Moving Long-Range Beam-Beam Encounters in Heavy-Ion Colliders

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Acceleration of “unequal” beams

- HI colliders are often operated with asymmetric beams. LHC: Pb-p; RHIC: Au-d, Al-p, ...
- Requires the collider being capable to accelerate two particle species with different charge-to-mass ratios

Acceleration with same energy and different magnetic rigidity

- Disadvantage: Requires the two beams to have different magnetic fields within the bending magnets
- Advantage: Both beams have the same revolution frequency (fixed LRBB encounter points)

Acceleration with same magnetic rigidity and unequal revolution frequency/energy

- Disadvantage: Requires independent RF systems, the beams have different velocities (moving LRBB encounters)
- Advantage: Applicable in the case of a 2-in-1 bending magnet design (like in the LHC)

Ramp with unequal revolution frequencies

LHC: p-Pb operation

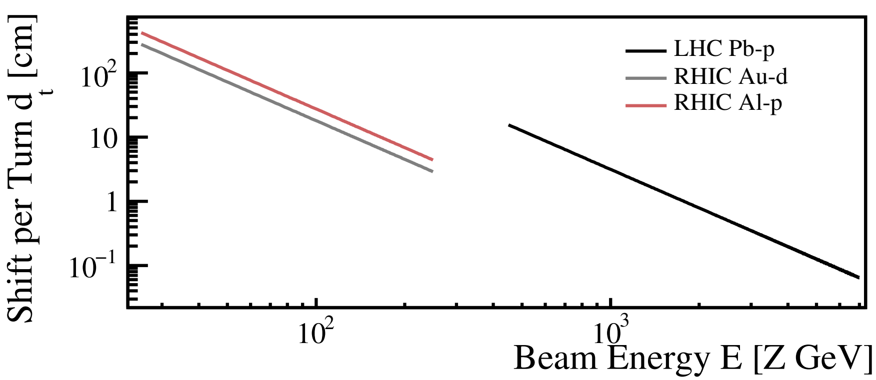
- At 450 Z GeV, the difference of the revolution frequencies is Δf_0=0.13 Hz
- RF frequencies are ramped separately

  → At target energy, cogging of the RF frequencies

  → Energy offset forces beams on dispersive orbits

RHIC: Au-d operation

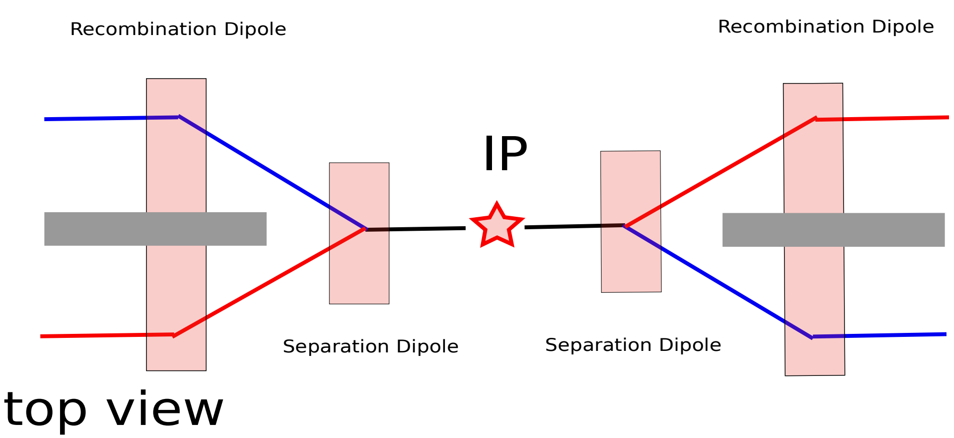
- Since the injection energy in RHIC is significantly lower (25.5 Z GeV), the beam-beam effect is much stronger
- At 25.5 Z GeV, the difference of the revolution frequencies is Δf_0=113 Hz



Moving BBLR effect

- Encounter points are moving due to unequal revolution frequencies
- Possible excitation of *overlap knock-out* (OKO) resonances

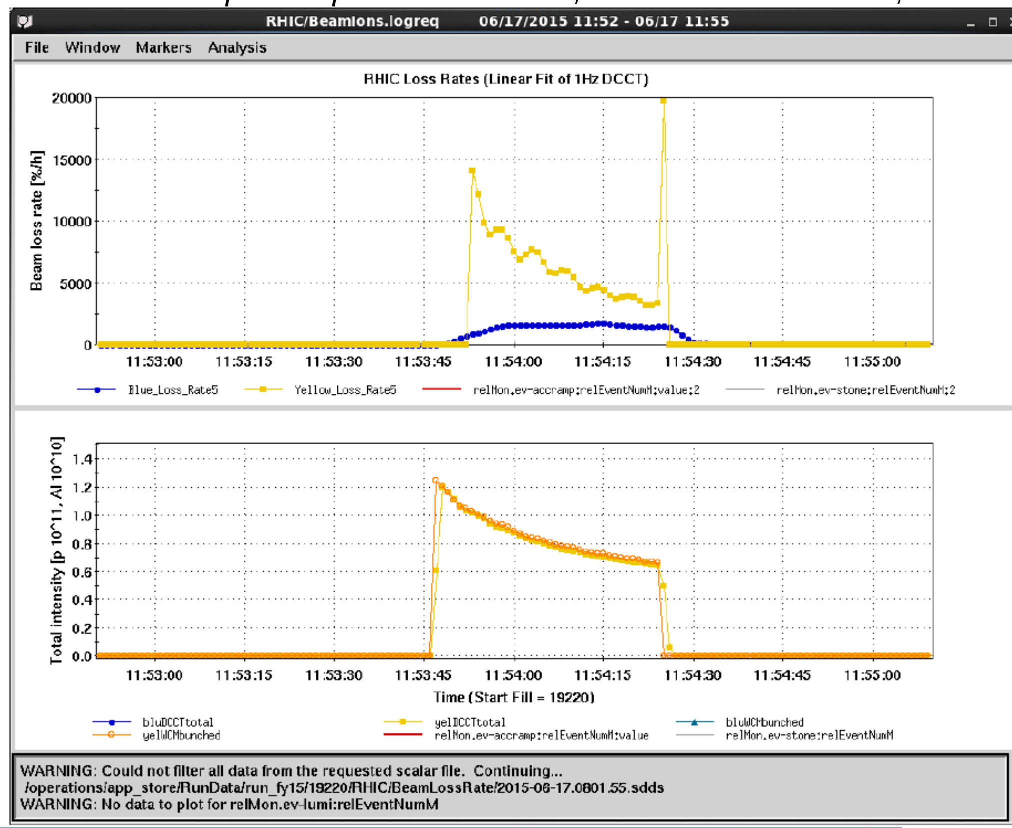
  → Overlap of the transverse betatron-frequency distribution with frequency components of longitudinal shift of encounter points



Motivation: RHIC not operable

- Au-d: encounter points shift by more than 2.5 m per turn at injection
- *Example:* Blue ring filled with 111 p bunches. Then, one Al bunch is injected into the yellow ring → substantial losses.
- RHIC is therefore ramping with equal frequencies (not possible in the LHC)
- p-Pb pilot run 2012 → LHC is operable with unequal revolution frequencies

C. Liu et al., *Attempt to accelerate asymmetric species with unequal frequencies in RHIC*, BNL-108493-2015-IR, 2015.



Previous diffusion model assumes normal-distributed random BB kicks. Does not explain difference between RHIC and the LHC too well. Issues in the case of HL-LHC?

Linear model

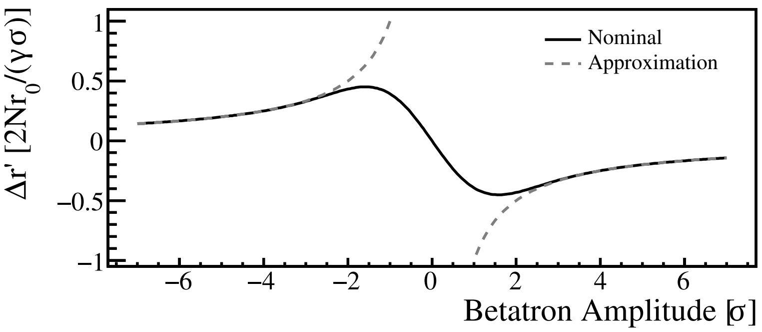
- weak-strong BB model

  LHC: p: strong, Pb: weak

  RHIC: d: strong, Au: weak

- Looking at a single bunch at injection energy: RF frequencies not cogged, no head-on collisions
- *1/r* approximation of the beam-beam force (large separations)
- linear model based on affine matrices → extended phase space

$$\vec{r} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ 1 \end{pmatrix}$$



$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} & c_1 \\ m_{21} & m_{22} & m_{23} & m_{24} & c_2 \\ m_{31} & m_{32} & m_{33} & m_{34} & c_3 \\ m_{41} & m_{42} & m_{43} & m_{44} & c_4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{\partial \Delta x'(d_x, d_y)}{\partial x} & 1 & \frac{\partial \Delta x'(d_x, d_y)}{\partial y} & 0 & \Delta x'(d_x, d_y) \\ 0 & 0 & 1 & 0 & 0 \\ \frac{\partial \Delta y'(d_x, d_y)}{\partial x} & 0 & \frac{\partial \Delta y'(d_x, d_y)}{\partial y} & 1 & \Delta y'(d_x, d_y) \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Linear model (II)

- linear transfer between BB encounters
- assuming <u>no tune spread in the IRs</u> → single matrix for whole IR including all LRBB encounters
- transfer in the arcs considers spread in tunes

$$M(s_0, s_1) = \begin{pmatrix} 1/\sqrt{\beta_1} & 0 \\ \alpha_1/\sqrt{\beta_1} & \sqrt{\beta_1} \end{pmatrix}^{-1} \begin{pmatrix} \cos(\mu) & \sin(\mu) \\ -\sin(\mu) & \cos(\mu) \end{pmatrix} \begin{pmatrix} 1/\sqrt{\beta_0} & 0 \\ \alpha_0/\sqrt{\beta_0} & \sqrt{\beta_0} \end{pmatrix}$$

