Charged pion masses under strong magnetic fields in the NJL model

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arXiv.org > hep-ph > arXiv:1802.08041

High Energy Physics - Phenomenology

Charged pion masses under strong magnetic fields in the NJL model

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MOTIVATION

- Strong Magnetic Fields: compact stars, heavy ion collisions
- In NJL, calculations for m_{π^0} but not for $m_{\pi^\pm}$ (not exact)
- LQCD calculations for m_{π^+} for **small** eB<0.4 GeV^2 (2013)
- Recent LQCD results (2017) for both m_π for **large** eB (<4 GeV^2) with **unrealistic** $m_\pi(B=0) = 415\ \mathrm{MeV}$
- Neutral Pion: Recent NJL calculation (2017) in full RPA with **G(eB)** in Fourier basis that agrees with LQCD but for a set with $f_\pi \sim 80\ \mathrm{MeV}$ in vacuum
- Charged Pion: Schwinger phases do not cancel, no translational invariance, can not use Fourier basis. Recent studies neglect this phases, or use a derivative expansion approximation (inexact for large eB or $m_{\pi^\pm}$)

THEORETICAL KEYS (local NJL with B)

- Quark Propagator: $$S^{\mathrm{MF},f}_{x,x'} = e^{i\Phi_f(x,x')} \int_p e^{i\,p\,(x-x')}\, \tilde{S}^f_p$$
  - $S^{\mathrm{MF},f}_{x,x'}$ → Flavor
  - $e^{i\Phi_f(x,x')}$ → Schwinger Phase
  - $\tilde{S}^f_p$ → Landau Levels $\sum_{k=0}^{\infty}(\ldots)$ / Proper time $\int_0^{\infty}(\ldots)$ — We use this one

- Polarization functions:
$$\begin{cases} J_{\pi^0}(x,x') = N_c \sum_f \mathrm{Tr}_{\mathrm{D}}\left[\mathcal{S}^{\mathrm{MF},f}_{x,x'}\ \gamma_5\ \mathcal{S}^{\mathrm{MF},f}_{x',x}\ \gamma_5\right] \\ J_{\pi^\pm}(x,x') = 2N_c\, \mathrm{Tr}_{\mathrm{D}}\left[\mathcal{S}^{\mathrm{MF},u}_{x,x'}\ \gamma_5\ \mathcal{S}^{\mathrm{MF},d}_{x',x}\ \gamma_5\right] \end{cases}$$

- Neutral Pion: $\Phi_f(x,x')\,\Phi_f(x',x) = 0 \;\Rightarrow\; \pi^0(x) = \int_q e^{iqx}\,\pi^0(q)$ → **Fourier**

- Charged Pion: $\Phi_u(x,x')\,\Phi_d(x',x) \neq 0 \;\Rightarrow\; \pi^+(x) = \sum_{\bar{q}} \mathbb{F}^+_{\bar{q}}(x)\,\pi^+_{\bar{q}}$ → **Ritus** → In each basis each $J_\pi(q,q')$ is diagonalized

- Regularization: MFIR scheme with 3D cutoff

RESULTS – QUARK CONDENSATE

- Three Parameters Sets: $M_0 = 350,\ 320,\ 380,\ G = cte,$ $\begin{cases} m_\pi = 138\ \mathrm{MeV} \\ f_\pi = 92.4\ \mathrm{MeV} \end{cases}$

