Precision Electroweak Theory and its Prediction of m(t), m(h)

M. E. Peskin
(contribution of Bryan Lynn)
SM@50
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disclaimer:

In the original program, Bryan Lynn was supposed to give the history of precision electroweak theory. However, Bryan is ill and cannot attend the meeting.

Bryan asked me to fill in for him (even though I have never done a precision calculation in electroweak theory). He gave me his notes, but this turned out to be a bibliography with 1008 entries.

Instead, I will give a very general outline of the progress and success of this theory. I apologize if your work is not mentioned here. I posted Bryan's article to the SM@50 indico site; please see this for proper citations.

The history of precision electroweak calculation begins, actually, before concept of the electroweak theory.

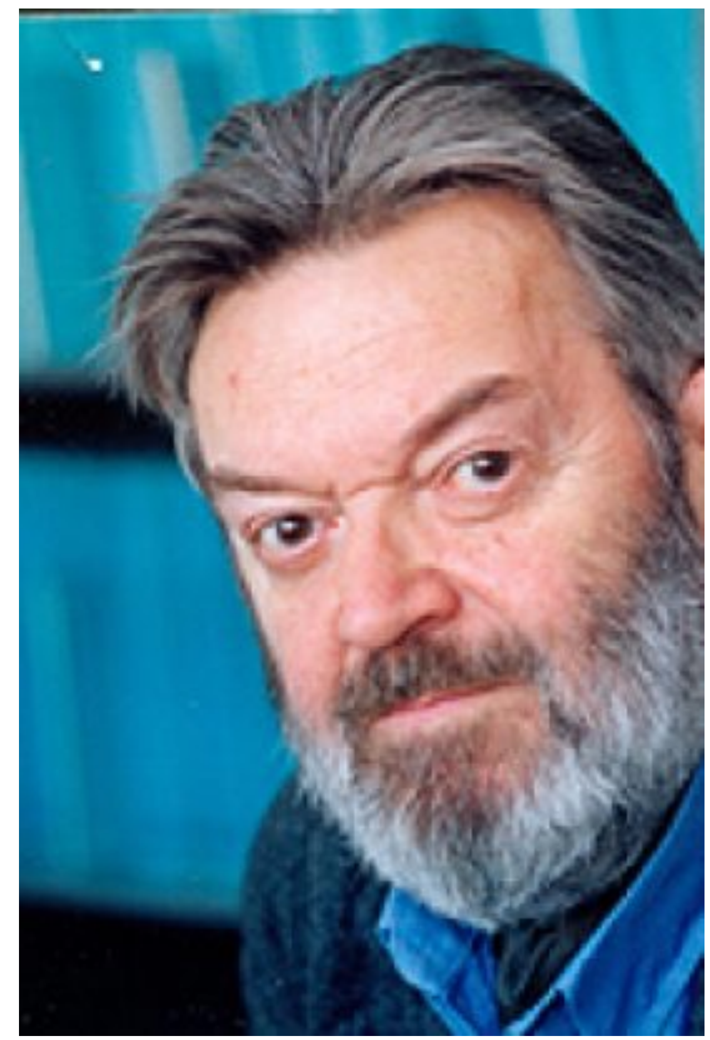
In 1957, Toichiro Kinoshita and Alberto Sirlin thought to improve the predictions of the new V-A theory of weak interactions by including QED radiative corrections. These corrections are IR divergent, with those divergences cancelling in the usual way. However, they found that the corrections are also enhanced by a UV logarithm. This log could not be controlled precisely because the theory was non-renormalizable.

Two decades later, after the invention of the SU(2)xU(1) gauge theory, its successful renormalization, and its order-1 vindication by experiment, Tini Veltman and Alberto Sirlin again picked up the dream of high-precision weak interaction calculation.

Veltman saw that this would require a complete technology for 1-loop Feynman integrals and proceeded to develop it.

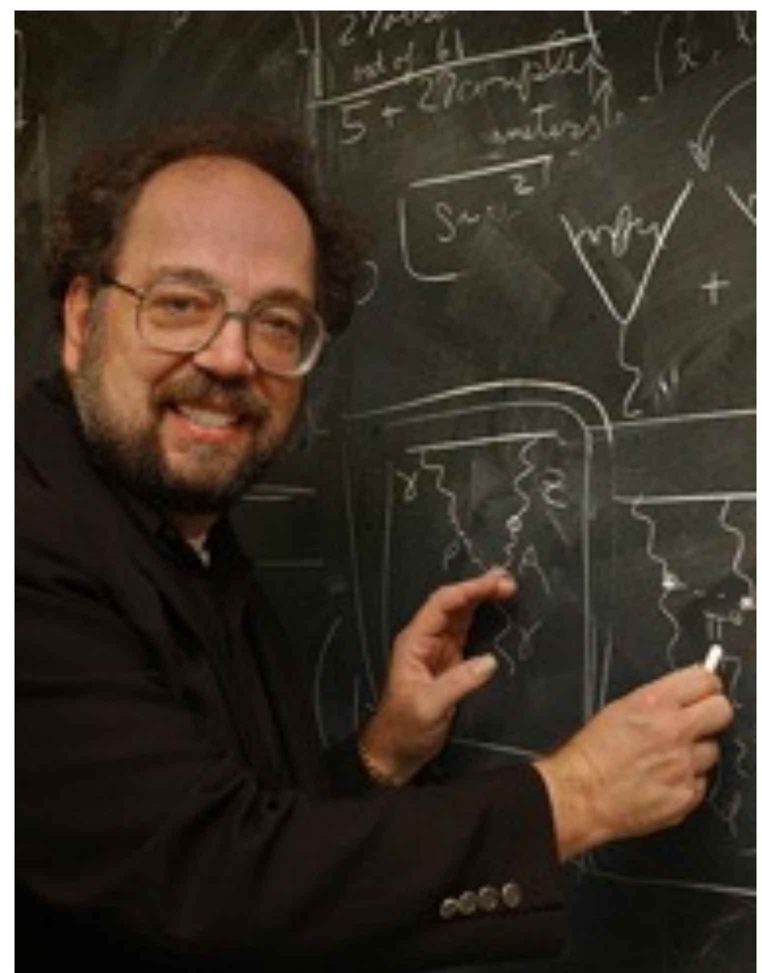
His student ’t Hooft (maybe not gainfully employed at the time) solved the final, most difficult, integrals.

The 1978 paper of Giampiero Passarino and Veltman gave the complete calculation of 1-loop corrections for the process $e^+e^- \rightarrow \mu^+\mu^-$. This paper introduced a reduction method for 1-loop Feynman diagrams that is still being used for precision calculations at the LHC.



In the 2000's, the Passarino-Veltman methods were enhanced by additional tricks due to Bern, Dixon, Dunbar, and Kosower and Ossola, Pittau, and Papadopoulos. These methods allow general reductions of diagrams for processes with many arbitrarily many external particles. The combination of methods is amenable to automation, and this is now achieved in codes such as MadGraph5_aMC@NLO. Even experimenters now have access to 1-loop accuracy!