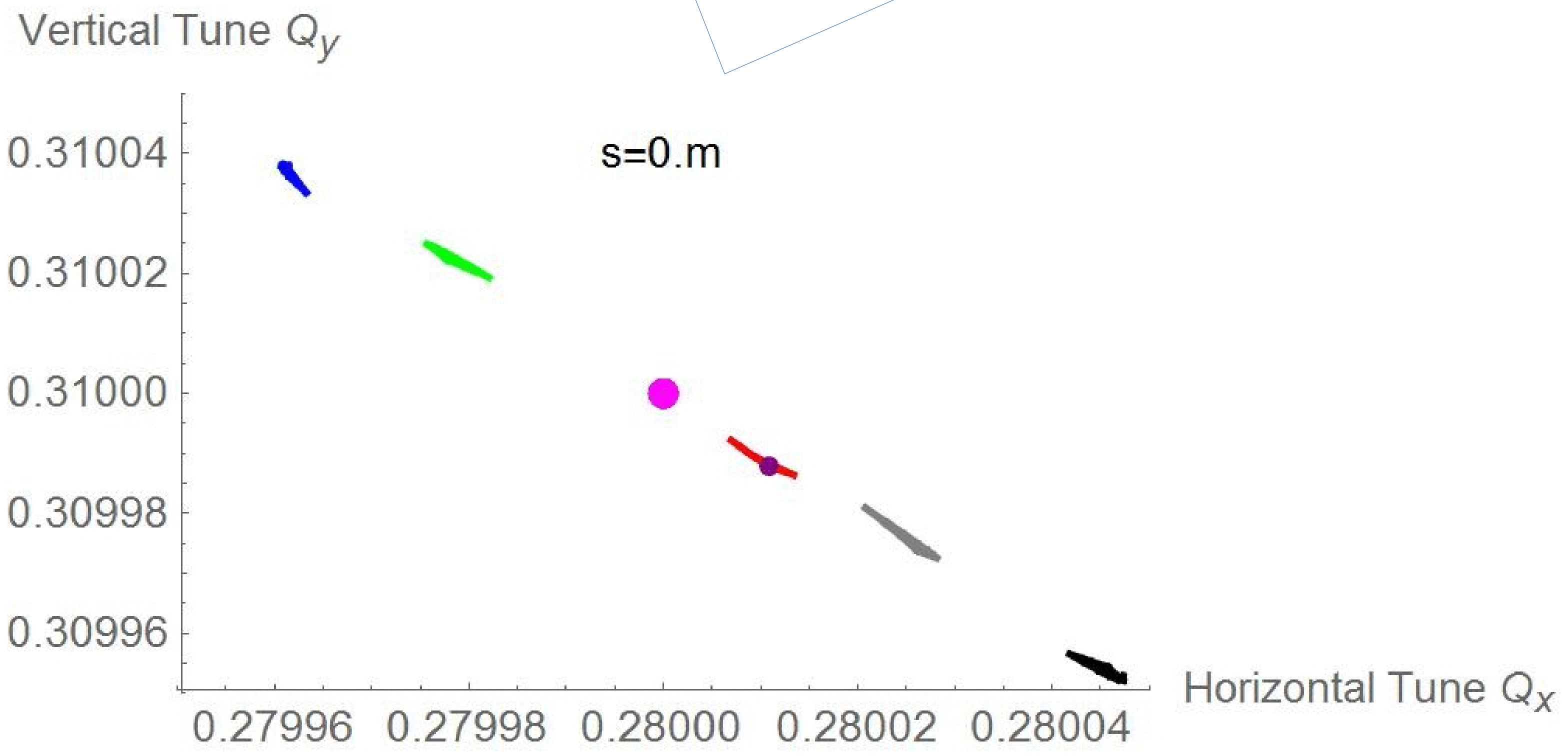
- one-turn transfer matrix of the form

$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} & c_1 \\ m_{21} & m_{22} & m_{23} & m_{24} & c_2 \\ m_{31} & m_{32} & m_{33} & m_{34} & c_3 \\ m_{41} & m_{42} & m_{43} & m_{44} & c_4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- easy tune calculation via eigenvalues $Q_i = arg(\lambda_i)/2\pi$
- shift of closed orbit obtained from eigenvector corresponding to $\lambda = 1$

Linear vs non-linear beam-beam model (Lie algebra)

The concept of a tune footprint does not really apply here…



100 ns uniform proton filling pattern; more than 60 long-range beam-beam encounter

Emittance growth

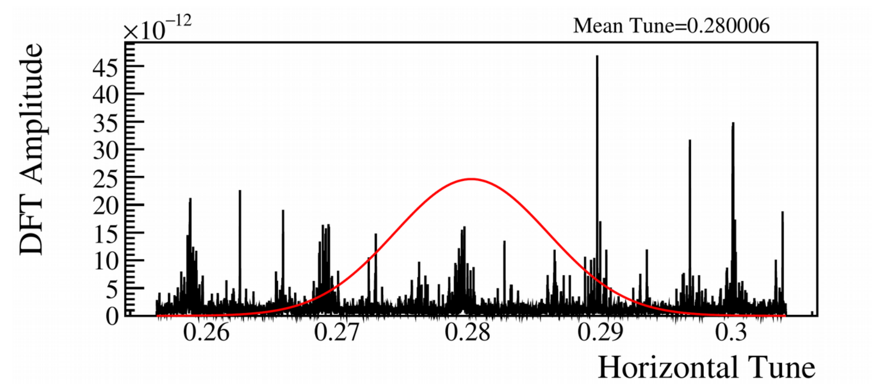
Time-modulated dipole-like kicks

- linear growth of emittance in time
- growth rate proportional to square of Fourier amplitudes

Time-modulated focusing strength

- non-linear growth (depends on autocorrelation of the quadrupole noise)
- tune oscillation introduces sidebands of existing resonances → potential sidebands close to nominal tune

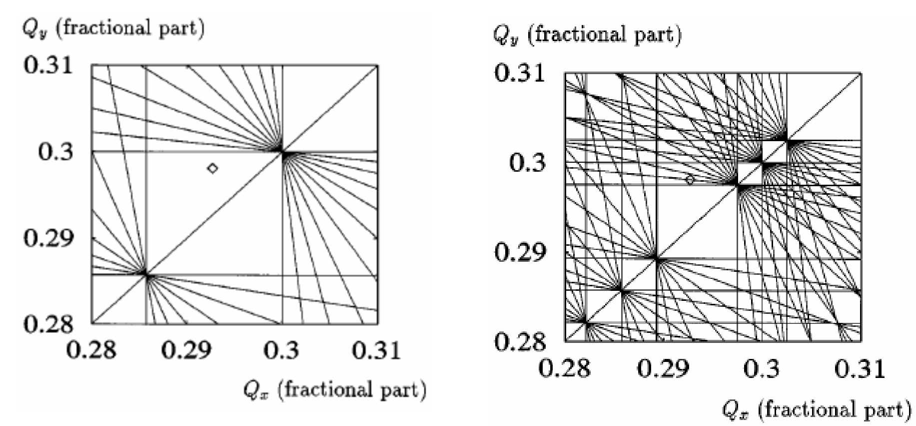
$$lQ_x + mQ_y + n\frac{f_{\mathrm{mod}}}{f_{\mathrm{rev}}} = r, \quad \text{with } l, m, n, r \text{ integers}$$



$$T_N = M\,D\,M^{-1}\,M\,D\,M^{-1}\,M\,D\,M^{-1}\,M\,D\,M^{-1}\ldots = M\,D^N\,M^{-1}$$

$$T_N = M_1 D_1 M_1^{-1} M_2 D_2 M_2^{-1} M_3 D_3 M_3^{-1} M_4 D_4 M_4^{-1} \ldots = ?$$

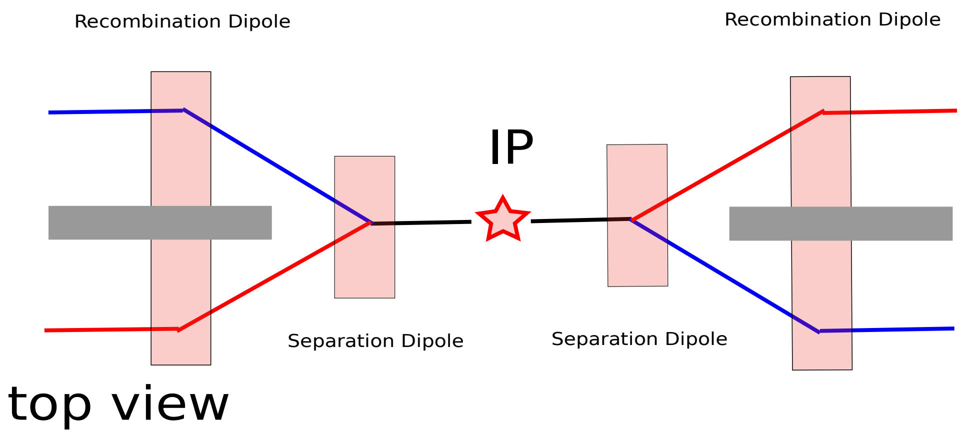
2nd case: eigenvalues not necessarily on complex unit circle

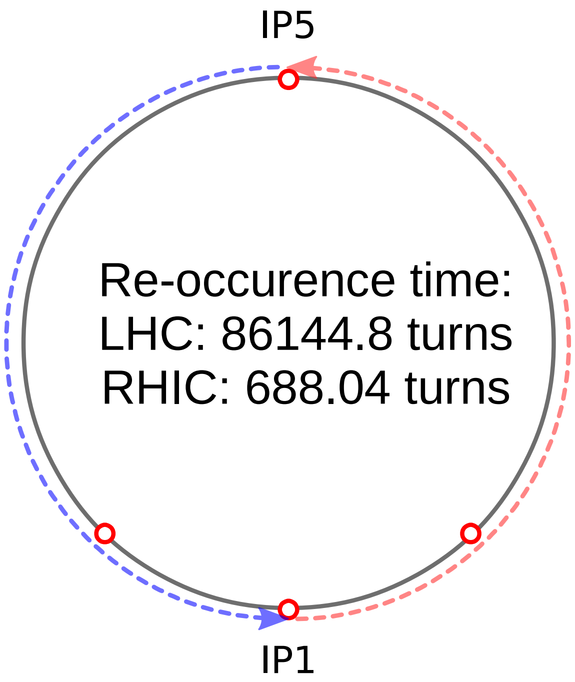


O. S. Brüning and F. Willeke, Reduction of Proton Losses in HERA by Compensating Tune Ripple due to Power Supplies, Phys. Rev. 76 (20), 1996.

