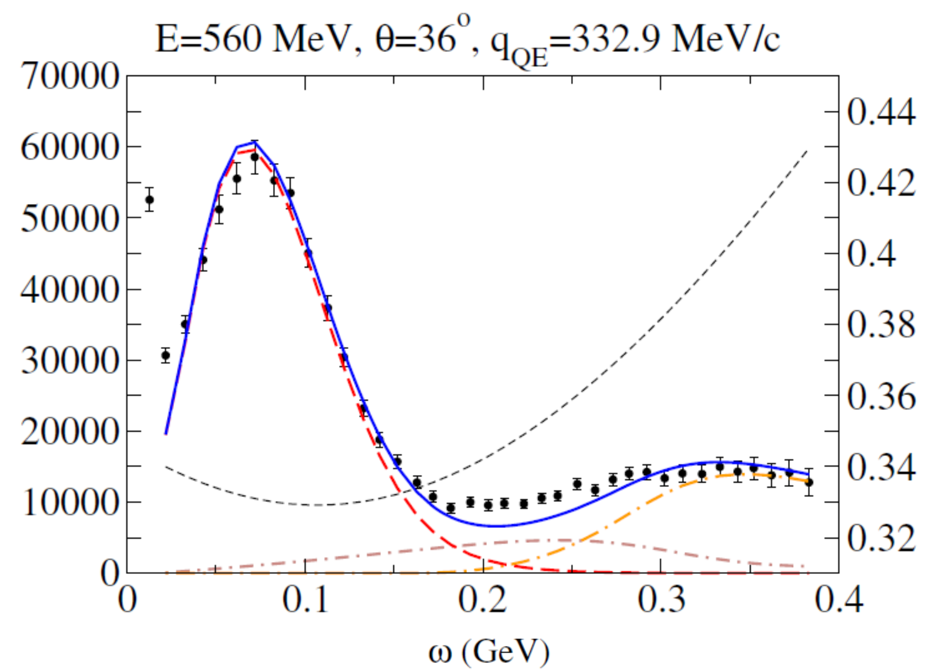
AMA *et al.*,
PRD **91**, 033005 (2015)



V. Pandey *et al.*,
PRC **92**, 024606 (2015)

Nuclear models are converging





J. E. Sobczyk,
PRC **96**, 045501 (2017)

G. D. Megias *et al.*,
PRC **94**, 013012 (2016)

Model differences

- Models developed to reproduce inclusive electron-scattering data may give similar results starting from different physics assumptions.
- Treating the initial states differently, they lead to different exclusive cross sections (hadron distributions).
- For long-baseline neutrino experiments, particularly those using calorimetric energy reconstruction, exclusive cross sections are essential.

Summary

- Next-generation appearance experiments require $\sigma(\nu_\mu)/\sigma(\nu_e)$ known with challenging precision.

- The ratio's precise measurement currently not possible. It may be necessary to rely on input from theory.

- For differential cross sections at low energies, different nuclear models may yield qualitatively different results.

- The $\sigma(\nu_\mu) - \sigma(\nu_e)$ difference small above 1 GeV, but significant at energies below ~600 MeV.

Backup slides

Charged-Current Cross section

Well-known dependence on the charged-lepton's mass

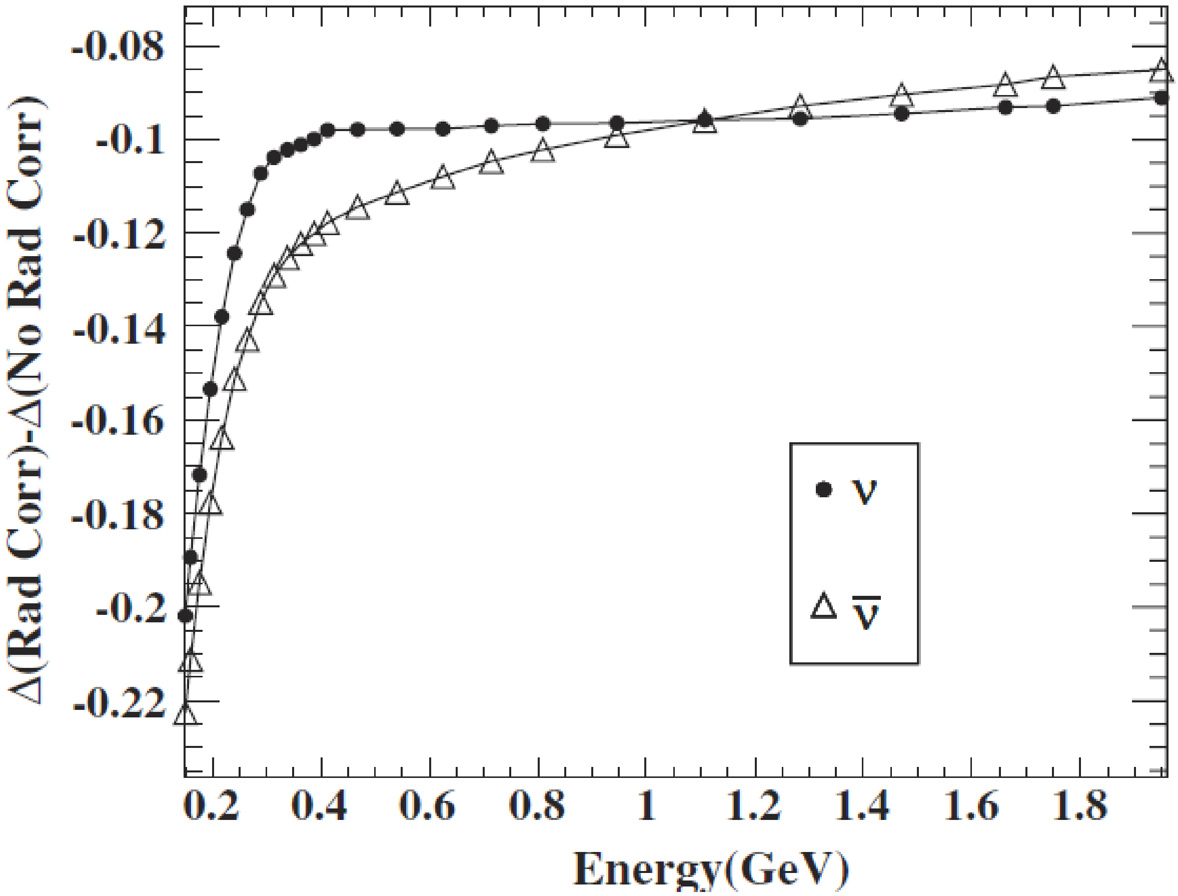
$$\frac{d\sigma}{d\omega d|\mathbf{q}|} = \frac{(G_F \cos\theta_C)^2}{2\pi} \frac{|\mathbf{q}|}{|\mathbf{k}|^2} \Big[v_{CC} R_{CC}(\omega, |\mathbf{q}|) + v_{CL} R_{CL}(\omega, |\mathbf{q}|) + v_{LL} R_{LL}(\omega, |\mathbf{q}|) + v_T R_T(\omega, |\mathbf{q}|) + v_{T'} R_{T'}(\omega, |\mathbf{q}|) \Big]$$

