The Nelson-Seiberg theorem, its extensions, string realizations, and possible machine learning applications

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Work in progress (James Brister, Zhengyi Li, ZS)

Outline

SUSY and SUSY breaking

R-symmetries, the Nelson-Seiberg theorem and revisions

String models

Extension to Non-Abelian R-symmetries

Possible machine learning applications

Summary

Supersymmetry (SUSY)

What is SUSY?

- Poincare $\rightarrow$ super-Poincare algebra, representations $\rightarrow$ fields.
- e.g., the chiral multiplet $\Phi = (\phi, \psi, F)$:

$$
\begin{aligned}
\text{the scalar boson } \phi, \quad & \delta_\xi \phi = \sqrt{2}\xi\psi, \\
\text{the fermion } \psi, \quad & \delta_\xi \psi = -i\sqrt{2}\sigma^\mu \bar{\xi}\partial_\mu \phi - \sqrt{2}\xi F, \\
\text{the auxiliary field } F, \quad & \delta_\xi F = -i\sqrt{2}\partial_\mu \psi \sigma^\mu \bar{\xi}.
\end{aligned}
$$

- Fermionic coordinates $\theta, \bar{\theta} \rightarrow$ superfields, e.g. the chiral fields:

$$\Phi(x^\mu, \theta, \bar{\theta}) = \phi + \sqrt{2}\theta\psi - \theta\theta F + \cdots .$$

- And the vector multiplet $V = (A_\mu, \lambda, \bar{\lambda}, D)$, etc.
- SUSY invariant action ($N = 1$, $d = 4$) $\rightarrow$ phenomenology.
- SUSY breaking in the hidden sector, mediation (gauge, gravity, anomaly) to the supersymmetric Standard Model.

SUSY breaking models

F-term SUSY breaking (Wess-Zumino or O'Raifeartaigh)

- The (holomorphic) superpotential $W(\Phi_i)$.
- The (real) Kähler potential $K(\bar{\Phi}_i, \Phi_j)$ (minimal: $\bar{\Phi}_i \Phi_i$).
- $S_W = \int d^4x d^2\theta W$ in the action $\Rightarrow$ the scalar potential:

$$V = K^{\bar{i}j} \bar{F}_i F_j, \quad \text{where } F_i = \partial_i W, \quad K^{\bar{i}j} = (\partial_{\bar{i}} \partial_j K)^{-1}.$$

- Spontaneous SUSY breaking $\Leftrightarrow$ $V > 0$ at the vacuum $\Leftrightarrow$ $F \neq 0$ at the vacuum, or no solution to $\partial_i W = 0$.

Extension to local SUSY: Supergravity (SUGRA)

- The scalar potential:

$$V = e^K (K^{\bar{i}j} \bar{F}_i F_j - 3 \bar{W} W), \quad \text{where } F_i = D_i W = \partial_i W + W \partial_i K.$$

- SUSY breaking $\Leftrightarrow$ $F \neq 0$. V: the vacuum energy density.

The pseudomodulous

Where is the pseudomodulus?

- With $K(\bar{\Phi}_i, \Phi_j) = \bar{\Phi}_i \Phi_i$, F-term SUSY breaking always gives a pseudomodulus, i.e., a flat direction at tree level. (Ray, 2006)
- The pseudomodulus $X \sim$ the goldstino $\psi \sim$ the F-term $F \neq 0$.
- Loop corrections may lift up the pseudomodulus.

Implications

- SUSY breaking $\Rightarrow$ a light field exists.
- W needs fine tuning, otherwise the "vacuum" is unstable.
- Non-minimal K or SUGRA may relax the fine tuning to a small region $\sim M_S^2/M_K^2$ or M_S^2/M_P^2.
- SUSY breaking is rare even considering metastable SUSY breaking. R-symmetries are needed. (Sun, 2011)

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- R-symmetries, the Nelson-Seiberg theorem and revisions
- String models
- Extension to Non-Abelian R-symmetries
- Possible machine learning applications
- Summary

R-symmetries

What is an R-symmetry?