LHC vs RHIC

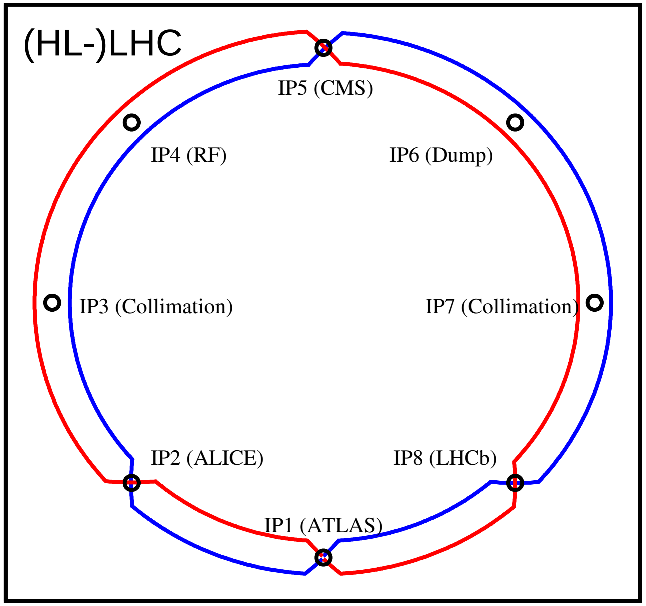
- LHC: Distance between ZDCs in ATLAS/CMS roughly 285 m
  → up to 20 LRBB encounter (assuming an uniform 100 ns bunch train)
- RHIC: Distance between beam-pipe crotches only 32 m
  → 1 (1 bunch and 55 bunches) or 2 (111 bunches) LRBB encounter
- **Important concept:** Recurrence time
  → Bunch encounters the same bunch of the other beam twice per revolution
  → *Some* periodicity when encounter points have shifted by half of the accelerator circumference

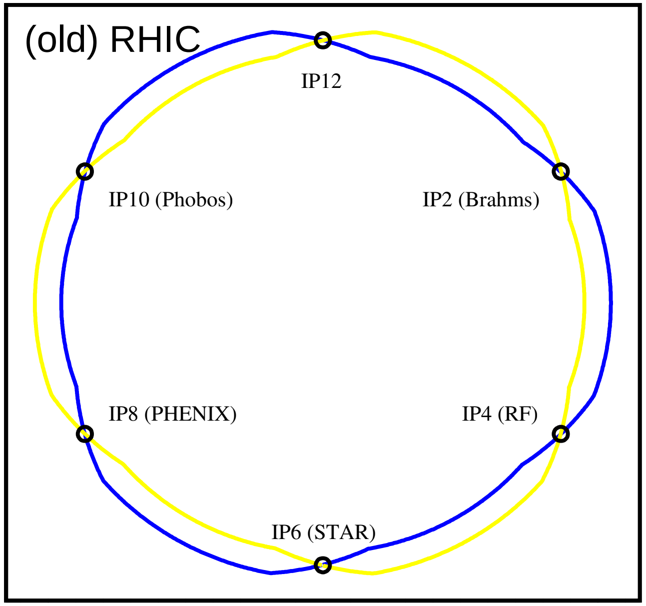




LHC vs RHIC at injection

Parameter	Symbol	Unit	LHC (Pb/p)	HL-LHC (Pb/p)	RHIC (Au/d)
Injection Energy	E_0	Z GeV	450	450	25.5
Circumference	C	m	26658.88	26658.88	3833.85
Relativistic Lorentz Factor	γ	1	(190,480)	(190,480)	(10.52, 13.00)
Harmonic Number	h	1	35640	35640	360
Bunch Harmonic (no gaps)	k	1	891	1782	1, 60, 120
Normalized Horizontal Emittance	ϵ_x	μm	(2.0,1.0)	(2.0,1.0)	(2.5,2.5)
Normalized Vertical Emittance	ϵ_y	μm	(1.5,1.0)	(1.5,1.0)	(2.5,2.5)
Horizontal Chromaticity	ξ_x	1	15	15	-2
Vertical Chromaticity	ξ_y	1	15	15	-2
Horizontal Tune	Q_x	1	64.28	62.27	28.277
Vertical Tune	Q_y	1	59.31	60.295	29.287
Relative Momentum Spread	$\Delta p/p_0$	10^{-4}	3.9	3.9	15.0
Revolution frequency difference	Δf_0	Hz	0.13	0.13	113
Particles per Bunch	N	10^8	(2,200)	(2,200)	(70,1000)
Shift per turn	d_t	m	0.15	0.15	2.78
Reoccurrence Time	T_m	turns	86144.8	86144.8	688.04

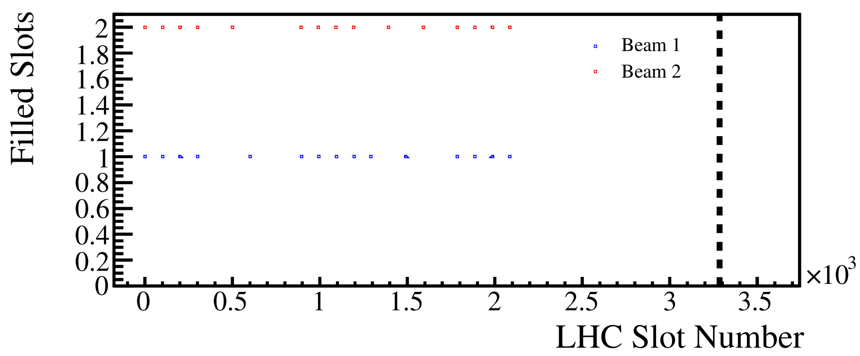




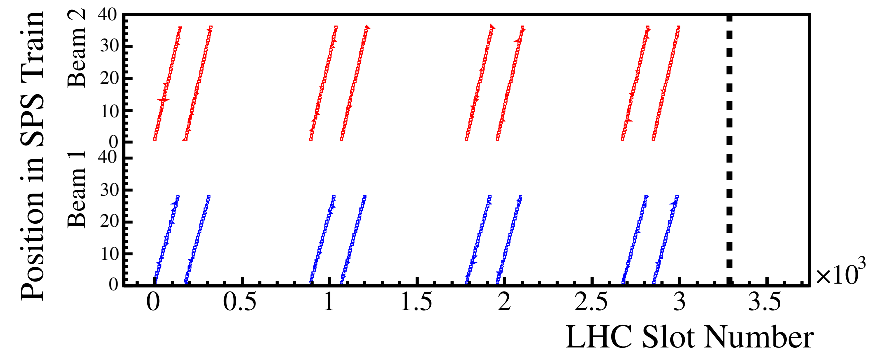
RHIC:
- 18 smaller gammas
- 7 times smaller circumference
- 6 IRs (vs 4 in LHC)
- Separations always vertical (alternating)

Parameter	Symbol	Unit	LHC IP1	LHC IP2	LHC IP5	LHC IP8	HL-LHC IP1	HL-LHC IP2	HL-LHC IP5	HL-LHC IP8	RHIC IP2/6/10	RHIC IP4/8/12
Separation/Crossing Plane			(h/v)	(h/v)	(v/h)	(v/h)	(h/v)	(h/v)	(v/h)	(v/h)	(v/h)	(v/h)
Total Separation	d	mm	-4	7	4	-7	4	-7	4	-7	-10	10
Beta function at IP	β^*	m	11	10	11	10	6	10	6	10	10	10
External Crossing Angle	θ	μrad	-170	170	170	-170	-295	170	295	-170	0	0
Considered Range from IP		m	142.75	112.58	142.75	112.58	127.135	112.58	127.135	112.58	16	16

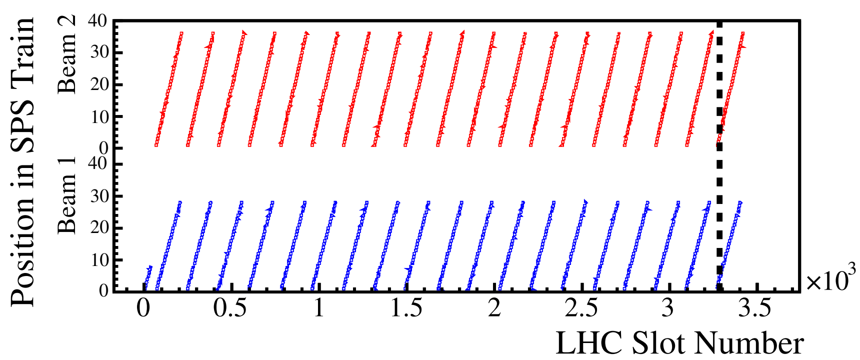
Filling patterns



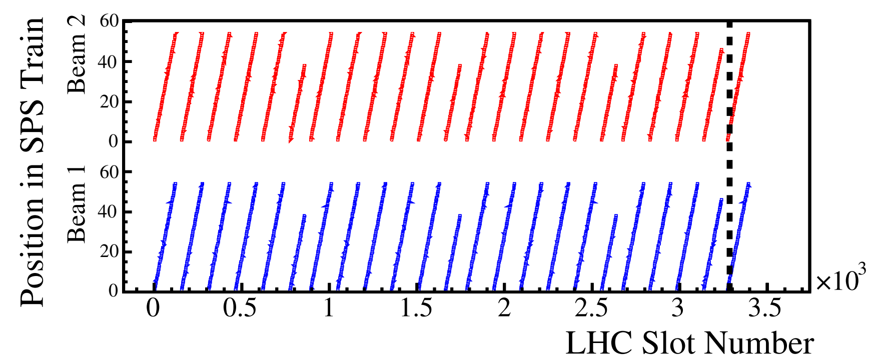
2012 p-Pb pilot filling scheme: 15 single bunches



