

NUCLEI IN THE STANDARD MODEL AND BEYOND

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“There are few problems in nuclear theoretical physics which have attracted more attention than that of trying to determine the fundamental interaction between two nucleons.

It is also true that scarcely ever has the world of physics owed so little to so many ...

... It is hard to believe that many of the authors are talking about the same problem or, in fact, that they know what the problem is.”

M. L. Goldberger

*Midwestern Conference on Theoretical
Physics, Purdue University, 1960*



Can we understand the emergence of nuclei within the Standard Model?

cf.
Bjorken's
talk

- nonperturbative physics in the simplest, nonrelativistic context
nontrivial fixed point, limit cycle, ...
- nuclear tests of physics **beyond** the Standard Model
neutrinoless double-beta decay, nuclear electric dipole moments, ...

Nuclear Effective Field Theory

implementation of the QFT paradigm in nuclear physics
solves the renormalization problems of the 50s
now the standard theory of nuclear physics



Outline

- Effective Field Theory
- Chiral EFT: Phenomenology, Fine-Tuning
- Pionless EFT: Unitarity, Lattice QCD
- Beyond the SM
- Conclusion



Standard Model: not a fundamental theory but an effective field theory

successes of SM
Weinberg '67

...

gauge coupling unification, ...
Georgi, Quinn + Weinberg '76

...



SM + higher-dimension ops

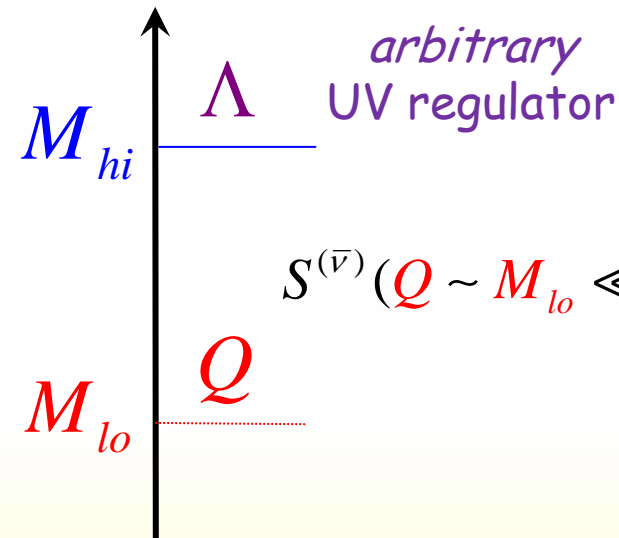
"Theorem"

Weinberg '79

The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general S matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and those symmetries, with no further physical content.



EFT

momentum
scalesnon-analytic functions,
from loops

$$S^{(\bar{\nu})}(Q \sim M_{lo} \ll M_{hi}) - 1 \propto \sum_{\nu=0}^{\bar{\nu}} \left[\frac{Q}{M_{hi}} \right]^{\nu} F^{(\nu)} \left(\frac{Q}{M_{lo}}, \frac{Q}{\Lambda}; \gamma_i^{(\nu)} \left(\frac{\Lambda}{M_{lo}}, \frac{M_{lo}}{M_{hi}} \right) \right)$$

$$+ \mathcal{O} \left(\frac{Q^{\bar{\nu}+1}}{M_{hi}^{\bar{\nu}+1}}, \frac{Q^{\bar{\nu}+1}}{M_{hi}^{\bar{\nu}} \Lambda} \right)$$

controlled

"low-energy
constants"
or "Wilson
coefficients"

RG invariance

$$\frac{\Lambda}{S^{(\bar{\nu})}} \frac{\partial S^{(\bar{\nu})}}{\partial \Lambda} = \mathcal{O} \left(\frac{Q^{\bar{\nu}+1}}{M_{hi}^{\bar{\nu}} \Lambda} \right)$$

to minimize cutoff errors, $\Lambda \gtrsim M_{hi}$
for realistic error estimate, $\Lambda \in [M_{hi}, \infty)$

model independent

(OTHERWISE, SENSITIVE TO HIGH-MOM DETAILS)



QC(+E)D (-LITE)

$$Q \ll M_{W,Z}$$

d.o.f.s

quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$

gluons: G_μ^a (+ photon: A_μ)

symmetries

SO(3,1) global, SU(3)_c (+U(1)_{em}) gauge

$$\mathcal{L}_{QCD} = \underbrace{\bar{q} (i\partial + g_s \mathbf{G}) q - \frac{1}{2} \text{Tr} G^{\mu\nu} G_{\mu\nu}}_{\text{quarks and gluons}} + \underbrace{\bar{m} \bar{q} (1 - \varepsilon \tau_3) q}_{\text{masses}} + \dots$$

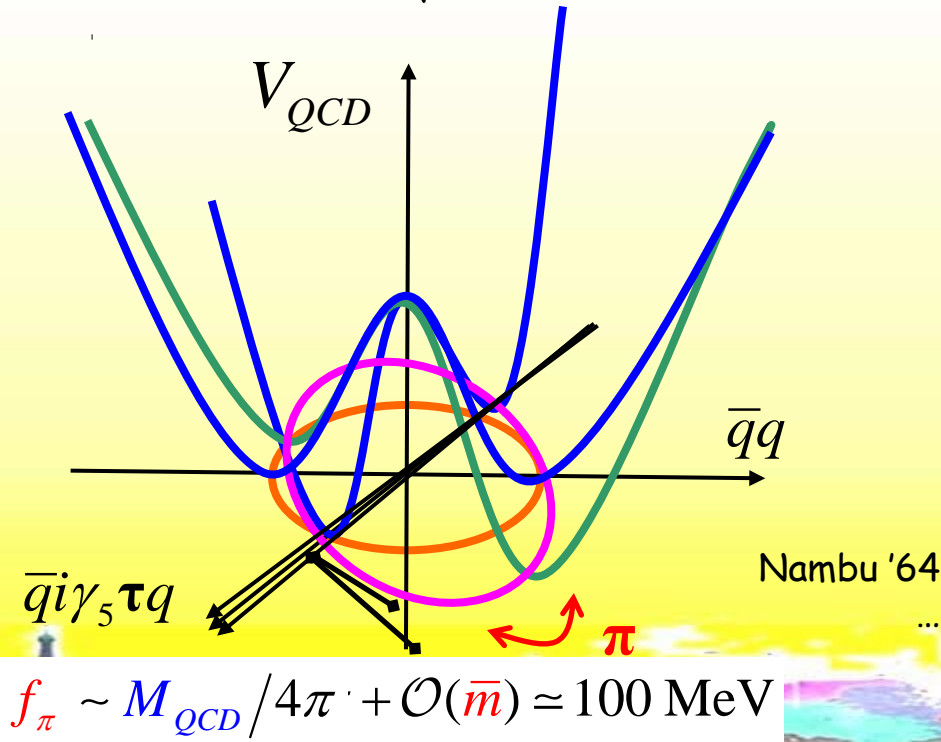
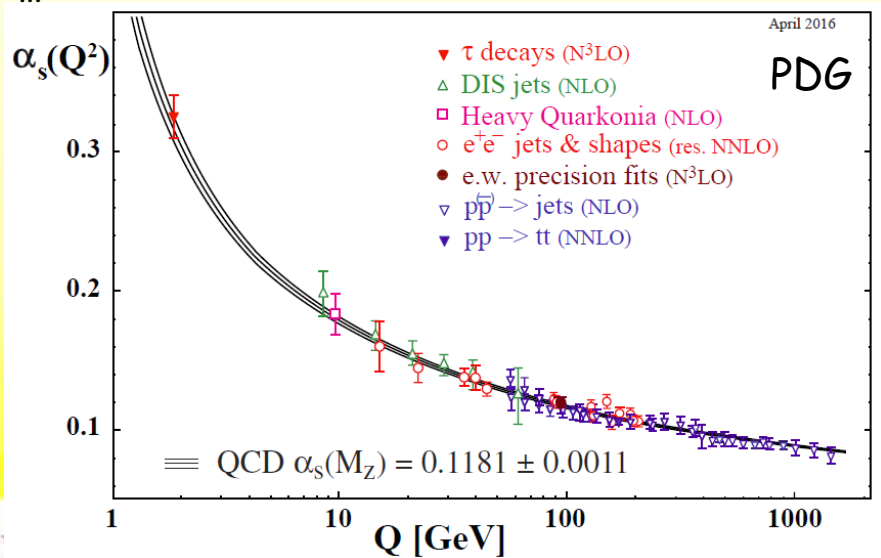
Basic