$$\begin{aligned} v_{CC} &= E_k E_{k'} + k_x k'_x + k_z k'_z, \\ v_{CL} &= -2(E_k k'_z + E_{k'} k_z), \\ v_{LL} &= E_k E_{k'} - k_x k'_x + k_z k'_z, \\ v_T &= E_k E_{k'} - k_z k'_z, \\ v_{T'} &= 2(E_{k'} k_z - E_k k'_z), \end{aligned}$$

$$\begin{aligned} k_x &= \frac{|\mathbf{k} \times \mathbf{k}'|}{|\mathbf{q}|} = k'_x, \\ k_y &= 0 = k'_y, \\ k_z &= \frac{|\mathbf{k} \cdot \mathbf{q}|}{|\mathbf{q}|}, \\ k'_z &= \frac{|\mathbf{k}' \cdot \mathbf{q}|}{|\mathbf{q}|}, \end{aligned}$$

See e.g. Amaro *et al.*, PRC 71, 015501 (2005)

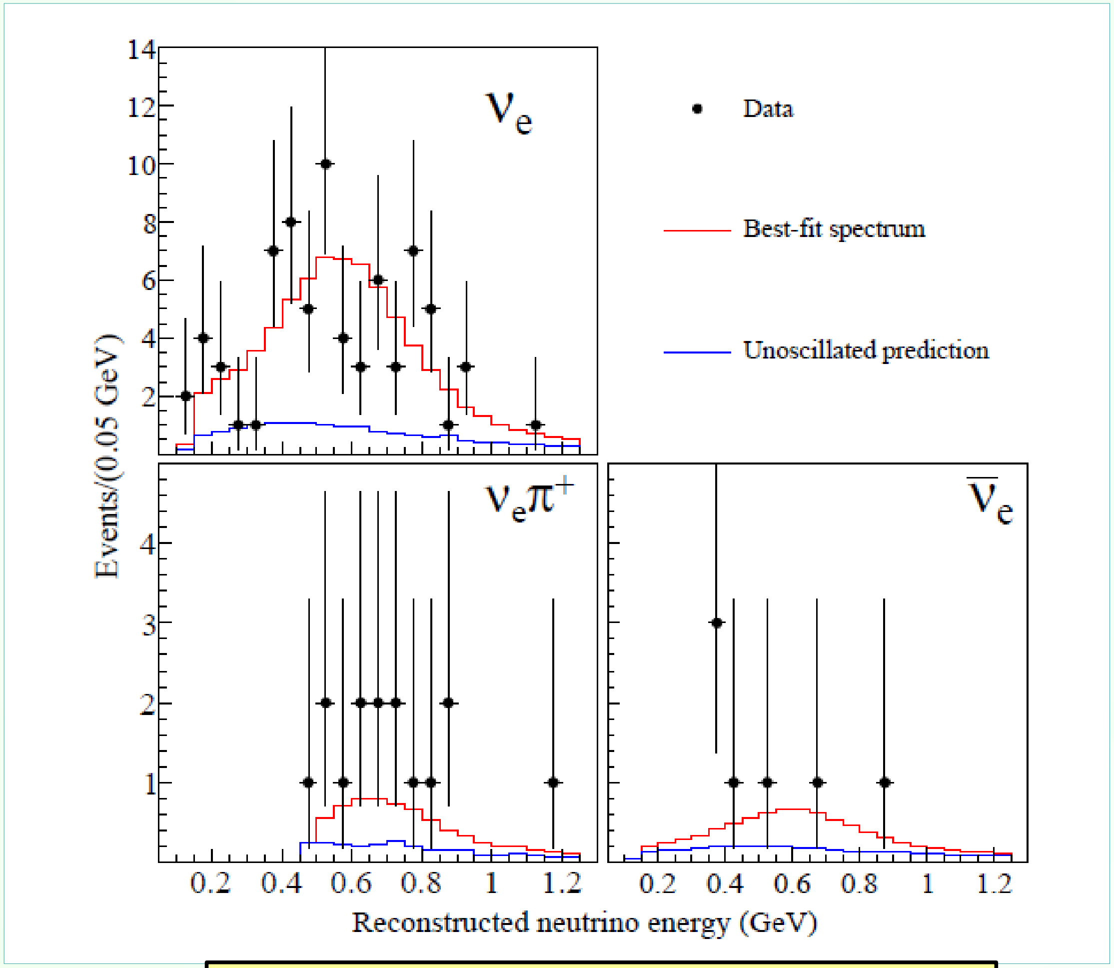
Radiative corrections



$$\Delta = \frac{\sigma(\nu_\mu) - \sigma(\nu_e)}{\sigma(\nu_e)}$$

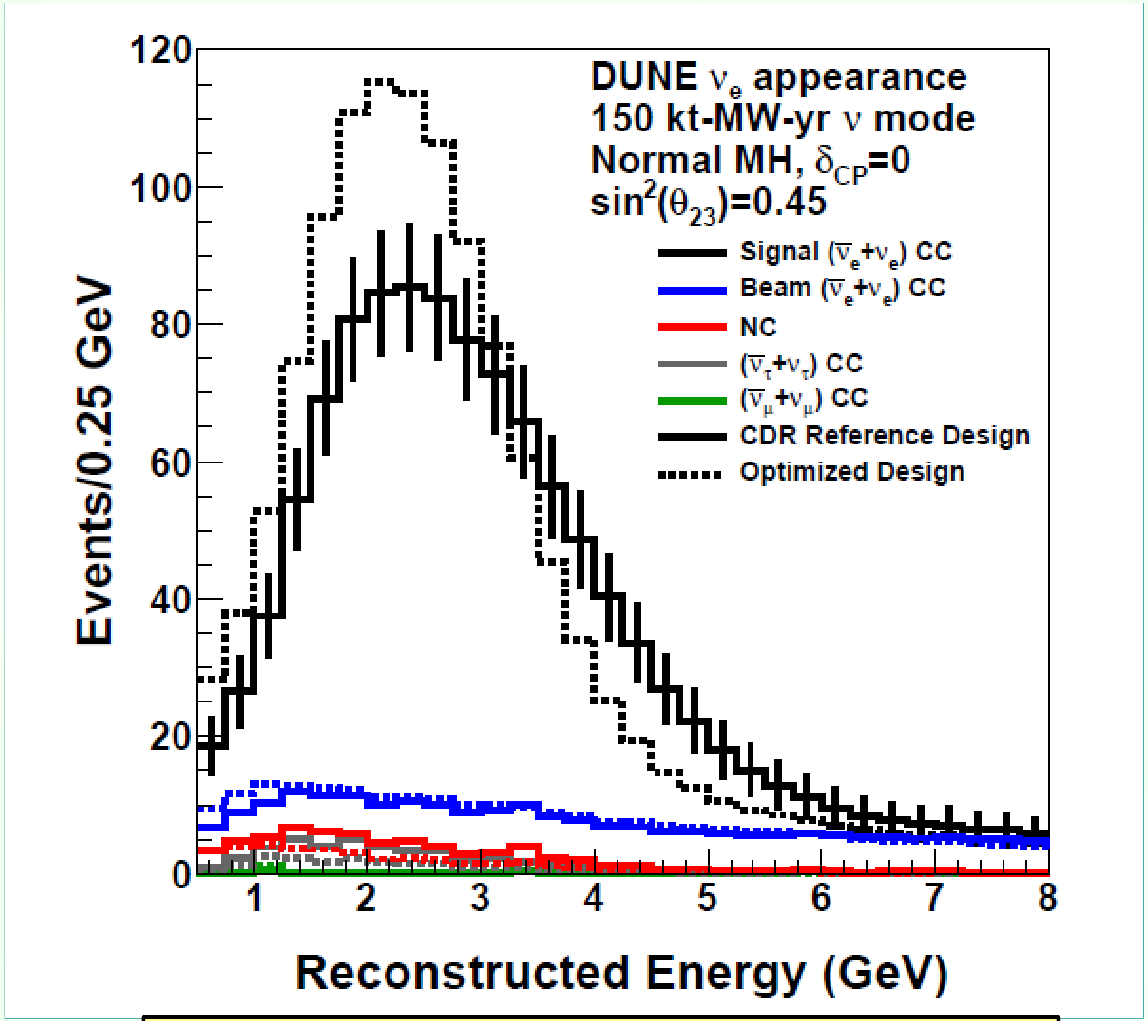
Day & McFarland,
PRD 86, 053003 (2012)