Fill 5546 (HI'16): 8 trains, B2: 36 p bunches per train, 100 ns spacing



Fill 5554 (HI'16): 19 trains, B2: 36 p bunches per train, 100 ns spacing

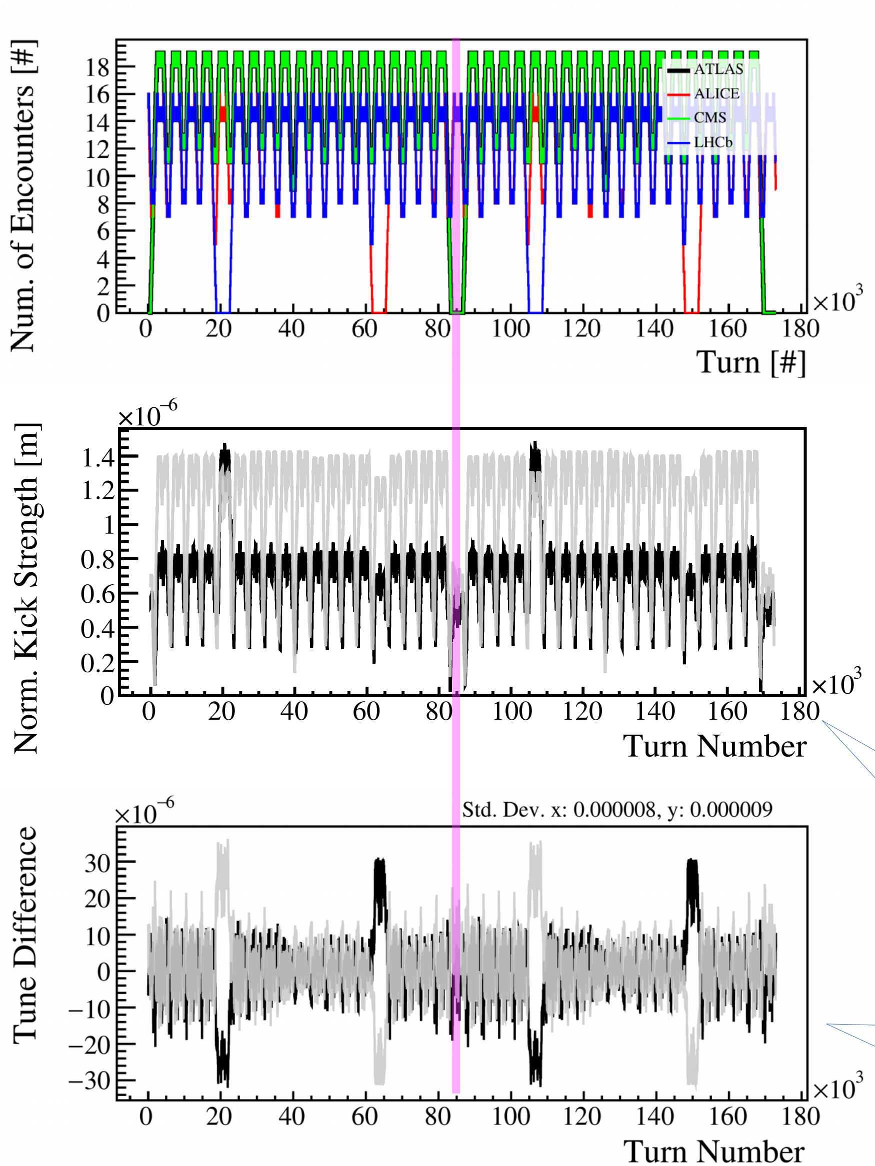


HL-Pattern by J. Jowett (Charmonix 2017) 23 trains, 56 bunches each, 50 ns spacing

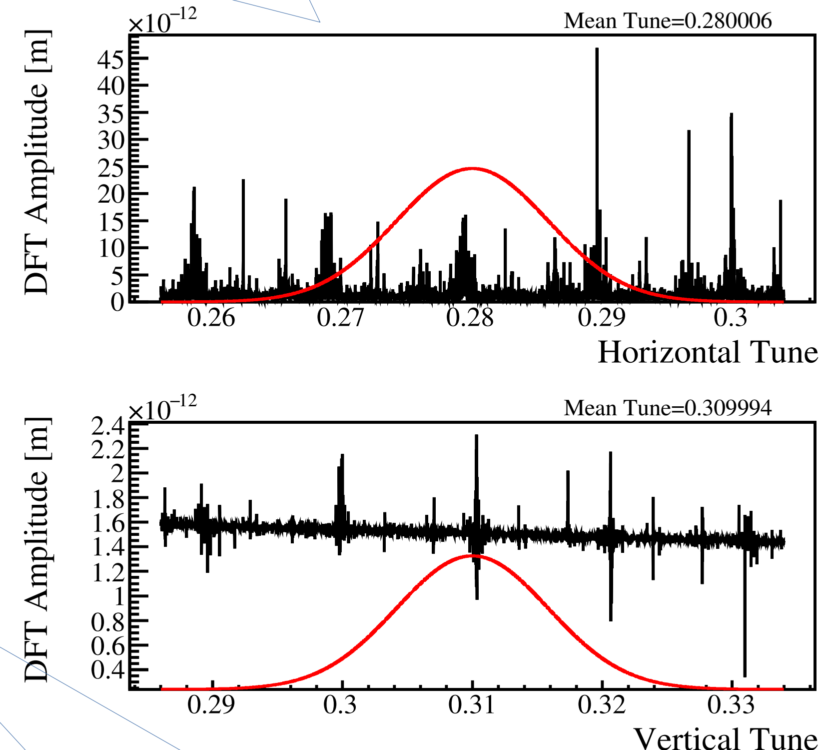
RHIC: Harmonic Number of 360.
Either 1 bunch, 55 bunches (AG: 5 empty slots) or 111 bunches (AG: 9 empty slots).

5554 filling scheme (19 trains)

