# Comments on Scaling Formulae for LPA-FEL

# operation

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(M. E. Couprie et al.

**Undulator design for Laser Plasma Based Free electron laser)**

# FEL SCALING FORMULAE

*Few Global parameters account form the FEL performances in SASE-Regime*

$$P(z) = P_b \frac{B(z)}{1 + \frac{P_b}{P_{F,1}} B(z)},$$

$$B(z) = 2\left[\cosh\left(\frac{z}{L_g}\right) - \exp\left(-\frac{z}{2L_g}\right)\cos\left(\frac{\pi}{3} + \frac{\sqrt{3}z}{2L_g}\right) - \exp\left(\frac{z}{2L_g}\right)\cos\left(\frac{\pi}{3} - \frac{\sqrt{3}z}{2L_g}\right)\right]$$

$$P_S \simeq \sqrt{2}\rho P_E$$

$$L_g = \frac{\lambda_u}{4\pi\sqrt{3}\rho} \longrightarrow$$

$$L_{g,3d}(\chi) = \chi L_g, \quad \chi \simeq 1 + \frac{0.185\sqrt{3}}{2}\tilde{\mu}_\epsilon^2$$

$$\tilde{\mu}_\epsilon = 2\frac{\sigma_\epsilon}{\rho}, \quad \sigma_\epsilon = \frac{\Delta E}{E}, \quad \Delta E = \sqrt{\langle E^2\rangle - \langle E\rangle^2}$$

$$\mu_D = \frac{\lambda\lambda_u}{(4\pi\sigma_T)^2\rho}, \quad \rho_D = \frac{\rho}{\sqrt[3]{1+\mu_D}}$$