Sirlin and William Marciano took the next important step, formalizing the renormalization program needed to use weak interaction data to make predictions of particle masses and couplings. (Actually, they hoped to control both the TeV scale and the GUT scale radiative corrections in SU(5) grand unification).



In addition to their important formal contributions, Marciano and Sirlin pointed out a large 1-loop correction (5% in m_W^2) due to the running of the QED coupling α from its conventional atomic-physics value to the weak interaction scale.

At tree level, the electroweak theory has three parameters: g g' v . Marciano and Sirlin suggested defining these by an “on-shell” renormalization method, fixing the values of

$$\alpha(m_Z^2)\ ,\ G_F\ ,\ \sin^2\theta_w = 1 - m_W^2/m_Z^2$$

This gives a very clean calculational method that can be applied to a wide range of electroweak observables.

This point deserves some further discussion.

Electroweak theory requires 3 renormalizations for its three parameters, but there is a great deal of freedom in the choice of these. It is convenient to fix the very well measured parameters α and GF. The third parameter can be taken to be $\sin^2\theta_w$.

There are many possible definitions of $\sin^2\theta_w$ in terms of experimental observables. All of these definitions agree at tree level. The equality of any two definitions is a "tree-level natural relation". In a renormalizable quantum field theory, such a relation obtains finite corrections at 1-loop order. These corrections are testable predictions of the theory.

For example,

$\sin^2\theta_w|_{MS}$ defined by $\cos^2\theta_w = m_W^2/m_Z^2$

$\sin^2\theta_w|_0$ defined by $\sin^2 2\theta_w = \dfrac{\alpha(m_Z^2)}{\sqrt{2}G_F m_Z^2}$

$\sin^2\theta_w|_*$ defined by

$$A_\ell = \frac{(1/4 - \sin^2\theta_w)}{1/4 - \sin^2\theta_w + 2(\sin^2\theta_w)^2} \approx 8(1/4 - \sin^2\theta_w)$$

Dallas Kennedy and Bryan Lynn extended the last definition to a gauge-invariant running $\sin^2\theta_{w*}(Q^2)$. With finite (and actually very small) corrections for each process, this quantity could predict

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to f\overline{f})$$

for arbitrary final-state fermions, through the Z^0 resonance.

The accelerators LEP and SLC, which were designed to sit on the Z resonance and measure individual Z boson decays, gave an opportunity to put these predictions to the test.

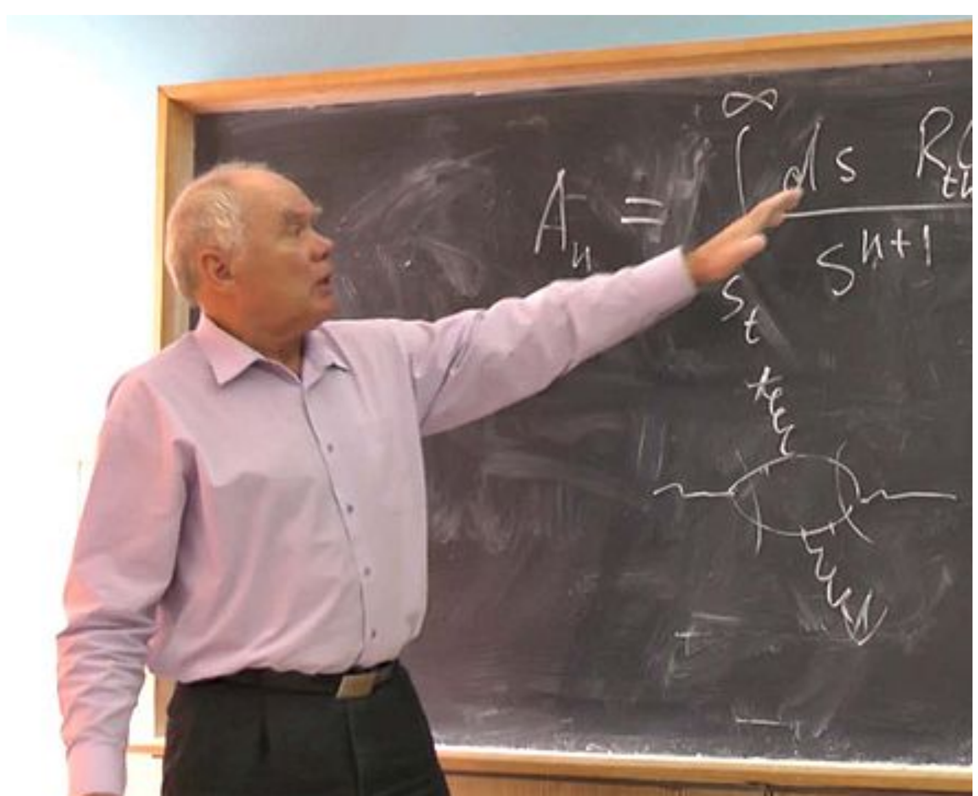
One more new ingredient was needed, a precise theory of the Z resonance lineshape. The Z shape is distorted by initial-state photon radiation, and this turns out to be a large effect. The magnitude of radiation is given by

$$\beta = \frac{2\alpha}{\pi}(\log\frac{s}{m_e^2} - 1) = 0.108 \quad \text{at } s = m_Z^2$$

But also, the Z is narrow, so the distortion of the resonance is actually proportional to

