Ab initio theoretical approaches

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V. Somà (Cea-Irfu)

Outline

GFMC →

😄 Virtually exact electroweak response of nuclei A ≤ 12, quasi elastic sector

😢 Non relativistic approximation in the kinematics and (partially) in the current operators

SCGF
CBF →

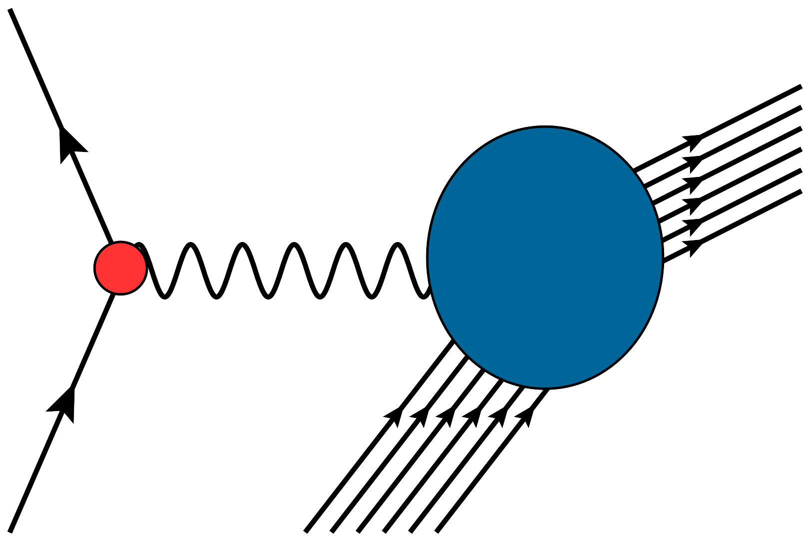
😄 Realistic Spectral Function + Impulse Approximation: Relativistic effects accounted for, accurate description of the initial state

😢 Approximation are made in the description of hadronic final state

Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus and the hadronic final state is undetected can be written as

$$\frac{d^2\sigma}{d\Omega_\ell dE_{\ell'}} = L_{\mu\nu} W^{\mu\nu}$$



- The Leptonic tensor is fully specified by the lepton kinematic variables. For instance, in the electron-nucleus scattering case

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k\, k') + i\epsilon_{\mu\nu\alpha\beta} k'^{\alpha} k^{\beta}$$

- The Hadronic tensor contains all the information on target response

$$W^{\mu\nu} = \sum_f \langle 0 | J^{\mu\dagger}(q) | f \rangle \langle f | J^\nu(q) | 0 \rangle \delta^{(4)}(p_0 + q - p_f)$$

Non relativistic nuclear many-body theory (NMBT) provides a fully consistent theoretical approach allowing for an accurate description of |0>, independent of momentum transfer.

Non relativistic Nuclear Many Body Theory

• Within NMBT the nucleus is described as a collection of A point-like nucleons, the dynamics of which are described by the non relativistic Hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

The nuclear energy spectrum can be accurately determined

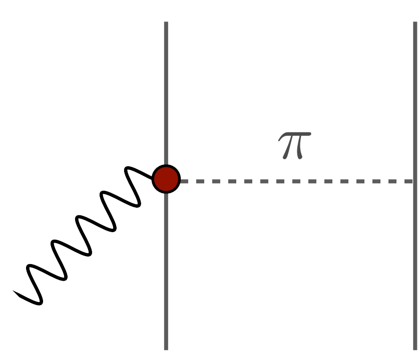
$$H\,|0\rangle = E_0\,|0\rangle \quad , \quad H\,|f\rangle = E_f\,|f\rangle$$

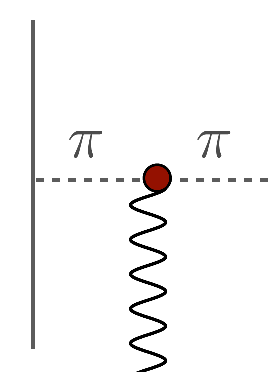
The nuclear electromagnetic current is constrained through the continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

• The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.

• Non relativistic expansion of $\mathbf{J}_{\rm EM}$, in powers $|\mathbf{q}|/m$



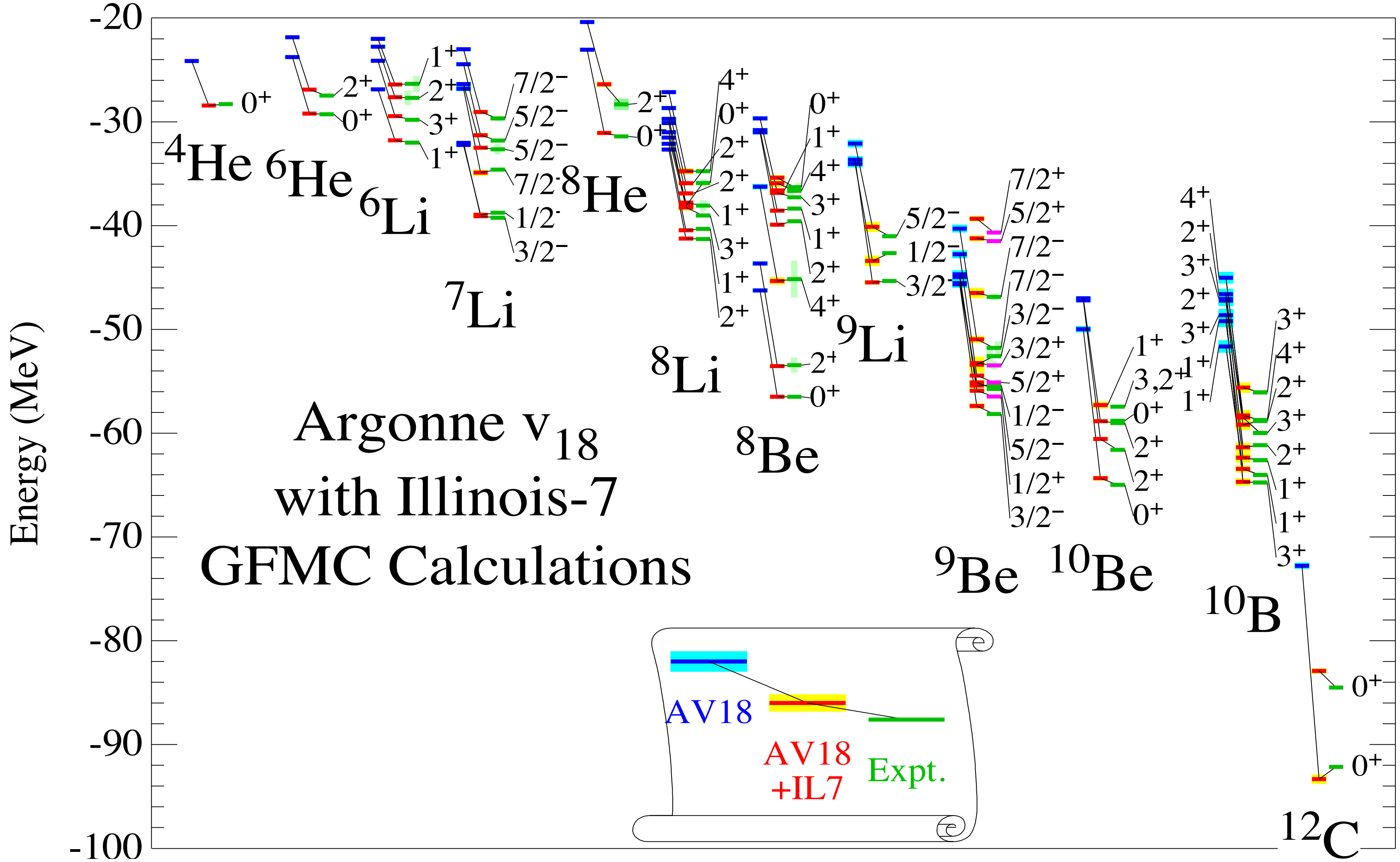


The Green's Function Monte Carlo approach

- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting (correlated) trial wave function.

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

- Suitable to solve A ≤ 12 nuclei with ~1% accuracy



J. Carlson et al. RMP 87, 1067 (2015)

The Green's Function Monte Carlo approach

✤ Accurate GFMC calculations of the electroweak responses of ^{4}He and ^{12}C have been recently performed: A. Lovato et al, Phys.Rev.Lett. 117 (2016), 082501, Phys.Rev. C97 (2018), 022502

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0 | J_\alpha^\dagger(\mathbf{q}) | f \rangle \langle f | J_\beta(\mathbf{q}) | 0 \rangle \delta(\omega - E_f + E_0)$$

- Valuable information on the energy dependence of the response functions can be inferred from the their Laplace transforms

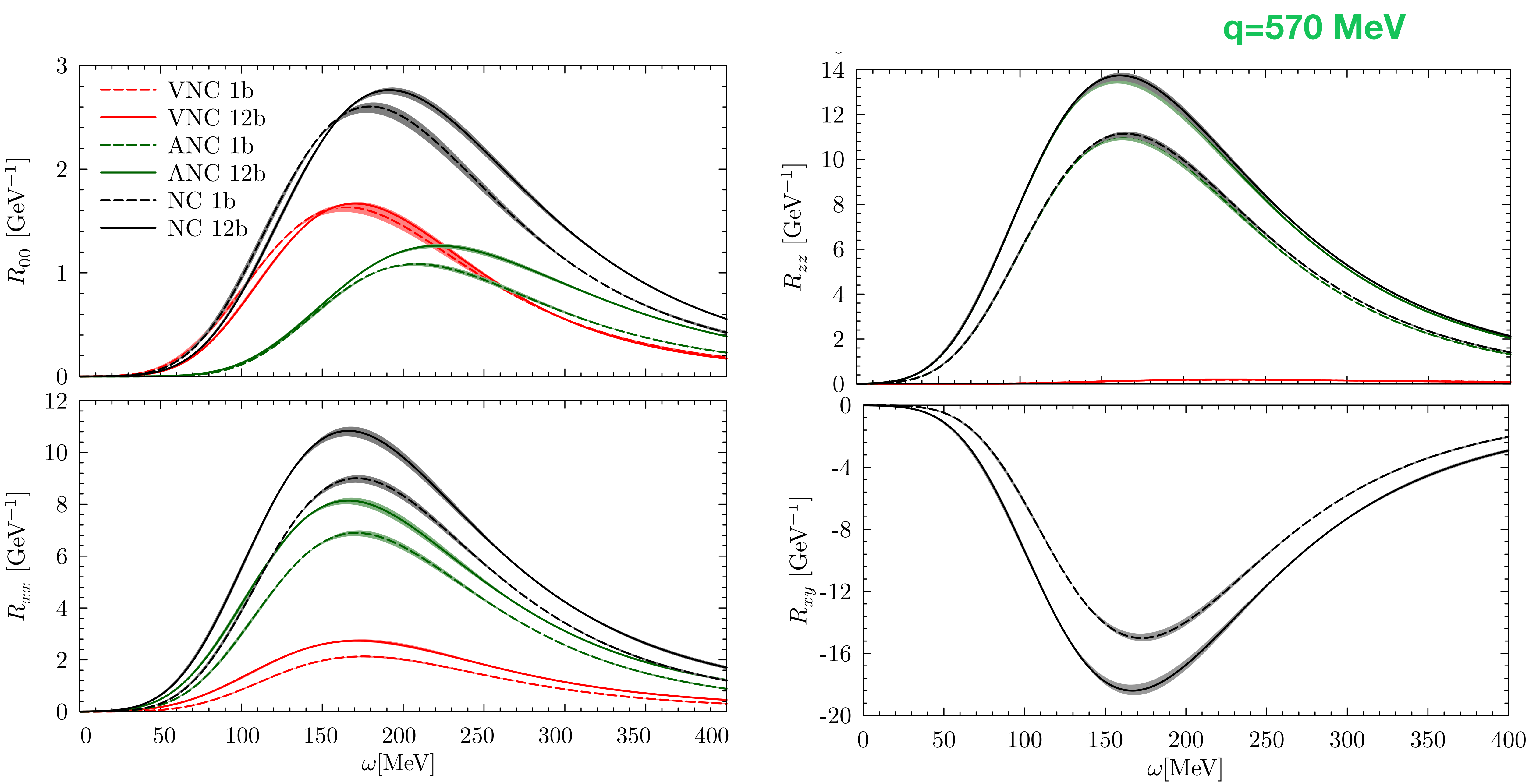
$$E_{\alpha\beta}(\mathbf{q}, \tau) = \int d\omega \, e^{-\omega\tau} R_{\alpha\beta}(\mathbf{q}, \omega) = \langle 0 | J_\alpha^\dagger(\mathbf{q}) e^{-(H-E_0)\tau} J_\beta(\mathbf{q}) | 0 \rangle$$

Using the completeness relation for the final states, we are left with ground-state expectations value

- Maximum entropy techniques are used perform the analytic continuation of the Euclidean response functions, corresponding to the "inversion" of the Laplace transforms

^{12}C neutral-current response

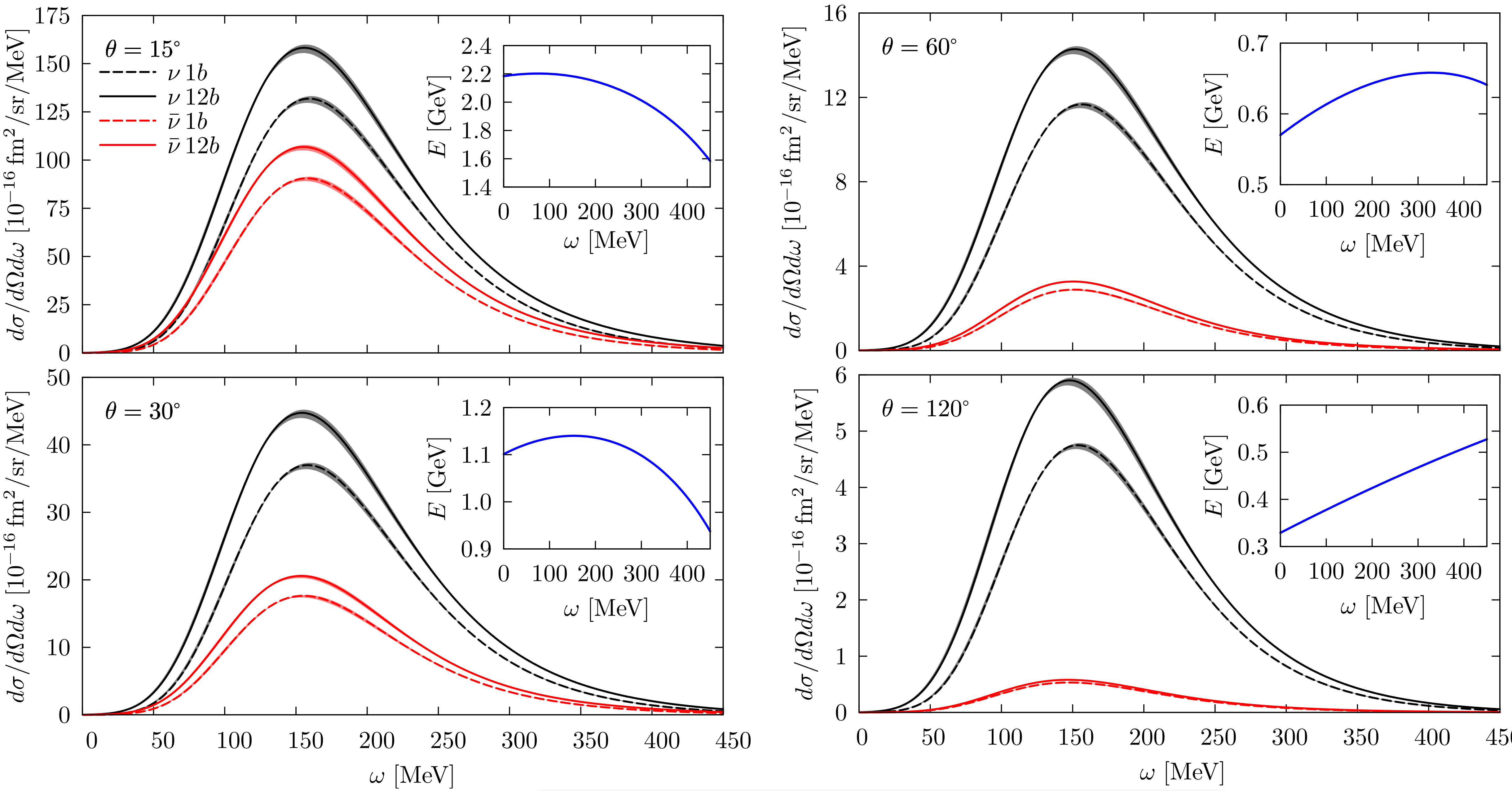
- The neutral-current response functions of ^{12}C have been recently computed



Alessandro Lovato et al. PRC 97 022502 (2018)