mass scales

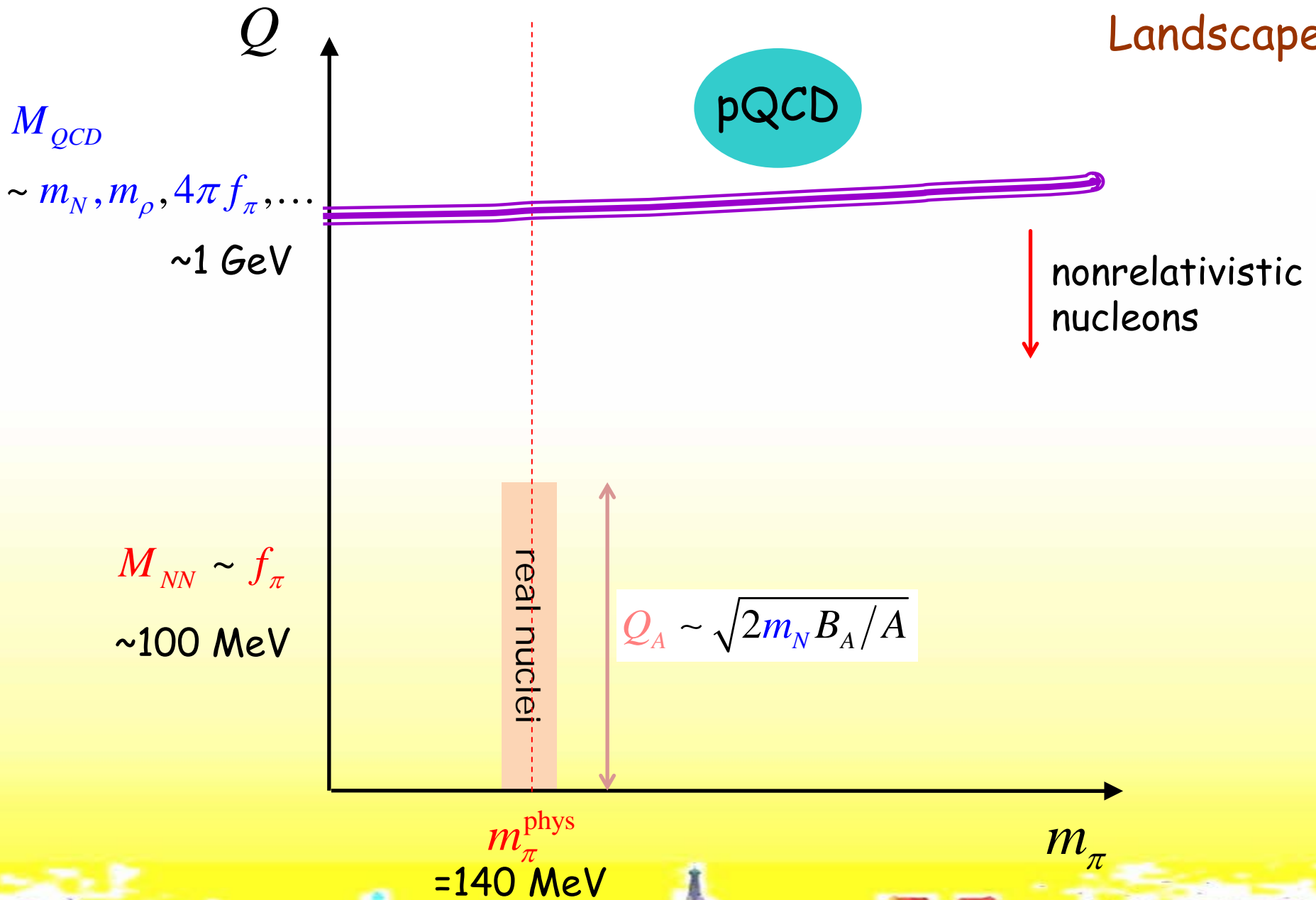
$$M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \dots \sim 1 \text{ GeV}$$