$$-\beta \cdot \log\frac{m_Z}{\Gamma_Z} = 40\%$$

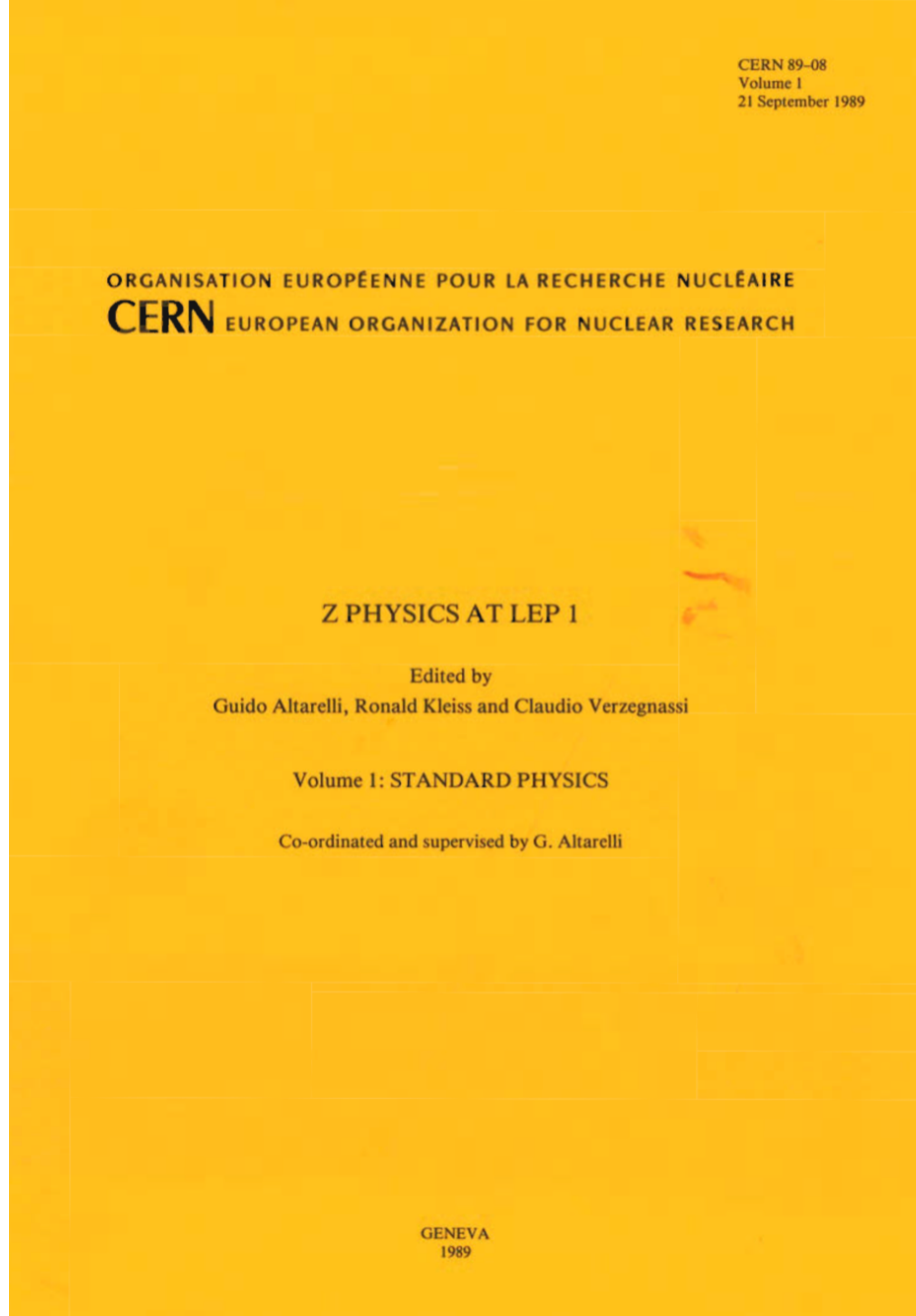
The missing piece was supplied by Victor Fadin and Valery Khoze in 1987.   They showed how the photon emissions could be resummed by viewing the radition as creating structure for the electron, in which hard electrons and photons are partons of the on-shell electron   The initial condition is given by QED, so the parton evolution equation can be solved without further input.



After this, there was the hard work of supplying precise computations for all of the aspects of Z physics. I do not have time here for a complete accounting of the contributions. Some names that should not be forgotten are

Dima Bardin, Frits Berends, Manfred Bohm,
Ansgar Denner, Wolfgang Hollik, Fred Jegerlehner,
Ronald Kleiss, Luca Trentadue, and Tord Riemann

Guido Altarelli made an essential contribution in coordinating the “Yellow Book” to insure that every needed aspect was prepared for the experiments.



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Z PHYSICS AT LEP 1

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Volume 1: STANDARD PHYSICS

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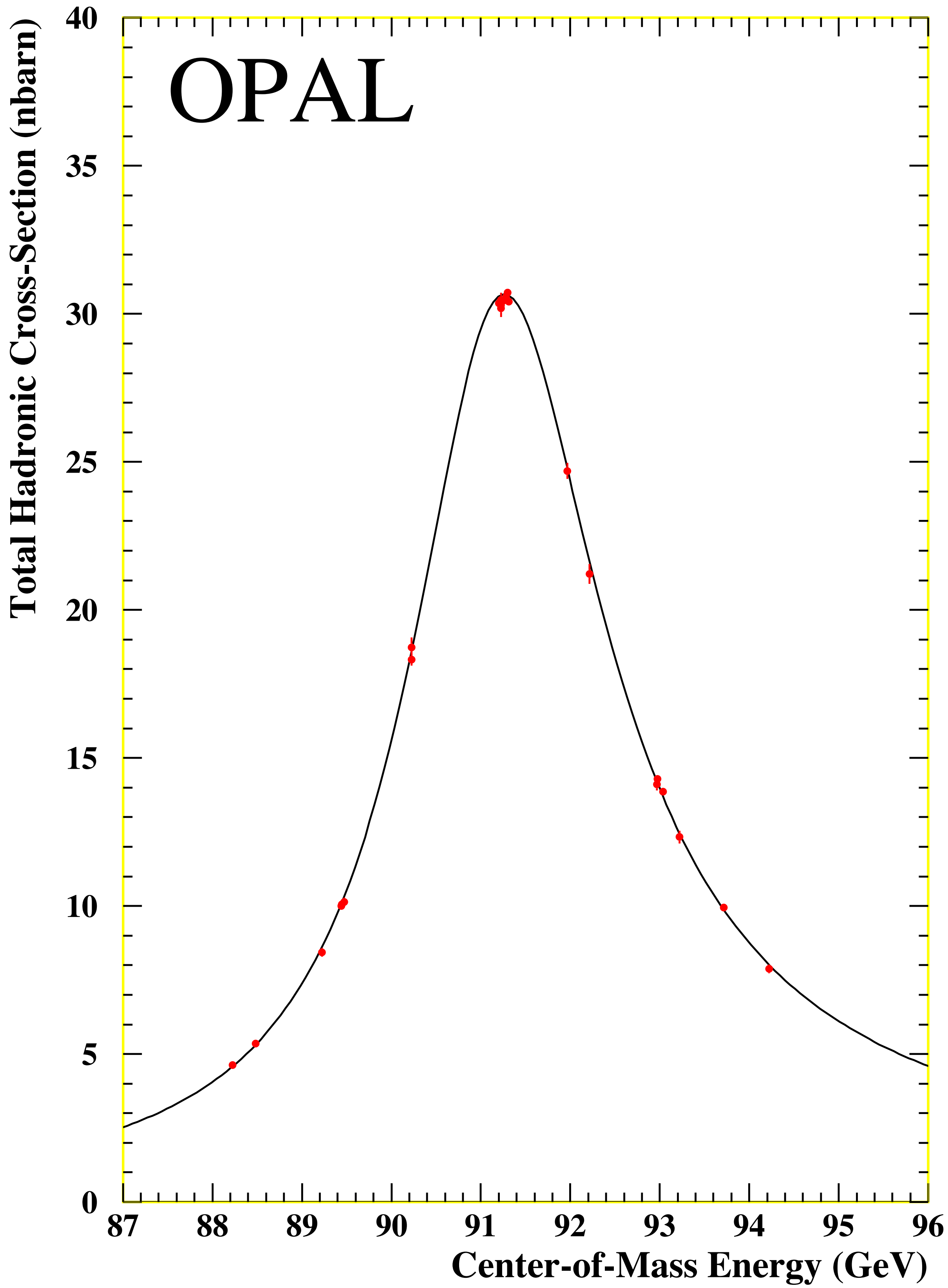
The event generators that implemented this theory for the experimenters played a crucial role. The two most important ones were