^{12}C neutral-current cross-section

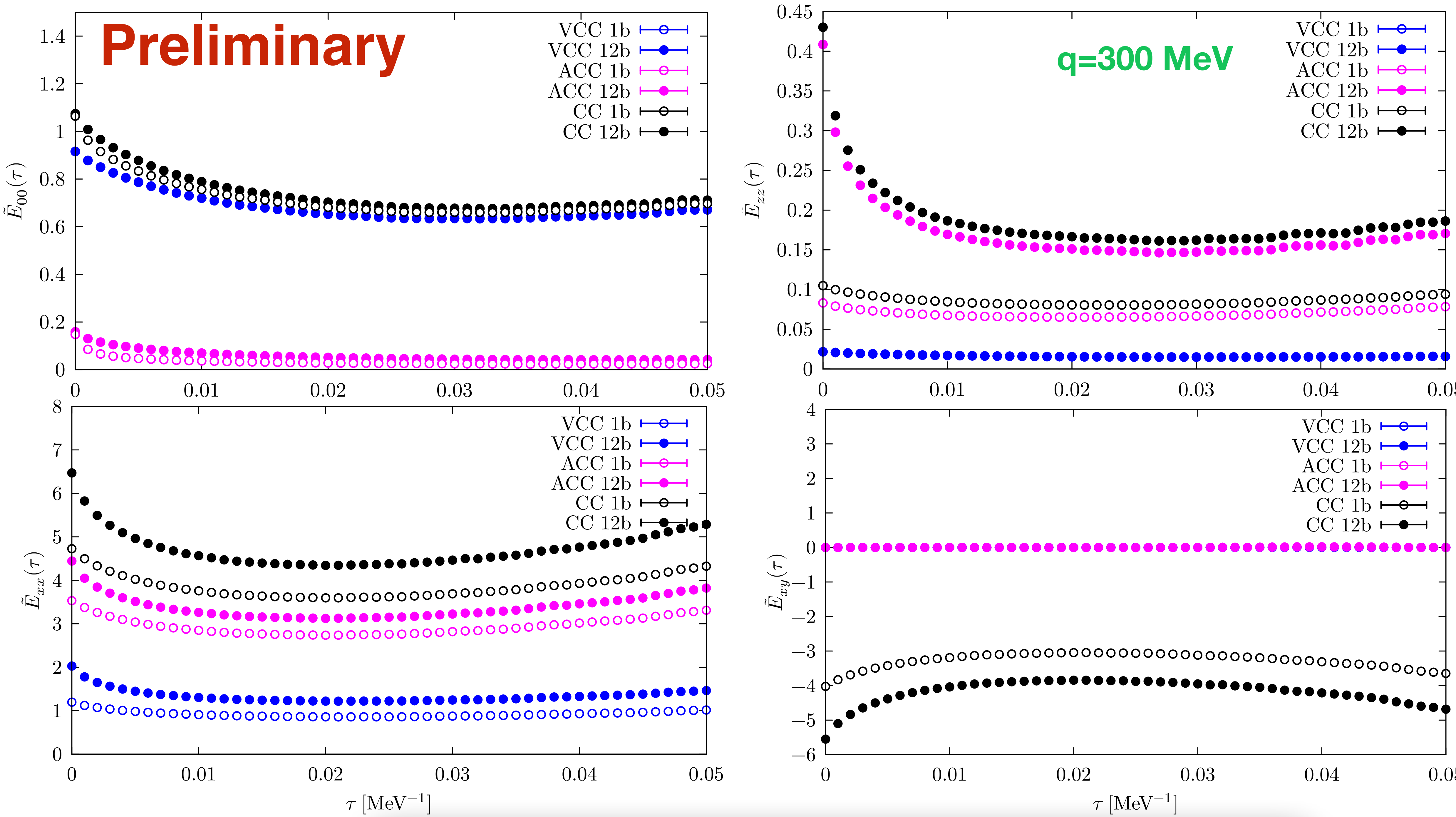
- Neutrino and anti-neutrino differential cross sections for a fixed value of the three-momentum transfer as function of the energy transfer for a number of scattering angles
- The anti-neutrino cross section decreases rapidly relative to the neutrino cross section as the scattering angle changes from the forward to the backward hemisphere



Alessandro Lovato et al. PRC 97 022502 (2018)

^{4}He charge-current Euclidean responses

- The charged-current Euclidean responses of ^{4}He have been recently computed

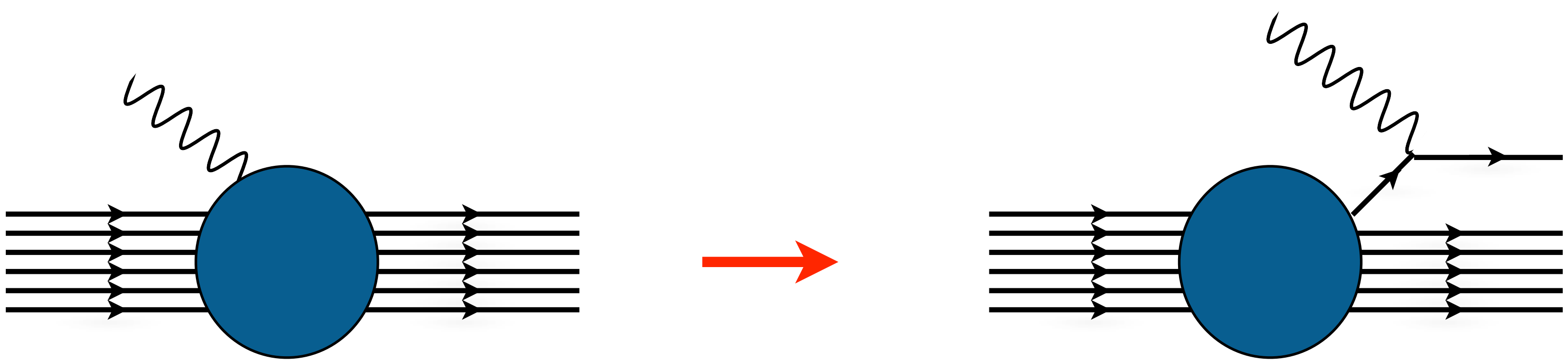


A. Lovato, S. Gandolfi, J. Carlson, E. Lusk, Steven C. Pieper, R. Schiavilla

The Impulse Approximation

- For sufficiently large values of |**q**|, the IA can be applied under the assumptions

$$|f\rangle \longrightarrow |p\rangle \otimes |f\rangle_{A-1} \qquad J_\alpha = \sum_i j^i_\alpha$$



- The matrix element of the current can be written in the factorized form

$$\langle 0|J_\alpha|f\rangle \to \sum_k \langle 0|[|k\rangle \otimes |f\rangle_{A-1}]\langle k|\sum_i j^i_\alpha|p\rangle$$

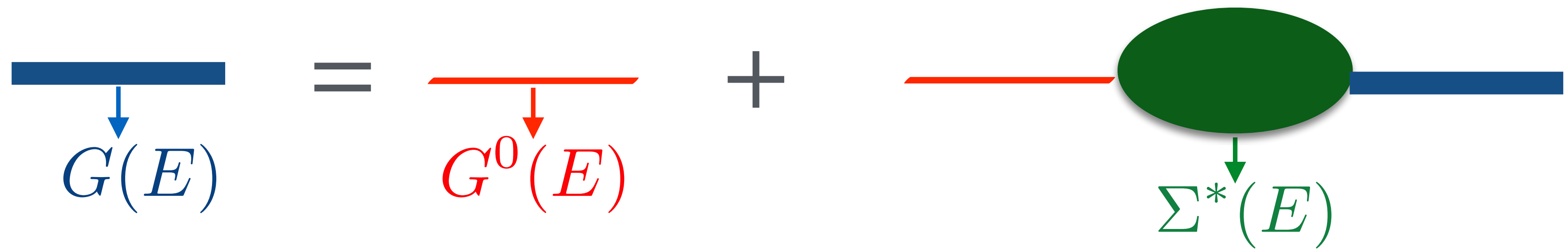
- The nuclear cross section is given in terms of the one describing the interaction with individual bound nucleons

$$\boxed{d\sigma_A = \int dE\, d^3k d\sigma_N P(\mathbf{k}, E)} \rightarrow \frac{1}{\pi}\mathrm{Im}\, G(\mathbf{k}, E)$$

- The intrinsic properties of the nucleus are described by the hole spectral function

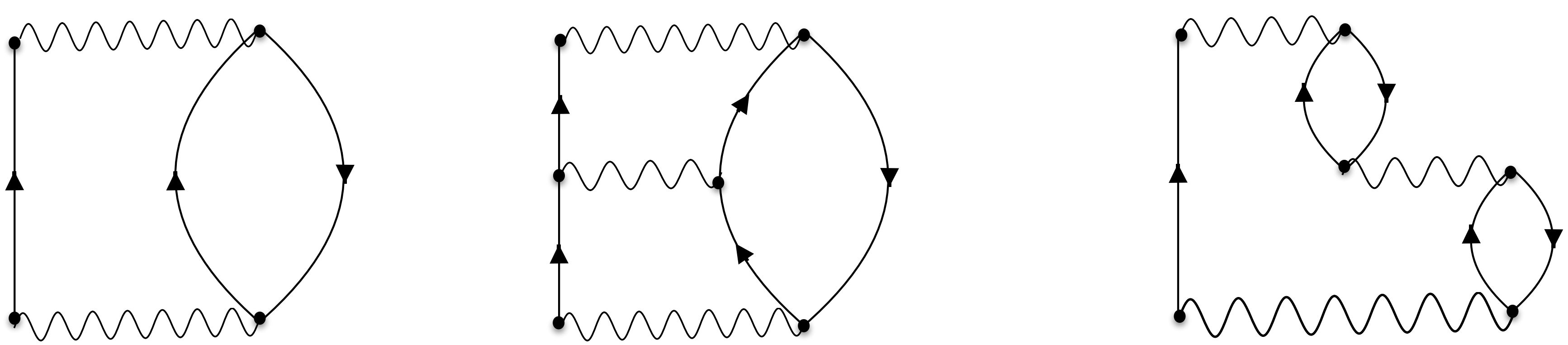
Self Consistent Green's Function

- Accurately solve the Dyson equation



- $\Sigma^* = \Sigma^*[G(E)]$, an iterative procedure is required to solve the Dyson equation self-consistently

- The self-energy is systematically calculated in a non-perturbative fashion within the Algebraic Diagrammatic Construction (ADC). The saturating chiral interaction at NNLO ($NNLO_{sat}$) is used.

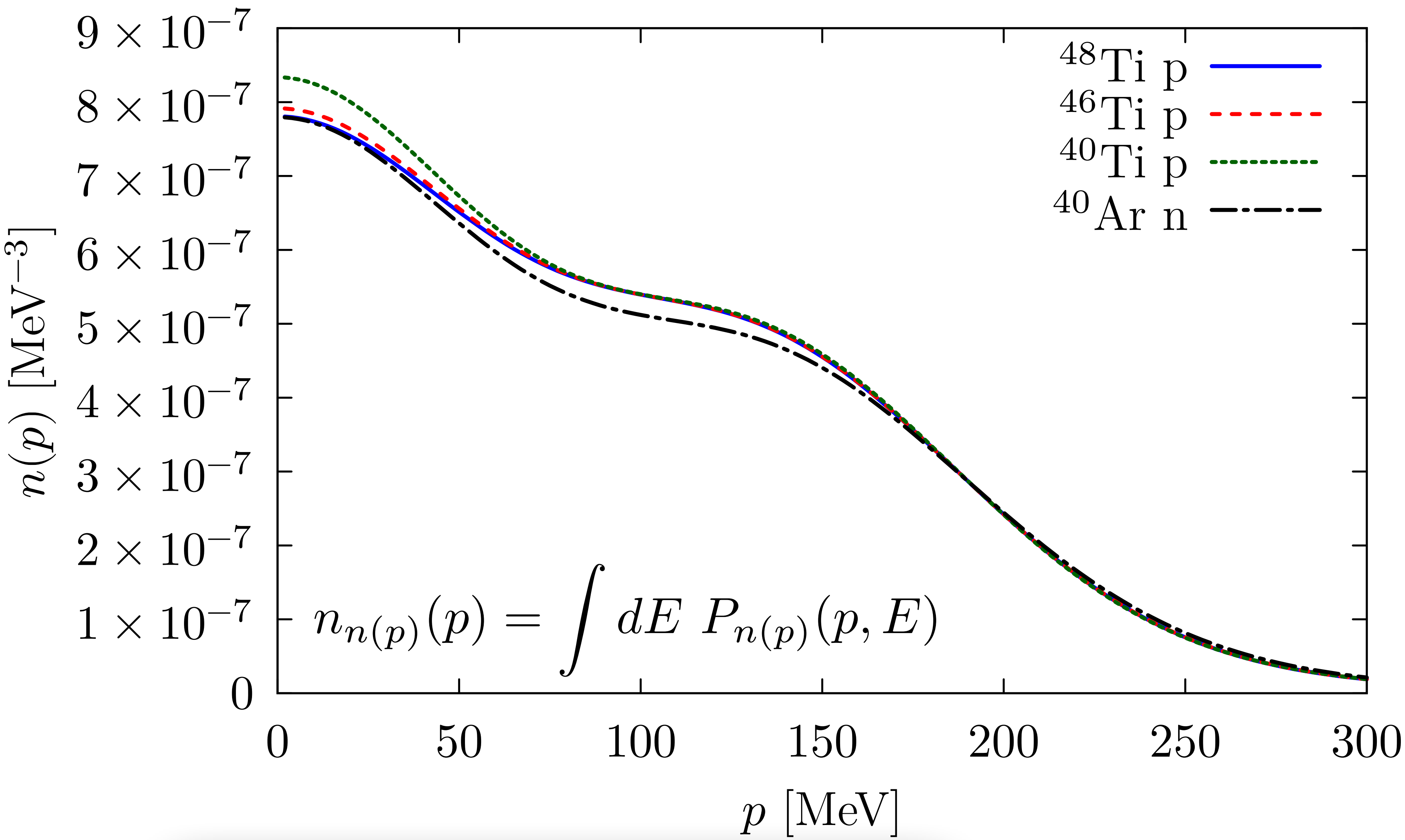


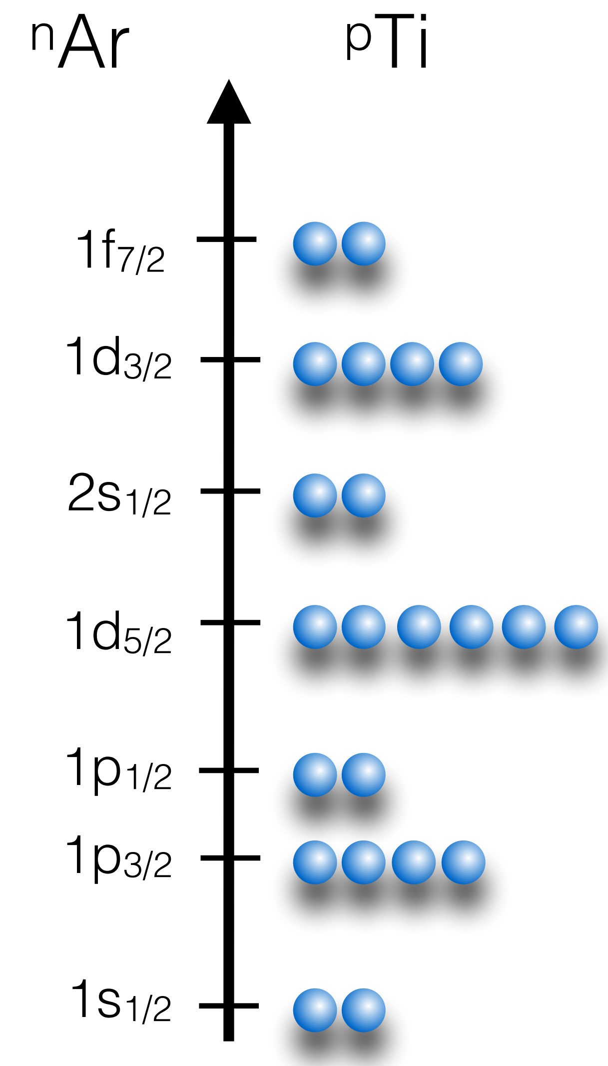
2nd and 3rd order diagrams with 2h1p (and 2p1h) intermediate configurations

✤ V. Somà et al, Phys.Rev. C87 (2013) no.1, 011303 : generalization of this formalism within Gorkov theory allows to describe open-shell nuclei such as Ar^{40}, Ti^{48} …

Understanding the differences between mirror nuclei

- Single n(p)-momentum distribution of ^{40}Ar (Ti)