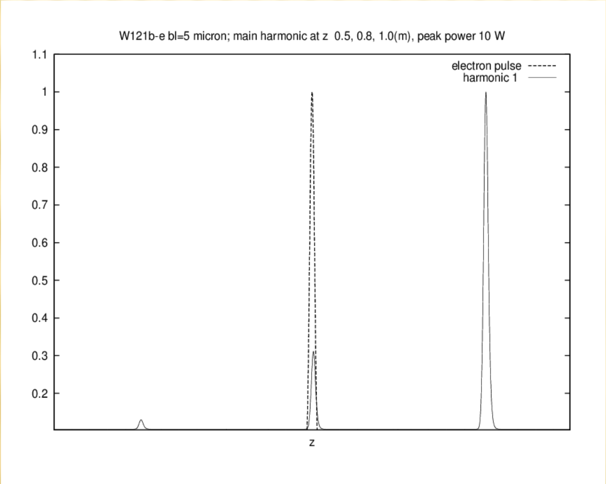
# SHORT BUNCH EFFECT

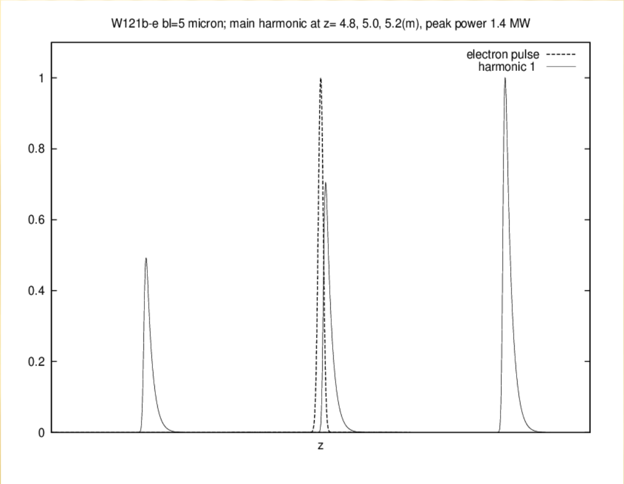
**Coherence length**

$$l_c = \frac{\lambda}{4\pi\sqrt{3}\rho}$$

**Bunch length**

$$\sigma_z >> \frac{\lambda}{\rho}$$



# GAIN DILUTION EFFECTS

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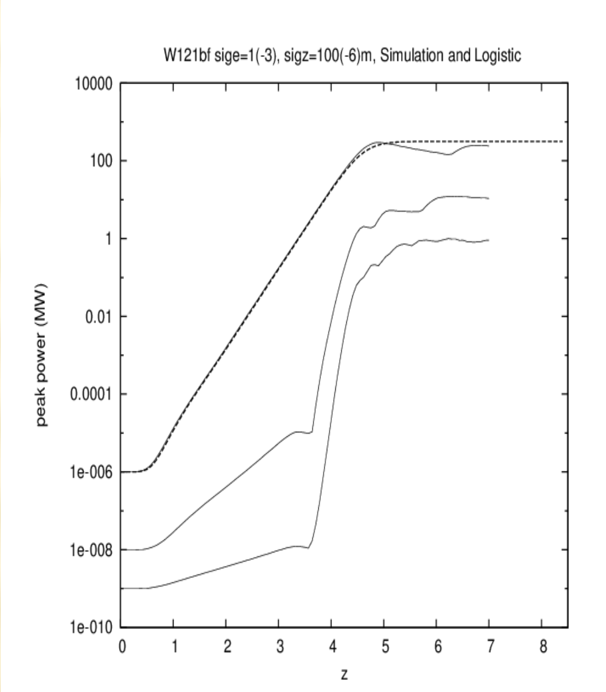


