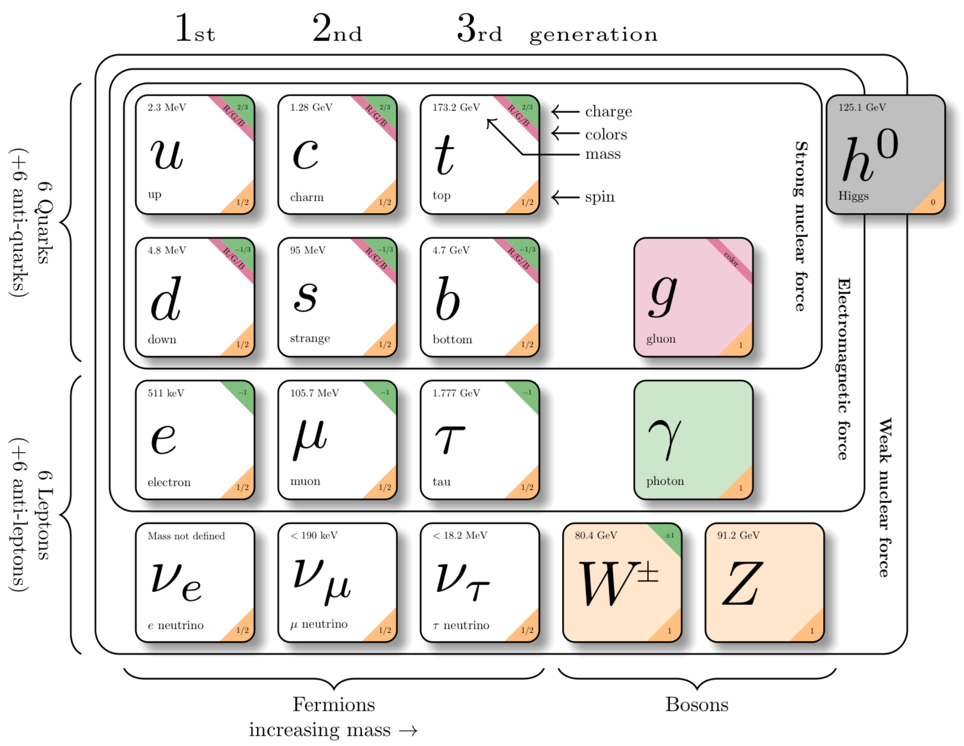
Electroweak Physics

An introduction

John Anders

The Standard Model



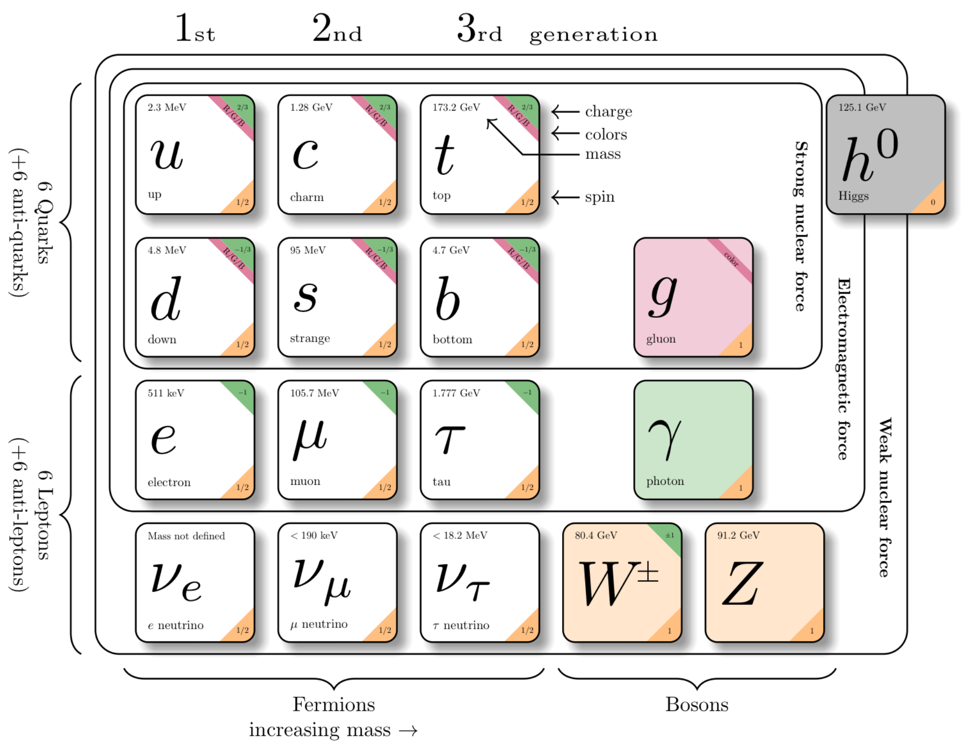
- The Standard Model (SM) contains everything we currently know about visible matter and forces (...except gravity)
  - Fermions: constituents of visible matter in the universe.
  - Gauge Bosons: mediate the interactions between fermions

- Electromagnetism: photon
- Weak nuclear force: $W^{\pm}$, Z

} This set of lectures

- Strong nuclear force: gluon – QCD Lectures: C. Doglioni
- Higgs boson: excitation of the higgs field, which “gives mass” to massive particles – Higgs Lectures: P. Francavilla

The Standard Model



Force	Strength	Range (m)	Mediator	Charge
Electromagnetic	10^{-2}	Infinite	Photon	Electric
Weak	10^{-13}	10^{-18}	$W^{\pm}$/Z Boson	Weak
Strong	10	10^{-15}	Gluon	Colour

Overview

- Introduction to electroweak physics
  - Key building block of the SM

- Lecture 1
  - Overview of Quantum Mechanics
  - Some important Langrangians
  - The Klein-Gordon equation
  - Dirac equation

- Lecture 2
  - Feynman Diagrams
  - Quantum Electrodynamics
  - Cross-section calculations

- Lecture 3
  - The Weak nuclear force
  - Lepton universality
  - CKM Matrix
  - Flavour physics

- Lecture 4
  - EW & Z Boson masses
  - Electroweak unification
  - Experimental verification

Electroweak physics: The Basics

Lecture 1

- Overview of basic mathematical tools required to construct electroweak theory:
- The Basics
    - Maxwell's equations
    - Special relativity and QM
    - Schrödinger → Born → KG Equation
    - Dirac Equation and anti-matter
    - Lagrangian formalism
    - Noether's theorem
    - Conservation Laws

Maxwell's Equations

- Maxwell (1895) proposed electric and magnetic fields are related by four equations...
  Electromagnetic unification

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \text{ - Gauss' Law (electrical)}$$

$$\nabla \cdot \vec{B} = 0 \text{ - Gauss' Law (magnetism)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{ - Faraday's Law}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \text{ - Ampere's Law}$$

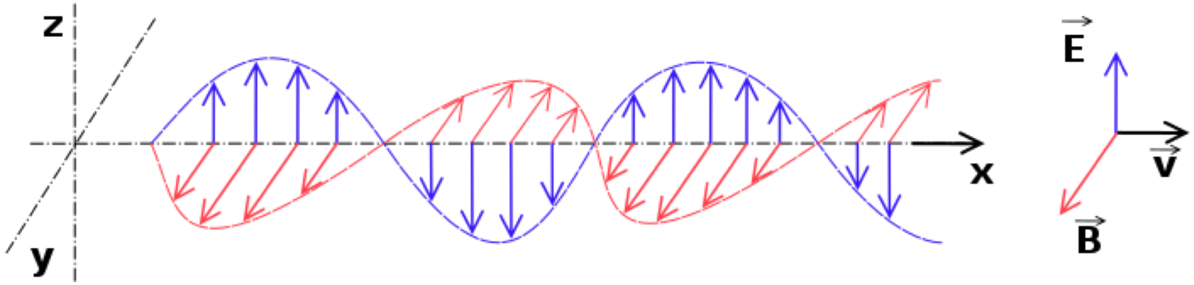
- Separates the object that feels the force, from the force itself

- Oscillating fields produce EM waves

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}, \; \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

- Light!



Special relativity

Introduction

- Maxwell realised his equations were dependent upon the observer's frame of reference
  - Also proposed a medium (luminiferous ether) that exists throughout the universe for the propagation of light
  - Michelson-Morley experiment disproved this

- Einstein (special relativity):
  - Postulate I: The laws of physics are identical in all inertial frames of reference (Lorentz invariance)
  - Postulate II: The speed of light in vacuum is constant in all inertial reference frames
    (no medium needed, resolves frame of reference issue)

  - Space and time can "mix", leading to 4-dimensional spacetime (4-vectors)
  - Different reference frames lead to length contraction and time dilation
  - Mass and energy equivalence: $E = mc^2$

Special relativity

Electromagnetism

- Maxwell's equations can be made relativistically invariant using potentials (V, A) instead of fields (and extending to 4-vectors)

$$\vec{E}(\vec{x},t) = -\nabla V(\vec{x},t) - \frac{\partial \vec{A}(\vec{x},t)}{\partial t}$$

$$\vec{B}(\vec{x},t) = \nabla \times \vec{A}(\vec{x},t)$$

Extended to 4D spacetime

$\mu = 0,1,2,3$ - Covariant indices

$A^\mu = (V, \vec{A})$ - Relativistic 4-vector potential

$J^\mu = (\rho, \vec{J})$ - Relativistic current

EM Field strength tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0, \partial_\mu F^{\mu\nu} = J^\nu$$

Covariant form of Maxwell's equations

Special relativity

Gauge Invariance

- Conservation of EM current

$$\Box = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial \rho}{\partial t} + \nabla J = 0 \rightarrow \partial_\nu J^\nu = 0$$

$$\partial_\nu \partial_\mu F^{\mu\nu} = 0$$

Can be rewritten as

$$\Box A^\nu - \partial^\nu (\partial_\mu A^\mu) = J^\nu$$

- The above equation can be satisfied by many different potentials, which will produce the same field strength tensor, provided:

$$A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \Lambda(\vec{x}, t)$$

Gauge invariance, we can add an arbitrary quantity to our potential resulting in the same physics

(Satisfies Einstein's second postulate)

- The Lorentz Gauge (chose a potential where $\partial_\mu A^\mu = 0$): $\Box A^\nu = J^\nu$

(We'll come back to Gauge invariance later....)

Quantum Mechanics

Wave Particle Duality

- Concurrently:
  - Planck quantised black-body radiation $E = h\nu = \frac{hc}{\lambda}$
  - Einstein produced the theory of the photoelectric effect (particles of light interacting with electrons)
  - Suggests light is a particle? But light as a wave is a robust theory (interference patterns etc)
- Furthermore, electrons (thought to be particles) exhibited wave-like behaviour (double slit experiment)
  - De Broglie: “Matter has a wavelength” $\lambda = \frac{\hbar}{p}$
- How to reconcile these two viewpoints?

The Schrödinger Equation

- Schrödinger (1926) proposed an approach which combined the wave-like and particle-like behaviour:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + V(\vec{r},t)\psi(\vec{r},t)$$

- Uses the classical Hamiltonian, with the mass-term from the kinetic energy of a particle, and the De Broglie terms for momentum of a ”matter-wave”

- Requires the introduction of $\psi(\vec{r},t)$?

Wavefunctions and Probabilities

- Born (1926) interpreted $\psi(\vec{r}, t)$, (the wavefunction) as related to the probability of finding an object (particle/wave) in a given state:

$$P(r + dr, t) = \int_r^{r+dr} |\psi(\vec{r}, t)|^2 dr$$

- Moves away from classical (deterministic) physics, and into quantum (probabilistic) physics

- But, the Schrodinger equation isn't relativistically invariant....

The Klein Gordon Equation

- We can get to a relativistically invariant form, starting from special relativity:

$$E^2 - |\vec{p}|^2 = m = p_\mu p^\mu$$

- Substitute in the QM (De Broglie) relations:

$$E \rightarrow i\hbar \frac{\partial}{\partial t}$$

$$\vec{p} \rightarrow -i\hbar \vec{\nabla}$$

- Which enables us to rewrite the momentum 4-vector as:

$$p_\mu \rightarrow -i\hbar \partial_\mu$$

- Subbing in to the first relation (and adding in the wavefunction), leads to the Klein Gordon equation:

$$(-\frac{\partial^2}{\partial t^2} + \nabla^2)\psi = m^2\psi$$

$$\partial^2\psi - m^2\psi = 0 \text{ with } \partial^2 = \Box$$

- With solution:

$$\psi(x^\mu) \propto e^{-ip_\mu x^\mu}$$

- This wavefunction has both positive and negative energy states?

The Dirac Equation

- Dirac (1928), can we remove the squared parts of the KG equation?

$$(\partial^2 - m^2)\psi \rightarrow (i\gamma^0 \frac{\partial}{\partial t} + i\vec{\gamma}\vec{\nabla} - m)\psi = 0$$

$$\rightarrow (i\gamma^\mu \partial_\mu - m)\psi = 0$$

- This can only be done if the gamma are matrices, as to get back to the original (KG) equation, they must satisfy

$$(-i\gamma^0 \frac{\partial}{\partial t} + i\vec{\gamma}\vec{\nabla} - m) \cdot (i\gamma^0 \frac{\partial}{\partial t} - i\vec{\gamma}\vec{\nabla} - m) = 0$$

The Dirac Matrices

- In order for the previous condition to be satisfied, the gamma matrices must satisfy:

$$(\gamma^0)^2 = 1, (\gamma^1)^2 = (\gamma^2)^2(\gamma^3)^2 = -1 \text{ - unitarity}$$

$$\gamma^i\gamma^j + \gamma^j\gamma^i = 0 \ \ (i \neq j) \text{ - anti-commutation}$$

$$\{\gamma^i, \gamma^j\} = 2g^{ij}$$

- It can be shown that the simplest solution is a set of 4X4 matrices (this 4 isn't to be confused with the 4D spacetime), it means that the wavefunction has 4-components: $\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \quad \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}$$

- I = 2x2 Identity matrix, σ^i = the Pauli matrices

Solutions to the Dirac Equation

- We've extended the the Schrodinger/KG equation from a single plane wave, wavefunction, to requiring a 4-component solution

- We can write the solutions as the plane wave solution, multiplied by a 4-component spinor (which is momentum dependent)

$$\psi \propto u(p)e^{-ip_\mu x^\mu}$$

- For ease of calculation, assume our particle is at rest (p = 0), the Dirac equation reduces to

$$(i\gamma^0 \frac{\partial}{\partial t} - m)\psi \rightarrow (i\gamma^0(-iE) - m)\psi = 0$$

$$Eu = \begin{bmatrix} mI & 0 \\ 0 & -mI \end{bmatrix} u$$

- Four eigenstates (u is a 4-component object), two with positive mass and two with negative mass?

Solutions to the Dirac Equation

- The spinor solutions are:

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, u_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

- Which, when subbed back into the (p = 0) Dirac equation, the individual components of the spinor are:

$$\psi_1 \propto e^{imt} u_1,\ \psi_2 \propto e^{imt} u_2,\ \psi_3 \propto e^{-imt} u_3,\ \psi_4 \propto e^{-imt} u_4$$

- Dirac's interpretation:

  Positive E solutions – Fermions travelling forwards in time

  Negative E solutions – Fermions travelling backwards in time

  alternatively, anti-fermions travelling forwards in time

Solutions to the Dirac Equation

- The spinor solutions for a particle in motion (using v_1 (p)= u_4(-p) and v_2 (p)= u_3(-p) as these are the anti-particle solutions that we still want to have positive momenta)

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ p_z/(E+m) \\ (p_x+ip_y)/(E+m) \end{bmatrix}, \; u_2 = \begin{bmatrix} 0 \\ 1 \\ (p_x-ip_y)/(E+m) \\ -p_z/(E+m) \end{bmatrix} \quad v_1 = \begin{bmatrix} (p_x-ip_y)/(E+m) \\ -p_z/(E+m) \\ 0 \\ 1 \end{bmatrix}, \; v_2 = \begin{bmatrix} p_z/(E+m) \\ (p_x+ip_y)/(E+m) \\ 1 \\ 0 \end{bmatrix}$$

- These solutions give us 4 states:

u_1 = electron with spin ↑

u_2 = electron with spin ↓

v_1 = anti-electron with spin ↑

v_2 = anti-electron with spin ↓

Helicity and Chirality

- While we have introduced spin, it is also useful to introduce the gamma-5 matrix, and helicity

$$\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ (\gamma_5)^2 = 1$$

- The chirality operators (for left-handed and right-handed components) are defined as:

$$P_L = \frac{1-\gamma_5}{2}, \ P_R = \frac{1+\gamma_5}{2}$$

and when they're applied to a wavefunction, it will decompose the wavefunction into right- and left-handed components

$$\psi(x) = [P_L + P_R]\psi(x) \equiv \psi_L(x) + \psi_R(x)$$

- The helicity operator $\mathcal{H} = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{\sigma}||\vec{p}|}$

Gives the projection of the particle's spin onto the direction of linear momentum (combines both QM and SR)

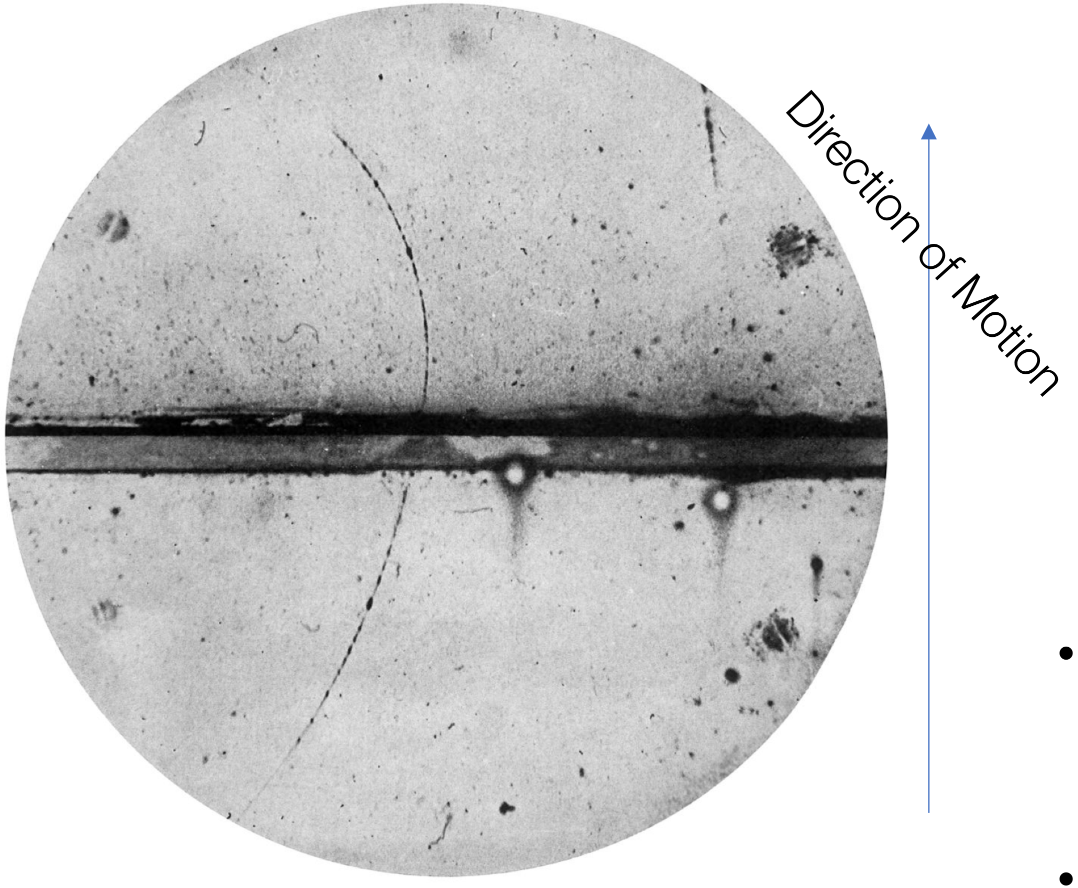
Anti-Matter

- With the previously introduced formalism, the Dirac equation provides solutions to electrons moving at high energies (magnetic properties/energy levels of Hydrogen)

- However the other solutions (anti-electron/positron) were not understood until a few years later, with Dirac himself suggesting, that the solutions should suggest a new particle, with the same mass as the electron, but opposite charge.

Anti-Matter

- Positron discovery in 1932 (Anderson)



- Cosmic rays passing through a cloud chamber and a lead plate (all immersed in a magnetic field)

- Positron curves in the opposite direction to electrons (opposite charge), but with the same mass-to-charge ratio

Lagrangian Formalism

- The KG and Dirac equations are equations of motion, and as such they can be derived from an associated Lagrangian (as in Classical Mechanics)

$$\mathcal{L}(\phi, \partial_\mu \phi) \rightarrow \; \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} = \frac{\delta \mathcal{L}}{\delta \phi}$$

- The Lagrangian which leads to the KG equation is: $\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$

- The corresponding Lagrangian which leads to the Dirac equation is:

$$\mathcal{L}_{Dirac} = \psi^\dagger \gamma^0 (i\gamma_\mu \partial^\mu - m)\psi$$

$$\rightarrow \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

Symmetries

- Symmetries are one of the most important ideas in physics.

- If we can operate on some “thing” and it appears the same after the operation, then it possesses a certain symmetry
  - Continuous symmetries
  - Discrete symmetries

Continuous Symmetries

Noether's Theorem

$$\phi(x) \to \phi'(x) = T(\alpha; \phi(x))$$

- Assume we have a Lagrangian which is invariant under some continuous transformation of the fields (**α**)

$$\phi(x) \to \phi'(x) = \phi(x) + \alpha\Delta(\phi(x)) \text{ for small } \alpha$$

- First order expansion

$$\mathcal{L} \to \mathcal{L}' = \mathcal{L} + \alpha\Delta\mathcal{L} = \mathcal{L}$$

- Lagrangian is invariant, (**α** is non-zero)

$$\partial_\mu j^\mu = 0$$

$$j_\mu = \frac{\delta\mathcal{L}}{\delta(\partial_\mu)}\Delta\phi$$

- Noether's theorem, j_μ, is a conserved current.

  For every continuous symmetry in nature, there is a corresponding conservation law

Discrete Symmetries

Charge, Parity, Time

- Some interesting discrete symmetries
  - Charge conjugation (q → -q)
  - Parity transformation (x → -x)
  - Time reversal (t → -t)

Lecture 1: Recap

- Moved from the formulation of electromagnetism through to special relativity and quantum mechanics
- Attempts to combine QM and special relativity (KG equation, Dirac equation)
- Predictions of the Dirac equation (anti-matter)
- Introduced the Lagrangian formalism
- Brief overview of symmetries

Electroweak physics: QED

Lecture 2

- Introduce Quantum Electrodynamics, Feynman diagrams and cross sections
    - Build the EM Lagrangian
    - Experimental observables: Cross sections, luminosity calculations, Branching ratios
    - How Feynman diagrams are related to the Lagrangian
    - Cross section calculations
    - Mandelstam Variables

Quantum Field Theory

- Combining quantum mechanics and relativity led to the Dirac equation, and with much additional work this eventually evolved into the full formulation of Quantum Field Theory (QFT)

- QFT describes how particles interact with each other, but it does this via the introduction of fields (ala – Maxwell), where the particles are merely the physical manifestation of the quantised fields

- QFT underpins all of modern particle physics.

- Next we will have a brief run-through of a QFT, in the context of Quantum Electrodynamics (QED)

QED

- The electromagnetic force can be described as an interaction between fields, the fermion field and the photon field (the photon which is the mediator of the EM interaction).

- The Lagrangian for QED can be built by requiring the Dirac Lagrangian be made gauge invariant

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$$\psi(x) \rightarrow \psi' x = e^{i\alpha(x)}\psi(x)$$

$$D_{\mu}\psi(x) \rightarrow D'_{\mu}\psi'(x) = e^{i\alpha(x)}D_{\mu}\psi(x)$$

- This requires that both the wavefunction, and the derivative of the wavefunction transform in the same manner

QED

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$$\psi(x) \rightarrow \psi' x = e^{i\alpha(x)}\psi(x)$$

$$D_{\mu}\psi(x) \rightarrow D'_{\mu}\psi'(x) = e^{i\alpha(x)}D_{\mu}\psi(x)$$

- And can be done by introducing the co-variant derivative $D_{\mu} = \partial_{\mu} + ieA_{\mu}$
- And the photon field, which transforms as: $A_{\mu} \rightarrow A'_{\mu} = A_{\mu} + \partial_{\mu}\alpha(x)$ (our gauge condition from earlier)
- By substituting these terms into the Dirac Lagrangian it can now be shown that it is gauge invariant

QED

- As we've added in the gauge field term (as part of the covariant derivative), we now need to add the kinetic energy term for the gauge field to produce the full EM Lagrangian

$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ with $F_{\mu\nu} = ie(\partial_\mu A_\nu - \partial_\nu A_\mu)$

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$

QED

- If we fully expand this out (covariant derivative term etc)

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - m\bar{\psi}\psi$$

$$\rightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi - m\bar{\psi}\psi$$

$$\rightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + ie\bar{\psi}\gamma^{\mu}A_{\mu}\psi - m\bar{\psi}\psi$$

$$\rightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + ie\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

Interaction between the fermion field and the photon field

QED

Massless Photons

- If our photon field would have a mass term, then it is introduced as in the KG Lagrangian ($m^2\phi^2$)

- We can replace the ϕ term with the photon field term

$$\mathcal{L}_{mass} = \frac{1}{2}m^2 A_\mu A^\mu$$

- Now we impose Gauge invariance, and try to reproduce the above mass term (just considering trying to make the mass term invariant)

$$\mathcal{L}'_{mass} = \frac{1}{2}m^2 A'_\mu A'^\mu$$

$$\rightarrow \frac{1}{2}m^2(A_\mu + \partial_\mu \Lambda(\vec{x},t))(A^\mu + \partial^\mu \Lambda(\vec{x},t))$$

$$\rightarrow \frac{1}{2}m^2(A_\mu A^\mu + A_\mu \partial^\mu \Lambda(\vec{x},t) + \partial_\mu \Lambda(\vec{x},t) A^\mu + \partial_\mu \Lambda(\vec{x},t)\partial^\mu \Lambda(\vec{x},t))$$

$$\rightarrow \frac{1}{2}m^2(A_\mu A^\mu + [A_\mu \partial^\mu + \partial_\mu A^\mu]\Lambda(\vec{x},t)) \neq \mathcal{L}_{mass}$$

An Aside: Cross-sections

Overview

- A cross-section is related to the probability that a certain process happens
- Contains the information about the dynamics of the process that we're interested in

- Say we have a beam of electrons, and a beam of positrons colliding head on, the number of events I expect depends upon the cross-section and the luminosity

$$N_{\text{events}} = \sigma(e^+e^- \to \mu^+\mu^-) \cdot \mathcal{L}dt$$

Total number of events measured → N_{events}

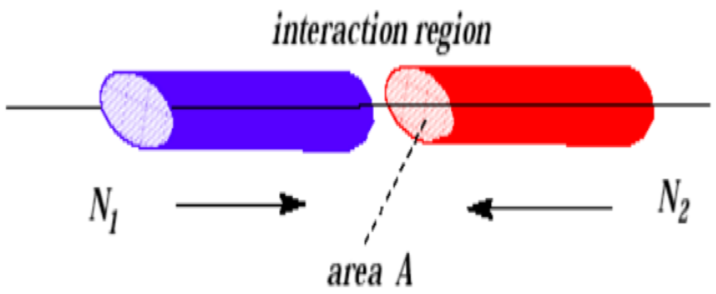
Integrated (wrt time) luminosity → $\mathcal{L}dt$

cross-section of e+e- collisions producting muon- anti-muon → $\sigma(e^+e^- \to \mu^+\mu^-)$

units are barns (b): 1 barn = 10^{-28}m^2

An Aside: Luminosity

- The luminosity contains information about the beams of particles that are colliding



For a circular collider (LHC) the luminosity is related to:

- the number of particles in each bunch (N_1, N_2)
- the revolution frequency f_{rev}
- The transverse area A covered by the beams ($A = 4\pi\sigma_x\sigma_y$)
- The number of colliding bunches n_b
- As we measure cross-section in barns, Luminosity is usually measured in inverse barns (b^{-1})

$$N_{\text{events}} = \sigma(e^+e^- \to \mu^+\mu^-) \cdot \mathcal{L}dt$$

An Aside: Decay Rate

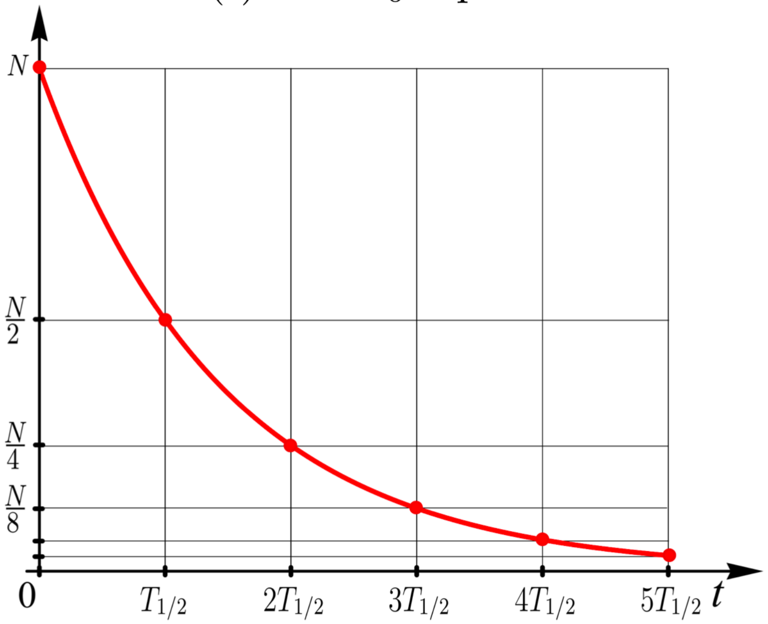
- The decay rate is the probability per unit time that a particle will decay

- After a time t, if we have N particles then:

$$N(t) - N(t + \Delta t) = -N\Gamma\Delta t$$

$$dN(t) = -N\Gamma dt$$

$$N(t) = -N_0 \exp^{-\Gamma t}$$



If there are multiple decay modes possible, then the total rate is the sum of all possible decay modes

$$\Gamma_{\text{Total}} = \Sigma_n \Gamma_n$$

The Branching ratio (BR) is the fraction an individual decay mode contributes to the total rate

$$\text{BR}_n = \frac{\Gamma_n}{\Gamma_{Total}}$$

Calculations

Theory vs Experiment

- The previously introduced concepts (cross-section, decay rate/mode) are experimentally observable, so we should be able to predict them using our theory....