- A $U(1)$ symmetry transforming supercharges.

$$\theta \to e^{i\alpha}\theta, \quad \phi_i \to e^{ir_i\alpha}\phi_i, \quad W \to e^{2i\alpha}W, \quad \alpha \in \mathbb{R}.$$

- Important for SUSY breaking and phenomenology.

The Nelson-Seiberg theorem

- Generically, F-type SUSY breaking at the global minimum $\Rightarrow$ W has an R-symmetry, $\Leftarrow$ the R-symmetry is spontaneously broken. (Nelson, Seiberg, 1993)
- "Generic" means that a small change of parameters does not destroy the vacuum, so no fine-tuning is needed.
- Metastable SUSY breaking at local minima needs approximate R-symmetries. (Intriligator, Seiberg, Shih, 2007)

Proof of the Nelson-Seiberg theorem

Proof.

- Solving $\partial_i W = 0 \Leftarrow N_{\text{equations}} = N_{\text{variables}}$.
- With R, the form of W is constrained (with field redefinition):

$$W = xf(y_1, \ldots, y_{d-1}), \quad x = \phi_d^{2/r_d}, \quad y_i = \phi_i / \phi_d^{r_i/r_d}.$$

- If $x \neq 0$ then solving $f = 0$, $\partial_{y_i} f = 0 \not\Leftarrow N_{\text{equations}} > N_{\text{variables}}$.
- $x = 0$ singular, existence of solutions is unclear.
- Notice $r_d \neq 0$, $x \neq 0$ breaks the R-symmetry.
- So SUSY breaking $\Rightarrow$ R, $\Leftarrow$ spontaneously broken R. □

Issues in the proof

- Field redefinition singular at $x = 0 \Rightarrow$ existence of solutions at $x = 0$ is unknown $\Rightarrow$ no "$\Leftrightarrow$" condition is proved.

A "$\Leftrightarrow$" condition for SUSY breaking

Proof.

- Without field redefinitions, W has the general form:

$$W = X_i f_i(Y_j) + W_1,$$
$$W_1 = \mu_{ijk} X_i X_j A_k + \nu_{ijk} X_i A_j A_k + A_i A_j (\kappa_{ij} + \lambda_{ijk} Y_k + \xi_{ijk} A_k),$$
$$r(X_i) = r(W)(=2), \quad r(Y_j) = 0, \quad r(A_k) \neq r(W) \text{ or } 0.$$

- SUSY $\Leftarrow f_i(Y_j) = 0$ and $X_i = A_k = 0$.
- $N_Y \geq N_X \Rightarrow$ SUSY and $W = 0$ (one should check all possible R-charge assignments to see whether a SUSY vacuum exists)
- $N_Y < N_X$ and $X = A = 0 \Rightarrow$ SUSY breaking.
- $X \neq 0$ or $A \neq 0 \Rightarrow$ R breaking $\Rightarrow$ SUSY breaking (by N-S).
- Without R $\Rightarrow$ SUSY (by N-S).
- So SUSY breaking $\Leftrightarrow$ R and $N_Y < N_X$. □

Results

The Nelson-Seiberg theorem revised

- F-type SUSY breaking at the global minimum $\Leftrightarrow$ W has an R-symmetry and $N_Y < N_X$ for any consistent R-charge assignment. (Kang, Li, ZS, 2012)
- W needs to be a generic polynomial.
- A's are needed for R-breaking in phenomenology.

R-symmetric SUSY vacua

- R-symmetries and $N_Y \geq N_X \Rightarrow$ SUSY with $W = 0 \Rightarrow$ $D_i W = 0$ and $V = 0$ in SUGRA. (ZS, 2011)
- The proof is valid for discrete $\mathbb{Z}_{n\geq 3} < U(1)$ R-symmetries.
- R-symmetries survive at vacua with $X = A = 0$ and $N_Y - N_X$ perturbatively flat directions.
- Realizations in string models.