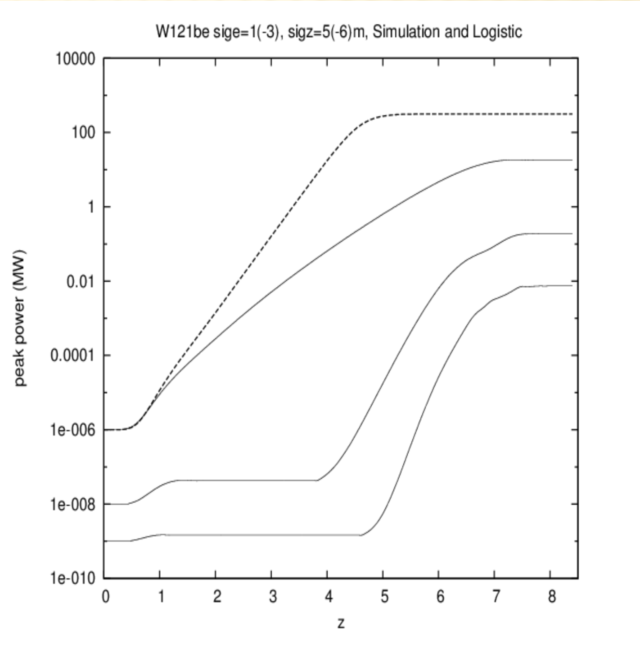

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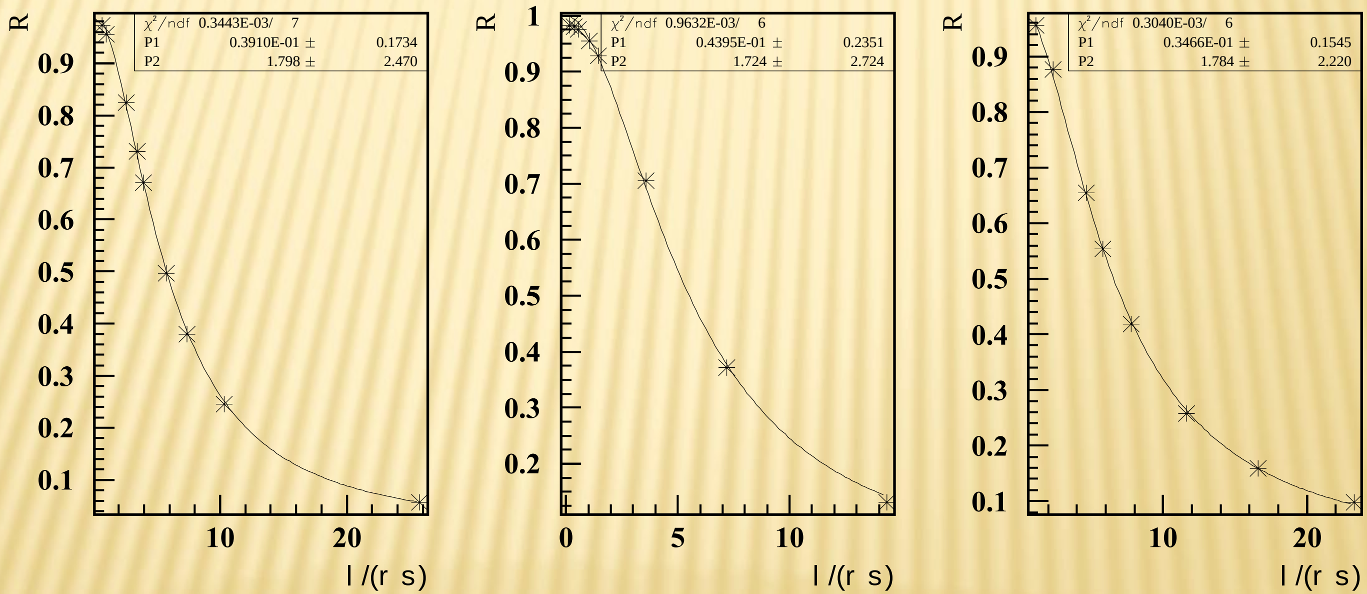
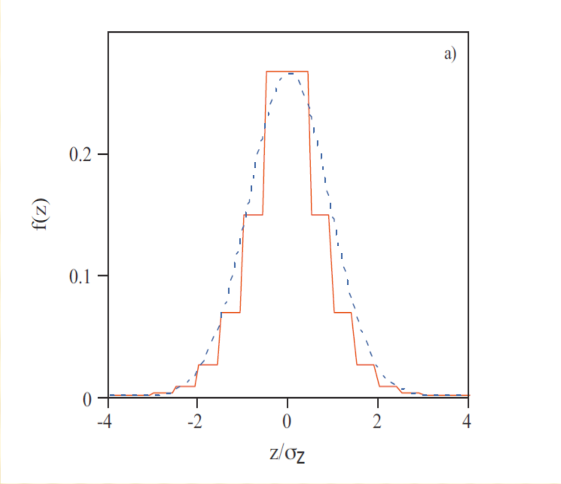
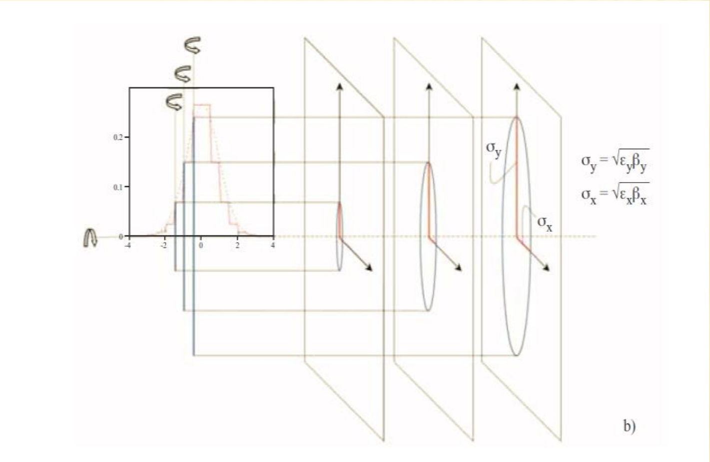