- We can, but we need two ingredients:
  - Matrix element (this contains the dynamics of the interaction) → Feynman diagrams
  - Phase space → contains the masses, momenta and energy of the particles in the interaction (forbids us from trying to produce outgoing particles with more energy than we put in)

Calculations

Decay Rates

- Suppose we have particle 1 at rest, and it decays to n identical particles

  1 → 2 + 3 + 4 ... n

- The decay rate is given by:

$$\Gamma = \frac{S}{2m_1} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 - (p_2 + p_3 + ...p_n)) \times \prod_2^n 2\pi\delta(p_j^2 - m_j^2)\theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$

Calculations

Decay Rates

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outgoing particles are on shell

Matrix element

4-momentum conservation

Requires $E/c^2 > 0$

Integrates over all kinematic combinations

Calculations

Scattering

- We can also perform something similar for scattering, (1 + 2 → 3 + 4 + 5...n)

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - (p_3 + ...p_n)) \times \prod_3^n 2\pi\delta(p_j^2 - m_j^2)\theta(p_j^0)\frac{d^4 p_j}{(2\pi)^4}$$

Calculations

Simplified decay rates, and simplified scattering

- To reduce this to a simpler decay scenario, lets just say we have 1 → 2 + 3

$$\Gamma = \frac{S|\vec{p}|}{8\pi\hbar m_1^2}|\mathcal{M}|^2$$

(p is the magnitude of the outgoing particle momenta), in this case M has units of E

- We can also perform something similar for scattering, (1 + 2 → 3 + 4)

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

(where p_f is the outgoing particle momenta and p_i is the incoming particle momenta) in this case M is dimensionless

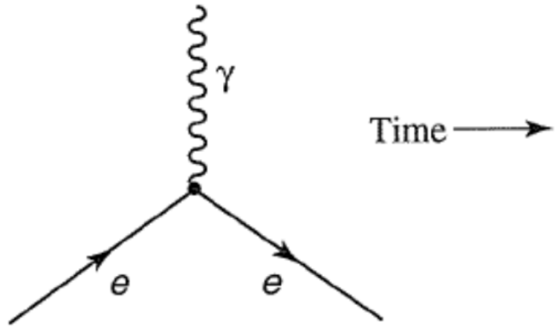
What is M?

Feynman Diagrams

- M represents a probability amplitude between an initial state and a final state:

- Contains the information about the interaction

- It is then integrated out, and summed over all possible polarizations etc.

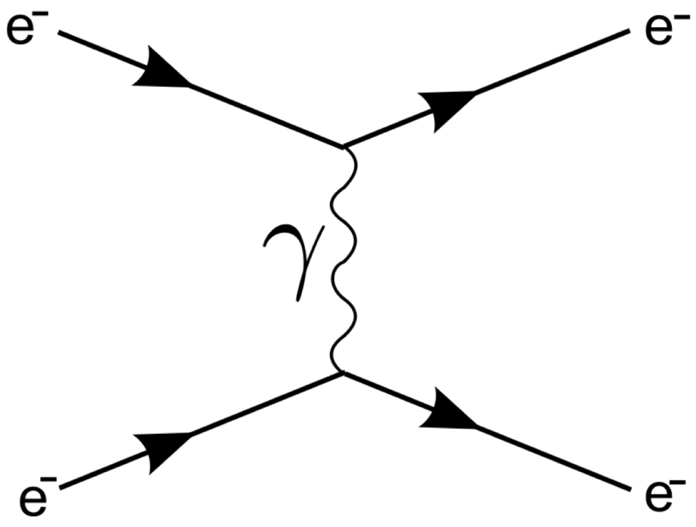


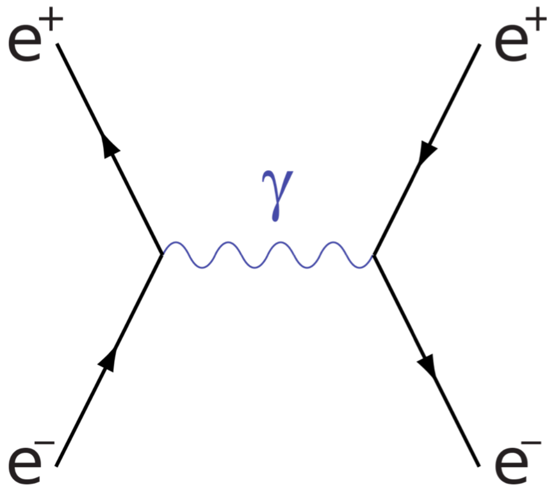
- We calculate M (known as the matrix element) using Feynman diagrams.

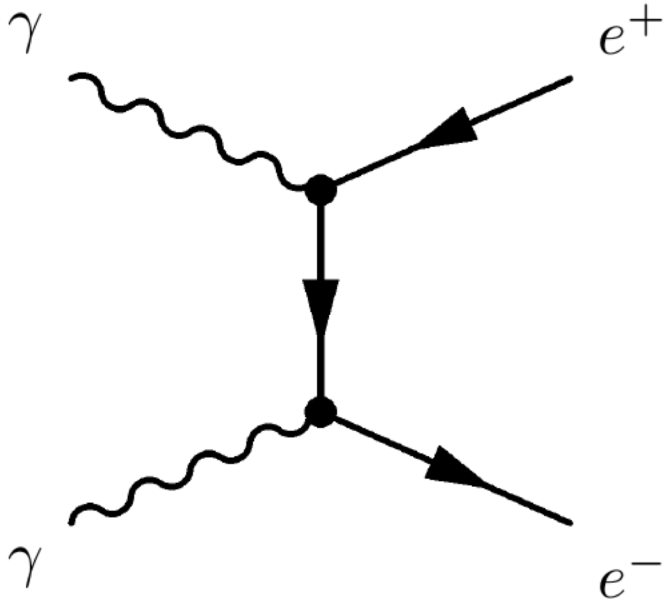
- The diagram is a tool to perform calculations and allows us to calculate the probability that a process occurs depending upon the kinematics of the initial and final state

Feynman Diagrams

Examples

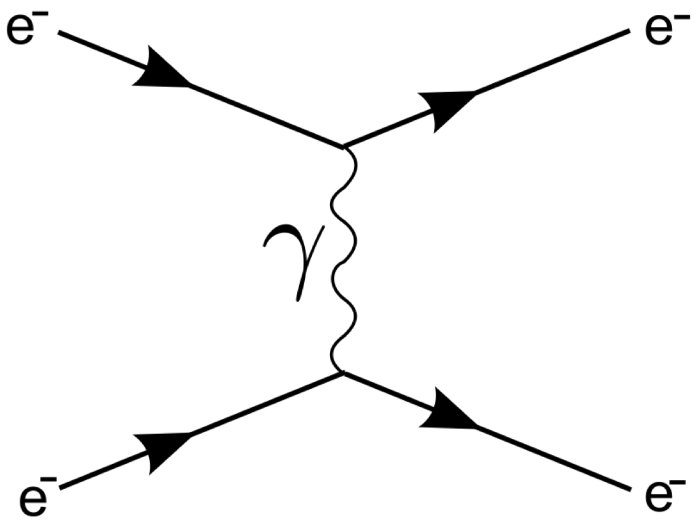


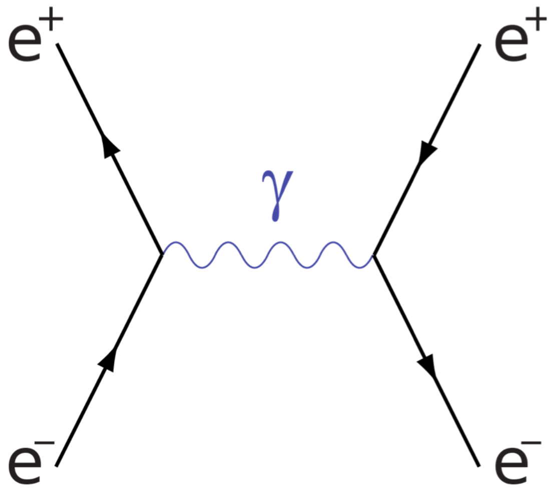


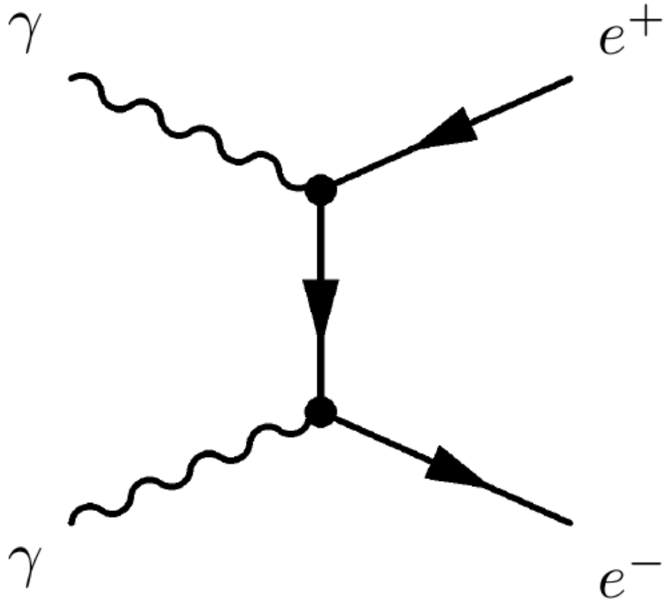


Feynman Diagrams

Examples







Time

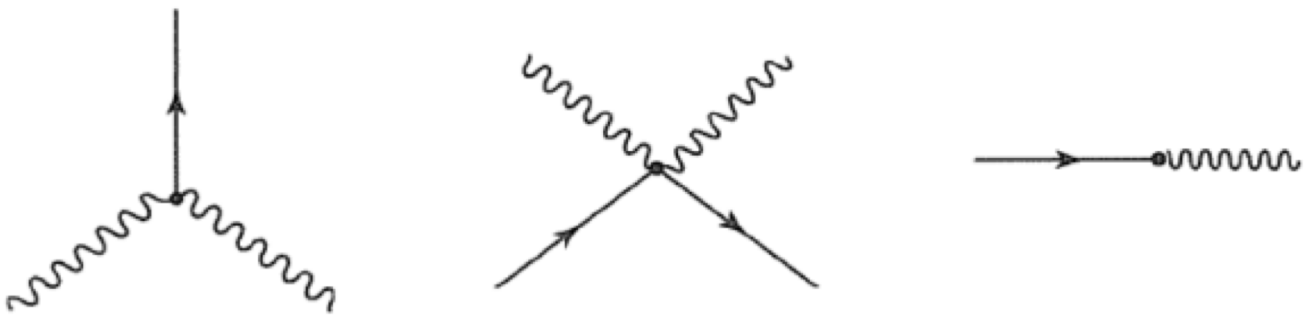
- Coulomb scattering
- e^+e^- scattering
- Pair production

- We can also exchange the electrons/positrons for muons/quarks...

Feynman Diagrams

Rules

- There are rules, and things that we cannot do with these diagrams

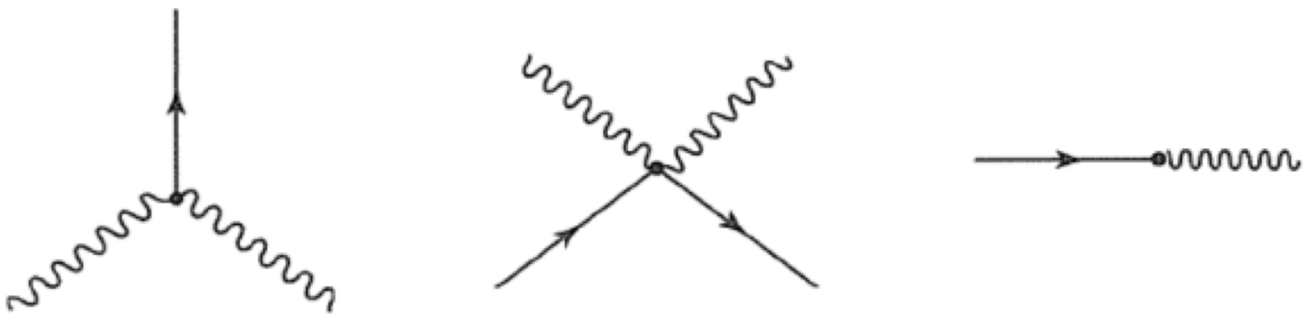


- These rules arise from the QED Lagrangian
- Feynman diagrams and rules are simple ways of visualizing what is allowed, for a given Lagrangian

Feynman Diagrams

Rules

- These interactions (vertices) are not allowed, as they do not appear in the QED Lagrangian



$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + ie\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

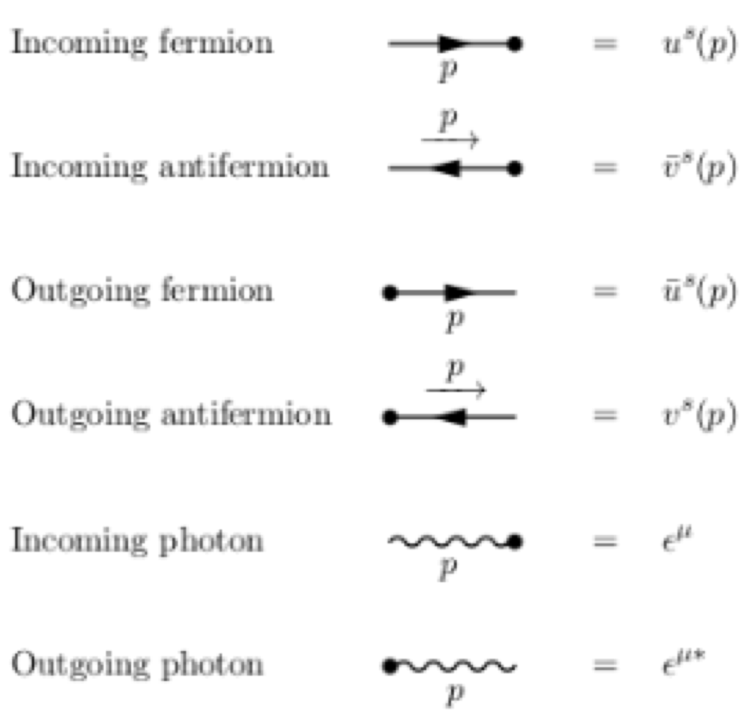
- If we look at the interaction term, we are only allowed to have an interaction which contains an anti-fermion+photon+fermion interaction
- The first two diagrams aren't allowed as the photon cannot interact with itself (no $A_{\mu}A^{\mu}$ terms)
- The final diagram isn't allowed as it doesn't conserve the Fermion current

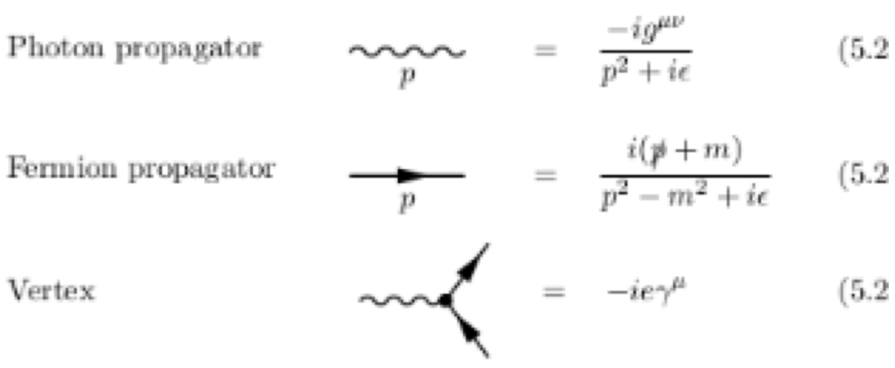
Feynman Diagrams

Rules

- There is a set of Feynman rules for translating a Feynman diagram to a Matrix element
  (You can almost think of the Feynman diagram as hiding all of the Maths of the Lagrangian)

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + ie\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$



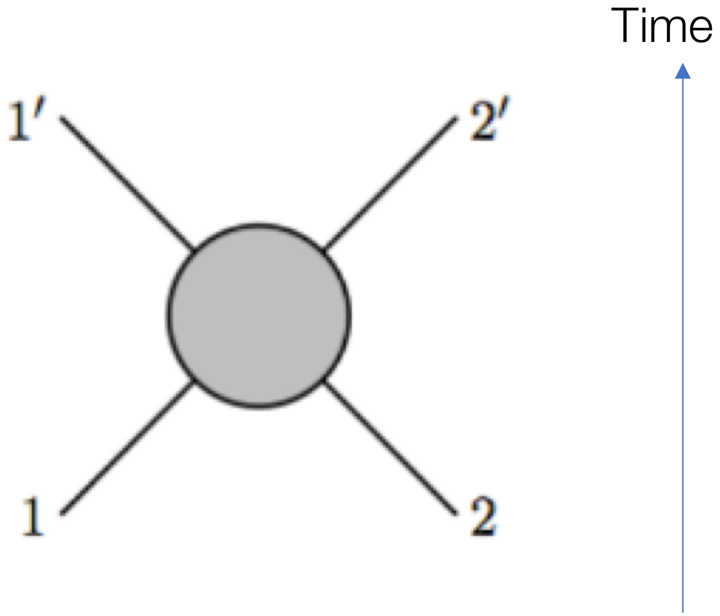


External (incoming/outgoing) represent a real particle

Internal (propagators) are virtual particles

Mandelstam Variables

- The Mandelstam variables contain the Energy and momentum of particles in a Lorentz invariant manner (p_1 and p_2 are initial state momenta, p'_1 p'_2 are final state momenta)



$$s = (p_1 + p_2)^2 = (p'_1 + p'_2)^2$$

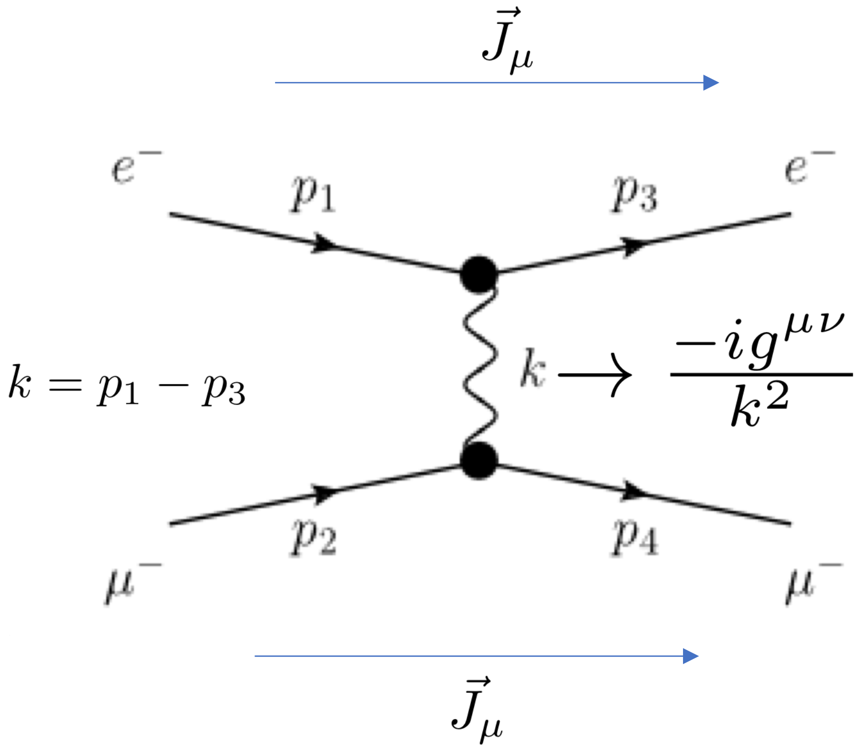
$$t = (p_1 - p'_1)^2 = (p'_2 - p_2)^2$$

$$u = (p_1 - p'_2)^2 = (p'_1 - p_2)^2$$

Matrix element calculation

- Using the Feynman rules, we can work out the matrix element for a process

- Consider there being three separate steps, the electron line, the muon line, and the photon propagator



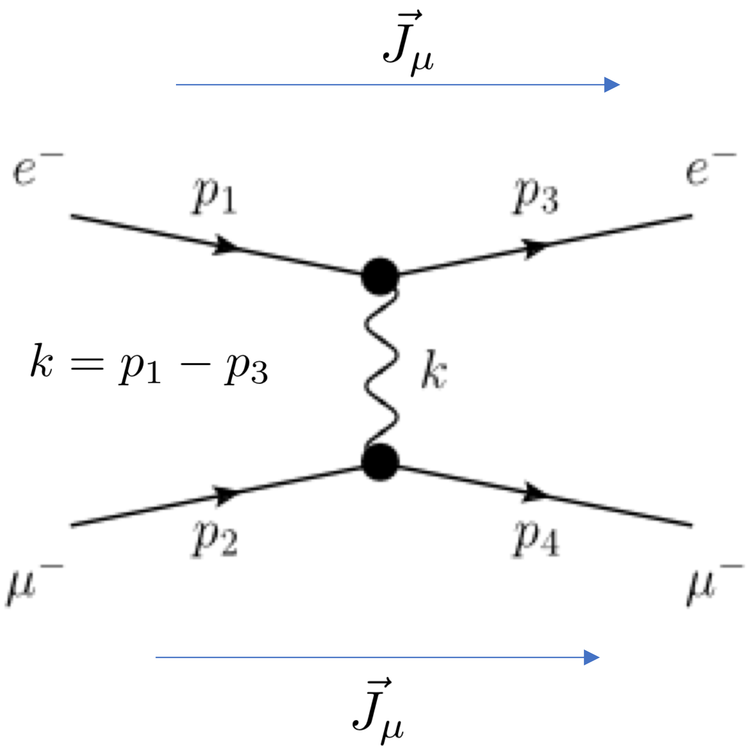
$$\rightarrow -ie[\bar{u}_{ele}(p_3)\gamma^\mu u_{ele}(p_1)]$$

$$k \rightarrow \frac{-ig^{\mu\nu}}{k^2}$$

$$\rightarrow -ie[\bar{u}_{mu}(p_4)\gamma^\nu u_{mu}(p_2)]$$

Matrix element calculation

- Using the Feynman rules, we can work out the matrix element for a process



$$\mathcal{M} \propto (-ie)^2 [\bar{u}_{ele}(p_3)\gamma^\mu u_{ele}(p_1)] \frac{-ig^{\mu\nu}}{k^2} [\bar{u}_{mu}(p_4)\gamma^\nu u_{mu}(p_2)]$$

$$\rightarrow \frac{e^2}{k^2}[\bar{u}_{ele}(p_3)\gamma^\mu u_{ele}(p_1)][\bar{u}_{mu}(p_4)\gamma_\mu u_{mu}(p_2)]$$

$$\mathcal{M} = -\frac{g_e}{k^2}[\bar{u}_{ele}(p_3)\gamma^\mu u_{ele}(p_1)][\bar{u}_{mu}(p_4)\gamma_\mu u_{mu}(p_2)]$$

$\bar{u}_{ele/mu} \rightarrow$ destroys an electron/muon or creates a positron/anti-muon

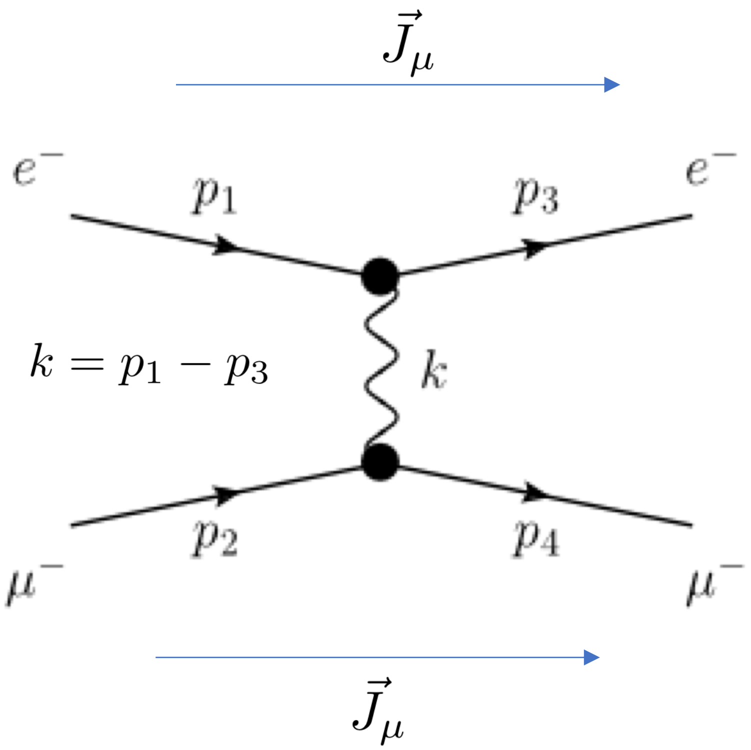
$u_{ele/mu} \rightarrow$ creates an electron/muon or destroys a positron/anti-muon

$g_e^2 \rightarrow$ electromagnetic gauge coupling, related to the fine structure constant

$\alpha = g_e^2/4\pi$

Matrix element calculation

- Using the Feynman rules, we can work out the matrix element for a process



$$\mathcal{M} = -\frac{g_e}{k^2}[\bar{u}_{ele}(p_3)\gamma^\mu u_{ele}(p_1)][\bar{u}_{mu}(p_4)\gamma_\mu u_{mu}(p_2)]$$