$$m_\pi \sim \sqrt{\bar{m} M_{QCD}} \approx 140 \text{ MeV}$$

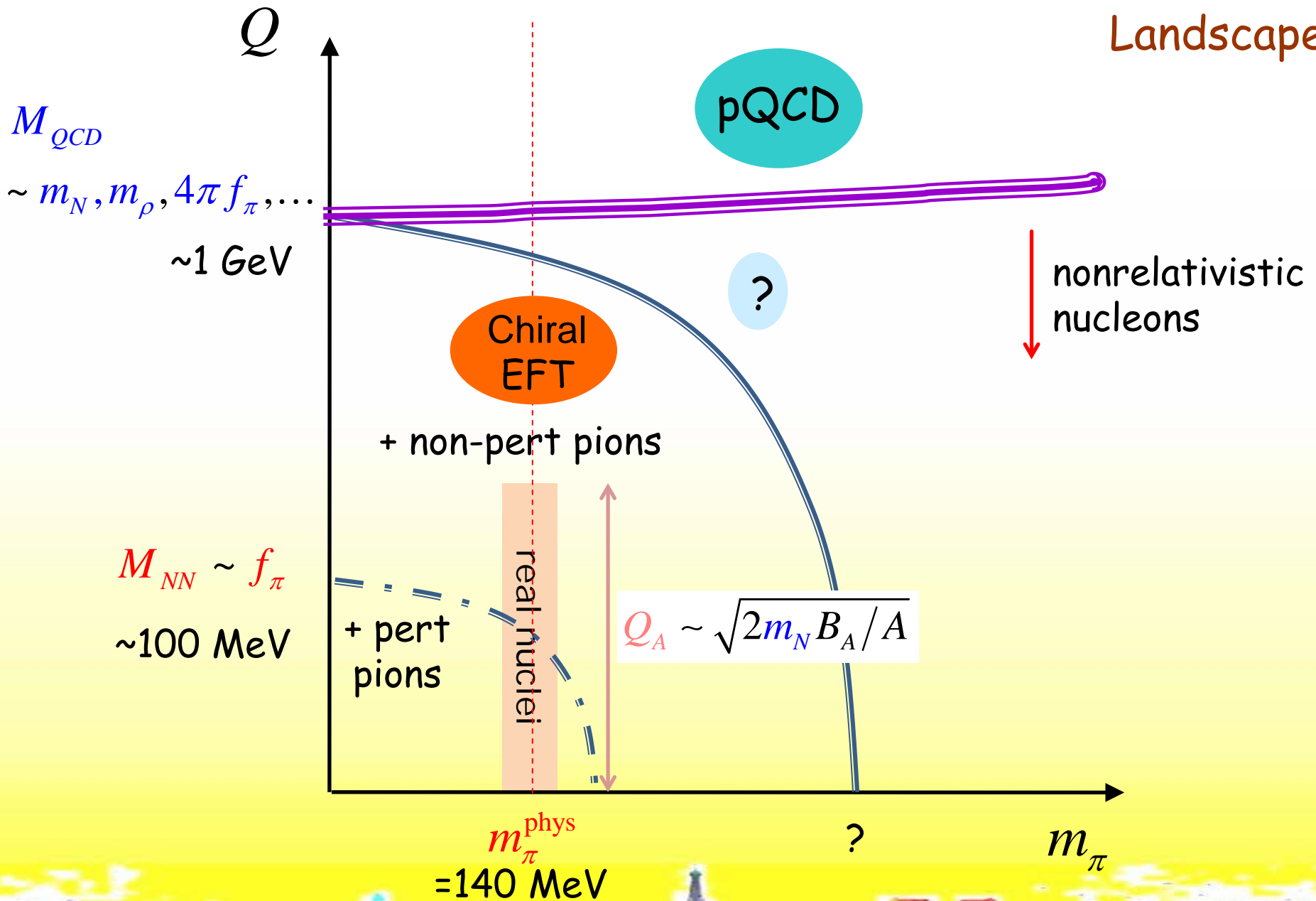
Gross + Wilczek '73
Politzer '73



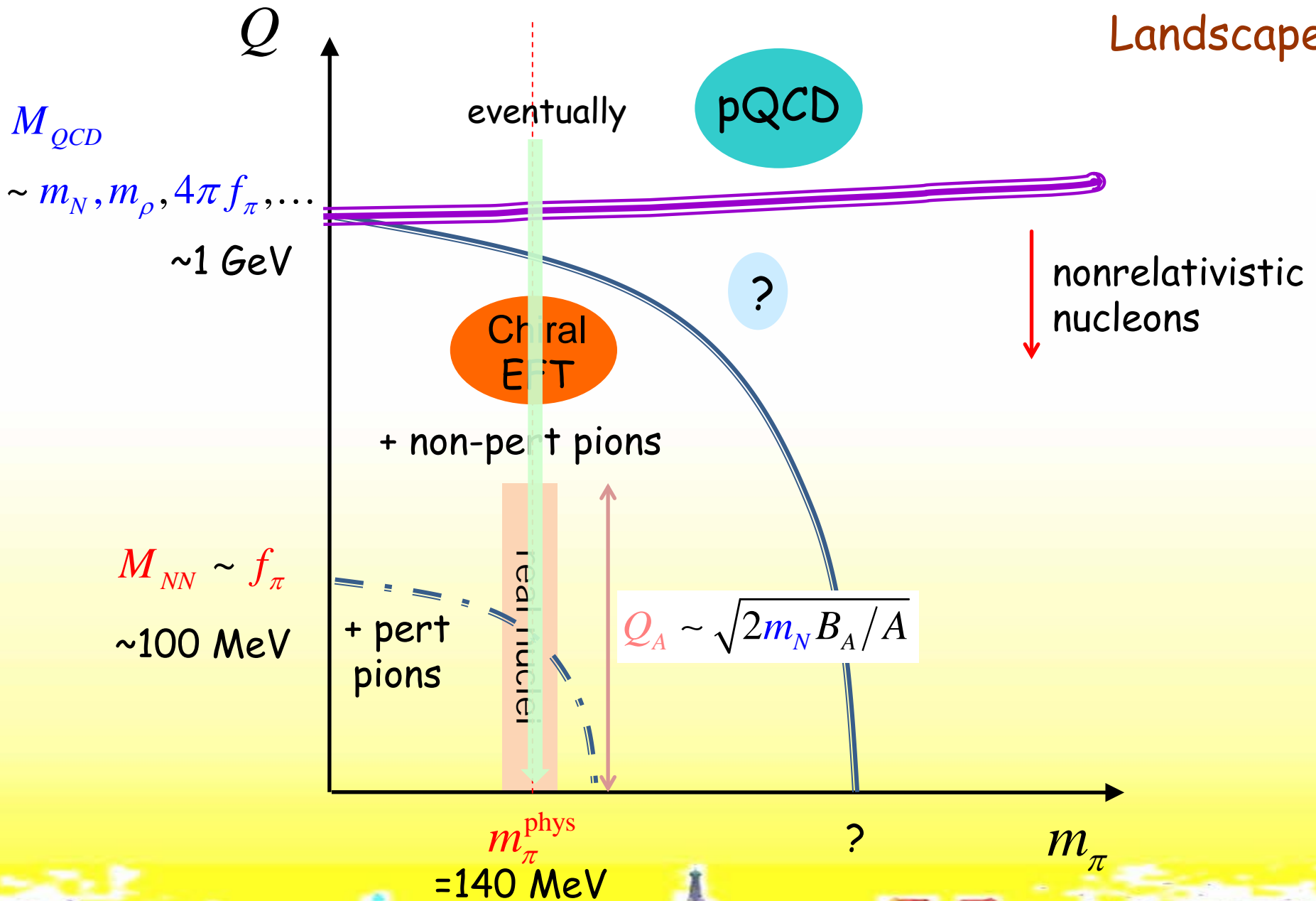
The Nuclear EFT Landscape



The Nuclear EFT Landscape



The Nuclear EFT Landscape



$$Q \sim m_\pi \ll M_{QCD}$$

Chiral EFT

d.o.f.s

nucleons: $N = \begin{pmatrix} p \\ n \end{pmatrix}$

pions: $\pi = \begin{pmatrix} (\pi^+ + \pi^-)/\sqrt{2} \\ i(\pi^+ - \pi^-)/\sqrt{2} \\ \pi^0 \end{pmatrix}$

(+ photon: A_μ)

+ Deltas + Roper + ...?

symmetries

SO(3,1) global, SU(3)_c (+U(1)_{em}) gauge, ~~SU(2) × SU(2) global~~
(trivial)

$$\mathcal{L}_{\chi EFT} = \frac{1}{2} \left(\partial_\mu \pi \cdot \partial^\mu \pi - m_\pi^2 \pi^2 \right) + N^+ \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right) N + \frac{g_A}{2f_\pi} N^+ \vec{S} \tau N \cdot \vec{\nabla} \pi$$

cf.
Adler's
talk

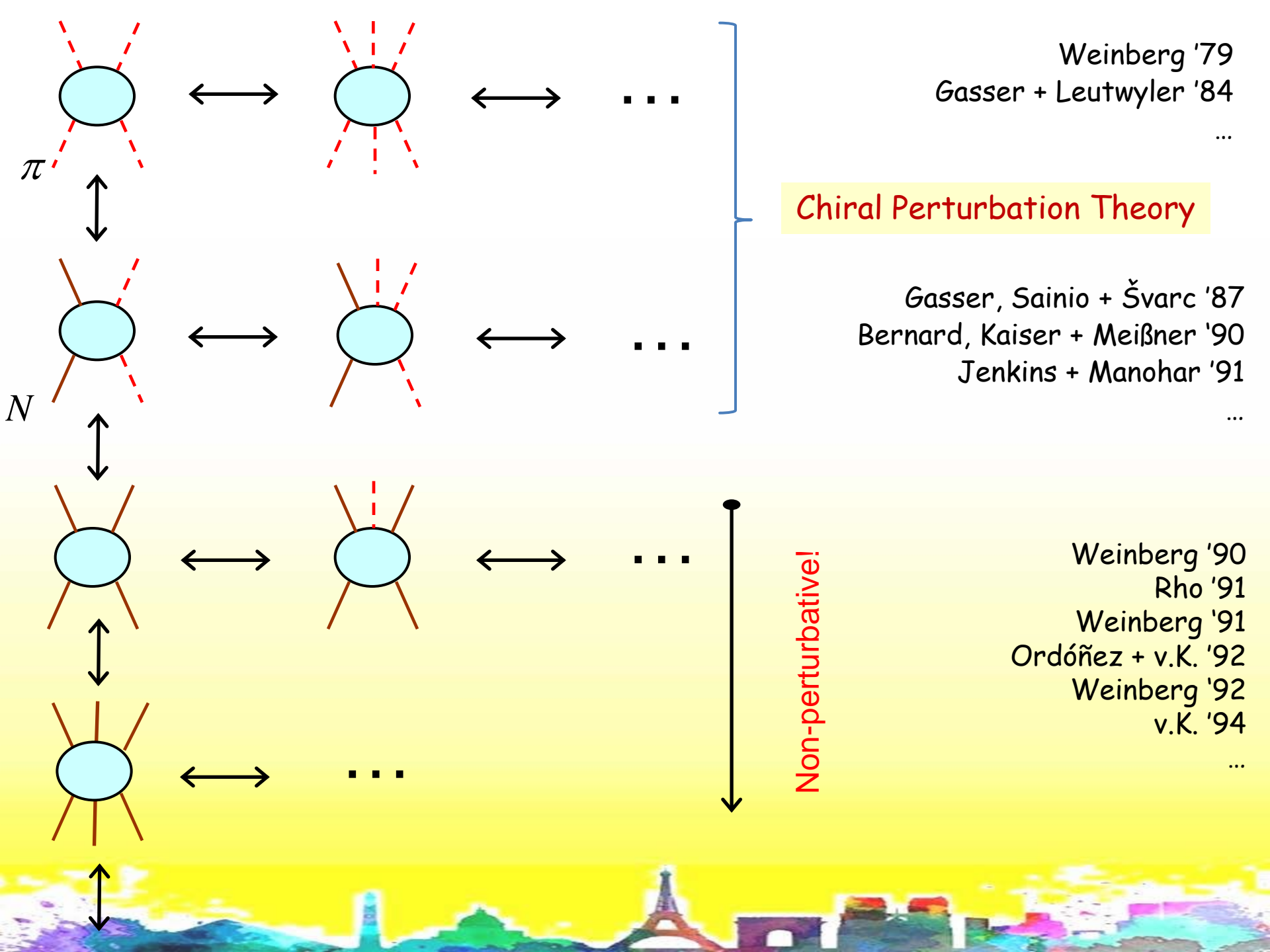
$$+ C_0 N^+ N N^+ N + \dots$$

expansion in:

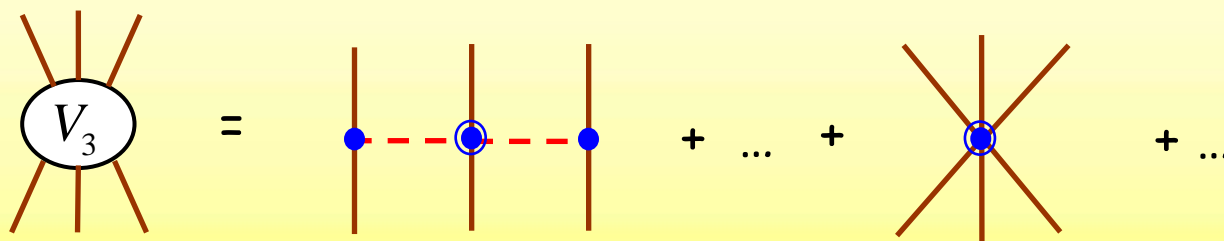
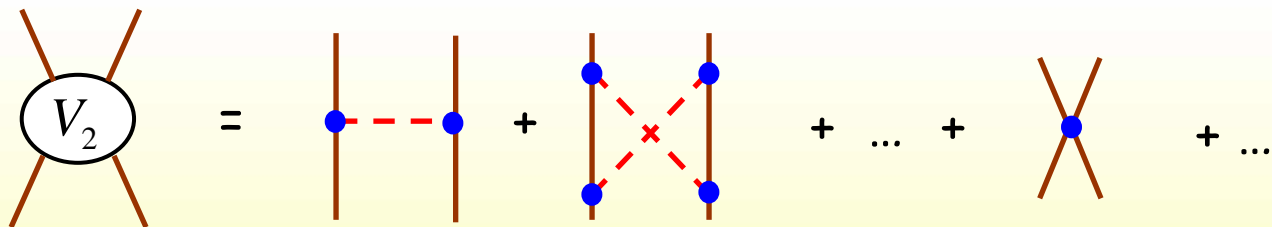
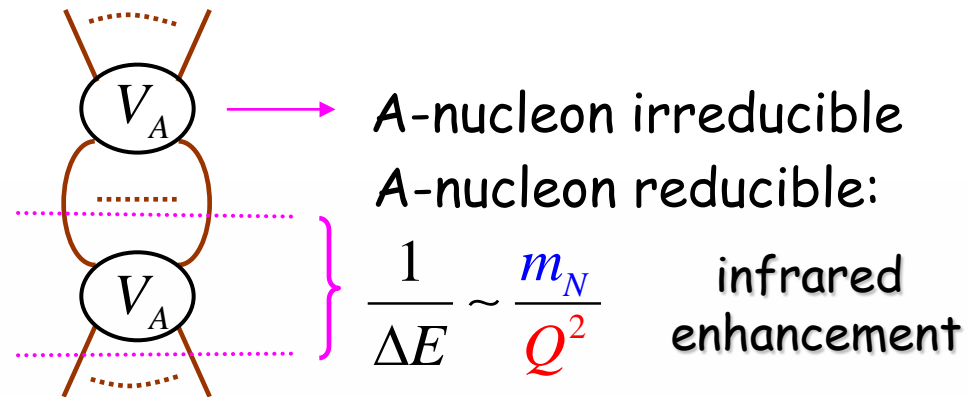
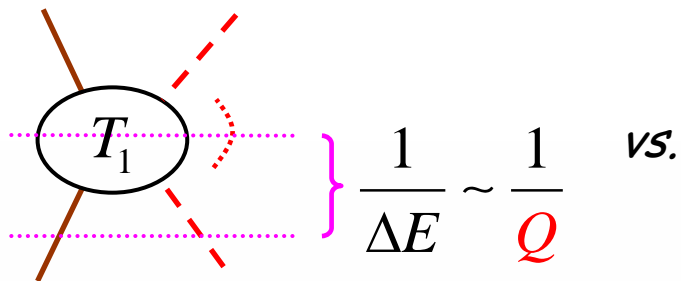
other spin/isospin,
chiral partners,
more derivatives and fields,
powers of pion mass

$$\frac{Q}{M_{QCD}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \dots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases}$$





chiral perturbation theory



etc.