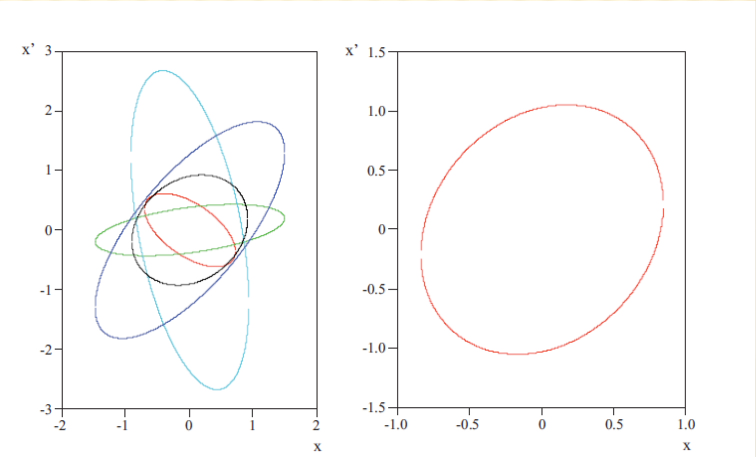
$$R(l_c, \tilde{\mu}_z) = \frac{1}{1+\frac{\alpha_1}{1+\tilde{\mu}_z^2}\left(4\pi\sqrt{3}\, l_c\right)^{\alpha_2(1+\tilde{\mu}_z^2)}}$$



