

Electroweak Theory

Georg Weiglein, DESY Split, 09 / 2018

Introduction

The Standard Model (SM) is incomplete (in particular, it describes only three of the four fundamental interactions, i.e. it does not contain gravity) and cannot be the ultimate theory

How to get access to physics beyond the SM?

- Searches for physics beyond the Standard Model (BSM): light / heavy new states
 The options proposed in models that are currently discussed span many orders of magnitude from extremely light (e.g. axion-like particles: WISPs, ...) to very heavy
- High-precision tests: high sensitivity to deviations from the SM SM vs. other explicit models
 Effective field theory (EFT) analyses: new physics is assumed to be heavy

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Electroweak effects at the hadron collider LHC Non-comprehensive list!

 γ, W, Z

k

- Large logarithmic contributions ~ ln²(Q²/M²_V), ln(Q²/M²_V), V = W, Z
 Sudakov logs"
- Photon radiation
- Longitudinal vector boson scattering: large unitarity cancellations
- Mixed QCD/EW contributions
- Electroweak enhancement factors: ~mt², mt⁴ (e.g. corrections in SUSY Higgs sector)
- Electroweak precision observables: electroweak effects are important for extracting the measured quantity, and per-mille level precision is needed to match the experimental accuracy

Example for impact of electroweak corrections

ASSOCIATED PRODUCTION OF TOP PAIRS $t\bar{t}W$ with full EW corrections

(Frederix, Pagani, Zaro, 2018)

[S. Forte '18]

- FULL NLO QCD-EW CORRECTIONS COMPUTED: $O(\alpha_s^4)$ (NLO₁), $O(\alpha_s^3\alpha)$ (NLO₂), $O(\alpha_s^2\alpha^2)$ (NLO₃) $O(\alpha^4)$ (NLO₄)
- NAIVE COUNTING $O(\alpha) = O(\alpha_s^2) = O(1/100)$ VIOLATED
- LARGE $tW \rightarrow tW$ scattering contributions, NLO₃=20% of NLO₁ at LHC13! (would be 70% at FCC!)



Data-driven methods: theory uncertainties from extrapolations; example Higgs \rightarrow invisible search



• Simultaneous fit to both signal region and $W(\rightarrow \ell \bar{\nu})$ +jets and $Z(\rightarrow \ell \bar{\ell})$ +jets control regions

$$\frac{\mathrm{d}\sigma^{\mathrm{QCD}+\mathrm{EW}}(Z)}{\mathrm{d}p_T} = \bigg[\frac{\mathrm{d}\sigma^{\mathrm{QCD}+\mathrm{EW}}(Z)/\mathrm{d}p_T}{\mathrm{d}\sigma^{\mathrm{QCD}+\mathrm{EW}}(W)/\mathrm{d}p_T}\bigg]_{\mathrm{theory}} \times \bigg[\frac{\mathrm{d}\sigma^{\mathrm{QCD}+\mathrm{EW}}(W)}{\mathrm{d}p_T}\bigg]_{\mathrm{meas.}}$$

- Effective extrapolation for the sum of QCD and EW production processes
- In the presence of nontrivial VBF cuts and veto on 3rd jet
- Uses common QCD scale and parton shower variations
 - ⇒ Should be very cautious to trust any substantially reduced scale dependence to provide meaningful uncertainty estimate

No easy recipe available; close interaction between theory and experiment needed! For V + 1 jet, see [arXiv:1705.04664]

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[F. Tackmann '17]

Lepton distributions: invariant mass, p_T , $\Delta \varphi$, ...

Can electroweak effects (within or beyond the SM) account for certain deviations observed in lepton distributions?



And what is this?



[CMS Collaboration '18]

[see Sasha's talk on Monday]

Electroweak precision physics: high-precision data vs. theory predictions

EW precision data: $M_{\rm Z}, M_{\rm W}, \sin^2 \theta_{\rm eff}^{\rm lept}, \dots$

Theory: SM, MSSM, ...