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R-symmetries in flux compactification

General plot

- Quantum gravity do not allow continuous global symmetries, but discrete symmetries are common.
- IIB String compactification on a Calabi-Yau (CY) 3-fold $\to$ $N = 1$, $d = 4$ SUGRA, fluxes $\to$ W for moduli stabilization:

$$W = \int_{M_6} (F_{(3)} - \phi H_{(3)}) \wedge \Omega_{(3)}, \quad \phi \text{ is the axiodilaton.}$$

- Dual picture: F-theory on CY 4-fold.
- CY with geometrical symmetries $\to$ R, fluxes respecting R $\to$ W, if $N_Y \geq N_X \Rightarrow$ SUSY with $W = 0$ at the string scale.
- Assuming generic fluxes, no calculation for the landscape.
- Non-perturbative effects $\to$ low scale SUSY breaking and $V \gtrsim 0$, e.g., racetrack (no KKLT or large volume scenario).

An orbifold compactification example

IIB on a $T^6/\mathbb{Z}_2$ orientifold

- T^6 coordinates $z^i = x^i + \tau_{ij} y^j$, $i = 1, 2, 3$, τ_{ij}'s are the complex structure, $x^i \sim x^i + 1$, $y^i \sim y^i + 1$.
- Orientifold with $\mathbb{Z}_2$: $x^i \to -x^i, y^i \to -y^i, i = 1, 2, 3$.
- $\mathbb{Z}_4$ symmetry: $x^1 \to y^1, y^1 \to -x^1$.
- $\tau = \begin{pmatrix} i+\tau'_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & i+\tau'_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & i+\tau'_{33} \end{pmatrix}$ expanded from a $\mathbb{Z}_4$ symmetric point.
- $X : \tau_{12}, \tau_{13}, \tau_{21}, \tau_{31}$; $Y : \tau'_{22}, \tau_{23}, \tau_{32}, \tau'_{33}, \phi$; $A : \tau'_{11}$; $N_Y > N_X$.
- Calculation: Turn on invariant fluxes a_{12}, a_{13}, a_{21}, a_{31}, etc.,

$$\begin{aligned} W &= (a_{12} - \phi c_{12})(\operatorname{cof} \tau)_{12} + (b_{12} - \phi d_{12})\tau_{12} + \{12 \leftrightarrow 13, 21, 31\} \\ &= \tau_{12} f_{12}(\tau_{22}, \tau_{23}, \tau_{32}, \tau_{33}, \phi) + \{12 \leftrightarrow 13, 21, 31\}. \end{aligned}$$

- $\tau_{12} = \cdots = 0$, solving $f_{12} = \cdots = 0 \Rightarrow$ SUSY with $W = 0$.

(Dine, ZS, 2005)

A CY compactification example

IIB on a quintic hypersurface in $\mathbb{CP}^4$

- $T^6/\mathbb{Z}_2$ has trivial holonomy except at the singular point.
- Smooth CY has full SU(3) holonomy, e.g., the quintic hypersurface in $\mathbb{CP}^4$: $P = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$.
- 1 axiodilaton, 101 complex structures (monomial deformations of P), W transforms like $z_1 z_2 z_3 z_4 z_5$.
- $\mathbb{Z}_5$ symmetry: $z_1 \to e^{2\pi i/5} z_1$.
- 31 X's (e.g. $z_1 z_2 z_3^3$), 40 Y's (e.g. $z_2^2 z_3^3$), 31 A's (e.g. $z_1^2 z_3^3$), $N_Y > N_X \Rightarrow$ SUSY with $W = 0$.
- This provides a tool to survey SUSY vacua in a large set of CY compactification models. No need for detailed calculation while studying statistics of the landscape or pre-selecting models before detailed calculation.
- Orientifolding is omitted, and should be imposed.