black:
horizontal
gray:
vertical



combined DFT of kicks from all IRs

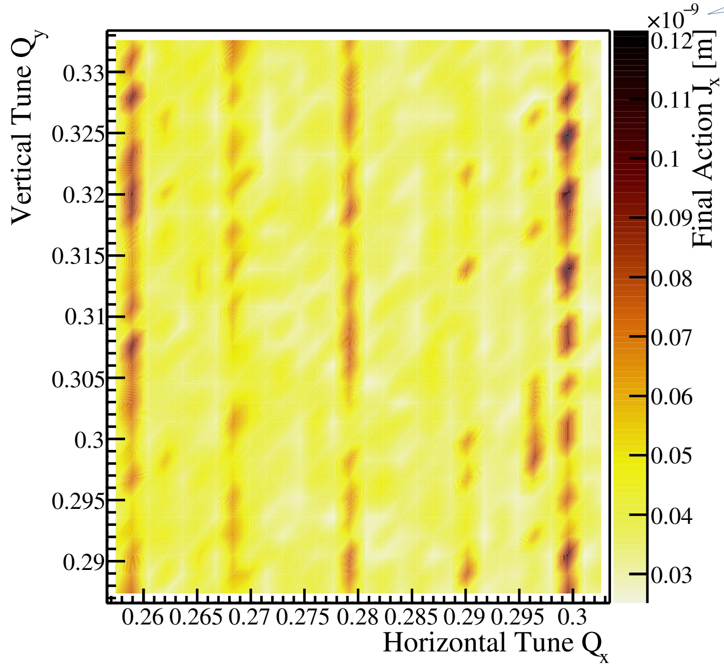


from one-turn matrix
of particle with nominal tune

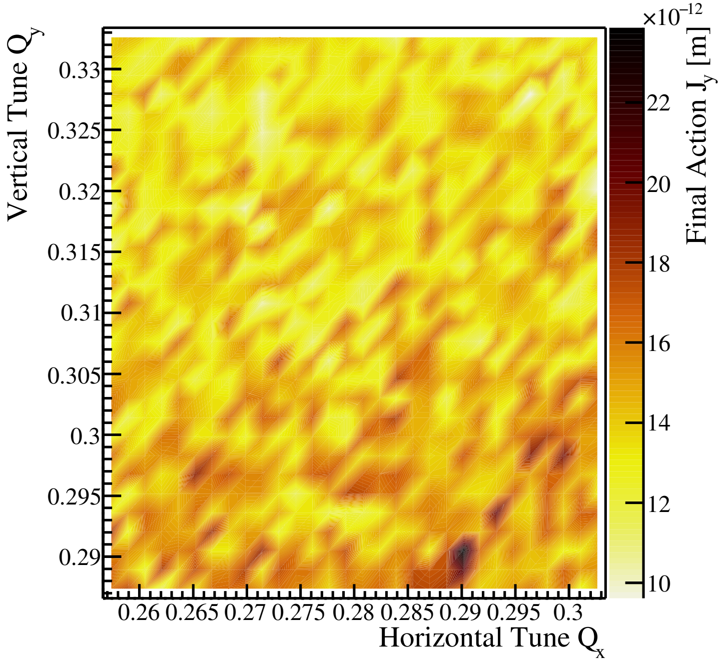
LHC: tune scan

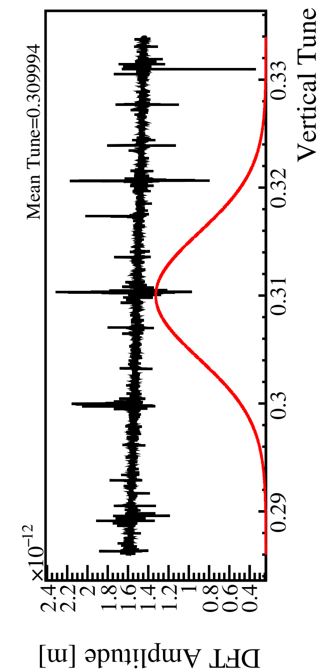
1% of real emittance

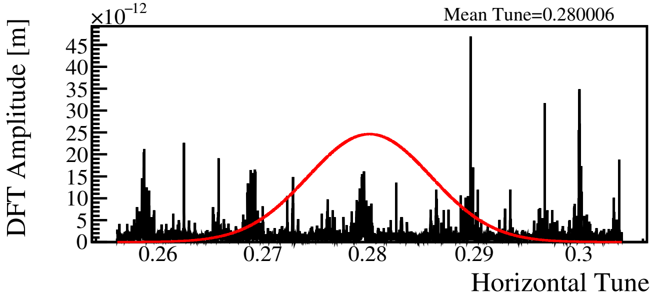
LHC - 5554 Pattern



LHC - 5554 Pattern





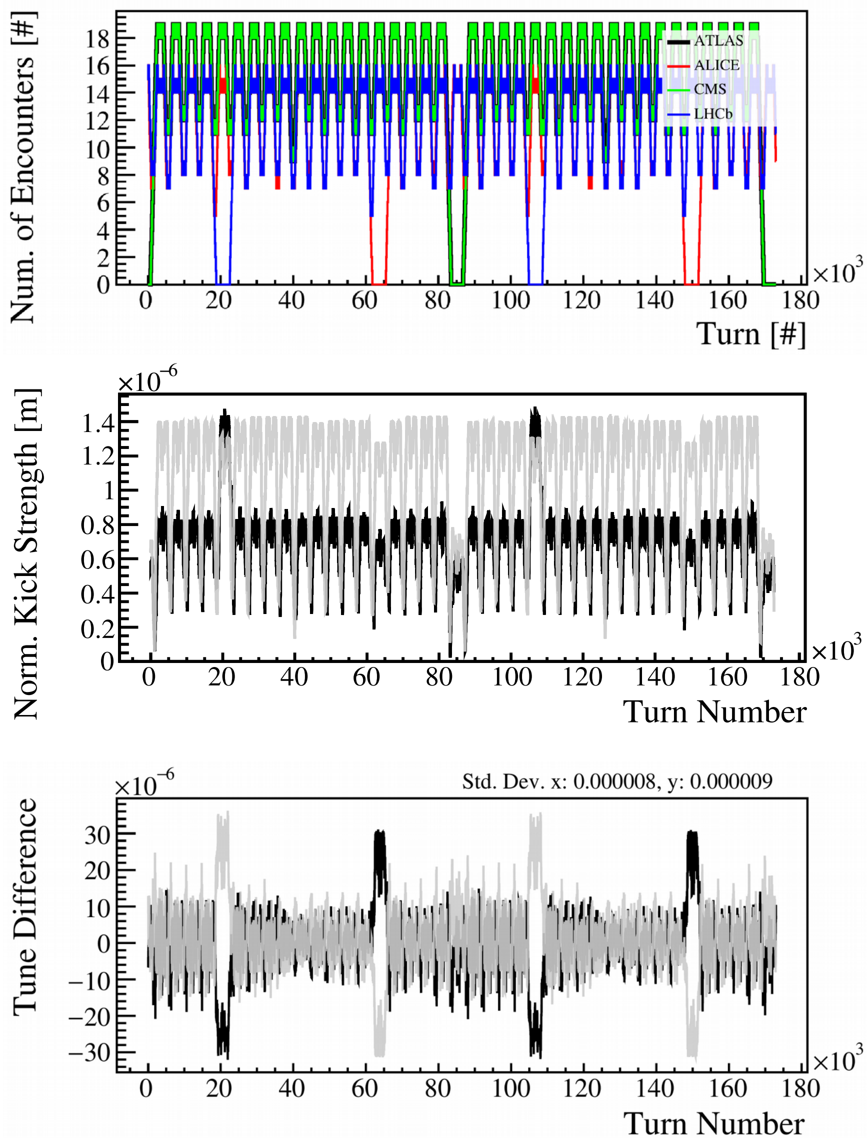


Distribution of 100k particles in “unperturbed” tune space. Simulation over 200k turns with zero starting conditions.

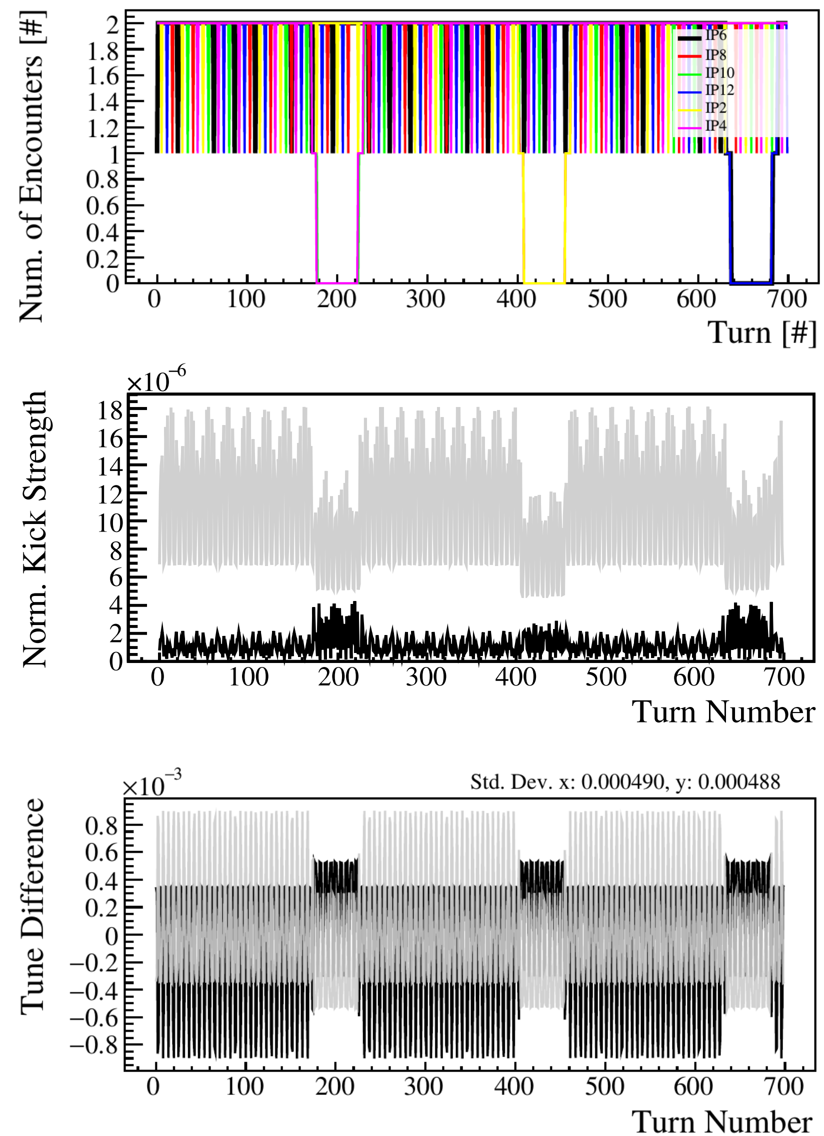
Growth seems to be untouched by tune modulation.

Comparison with RHIC (111 b)