# SLICE EMITTANCES



# SLICE EMITTANCE POWER GROWTH

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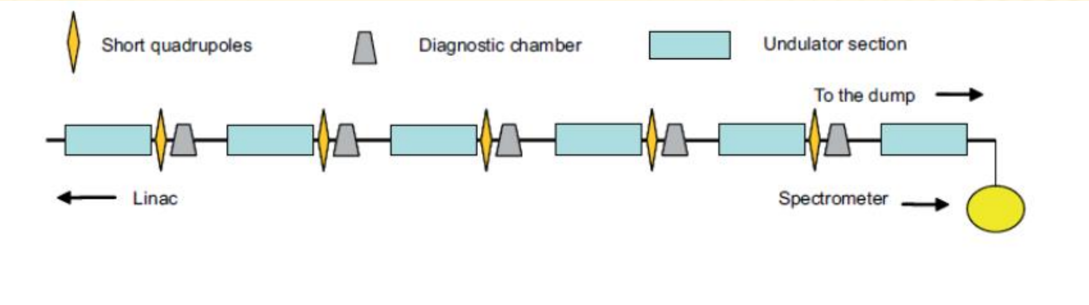


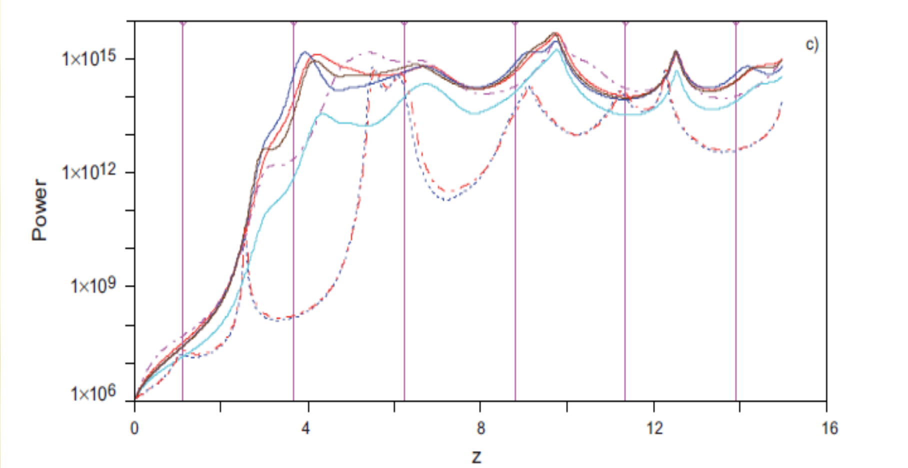

# COLLECTIVE DIVERGENCE

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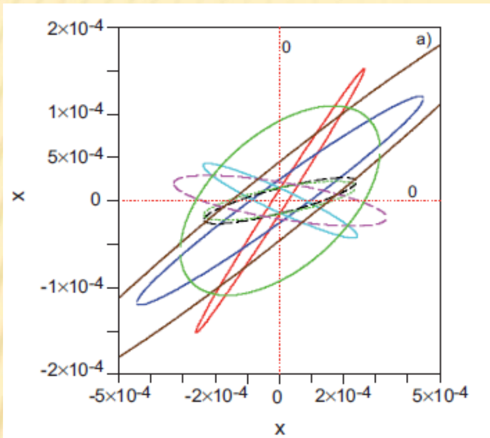
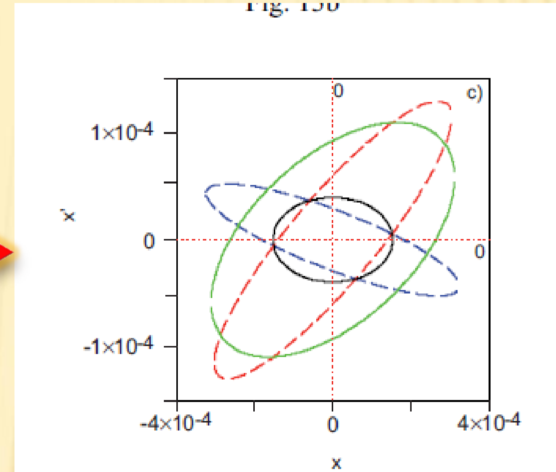
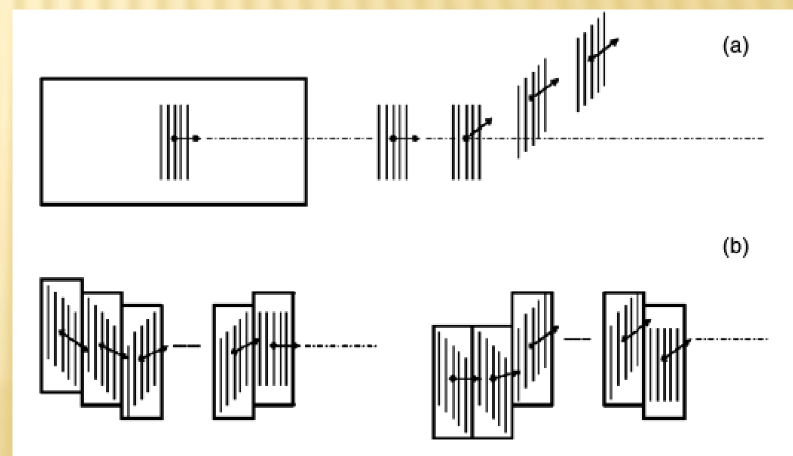



$$\tilde{\mu}_{coll} \cong \frac{\left\langle (\gamma\, x')^2 \right\rangle}{2\pi\,\rho}$$

$$L_{g,coll} = L_g\left(1 + \varsigma\, \tilde{\mu}_{coll}^2\right)$$



•The effective accelerating length is

# LPA-PARAMETERS

$$\Delta E_{Acc}\left[\frac{GeV}{m}\right] \cong 96\sqrt{\frac{n_0\left[cm^{-3}\right]}{10^{18}}},$$

$$n_0\left[cm^{-3}\right] \equiv \text{plasma density}$$

$$\Delta E\left[GeV\right] \cong 3.23 \cdot 10^{-3}\left(\frac{\omega_l}{\omega_p}\right)^2$$

$$L_{eff} \cong \gamma_p^2 \lambda_p [m]$$

$$\lambda_p [m] \cong 33.7 \cdot 10^{-6}\sqrt{\frac{10^{18}}{n_0\left[cm^{-3}\right]}},$$

$$\lambda_p \equiv \text{plasma wavelength}$$

$$\omega_\mathrm{p}[Hz] \equiv 5.6 \cdot 10^{13}\sqrt{\frac{n_0[cm^{-3}]}{10^{18}}}$$

$$\omega_\mathrm{p} \equiv \text{plasma frequency}$$

$$\gamma_p = \frac{1}{\sqrt{1-\beta_p^2}} = \frac{\omega_l}{\omega_p}$$

$$\beta_p = \left(1-\left(\frac{\omega_p}{\omega_l}\right)^2\right)^{\frac{1}{2}}$$