Outline

Non-Abelian R-symmetries

Transformation of fields

- For a non-Abelian R-symmetry G, θ still transforms as a 1-dimensional representations (singlet) of the R-symmetry.
- Fields may transform as higher dimensional representations, i.e., $\phi \to \Gamma^{\phi}(g)\phi$ where $\Gamma^{\phi}(g)$ is the representation matrix.

Building up W

- W transforms like θ^2. Fields must combine to singlets for all terms of W.
- Singlets must appear in the Clebsch-Gordan series of the tensor product of Γ^{ϕ}'s whose ϕ's appear in one term of W.
- e.g., $W = c_{ij}\phi_i^{(1)}\phi_j^{(2)} + \cdots \Rightarrow \Gamma^{(1)} \otimes \Gamma^{(2)} = \mathbf{1_2} \oplus \cdots$.

Possible degenerated Abelian R-symmetries

Non-Abelian continuous R-symmetries are trivial

- $\Gamma^\theta : G \to U(1) \Rightarrow G/\,\mathrm{Ker}(\Gamma^\theta) \cong U(1)$.
- Levi decomposition: A compact Lie algebra $\mathfrak{L} = \mathfrak{L}_S \oplus n\,\mathfrak{u}(1)$, and the semi-simple $\mathfrak{L}_S = \bigoplus_i \mathfrak{L}_i$ where $\mathfrak{L}_i$'s are simple.
- One combination of $U(1)$'s rotates θ, and can be identified as $U(1)^{(\mathrm{R})}$. The rest are just flavor symmetries.
- The Nelson-Seiberg theorem and previous extensions are unchanged if we consider the $U(1)^{(\mathrm{R})}$ subgroup.

Non-Abelian discrete R-symmetries

- $\Gamma^\theta : G \to \mathbb{Z}_n \Rightarrow G/\,\mathrm{Ker}(\Gamma^\theta) \cong \mathbb{Z}_n$.
- Non-Abelian discrete R-symmetries may have no degenerated $\mathbb{Z}_n^{(\mathrm{R})}$ R-symmetry. (Chen, Ratz, Trautner, 2013)

R-charges and possible effective R-charges

$U(1)$ and $\mathbb{Z}_n$ R-charges

- $\phi \to \Gamma^\phi(g)\phi = e^{ir_\phi \alpha}\phi \Rightarrow r_\phi = \log \Gamma^\phi(g)/\log \Gamma^\theta(g)$.
- Same for discrete R-symmetries $\mathbb{Z}_n$ by assuming $r_\theta = 1$.

Non-Abelian cases

- Analogous to Abelian cases, the proposed effective R-charge is

$$r_\phi = (1/\dim \Gamma^\phi) \log \det \Gamma^\phi(g)/\log \Gamma^\theta(g).$$

- In the (trivial) example $U(N) = PSU(N) \times U(1)$,

$$\Gamma^\phi(g) = U = (\det U)^{1/N} \mathbf{1}_N \cdot U' = (\det U)^{1/N} \otimes U'.$$

  The effective R-charge is the same as the $U(1)$ R-charge.
- Non-Abelian discrete cases may be non-trivial and the effective R-charge may not be consistently defined.

A trivial example

The O'Raifeartaigh model extended

- The original O'Raifeartaigh model:

$$W = fX_1 + gX_1Y^2 + hX_2Y,$$

$N_Y = 1 < N_X = 2 \Rightarrow$ SUSY breaking.

- Extend to an $U(N)$ R-symmetry:

$$W = fX_1 + g_{ij}X_1Y_1^iY_2^j + h_{ij}X_2^iY_2^j + k_{ij}X_3^iY_1^j,$$

where $X_1 \in \mathbf{1_2},\ X_2 \in \mathbf{N_2},\ X_3 \in \bar{\mathbf{N}}_\mathbf{2},\ Y_1 \in \mathbf{N_0},\ Y_2 \in \bar{\mathbf{N}}_\mathbf{0}$.