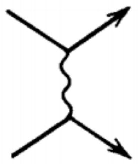
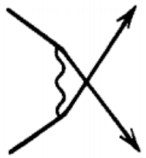
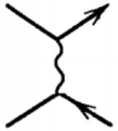
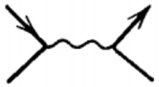
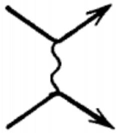
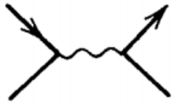
$$\sigma \propto |\mathcal{M}|^2$$

$$|\mathcal{M}|^2 = 2e^4\frac{s^2+u^2}{t^2}$$

Leading order QED

Shortcuts

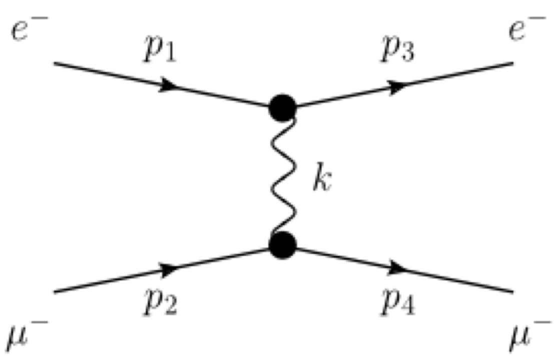
TABLE 6.1
Leading Order Contributions to Representative QED Processes

	Feynman Diagrams		$\overline{\|\mathfrak{M}\|^2}/2e^4$		
	Forward peak	Backward peak	Forward	Interference	Backward
Møller scattering $e^-e^- \to e^-e^-$			$\frac{s^2+u^2}{t^2}$ +	$\frac{2s^2}{tu}$ +	$\frac{s^2+t^2}{u^2}$
				$(u \leftrightarrow t$ symmetric)	
(Crossing $s \leftrightarrow u$) ↓	Forward	"Time-like"	Forward	Interference	Time-like
Bhabha scattering $e^-e^+ \to e^-e^+$			$\frac{s^2+u^2}{t^2}$ +	$\frac{2u^2}{ts}$ +	$\frac{u^2+t^2}{s^2}$
$e^-\mu^- \to e^-\mu^-$			$\frac{s^2+u^2}{t^2}$		
(Crossing ↓ $s \leftrightarrow t$) $e^-e^+ \to \mu^-\mu^+$					$\frac{u^2+t^2}{s^2}$

Halzen & Martin p129

NLO QED

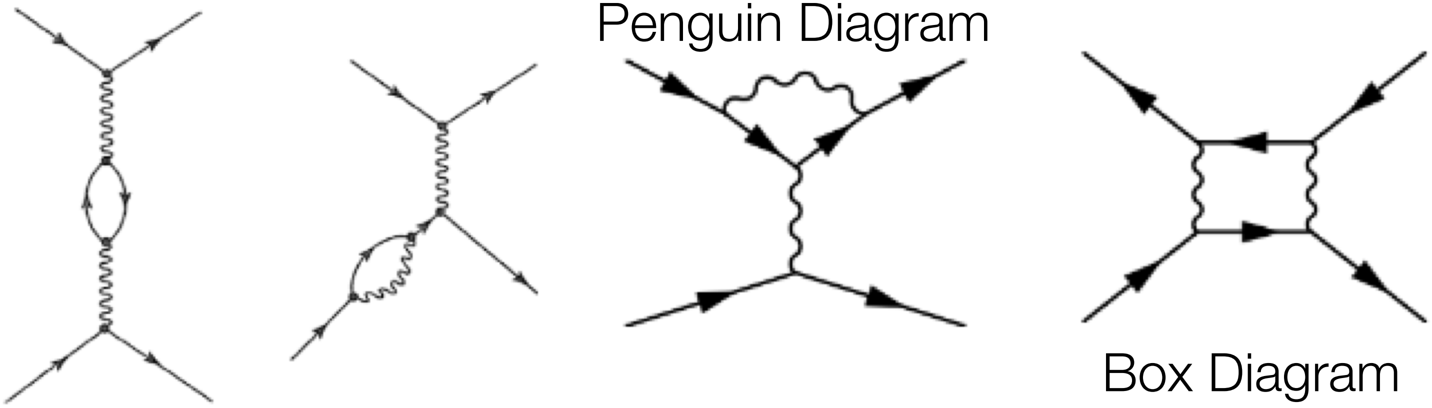
- The previous diagrams were all at Leading Order (or Tree-level)
- We can introduce (closed) loops into the Feynman diagrams, which add higher order corrections to the original process we are considering



Leading Order (LO)

(2 Vertices)

Penguin Diagram



Box Diagram

Next-to Leading Order (NLO)

(4 Vertices)

- The addition of each pair of vertices adds a factor of $\alpha = g_e^2/4\pi$

Renormalisation

- In order to calculate the cross section then, we need to take into account all of these higher orders?
- No...
- The higher order diagrams with very high (virtual) 4-momentum transfer give divergent integrals when calculating the cross-section
- The solution is to introduce a “cut-off” (called Renormalisation), which redefines the couplings, masses etc
- However, changing the coupling in this manner introduces a dependence upon the energy scale of the interaction

Running coupling

- The dependence of the coupling is:

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \ln(\frac{q^2}{\mu^2})}$$

The μ term is just a momentum scale to remove the dependence upon the cut-off scale

- At low energies, **α** = 1/137 (fine structure constant for energy-levels of hydrogen)
- At the Z-boson mass (q2 = mZ), **α** = 1/128

- Can be thought of as a “screening” of the bare charge by the virtual photons (higher order diagrams)

Anomalous Magnetic Moment

Electron

- Higher order corrections are important though

- Correction to the gyromagnetic moment of the electron (the ratio between the magnetic moment and the spin)

- Dirac equation predicts g = 2

- Higher order corrections from QED predict slightly different

- As we expect these values to be close, we define a → the anomalous magnetic moment

$$a = \frac{g-2}{2}$$

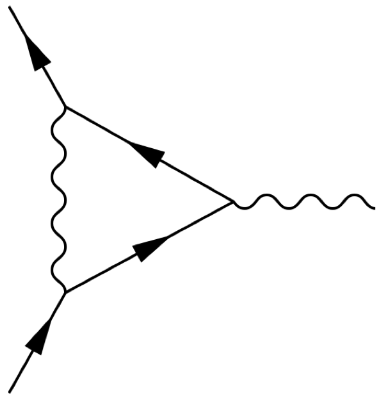
(if Dirac is correct, this is 0)

Anomalous Magnetic Moment

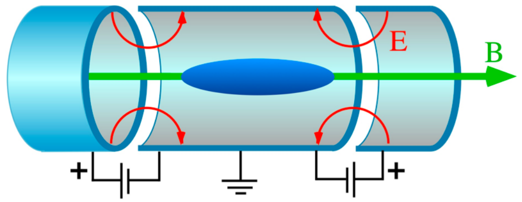
Electron

- Higher order QED corrections (such as the Schwinger correction) predict:

  a = 0.0011596521869 (41)



- Experimental measurement:



  Catch a single electron in a Penning trap, measure the difference between the cyclotron frequency and the spin precession frequency

  a = 0.00115965213(3)

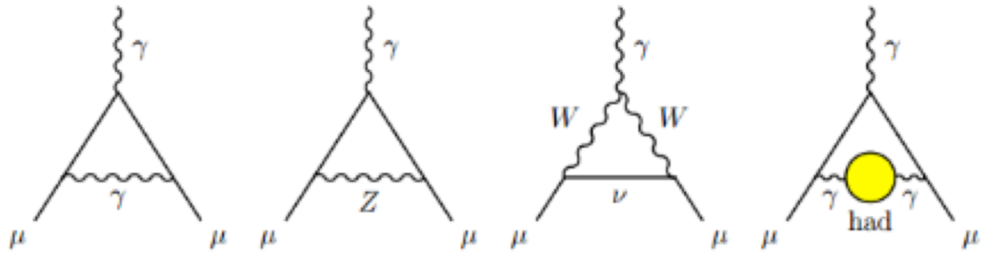
- Most precise test of QED, proof that higher order corrections are required

Anomalous Magnetic Moment

Muon

- Let's do the same for the Muon?

- The theoretical calculation is much more complex (the fact the muon is around ~200 times more massive means the contribution from hadronic particles in loops is more relevant)



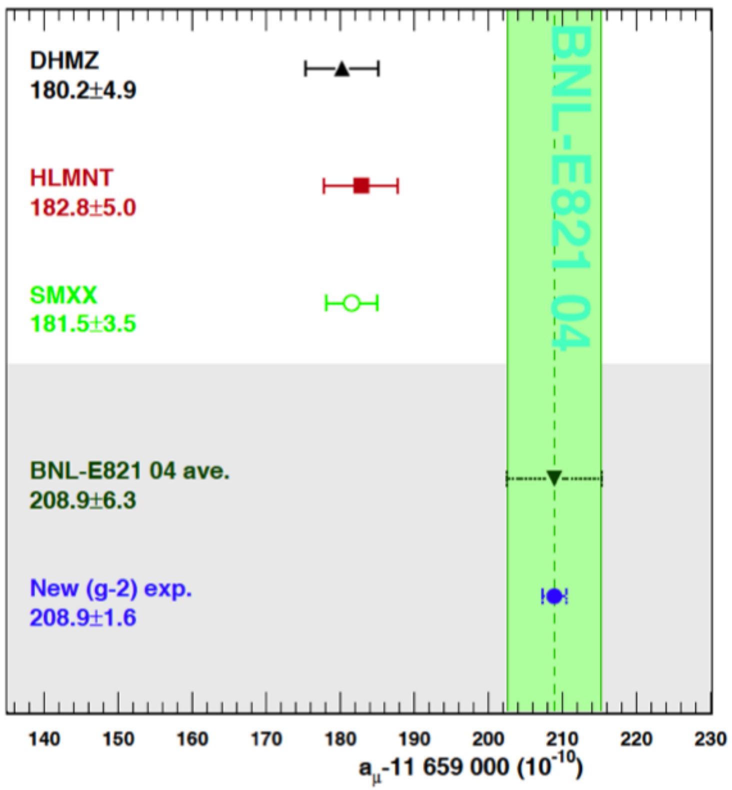
Current prediction a = 0.0011659203(20)

Anomalous Magnetic Moment

Muon

- Experimental measurement

  Produce a beam of muons and filter into a storage ring (under magnetic field)
  Again measure the difference between the precession and cyclotron frequency
  (this is more difficult with muons, as they decay, but the spin precession frequency is measured by the positrons in the decay)



- Current world's best limit from Brookhaven a = 0.0011659106(6)

  (more than 3σ discrepancy between theory and experiment)

- The Fermilab g-2 experiment is preparing to take data next year, with lower predicted experimental uncertainties, to further investigate the discrepancy

Lecture 2 - Recap

- Reviewed
  - QED
  - Experimental measurements (decay rate, branching ratio, cross sections)
  - Calculating the above properties using theory
  - Feynman Diagrams
  - Matrix elements
  - Mandelstam Variables
  - Tree-level diagrams
  - Next to leading order and Renormalisation

Electroweak physics: The weak force & fermions

Lecture 3

- Introduce the weak nuclear force, some flavour physics, neutrinos
    - The nuclear decay problem, Fermi theory
    - Parity violation
    - The weak force
    - Lepton universality
    - Quarks and the CKM matrix
    - CP Violation

Ordinary Matter

- All of our “ordinary” matter is made up of of the first generation (1st column) of particles

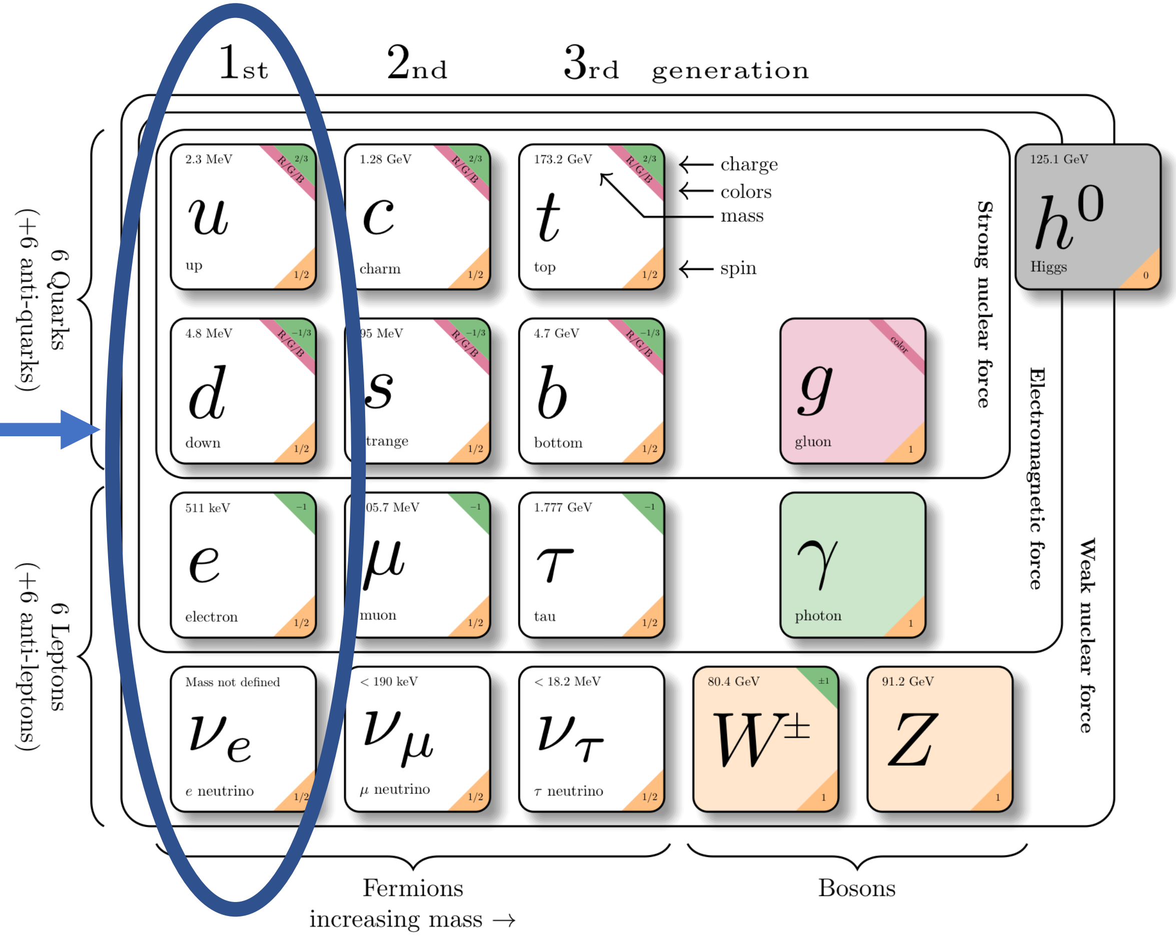
Protons

Neutrons

Pi+

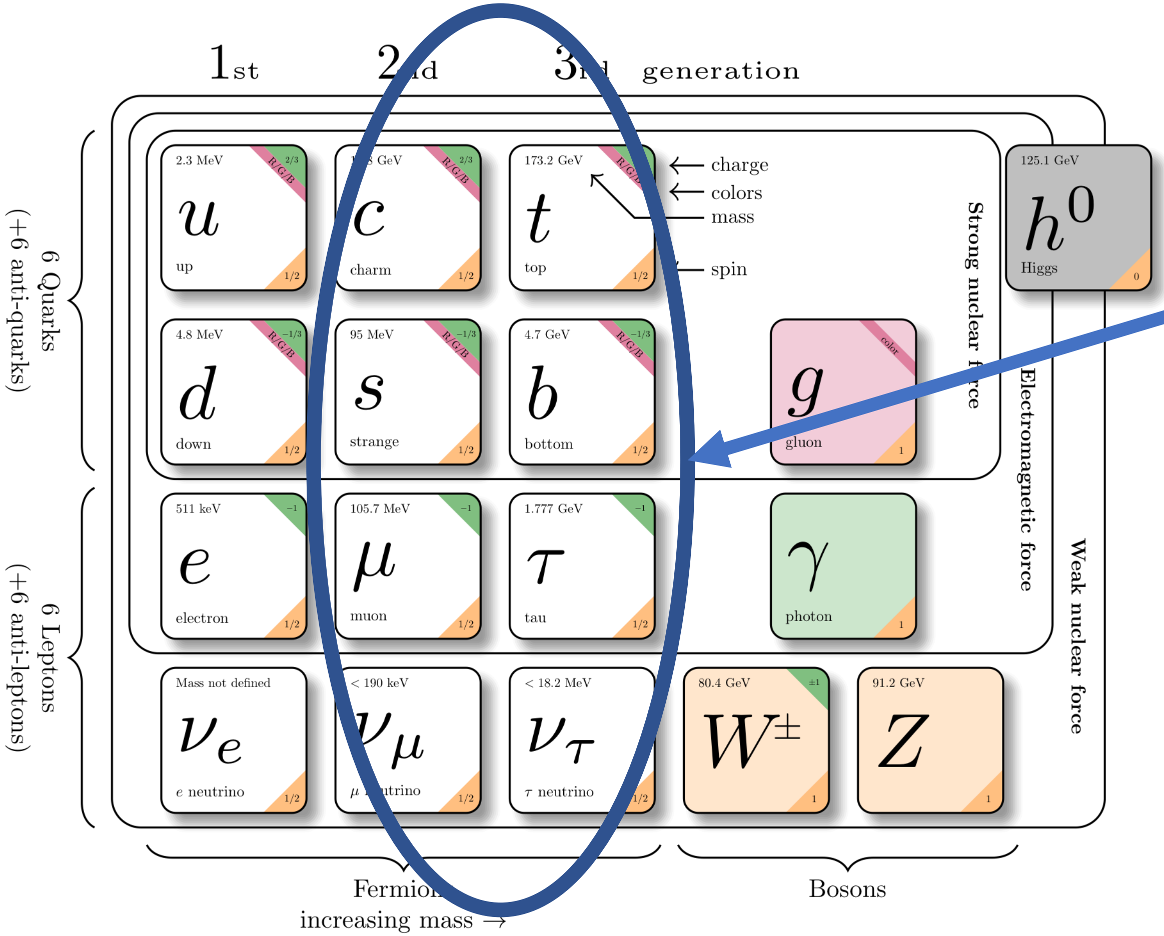
Pi0

Atoms



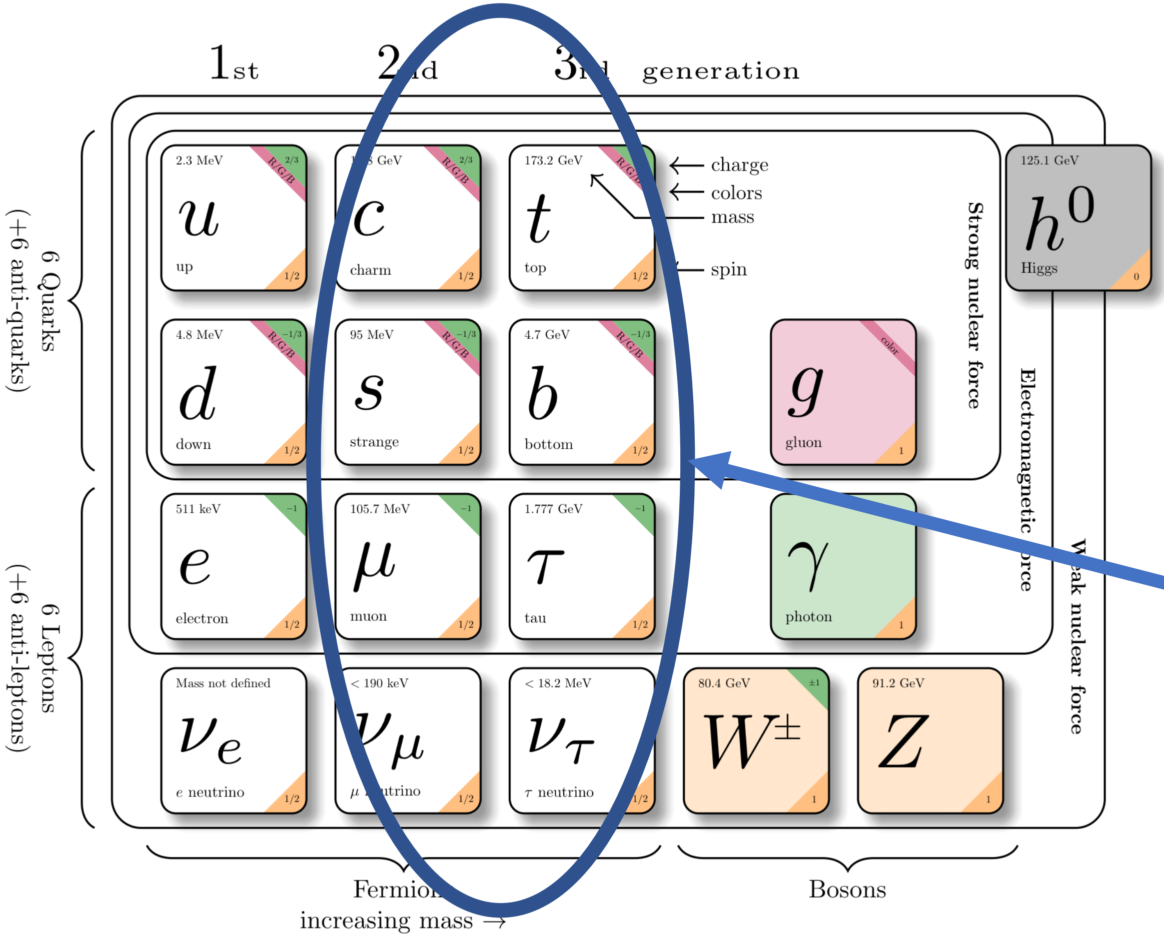
Exotic Matter

- What about the 2nd and 3rd columns?



Any ideas?

Exotic Matter



Kaons, Sigma Baryons (strange quarks)

D-mesons (charm quarks)

B-mesons (bottom quarks)

Muonium

(there are no bound states containing the top-quark)

- Why are these particles not “commonly” found in nature?

Decays and conservation laws

- Generally a more massive particle will decay into lighter particles, unless there is some kind of conservation law forbidding the decay from occurring

- All of the matter that we refer to as “ordinary” matter is **stable** (for a variety of reasons):
  - The electron is stable → it is the lightest charged particle
  - The proton is stable → it is the lightest baryon, and baryon number is conserved
  - Neutrons are “somewhat” stable (when they’re in a low atomic number nucleus)

- More exotic particles can be produced, but they decay, eventually into the above particles

Conservation laws

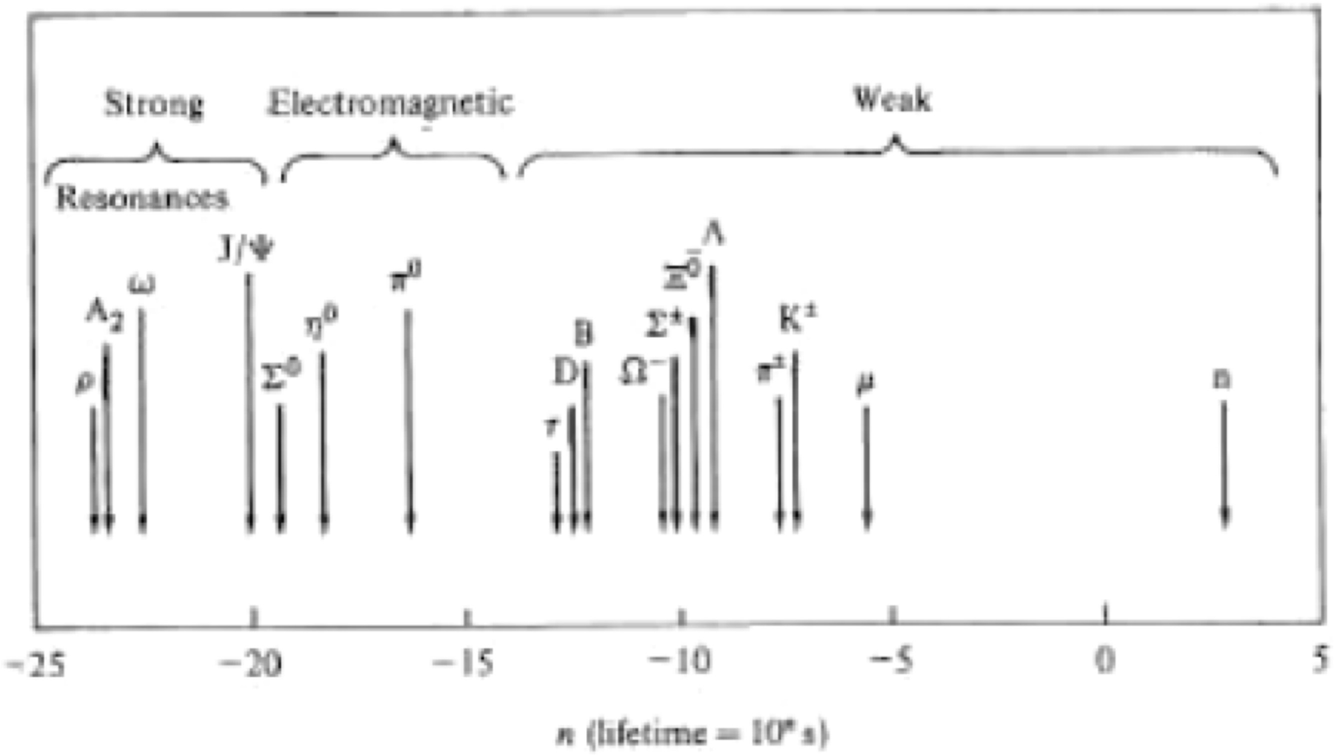
Property	Strong force	EM Force	Weak Force	Comment
Charge	Conserved	Conserved	Conserved	
Colour	Conserved	Conserved	Conserved	
Baryon Number	Conserved	Conserved	Conserved	Related to conservation of quark current.
Lepton Number	Conserved	Conserved	Conserved	
Flavour	Conserved	Conserved	Not Conserved	(See Below)

In weak interactions, if no other decay is available/more favoured (strong or EM), then the flavour of a particle can change:

A muon can decay to an electron
A strange quark can decay to a down quark
.... and so on

Lifetimes and interactions

- The lifetime of a particle's decay is related to the strength of the interaction which enables the decay

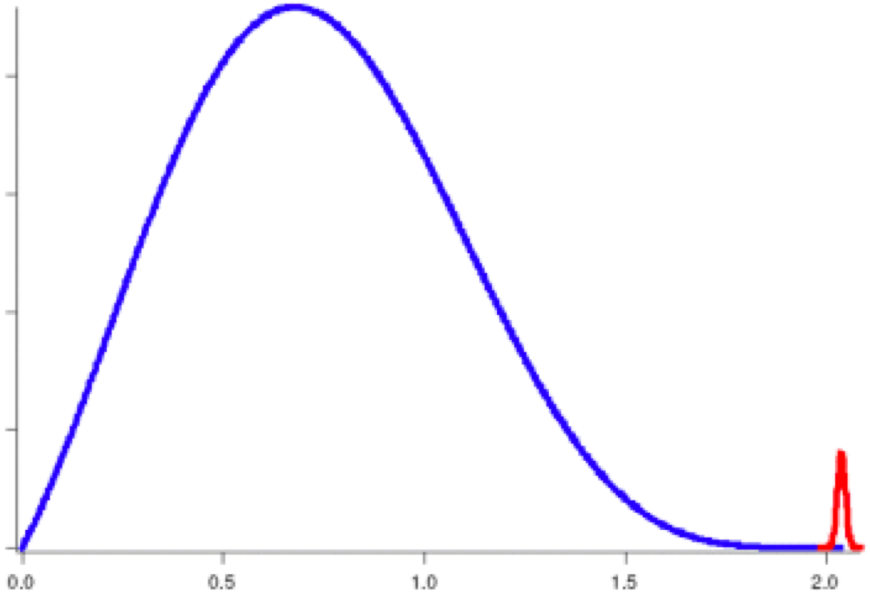


Particle	Lifetime (s)
Muon	2.2 $\times 10^{-6}$
$\pi^{\pm}$	2.8 $\times 10^{-8}$
π^{0}	8.3 $\times 10^{-17}$
n	881.5