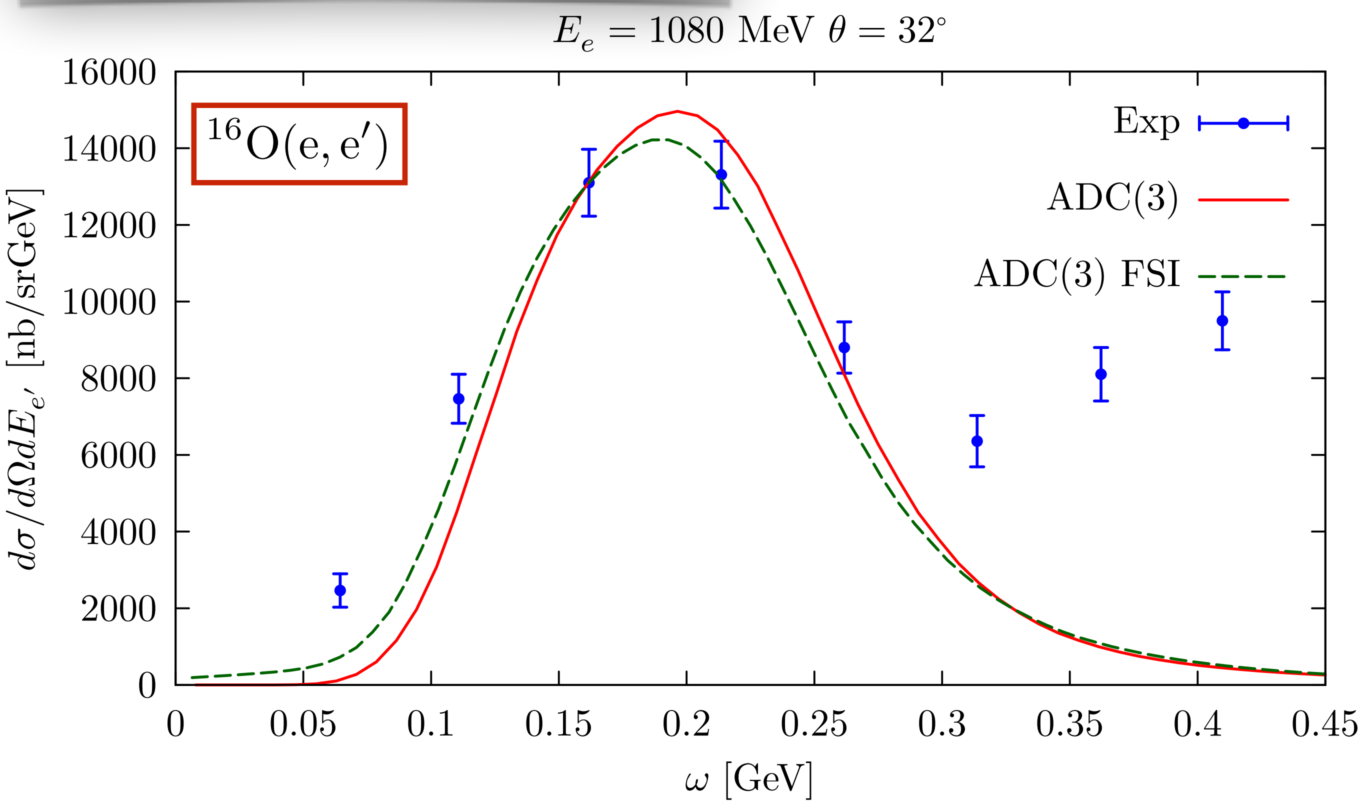


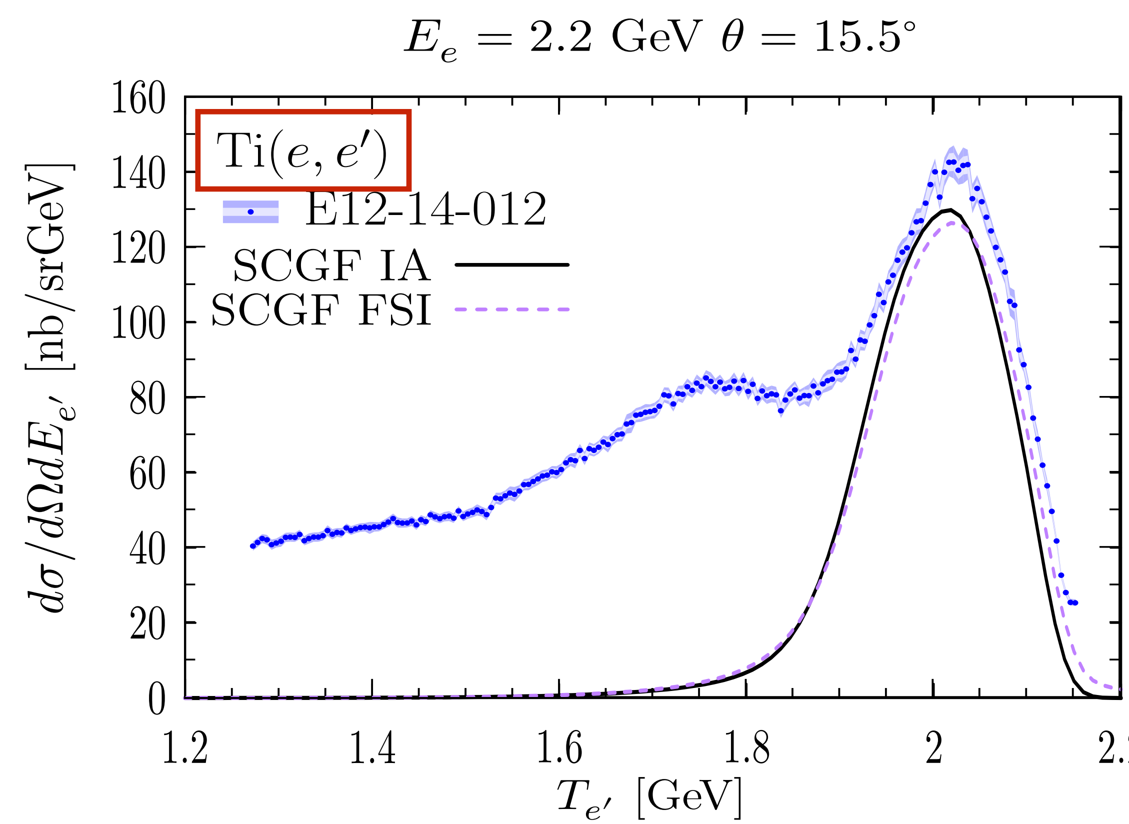
V. Somà, C. Barbieri, NR, in preparation

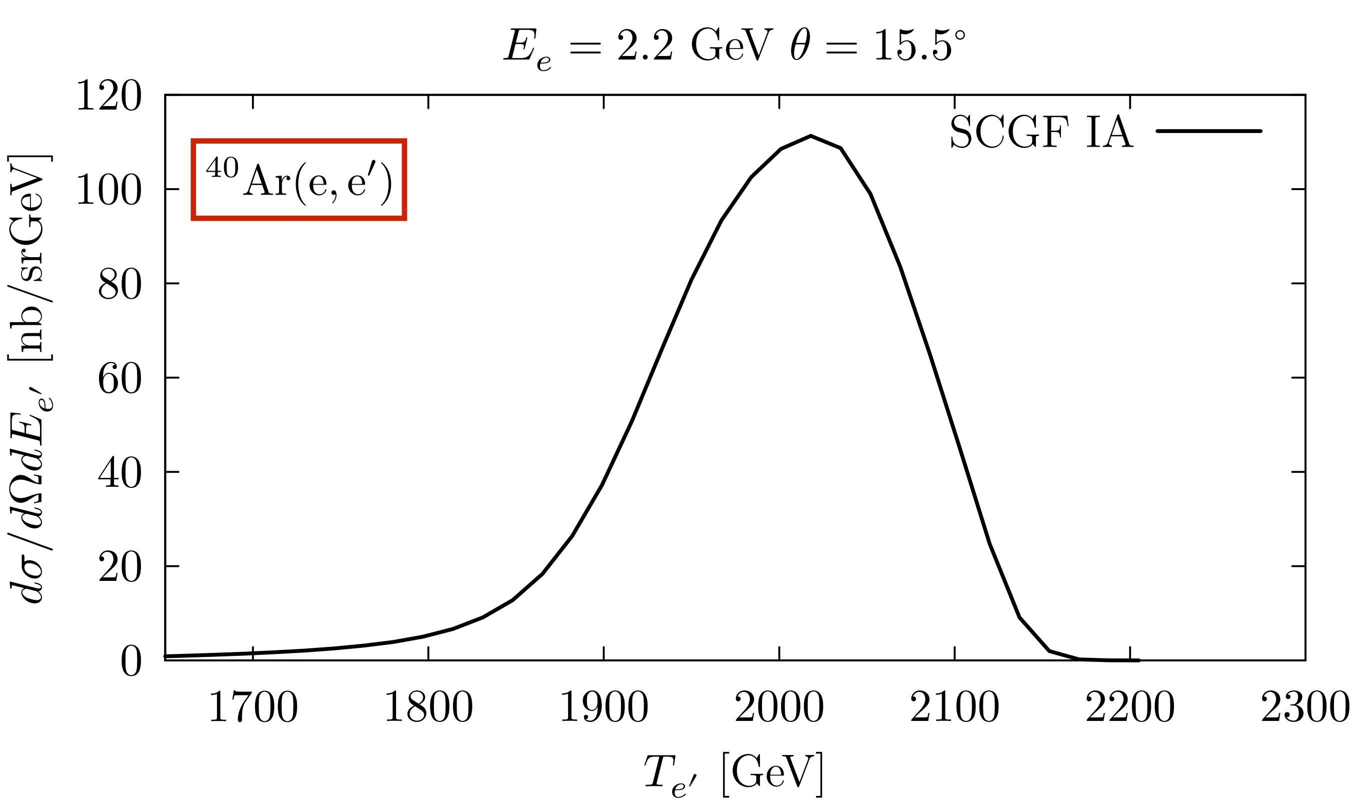
- The neutron structure of ^{40}Ar corresponds to the proton Ti isotopic chain

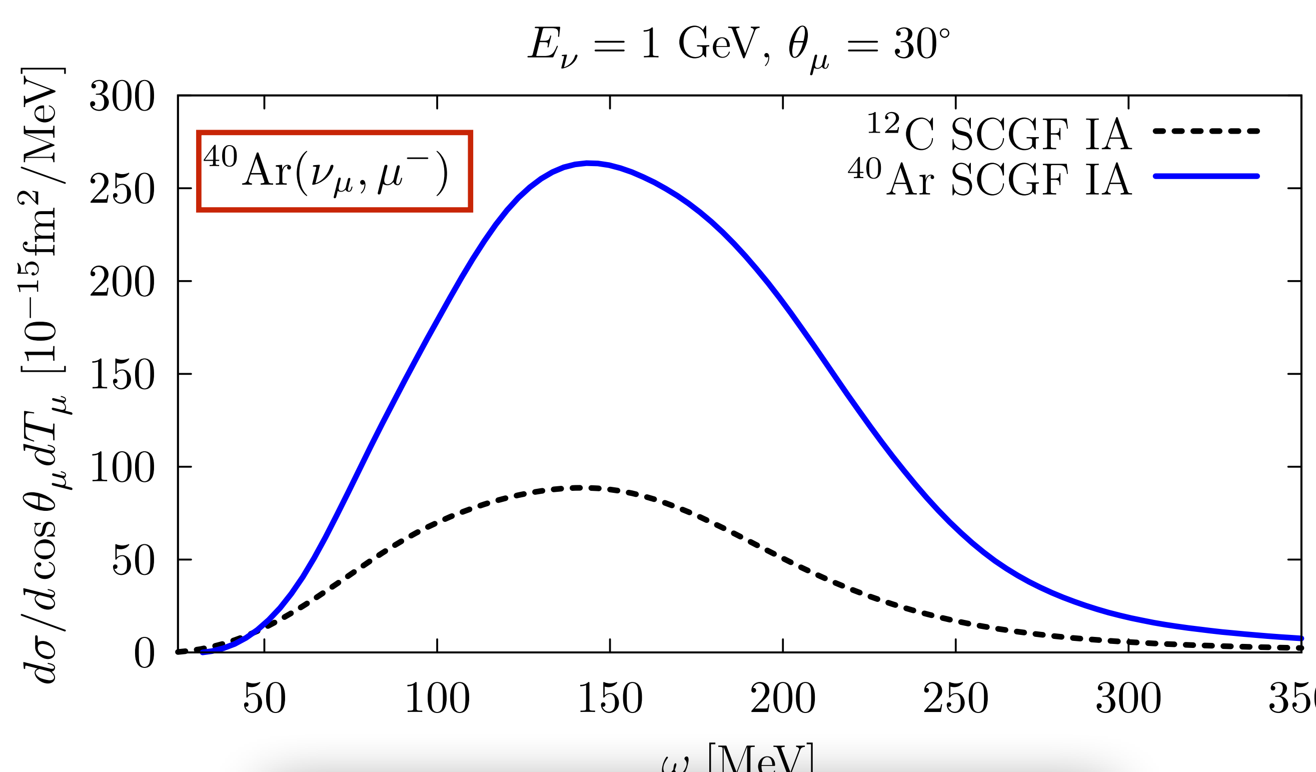
Electroweak scattering cross sections: 1b

NR, C. Barbieri, PRC98 (2018) 025501









V. Somà, C. Barbieri, NR in Preparation

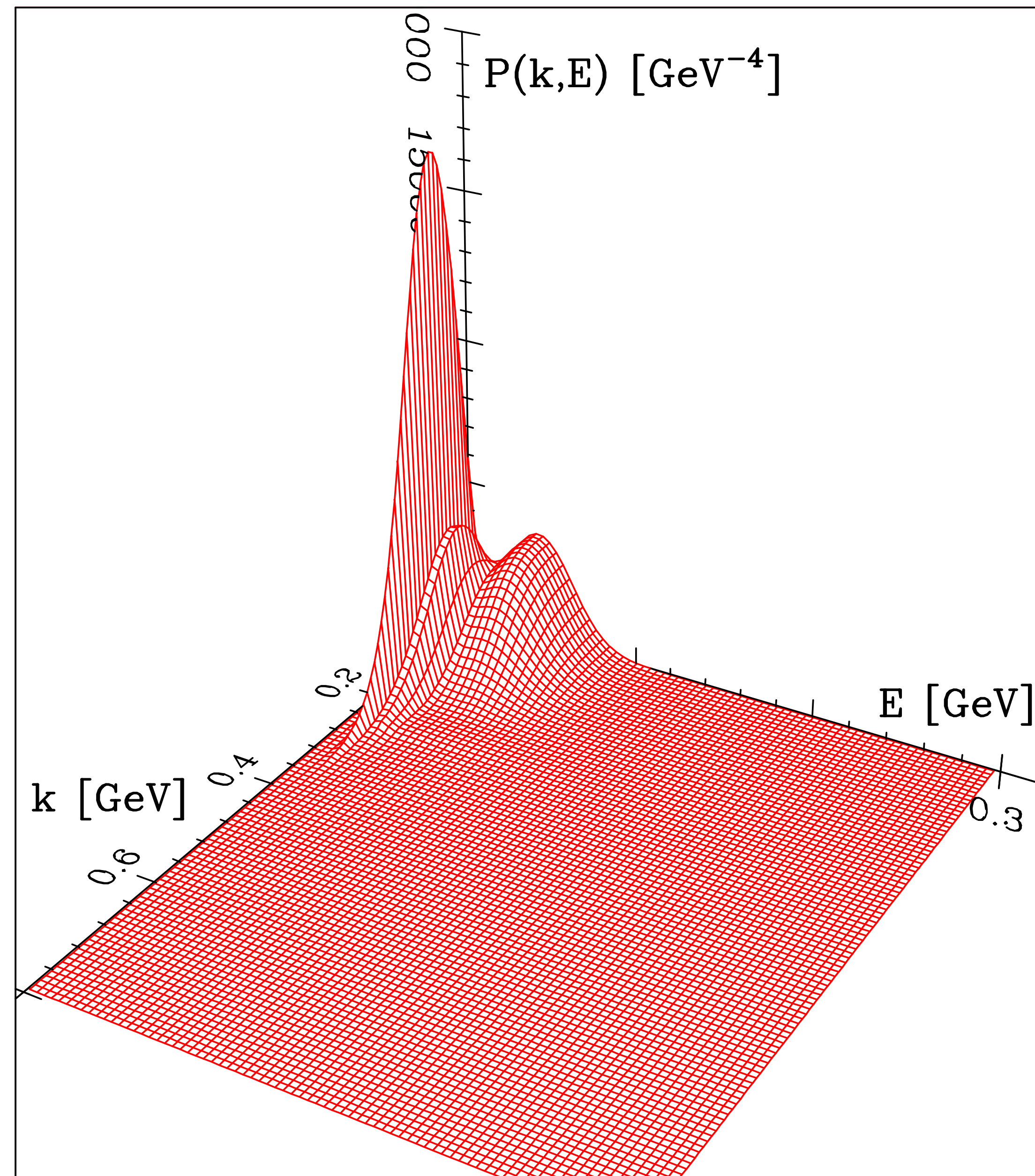
The CBF one-body Spectral Function of finite nuclei

- ^{12}C Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k},E) = P_{MF}(\mathbf{k},E) + P_{corr}(\mathbf{k},E)$$

$$P_{MF}(\mathbf{k},E) \rightarrow \boxed{\sum_n Z_n |\phi_n(\mathbf{k})|^2 F_n(E-E_n)}$$

$$P_{corr}(\mathbf{k},E) \rightarrow \boxed{\int d^3r P^{NM}_{corr}(\mathbf{k},E;\rho=\rho_A(r))}$$



- Z_n : spectroscopic factor extracted from (e, e'p)
- F_n : finite width function accounting for residual interactions not included in a MF picture