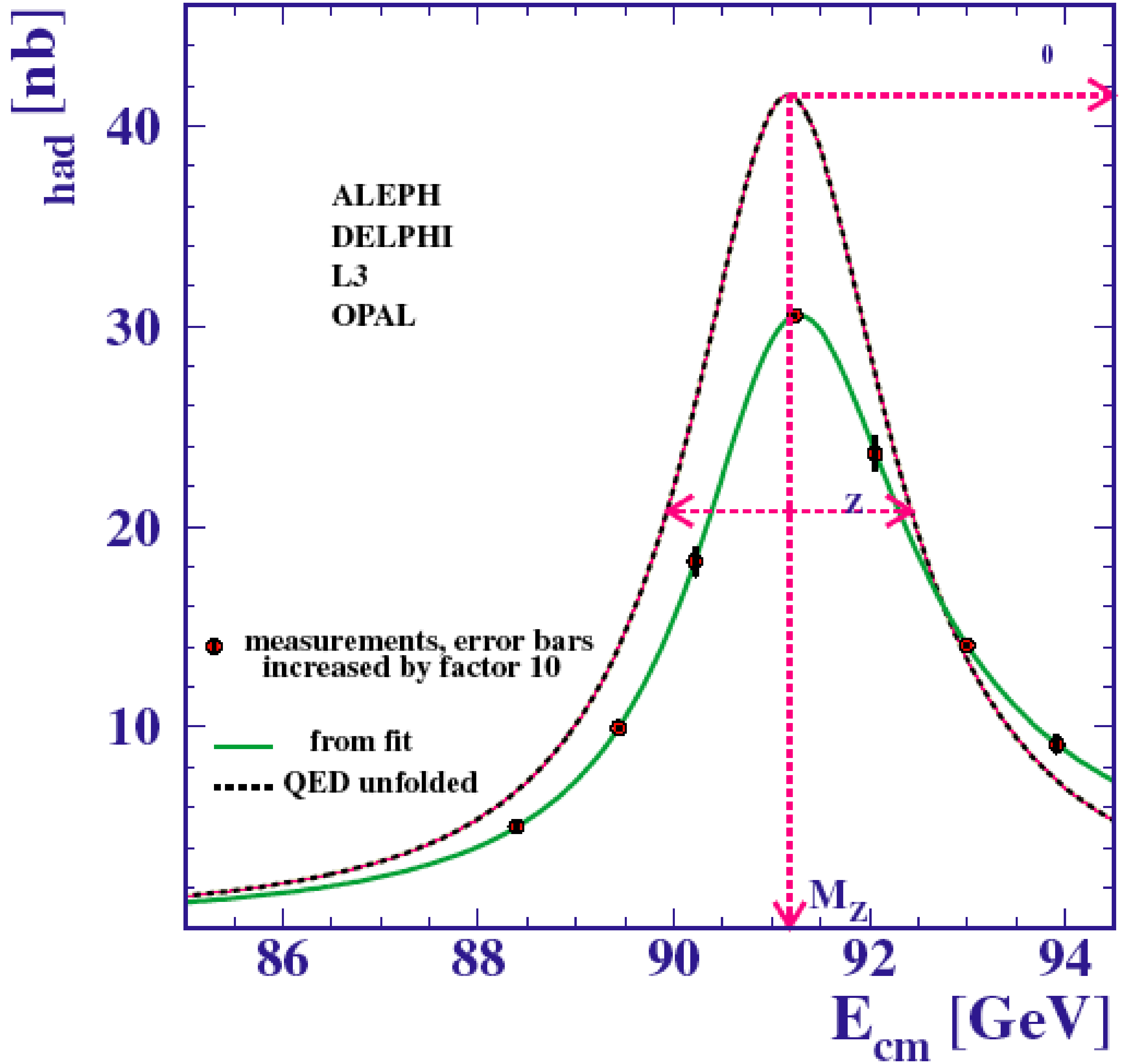
ZFITTER : Lida Kalinovskaya, Pena Christova, Dima Bardin, Tord Riemann, Sabine Riemann, Andrej Arbuzov

KORALZ: Stanislaw Jadach, Bennie Ward

Alain Blondel will describe the precision electroweak experiments tomorrow.   But excuse me if I preview a few of the most important results.

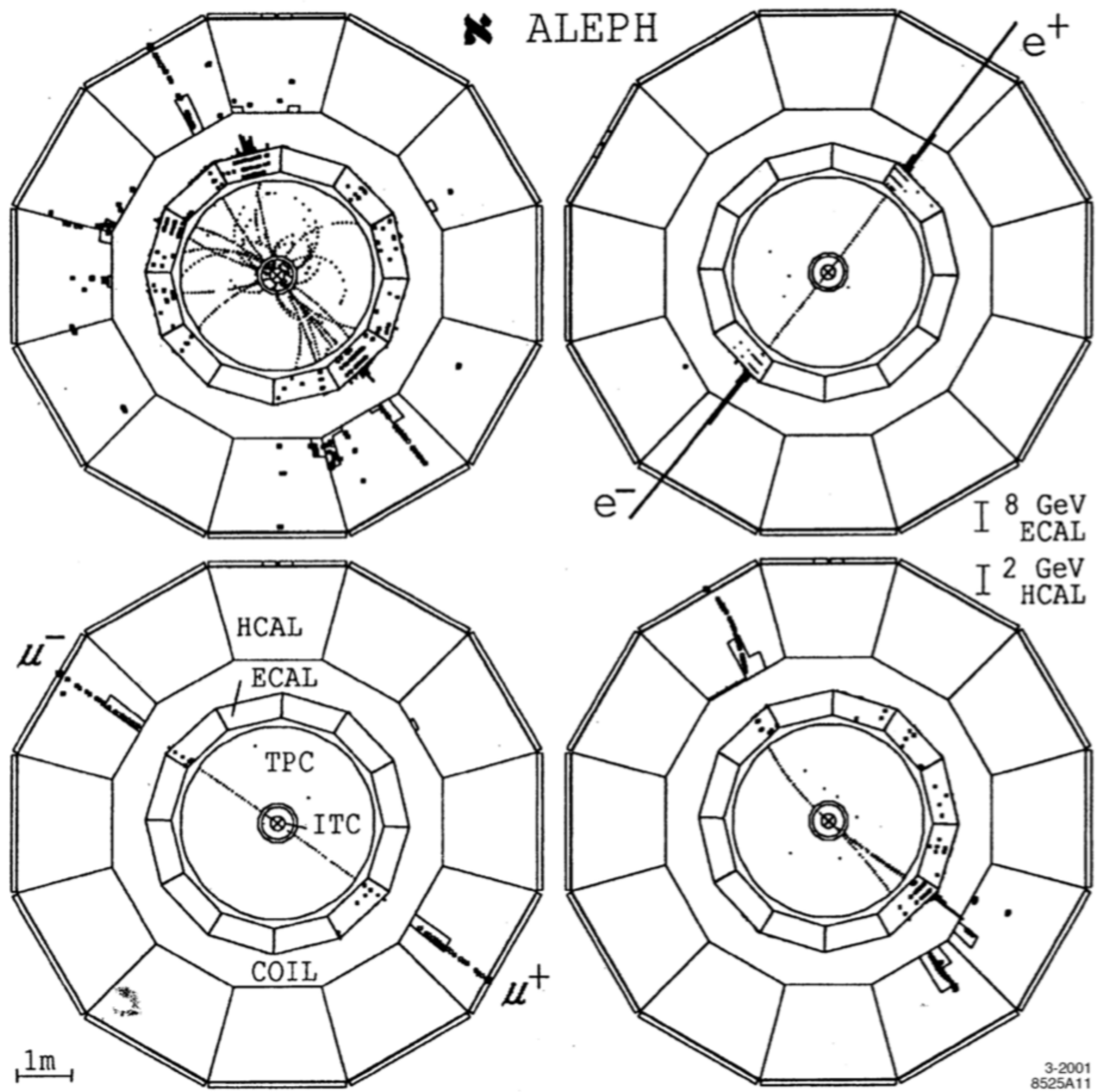
Here is the Z lineshape measurement by the OPAL experiment:



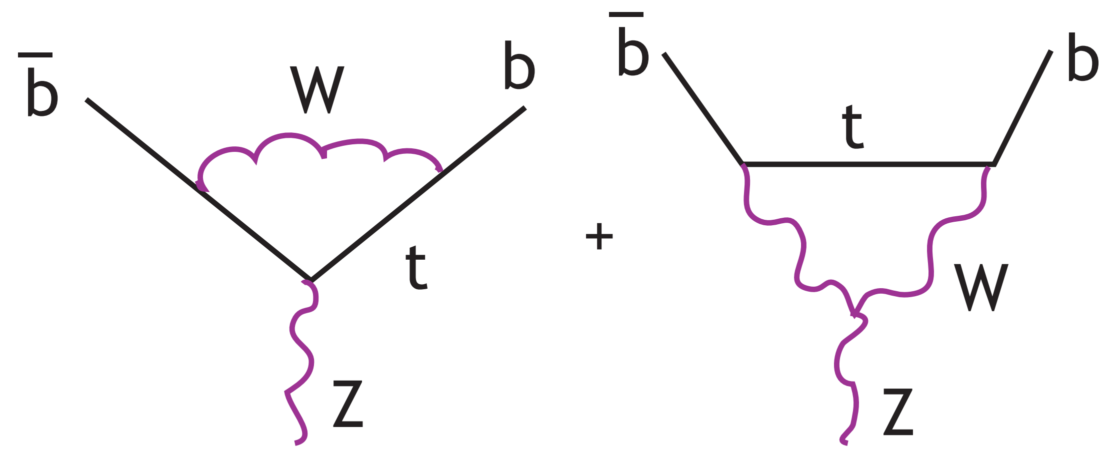


and a composite of the four LEP experiments, showing the effect of ISR

In general, the various classes of Z decays are cleanly separable from one another:



A special process is the decay $Z \to b\bar{b}$. This obtains a special (finite) correction due to the top quark, enhanced by m_t^2/m_W^2 , as pointed out by Bardin, Bilenky, Fedorenko, and Riemann.



This gives a -2% correction, most visible in the ratio

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{ hadrons})}$$

The LEP measurements require this correction.

Another aspect of the Z profile is the pattern of parity asymmetries

$$A_f = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}$$

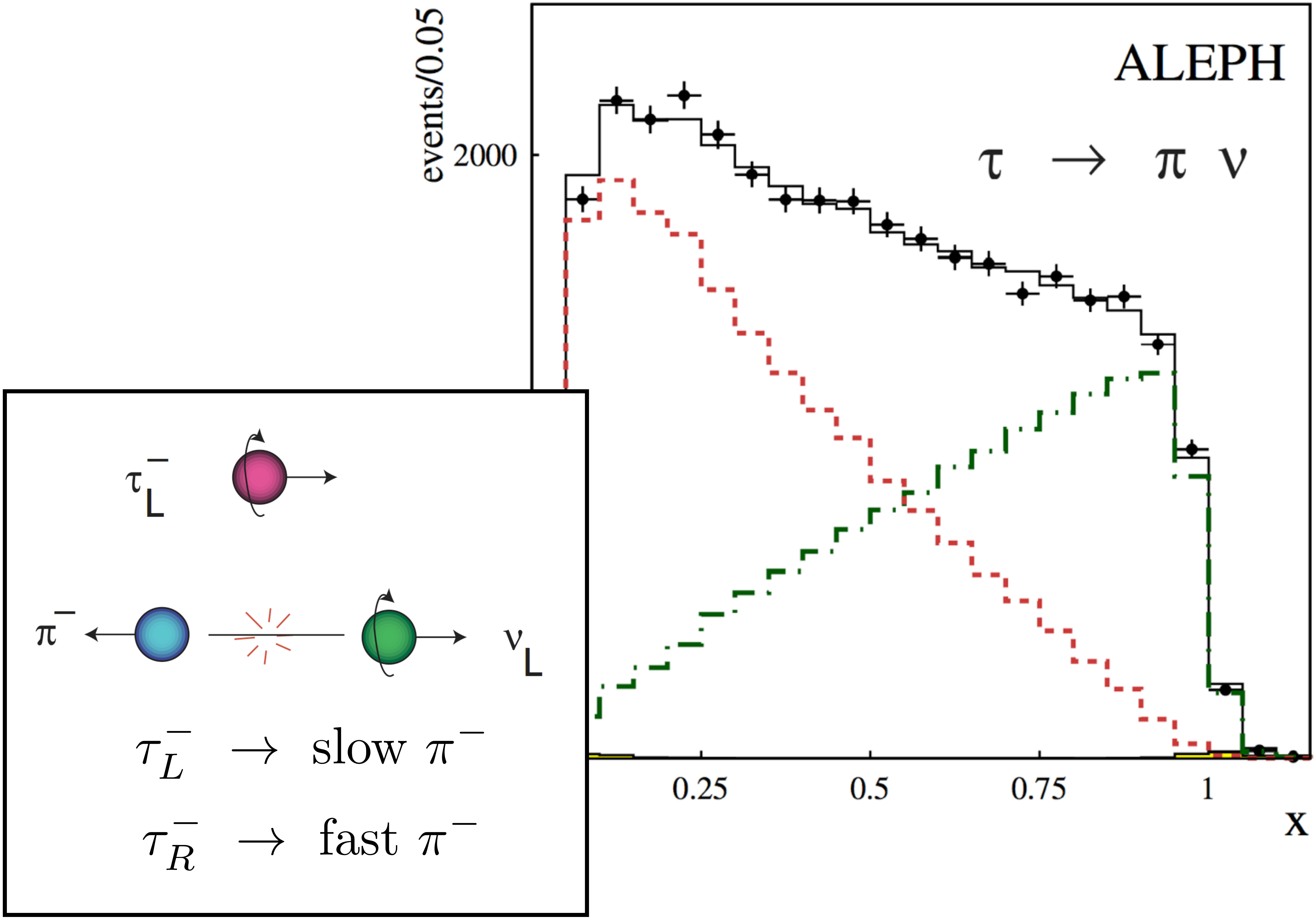
$$= 0.15 \text{ for } \ell \ , \quad 0.63 \text{ for } u \ , \quad 0.94 \text{ for } d$$