Test of theory at quantum level: sensitivity to loop corrections



Electroweak precision observables

In addition to the ``traditional" electroweak precision observables, the mass of the detected Higgs boson is meanwhile also a high-precision observable

The achievable accuracy at the LHC in comparison with former (LEP, SLC) and possibly future e⁺e⁻ colliders depends on the type of observable. Statistics, systematics and also the collider energy (some observables profit from higher energy) play an important role.

In order to extract the quantity that is called precision observable, which is in fact a "pseudo-observable", from what is actually measured, effects of both the strong and the electroweak interaction need to be taken into account at a sufficient level of accuracy.

⇒ Extraction of pseudo-observables is affected by experimental and theoretical uncertainties What is actually meant by a "measurement" of M_W , $\sin^2\theta_{eff}$, ...?

Particle masses are not directly physical observables

Can only measure cross sections, branching ratios, kinematical distributions, ...

⇒ masses are "pseudo-observables"

Need to define what is meant by M_Z , M_W , m_t , ...:

MS mass, pole mass (real pole, real part of complex pole, Breit–Wigner shape with running or constant width), ...

⇒ Determination of M_Z, M_W, m_t, ... involves deconvolution procedure (unfolding)
 Mass obtained from comparison data – Monte Carlo
 ⇒ M_Z, M_W, m_t, ... are not strictly model-independent

What is / was experimentally measured?

- LEP: e+e- → W+W- in the continuum and at threshold (small amount of data); impact of fully hadronic final state suffered from uncertainties due to BE correlations, colour reconnections
- Tevatron, LHC: transverse mass distribution

How is the measured parameter (Monte Carlo mass) related to the theoretically well-defined quantity M_W ?

Similar question as for top-quark mass, where the latter is conceptually much more difficult (coloured object, renormalon ambiguities, ...), but here we are aiming for a two orders of magnitude higher accuracy

Mass of an unstable (elementary) particle

For an unstable particle:

 $\Sigma(\mathcal{M}^2)$ is complex \Rightarrow Pole in the complex plane

$$\mathcal{M}^2 - m^2 + \Sigma(\mathcal{M}^2) = 0, \quad \mathcal{M}^2 = M^2 - iM\Gamma$$

M: physical mass, Γ : decay width of the unstable particle

 \Rightarrow The mass of an unstable (elementary) particle is defined according to the real part of the complex pole

Example:

resonant production

of the Z boson and its decay

(point-like particle!)



Expansion around the complex pole for a single resonance

$$p^{2} - m^{2} + \hat{\Sigma}(p^{2}) = \underbrace{\left(p^{2} - \mathcal{M}^{2}\right)}_{\text{with fixed width}} \left\{ 1 + \frac{d\hat{\Sigma}}{dp^{2}} \right\}_{p^{2} = \mathcal{M}^{2}}^{p^{2} + \dots} + \dots$$

$$\xrightarrow{\text{Breit-Wigner factor}}_{\text{with fixed width}} \xrightarrow{\text{Field renormalisation}}_{\text{and wave function}}_{\text{normalisation factor}}^{p^{2} + \dots}$$

Note:

of unstable particle

Wave-function normalisation factor needs to be evaluated at the complex pole

One-loop field renormalisation: $\delta Z^{(1)} = -$ Complex quantity, no restriction to Re(...)

$$\frac{\partial \Sigma(p^2)}{\partial p^2} \bigg|_{p^2 = m^2}$$

Expansion of amplitude around complex pole:

$$\mathcal{A}(e^+e^- \to f\bar{f}) = \frac{R}{s - \mathcal{M}_Z^2} + S + (s - \mathcal{M}_Z^2)S' + \cdots$$
$$\mathcal{M}_Z^2 = \overline{\mathcal{M}}_Z^2 - i\overline{\mathcal{M}}_Z\,\overline{\Gamma}_Z$$