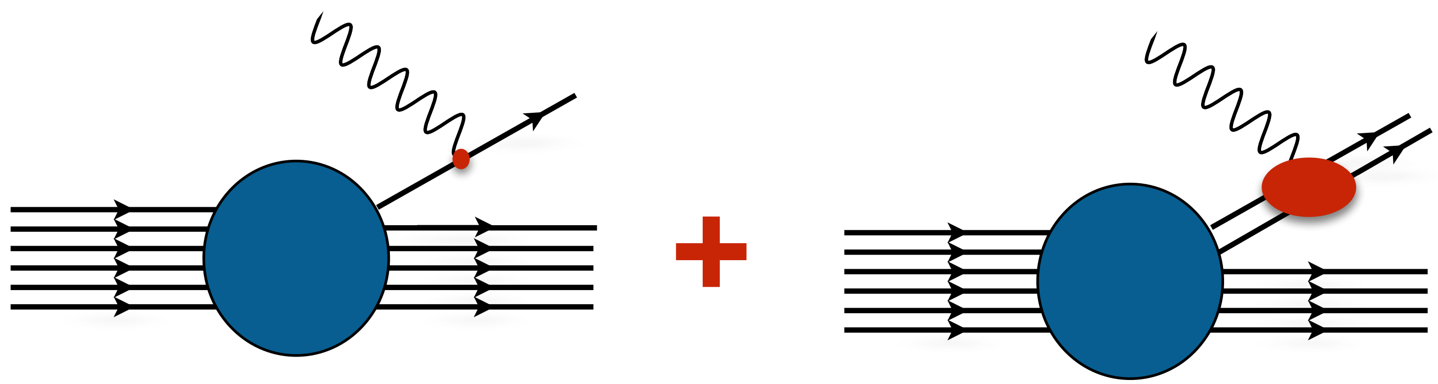
- The one-body Spectral function of nuclear matter:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

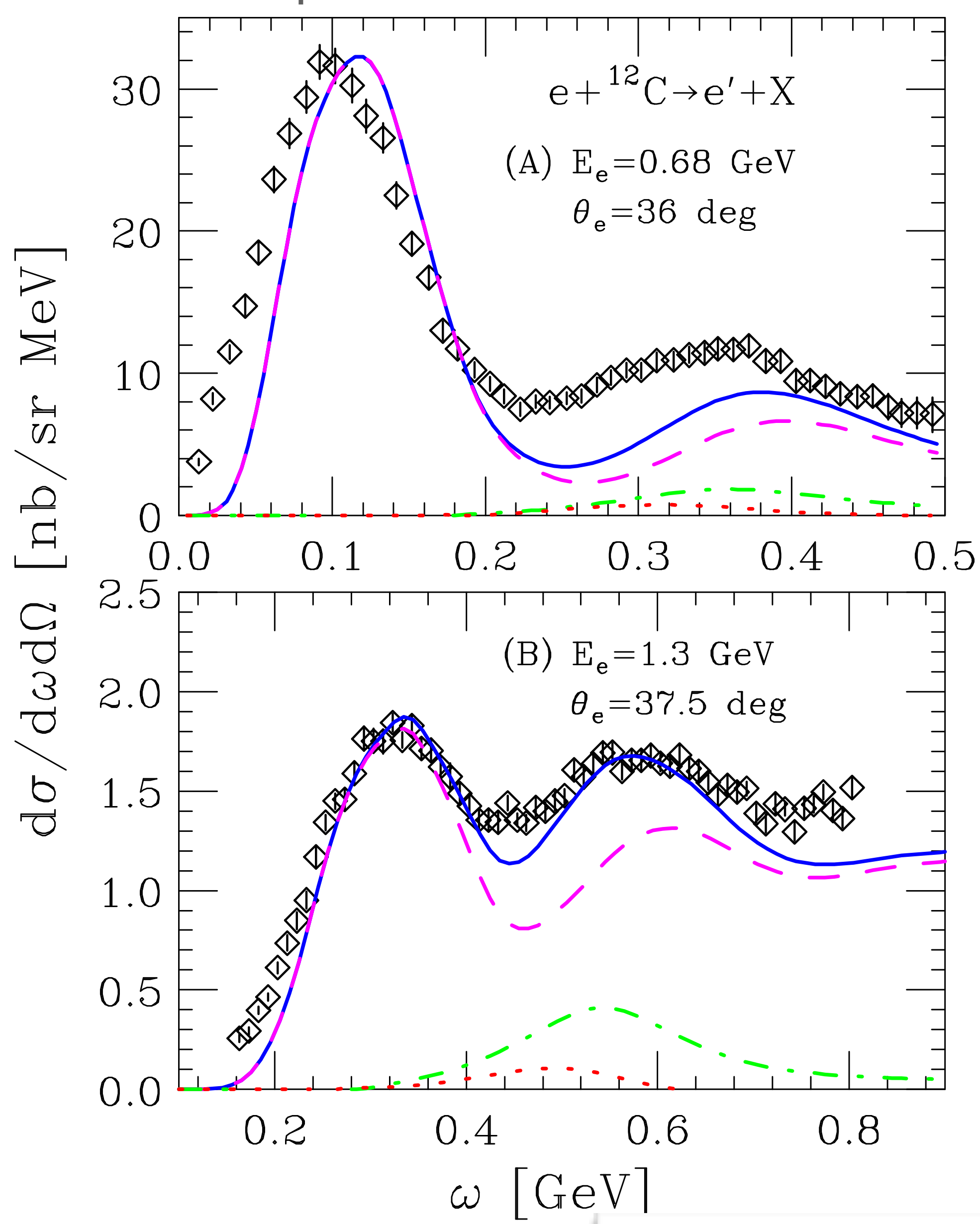
v_{ij} → Argonne v18

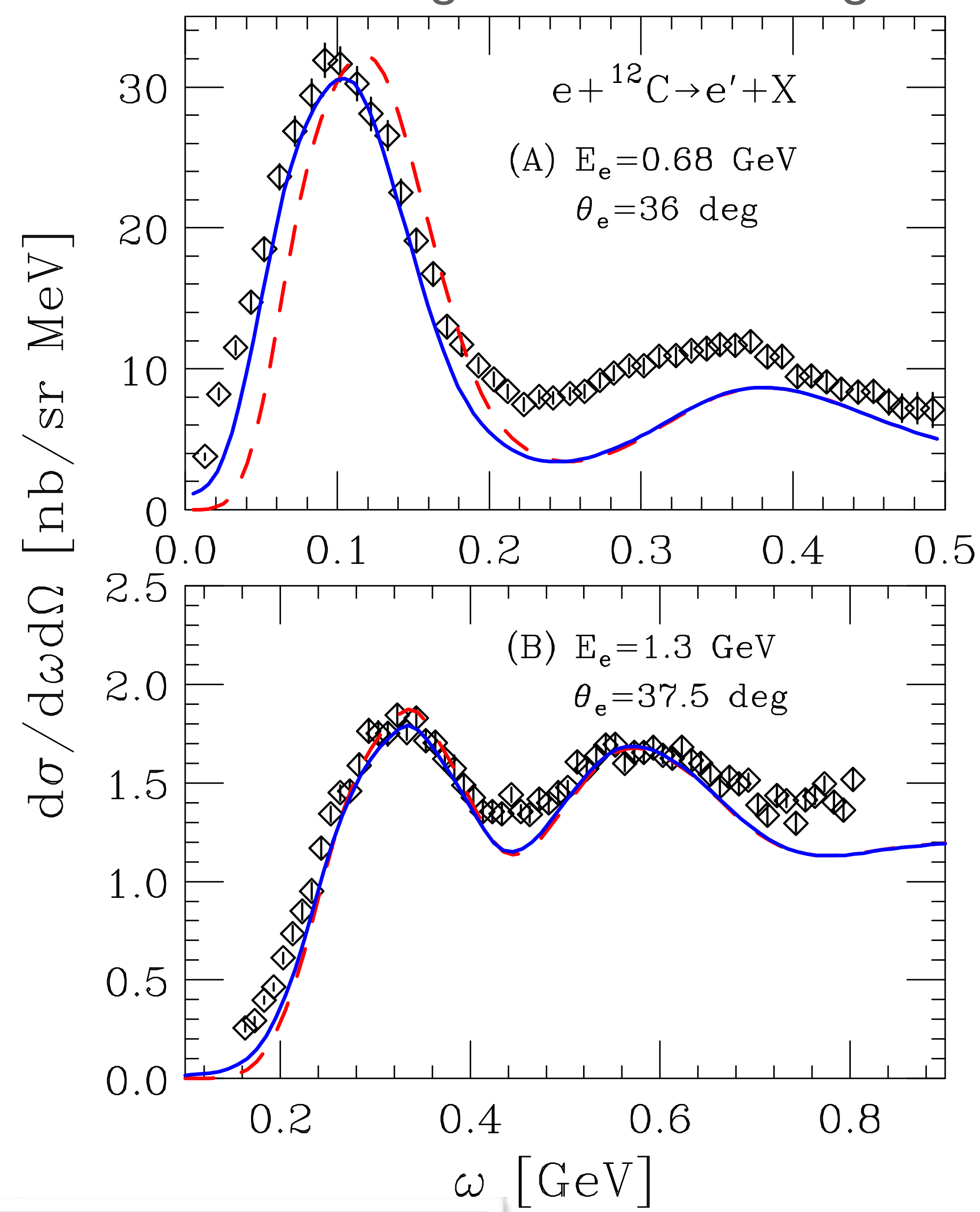
V_{ijk} → UIX, IL7

Results for $^{12}C(e,e')$ cross sections



- Separate contributions: IA
- Including FSI in the QE region



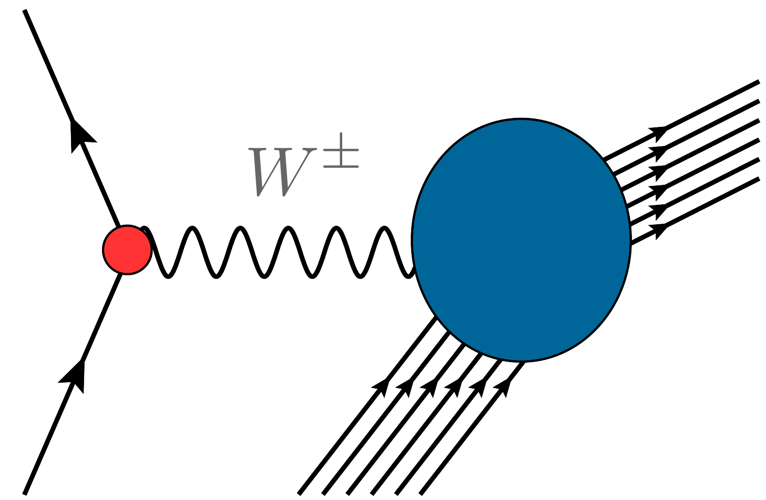


NR, A.Lovato, O.Benhar, PRL 116, 192501 (2016)

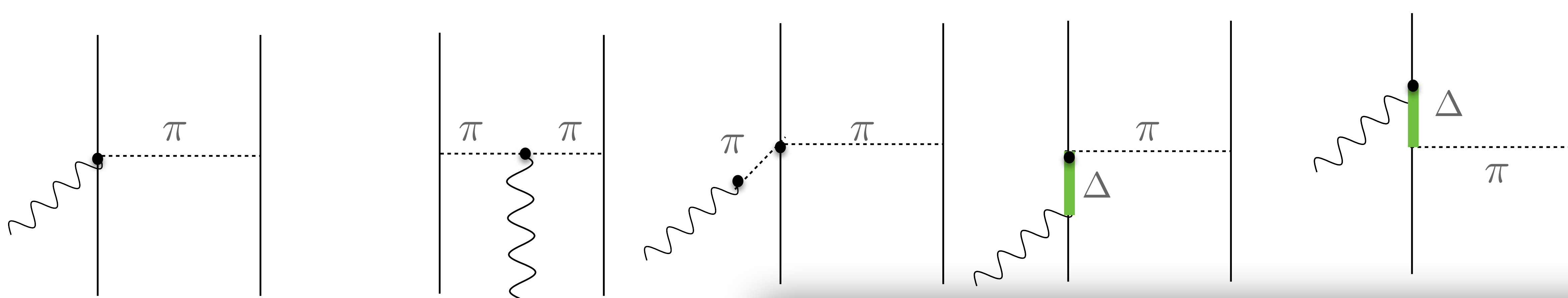
(Anti)neutrino -^{12}C scattering cross sections

The inclusive cross section of the process in which a neutrino or antineutrino scatters off a nucleus can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'}d\Omega_\ell} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$



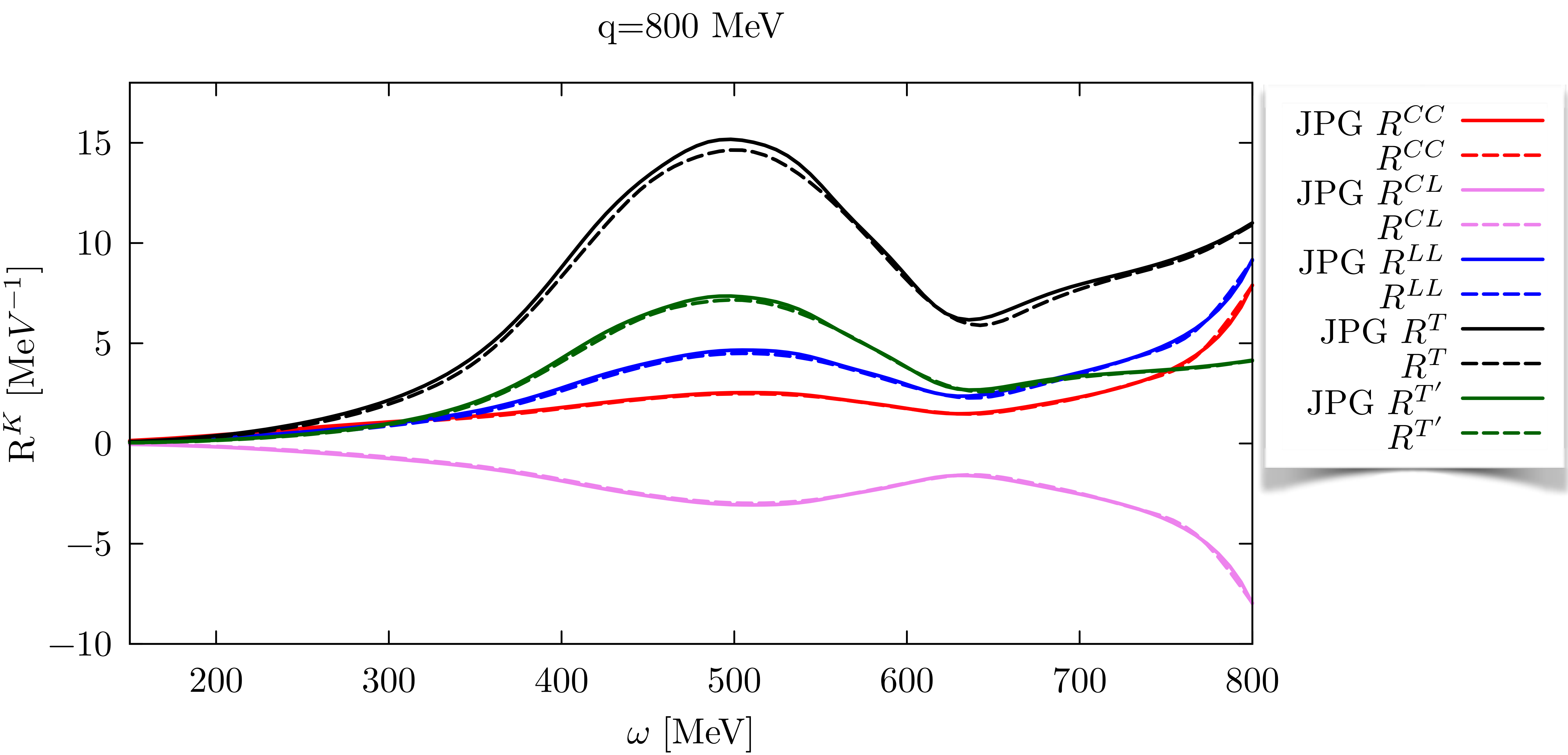
- We generalized the SF formalism to include vector and axial vector relativistic two-body currents



NR, C.Barbieri, O. Benhar, A. De Pace, A. Lovato, in Preparation

- The calculation of the MEC current matrix is carried out automatically
- 9d-integral + use of realistic SFs implies dealing with a broader phase space: we developed an highly parallel Monte Carlo code, importance sampling procedure

Two-body CC response functions of ^{12}C



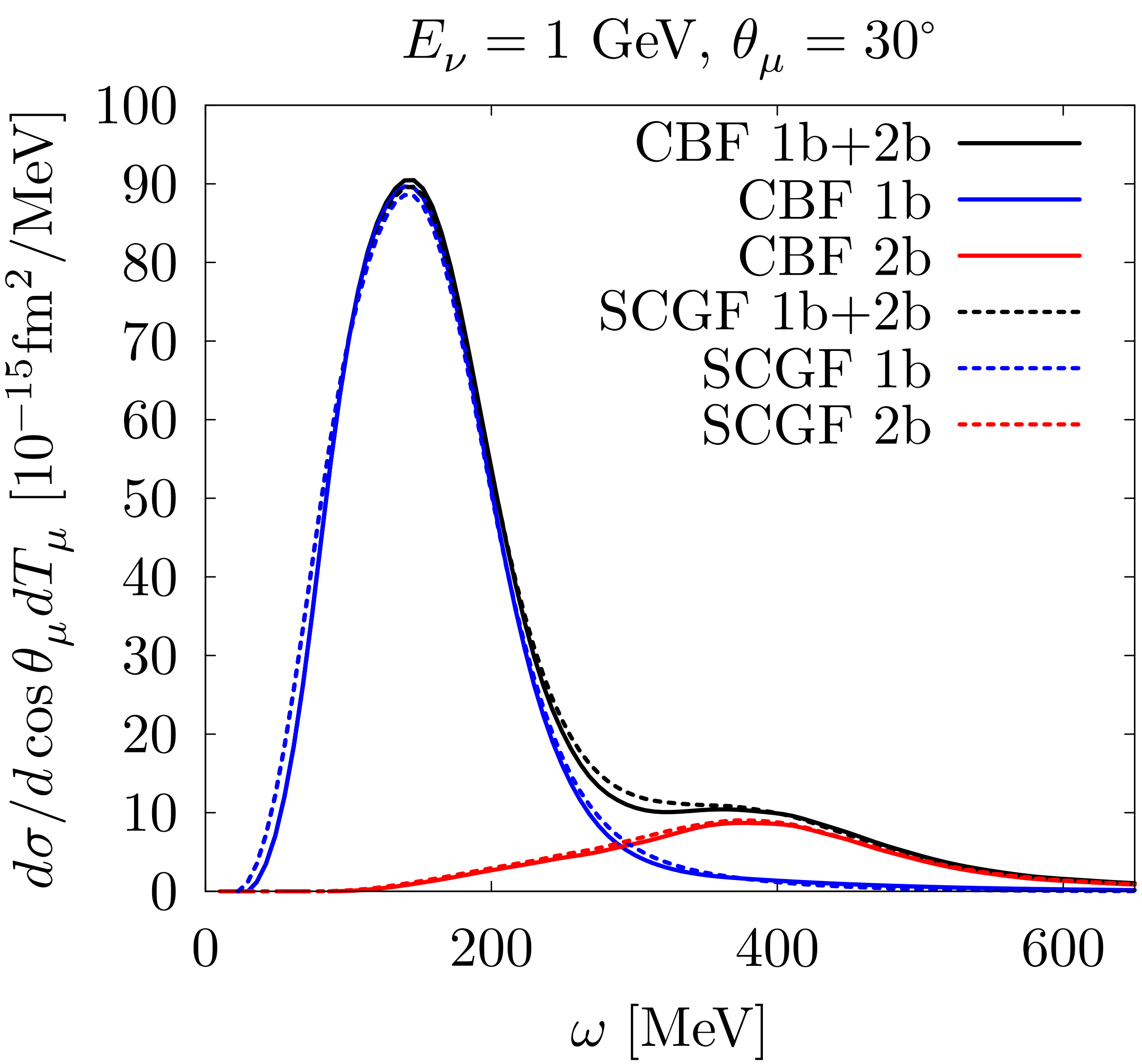
- Comparison of the five CC response functions of ^{12}C with the results of I. Ruiz Simo, et. al, Journal of Phys. G 44, no. 6 (2017).