more nucleons
↓
higher powers of Q



From naïve dimensional analysis (Georgi + Manohar '84)

$$V_2^{(0)} = \text{diagram 1} + \text{diagram 2} \sim \frac{4\pi}{m_N M_{NN}}$$

$$M_{NN} \equiv \frac{1}{g_A^2} \frac{4\pi f_\pi}{m_N} f_\pi \sim f_\pi$$

Kaplan, Savage + Wise '98

$$T_2^{(0)} = \text{diagram 1} + \text{diagram 2} + \dots \sim \frac{4\pi}{m_N M_{NN}} \left[1 + \frac{m_N Q}{4\pi} \frac{4\pi}{m_N M_{NN}} + \dots \right]$$

loop

$$\sim \frac{4\pi}{m_N M_{NN}} \frac{1}{1 - \frac{Q}{M_{NN}}}$$

$$M_{lo} = M_{NN} \sim f_\pi \ll M_{QCD}$$

bound state

$$Q \sim M_{NN} \sim 100 \text{ MeV}$$

$$-E \sim \frac{M_{NN}^2}{M_{QCD}} \sim 10 \text{ MeV}$$

Nuclear scale arises naturally in QCD due to spontaneous chiral symmetry breaking

cf. Friar, Madland + Lynn '96



add more derivatives, many-nucleon interactions

$$M_{QCD}^{-1} \vec{\nabla} \quad \left(f_{\pi}^{-2} M_{QCD} \right)^{-1} N^{\dagger} N$$

Current standard in nuclear physics:
solve the many-body Schrödinger equation numerically
with a truncated "chiral potential"

Ordóñez, Ray
+ v.K. '94'96
Epelbaum Glöckle
+ Meißner '98

...
Entem + Machleidt
'03

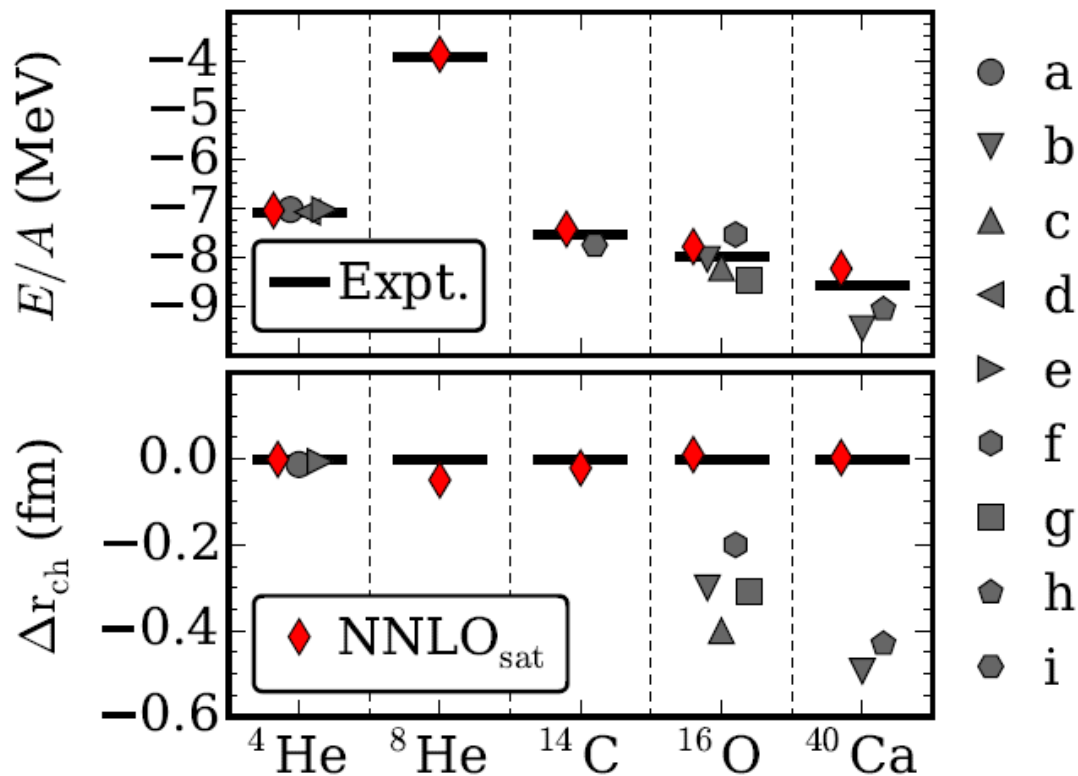
...

One of countless examples:

Ekström *et al.* '15

binding energy
per nucleon

expt - th
charge radius
difference



various
chiral
potentials

Problem

no RG invariance

-- existing results for fixed regulator

Kaplan, Savage + Wise '96
Nogga, Timmermans + v.K. '05
Pavón Valderrama + Ruiz Arriola '06
Birse '06
...

Partial solution

Pions in perturbation theory

-- only works for $Q \lesssim M_{NN}$

Subleading potentials in perturbation theory

Kaplan, Savage + Wise '98
...
Fleming, Mehen + Stewart '00

Solution

Correct NDA

-- running of some LECs: $M_{QCD}^{-1} \rightarrow f_{\pi}^{-1}$

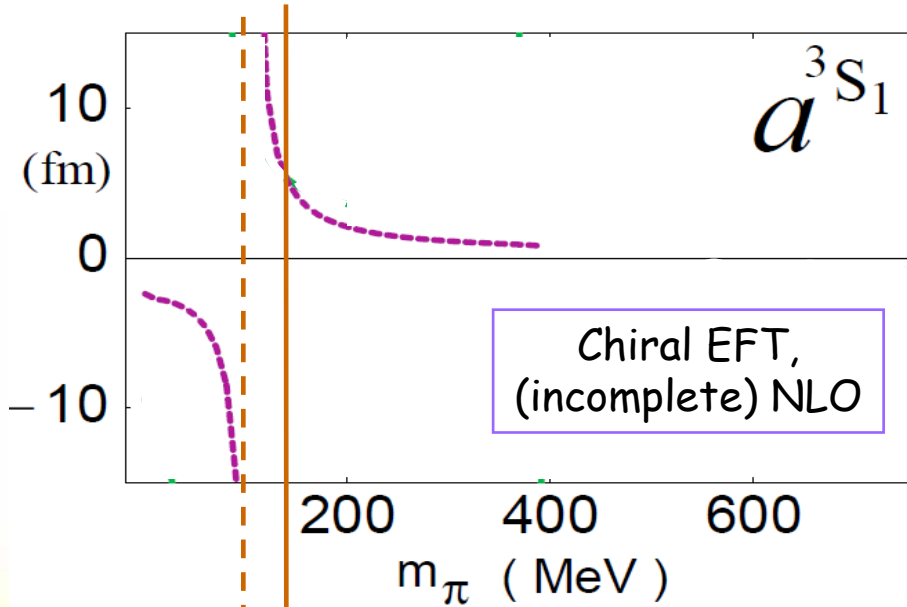
Subleading potentials in perturbation theory

Nogga, Timmermans + v.K. '05
Birse '06
Long + v.K. '08
Pavón Valderrama '11'12
Long + Yang '12'13
...