Spectra in T2K



Abe *et al.* (T2K), arXiv:1807.07891

Expected spectra in DUNE



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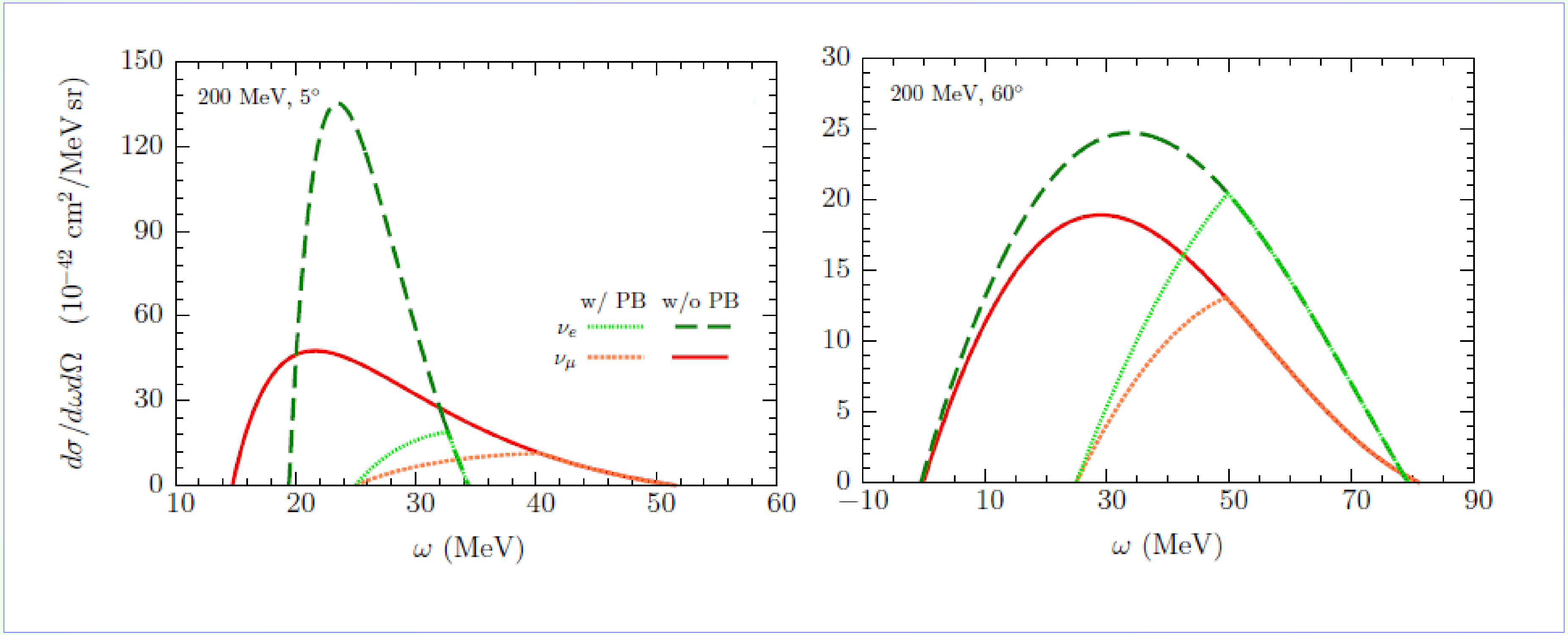
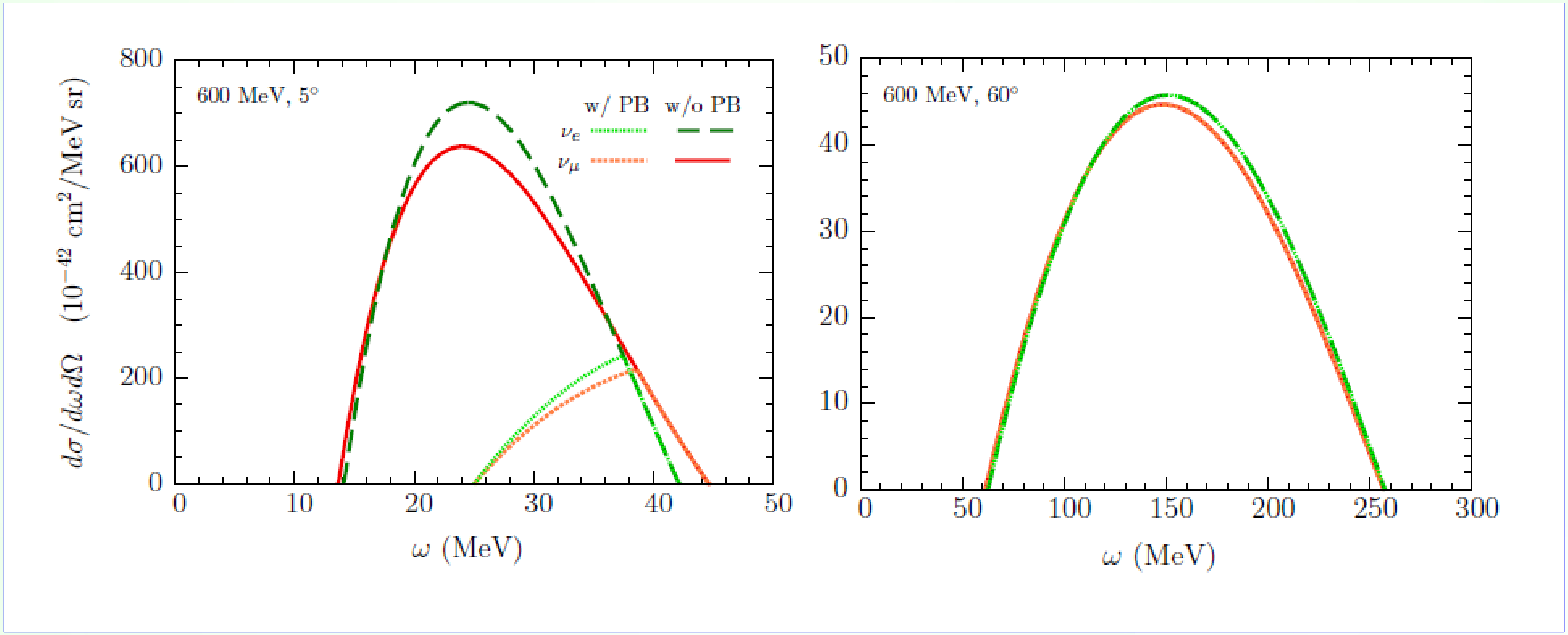
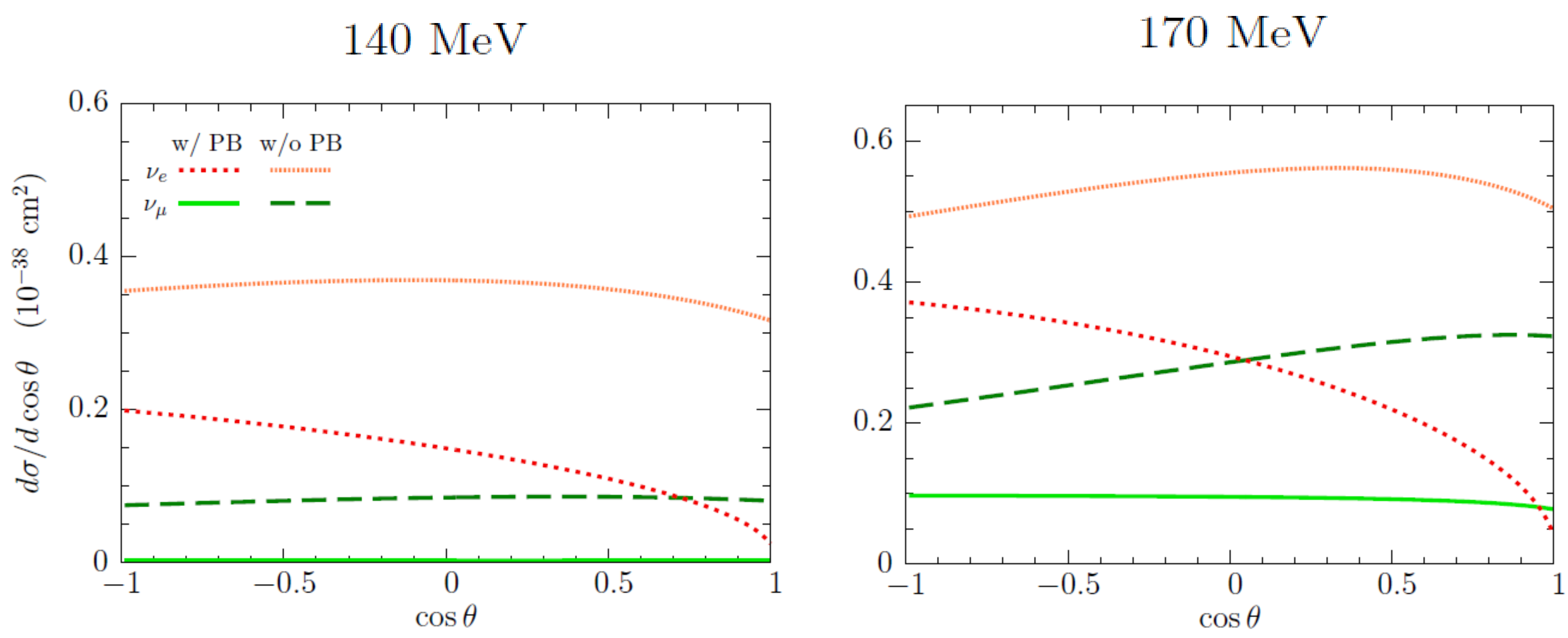
CCQE scattering at 200 MeV