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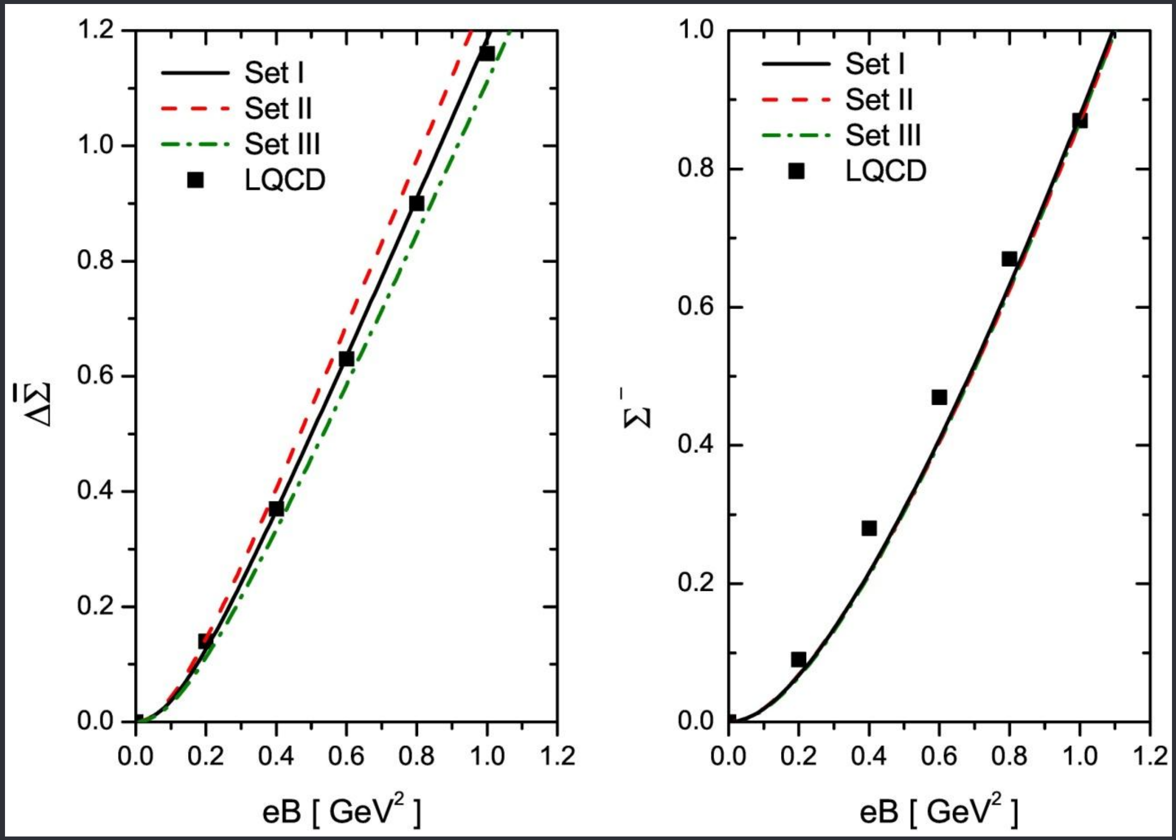
- In *Avancini* '17 they use G(eB) to fit LQCD, G=*const.* does not agree



$$\Delta\bar{\Sigma}(B) \equiv \frac{\Delta\Sigma_u(B) + \Delta\Sigma_d(B)}{2}$$

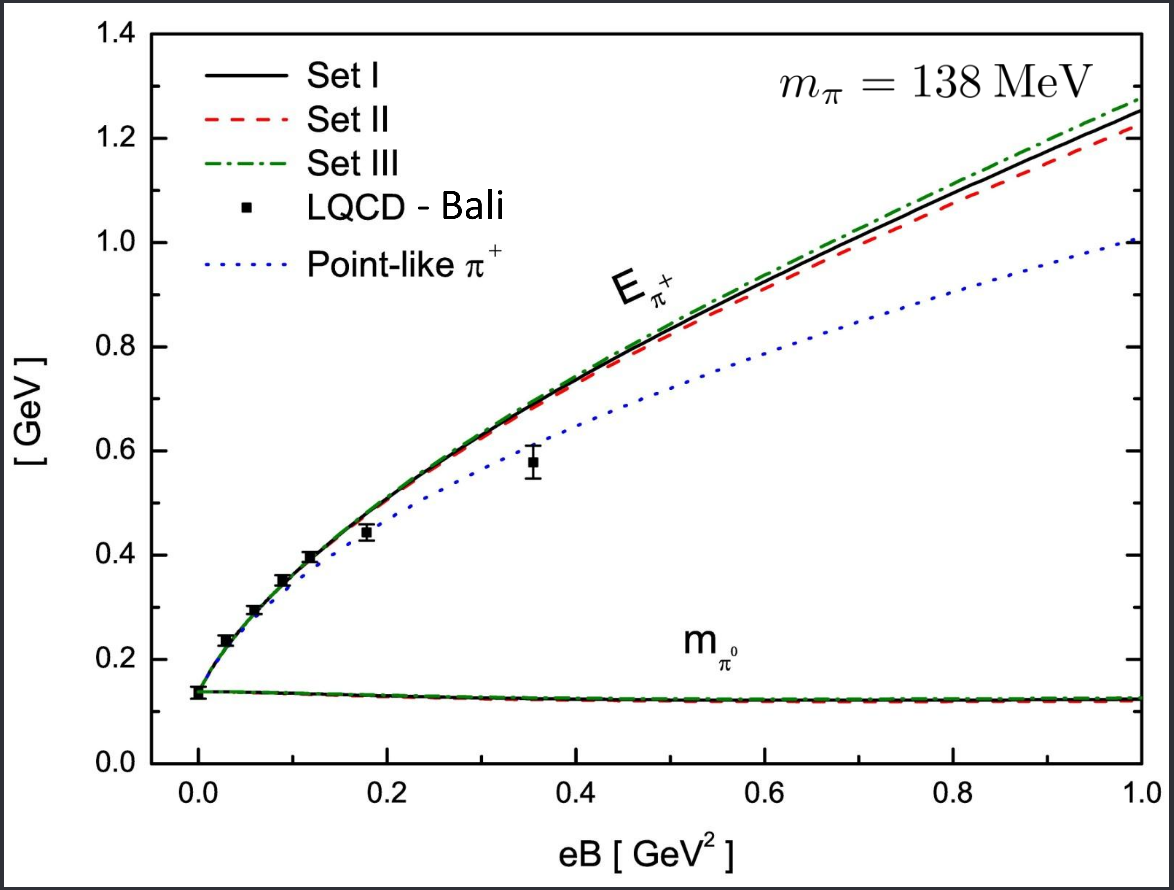
$$\Sigma^-(B) = \Delta\Sigma_u(B) - \Delta\Sigma_d(B)$$