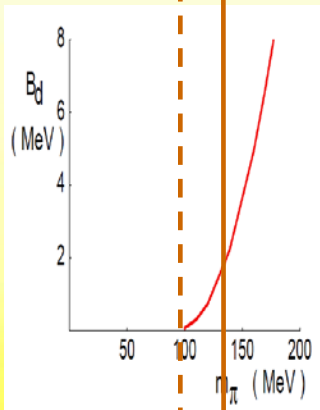
Fine-Tuning

unitarity limit



$$m_\pi^* (M_{QCD})$$

$$m_\pi \approx 140 \text{ MeV}$$

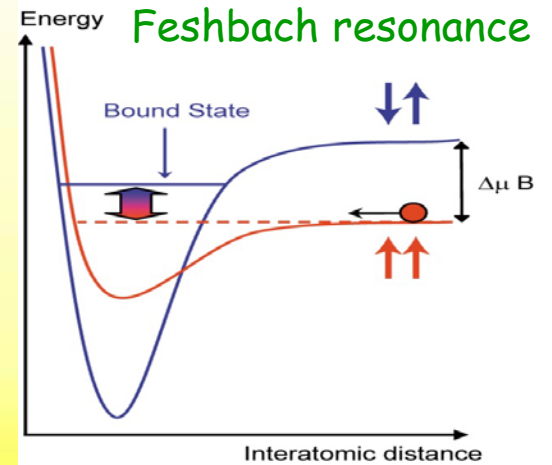
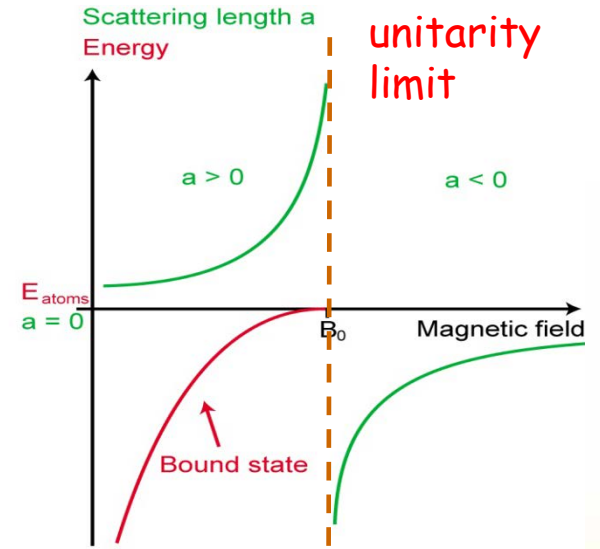


Beane, Bedaque, Savage + v.K. '02
Braaten + Hammer '03
...

New scale emerges

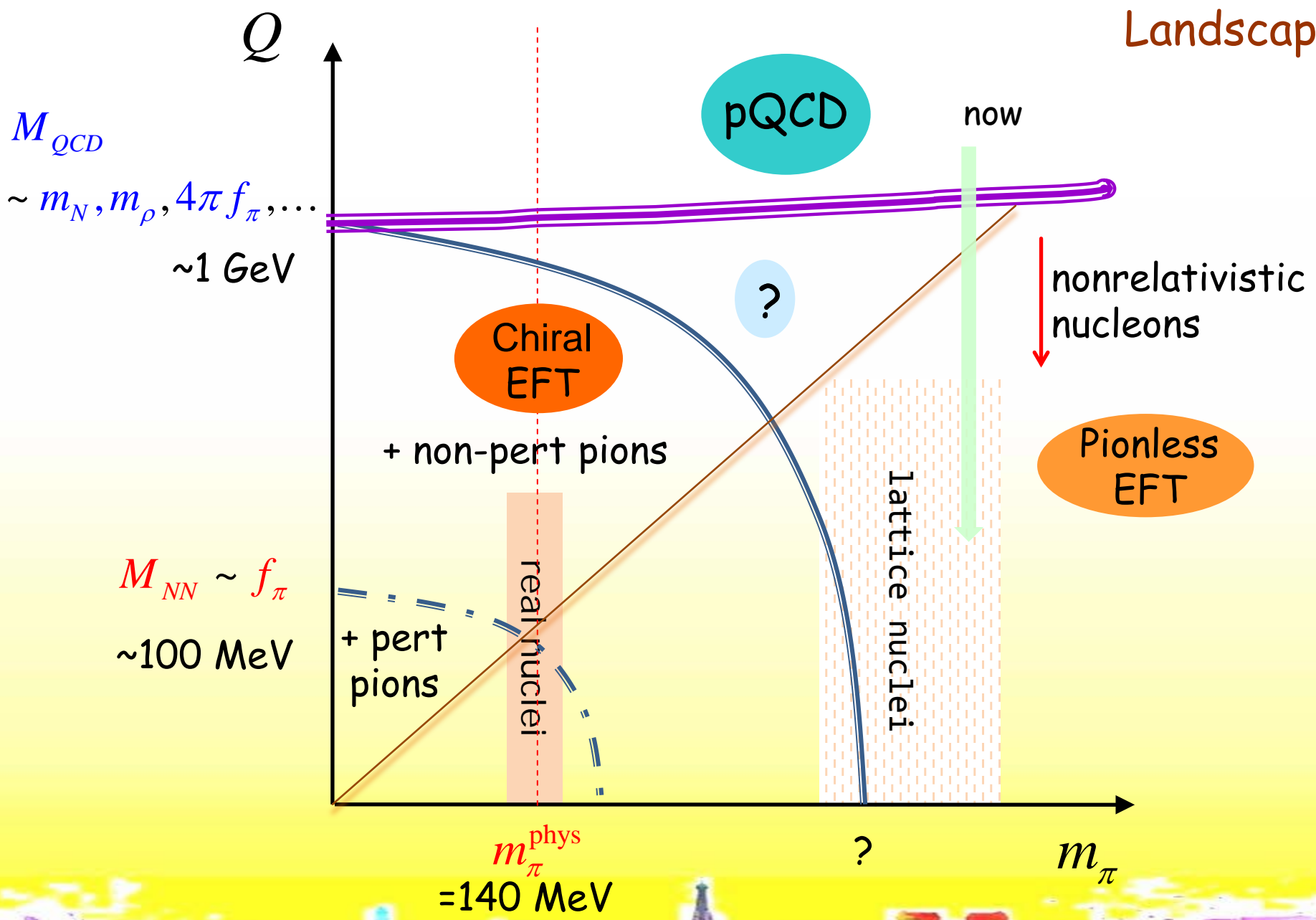
$$\mathcal{L} \sim \frac{m_\pi - m_\pi^*}{m_\pi^*} M_{NN}$$

cf.



or "accidentally", e.g. ^4He atoms

The Nuclear EFT Landscape



$$Q \ll m_\pi$$

Pionless EFT

d.o.f.s

nucleons: $N = \begin{pmatrix} p \\ n \end{pmatrix}$ (+ photon: A_μ)

symmetries

$SO(3,1)$ global, $SU_c(3)$ (+ $U_{em}(1)$) gauge
(trivial)

$$\mathcal{L}_{\pi EFT} = N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right) N + C_0 N^\dagger N N^\dagger N + D_0 N^\dagger N N^\dagger N N^\dagger N + \dots$$

more derivatives
and fields,
isospin violation

Universality:
first orders
apply also to
neutral atoms

$$m_\pi \rightarrow 1/l_{\text{vdW}} \text{ where } V(r) = -\frac{l_{\text{vdW}}^4}{2mr^6} + \dots$$

Bedaque, Hammer
+ v.K. '99'00
Bedaque, Braaten
+ Hammer '01
...



$$V_2^{(0)} = \text{diagram} \sim \frac{4\pi}{m_N \mathcal{S}} \approx \frac{m_\pi - m_\pi^*}{m_\pi^*} M_{NN}$$

$$T_2^{(0)} = \text{diagram} + \text{diagram} + \dots \sim \frac{4\pi}{m_N \mathcal{S}} \frac{1}{1 - \frac{Q}{\mathcal{S}}}$$

bound state

$Q \sim \mathcal{S} \sim 30 \text{ MeV}$

$-E \sim \frac{\mathcal{S}^2}{M_{QCD}} \sim 1 \text{ MeV}$

$$\gamma_0 \equiv \frac{m_N \Lambda}{4\pi} C_0 \quad \Lambda \frac{d\gamma_0}{d\Lambda} = \gamma_0 (1 + \gamma_0)$$

fixed points: $\gamma_0 = \begin{cases} 0 & \text{trivial (perturbative)} \\ -1 & \text{nontrivial: unitarity} \end{cases}$

Weinberg '91

$$\mathcal{S} = 0$$

Mehen, Stewart + Wise '99

no physical scale:
scale invariance

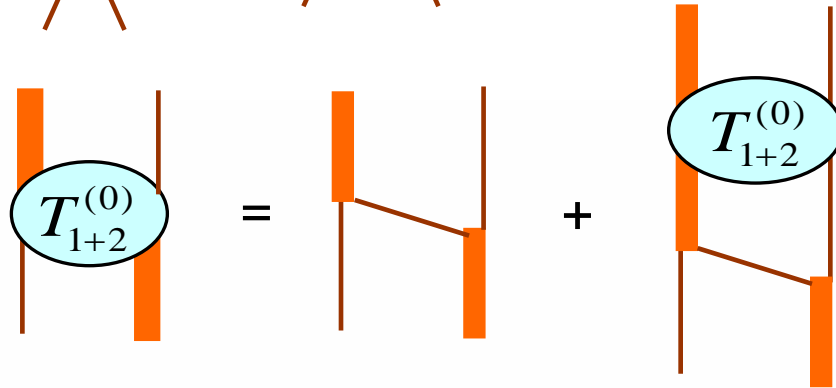
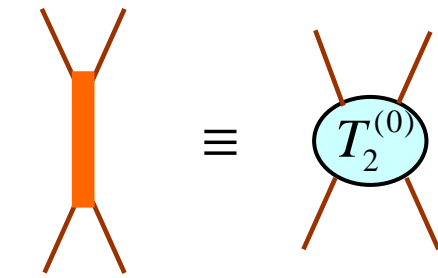


Bedaque, Hammer + v.K. '99 '00

Hammer + Mehen '01

Bedaque *et al.* '03

...



$$\Rightarrow T_{1+2} \xrightarrow{p \gg \hbar} A \cos \left(s_0 \ln \frac{p}{\Lambda} + \delta \right)$$