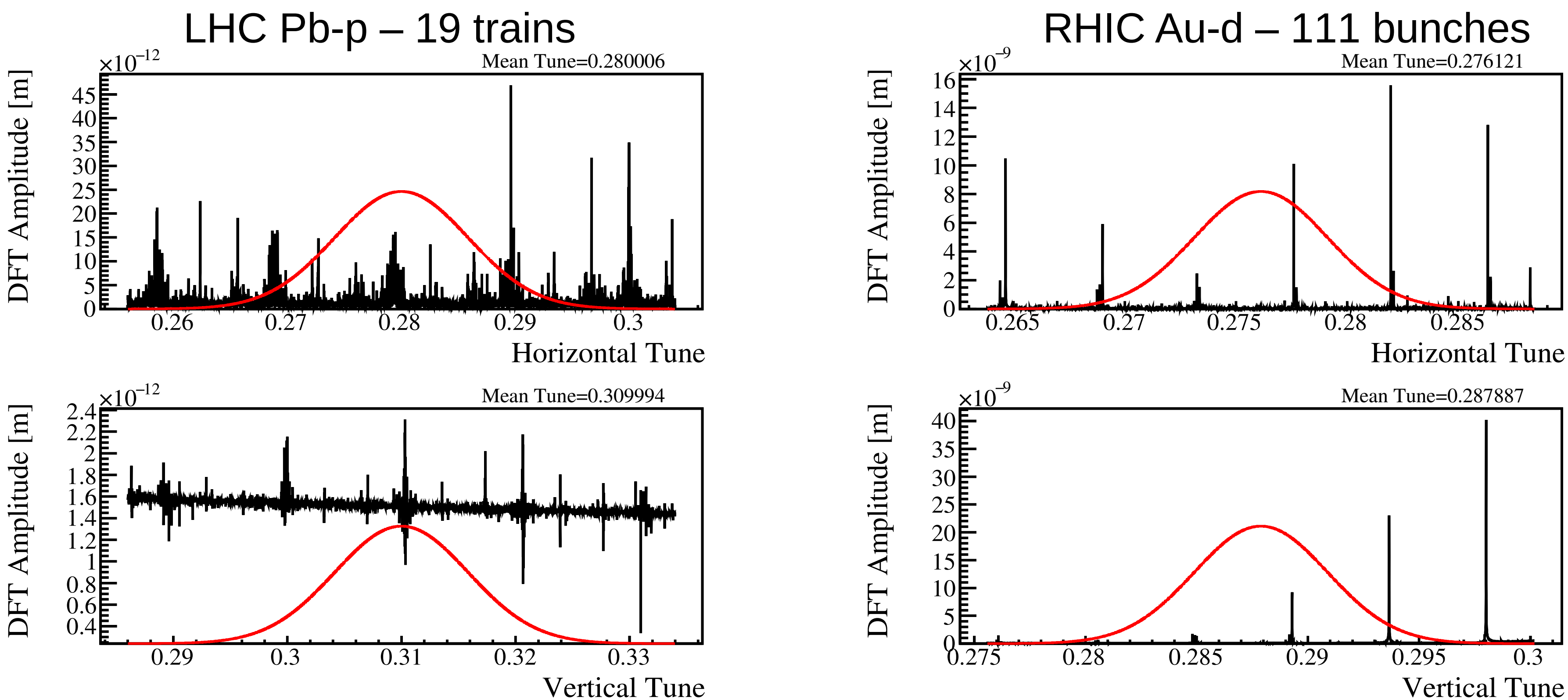
LHC Pb-p – 19 trains



RHIC Au-d – 111 bunches

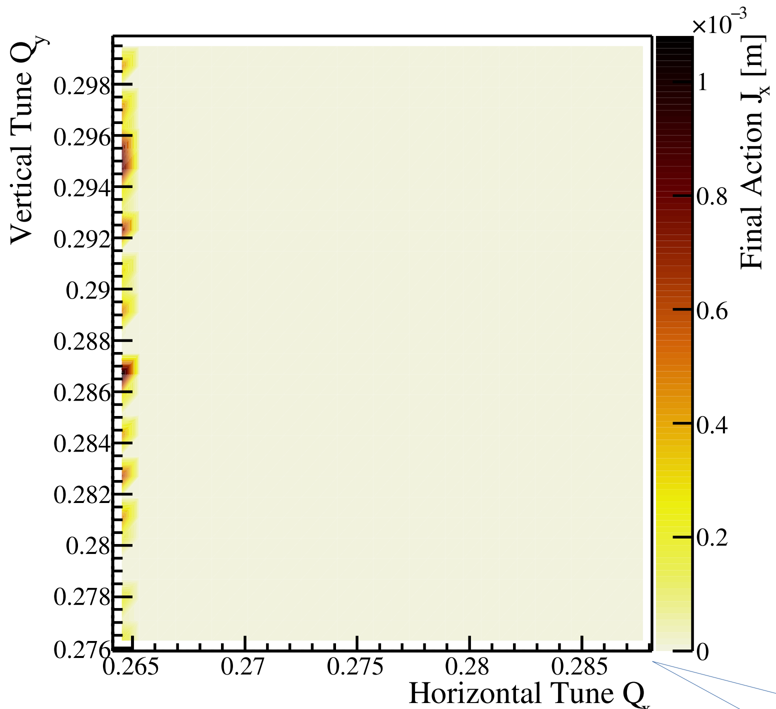


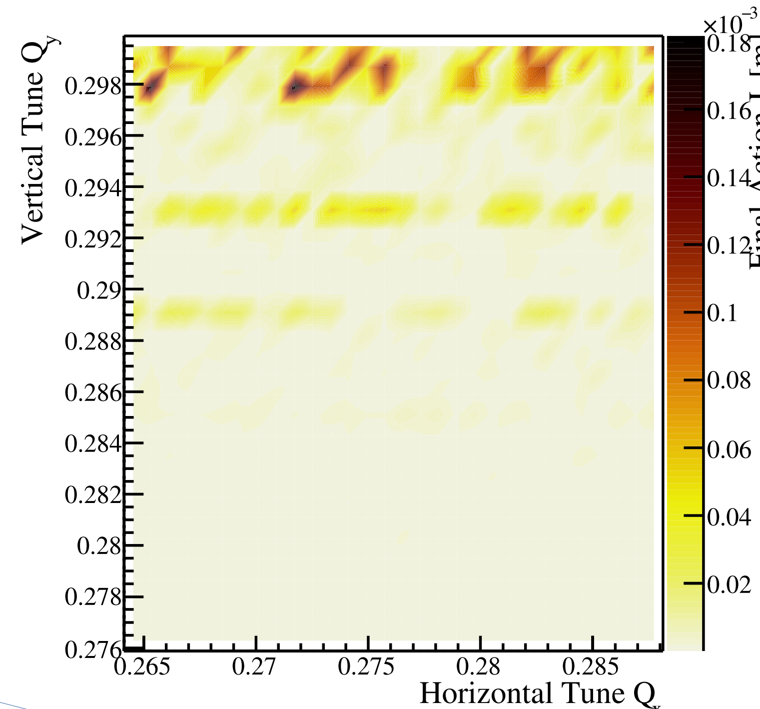
Comparison with RHIC (111 b)

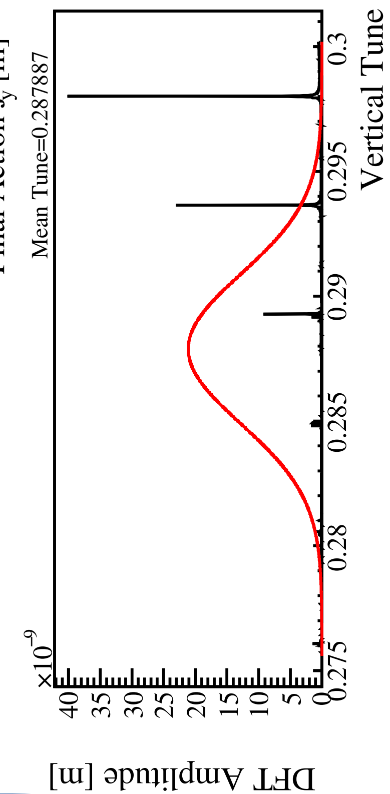


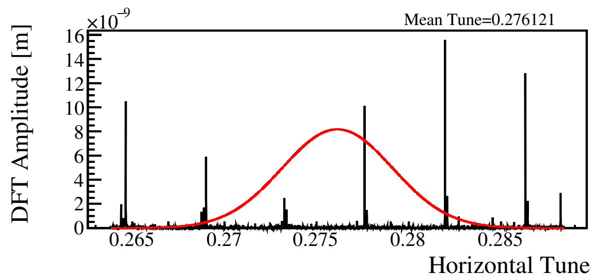
Expect approx. 125k faster horizontal growth per turn in RHIC than in the LHC (875k times faster in time) due to time-modulated dipole-like kicks.

Tune scan of RHIC with 111 bunches





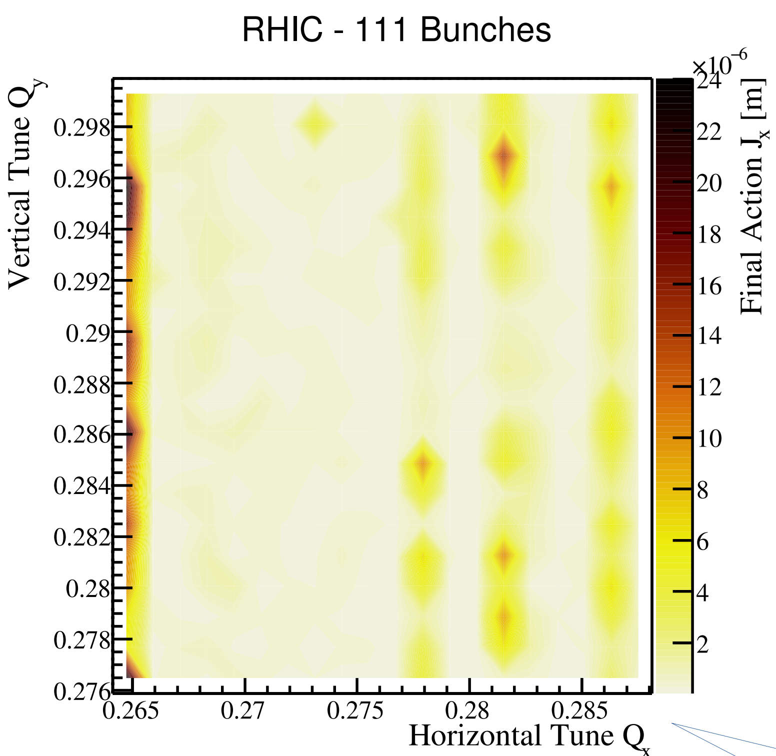


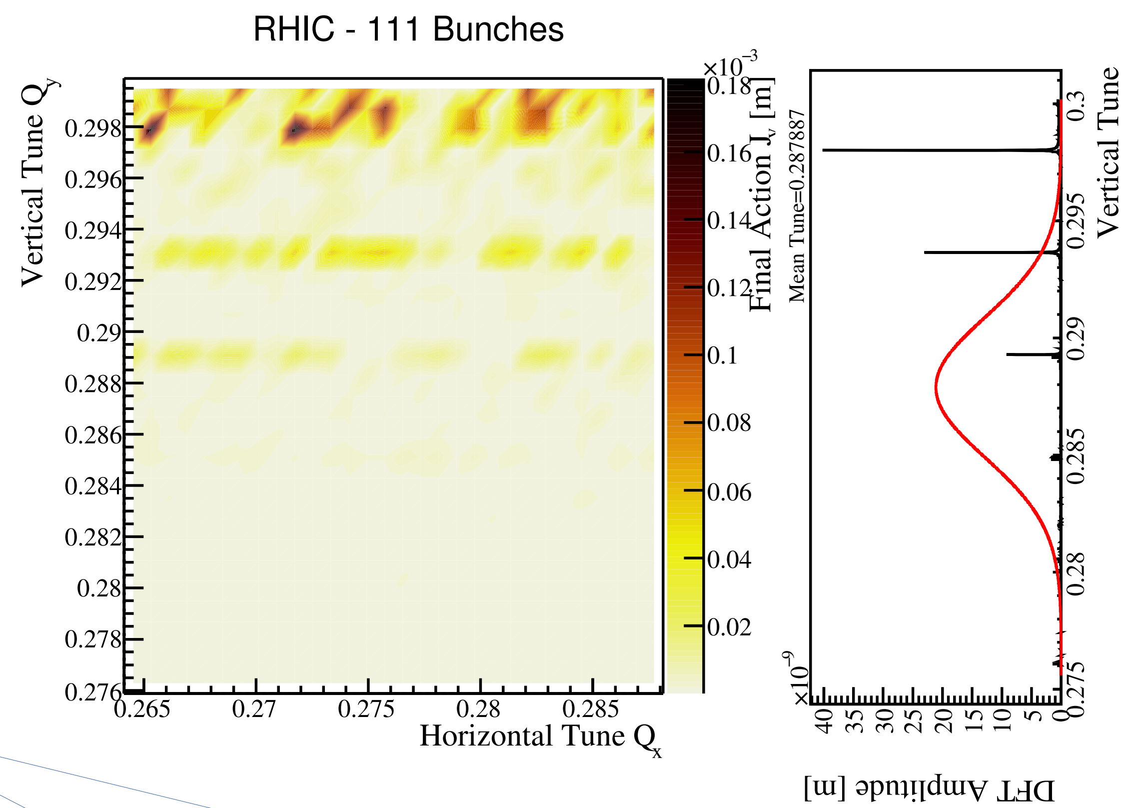


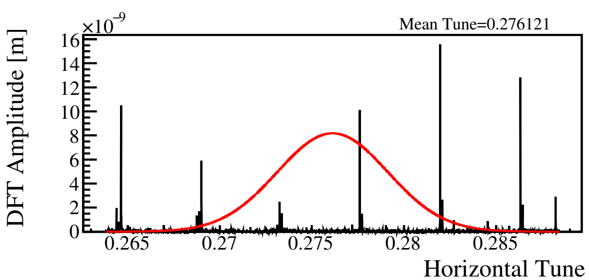
No corresponding resonance lines visible

Tune modulation dominates.

Tune scan of RHIC with 111 bunches







Tune modulation dominates.