With unpolarized beams, the forward backward asymmetries are

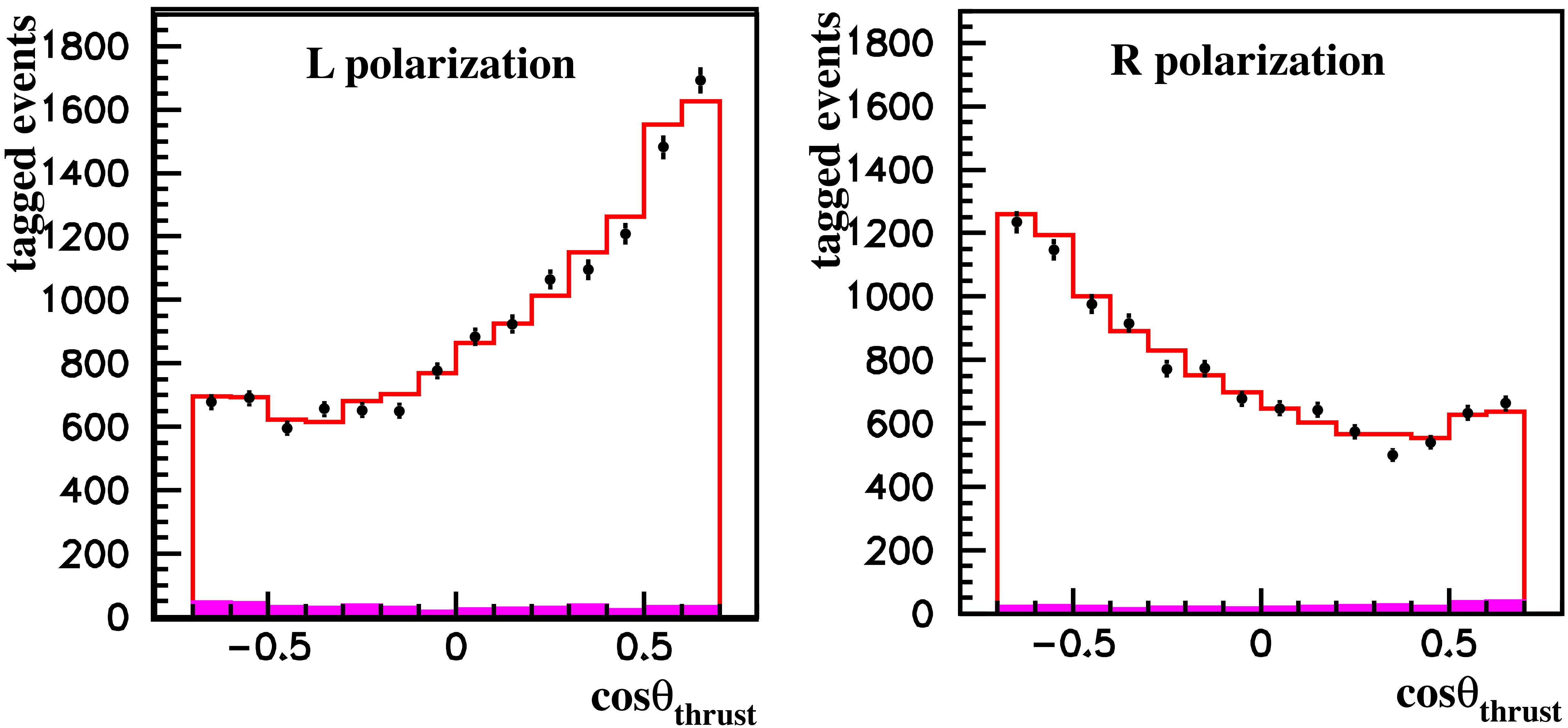
$$A_{FB} = \frac{3}{4} A_e \ A_f$$

This striking pattern is observed in the experiments.

$$e^+e^- \to Z^0 \to \tau^+\tau^-$$



Ab was determined at SLAC using polarized electron beams.



SLD

For a full accounting, one more important effect must be included.   The top quark corrects the difference of the W and Z vacuum polarization amplitudes by an effect proportional to

$$m_t^2/m_W^2$$

The Higgs boson of the Standard Model also contributes a negative correction proportional to

$$\log(m_h^2/m_W^2)$$

The top quark effect is absolutely necessary to reconcile precision electroweak theory and experiment.

There is a complicated history of whether precision electroweak predicted the top quark mass in advance of the CDF/D0 discovery. Certainly a low top quark mass, below about 110 GeV, was inconsistent with the LEP data.   In his notes, Lynn quotes the 1993 ALEPH result