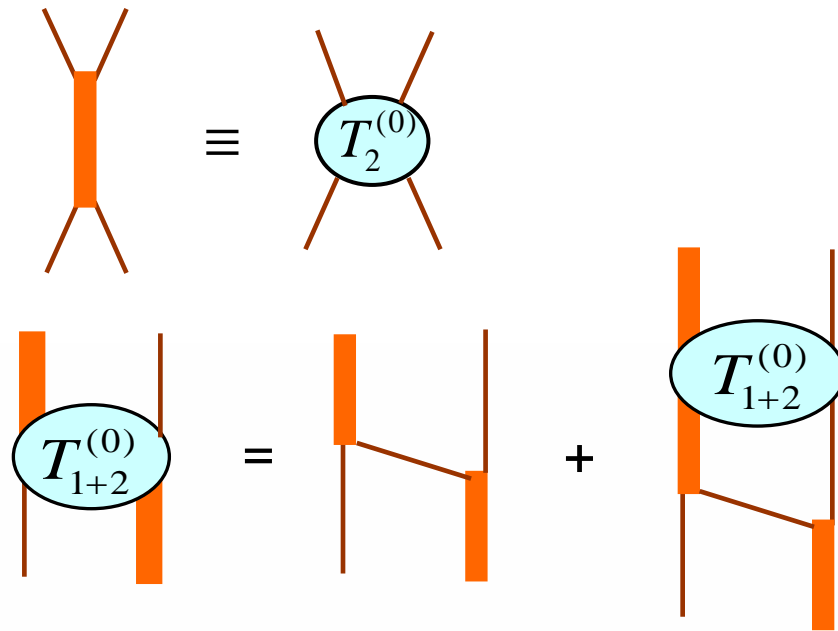


Bedaque, Hammer + v.K. '99 '00

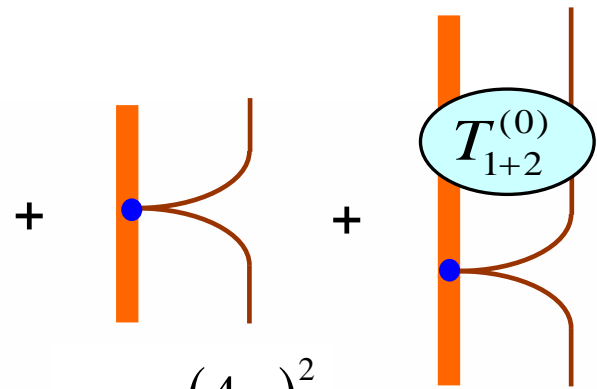
Hammer + Mehen '01

Bedaque *et al.* '03

...



$$\rightarrow T_{1+2} \xrightarrow{p \gg \hbar} A \cos \left(s_0 \ln \frac{p}{\Lambda_*} + \tilde{\delta} \right)$$



$$D_0 \sim \frac{(4\pi)^2}{m_N \hbar^4}$$

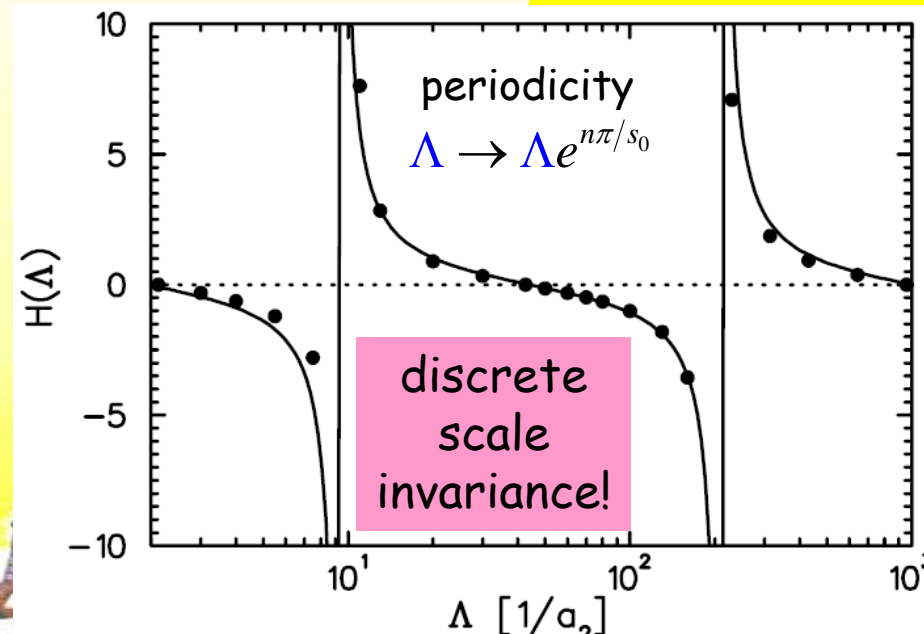
cf. Wilson '71

RG limit cycle!

dimensionful parameter
(dimensional transmutation)

$$H(\Lambda) \equiv \frac{\Lambda^2 D_0(\Lambda)}{m_N C_0^2(\Lambda)} \quad s_0 = 1.00624\dots$$

$$\approx \frac{\sin(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0))}{\sin(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0))}$$

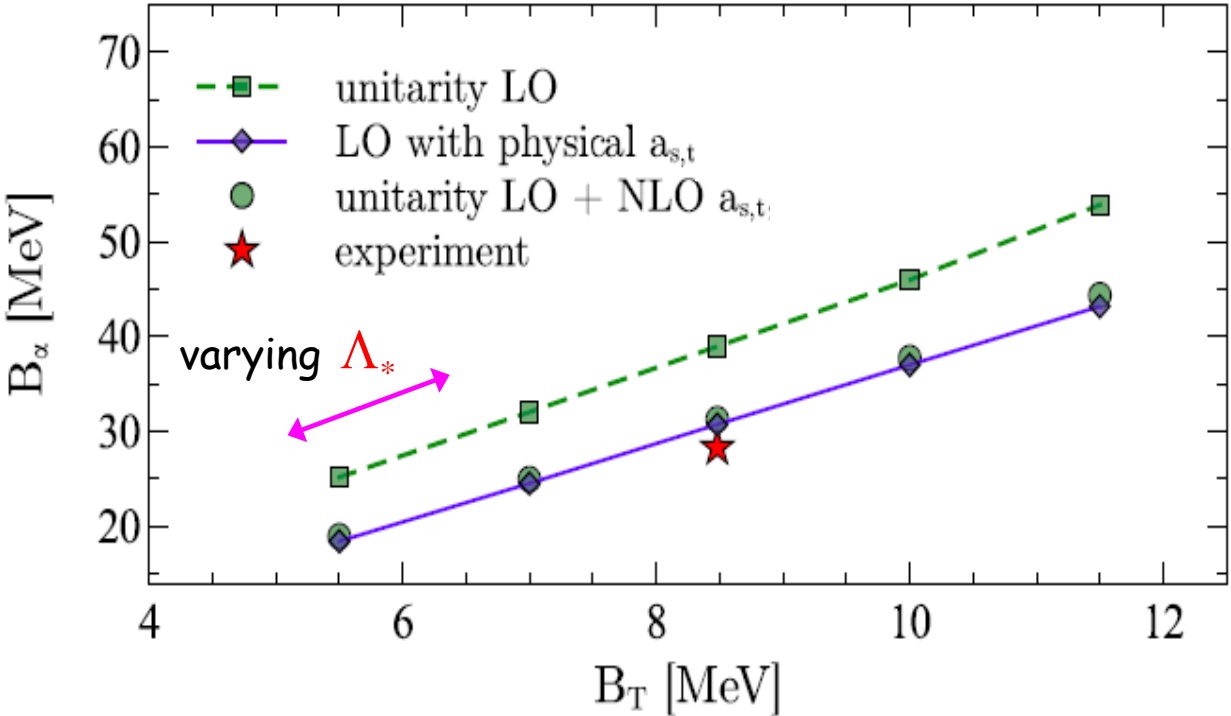


Physical quark masses:

expansion around two-body unitarity limit \Rightarrow **ONE** parameter at LO
 in \mathcal{N}/Λ_* Λ_*

geometric towers of states (Efimov '71)
 ground states: $\frac{B_A^{(0)}(\Lambda_*)}{A} = \kappa_A \frac{B_3(\Lambda_*)}{3}$
universal numbers