Interaction	Typical Lifetime (s)
Strong	10^{-23}
Electromagnetic	10^{-16}
Weak	10^{-13} - 900

Neutron Decay

- In 1930, the neutron radioactive decay was the first process which the ideas behind the weak force arose
- The observed decay products suggested the decay proceeded as: n → e + p



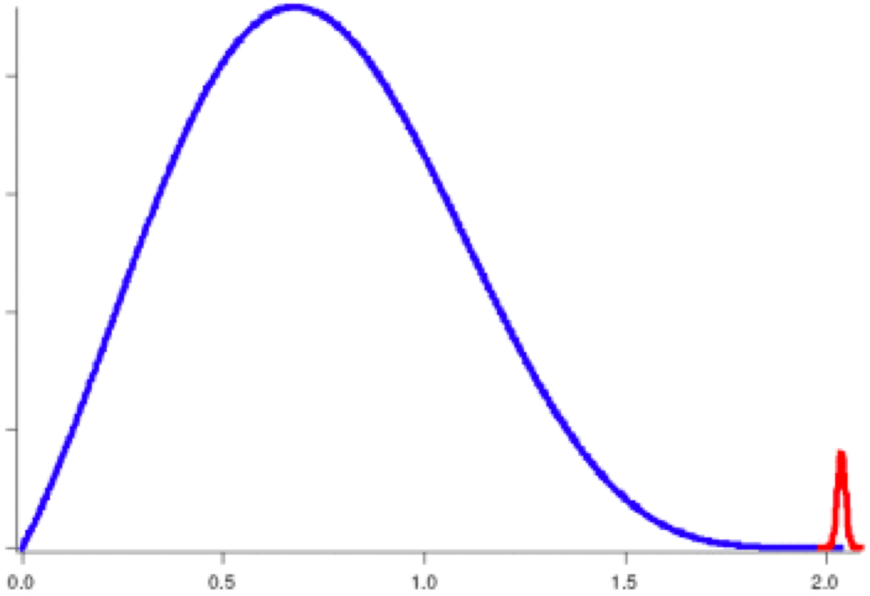
If this was the case, the energy spectrum of the electron from the decay should be

$E_e = E_n - E_p$ (a discrete energy spectrum)

However the observed energy spectrum did not confirm this

Neutron Decay

- In 1930, the neutron radioactive decay was the first process which the ideas behind the weak force arose
- The observed decay products suggested the decay proceeded as: n → e + p



Either, we suggest that energy conservation is broken, or

This continuous spectrum suggests a 3 body decay

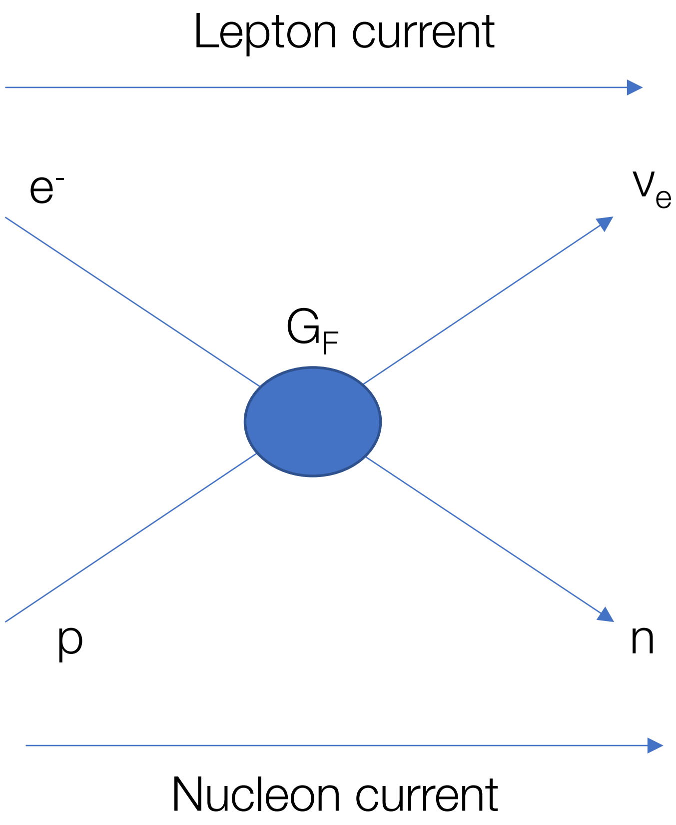
Pauli suggested that a 3rd particle was also produced in the decay:

n → e + p + $\bar{\nu}_e$

Pauli's hypothesis: the undetected neutrino has a very small (possibly 0) mass and no electric charge

But, the massless, chargeless photon is still detectable, so why isn't the neutrino?

Fermi Theory



- Fermi proposed a transition, one which the neutron transforms into a proton, with the emission of an electron and a neutrino

- The transition must be due to a new type of interaction, much weaker than the EM interaction (due to the lifetime of the decay) and with a very short range

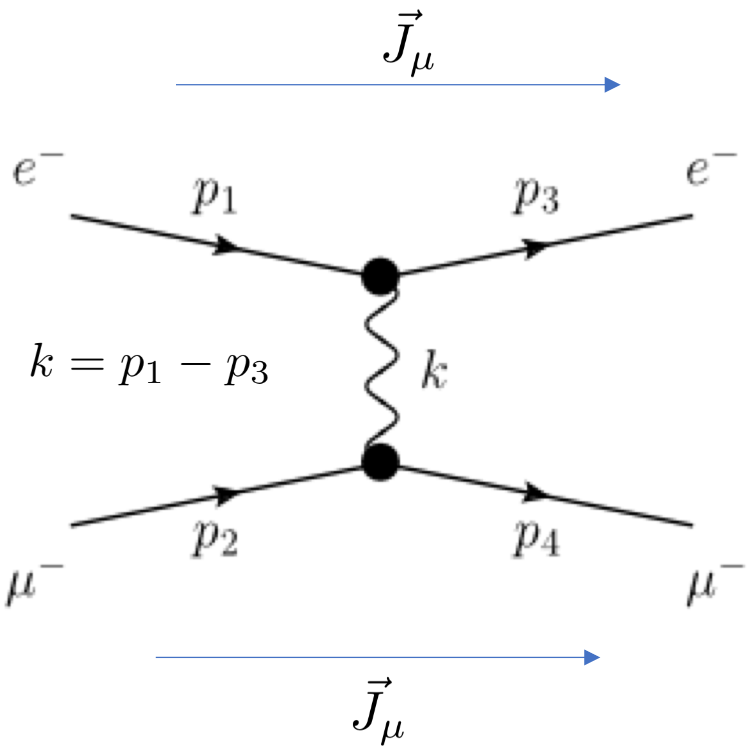
- The interaction term (G_F) isn't really known, but we can proceed by treating it as an Effective Field Theory (EFT)

  We don't really know what is happening at the vertex (unlike the EM force where we know a photon is exchanged)
  → Contact interaction

  We can proceed in a similar manner as for EM

Recall: Matrix element calculation

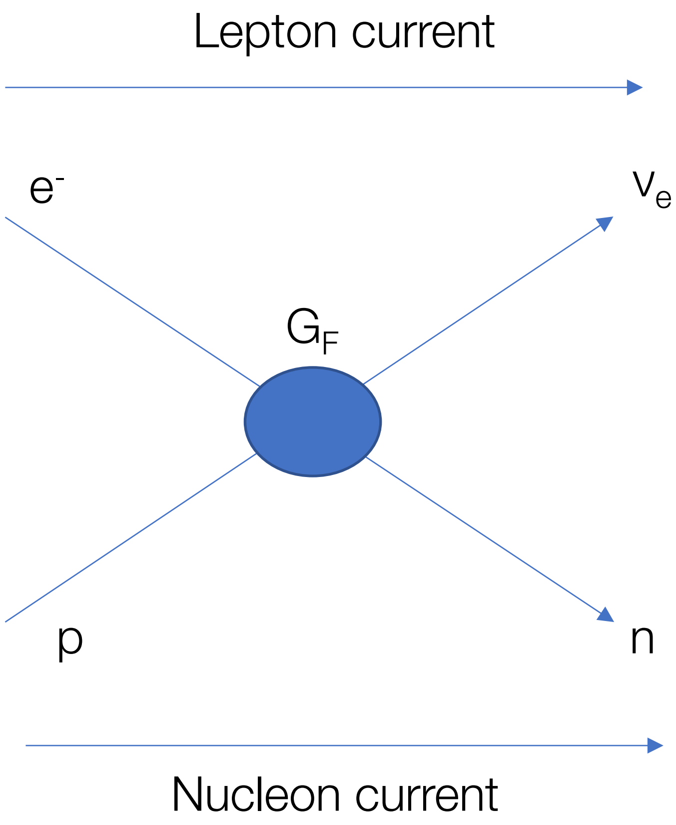
- Using the Feynman rules, we can work out the matrix element for a process



$$\mathcal{M} = -\frac{g_e}{k^2}[\bar{u}_{ele}(p_3)\gamma^\mu u_{ele}(p_1)][\bar{u}_{mu}(p_4)\gamma_\mu u_{mu}(p_2)]$$

Let's apply the same kind of rules to the Fermi Contact interaction

Fermi Theory



$$\mathcal{M} = -\frac{g_e}{k^2}[\bar{u}_{ele}(p_3)\gamma^{\mu}u_{ele}(p_1)][\bar{u}_{mu}(p_4)\gamma_{\mu}u_{mu}(p_2)]$$

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}}[\bar{u}(p)\gamma^{\mu}u(n)][\bar{u}(e)\gamma_{\mu}u(\nu)]$$

- Still a vector current
- G_F is characterising the strength of the interaction
  $G_F = 1.17 \times 10^{-5}$ GeV^{-2}
- This current is changing the electric charge of the ingoing/outgoing particles → Weak charged current

- The cross section diverges as E_{ν}^2 increases (only an EFT)

Need to introduce a cut-off scale to avoid this

Parity Violation

- Fermi's proposal is ok, however the discovery of parity violation in weak decays requires it to be modified

- Parity conservation implies switching x → -x, should give the same physical outcome

- Wu's experiment (Beta decay of Co60)

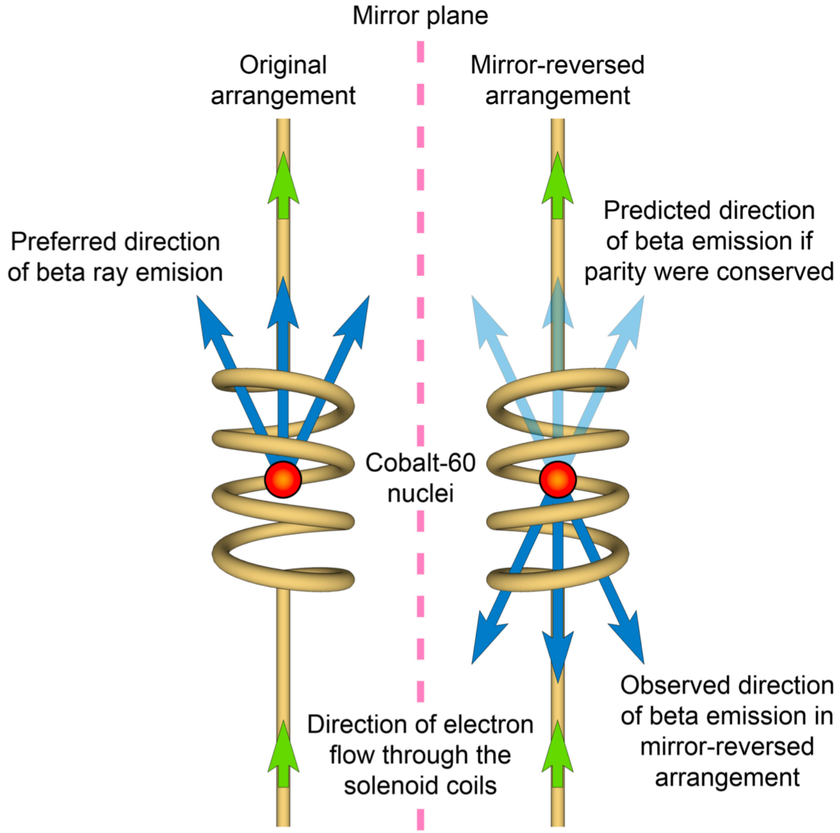
  ^{60}Co in magnetic field, with spin aligned with field will emit electrons (beta decay) in a given direction

  Then with anti-aligned spin, the emission direction depends on if parity is conserved or not

  The emission direction is different in each case

- Parity is maximally violated in weak decays

  → needs to be introduced in Fermi's theory

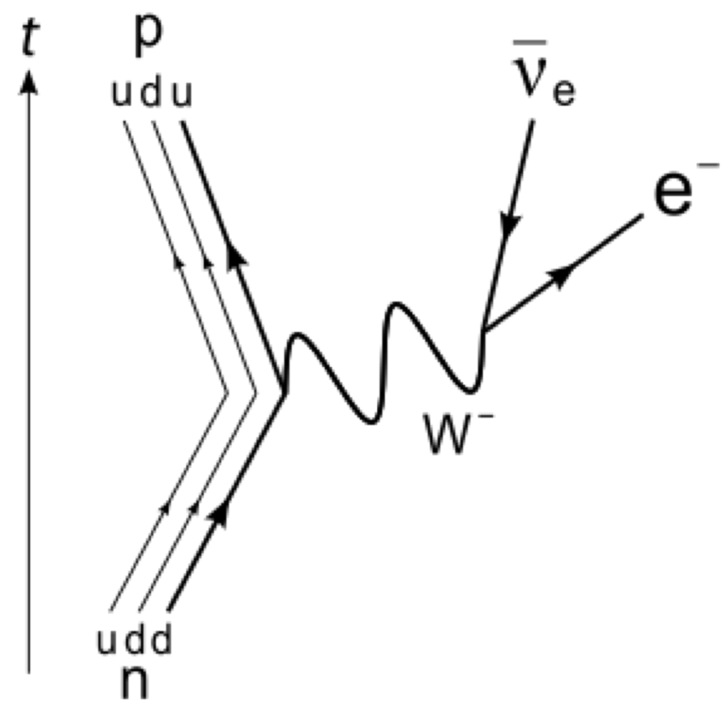


Range of the Weak force

- The weak force has finite range (10^{-16}m)

- When we look at the EM force (infinite range) the propagator in this case is massless, so this suggests that whatever is propagating the weak force, has mass (which is what we referred to earlier as the cut-off scale for the Fermi theory)

- Using the parity violation measurement and the requirement of a massive propagator we can have another attempt at the theory behind weak interactions



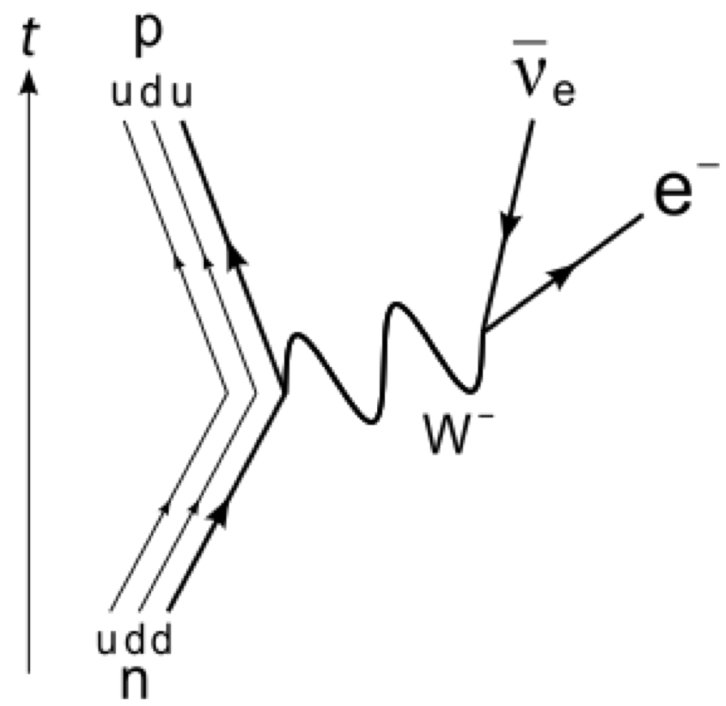
Introduced the weak coupling/charge (g_W)

$$\mathcal{M} = [\frac{g_w}{\sqrt{2}}\bar{u}(u)\gamma^{\mu}\frac{1}{2}(1-\gamma^5)u(d)]\frac{1}{M_W^2-q^2}[\frac{g_w}{\sqrt{2}}\bar{u}(\nu_e)\gamma_{\mu}\frac{1}{2}(1-\gamma^5)u(e)]$$

Massive mediator

Right handed parity operator
(contains Vector and axial component)

- If this is the correct matrix element describing the full QFT, then the EFT we introduced before, is related to it



$$\mathcal{M} = [\frac{g_w}{\sqrt{2}}\bar{u}(u)\gamma^\mu \frac{1}{2}(1-\gamma^5)u(d)]\frac{1}{M_W^2-q^2}[\frac{g_w}{\sqrt{2}}\bar{u}(\nu_e)\gamma_\mu \frac{1}{2}(1-\gamma^5)u(e)]$$

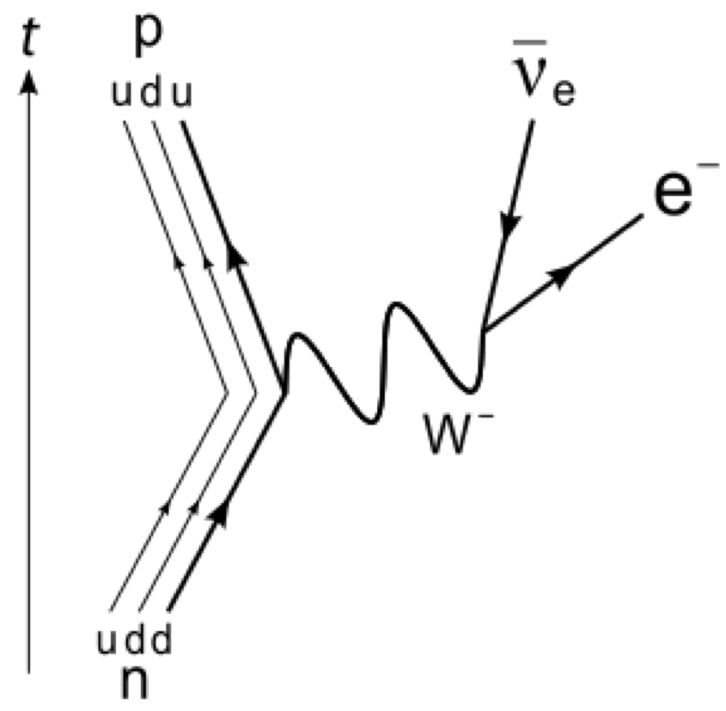
$$\frac{G_F}{\sqrt{2}} = \frac{g}{\sqrt{2}} \times \frac{g}{\sqrt{2}} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{M_W^2-q^2}$$

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}}[\bar{u}(p)\gamma^\mu u(n)][\bar{u}(e)\gamma_\mu u(\nu)]$$

Intensity of the weak force

Weak interactions

- If this is the correct matrix element describing the full QFT, then the EFT we introduced before, is related to it



$$\frac{G_F}{\sqrt{2}} = \frac{g}{\sqrt{2}} \times \frac{g}{\sqrt{2}} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{M_W^2 - q^2}$$

let $M_W >> q$ ($q \rightarrow 0$)

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \quad \rightarrow g = 0.65$$

$G_F = 10^{-5}[GeV^{-2}]$, $M_W = 80GeV$

$$\alpha_{EM} = \frac{e^2}{4\pi} = \frac{1}{137} \qquad \alpha_W = \frac{g^2}{4\pi} = \frac{1}{29.5}$$

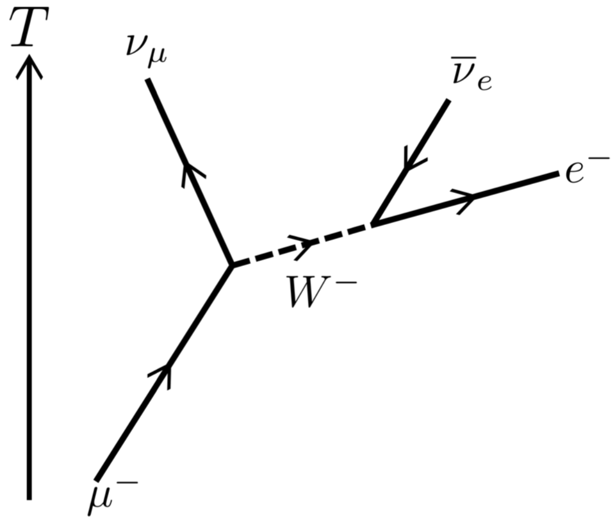
The couplings between the weak and EM force are comparable.
The weak force is "weak" because of the mass of the mediator, **not because of the coupling**

Measurement of G_F

Muon Decay

- We now have our method to construct the matrix element for the weak current.

- We can perform a more precise (both experimentally and theoretically) measurement of G_F using muon decay



From the previous lecture

$$\Gamma = \frac{S}{2m_1}\int |\mathcal{M}|^2(2\pi)^4\delta^4(p_1-(p_2+p_3+...p_n)) \times \prod_2^n 2\pi\delta(p_j^2-m_j^2)\theta(p_j^0)\frac{d^4p_j}{(2\pi)^4}$$

$$\Gamma = \frac{1}{2(2\pi^5)}\frac{|\mathcal{M}|^2}{2|m_\mu|}\delta^4(q_e+k'_{\overline{\nu}_e}+k_{\nu_e}+p_\mu)\frac{d^3k}{2E_k}\frac{d^3q}{2E_q}\frac{d^3k'}{2E'_k}$$

$$\Gamma_\mu \propto G_F^2 m_\mu^x$$

Lifetime has units [E^{-1}], G_F^2 as there are two vertices
Dimensional analysis infers: m_μ^5

Measurement of G_F

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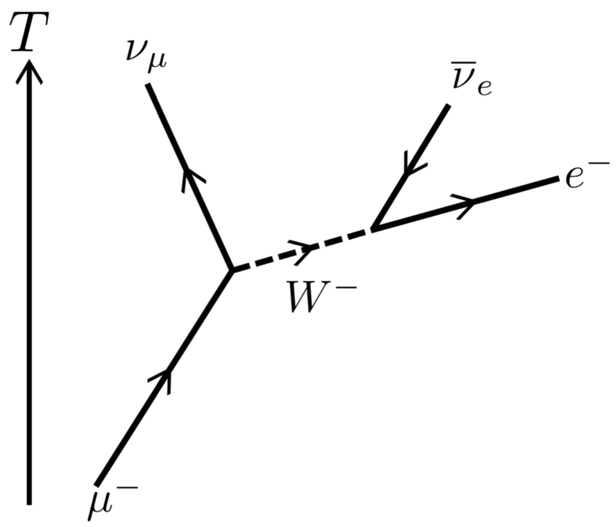


Full calculation gives (ε are higher order corrections):

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^2}(1+\epsilon)$$

muon mass and lifetime measurement leads to:

$G_F = (1.16667 \pm 0.00001) \times 10^{-5}$

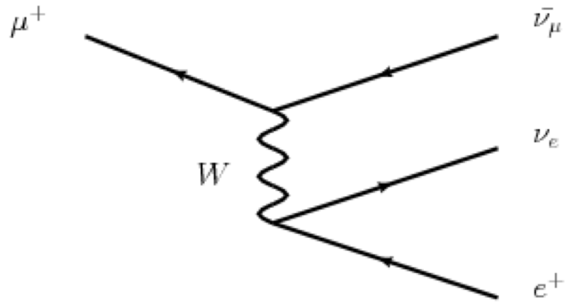
Lepton Universality

- Is the coupling constant (g) universal?
  First check: Is the value for g at an tau/tau neutrino vertex, the same as at a muon/muon neutrino vertex?