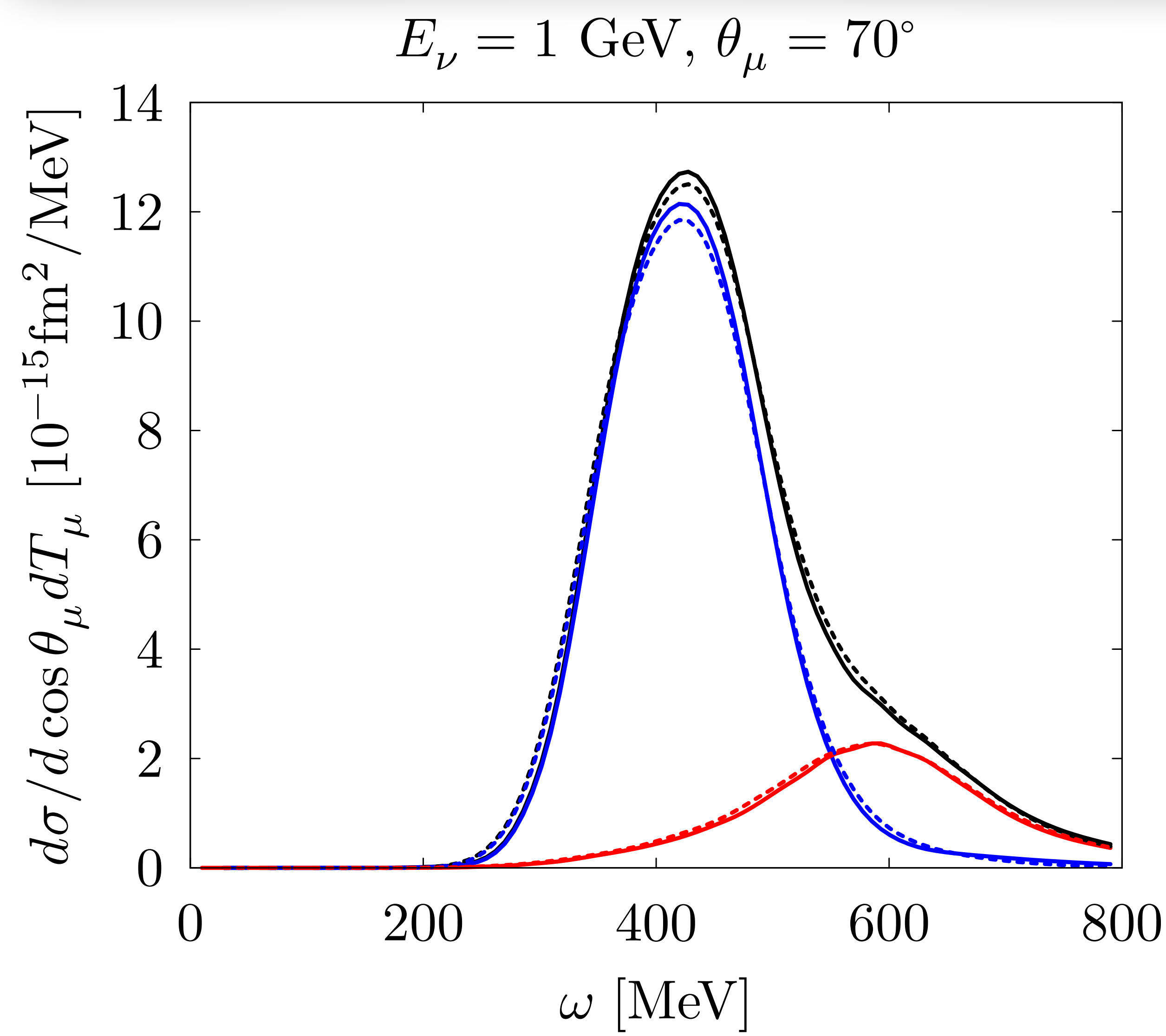
- In this case, we approximated the two-body spectral function with that of the global relativistic Fermi gas model

CCQE neutrino -^{12}C cross sections

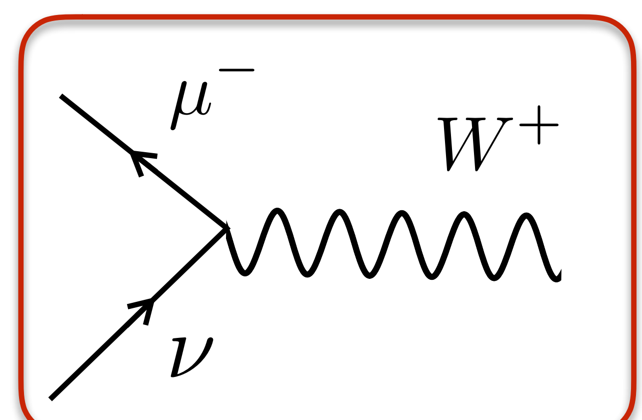
NR, C.Barbieri, O. Benhar, A. De Pace, A. Lovato, in Preparation

$$\nu_\mu + {}^{12}\mathrm{C} \rightarrow \mu^- + \mathrm{X}$$





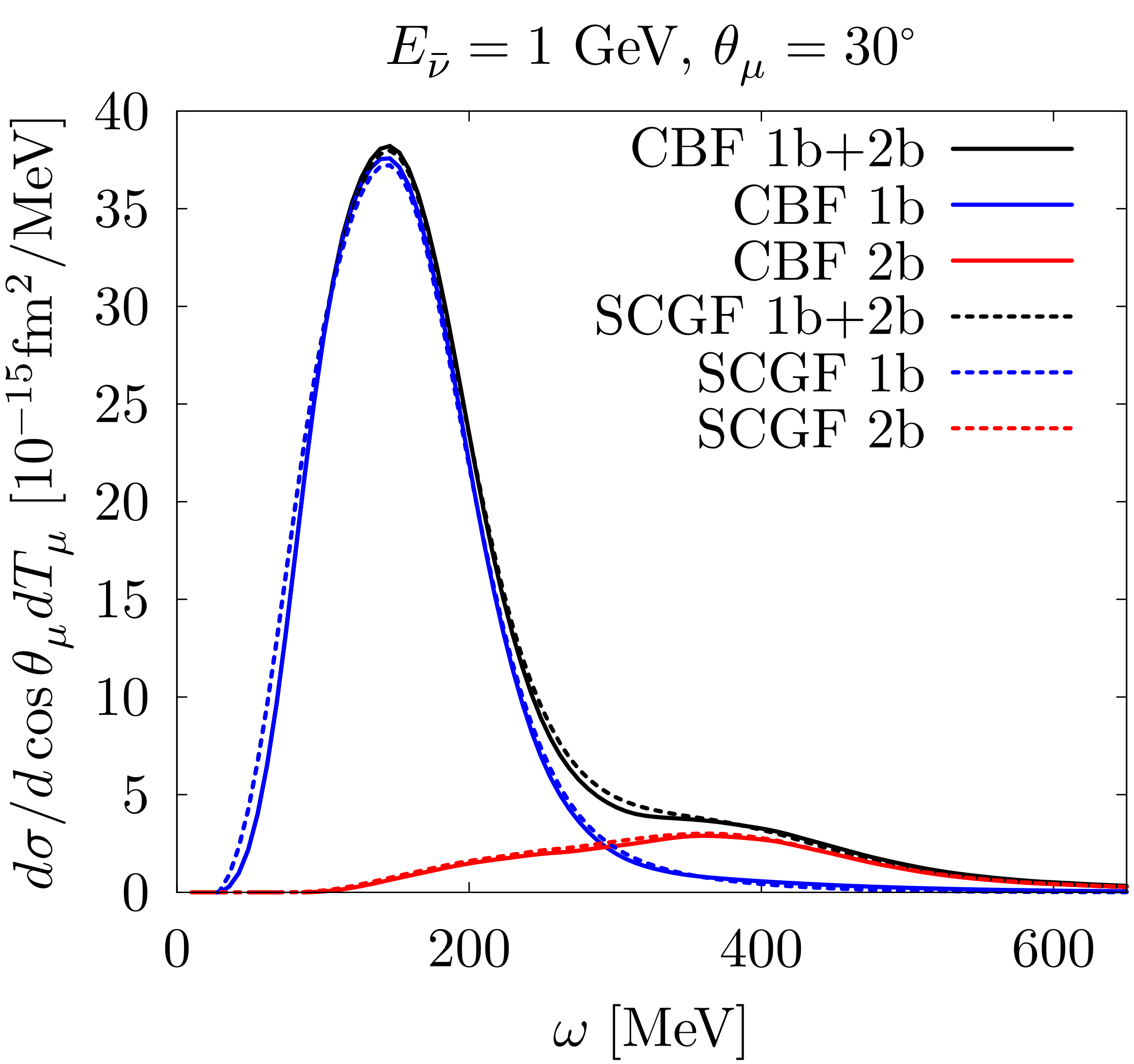
- The 2b contribution mostly affects the ‘dip’ region, in analogy with the electromagnetic case
- Meson exchange currents strongly enhance the cross section for large values of the scattering angle

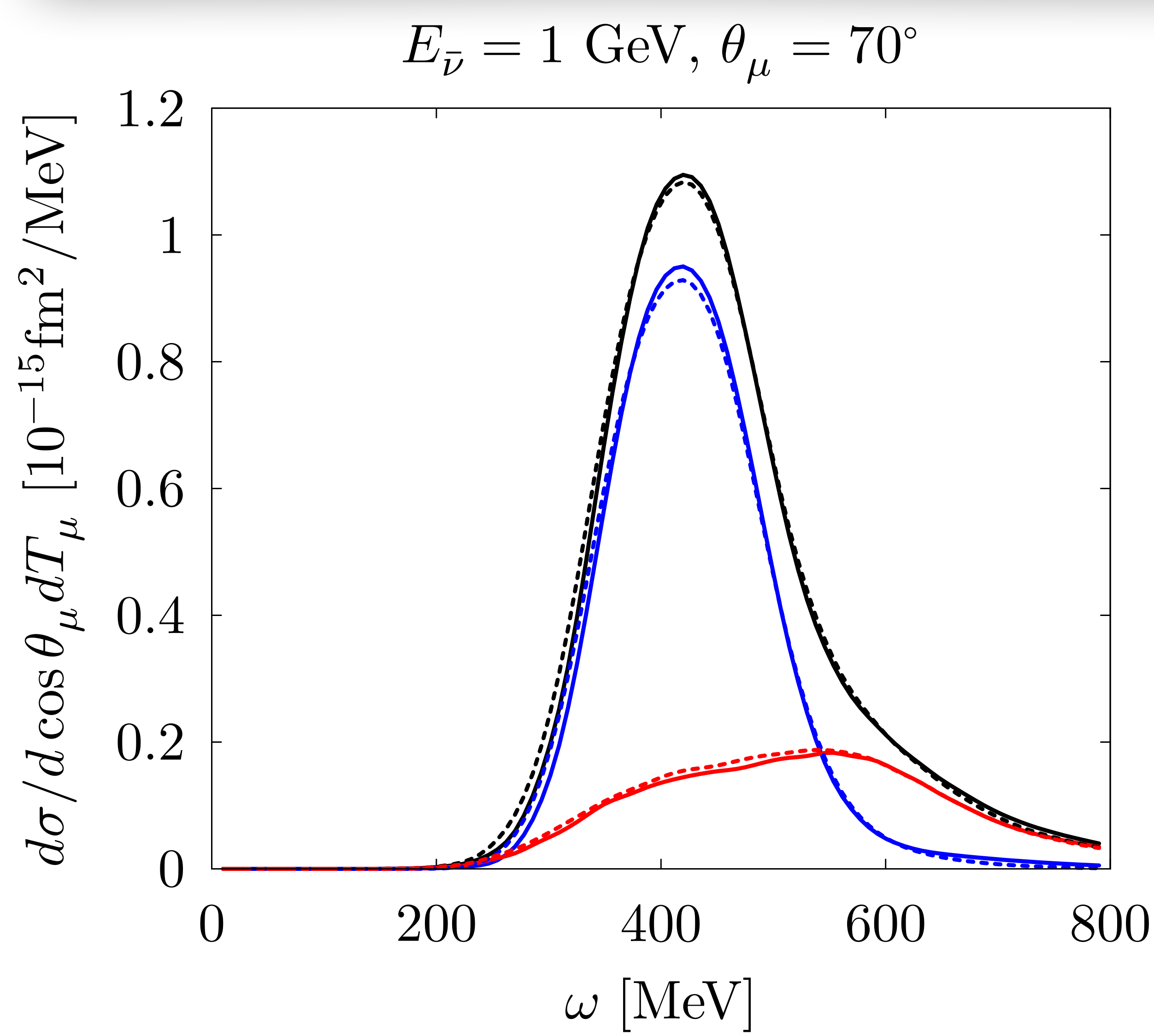


CCQE antineutrino -^{12}C cross sections

$\bar{\nu}_\mu + {}^{12}\mathrm{C} \to \mu^+ + \mathrm{X}$

NR, C.Barbieri, O. Benhar, A. De Pace, A. Lovato, in Preparation





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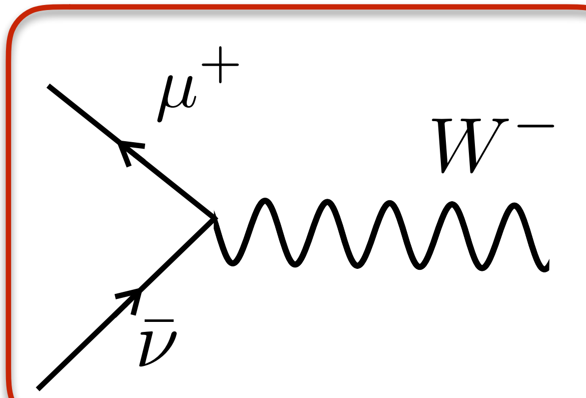