$$\omega_l \equiv \text{laser frequency}$$

# LPA....

## *Accelerated bunch charge*

$$Q_b[C] \cong 7.34 \cdot 10^{-3} \frac{(k_p \sigma_r)^2}{\sqrt{n_0[cm^{-3}]}}$$

## *Bunch Length*

$$\tau_b \cong \frac{\lambda_p}{2c}$$

## *Accelerated Bunch current*

$$I_b[A] \cong 5.16 \cdot 10^3 \left( \frac{\sigma_r}{\lambda_p} \right)^2$$

# POUR MAN COMPUTATION OF LPA INDUCED ENERGY SPREAD EFFECT

$$\Delta E[GeV] \cong 3.23 \cdot 10^{-3} \left( \frac{\omega_l}{\omega_p} \right)^2$$

$$\sigma_\varepsilon \cong \sqrt{2} \sqrt{\left( \frac{\Delta\omega_l}{\omega_l} \right)^2 + \left( \frac{\Delta\omega_p}{\omega_p} \right)^2} \cong \sqrt{2} \left( \frac{\Delta\omega_p}{\omega_p} \right) = \frac{1}{\sqrt{2}} \frac{\Delta n_0}{n_0}$$

$$\left( \frac{\Delta\omega_l}{\omega_l} \right) << \left( \frac{\Delta\omega_p}{\omega_p} \right), \omega_p \propto \sqrt{n_0}$$

$$\rho \cong \frac{9.272 \cdot 10^{-3}}{\gamma} \left( \frac{\sigma_r}{\lambda_p \sigma_T} \right)^{2/3},$$

$$K \cong 1.4, \lambda_u = 2.4\, cm \qquad \sigma_\varepsilon < 2\rho \qquad \frac{\Delta n}{n} < \frac{6.5 \cdot 10^{-3}}{\gamma} \left( \frac{\sigma_r}{\lambda_p \sigma_T} \right)^{2/3}$$

- Furthermore if we set

$$\gamma \cong 6.3\left(\frac{\lambda_p}{\lambda_l}\right)^2$$

$$\left(\frac{\sigma_r}{\sigma_T}\right) > 3.16\cdot 10^4 \frac{\lambda_p^4}{\lambda_l^3}\left(\frac{\Delta n}{n}\right)^{\frac{3}{2}}$$

# THAT'S NOT THE END OF THE STORY

- Laser Intensity

$$a_0 \cong 0.855\lambda_l[\mu m]\sqrt{\frac{I_l[W\cdot cm^{-2}]}{I_0}}$$

$$I_0 = 10^{18} W\cdot cm^{-2}$$

$$\Delta E^* \cong \frac{a_0^2}{\sqrt{1+\frac{a_0^2}{2}}}\Delta E \longrightarrow a_0^2 < 1 \longrightarrow \sigma_\varepsilon \cong \sqrt{\frac{1}{2}\left(\frac{\Delta n_0}{n_0}\right)^2 + 4\left(\frac{\Delta a_0}{a_0}\right)^2}$$