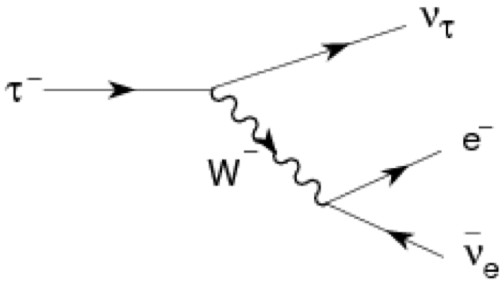
If we take the ratio of the decay rates of these processes we can discern any differences in g

The muon can only decay via anti-neutrino, electron and neutrino

The tau can decay via more channels (due to it's mass)



$$\Gamma(\mu \rightarrow e\bar{\nu}_\mu\nu) = \frac{1}{\tau_\mu}$$



$$\Gamma(\tau \rightarrow e\bar{\nu}_\tau\nu) = \frac{\mathrm{BR}(\tau \rightarrow e\bar{\nu}_\tau\nu)}{\tau_\tau}$$

Lepton Universality

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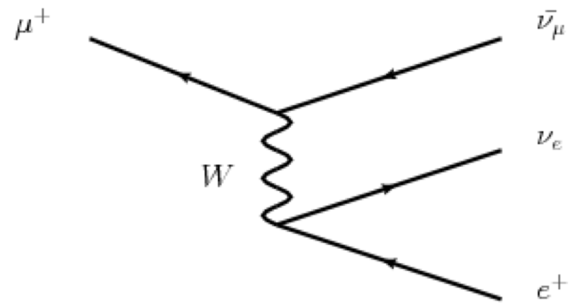
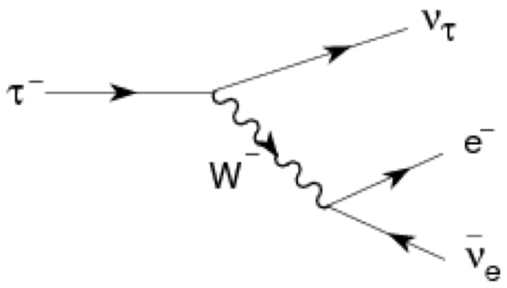
$$\Gamma(\tau \to e\bar{\nu}_\tau \nu) = \frac{\mathrm{BR}(\tau \to e\bar{\nu}_\tau \nu)}{\tau_\tau}$$





$$\frac{\Gamma(\mu \to e\bar{\nu}_\mu \nu)}{\Gamma(\tau \to e\bar{\nu}_\tau \nu)} = \frac{1}{\tau_\mu} \frac{\tau_\tau}{\mathrm{BR}(\tau \to e\bar{\nu}_\tau \nu)}$$

From theory, we get:

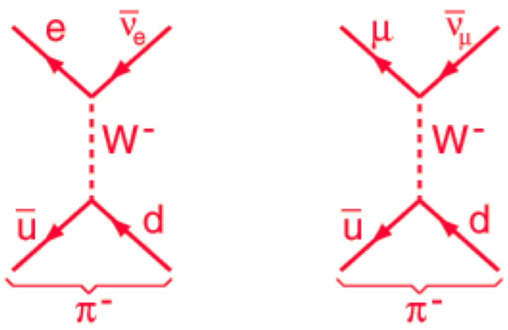
$$\frac{\Gamma(\mu \to e\bar{\nu}_\mu \nu)}{\Gamma(\tau \to e\bar{\nu}_\tau \nu)} = \frac{g_e^2 g_\mu^2 m_\mu^5}{g_e^2 g_\tau^2 m_\tau^5} \cdot \frac{\rho_\mu}{\rho_\tau} \to \frac{g_\mu^2 m_\mu^5}{g_\tau^2 m_\tau^5} \cdot \frac{\rho_\mu}{\rho_\tau}$$

just takes into account phase space differences

$$\frac{g_\mu}{g_\tau} = 1.001 \pm 0.003$$

Pion Decay

- Now, lets consider charged pion decay



We don't know how the pion couples with our W-boson, but we can describe it with a form factor $F^\mu = f_\pi p^\mu$

- The lifetime (for either electron or muon) is given by:

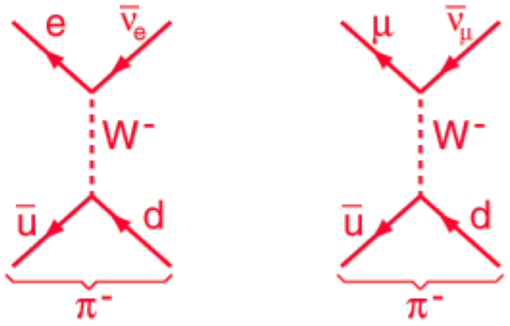
$$\Gamma = \frac{|p_\nu|}{8\pi m_\pi^2}(|\mathcal{M}|^2)$$

$$|\mathcal{M}|^2 = (\frac{g_w}{2M_W})^4 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$\Gamma = \frac{f_\pi^2}{\pi m_\pi^3}(\frac{g_w}{4M_W})^4 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

Pion Decay

- Now, lets consider charged pion decay



We don't know how the pion couples with our W-boson, but we can describe it with a form factor $F^\mu = f_\pi p^\mu$

- We don't know the form factor, but again we can take the ratio:

$$\Gamma = \frac{f_\pi^2}{\pi m_\pi^3}\left(\frac{g_w}{4M_W}\right)^4 m_\ell^2 (m_\pi^2 - m_\ell^2)$$

$$\frac{\Gamma(\pi \to e\nu_e)}{\Gamma(\pi \to \mu\nu_\mu)} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} = 1.283 \times 10^{-4}$$

Pion Decay

$$\frac{\Gamma(\pi \to e\nu_e)}{\Gamma(\pi \to \mu\nu_\mu)} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} = 1.283 \times 10^{-4}$$

- The decay to muons is highly favoured (goes against the naive idea that there would be more phase space in the case of the electron decay)

- This difference is due to spin conservation

  The pion is spin 0, so the lepton and anti-neutrino must emerge with opposite spin directions.
  The neutrino is always right-handed (parity violation) so only the right-handed component of the lepton can contribute to the decay

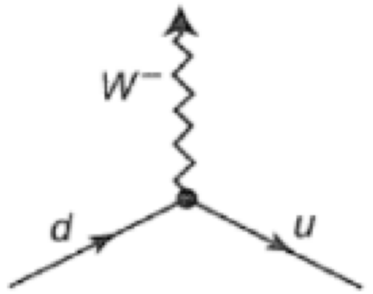
  This is related to the helicity, hence the more massive muon has a larger phase space for the decay, hence this decay is more favoured

Weak interactions for Quarks

The Cabibbo Angle

- We've already introduced neutron decay, which is basically the decay d → uW$^-$, we can also have the decau s → uW-

- It would be nice to have universality for quarks in addition to lepton, simply replace the lepton/anti-lepton with a up-type and associated down-type quark.
  From this we can get predicted decay rates in a similar manner as before



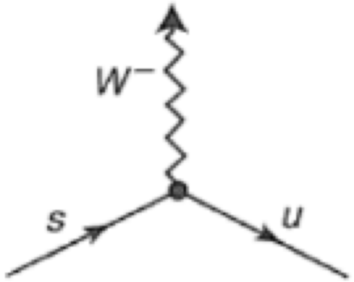
$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\cos\theta_C$$

This however doesn't work, as the calculated decay rates are not the same as the predicted rates...

The coupling for d→uW and s→uW are found to need a modification

We introduce the Cabibbo angle ($\boldsymbol{\theta}_C$ = 12.7 degrees)

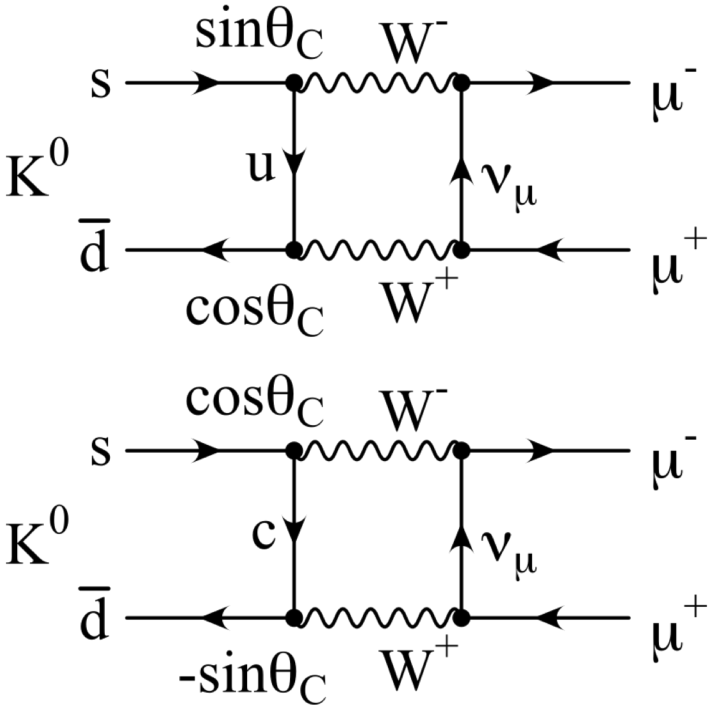
sin($\boldsymbol{\theta}_C$) = 0.220 → Cabibbo suppressed
cos($\boldsymbol{\theta}_C$) = 0.976 → Cabibbo allowed



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\sin\theta_C$$

GIM Mechanism

Suppression of Kaon decays



- The absence of the $K^0 \rightarrow \mu\mu$ decay was solved by the proposal by the GIM mechanism (Glashow, Iliopoulous and Maiani).
- Flavour changing neutral currents (FCNCs) are suppressed by the Cabibbo angle (the diagram proceeds via a loop, and Cabibbo supression occurs)
- This required the introduction of a 4^{th} quark – the Charm
- Also suggests that quarks are placed in doublets (like leptons and neutrinos)

$$\begin{bmatrix} u \\ d \end{bmatrix}, \begin{bmatrix} c \\ s \end{bmatrix}$$

The CKM Matrix

- The GIM mechanism is extended to 3 generations of quarks by the CKM matrix (the d', s', b' are the weak eigenstates of the down-type quarks, we get them from the mass eigenstates using the CKM matrix)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

The values in the matrix cannot be predicted and must all be measured experimentally

$$V_{CKM} = \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

We can also introduce CP violation:

2 generations can be explained by one real parameter (the Cabibbo angle)

3 generations require 3 parameters and one phase

Unitarity Triangles

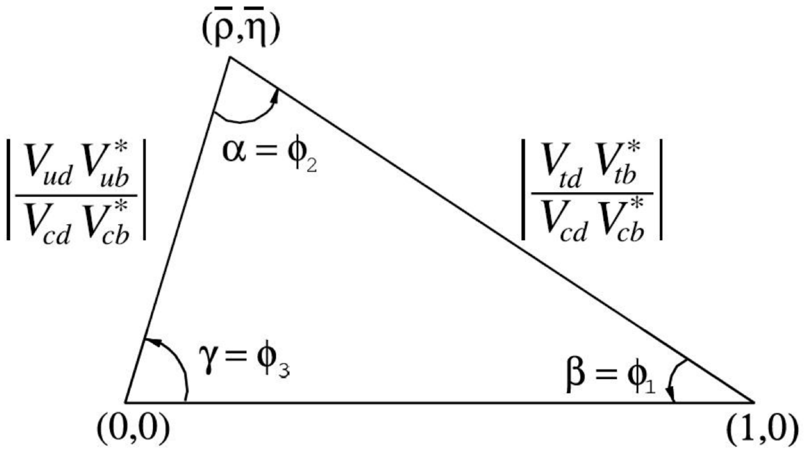
- Conservation of current requires that

$$V_{CKM}V_{CKM}^{\dagger} = V_{CKM}^{\dagger}V_{CKM} = 1$$

- Which leads to (among other combinations):

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

- Can be represented as a triangle:



Unitarity Triangles

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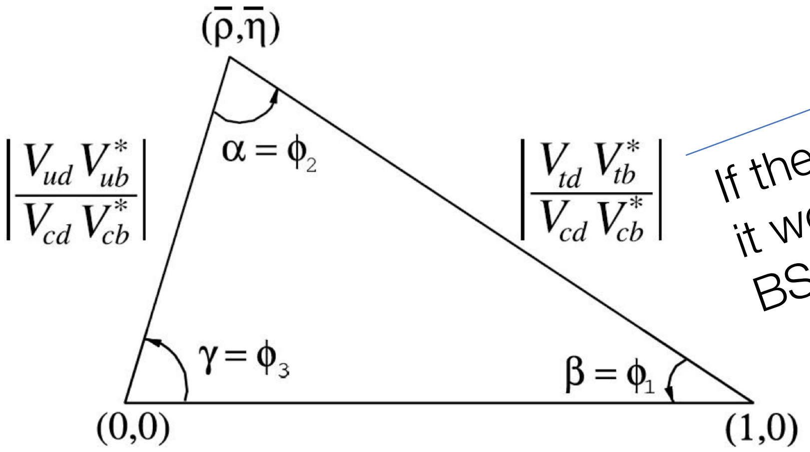
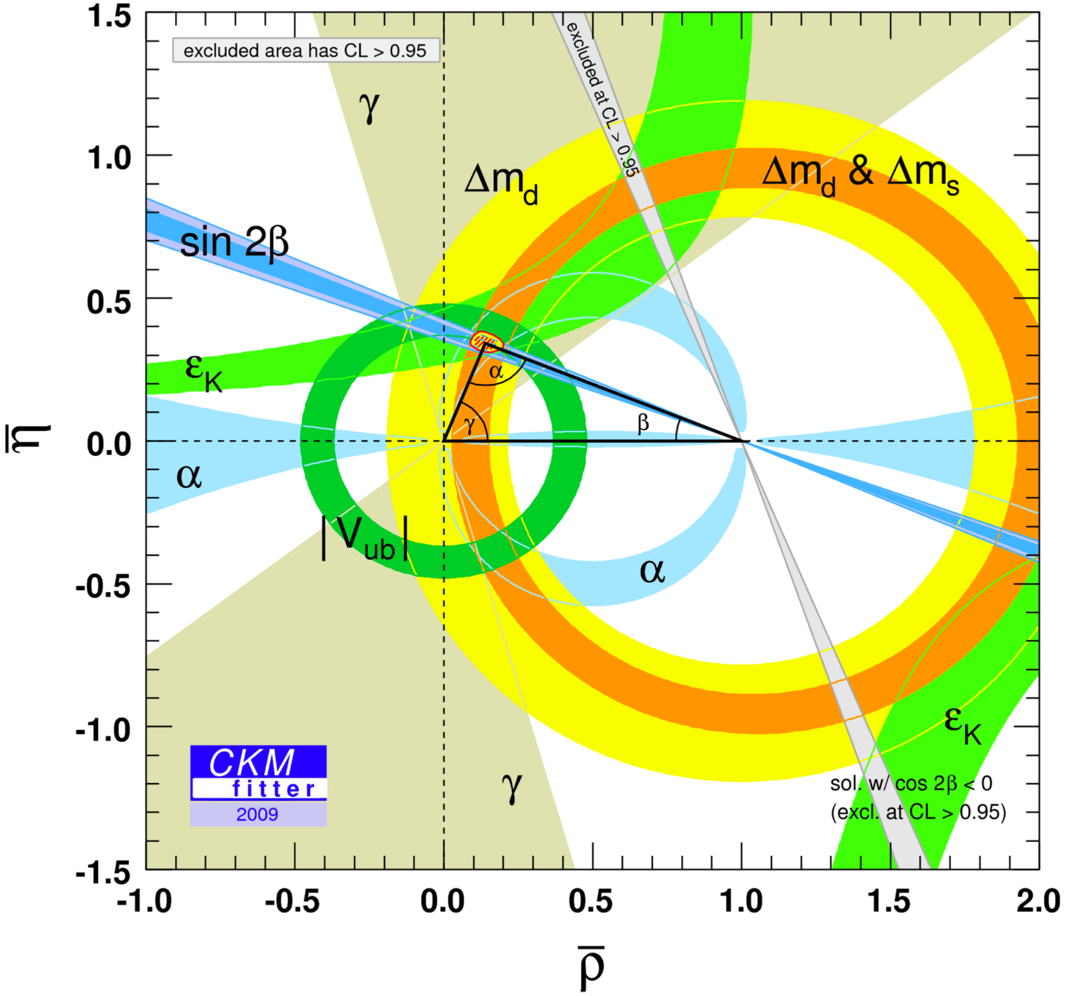


If the triangle didn't close it would suggest signs of BSM physics



Lecture 3 - Recap

- Reviewed
  - Conservation laws
  - Neutron decay
  - Fermi theory
  - Parity violation in Weak decays
  - Weak theory
  - Lepton Universality
  - Flavour physics (Cabibbo, CKM)

Electroweak physics: The weak force & fermions

Lecture 4

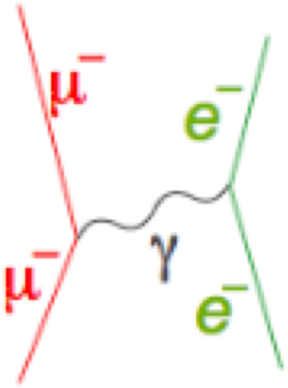
- Unification of the EM and Weak force, discoveries, unanswered questions
    - Discovery of the W/Z bosons
    - Weak neutral currents
    - Unification
    - Weinberg angle
    - Experimental measurements

Unification?

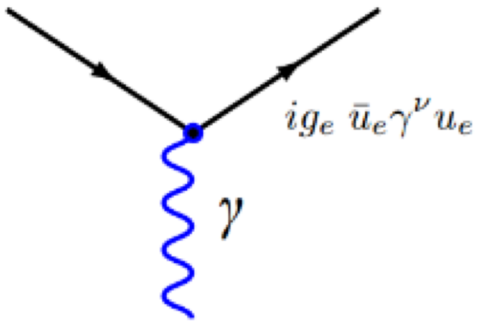
We have the EM force:

Exchange of a spin 1 particle

long range



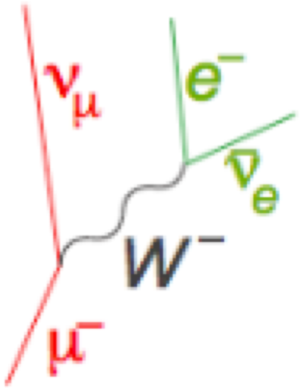
Parity conserving



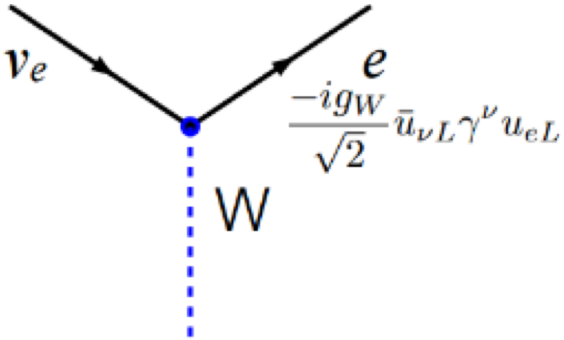
and the weak force:

Exchange of a spin 1 particle

short range



Parity violating

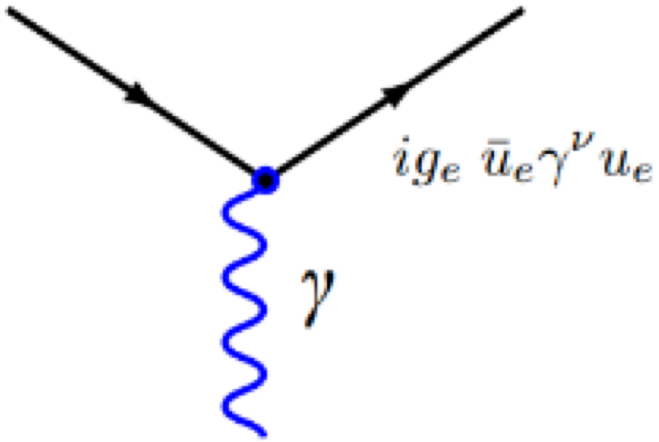


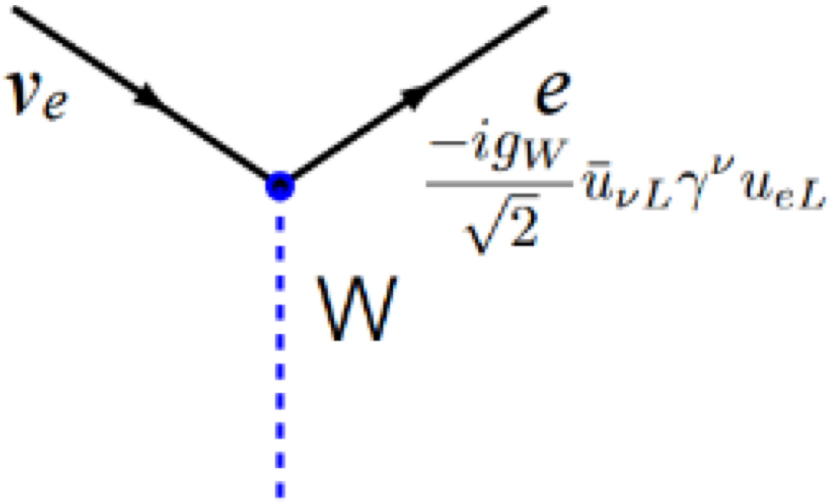
Is there some kind of symmetry, relating the propagators? If so it must be broken

But if there is, it means we can unify the forces

Unification?

- Glashow, Weinberg and Salam (1961) attempted this unification, based upon the gauge symmetry, but in Gauge theory (as we worked through earlier) the force mediators must be massless
- But we've seen that the mediator of the weak force is not massless (the strength of the weak interaction is such, due to the mediator mass)
- How to reconcile this?
  Glashow: "This is a stumbling we must overlook"
  (aka: let's ignore this for now and see what happens)
- So, let's try to construct this theory





Electroweak Theory

- Fermions can be placed into doublets: $\chi_L = \begin{bmatrix} \nu_{e_L} \\ e_L \end{bmatrix}$

- We can define two operators: $\tau_+ = \frac{1}{2}(\tau_1 + i\tau_2) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\quad \tau_- = \frac{1}{2}(\tau_1 - i\tau_2) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

(here $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are the 1st and 2nd Pauli matrices)

- We can write the exchange of a W^+ or W^- as:

$$j_\mu^- = \bar{\nu}_L \gamma_\mu e_L = \bar{\chi}_L \gamma^\mu \tau^+ \chi_L \qquad j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L = \bar{\chi}_L \gamma^\mu \tau^- \chi_L$$

Electroweak Theory

- Can we do anything with the third Pauli matrix?

$$\tau^3 = \tfrac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- Multiply this by the fermion doublet

$$\bar{\chi}_L \gamma^\mu \tau^3 \chi_L = \begin{bmatrix} \bar{\nu}_L & \bar{e}_L \end{bmatrix} \gamma^\mu \tfrac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$$

$$\rightarrow \boxed{\frac{1}{2}\bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma^\mu e_L}$$

A new current?

Electroweak Theory

- What is this current?

$$\rightarrow \frac{1}{2}\bar{\nu}_L\gamma^\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma^\mu e_L$$

- It isn't the EM current

$$j_\mu^{em} = -\bar{e}_L\gamma^\mu e_L - \bar{e}_R\gamma^\mu e_R$$

- We can also write the "orthogonal" current to our new current

$$-(\bar{\nu}_L\gamma^\mu\nu_L + \bar{e}_L\gamma^\mu e_L)$$

We can group the components of the currents into a triplet, and two singlets

Triplet (spin 1)

$$j_\mu^- = \bar{\nu}_L\gamma_\mu e_L = \bar{\chi}_L\gamma^\mu\tau^+\chi_L$$
$$j_\mu^+ = \bar{e}_L\gamma_\mu\nu_L = \bar{\chi}_L\gamma^\mu\tau^-\chi_L$$
$$j_\mu^3 = \bar{\chi}_L\gamma^\mu\tau^3\chi_L$$

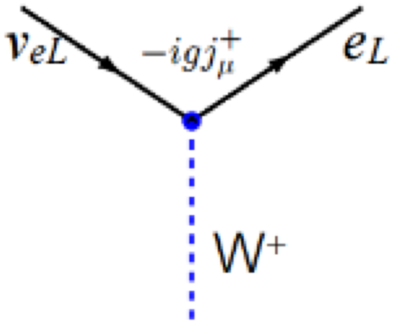
Singlet (spin 0)

$$-(\bar{\nu}_L\gamma^\mu\nu_L + \bar{e}_L\gamma^\mu e_L)$$

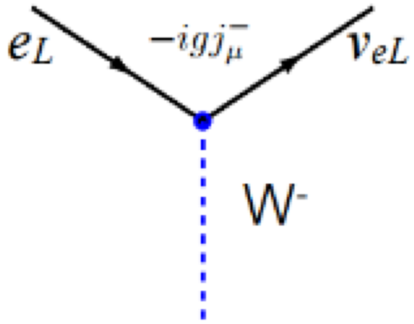
Singlet (from EM, also spin 0)

$$-\bar{e}_R\gamma^\mu e_R$$

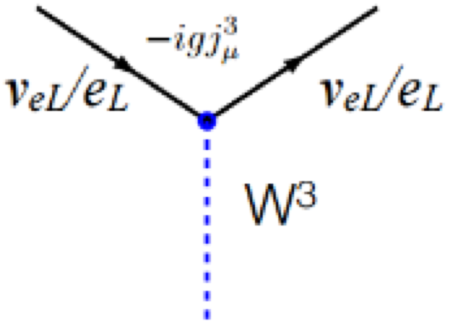
Interactions corresponding to the triplet:



$j_\mu^+ = \bar{\chi}_L \gamma_\mu \tau^+ \chi_L$

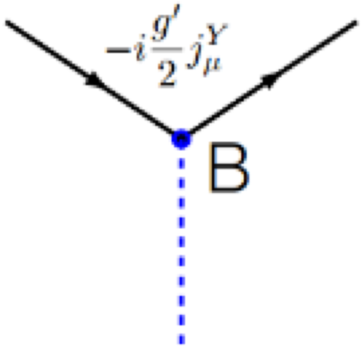


$j_\mu^- = \bar{\chi}_L \gamma_\mu \tau^- \chi_L$



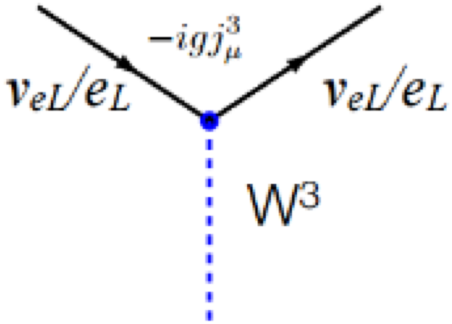
$j_\mu^3 = \frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma_\mu e_L$

And the singlet (we've basically grouped all of the singlet terms together)

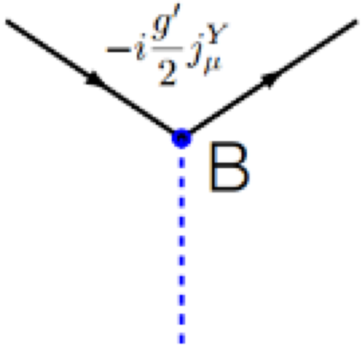


$$j_\mu^Y = -2\bar{e}_R \gamma_\mu e_R - \bar{e}_L \gamma_\mu e_L - \bar{\nu}_L \gamma_\mu \nu_L$$

Electroweak Theory



$$j_\mu^3 = \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L$$



$$j_\mu^Y = -2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L$$

We don't have anything that corresponds to these couplings or propagators, but maybe we can do something to fix this.