NCQE neutrino -^{12}C cross sections

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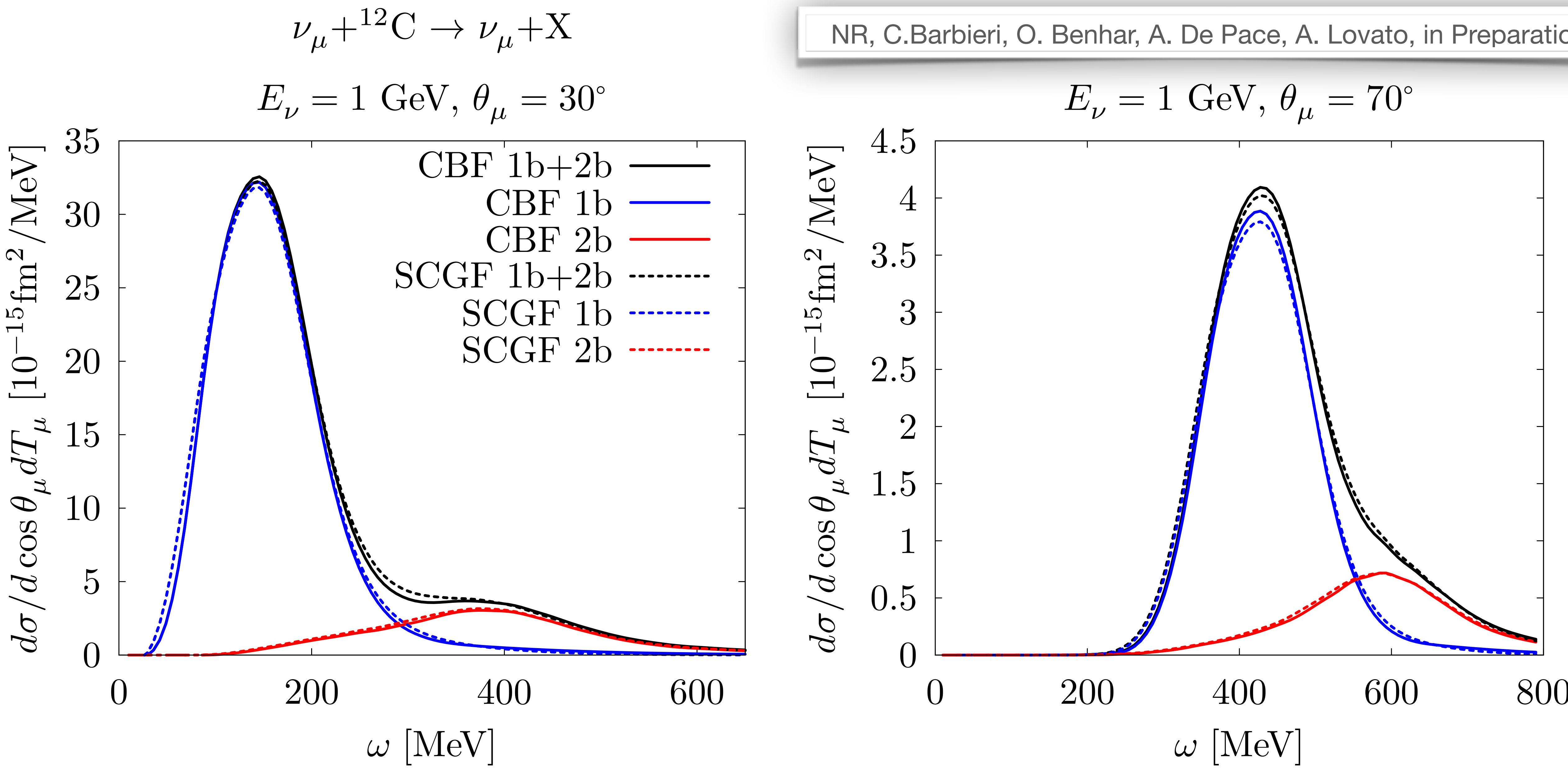

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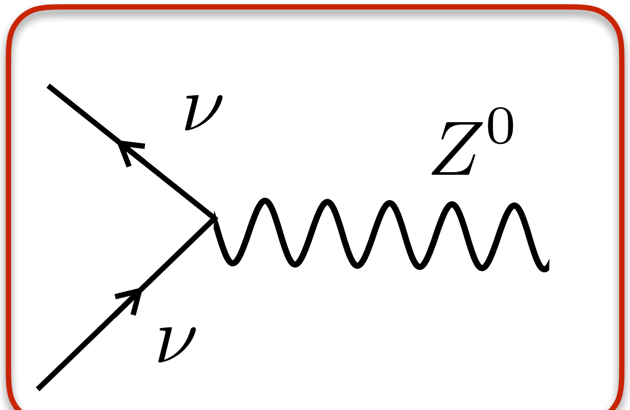

NCQE antineutrino -^{12}C cross sections

$\bar{\nu}_\mu + {}^{12}\mathrm{C} \rightarrow \bar{\nu}_\mu + \mathrm{X}$

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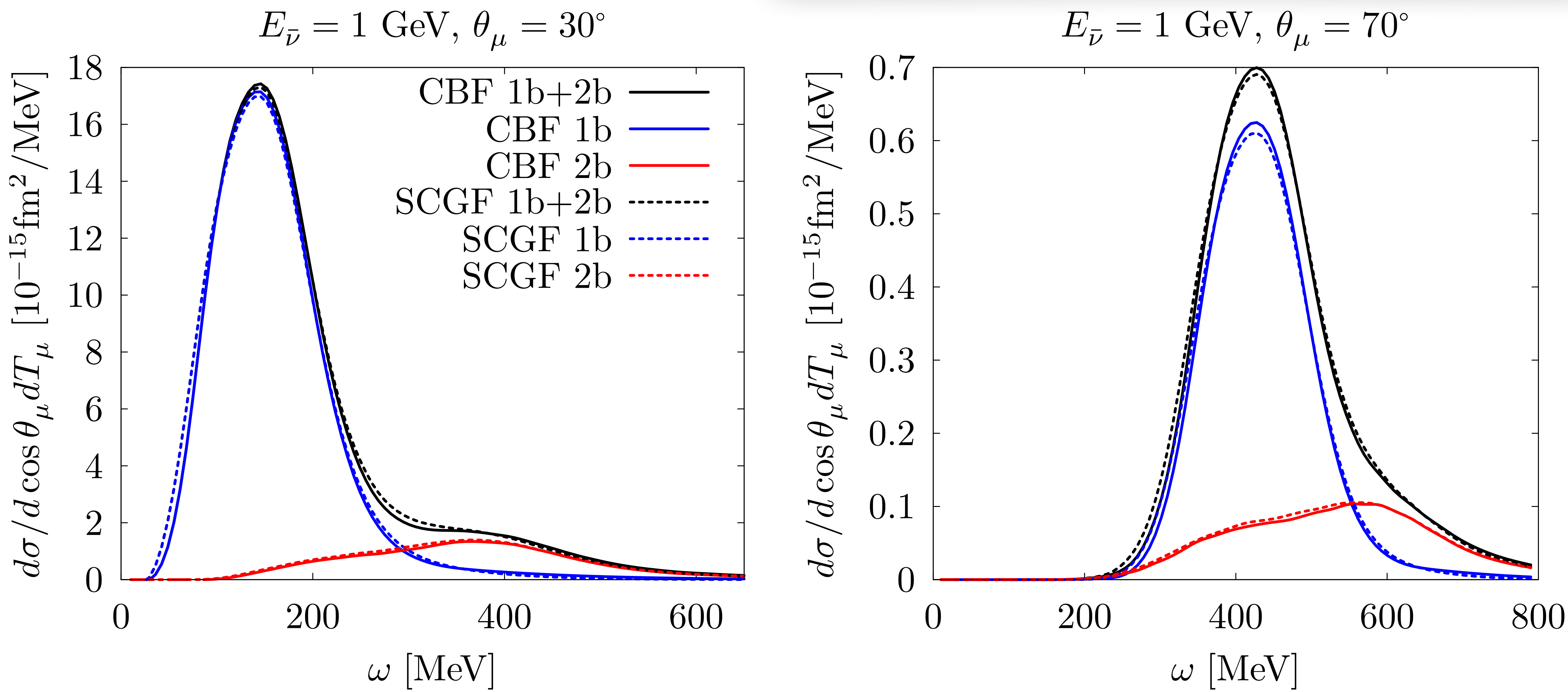

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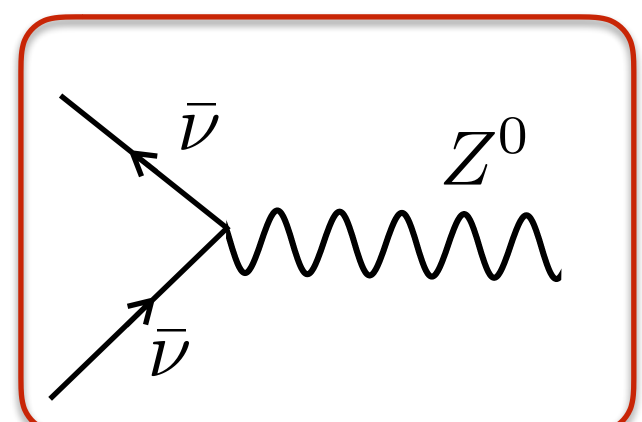
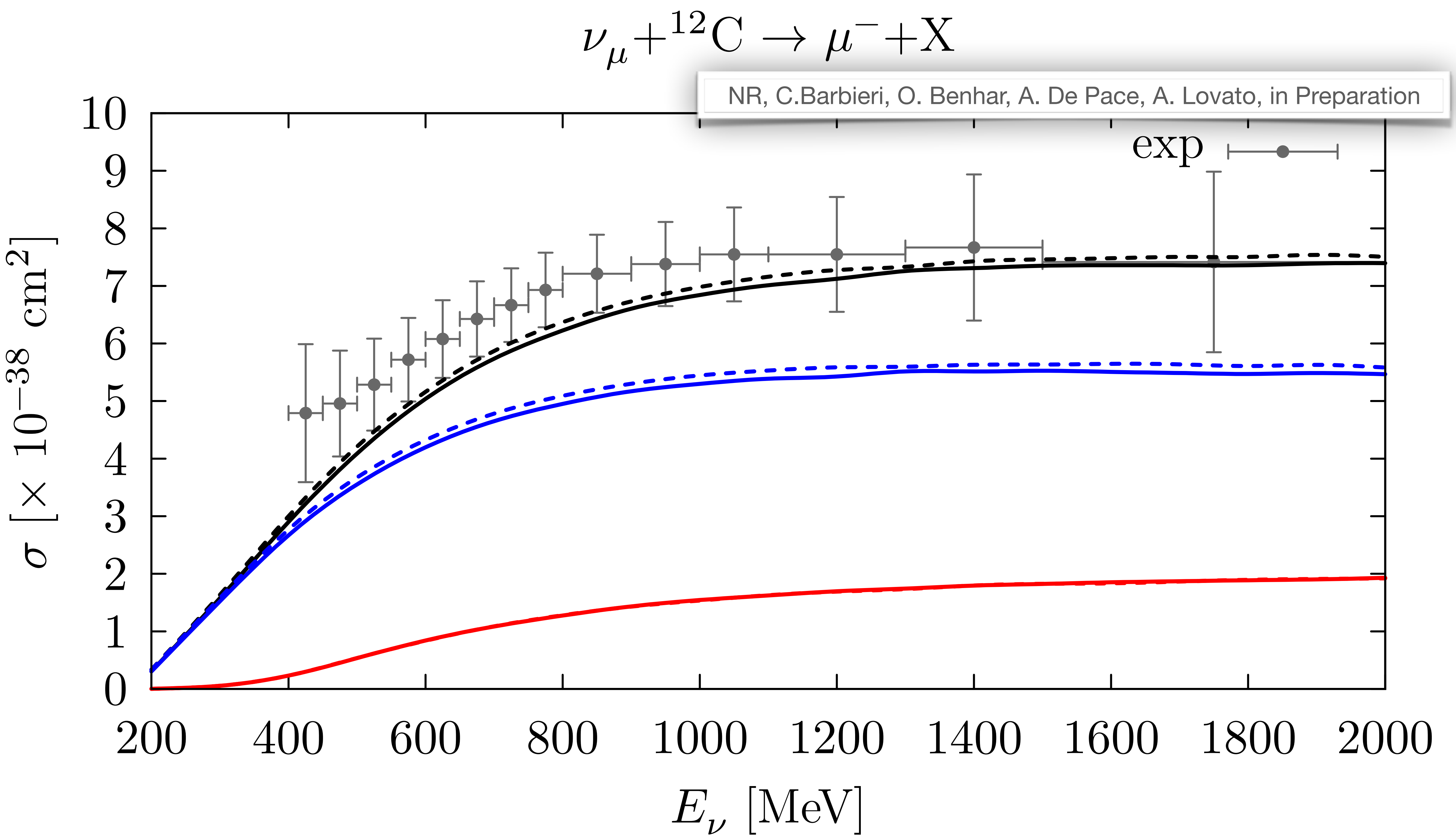
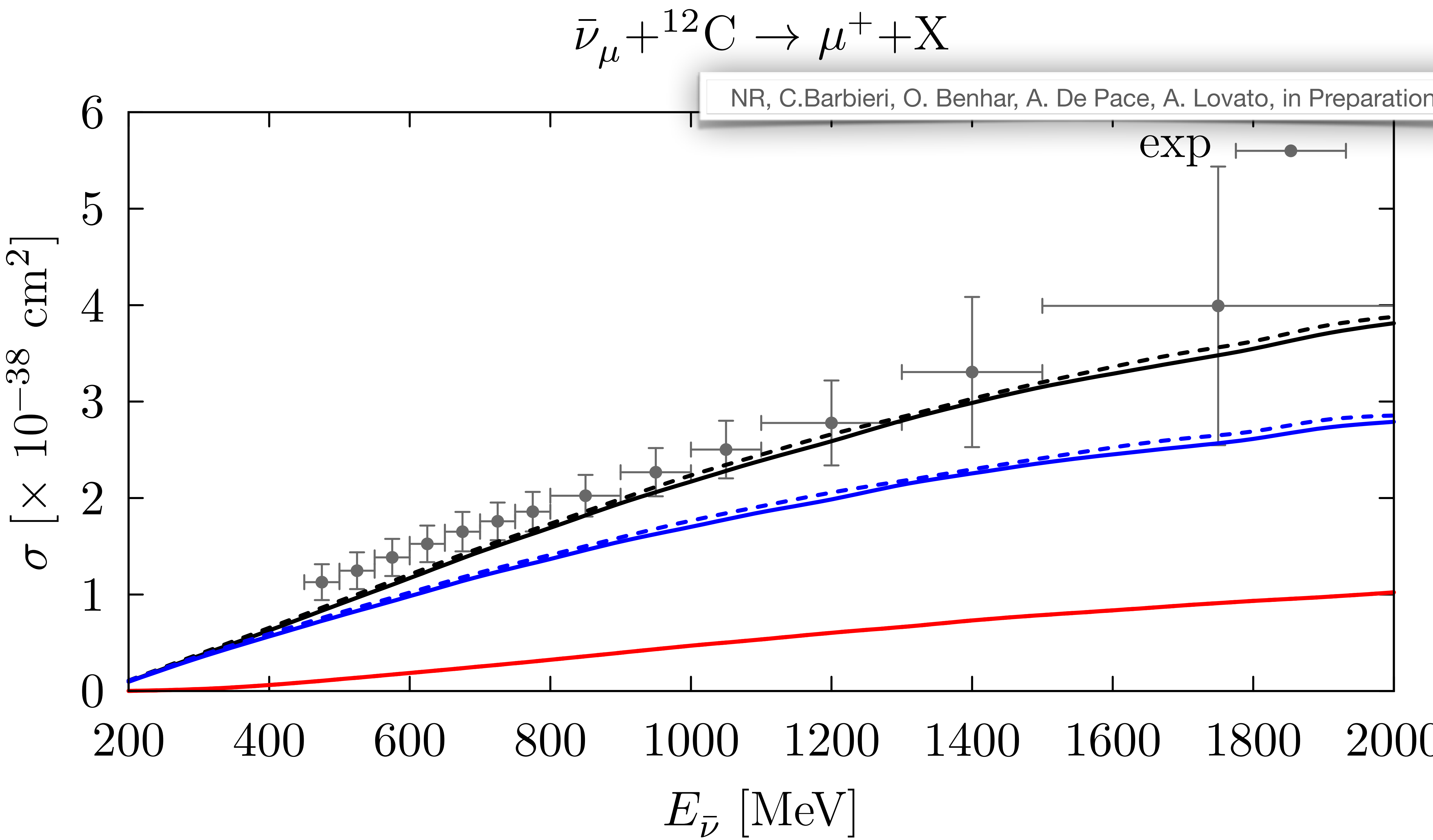


CCQE neutrino -^{12}C total cross section



- The 2p2h contribution is needed to explain the size of the measured cross section

CCQE antineutrino -^{12}C total cross section



NR, C.Barbieri, O. Benhar, A. De Pace, A. Lovato, in Preparation

- The 2p2h contribution is needed to explain the size of the measured cross section

Prospects

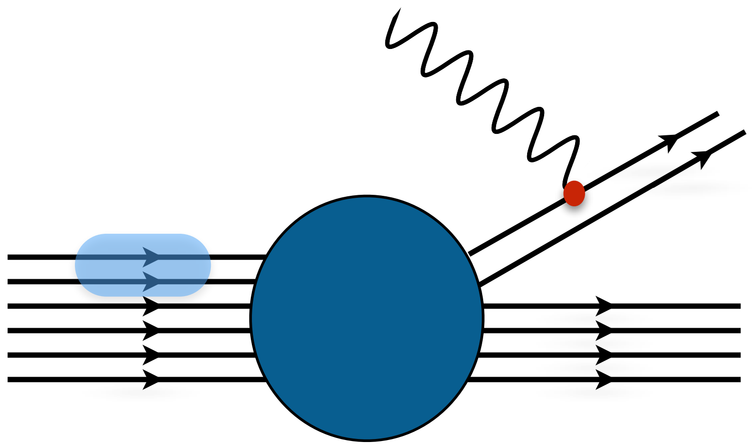
- Correlated Basis Function and Self Consistent Green's Function approach :
  - ❖ Flux-folded double differential inclusive cross sections for CC and NCQE processes
  - ❖ Inclusion of the interference between one- and two-body currents: benchmark with GFMC
  - ❖ Describe the resonance production region for CC and NC processes
  - ❖ Including the contribution of two-body currents in the ^{40}Ar and ··Ti scattering cross sections results

- Green's Function Monte Carlo approach :
  - ❖ Inverting the Euclidean responses for CC processes
  - ❖ Spectral Function calculation of light nuclei within GFMC with both phenomenological and chiral Hamiltonians
  - ❖ The use of different potentials can provide an estimate of the theoretical uncertainty of the calculation

