

# Time-dependent GL approach to the dynamics of inhomogeneous chiral condensates\*

Juan Pablo Carlomagno

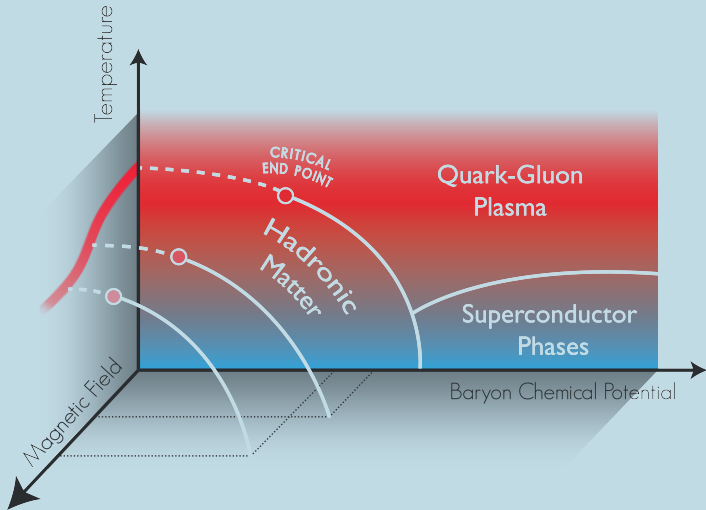
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arXiv:1804.05724



PHENOEXP 2018:  
LHC Physics and beyond  
May 10, 2018

# QCD phase diagram



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- Model calculations predict that different kind of inhomogeneous phases exist at finite density

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Formation of inhomogeneous spatial domains

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Formation of inhomogeneous spatial domains

**ANALYZE THE TIME EVOLUTION OF THE ORDER  
PARAMETER FOR THE CHIRAL SYMMETRY**

## What we need?

- An effective model that describes inhomogeneous quark condensates
- A formalism to study the time dynamics of an order parameter

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- An effective model that describes inhomogeneous quark condensates

⇒ **Nambu-Jona-Lasinio models**  
(with VEV's that break translational invariance)

- A formalism to study the time dynamics of an order parameter

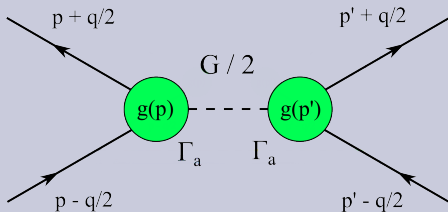
⇒ **Time dependent Ginzburg-Landau equations**  
(model A - non conserved order parameter)

## nonlocal NJL

$$\mathcal{L}_{NJL} = \bar{\psi} i \gamma_{\mu} \partial^{\mu} \psi + \frac{G}{2} j_{\alpha}(x) j_{\alpha}(x)$$

$$j_{\alpha}(x) = \int d^4 z \mathcal{G}(z) \bar{\psi} \left( x + \frac{z}{2} \right) \Gamma_{\alpha} \psi \left( x - \frac{z}{2} \right)$$

$$\phi_{\alpha}(\vec{x}) = \bar{\phi}_{\alpha}(\vec{x}) + \delta \phi_{\alpha}(\vec{x})$$

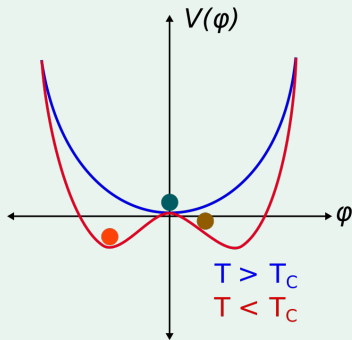




## Ginzburg-Landau equations (an example)

- free energy:

$$\begin{aligned}\Omega[\varphi] &= \int d^3x \omega[\varphi] \\ &= \int d^3x [(\nabla\varphi)^2 + V(\varphi)]\end{aligned}$$



$\varphi(x)$  is the order parameter

## Ginzburg-Landau equations (an example)

- free energy:

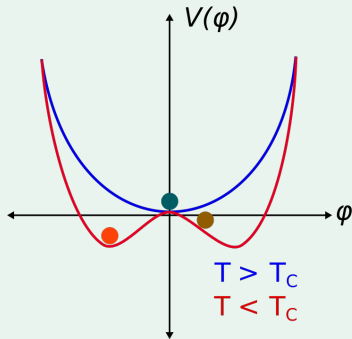
$$\begin{aligned}\Omega[\varphi] &= \int d^3x \omega[\varphi] \\ &= \int d^3x [(\nabla\varphi)^2 + V(\varphi)]\end{aligned}$$

- equilibrium:

$$\frac{\delta\Omega[\varphi]}{\delta\varphi(x,t)} = 0$$

- close to equilibrium:

$$\frac{\delta\Omega[\varphi]}{\delta\varphi(x,t)} = -\Gamma \frac{\partial\varphi(\vec{x},t)}{\partial t}$$



$\varphi(x)$  is the order parameter

## NJL (Nambu–Jona-Lasinio)

$$\omega(T, \mu, \phi) = \frac{\alpha_2}{2} \phi^2 + \frac{\alpha_4}{4} (\phi^2)^2 + \frac{\alpha_{4b}}{4} (\nabla\phi)^2 + \frac{\alpha_6}{6} (\phi^2)^3 + \frac{\alpha_{6b}}{6} (\phi, \nabla\phi)^2 + \frac{\alpha_{6c}}{6} [\phi^2(\nabla\phi)^2 - (\phi, \nabla\phi)^2] + \frac{\alpha_{6d}}{6} (\Delta\phi)^2$$