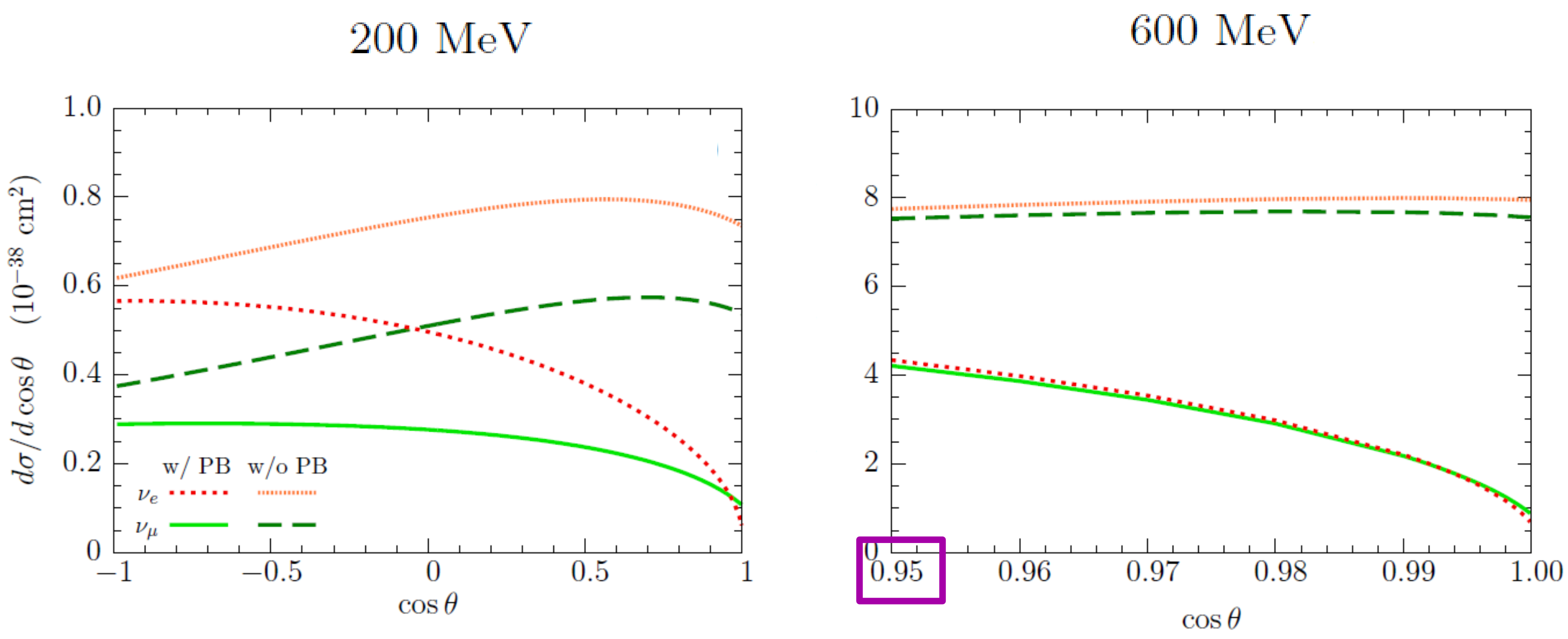
CCQE scattering at 600 MeV



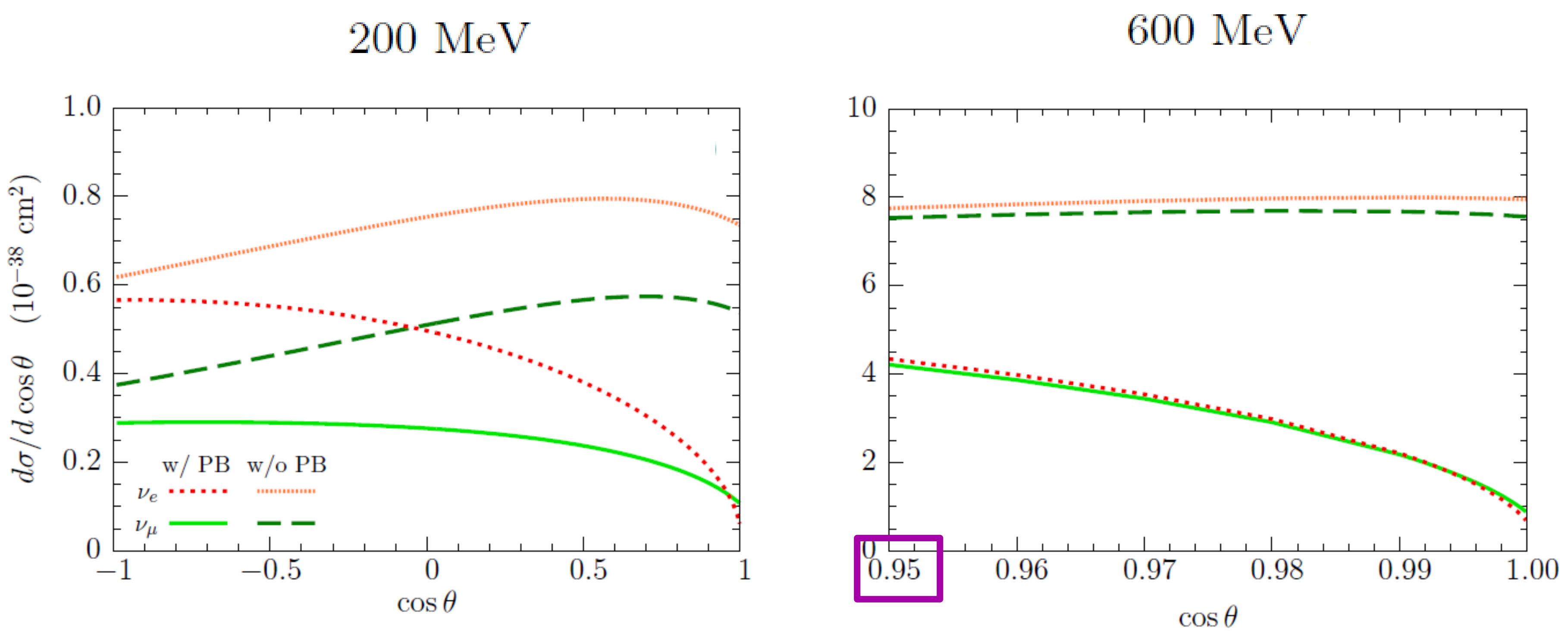
Differential cross section *dσ/d*cos*θ*



Differential cross section $d\sigma/d\cos\theta$



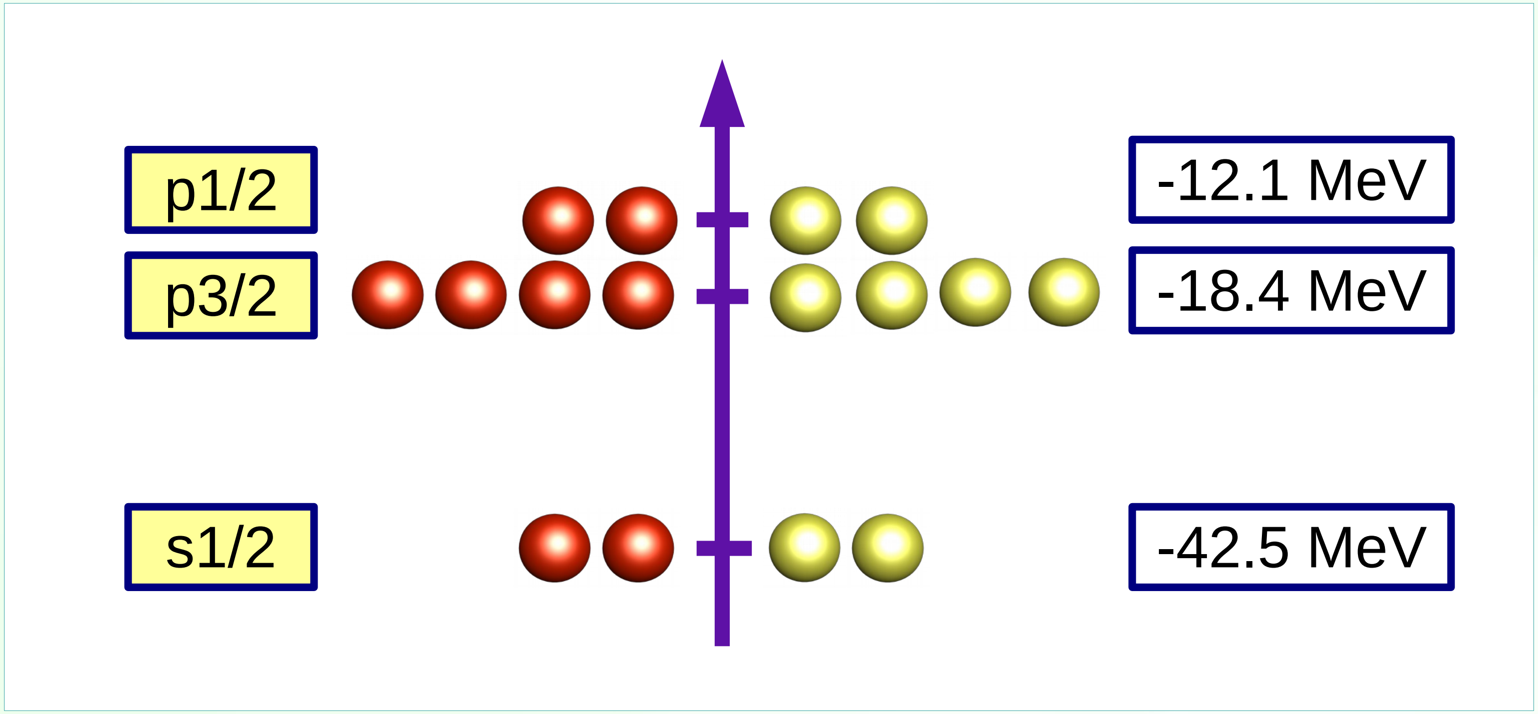
Differential cross section *dσ/d*cos*θ*