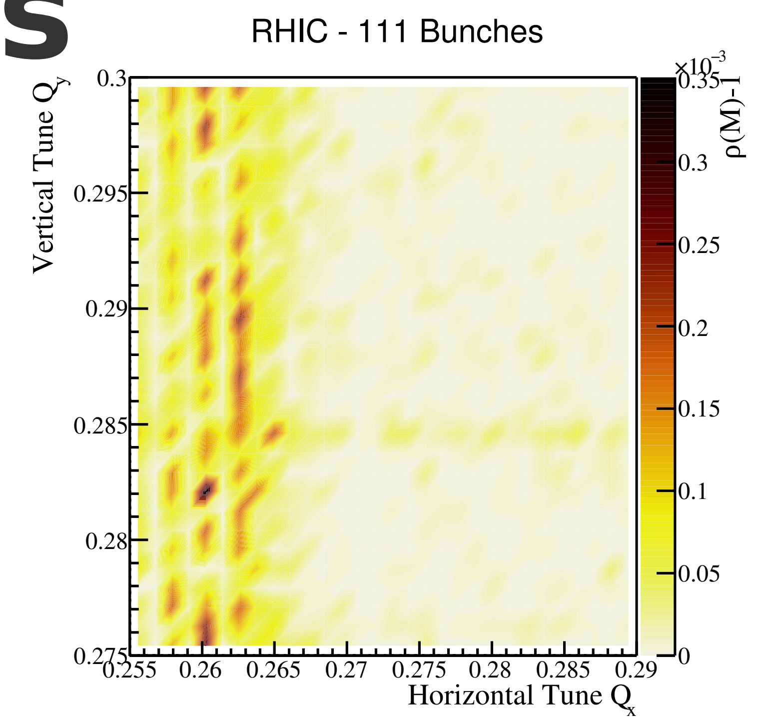
Limit max. final action: Resonances are still there.

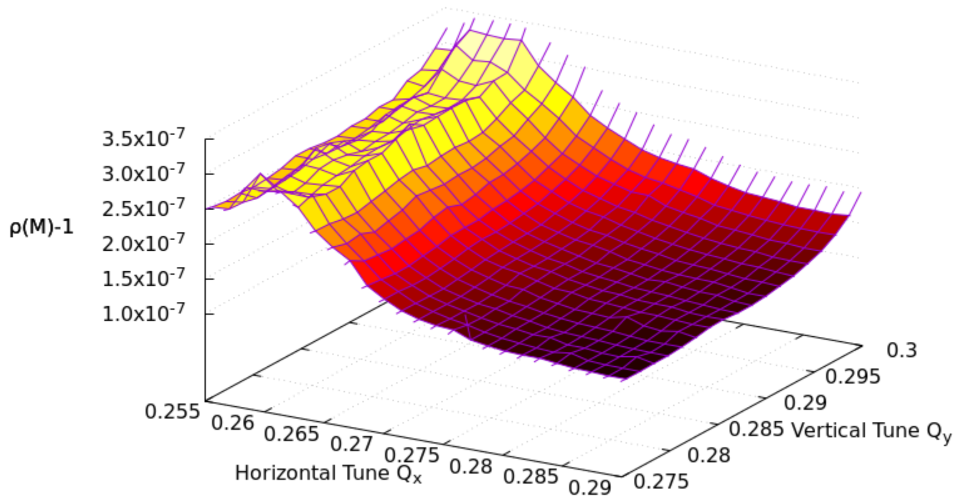
Spectral radius

- Multiply all 200k one-turn matrices for respective unperturbed tune (Q_x,Q_y) and calculate spectral radius

$$\rho(M) = max\{|\lambda_1|, |\lambda_2|, ...\}$$

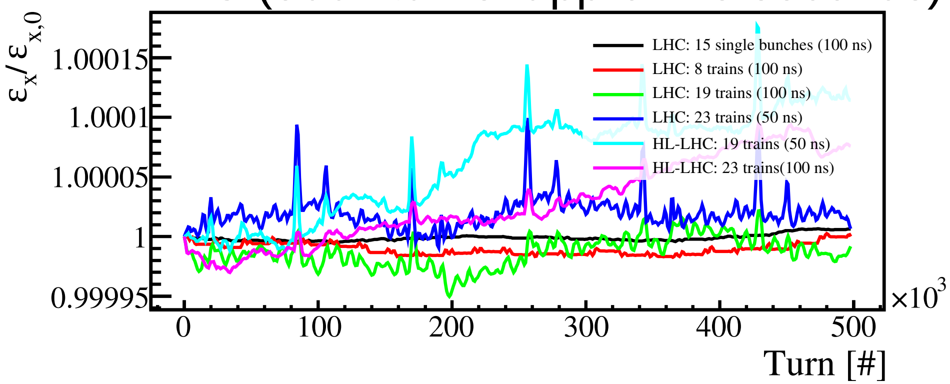
- ρ(M) should be 1 for stable motion
- ρ(M) gives exponential growth of fastest growing mode if matrix is applied multiple times

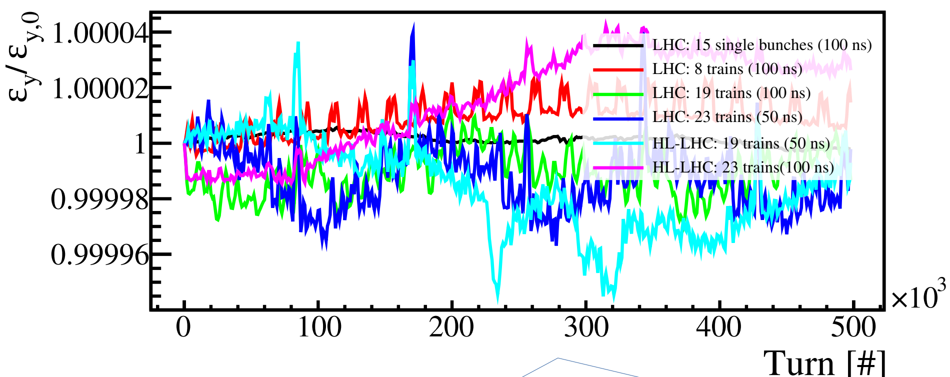




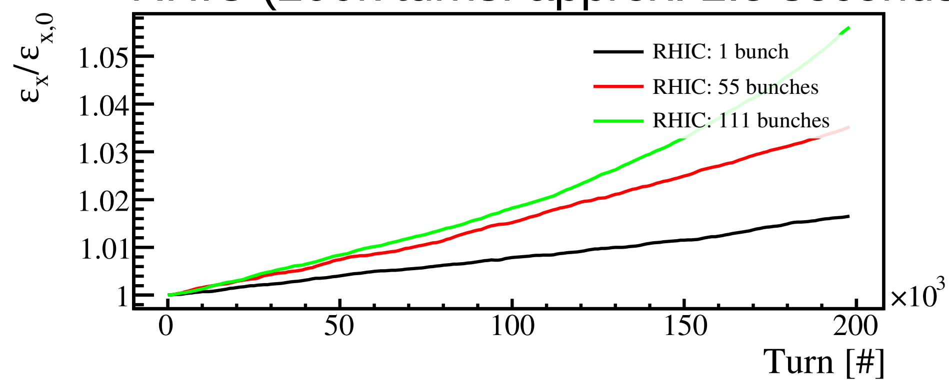
Emittance growth in the LHC and RHIC

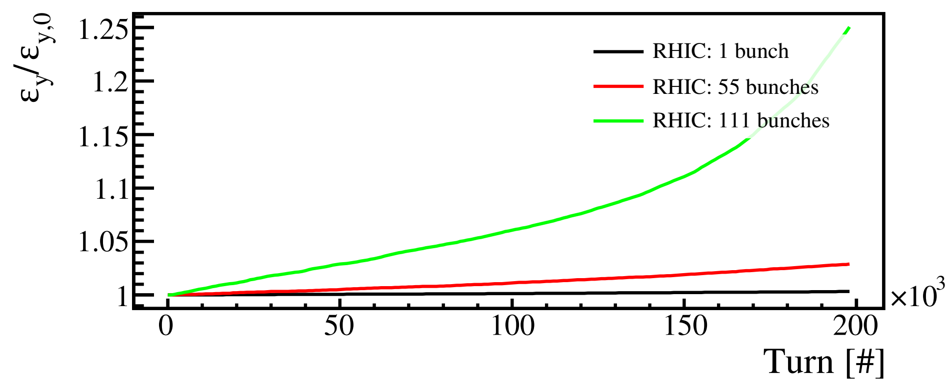
LHC (500k turns: approx 45 seconds)





RHIC (200k turns: approx. 2.5 seconds)





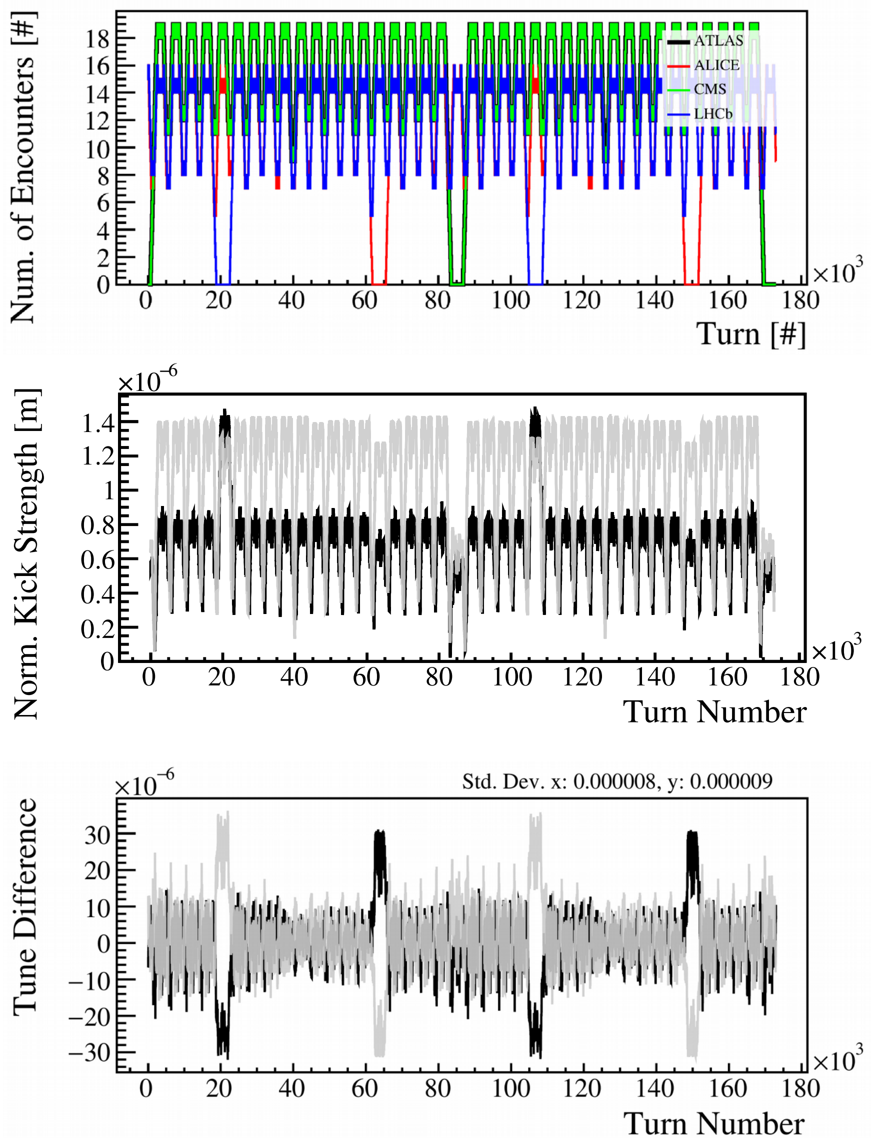
transverse IBS growth time: 6.5 h
HL-LHC growth time approx. 100 h