We can define the photon (A), and a new vector boson (Z) as linear combinations of W^3 and B (rotations of the states)

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$

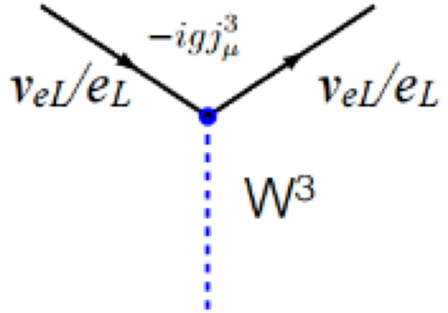
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

Electroweak Theory

We can then write the vertex terms for the A and Z in terms of B and W³

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$

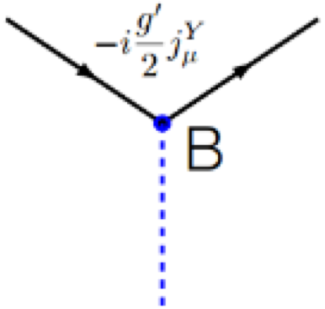
$$-i\tfrac{g'}{2}\cos\theta_W(-2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L) - ig\sin\theta_W(\tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L)$$



$$j_\mu^3 = \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L$$

But we want to reduce this to the photon vertex term: $ig_e(\bar{e}\gamma_\mu e)$

Which implies: $g\sin\theta_W = g'\cos\theta_W = g_e$



$$j_\mu^Y = -2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L$$

Electroweak Theory

We can then write the vertex terms for the A and Z in terms of B and W^3

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$$

$$-i\frac{g'}{2}\sin\theta_W(-2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L) - ig\cos\theta_W(\tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L)$$

We define the coupling for the Z: $g_Z = \frac{g_e}{\sin\theta_W\cos\theta_W} = \frac{g}{\cos\theta_W} = \frac{g'}{\sin\theta_W}$

$$ig_Z\sin^2\theta_W(\bar{e}_R\gamma_\mu e_R - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L - \tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L) - ig_Z\cos^2\theta_W(\tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L)$$

$$-ig_Z(\tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L + \sin^2\theta_W(\bar{e}_R\gamma_\mu e_R + \bar{e}_L\gamma_\mu e_L))$$

Unification

- We have just unified the EM and Weak fields, using a single parameter, the Weinberg angle ($\boldsymbol{\theta}_W$)
  $\sin^2(\boldsymbol{\theta}_W) = 0.22$
- The left handed fermions are in doublets and interact via weak Charged Current (W-Boson).
  The right handed fermions are singlets and do not.
- The Pauli matrices are the **Generators** of the SU(2) symmetry. EM is a gauge theory with the symmetry group U(1)
- Electroweak theory is SU(2) x U(1), and the currents are just Noether conserved currents from gauge invariance

Unification

- The Z has a left handed coupling (from both J3 and JEM) and a right handed coupling (from JEM)

$$g_L = I_3 - Q^2 \sin^2\theta_W \qquad g_R = -Q^2 \sin^2\theta_W$$

- This is usually expressed as a vector component and an axial component (V-A theory)

$$c_V = g_L + g_R = I_3 - 2Q\sin^2\theta_W \qquad c_A = g_L - g_R = I_3$$

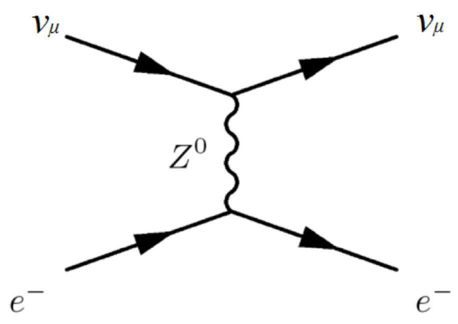
- The predictions of the (Z) couplings for the fermions are:

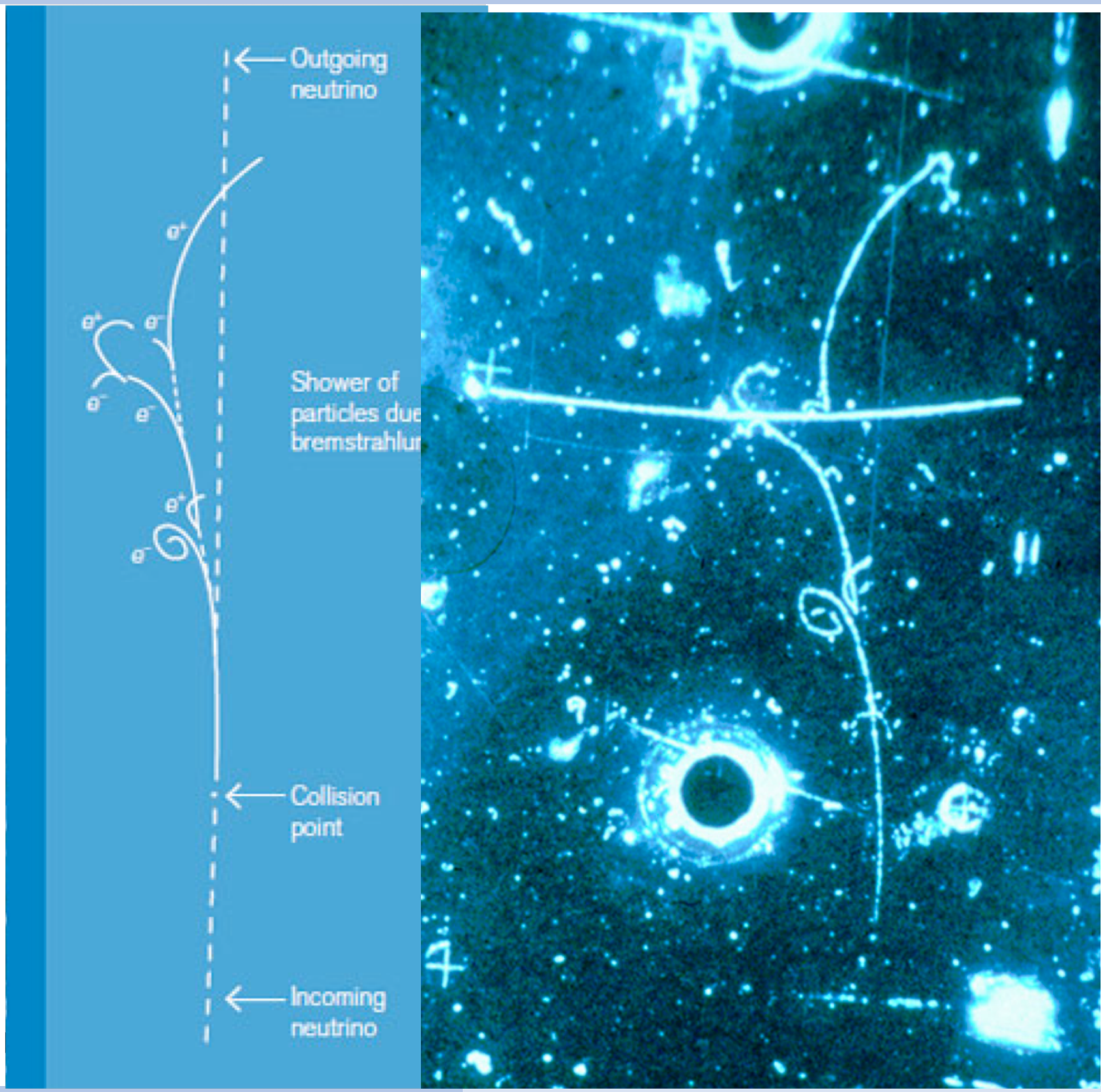
Lepton	$2c_V$	$2c_A$	Quark	$2c_V$	$2c_A$
ν_e, ν_μ, ν_τ	1	1	u, c, t	$1 - \frac{8}{3}\sin^2\theta_W$	1
e, μ, τ	$-1 + 4\sin^2\theta_W$	-1	d, s, b	$-1 + \frac{4}{3}\sin^2\theta_W$	-1

- In performing this unification, we've made a large prediction, that there is a new, neutral boson. If we can find this, we can provide evidence for the theory

Weak neutral currents

- Evidence for the Z-boson was found in 1973 with the Gargamelle experiment
- Protons (PS) incident on a target produce a beam of neutrinos (via Kaon and Pion decay)
- Neutrinos then interact in the bubble chamber with the electrons from nuclei, which leaves tracks





Boson Masses

- The masses of the W and Z can be written as functions of $\boldsymbol{\theta}_W$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \rightarrow M_W = \sqrt{\frac{e^2\sqrt{2}}{8G_F \sin^2\theta_W}} = \frac{37.4}{\sin\theta_W}[\mathrm{GeV}]$$

This just comes from EWK theory by rewriting the vertex terms

$$M_Z = \frac{M_W}{\cos\theta_W} \rightarrow \frac{75}{\sin 2\theta_W}[\mathrm{GeV}]$$

This comes from the Higgs mechanism (won't be discussed here)

- Measurements of the weak mixing angle suggested the boson masses must be around 80-90GeV

Discovery of the W-Boson

- In the early 1980's most experiments were fixed target experiments, where the center of mass energy is given by:

$$E_{cm} = \sqrt{2m_N E_{beam}}$$

- Instead, if both beams are accelerated (to the same E), the total energy available is:

$$E_{cm} = 2E_{beam}$$

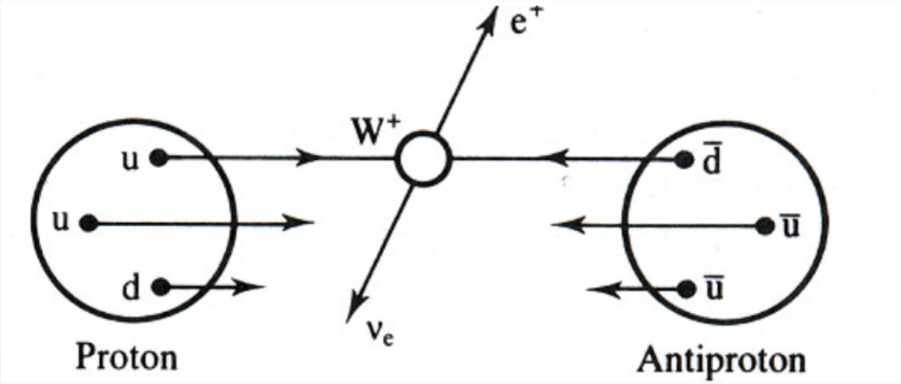
- The SPS accelerator was modified → SppS, from fixed target experiments with E_{beam} = 900GeV (E_{cm} = 30GeV), to a colliding experiment with E_{beam} = 450GeV (E_{cm} = 900GeV)

- This is enough to produce our W/Z-bosons.
  Caveat, not all of the energy of the proton is used in the collision (as protons are composite particles)

Discovery of the W-Boson

- To produce a W-boson, we use the interactions:

$$u + \bar{d} \rightarrow W^+ \rightarrow e^+, \mu^+ + \nu_e, \nu_\mu$$
$$\bar{u} + d \rightarrow W^- \rightarrow e^-, \mu^- + \bar{\nu}_e, \bar{\nu}_\mu$$



- We chose the lepton decay mode of the W (either electron, or muon and the associated neutrino) as in proton-(anti) proton collisions, the hadronic background is large, so requiring the presence of a lepton reduces the background

- This leaves us with a “high” p_T lepton, and a high p_T neutrino in the final state.
- But the neutrino escapes detection...

How can we measure the mass if we miss one of the decay products?

Discovery of the W-Boson

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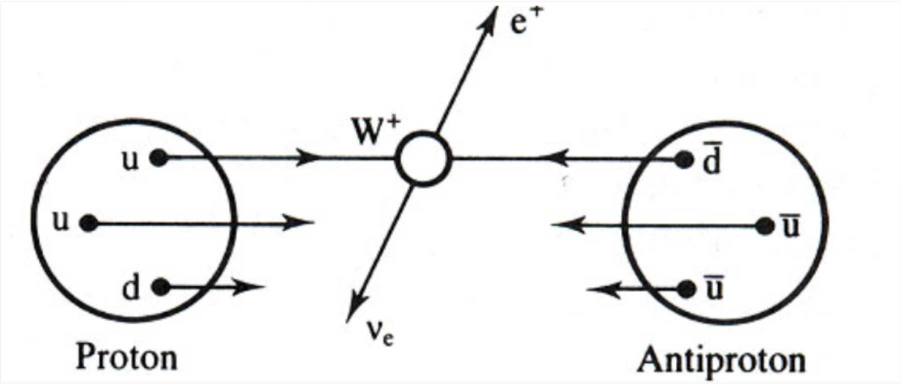

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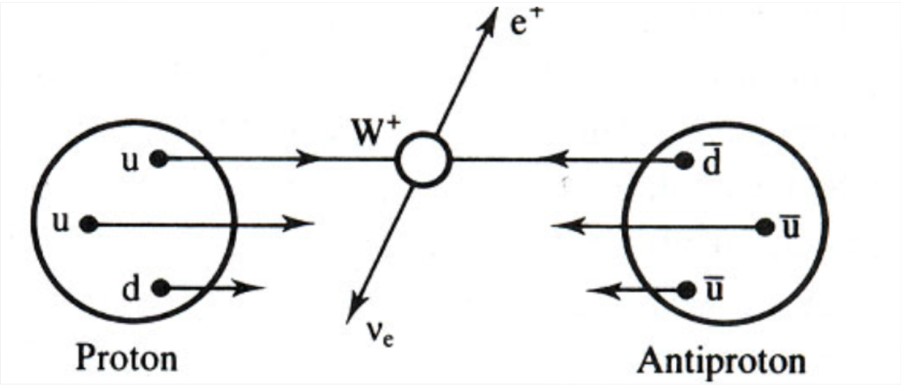
Discovery of the W-Boson

- When colliding protons we don't know the longitudinal input momentum into the collision



- But we can use the conservation of momentum/energy in the transverse plane (to the beam), in order to reconstruct the W-boson mass

- The neutrino in the decay will result in a momentum imbalance in the transverse plane, referred to as E_T^{miss}

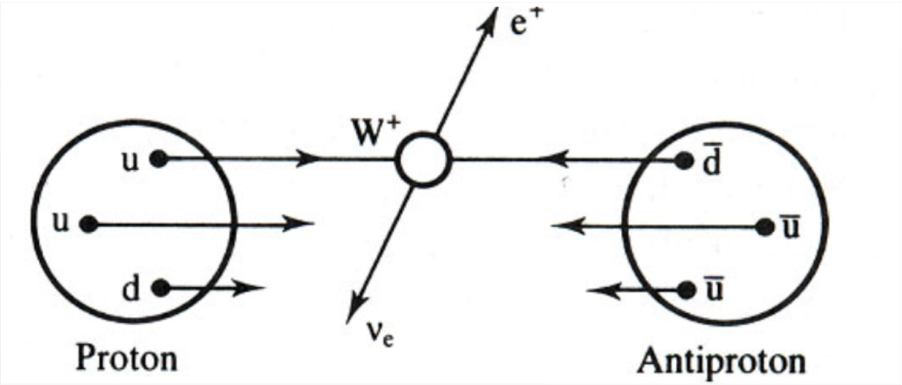
- We use the transverse mass, m_T, to infer the W-mass: $$m_{\mathrm{T}} = \sqrt{2 p_{\mathrm{T}}^{\ell} E_{\mathrm{T}}^{\mathrm{miss}} (1 - \cos\phi)}$$

(where ϕ is the angle between the lepton and the E_T^{miss} vector)

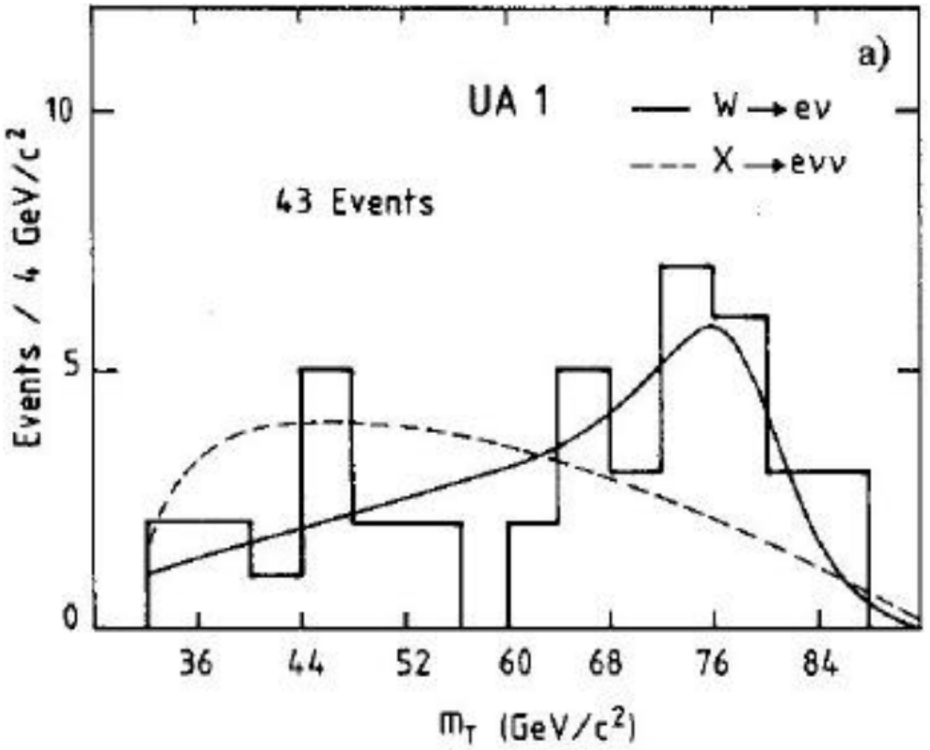
Discovery of the W-Boson

- The UA1 and UA2 experiments both took proton-anti-proton collision data to discover the W-boson

$$m_{\mathrm{T}} = \sqrt{2p_{\mathrm{T}}^{\ell} E_{\mathrm{T}}^{\mathrm{miss}} (1 - \cos\phi)}$$



$M_W = 80.9 \pm 1.5$ GeV



Discovery of the W-Boson

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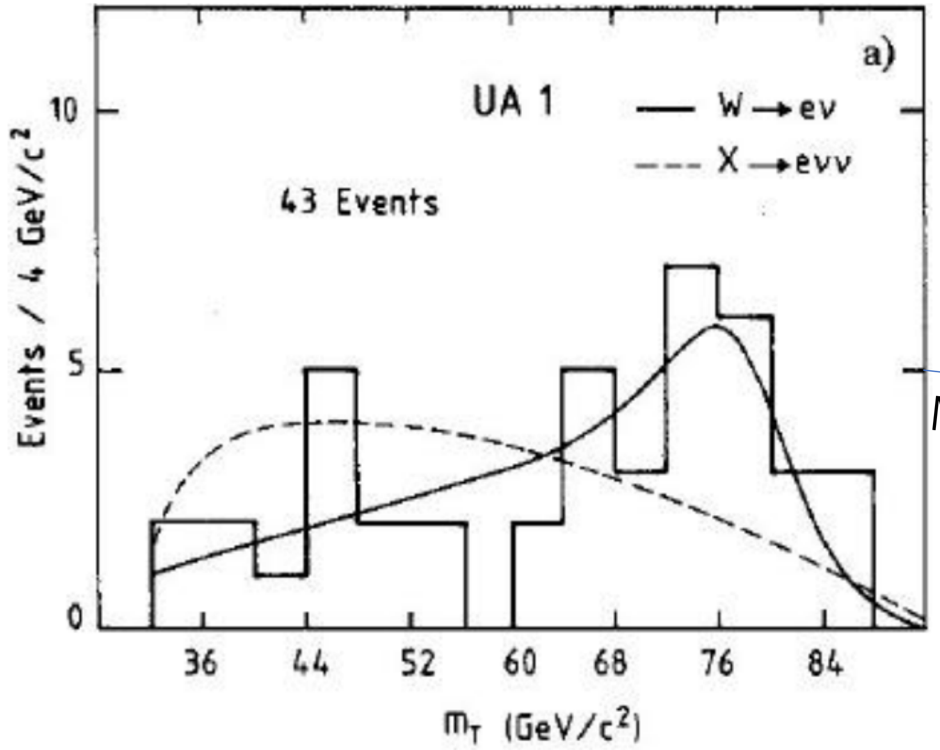
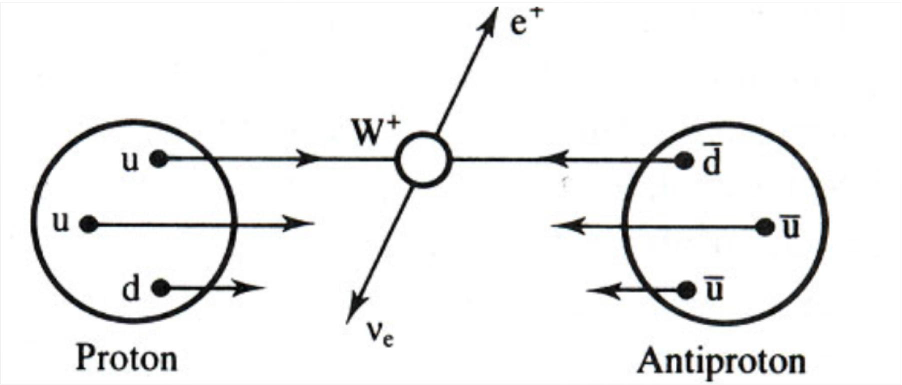
$$m_{\mathrm{T}} = \sqrt{2p_{\mathrm{T}}^{\ell} E_{\mathrm{T}}^{\mathrm{miss}}(1-\cos\phi)}$$

$M_W = 82.1 \pm 1.7$ GeV

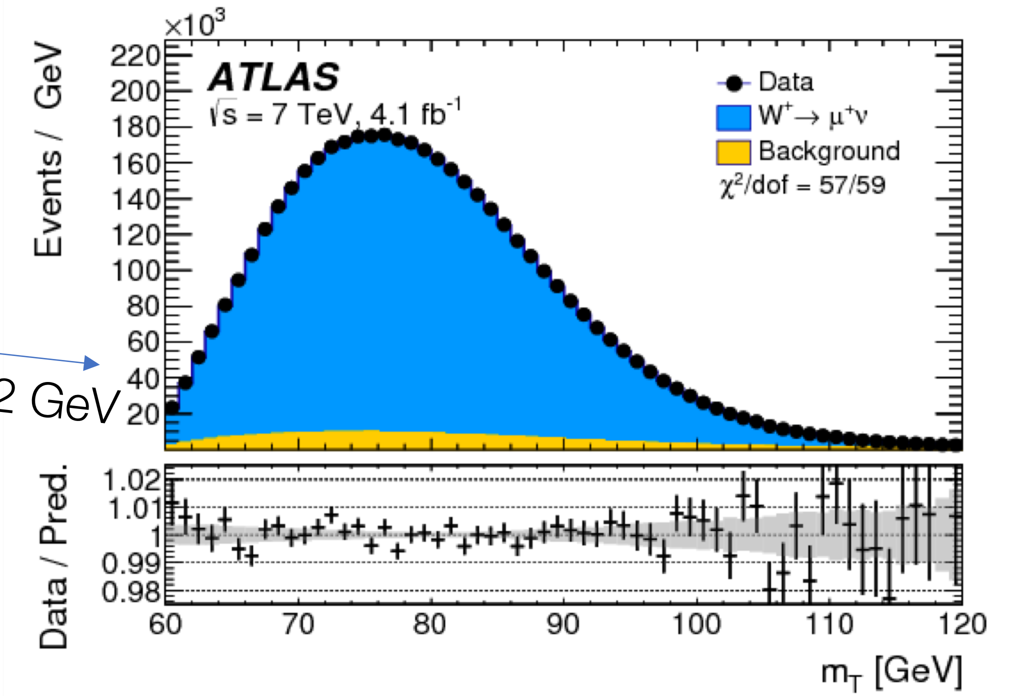




+ 30 Years

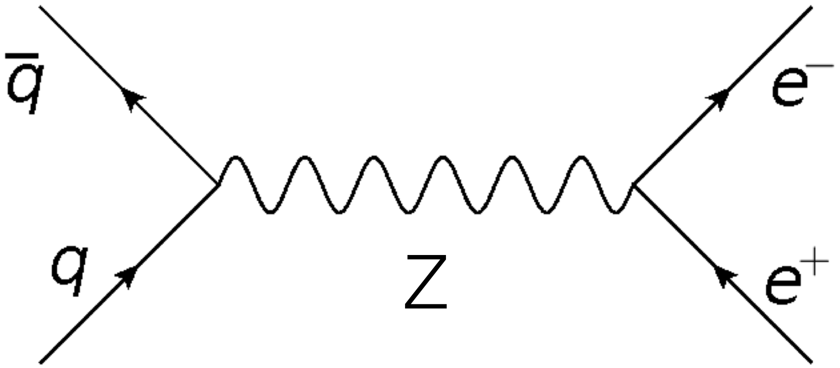
$M_W = 80.379 \pm 0.012$ GeV

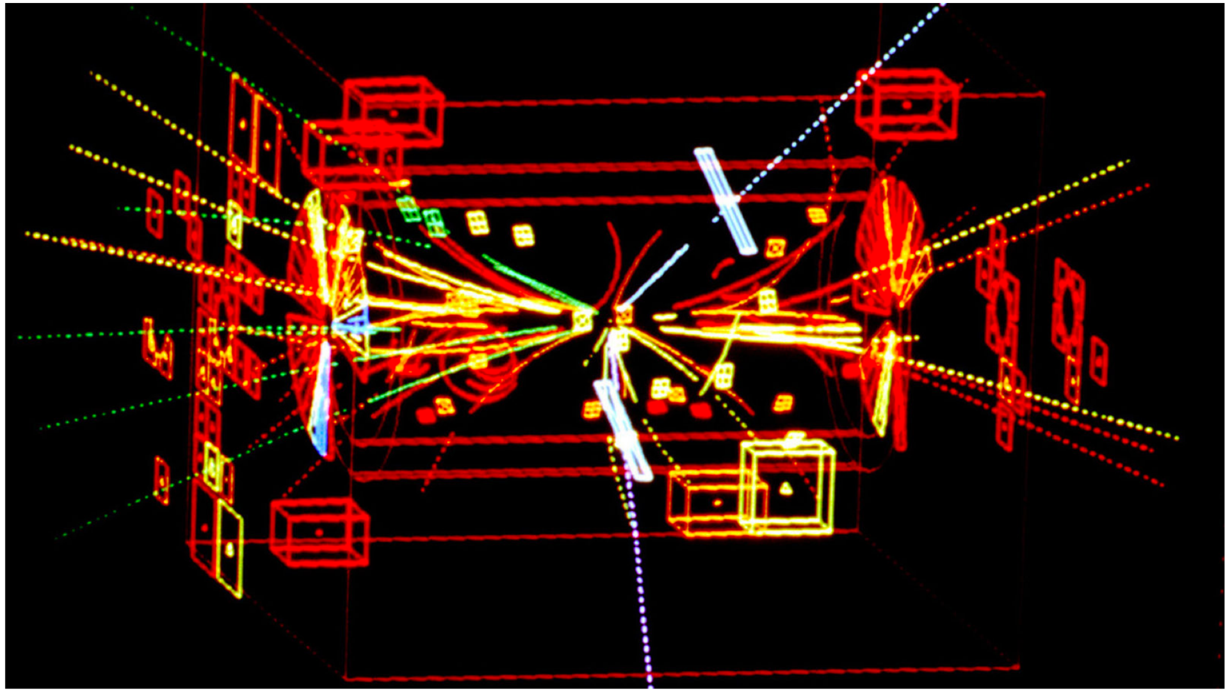
PDG Fit



Discovery of the Z-Boson

- The Z-boson was discovered later than the W (the BR for Z → leptons is lower for the Z than the W, hence required a higher luminosity)





M_Z = 93.0 ± 1.7 GeV

Discovery of the Z-Boson

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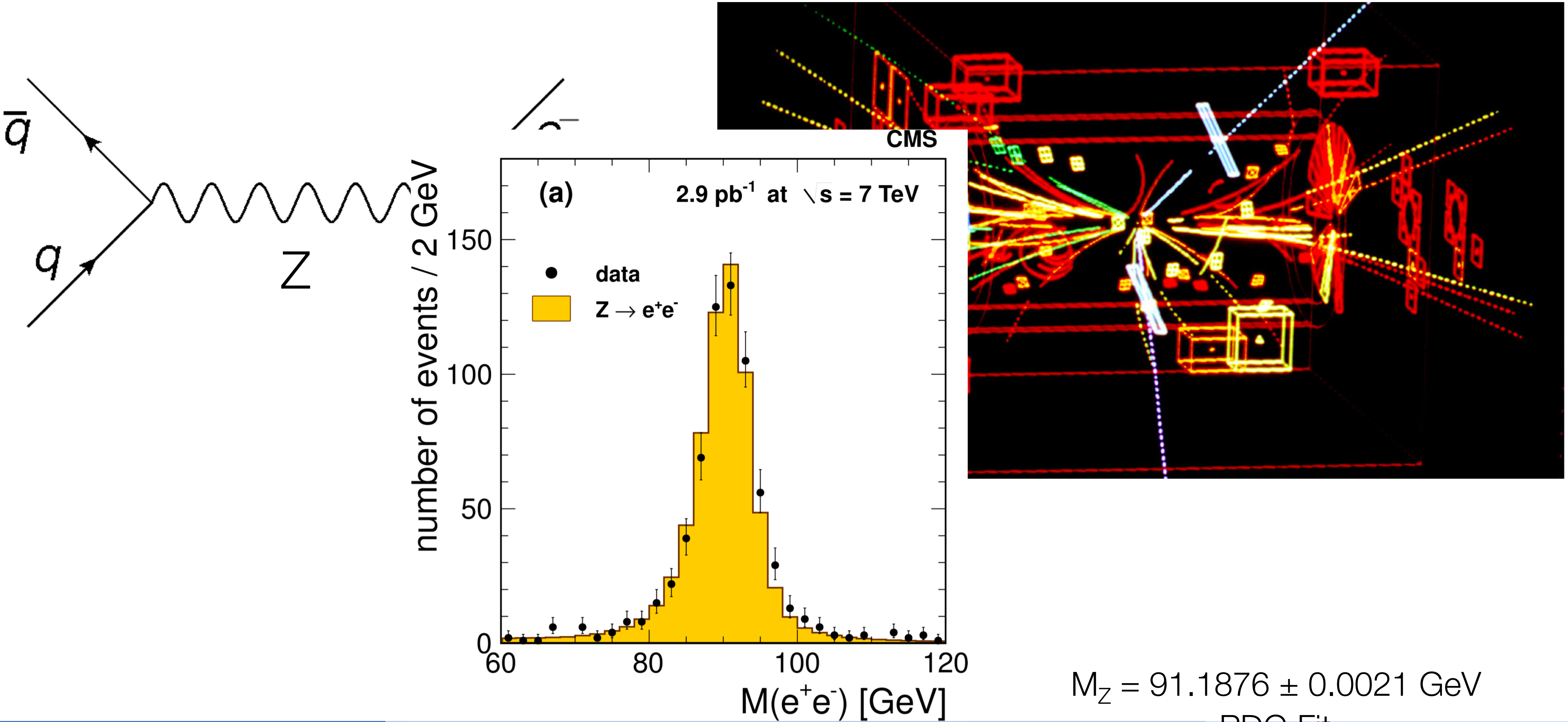
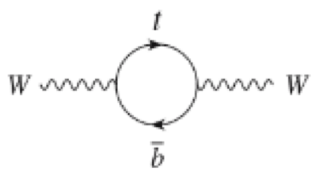
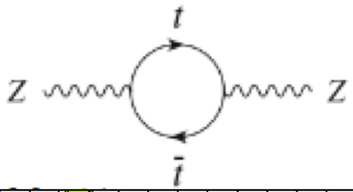