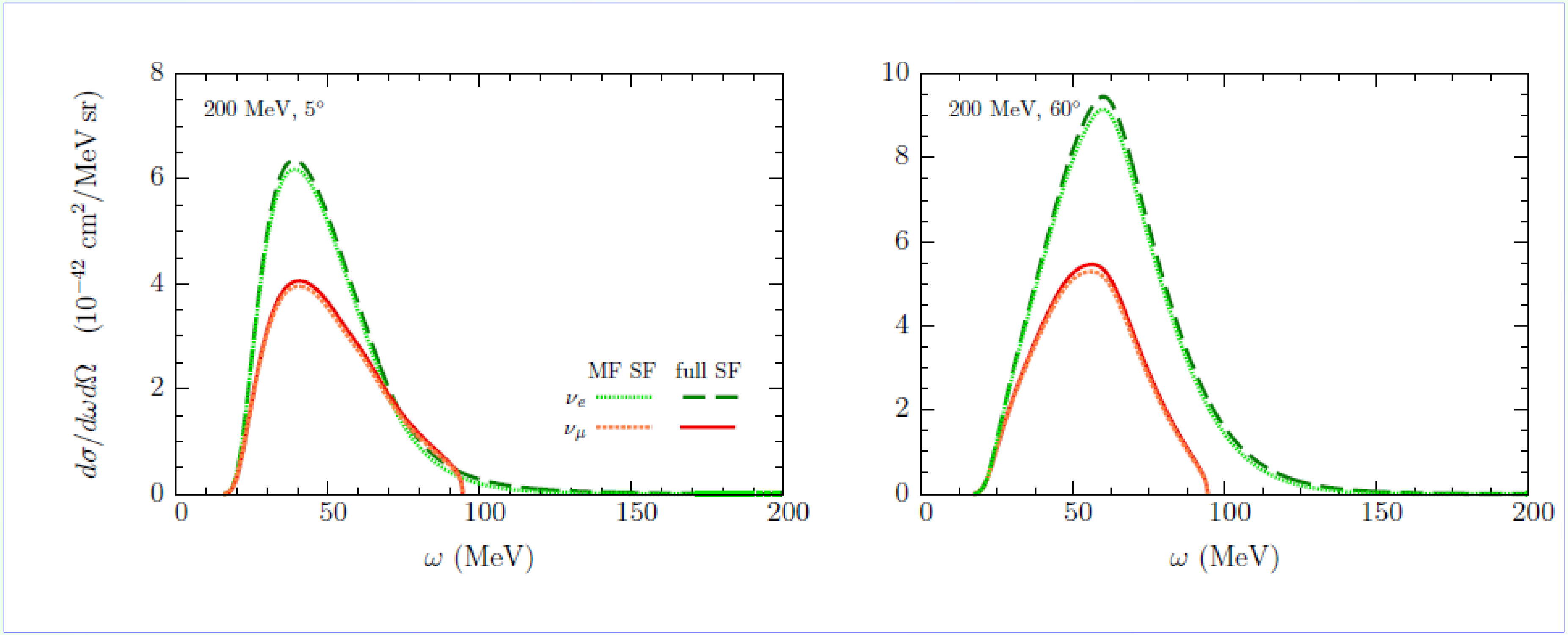
Shell model

In a spherically symmetric potential, the eigenstates correspond to definite values of the total angular momentum.

No correspondence between nucleon momentum and energy, unlike in the Fermi gas model.