A = 4



Tjon line

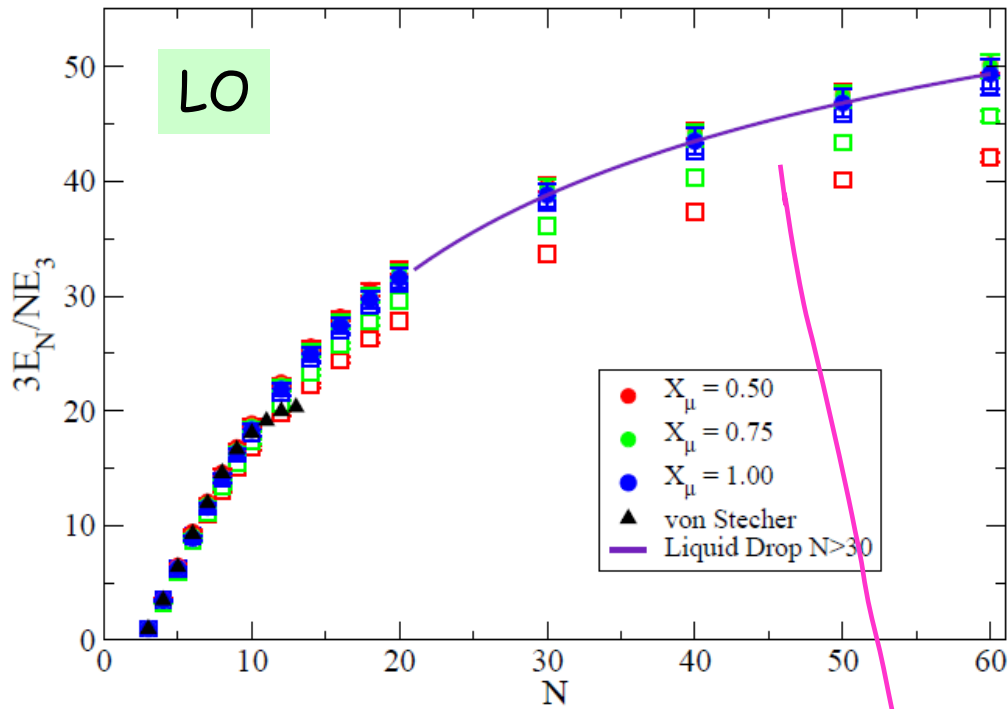
LO $\kappa_4 \approx 3.5$

incomplete
NLO

(Faddeev-Yakubovski equations)



Bosons at unitarity



increasing Λ

Von Weizsäcker '35

cf. semi-empirical mass formula

(Variational and Diffusion Monte Carlo)

saturation!

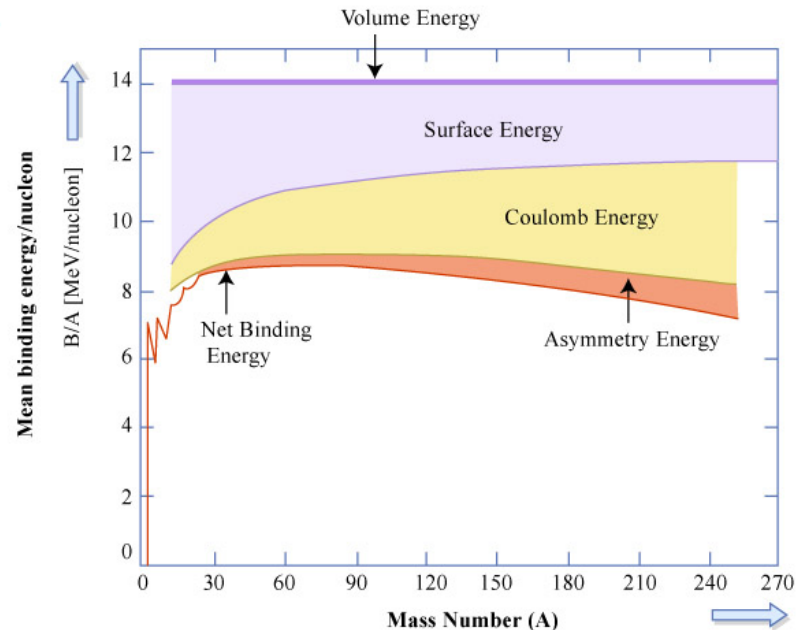
$$\kappa_N = \kappa_\infty \left[1 + \eta N^{-1/3} + \mathcal{O}(N^{-2/3}) \right]$$

$$\kappa_\infty = 90 \pm 10 \quad \eta = -1.7 \pm 0.3$$

volume

surface

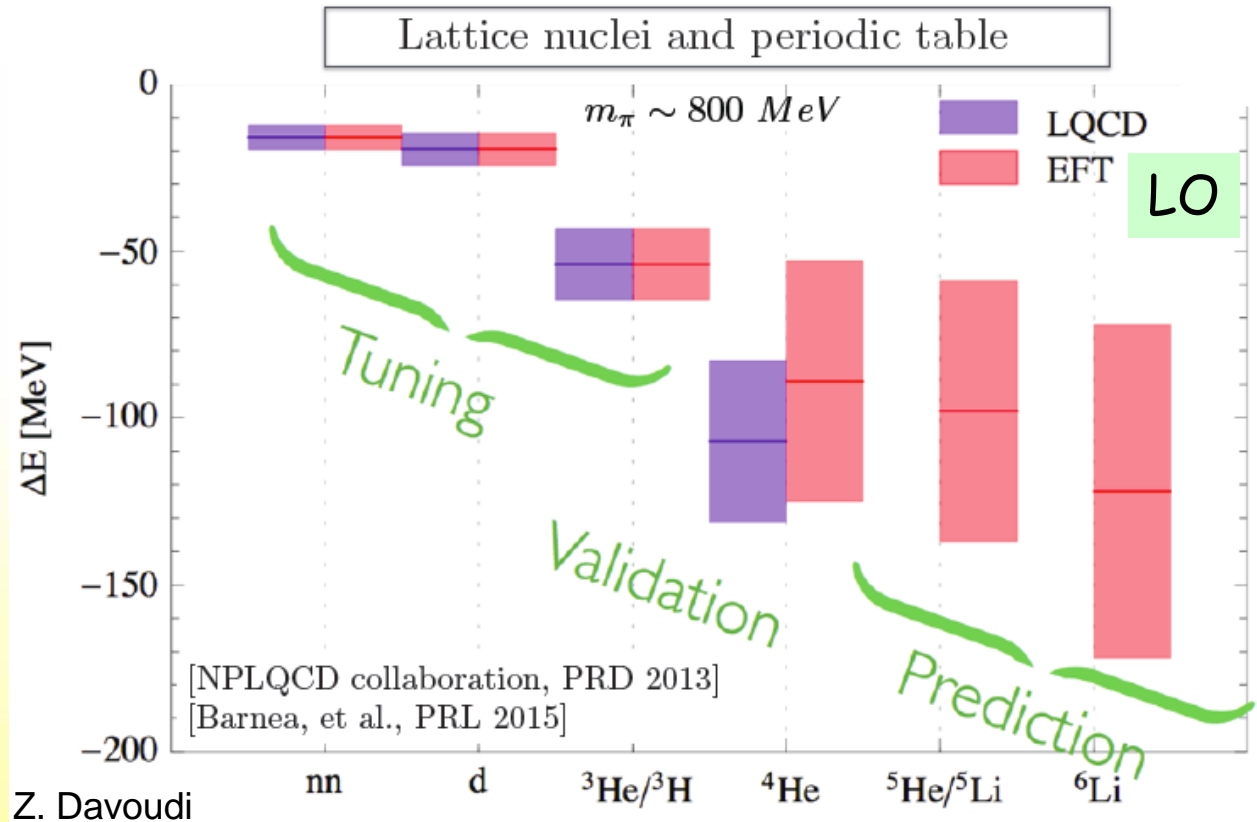
awaiting atomic experiments...



Unphysical quark masses: three parameters at LO

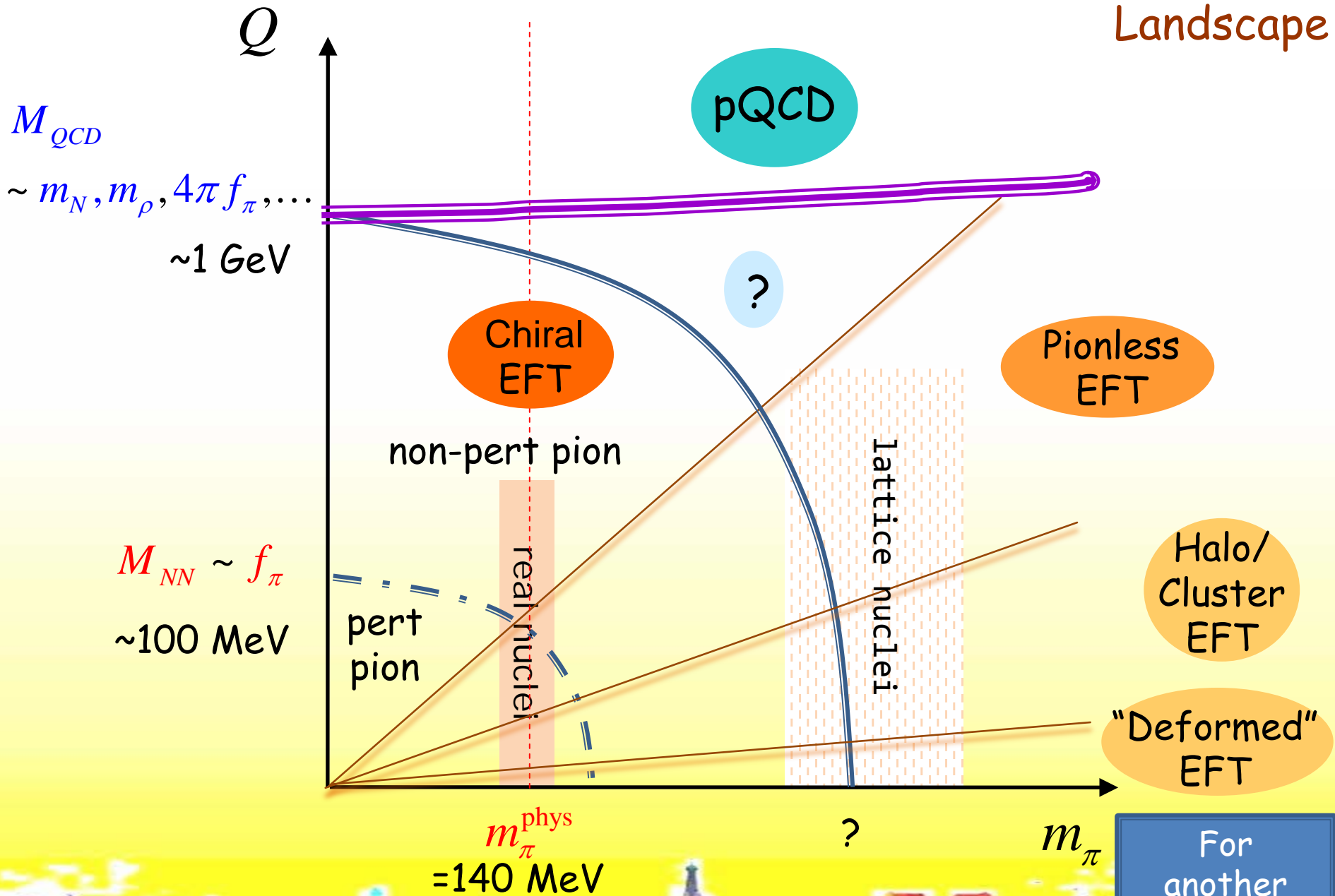
Barnea, Contessi,
Gazit, Pederiva
+ v.K. '15