$$\Delta\Sigma_f(B) = -\frac{2\,m_0}{D^4}\left[\langle\bar{f}f\rangle_B - \langle\bar{f}f\rangle_0\right]$$

- This sets agree with LQCD
- Rather independent of the set

RESULTS – PION MASSES (138 MeV)

- Instead of m_{π^+}: $E_{\pi^+}(eB) = \sqrt{m_{\pi^+}^2 + (2k+1)\,eB + q_3^2}\,\Big|_{\substack{q_3=0\\k=0}} = \sqrt{m_{\pi^+}^2 + eB}$



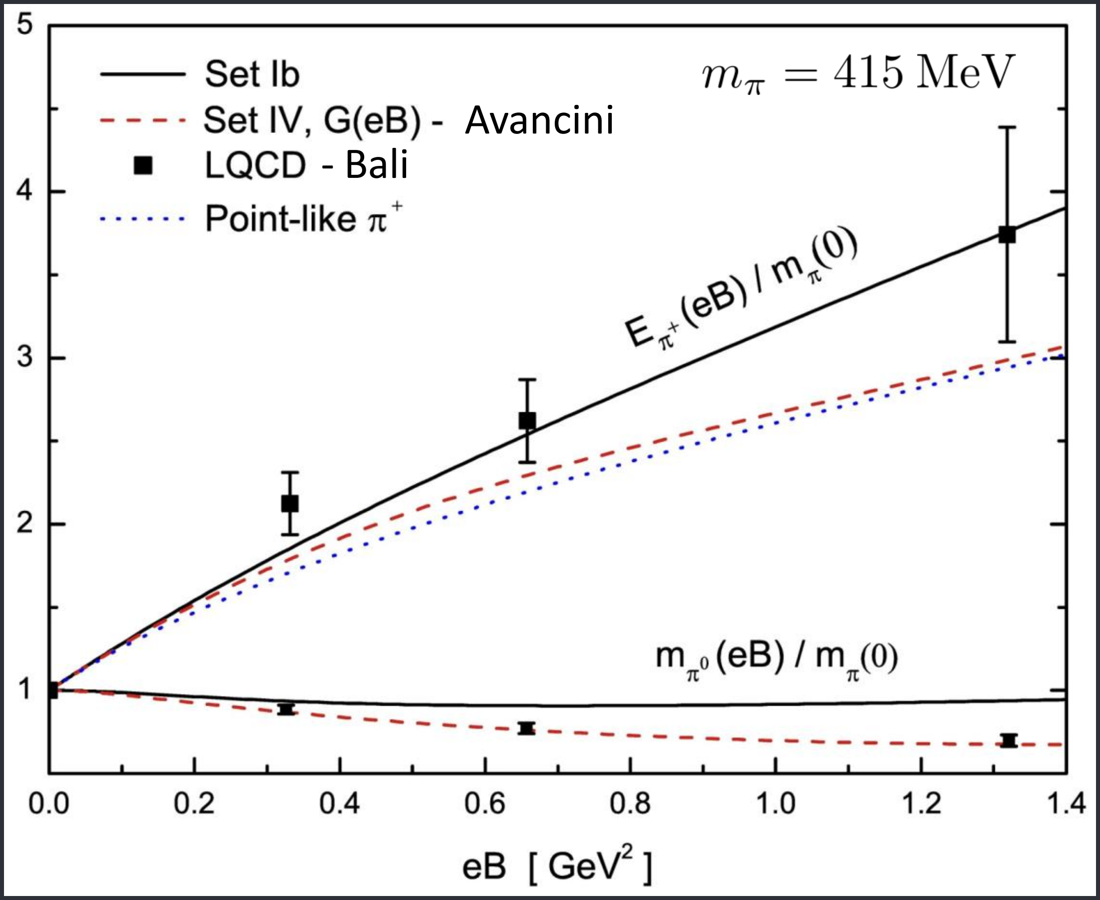
- Realistic vacuum pion mass
- Agreement with LQCD for eB<0.15 GeV^2
- Neutral Pion agrees with previous results