CCQE scattering at 200 MeV



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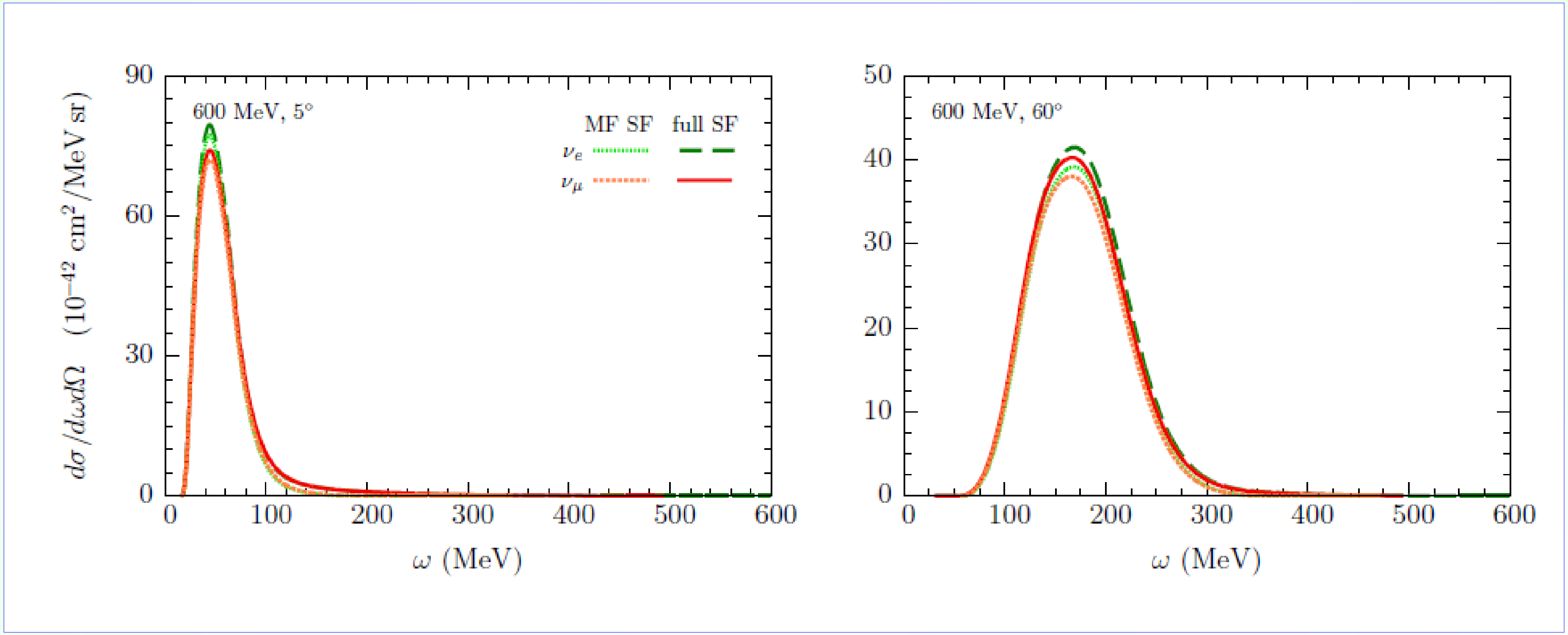

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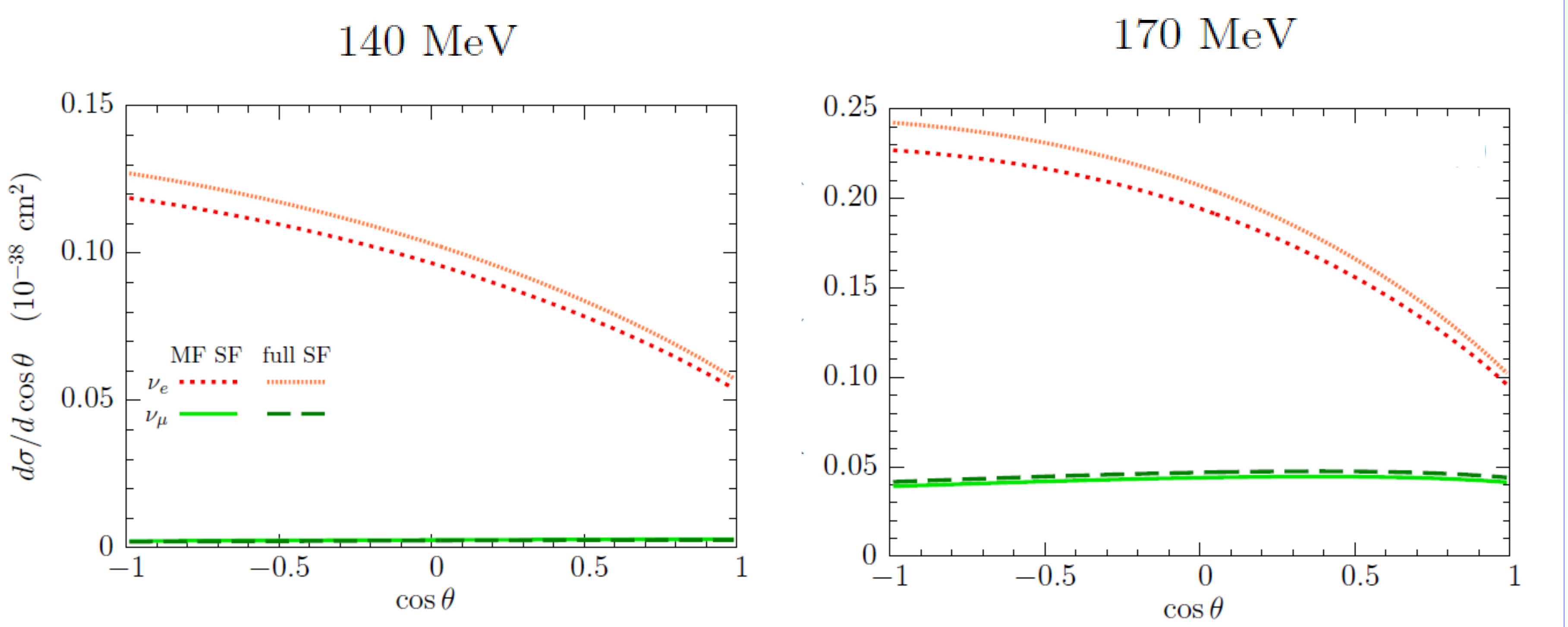

Differential cross section *dσ/d*cos*θ*