Expanding up to $\mathcal{O}(\alpha^2)$ using $\mathcal{O}(\overline{\Gamma}_{\rm Z}/\overline{M}_{\rm Z})=\mathcal{O}(\alpha)$

From 2-loop order on:

real part of complex pole, $\overline{M}_Z \neq \text{ pole of real part, } \widetilde{M}_Z^2$

$$\delta \overline{M}_{(2)}^2 = \delta \widetilde{M}_{(2)}^2 + \operatorname{Im}\left\{\Sigma_{\mathrm{T},(1)}^\prime(M^2)\right\} \operatorname{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}$$

gauge-parameter dependent!

Physical mass of unstable particles: real part of complex pole

 \Rightarrow Only the complex pole is gauge-invariant

Expansion around the complex pole leads to a Breit–Wigner shape with constant width

For historical reasons, the experimental values of M_Z , M_W are defined according to a Breit–Wigner shape with running width

Need to correct for the difference in definition when comparing theory with experiment

Fixed width / running width can be adjusted in the Monte Carlo code, but how about the renormalisation scheme for M_W ?

W-mass measurement at the LHC



[ATLAS Collaboration '17]

 $m_W = 80369.5 \pm 6.8 \text{ MeV(stat.)} \pm 10.6 \text{ MeV(exp. syst.)} \pm 13.6 \text{ MeV(mod. syst.)}$

 $= 80369.5 \pm 18.5$ MeV,



Extrapolation from Z to W



- There is no direct resummation for ratio, it is always a derived quantity
- Relies on ratio being more precise than individual processes, which relies on theory uncertainties being strongly correlated between processes
- More general: Use common theory framework and fit to Z data
 - Not restricted to a specific combination (like ratio)
 - Tuning Pythia on Z data is one example of this
 - Requires explicit information on correlations between processes

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The role of the W-boson mass as a precision observable

- Very accurately known both experimentally and theoretically
- Highly sensitive to quantum corrections of new physics
- Global fits in the Standard Model: dominated by the two observables M_W and $sin^2\theta_{eff}$

Note:

- Prospects for further experimental improvements of M_W from analysis of Tevatron data, LHC, future e⁺e⁻ collider
- Interpretation of constraints from sin² θ_{eff} is complicated by the fact that the two most precise individual measurements differ from each other by more than 3 σ

Theoretical prediction for the W-boson mass from muon decay: relation between M_W , M_Z , α , G_μ



 \Rightarrow Theo. prediction for $M_{\rm W}$ in terms of $M_{\rm Z}$, α , G_{μ} , $\Delta r(m_{\rm t}, m_{\tilde{\rm t}}, \ldots)$

Tree-level prediction: $M_W^{\text{tree}} = 80.939 \text{ GeV}$, $M_W^{\text{exp}} = 80.385 + 0.015 \text{ GeV}$ $\Rightarrow \text{ off by many } \sigma$ (accuracy of 2 x 10⁻⁴)

W-mass prediction within the SM:

one-loop result vs. state-of-the-art prediction



⇒Pure one-loop result would imply preference for heavy Higgs, $M_h > 400$ GeV Corrections beyond one-loop order are crucial for reliable prediction of M_W

[L. Zeune, G. W. '14]

Sources of theoretical uncertainties

From experimental errors of the input parameters

 $\delta m_{\rm t} = 0.9 \text{ GeV} \implies \Delta M_{\rm W}^{\rm para} \approx 5.4 \text{ MeV}, \ \Delta \sin^2 \theta_{\rm eff}^{\rm para} \approx 2.8 \times 10^{-5}$ $\delta(\Delta \alpha_{\rm had}) = 0.00014 \implies \Delta M_{\rm W}^{\rm para} \approx 2.5 \text{ MeV}, \ \Delta \sin^2 \theta_{\rm eff}^{\rm para} \approx 4.8 \times 10^{-5}$