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## GL (Ginzburg-Landau)

$$\Gamma \frac{\partial \phi_a(x, t)}{\partial t} = - \frac{\delta \Omega[\phi_a]}{\delta \phi_a(x, t)}$$

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### Recipe:

1- Choose  $(T, \mu)$  in the hadronic phase, close to the critical points

2- Set initial conditions out of equilibrium:  $\langle \phi_a(x, t = 0) \rangle_x \sim 0$

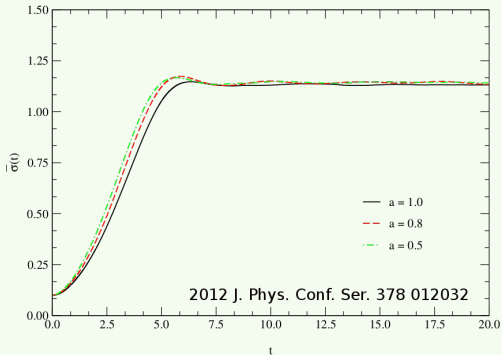
3- Study the time dynamics of the order parameters

# Volume average of the order parameter

random initial state with restored chiral symmetry (out of equilibrium)

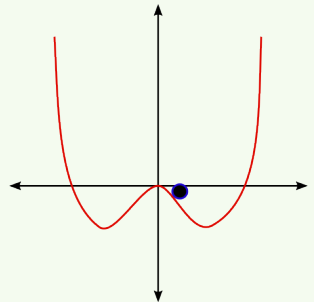


final homogeneous chirally broken state



Evolution of a homogeneous scalar field

CARTOON:

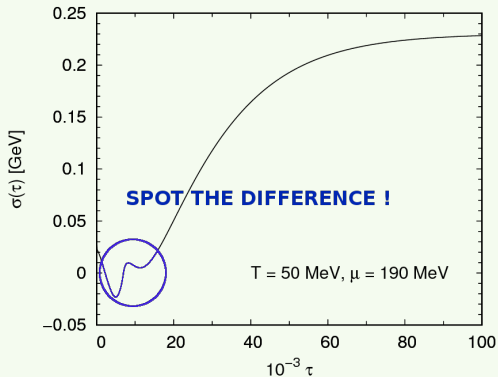


## Volume average of the order parameter

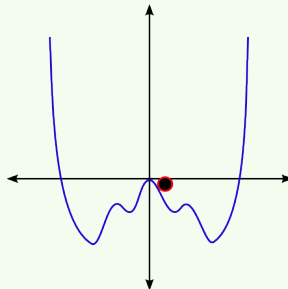
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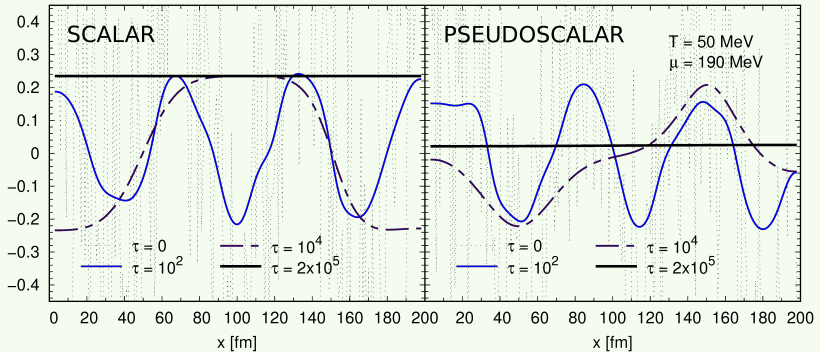


CARTOON:



Evolution of the scalar field when the vacuum has a nontrivial spatial dependence

## Snapshots of evolution



**LONG-LIVED METASTABLE CONFIGURATIONS OF THE ORDER PARAMETERS IN THE COURSE OF THE EVOLUTION (IN THE HADRONIC REGION)**

**EVEN IF AT EQUILIBRIUM THE CHIRAL PARAMETERS HAVE NO SPATIAL MODULATION, THE SYSTEM CAN LEAVE TRACES OF THE INHOMOGENEITIES IN OBSERVABLES**



## Summary (if I have time)

- ▶ **MOTIVATION** : UNDERSTAND THE DYNAMICS INVOLVED IN THE CHIRAL SYMMETRY RESTORATION.
- ▶ **WHAT TO DO** : STUDY THE TIME EVOLUTION OF THE CORRESPONDING ORDER PARAMETER.
- ▶ **HOW TO DO IT** :
  - EXPAND THE MEAN-FIELD THERMODYNAMIC POTENTIAL IN POWERS OF THE ORDER PARAMETERS AND THEIR SPATIAL GRADIENTS.
  - USE THE TIME DEPENDENT GINZBURG-LANDAU EQUATIONS OF MOTION
- ▶ **RESULTS** : IN THE HADRONIC REGION OF THE QCD PHASE DIAGRAM WE FOUND LONG-LIVED METASTABLE CONFIGURATIONS OF THE ORDER PARAMETERS IN THE COURSE OF THE EVOLUTION.
- ▶ **PERSPECTIVES** : 3D + EXPANSION + EXACT FREE ENERGY

thanks!