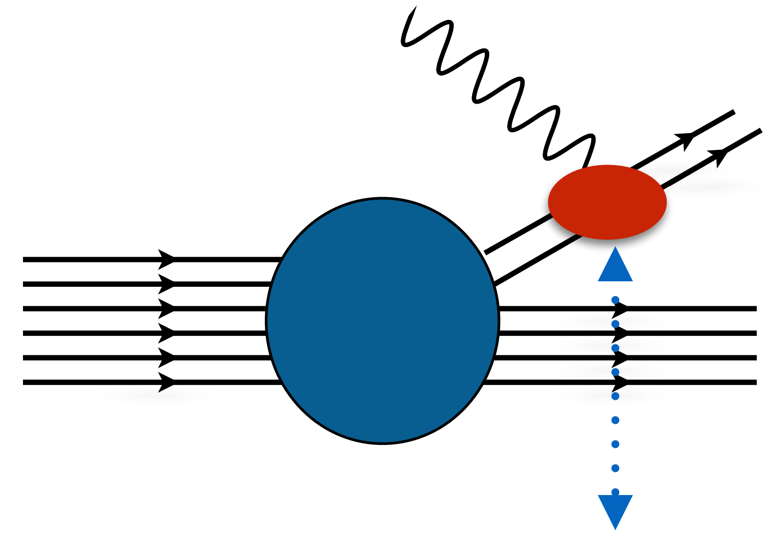
Back up slides

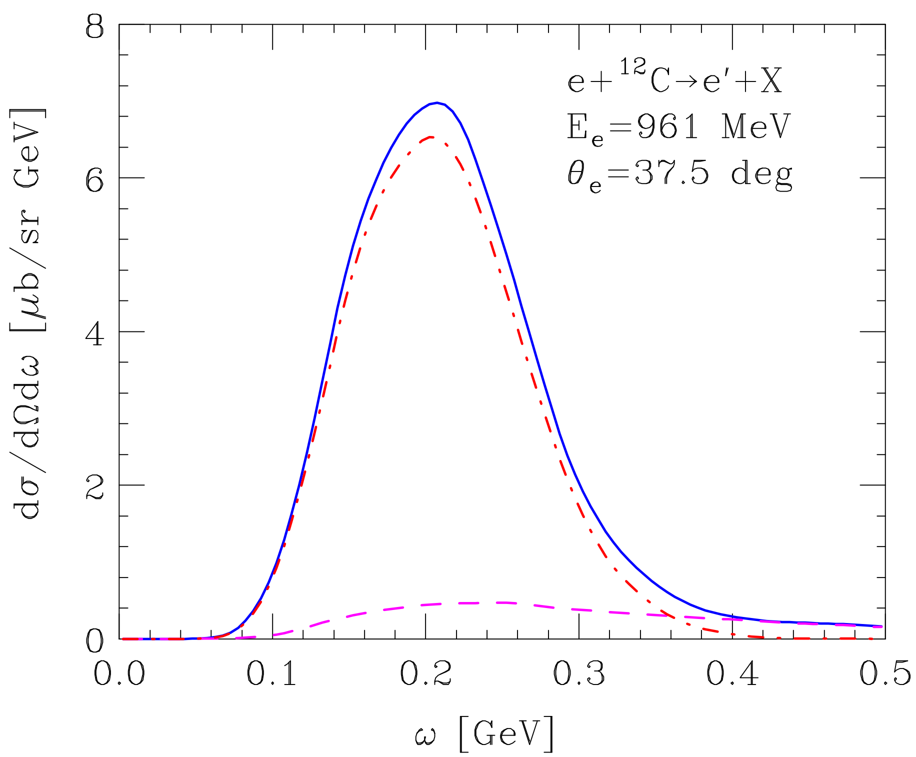
Production of two particle-two hole (2p2h) states

- Initial State Correlations

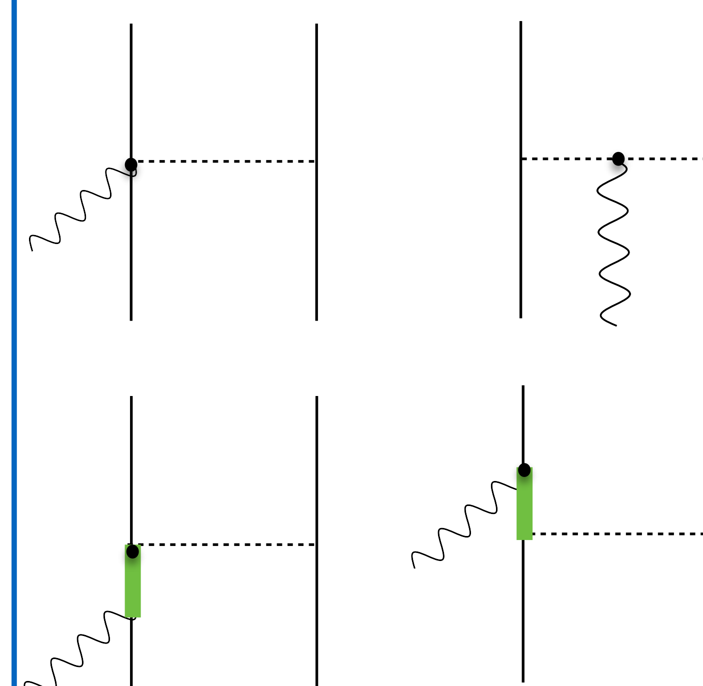


- Meson Exchange currents





- $P_{corr}(\mathbf{k},E)$ accounts for the presence of strongly correlated pairs. Its contribution to the cross section is clearly visible: appearance of a tail in the large energy transfer region



Different contributions to the relativistic two-body currents

- The Impulse Approximation has been generalized:

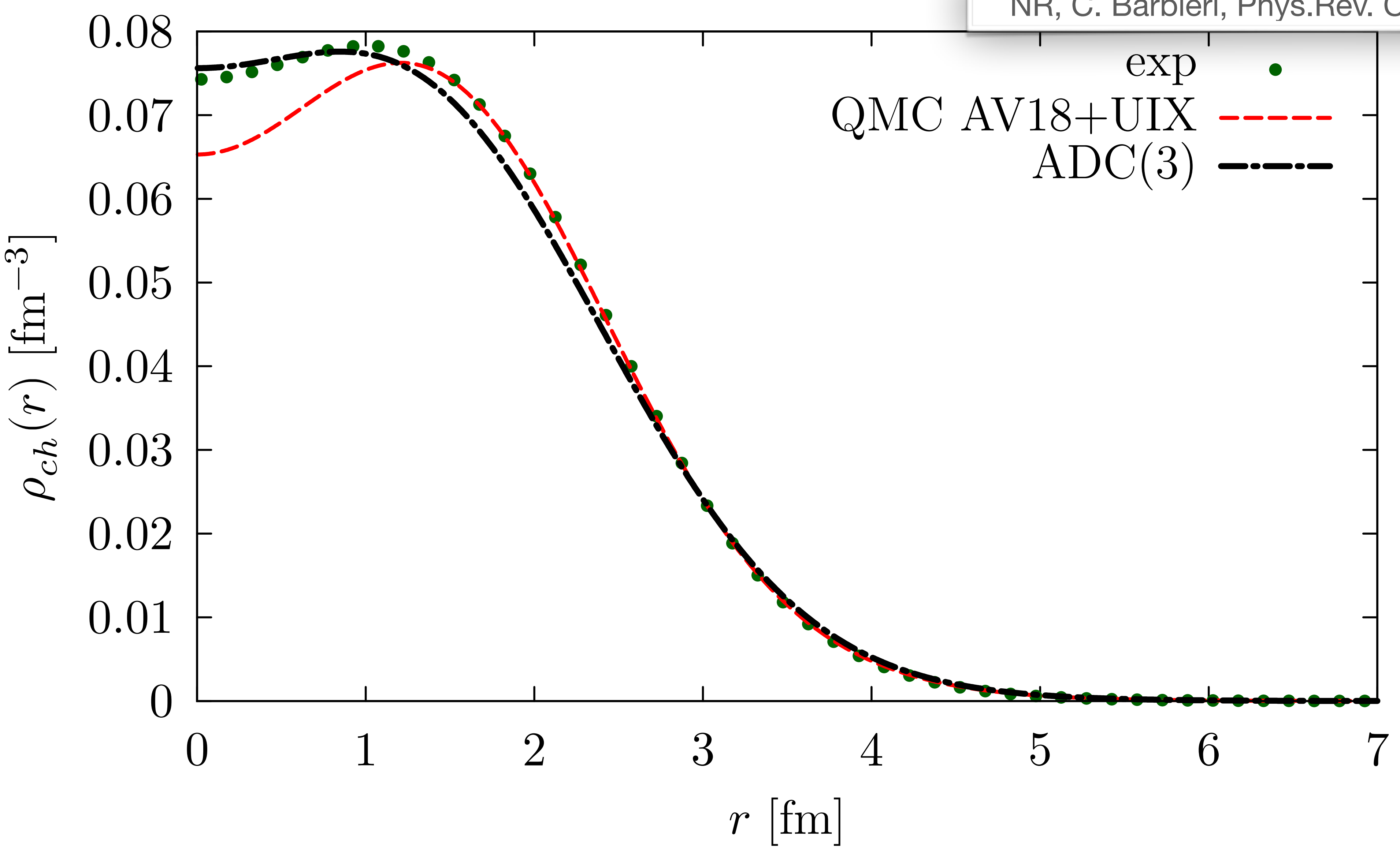
$$W^{\mu\nu}_{2p2h} = W^{\mu\nu}_{ISC} + W^{\mu\nu}_{MEC} + W^{\mu\nu}_{int}$$

Benchmark the nuclear model: ^{16}O charge density distribution

- The nuclear charge density distribution is the Fourier transform of the charge elastic form factor:

$$\rho_{ch}(r') = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}'} F_L(\mathbf{q})$$

NR, C. Barbieri, Phys.Rev. C98 (2018) 025501

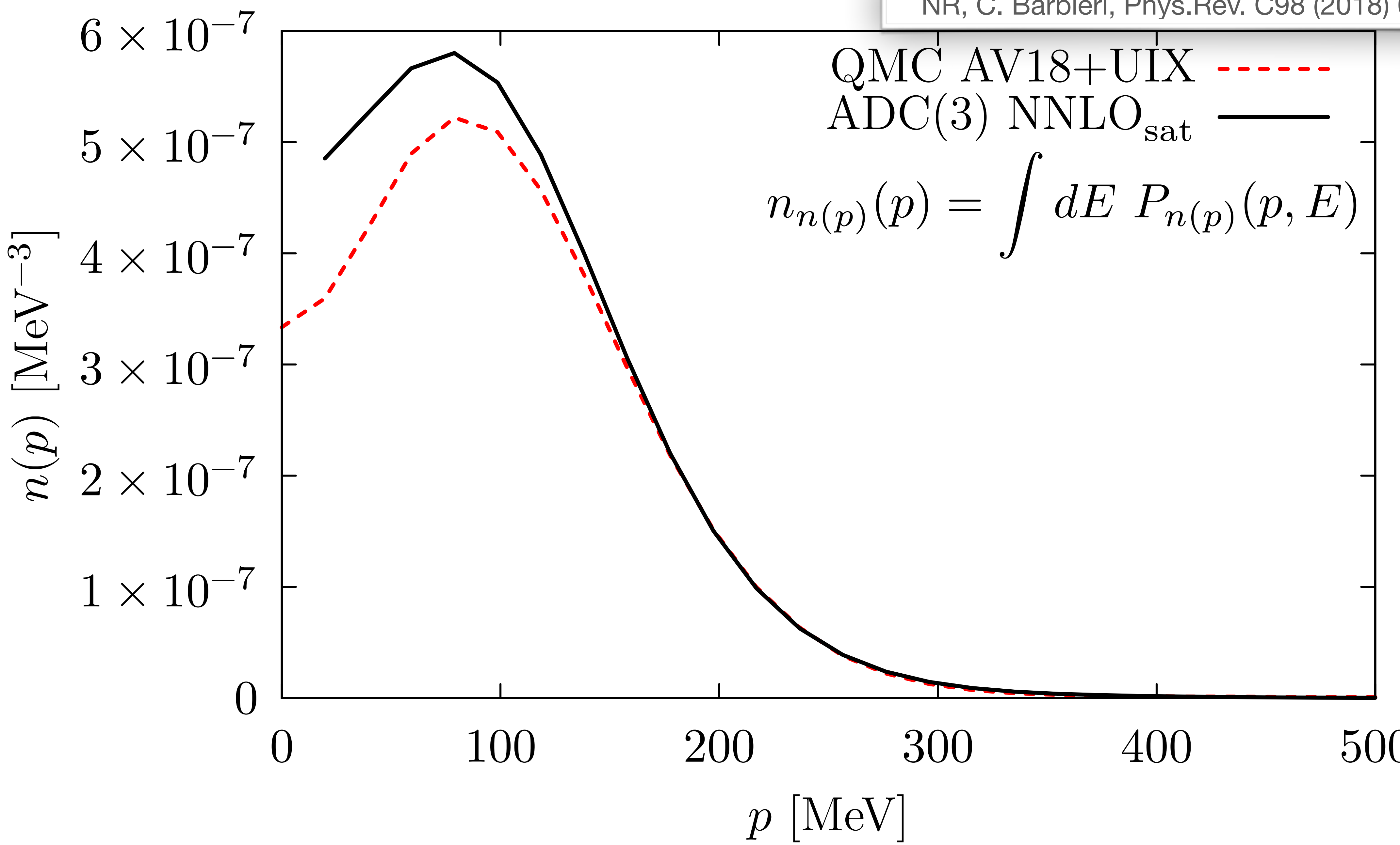


- Nice agreement between the SCGF and QMC calculations
- SCGF results agree with experiments (corroborates the goodness of $NNLO_{sat}$)

Benchmark the nuclear model: ^{16}O momentum distribution

- Single particle momentum distribution of ^{16}O

NR, C. Barbieri, Phys.Rev. C98 (2018) 025501

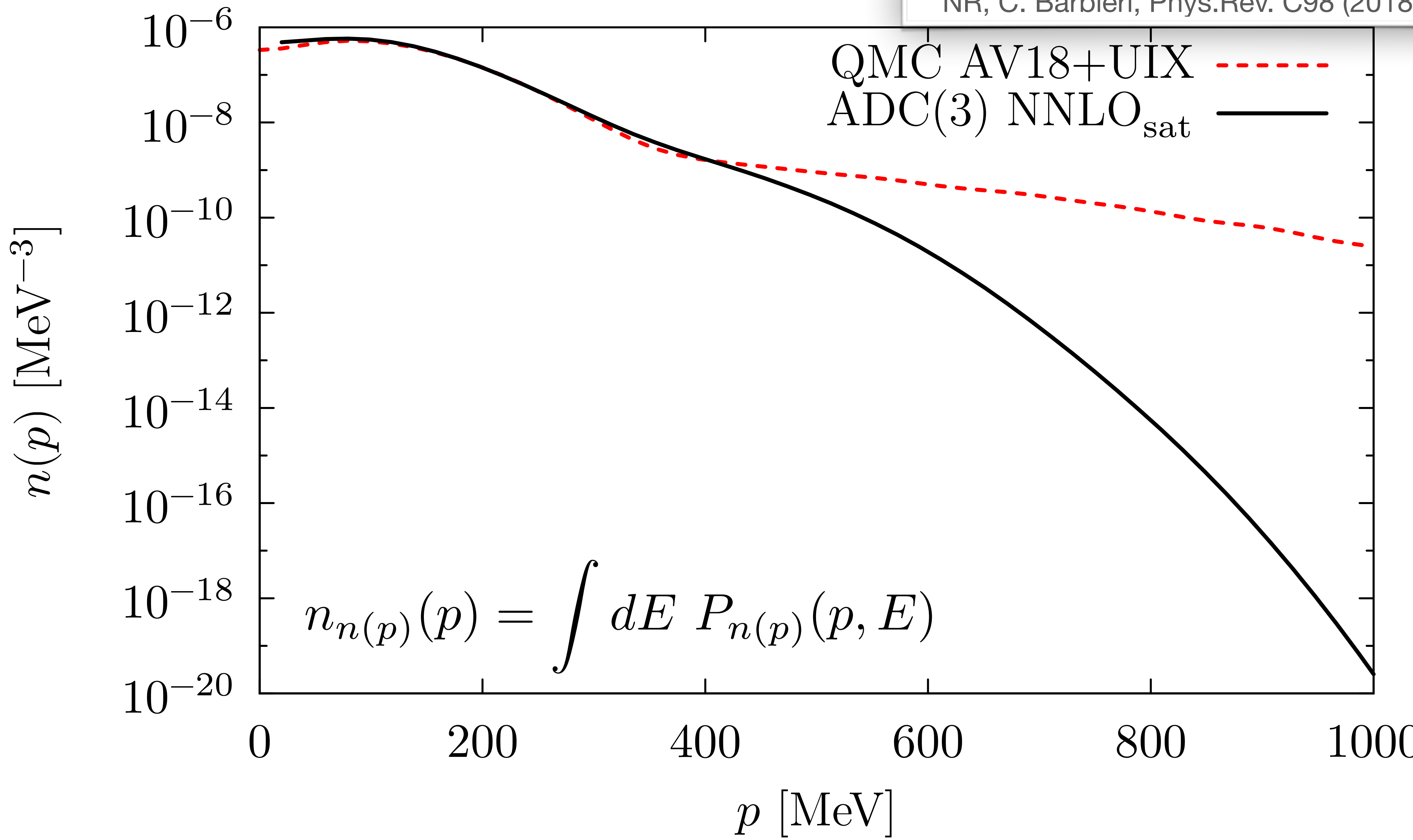


- The momentum distribution reflects the fact that NNLO$_{sat}$ is softer the AV18+UIX.