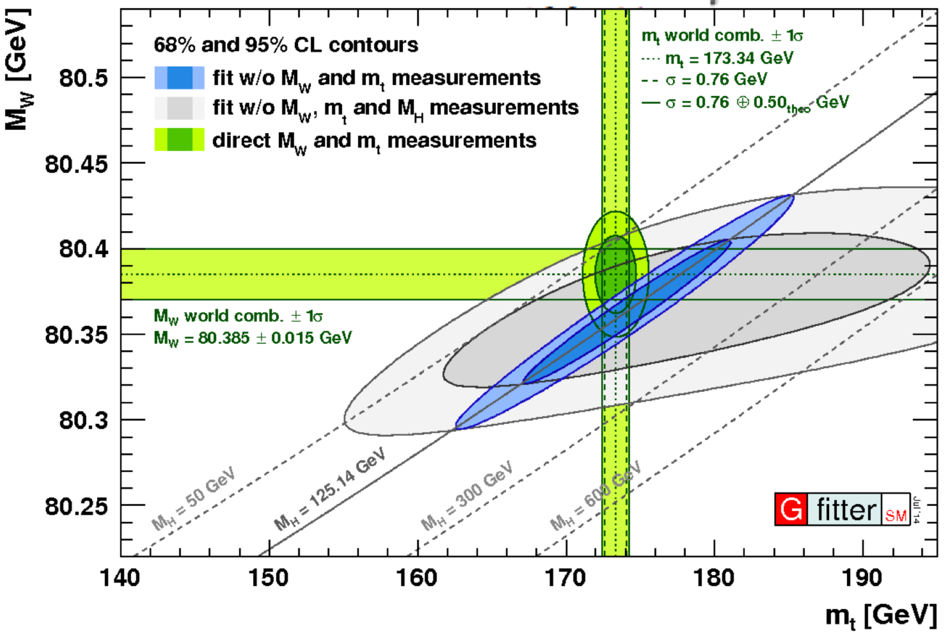
$M_Z = 91.1876 \pm 0.0021$ GeV
PDG Fit

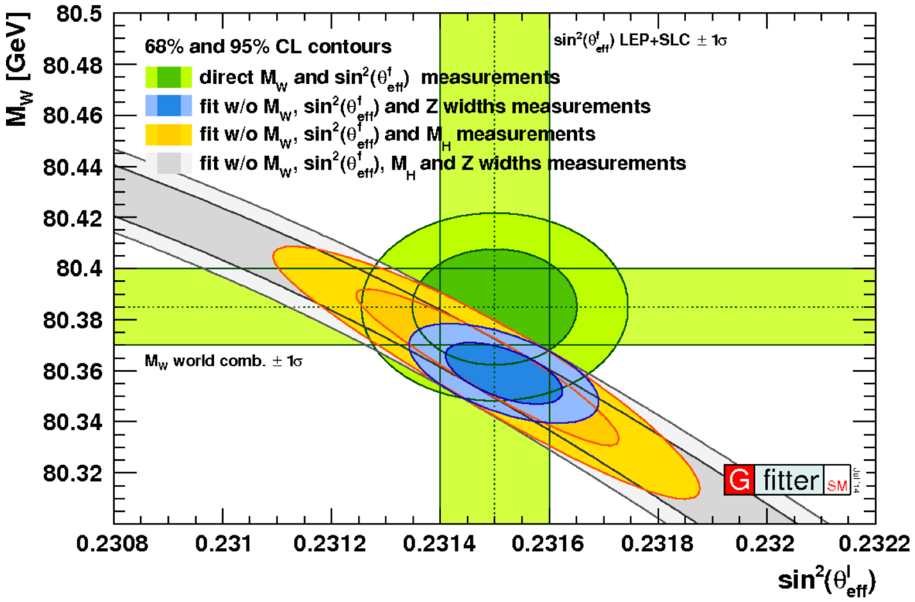
Electroweak Fits

- The current measurement of the W-mass is M_W = 80.379 ± 0.012 GeV (error of ~MeV)
- We can probe the SM relationships using loop processes and precision measurements









- If there are any inconsistencies, this gives hits at beyond the standard model physics

Number of Neutrino Generations

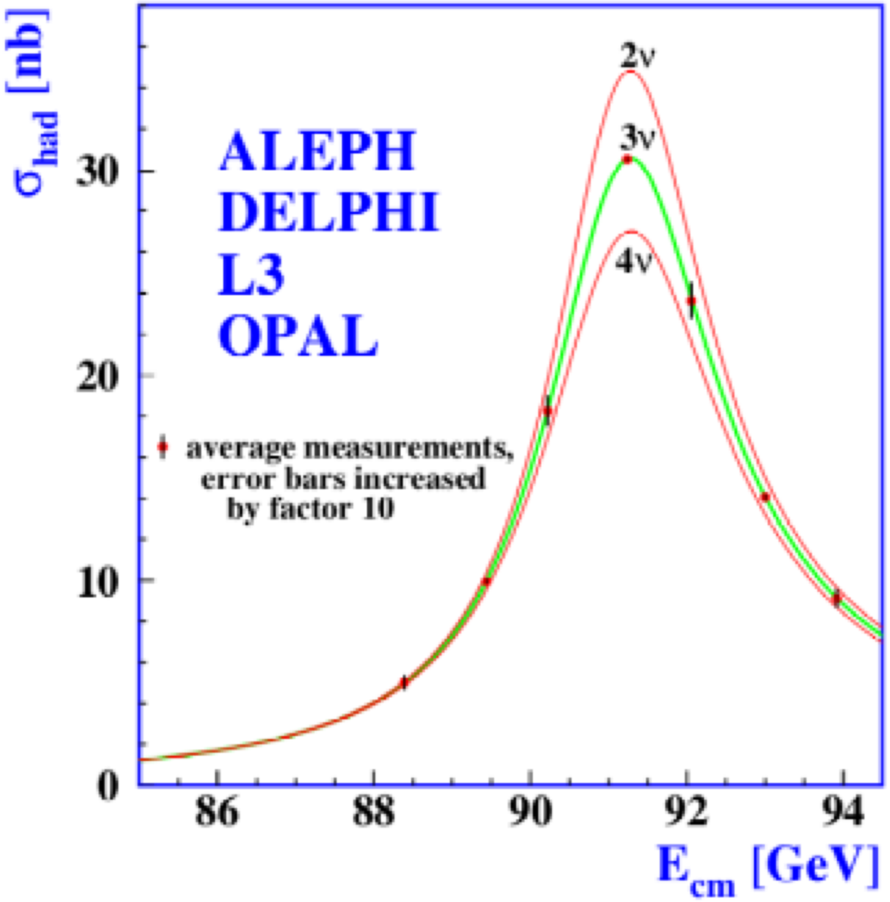
- With the discovery of the Z-boson, we also now have a method of discerning the number of neutrino generations

- We can calculate the partial widths of the Z decays:

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{hadrons} + \Gamma_{inv}$$

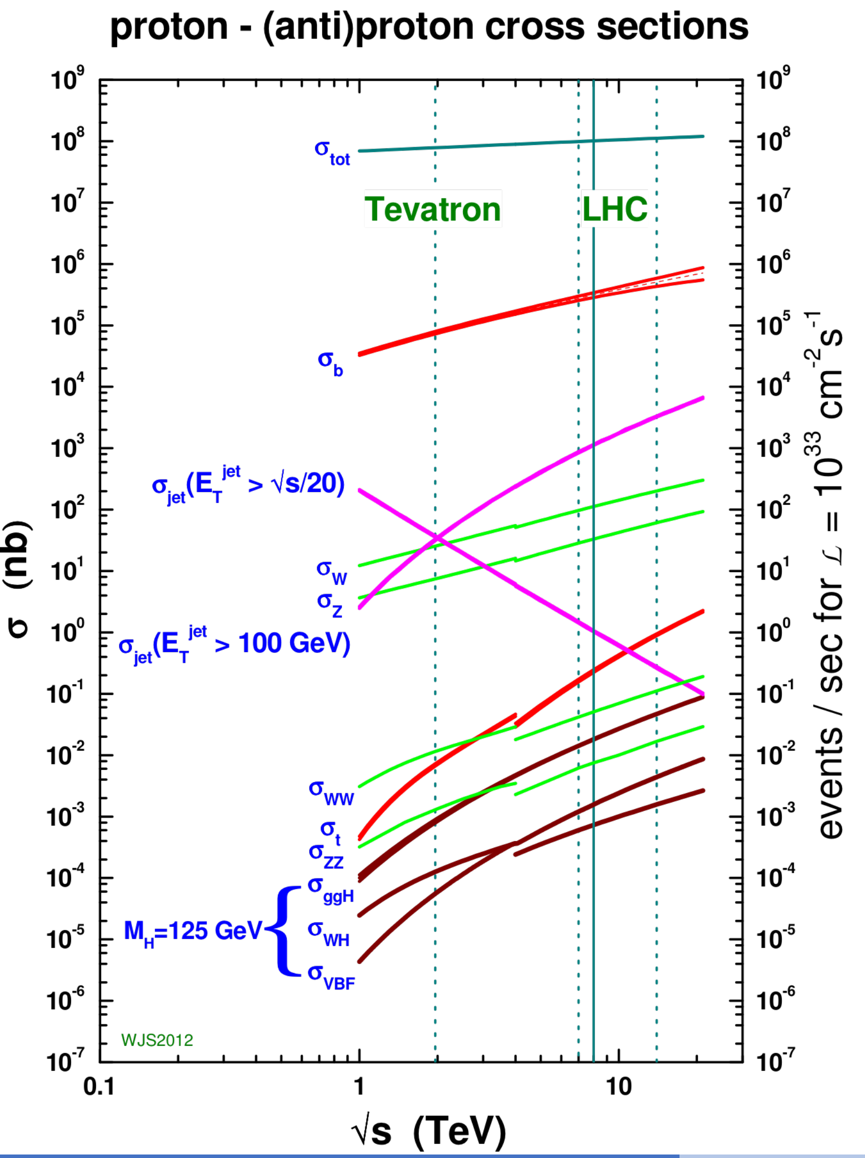
- Measure the hadronic cross section, and fit the value used for the total width, depending on neutrino generations.

$$\sigma_{had} = \frac{12\pi\Gamma_{ee}\Gamma had}{m_Z^2\Gamma_Z^2}$$



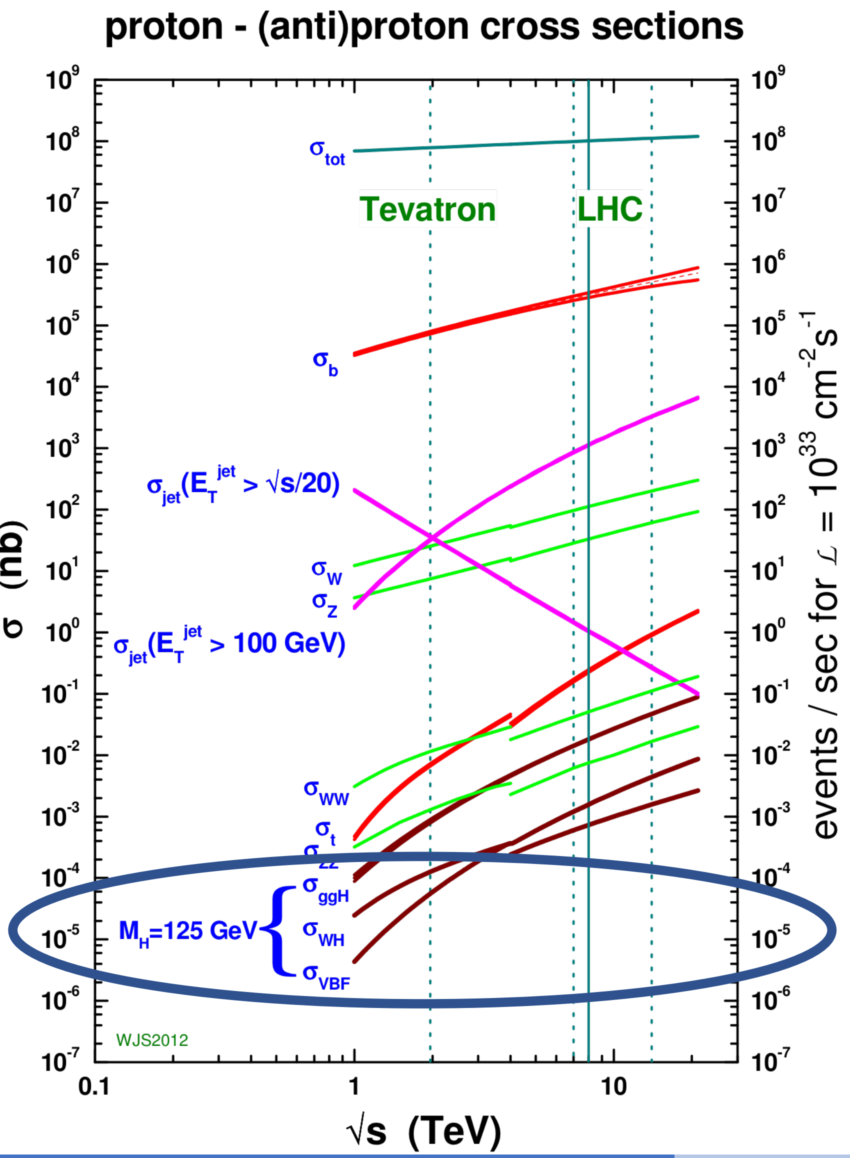
Electroweak Physics at the LHC

Overview



- Huge program of precision SM measurements from ATLAS, CMS and LHCb
- Measuring EWK physics to a high precision is needed for two reasons
- As previously discussed, probing the SM at higher and higher energies to investigate if the theory is still valid

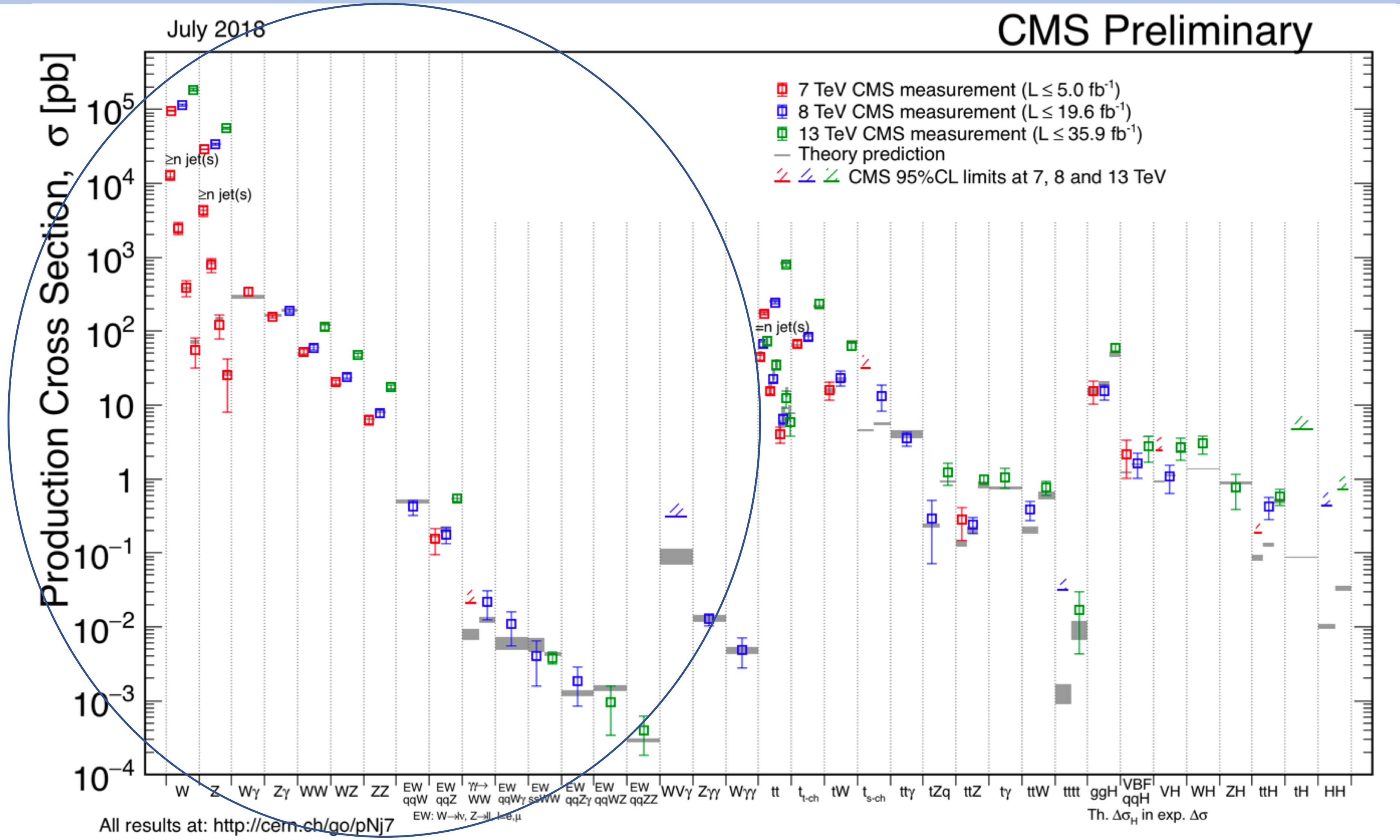
Overview



- Huge program of precision SM measurements from ATLAS, CMS and LHCb
- Measuring EWK physics to a high precision is needed for two reasons
- As previously discussed, probing the SM at higher and higher energies to investigate if the theory is still valid
- Also, the EWK processes are also at a relatively high cross-section, (100's are produced every second at the LHC), they've become the **background** for searches for new physics

Need to understand them at a very high precision to claim discovery of any new physics

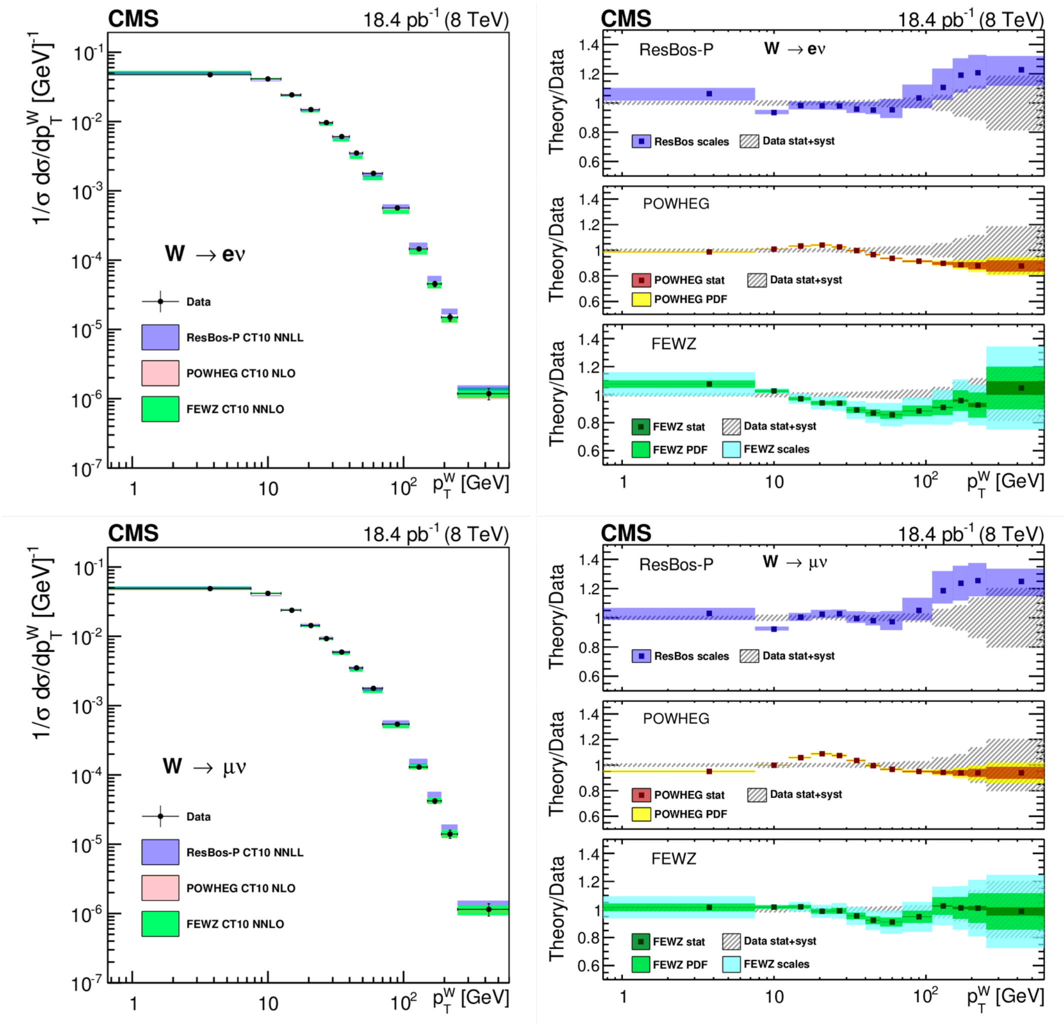
Overview



- Huge program of precision SM measurements from ATLAS, CMS and LHCb

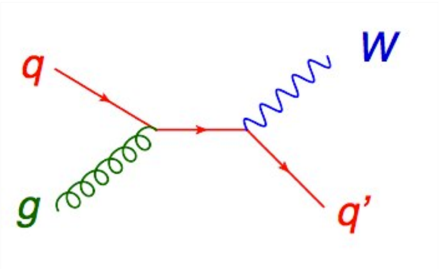
W-boson production

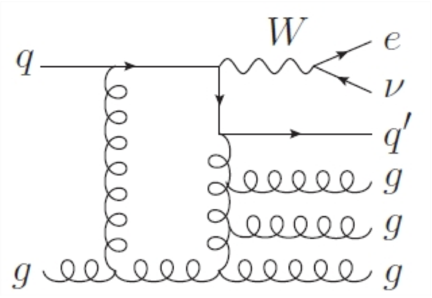
- Very high statistics available for W production allows a detailed comparison between the SM prediction and experimental measurement
  - Can investigate a large kinematic range
- Distributions of $p_T(W)$ vs a selection of MC generators
  - Allows us to tune our MC to the data

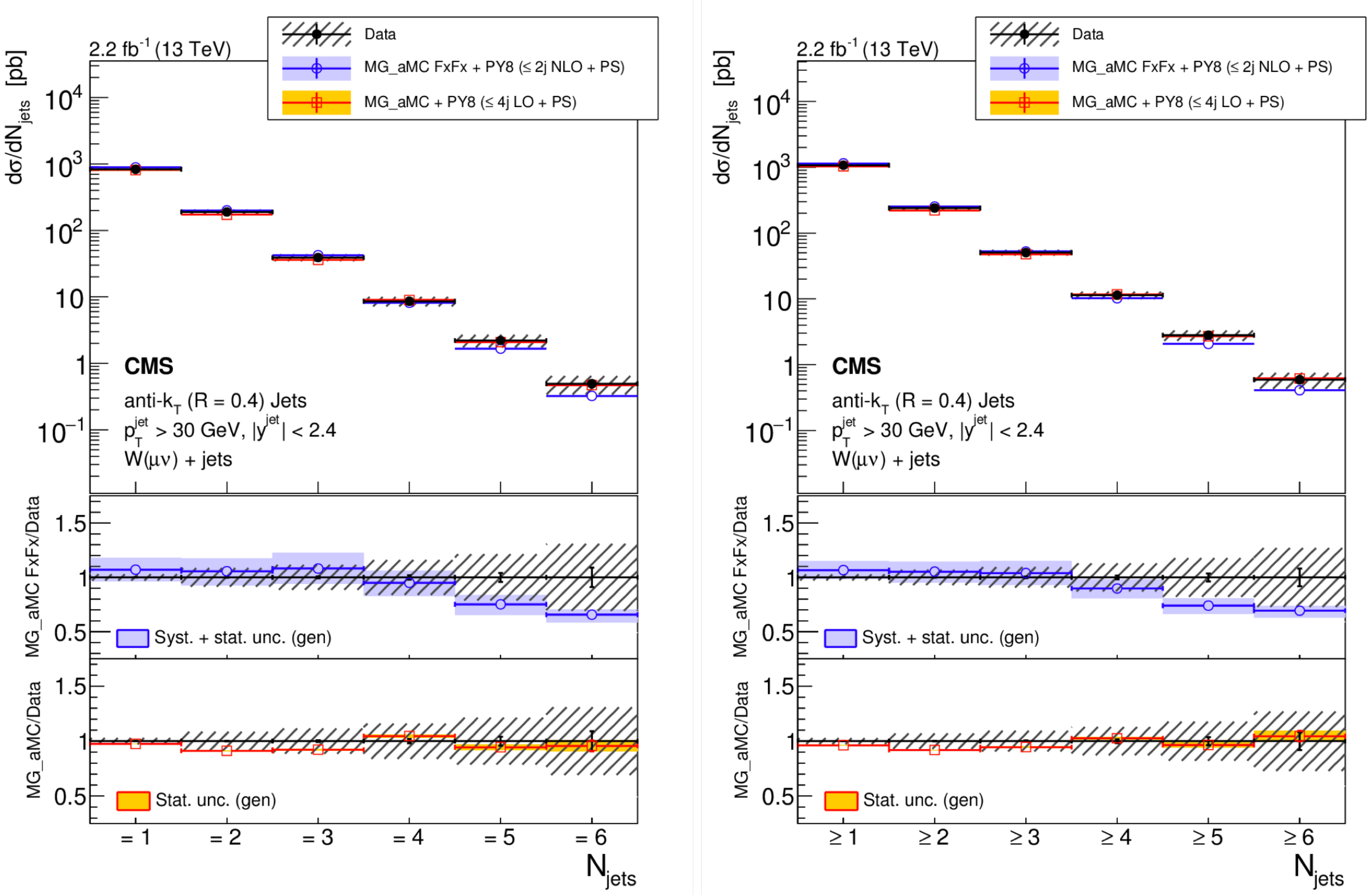


W+Jets production

- The W-jets process requires complex calculations involving NLO, resummation, parton showers etc
- Differential distributions allow to further probe the SM predictions



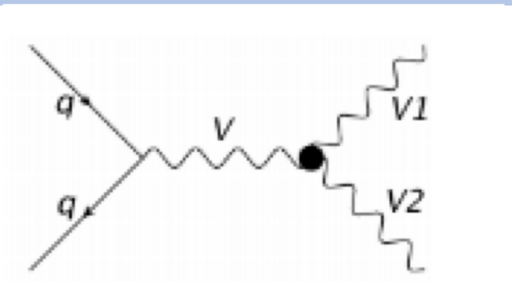




Differential production up to ≥ 7 jets is well produced by MC

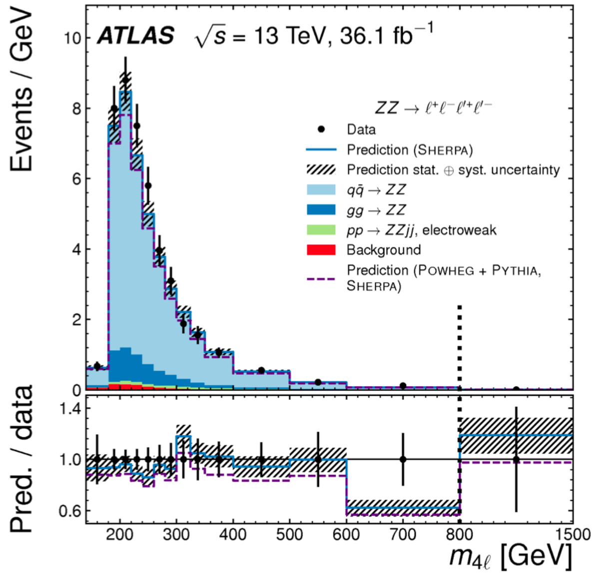
Diboson Production

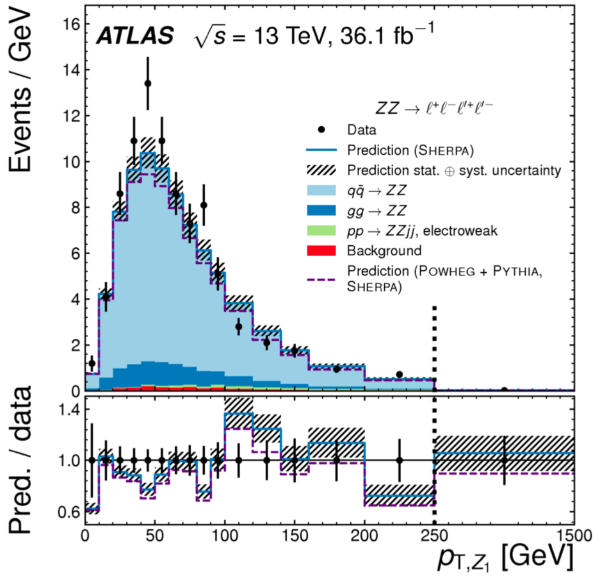
- Diboson production allows us to investigate the non-Abelian nature of SU(2)
  - Non-Abelian: the force carriers can interact with each other