# …AND MORE

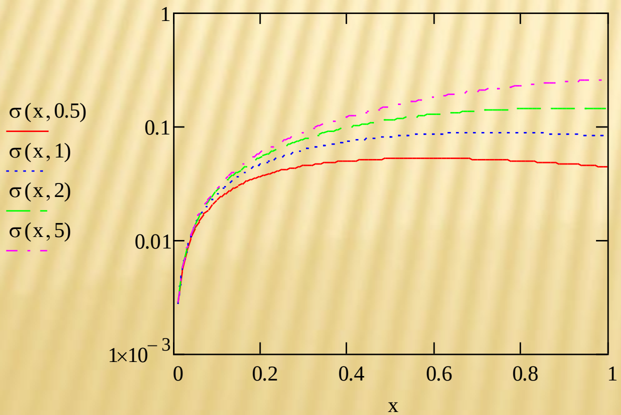
- **INCLUSION OF BUNCH LENGTH**

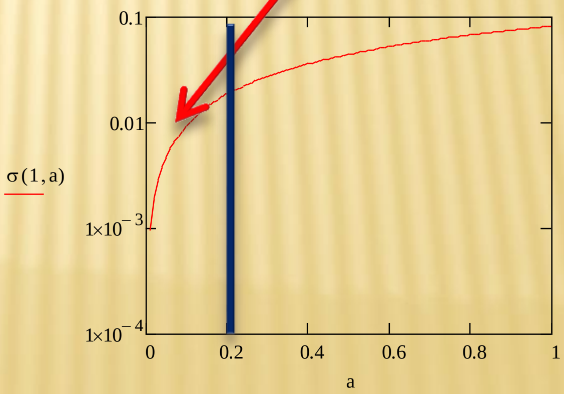
$$\sigma_\varepsilon(x) = \frac{a_0}{\sqrt{3}} \frac{(x-2)\,x}{6x + a_0(x-2)^2}$$

$$x = \frac{\sigma_b}{R},$$

$$R = bubble\ radius \cong L_{eff}$$

$$a_0 < \frac{6\sqrt{3}\rho}{2-\sqrt{3}\rho}$$

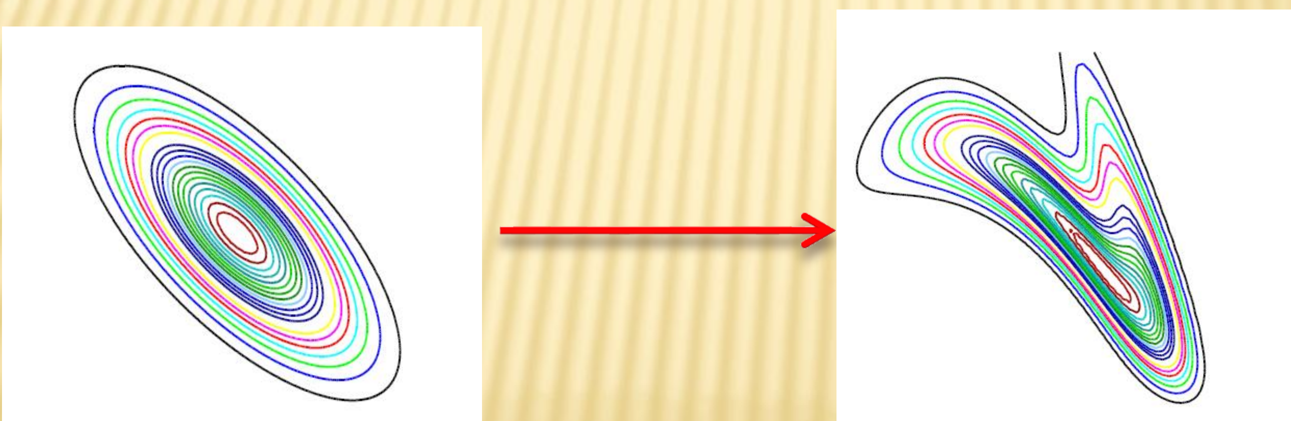


# WORK IN PROGRESS

- Longitudinal phase space evolution

$$H(\zeta, p_z) = \sqrt{\gamma_\perp^2 + p_z^2} - \beta_\phi p_z - \phi(\zeta)$$



# HOWEVER

- Toward low energy spread in plasma accelerators in quasilinear regime
- Xiangkun Li,* Phu Anh Phi Nghiem, and Alban Mosnier
- PHYSICAL REVIEW ACCELERATORS AND BEAMS 21, 111301 (2018)
- As a study case, 3D
- simulations for the 5 GeV laser-plasma acceleration stage of the European Plasma Research Accelerator
- with eXcellence in Applications project have been performed. Careful optimization of the parameters
- allows one to obtain an energy spread of ≤1% and a slice energy spread of ≤0.1%, with good agreement
- between theories and simulations.