Benchmark the nuclear model: ^{16}O momentum distribution

- Single particle momentum distribution of ^{16}O, log scale

NR, C. Barbieri, Phys.Rev. C98 (2018) 025501



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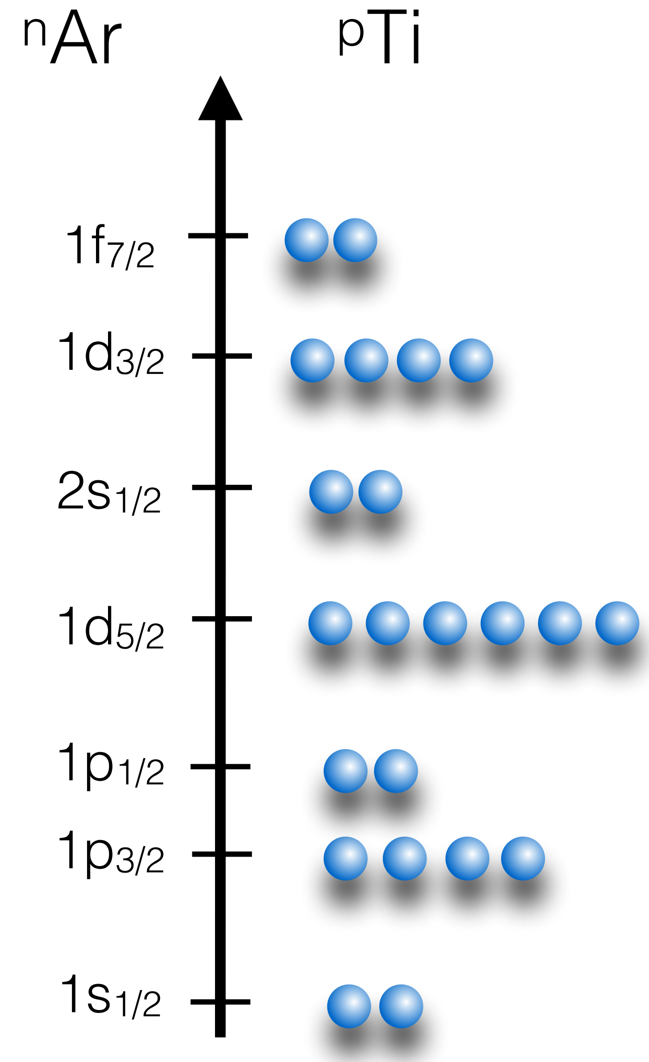
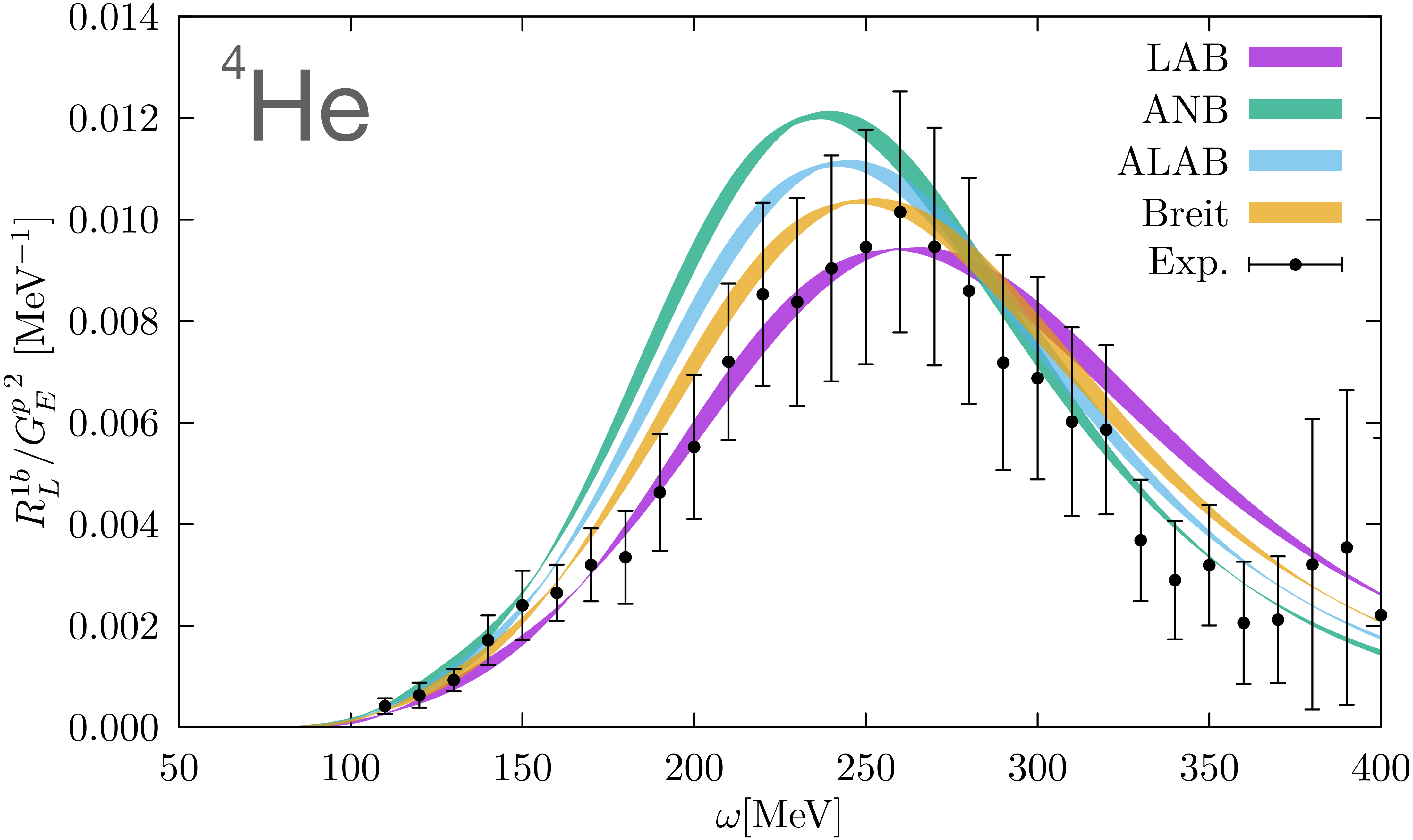


^{n}Ar
^{p}Ti
$1f_{7/2}$
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$

Relativistic effects in a correlated system



- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The curves show differences in both peak positions and heights.

Relativistic effects in a correlated system

- The frame dependence can be drastically reduced if one assumes a two-body breakup model with relativistic kinematics to determine the input to the non relativistic dynamics calculation

$$p^{fr} = \mu\Big(\frac{p_N^{fr}}{m_N} - \frac{p_X^{fr}}{M_X}\Big) \qquad \longleftrightarrow \qquad \mu = \frac{m_N M_X}{m_N + M_X}$$

$$P_f^{fr} = p_N^{fr} + p_X^{fr}$$

- The relative momentum is derived in a relativistic fashion

$$\omega^{fr} = E_f^{fr} - E_i^{fr}$$

$$E_f^{fr} = \sqrt{m_N^2 + [\mathbf{p}^{fr} + \mu/M_X \mathbf{P}_f^{fr}]^2} + \sqrt{M_X^2 + [\mathbf{p}^{fr} - \mu/m_N \mathbf{P}_f^{fr}]^2}$$

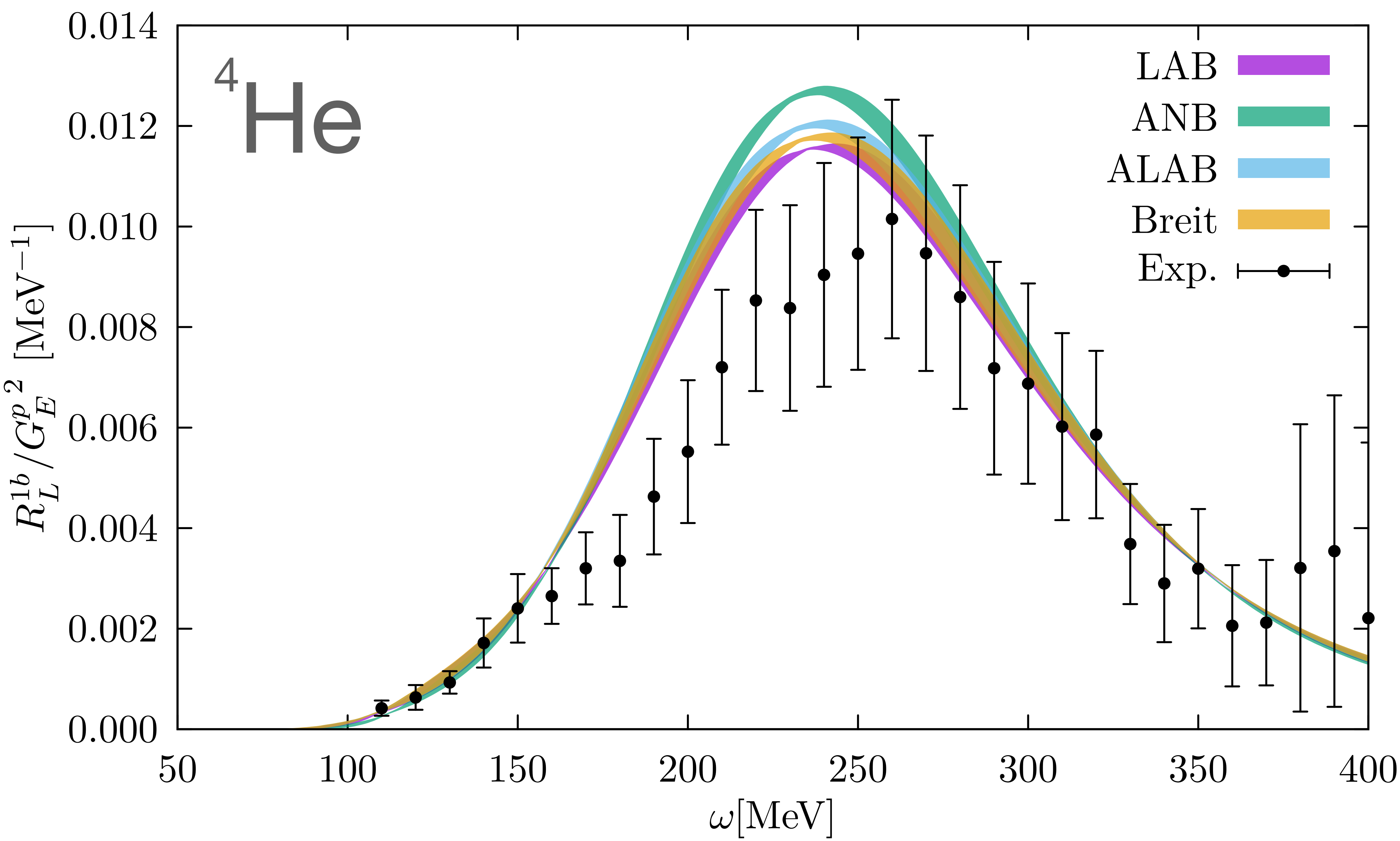
- And it is used as input in the non relativistic kinetic energy

$$e_f^{fr} = (p^{fr})^2/(2\mu)$$

- The energy-conserving delta function reads

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) = \delta(F(e_f^{fr}) - \omega^{fr}) = \Big(\frac{\partial F^{fr}}{\partial e_f^{fr}}\Big)^{-1} \delta[e_f^{fr} - e_f^{rel}(q^{fr}, \omega^f)]$$

Relativistic effects in a correlated system



- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The different curves are almost identical.

Extension of the factorization scheme to two-nucleon emission amplitude

$$|X\rangle \longrightarrow |\mathbf{p}\ \mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)}; \mathbf{p}\ \mathbf{p}'\rangle \ ,$$

We can introduce the two-nucleon Spectral Function...

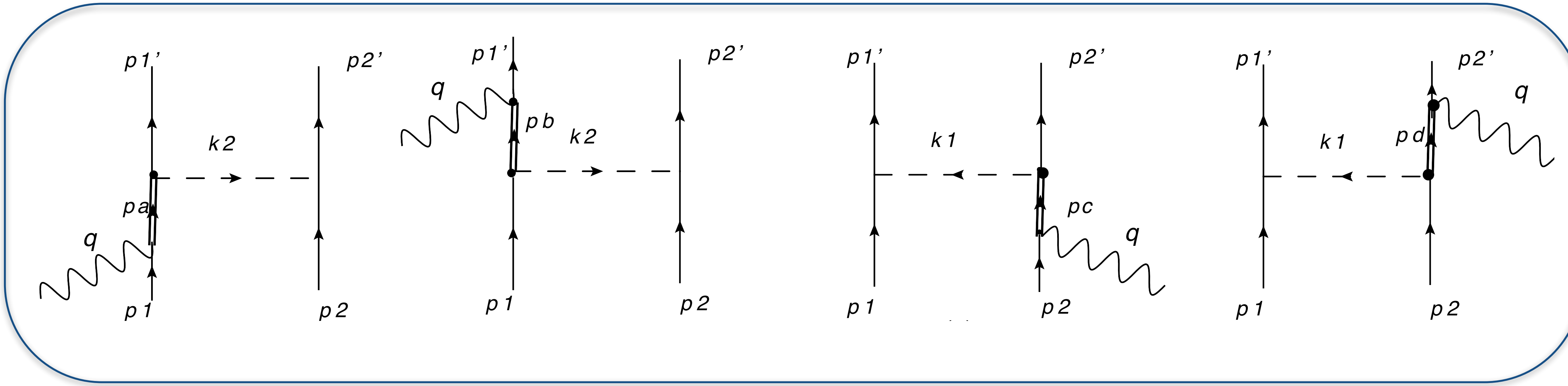
$$P(\mathbf{k}, \mathbf{k}', E) = \sum_n |\langle n_{(A-2)}; \mathbf{k}\ \mathbf{k}'|0\rangle|^2 \delta(E + E_0 - E_n)$$

probability of removing two nucleons leaving the A-2 system with energy E

The pure 2-body & the interference contribution to the hadron tensor read

$$W^{\mu\nu}_{2p2h,22} \propto \int d^3k d^3k' d^3p d^3p' \int dE\ P_{2h}(\mathbf{k}, \mathbf{k}', E) \langle \mathbf{k}\mathbf{k}'|j^{\mu}_{12}|\mathbf{p}\mathbf{p}'\rangle \langle \mathbf{p}\mathbf{p}'|j^{\nu}_{12}|\mathbf{k}\mathbf{k}'\rangle$$

$$W^{\mu\nu}{}_{2p2h,12} \propto \int d^3k\ d^3\xi\ d^3\xi'\ d^3h\ d^3h' d^3p\ d^3p' \phi^{hh'\,*}_{\xi\xi'} \langle \mathbf{p}, \mathbf{p}'|j^{\nu}_{12}|\boldsymbol{\xi}, \boldsymbol{\xi}'\rangle \left[\Phi^{hh'p'}_{k} \langle \mathbf{k}|j^{\mu}_{1}|\mathbf{p}\rangle + \Phi^{hh'p}_{k} \langle \mathbf{k}|j^{\mu}_{2}|\mathbf{p}'\rangle\right]$$



The Rarita-Schwinger (RS) expression for the Δ propagator reads

$$S^{\beta\gamma}(p, M_\Delta) = \frac{\not{p} + M_\Delta}{p^2 - M_\Delta^2}\left(g^{\beta\gamma} - \frac{\gamma^\beta\gamma^\gamma}{3} - \frac{2p^\beta p^\gamma}{3M_\Delta^2} - \frac{\gamma^\beta p^\gamma - \gamma^\gamma p^\beta}{3M_\Delta}\right)$$

WARNING

If the condition $p_\Delta^2 > (m_N + m_\pi)^2$ the real resonance mass has to be replaced by $M_\Delta \longrightarrow M_\Delta - i\Gamma(s)/2$ where $\Gamma(s) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{k^3}{\sqrt{s}}(m_N + E_k)$.

Hadronic monopole form factors

$$F_{\pi NN}(k^2) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - k^2}$$

$$F_{\pi N\Delta}(k^2) = \frac{\Lambda_{\pi N\Delta}^2}{\Lambda_{\pi N\Delta}^2 - k^2}$$

and the EM ones

$$F_{\gamma NN}(q^2) = \frac{1}{(1 - q^2/\Lambda_D^2)^2} \ ,$$

$$F_{\gamma N\Delta}(q^2) = F_{\gamma NN}(q^2)\Big(1 - \frac{q^2}{\Lambda_2^2}\Big)^{-1/2}\Big(1 - \frac{q^2}{\Lambda_3^2}\Big)^{-1/2}$$

where $\Lambda_\pi = 1300$ MeV, $\Lambda_{\pi N\Delta} = 1150$ MeV, $\Lambda_D^2 = 0.71\mathrm{GeV}^2$, $\Lambda_2 = M + M_\Delta$ and $\Lambda_3^2 = 3.5\ \mathrm{GeV}^2$.