$$m_t = \ 156^{+22+17}_{-25-22} \quad \mathrm{GeV}$$

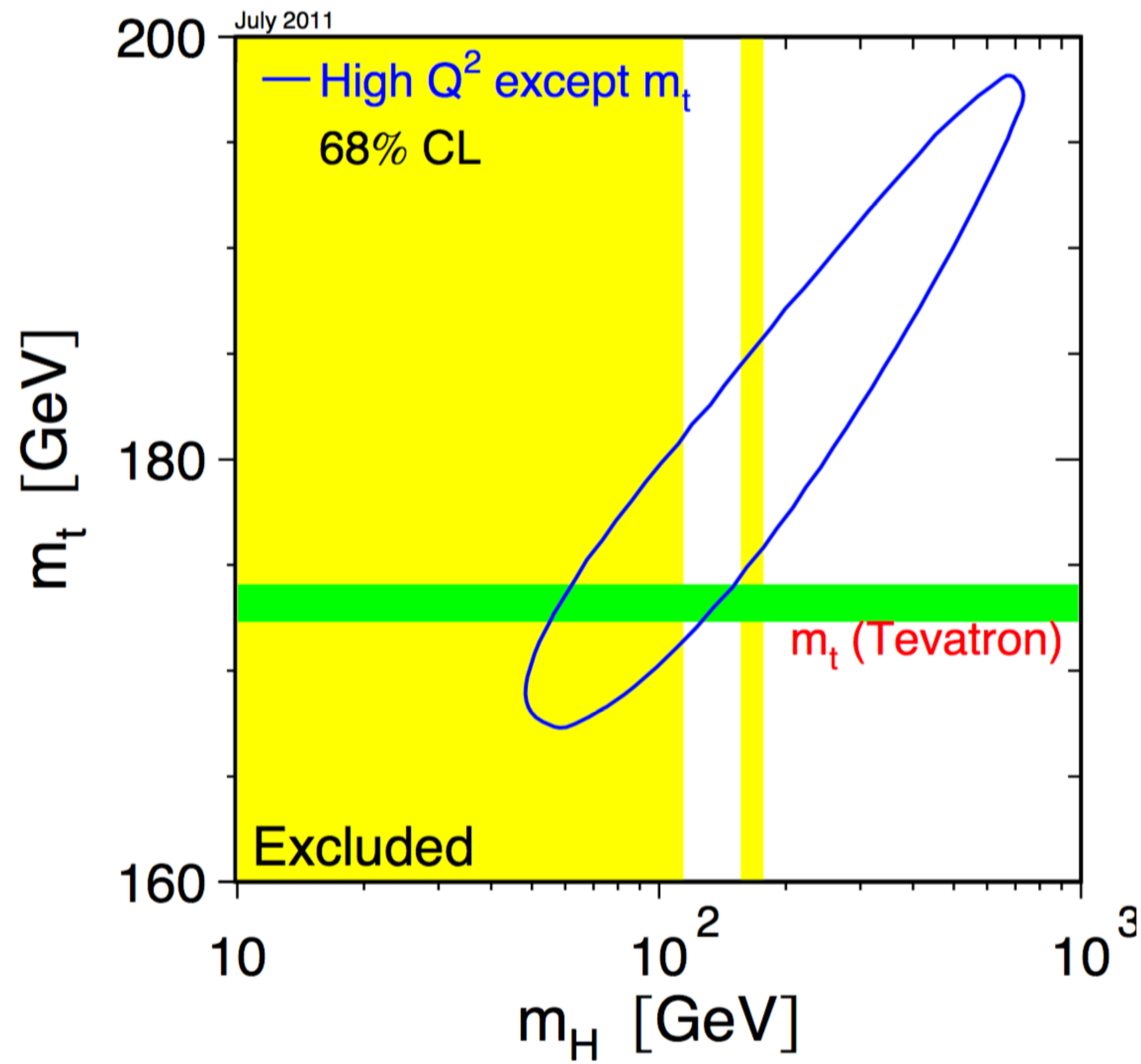
along with many others. The large top masses required in some theories of the Higgs sector were also excluded.

Sid Drell contested my certainty that

$$m_t < 200 \ \mathrm{GeV}$$

We made a bet (in the SLAC bet book) and I won dinner for 4.

Here is a more current reckoning (July 2011):

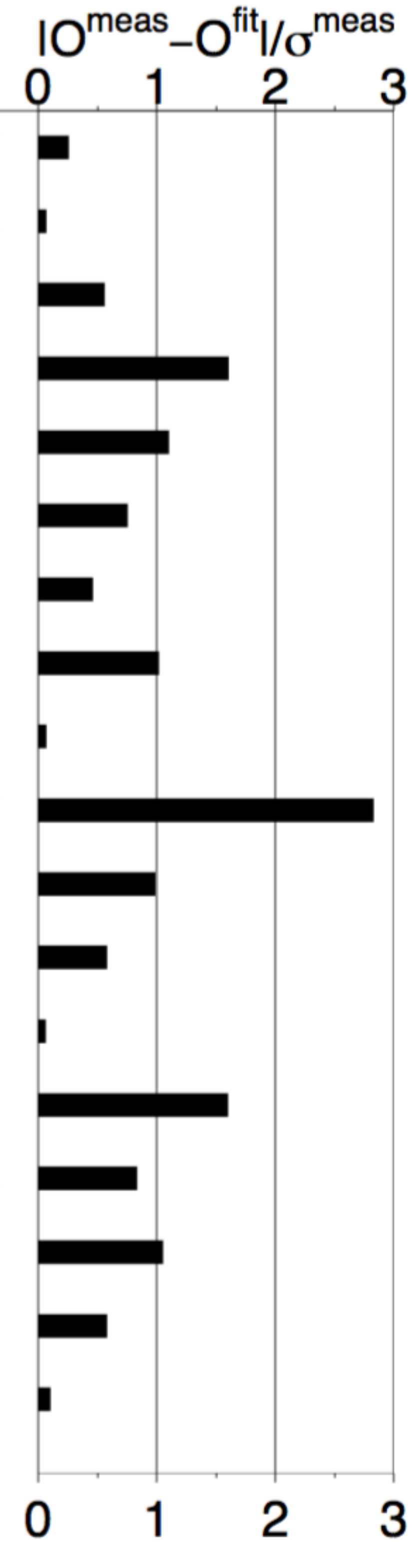


Note also the effect of m_h. The consistency of the data with the measured value of m_t required