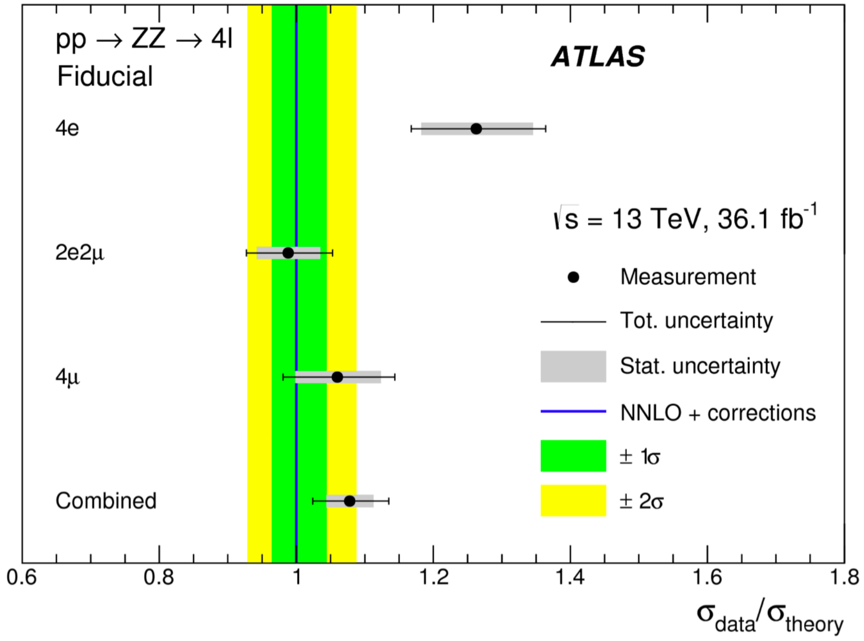


- Is also a very important background in Higgs searches
  (ZZ, WW)

- Multiple channels are investigated (only considering here ZZ → 4l)







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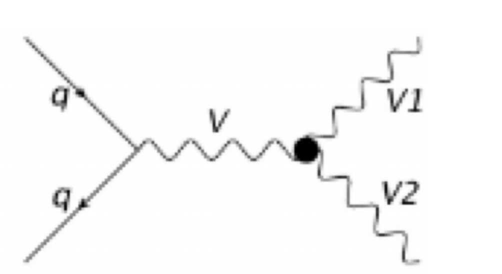

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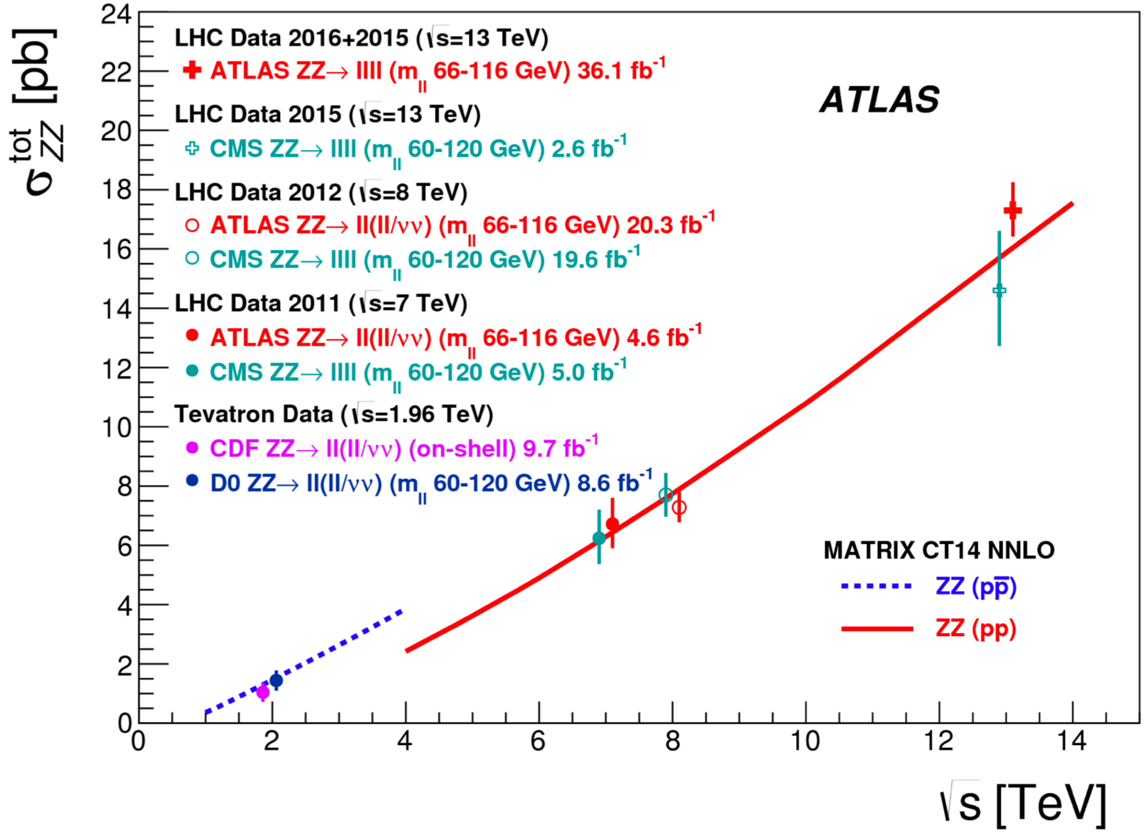
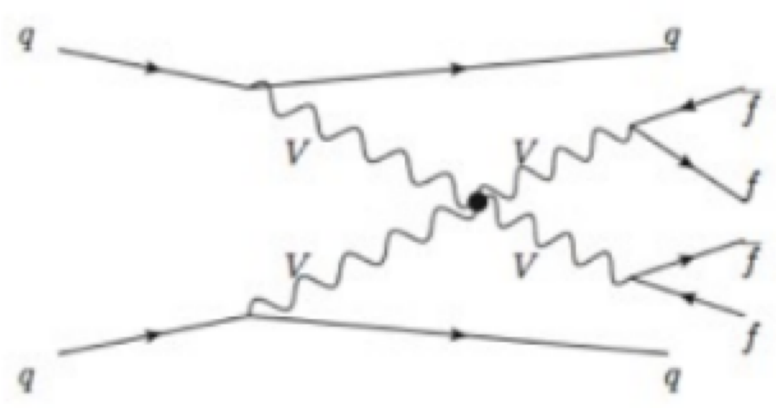


- ring here ZZ → 4l)

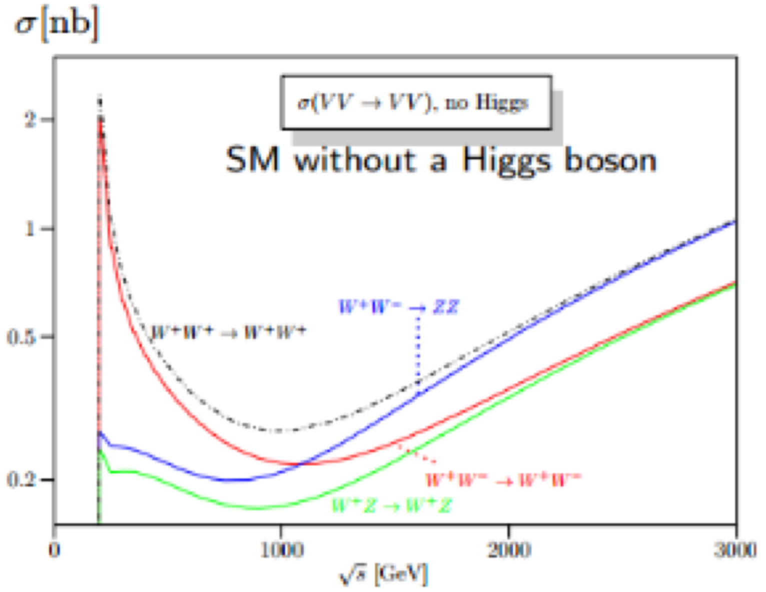
- We can also check that our predictions scale with the E_{cms} as they should

Vector boson scattering

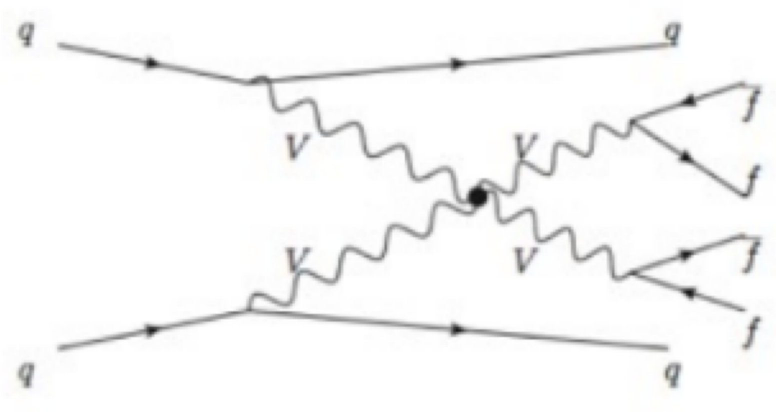


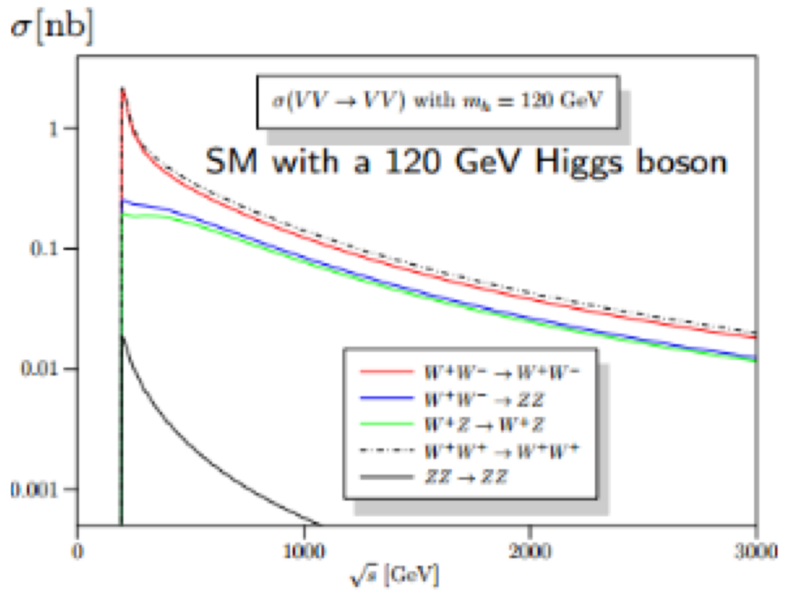
- Vector boson scattering is essentially using the LHC beam to provide a source of W/Z bosons (which then scatter)

- The cross section for this can be calculated using EWK theory… and it's divergent wrt E_{cms}



Vector boson scattering





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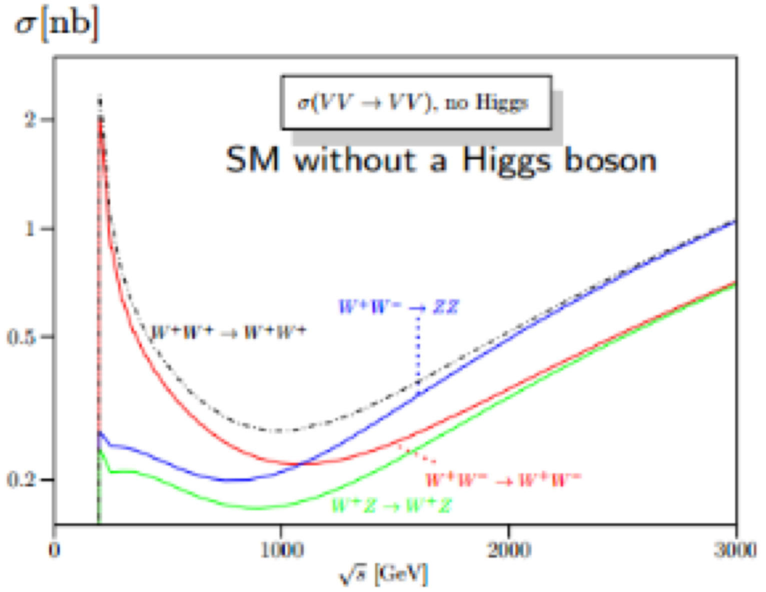


- Clear suggestion that the Higgs **should** exist (contributes extra diagrams to prevent the divergence)

Anomalous Gauge Couplings

- In the same manner that Fermi used an effective field theory (EFT) to model the neutron decay, we can do something similar to probe new physics (via EW), Λ is the energy scale of the new physics

$$\mathcal{L}_{eff} = \sum_n \frac{1}{\Lambda^n} \sum_i \alpha_i^{(n)} \mathcal{O}_i^{(n)}$$

$\alpha_i^{(n)}$ - coupling coefficients

$\mathcal{O}_i^{(n)}$ - operators of dimension mass^{4+n}

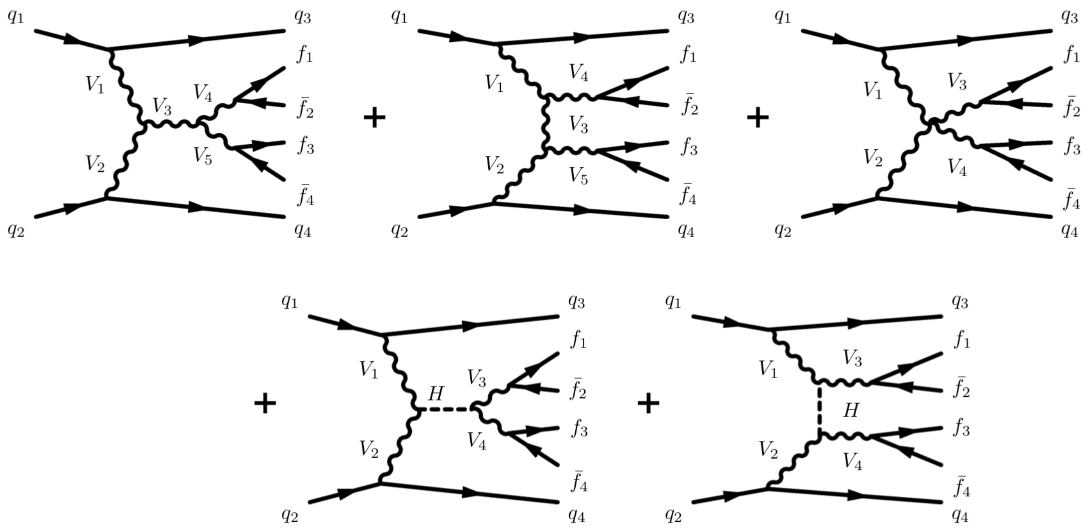
- We can produce an effective Lagrangian that introduces anomalous gauge couplings which are forbidden in the SM

$$\mathcal{L} = -ig_{WWV}[g_1^V(W^\dagger_{\mu\nu}W^\mu - W^{\dagger\mu}W_{\mu\nu})V^\nu + \kappa^V W^\dagger_\mu W_\nu V^{\mu\nu} + \frac{\lambda^V}{m_W^2} W^\dagger_{\rho\mu} W^\mu_\nu V^{\nu\rho}]$$

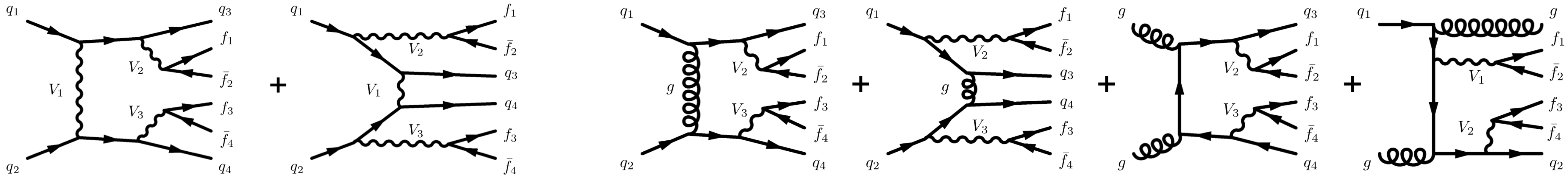
$$\mathbf{SM: g_1^V = \kappa_V = 1; \lambda_V = 0}$$

Anomalous Gauge Couplings

- Lots of processes to calculate to investigate this (EWK VBS):

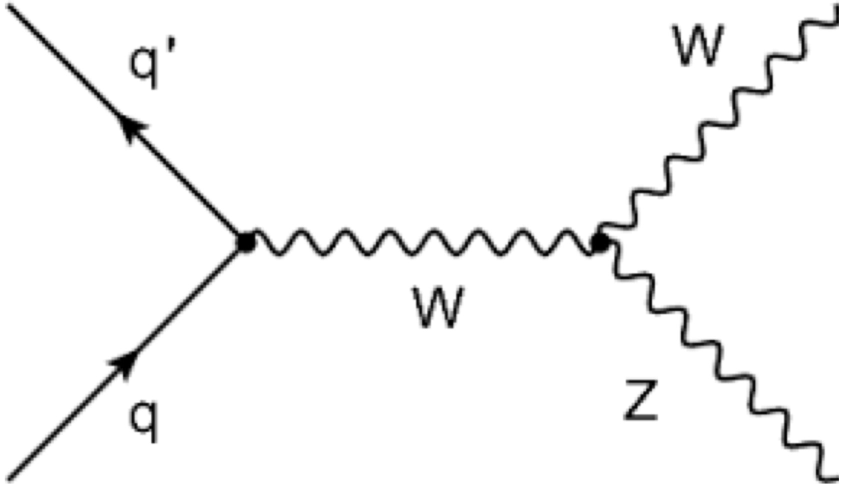


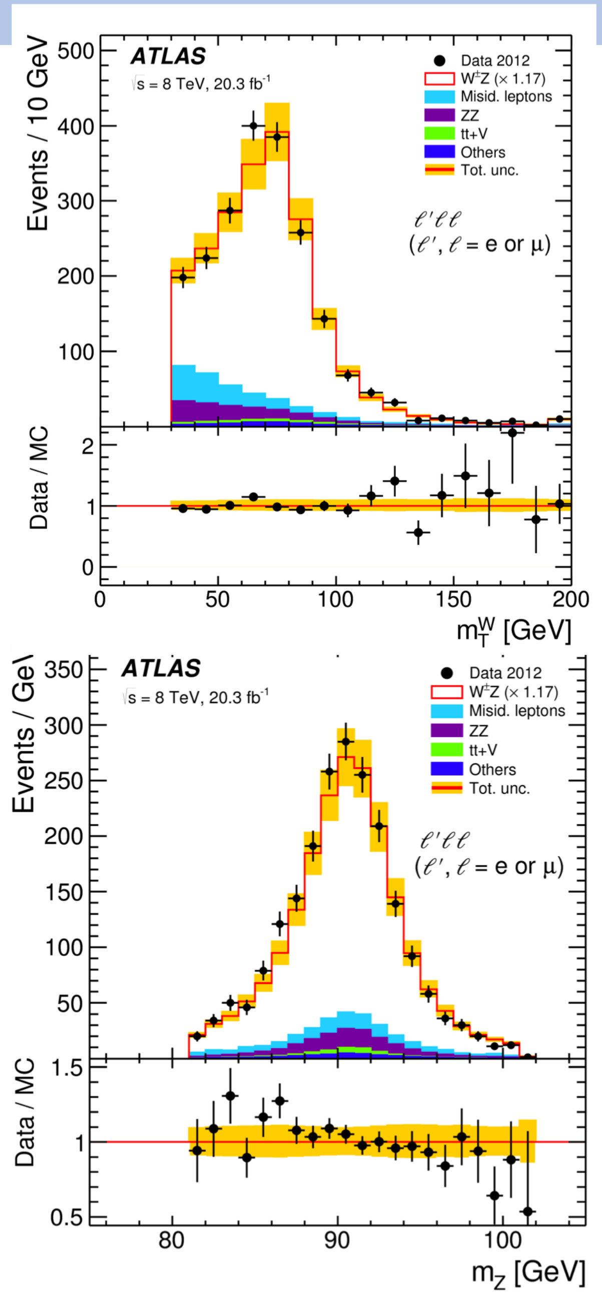
- And also a large contribution from processes that aren't EWK VBS diagrams

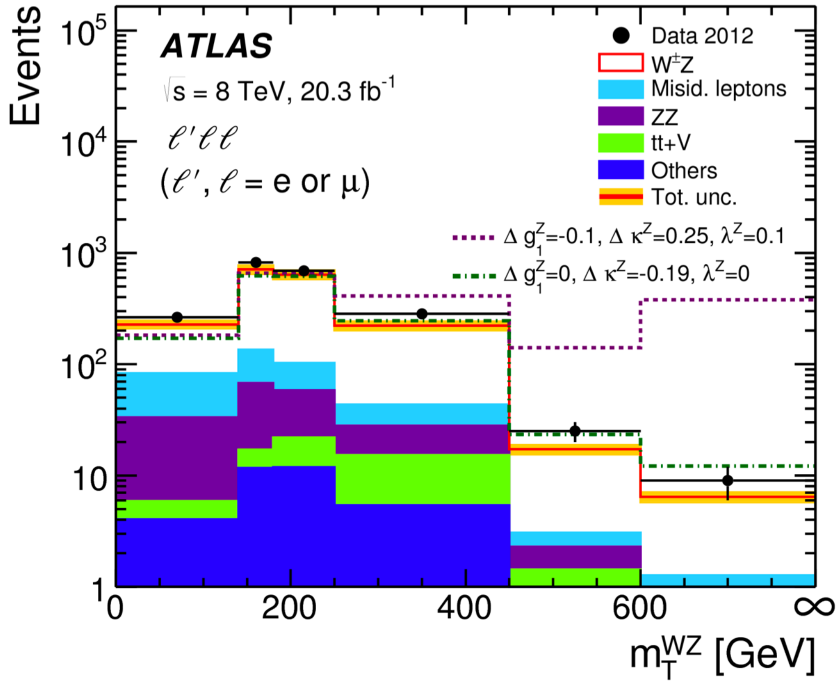


Anomalous Gauge Couplings

- Can be investigated via WZ production



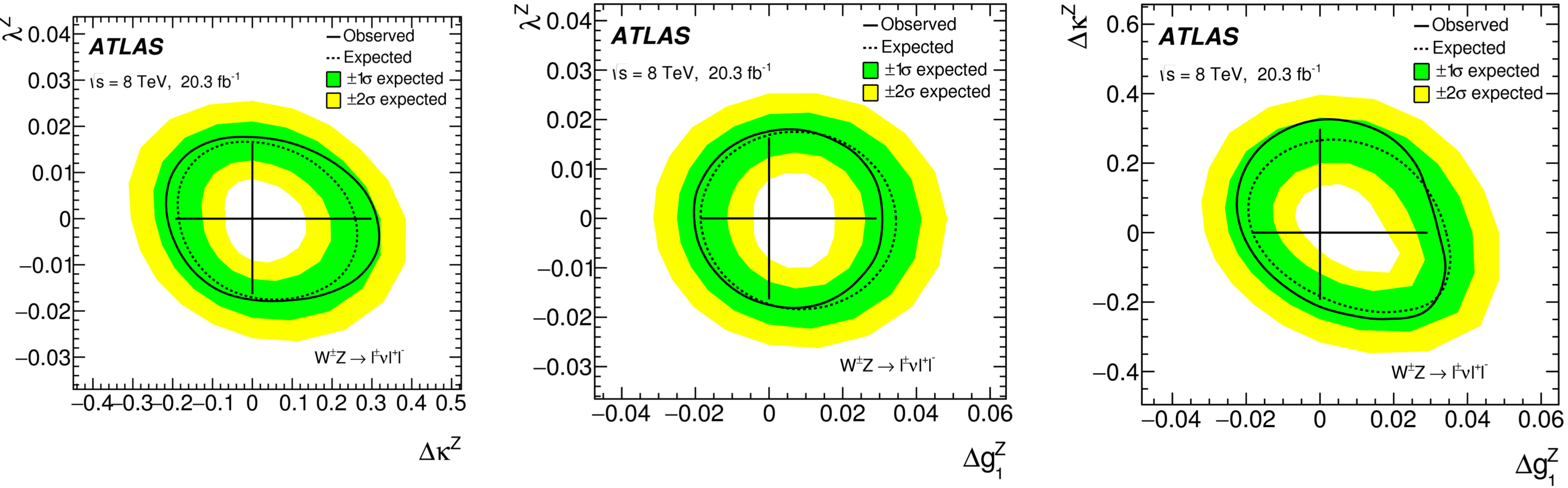




Anomalous gauge couplings overlaid

Anomalous Gauge Couplings

- No Significant excesses, hence limits on the terms in our EFT are calculated



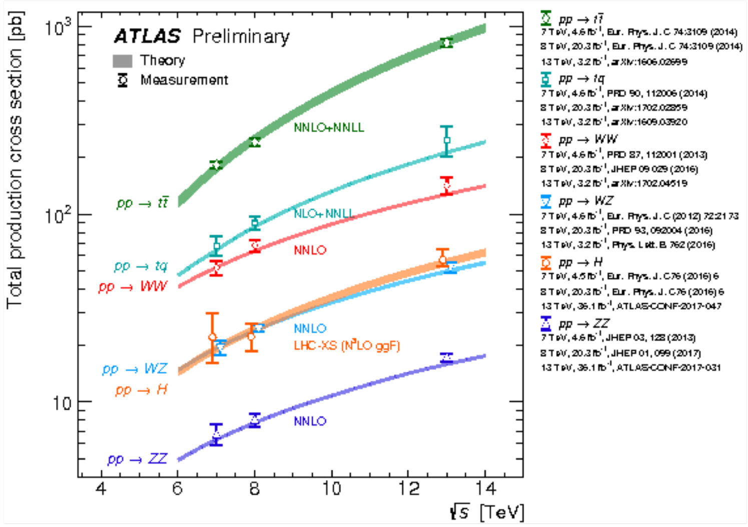
All very much consistent with 0

Lecture 4 - Recap

- Unified EM and Weak into the Electroweak force
- Prediction of the Z-boson, weak neutral current interactions
- Discovery of the Z-boson and W-boson (including their masses)
- Electroweak physics at the LHC

Course - Recap

- By now you should know the basics of the construction of the electroweak theory
  - How we moved from Maxwell's equations, using special relativity and quantum mechanics to the Dirac equation
  - Moving from the Dirac equation, using QFTs to build the QED Lagrangian
  - How to read a Feynman diagram to construct the matrix element (given the Feynman Rules)
  - The introduction of the weak force. The Fermi EFT method, and moving from this to the full weak force description
  - The unification of the EM and Weak force, into the electroweak theory
  - Some LHC physics, and why the EWK force is still very much relevant today.



- Thank you for your attention!!