NORMALIZED PION MASSES (415 MeV)

Set IV

$G(eB) = \alpha + \beta\, e^{-\gamma\,(eB)^2}$

- LQCD results for unrealistic $m_\pi(B=0) = 415\,\mathrm{MeV}$
- Set Ib = Set I with m_0 increased, as in Set IV of *Avancini* ‘17



<u>Neutral Pion</u>

- Low error bars in LQCD
- Set Ib above LQCD
- Set IV fitted to LQCD but with $f_\pi \simeq 80\,\mathrm{MeV}$

<u>Charged Pion</u>

- High error bars in LQCD
- Set Ib fits results from spacing extrapolation to the continuum
- Set IV fits results from finite lattice spacing

CONCLUSIONS

- <u>Quark Condensate</u>: we found Sets with G=*const.* that agree with LQCD

- <u>Neutral Pion</u>: Set IV with G(eB) fits LQCD ($m_\pi = 415\,\mathrm{MeV}$). Set Ib with G=*const.* lies above LQCD for large eB. We verified that using proper time or Landau levels (as in *Avancini* '17) for $\tilde{S}_p^f$ lead to the same results.

- <u>Charged Pion</u>: mass results agree with LQCD for eB<0.15 GeV^2, deviation for larger eB. Normalized mass agrees for Set Ib ($m_\pi = 415\,\mathrm{MeV}$), but error bars of LQCD are too large to be conclusive.

- **Development of the Ritus method for charged pions**. Fully takes into account the translational-breaking effect of the Schwinger phases. Can calculate $f_\pi(eB)$ or $\Gamma_\pi(eB)$, pion properties at finite T, etc.

Thanks!

EXTRAS

➢ Quark propagator

$$\tilde{S}_p^f = \int_0^\infty d\tau \; \exp\left[-\tau\left(M^2 + p_\parallel^2 + p_\perp^2 \frac{\tanh \tau B_f}{\tau B_f}\right)\right] \times$$
$$\left\{(M - p_\parallel \cdot \gamma_\parallel)\left[1 + i s_f \; \gamma_1\gamma_2 \; \tanh \tau B_f\right] - \frac{p_\perp \cdot \gamma_\perp}{\cosh^2 \tau B_f}\right\}$$

Proper time

$$\tilde{S}_p^f = 2 \exp(-p_\perp^2/|q_f B|) \sum_{k=0}^{\infty} \sum_{\lambda=\pm} \frac{1}{d^f_{k,p_\parallel}} \times$$
$$\left[(-1)^{k_\lambda}(M - p_\parallel \cdot \gamma_\parallel) L_{k_\lambda}(2p_\perp^2/B_f) + 2\,(-1)^k\, p_\perp \cdot \gamma_\perp \, L^1_{k-1}(2p_\perp^2/B_f)\right] \Delta^\lambda$$

Sum over Landau Levels

➢ Polarization Functions in coordinate space

$$S^{\text{quad}}_{\text{bos}} = \frac{1}{2}\int_{x,x'} \delta\pi(x)^* \left[\frac{1}{2G}\,\delta^{(4)}(x - x') - J_\pi(x,x')\right] \delta\pi(x')$$

$$J_{\pi^0}(x,x') = N_c \sum_f \text{Tr}_{\text{D}}\left[\mathcal{S}^{\text{MF},f}_{x,x'}\;\gamma_5\;\mathcal{S}^{\text{MF},f}_{x',x}\;\gamma_5\right], \quad J_{\pi^\pm}(x,x') = 2N_c\,\text{Tr}_{\text{D}}\left[\mathcal{S}^{\text{MF},u}_{x,x'}\;\gamma_5\;\mathcal{S}^{\text{MF},d}_{x',x}\;\gamma_5\right]$$