$$\Lambda_*, \mathcal{N}_{3S_1}, \mathcal{N}_{1S_0}$$



Qualitatively similar to physical pion mass,
 just more bound by a factor ~ 5

The Nuclear EFT Landscape



For another talk

Beyond the SM

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\text{dim}=5} + \mathcal{L}_{\text{dim}=6} + \dots$$

Weinberg '79



$\Delta L = 2$

\mathcal{T} violation

neutrinoless
double-beta
decay

nuclear
electric dipole
moments



Two examples

Buchmüller + Wyler '86

Weinberg '89

de Rújula *et al.* '91

...
Ng + Tulin '11

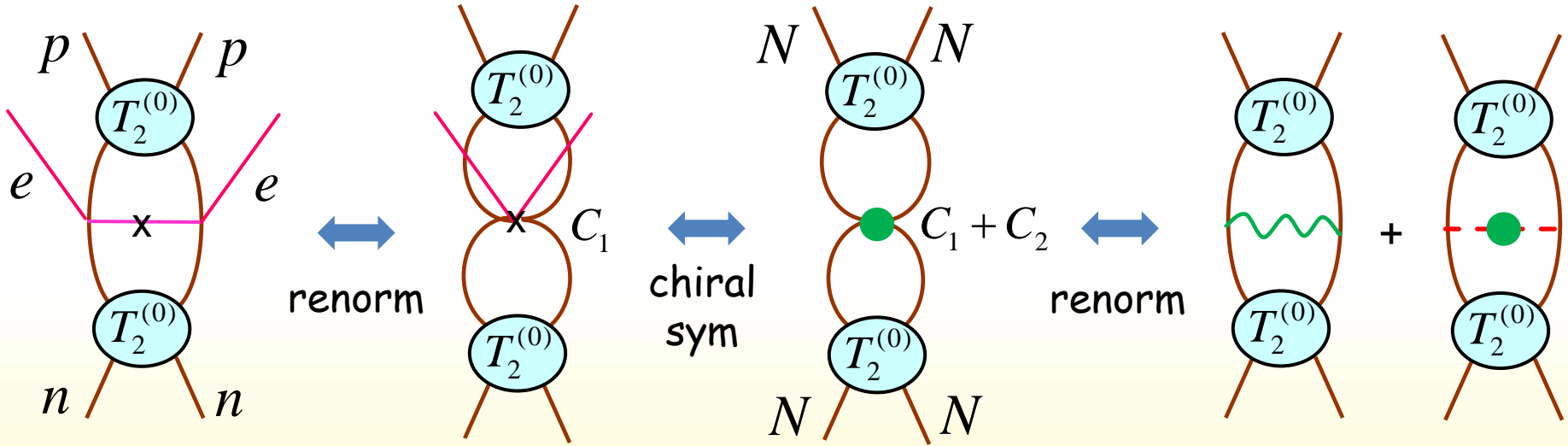


Neutrinoless double-beta decay

$$Q \ll M_{W,Z}$$

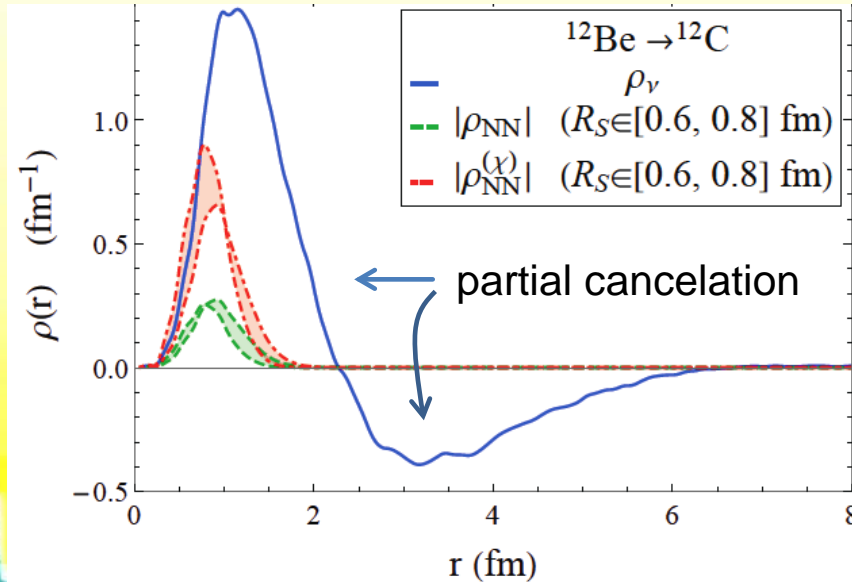
$$\mathcal{L}_{\text{dim}=5} = -\frac{m_{\beta\beta}}{2} \nu_{eL}^T C \nu_{eL} + \dots$$

$$m_{\beta\beta} \equiv \sum_{i=1}^3 U_{ei}^2 m_{\nu i}$$



lattice QCD needed
to disentangle
 C_1 and C_2

determining $C_1 + C_2$
from NN data
and assuming $C_1 \sim C_2$



$$A_{0\nu 2\beta} = \int dr \rho(r)$$

$$\frac{A_{NN}}{A_\nu} \simeq 0.3 - 0.6$$

$$Q \ll M_{W,Z}$$

Nuclear electric dipole moments

De Vries *et al.* '10'11'13
 Bsaisou *et al.* '12'13'15
 Dekens *et al.* '14

$$\begin{aligned} \mathcal{L}_{\text{dim}=4,6} = & \dots + \frac{\bar{m}}{2} (1 - \varepsilon^2) \bar{\theta} \bar{q} i \gamma_5 q - \frac{1}{2} \bar{q} (c_q^{(0)} + c_q^{(1)} \tau_3) \sigma_{\mu\nu} \tilde{G}^{\mu\nu} q - \frac{1}{2} \bar{q} (d_q^{(0)} + d_q^{(1)} \tau_3) \sigma_{\mu\nu} q \tilde{F}^{\mu\nu} \\ & + \frac{C_G}{6} f^{abc} G_{\mu\nu}^a \tilde{G}^{b\nu\rho} G_\rho^{c\mu} + \frac{C_1}{4} (\bar{q} q \bar{q} i \gamma_5 q - \bar{q} \boldsymbol{\tau} q \cdot \bar{q} i \gamma_5 \boldsymbol{\tau} q) + \frac{C_8}{4} (\bar{q} \lambda^a q \bar{q} i \gamma_5 \lambda^a q - \bar{q} \boldsymbol{\tau} \lambda^a q \cdot \bar{q} i \gamma_5 \boldsymbol{\tau} \lambda^a q) \\ & + \frac{D_1}{4} \varepsilon_{3ij} \bar{q} \tau_i \gamma^\mu q \bar{q} \tau_j \gamma_\mu \gamma_5 q + \frac{D_8}{4} \varepsilon_{3ij} \bar{q} \tau_i \gamma^\mu \lambda^a q \bar{q} \tau_j \gamma_\mu \gamma_5 \lambda^a q + \dots \end{aligned}$$

Neutron EDM d_n can be produced by any source, **but**

different chiral properties of sources \rightarrow different magnitudes for nuclear EDMs

		θ term	qCEDM	qEDM	gCEDM, PS4QO	LR4QO
^1H	d_p/d_n	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
^2H	d_d/d_n	$\mathcal{O}(1)$	$\mathcal{O}(M_{QCD}^2/m_\pi^2)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(M_{QCD}^2/m_\pi^2)$
^3He	d_h/d_n	$\mathcal{O}(M_{QCD}^2/m_\pi^2)$	$\mathcal{O}(M_{QCD}^2/m_\pi^2)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(M_{QCD}^2/m_\pi^2)$
^3H	d_t/d_h	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$

Farley *et al.* '04

+ specific relations

storage-ring measurements (CERN?) could teach us about sources!

Conclusion

- ◆ Effective field theory allows us to connect the Standard Model and nuclear physics in a controlled and systematic way
- ◆ We are in position to use nuclei as a laboratory to test physics beyond the SM