$$m_h \sim 100 \text{ GeV}$$

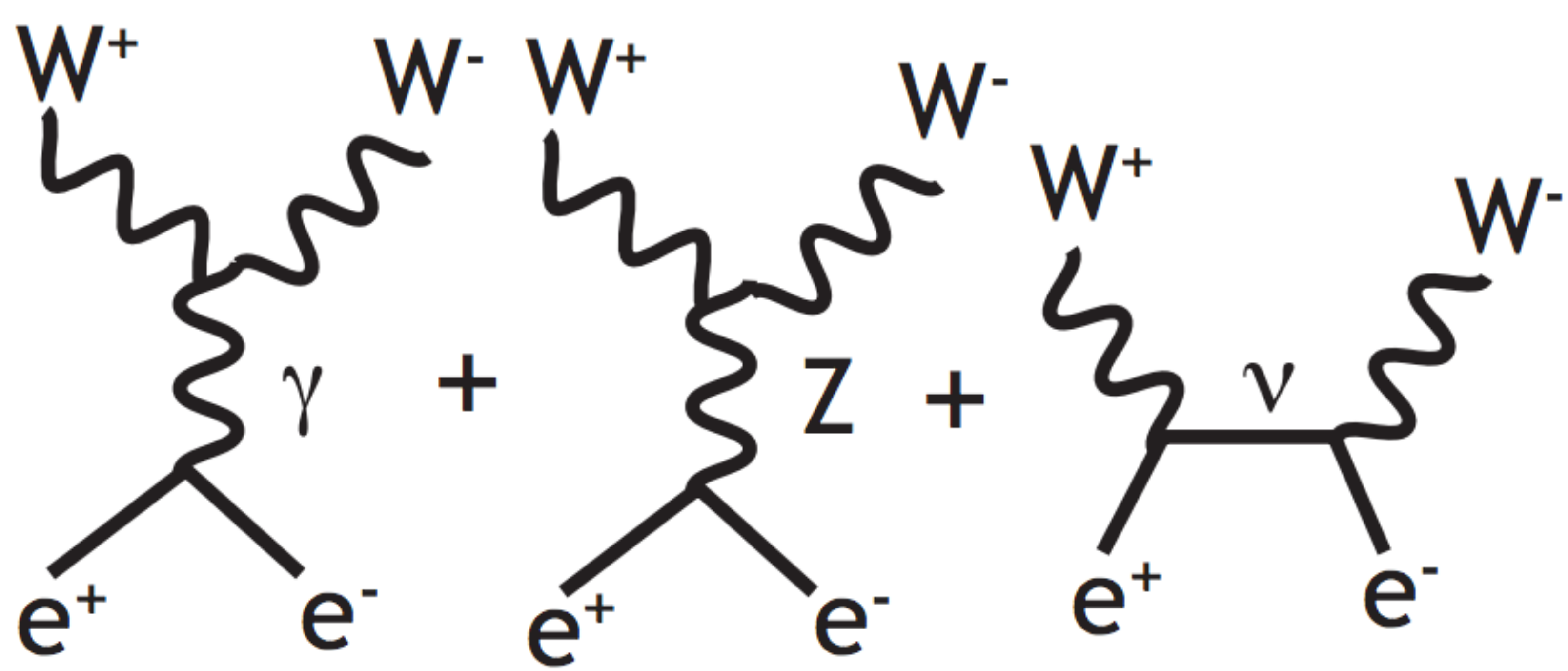
Here is the status of the precision measurements, in relation to the best fit set of SM parameters, as of 2006:

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^0 [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



I cannot leave this subject without discussing one more reaction studied with precision at LEP: $e^+e^- \to W^+W^-$

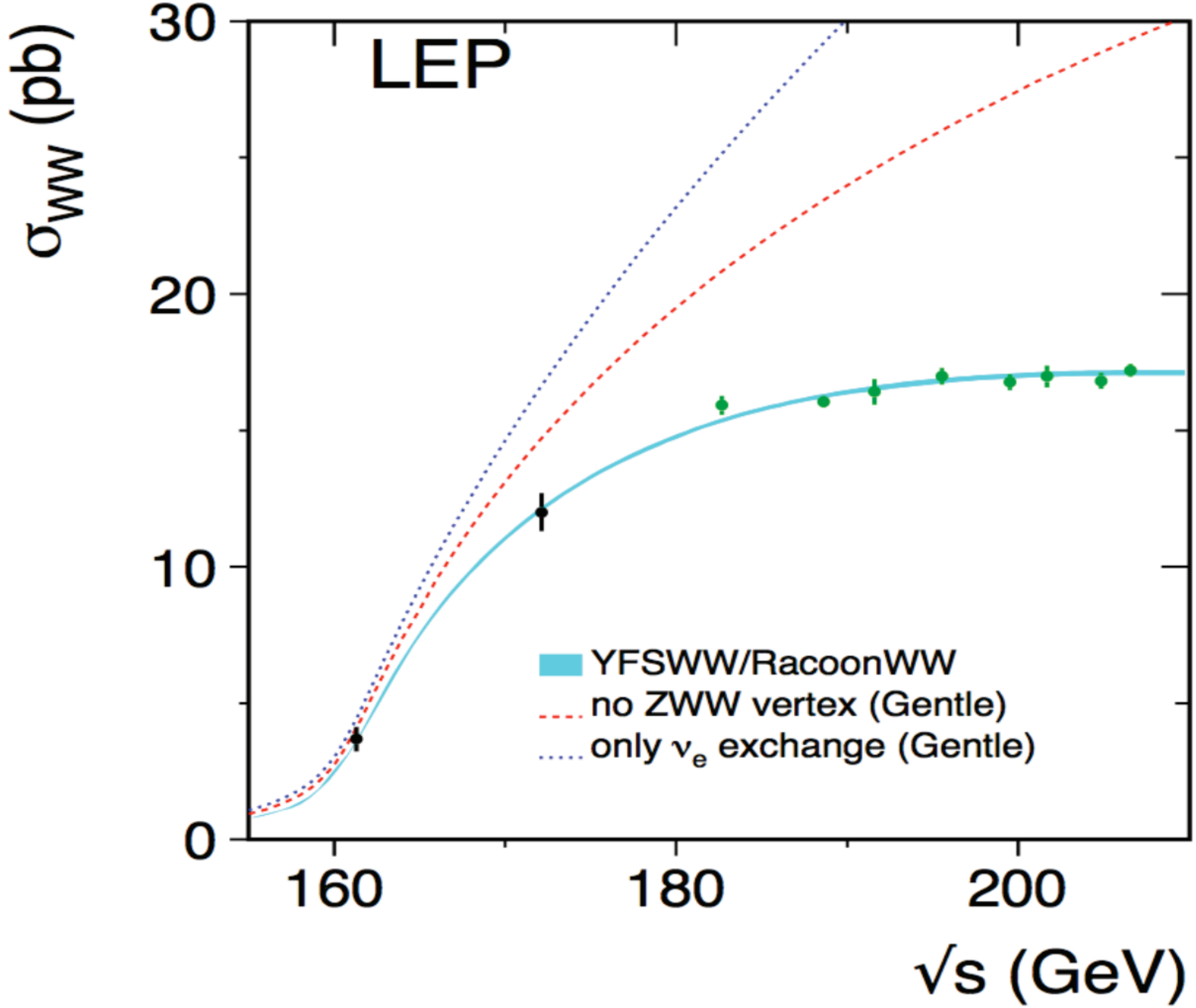
The amplitudes for this process threaten to violate unitarity at high energy. This is avoided in the Standard Model by a delicate cancellation of diagrams, requiring the precise form of the Yang-Mills vertex. The tree level result was found in 1977 by Alles, Boyer, and Buras.



The radiative corrections to this process were first computed in 1987 by Bohm, Denner, Sack, Beenakker, Berends, and Kuijf . However, this calculation assumed stable Ws. To reach the 1% level of accuracy, one must take into account that the W decays, and one must include all $e^+e^- \to 4f$ processes that interfere with the off-shell W decays.

This was done in the RACOON event generator (1999), by Ansgar Denner, Stefan Dittmaier, Markus Roth, and Doreen Wackeroth.

The cross section for $e^+e^- \to W^+W^-$ was measured by the LEP experiments, with this result:



So, the Standard electroweak model is understood now to very high precision, below 1% for all important observables. It passes many nontrivial tests.

Shouldn’t we believe it is exactly correct ?