- $N_Y = 2N < N_X = 2N+1 \Rightarrow$ SUSY breaking.
- This example is trivial since $U(N) = PSU(N) \times U(1)^{(R)}$. The effective R-charges is the same as the U(1) R-charge.
- Non-trivial example must have discrete non-abelian R-symmetries.

A non-trivial discrete non-abelian example

$Dic_3 = Q_{12} = \mathbb{Z}_3 \rtimes \mathbb{Z}_4$

- $\Gamma^\theta : \mathbb{Z}_3 \rtimes \mathbb{Z}_4 \to \mathbb{Z}_4$. Obviously $\mathbb{Z}_4 \ntriangleleft \mathbb{Z}_3 \rtimes \mathbb{Z}_4 \Rightarrow$ no $\mathbb{Z}_4^{(R)}$.
- Irreducible representations of $\mathbb{Z}_3 \rtimes \mathbb{Z}_4$: $\mathbf{1}^{(i=1,\ldots,4)}$, $\mathbf{2}^{(j=1,2)}$.
- Branching rules for $\mathbb{Z}_3 \rtimes \mathbb{Z}_4 \to \mathbb{Z}_4$:

$$\mathbf{1}^{(1)} \to \mathbf{1_0}, \quad \mathbf{1}^{(2)} \to \mathbf{1_1}, \quad \mathbf{1}^{(3)} \to \mathbf{1_2}, \quad \mathbf{1}^{(4)} \to \mathbf{1_3},$$
$$\mathbf{2}^{(1)} \to \mathbf{1_0} \oplus \mathbf{1_2}, \quad \mathbf{2}^{(2)} \to \mathbf{1_1} \oplus \mathbf{1_3}.$$

- Fields combine to singlets for W, but the $\mathbb{Z}_4$ charge depends on contraction details. e.g., $\mathbf{2}^{(2)} \otimes \mathbf{2}^{(2)} = \mathbf{1}^{(1)} \oplus \mathbf{1}^{(3)} \oplus \mathbf{2}^{(1)}$. Effective R-charges can not be defined.
- The proof for R-symmetric SUSY vacua can go through by treating non-singlets as A's, $N_Y > N_X \Rightarrow$ SUSY with $W = 0$.

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Possible machine learning (ML) applications

Motivations

- Construct SUSY models with certain vacuum properties.
- Validity check of previous theorems.
- Counter examples with non-generic W because of renormalizability or extra symmetries.
- Explore string constructions.

ML ideas

- $W = \sum_{i \leq j \leq k} c_{ijk} \phi_i \phi_j \phi_k$ with $\phi_0 = 1$, encoded as a sequence.
- $W \to$ R and non-R charges, $\min(V)$, $\langle \partial W \rangle$, $\langle W \rangle$, etc..
  Classical algorithm $\to$ ML equation solvers (RNN).
- SUSY breaking or SUSY model generators (RL or GAN).
- Flux vacuum structure, e.g., with CY echanced symmetry (?).

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The Nelson-Seiberg theorem and revisions

- SUSY breaking $\Rightarrow$ R, $\Leftarrow$ spontaneous broken R.
- SUSY breaking $\Leftrightarrow$ R and $N_Y < N_X$ (with polynomial W).
- R ($U(1)$ or $\mathbb{Z}_{n\geq3}$) and $N_Y \geq N_X \Rightarrow$ SUSY with $W = 0$.

Non-Abelian R-symmetries

- Continuous R $\Rightarrow U(1)^{(R)}$, but discrete R may not $\Rightarrow \mathbb{Z}_n^{(R)}$.
- Previous results are unchanged if $U(1)^{(R)}$ or $\mathbb{Z}_n^{(R)}$ exists.
- Even if without $\mathbb{Z}_n^{(R)}$, by treating non-singlets as A's, discrete non-Abelian R and $N_Y \geq N_X \Rightarrow$ SUSY with $W = 0$.

Possible machine learning applications

- $W \rightarrow$ vacua with certain properties.
- Equation solvers, model generators, vacuum structure, etc..

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