EXTRAS

- Polarization Functions diagonalized in momentum space

$$S^{\rm quad}_{\pi^0} = \frac{1}{2}\int_q \delta\pi^0(-q)\left[\frac{1}{2G} - J_{\pi^0}(q_\perp^2, q_\parallel^2)\right]\delta\pi^0(q) \longrightarrow \frac{1}{2G} - J^{(reg)}_{\pi^0}(0, -m^2_{\pi^0}) = 0$$

$$\downarrow$$

$$q_\perp^2 = 0,\ \ q_\parallel^2 = m^2_{\pi^0}$$

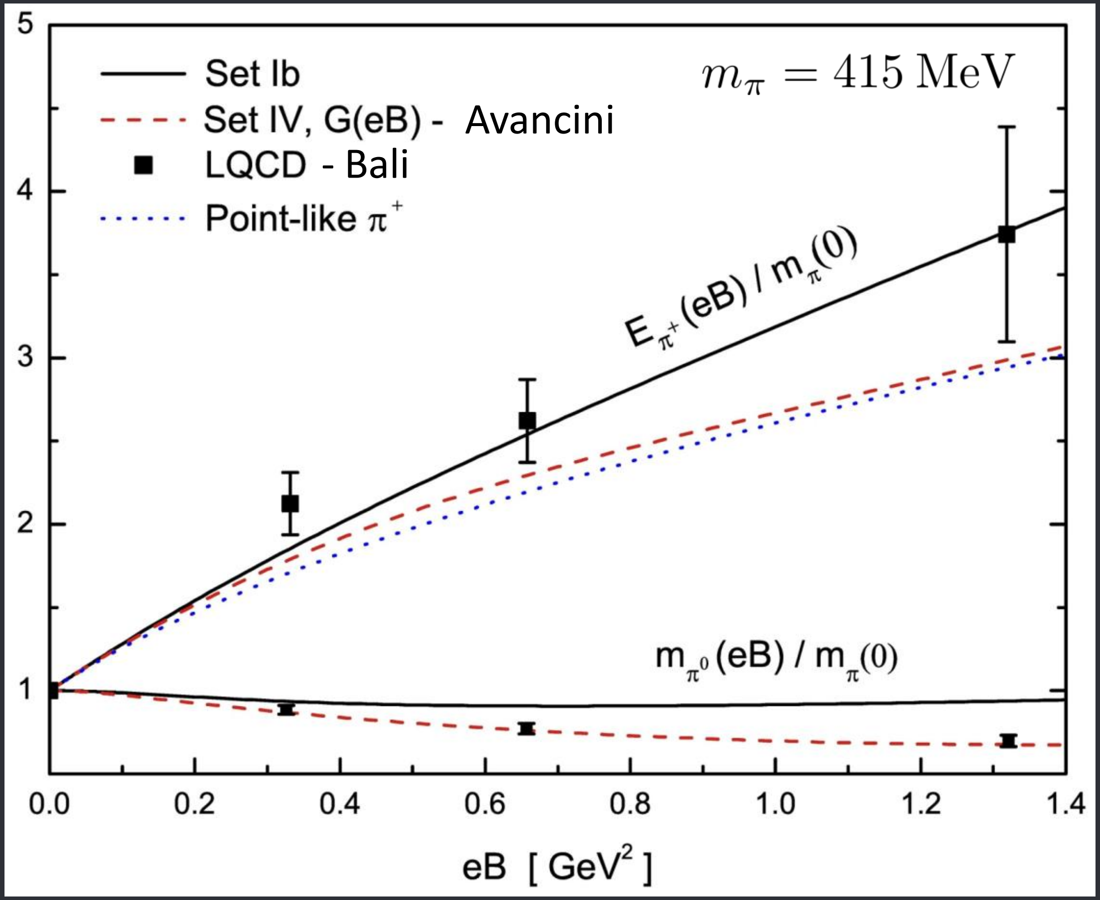
$$S^{\rm quad}_{\pi^+} = \frac{1}{2}\sum_{\bar{q}} (\delta\pi^+_{\bar{q}})^* \left[\frac{1}{2G} - J_{\pi^+}(k, \Pi^2)\right]\ \delta\pi^+_{\bar{q}} \longrightarrow \frac{1}{2G} - J^{(\rm reg)}_{\pi^+}(k, -m^2_{\pi^+}) = 0$$

$$\downarrow$$

$$\Pi^2 = (2k+1)\, B_{\pi^+} + q_\parallel^2 = m^2_{\pi^+}$$

NORMALIZED PION MASSES (415 MeV)

- LQCD results from lattice spacing extrapolation to the continuum (the neutral pion fits finite lattice spacing)



NORMALIZED PION MASSES (415 MeV)

- LQCD results for the meson masses from the lines of constant physics