Settings		Linear Growth Rates		Linear Growth Times	
Collider	Pattern	$\dot{\epsilon}_{n,x}$ [m/s]	$\dot{\epsilon}_{n,y}$ [m/s]	$\epsilon_x(0)/\dot{\epsilon}_x$ [h]	$\epsilon_y(0)/\dot{\epsilon}_y$ [h]
RHIC	1 bunch	$1.579 \cdot 10^{-8}$	$3.220 \cdot 10^{-9}$	0.04	0.2
RHIC	55 bunches	$3.478 \cdot 10^{-8}$	$2.805 \cdot 10^{-8}$	0.02	0.02
RHIC	111 bunches	$5.190 \cdot 10^{-8}$	$2.015 \cdot 10^{-7}$	0.01	0.003

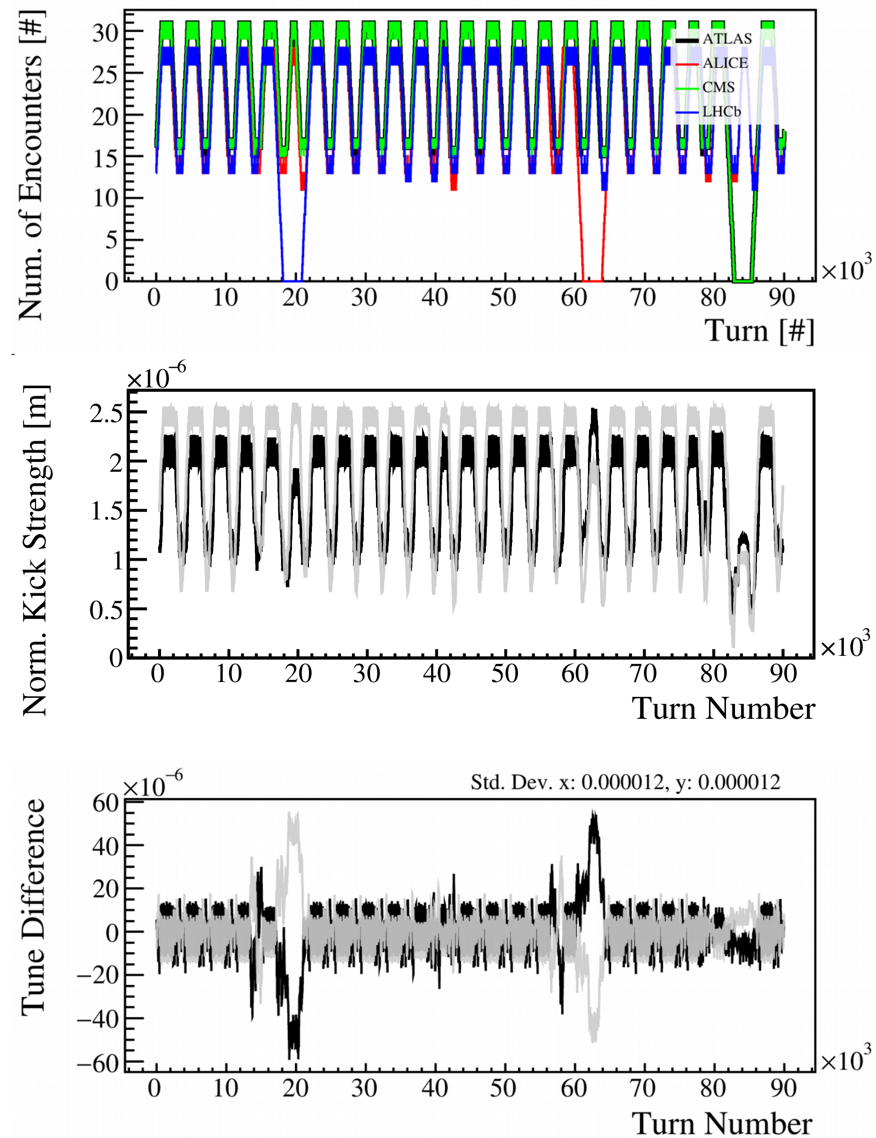
emittance doubling time

Comparison with HL-LHC

LHC Pb-p – 19 trains

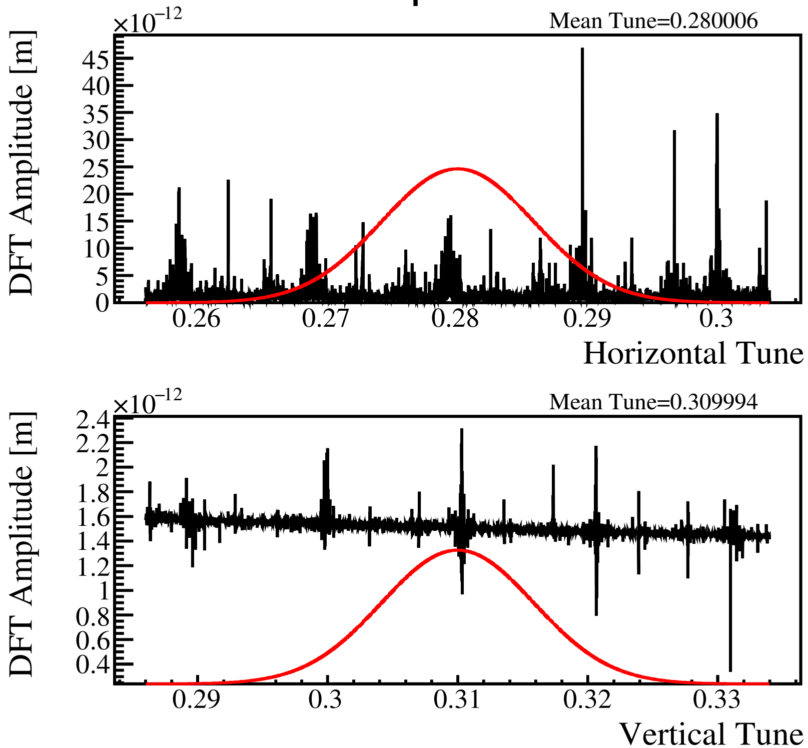


HL-LHC Pb-p – 23 trains

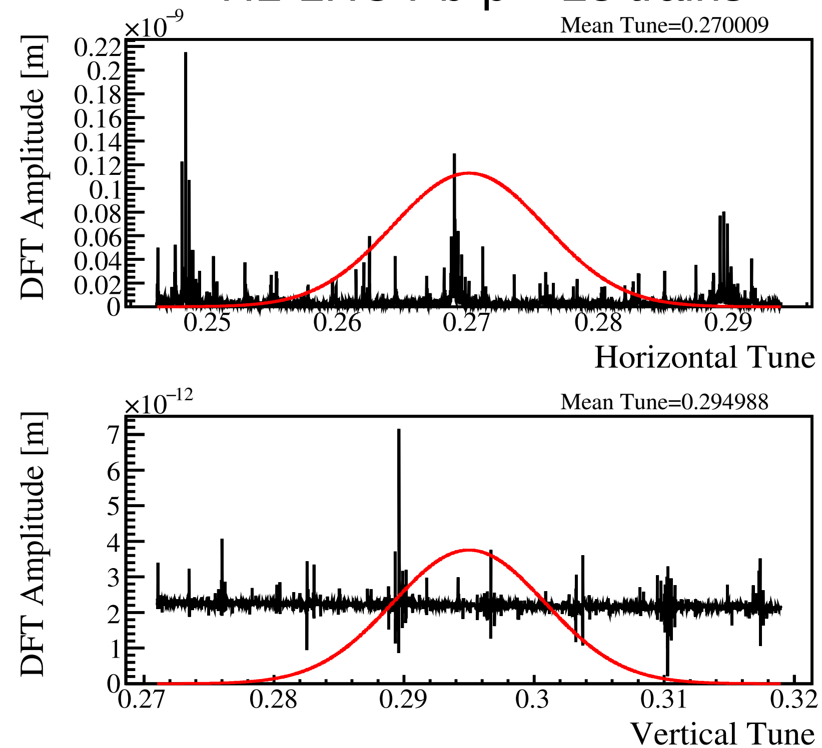


Comparison with HL-LHC

LHC Pb-p – 19 trains



HL-LHC Pb-p – 23 trains



Expect two order of magnitude faster growth horizontally in the case of HL-LHC.
Does not mean too much, since growth is small compared to IBS anyway

Conclusion

- Model is able to reproduce RHIC being *problematic*.
- Model shows a stable linear LHC
- First estimates show HL-LHC to be stable in the linear 4D case

Outlook

- Include synchrotron motion, amplitude detuning and ADT in simulation
- Idea: Since the effective effects within the IRs are now known, may be include a multipole in each IR doing the same time-varying effect in Sixtrack to allow for complete non-linear tracking.
- Estimate potential intensity limits for the HL-LHC (if there are any)

Thanks for listening.

Backup slides...

Combine DFTs

- Calculate $\delta_N = u_N - i\,u'_N$ from normalized kicks of IR matrix
- Calculate the DFT

$$DFT(\delta_N)_j = \sum_{q=0}^{N-1} \delta_{N,q} \exp(i\,2\pi q\,j/N)$$

- Assuming IR is positioned at the linear phase advance μ_{IR}. Calculate DFT with correct complex phase

$$DFT(\delta_N)_j \rightarrow DFT(\delta_N)_j \exp(2\pi\epsilon\,j/N)$$
$$\epsilon = \mu_{IR}/(2\pi Q)$$

- Simply superimpose DFTs of the different IRs to get total DFT

$$DFT_{total,j} = \sum_{i=1}^{N} DFT_{IRi,j} \exp(2\pi\epsilon_i\,j/N)$$
$$\epsilon_i = \mu_{IRi}/(2\pi Q)$$

- Only works if the detuning in the arcs is proportional to linear phase advance