• From unknown higher-order corrections ("intrinsic") SM: Complete 2-loop result + leading higher-order corrections known for $M_{\rm W}$ and $\sin^2 \theta_{\rm eff}$

⇒ Remaining uncertainties:
[M. Awramik, M. Czakon, A. Freitas, G.W. '03, '04]
[M. Awramik, M. Czakon, A. Freitas '06]

 $\Delta M_{\rm W}^{\rm intr} \approx 4 \,\,{\rm MeV}, \quad \Delta \sin^2 \theta_{\rm eff}^{\rm intr} \approx 5 \times 10^{-5}$

Prediction for M_W in the SM and the MSSM vs. experimental results for M_W and m_t

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]



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Prediction for M_W (parameter scan): SM vs. MSSM



The effective leptonic weak mixing angle: $\sin^2 heta_{ m eff}$

Of particular importance: effective leptonic weak mixing angle at the Z resonance, $\sin^2\theta_{\rm eff}$

Observable with the highest sensitivity to SM Higgs mass, ...

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left(1 - \operatorname{Re} \frac{g_V}{g_A} \right) = \left(1 - \frac{M_W^2}{M_Z^2} \right) \left(1 + \Delta \kappa \right)$$

Current experimental value from LEP and SLD: $\sin^2 \theta_{\text{eff}} = 0.23153 \pm 0.00016 \Rightarrow \text{Accuracy of } 0.07\%$

However: the small experimental error of the world-average is driven by two measurements that are not well compatible with each other: A_{LR} (SLD) and A_{FB} (LEP)

 $\sin^2 \theta_{\rm eff}(A_{\rm LR}) = 0.23098 \pm 0.00026, \quad \sin^2 \theta_{\rm eff}(A_{\rm FB}) = 0.23221 \pm 0.00029$

$\sin^2 heta_{ m eff}$: unclear experimental situation



[LEPEWWG '07]

 $\sin^2 \theta_{\rm eff}$ has a high sensitivity to $M_{\rm H}$ and effects of new physics

But: large discrepancy between $A_{\rm LR}$ (SLD) and $A_{\rm FB}$ (LEP),

has big impact on constraints on new physics

Extraction of $sin^2\theta_{eff}$: improved Born approx. (IBA)

[F. Piccinini, LHCEWG Meeting '18]

preliminary draft by S. Dittmaier, D. Wackeroth, A. Vicini

- to give recommendations for a solid theoretical recipe for $\sin^2 \vartheta_{\text{eff}}^{\ell}$ extraction, based on the pole expansion which allowd to define an IBA
- key observation: at the Z^0 pole

$$\mathcal{M}_{ij,\text{weak}}^{\text{vert}} = \mathcal{M}_Z^0 \Big|_{v_q \to \bar{g}_{\mathcal{V},q}, a_q \to \bar{g}_{\mathcal{A},q}} + \mathcal{M}_Z^0 \Big|_{v_\ell \to \bar{g}_{\mathcal{V},\ell}, a_\ell \to \bar{g}_{\mathcal{A},\ell}}$$

with the corrected ("effective") vector and axial-vector couplings

$$\overline{g}_{\mathrm{V},f} = v_f \left(1 + \hat{F}_{\mathrm{Z}ff,\mathrm{weak}}^{\mathrm{V}}(M_{\mathrm{Z}}^2) \right),$$

$$\overline{g}_{\mathrm{A},f} = a_f \left(1 + \hat{F}_{\mathrm{Z}ff,\mathrm{weak}}^{\mathrm{A}}(M_{\mathrm{Z}}^2) \right).$$

$$\bar{s}_{\text{eff},f}^2 = \frac{1}{4|Q_f|} \left(1 - \frac{\text{Re}\bar{g}_{\text{V},f}}{\text{Re}\bar{g}_{\text{A},f}}\right).$$

 \bullet Outside the Z peak the form factors are not gauge invariant

• the reliability of the IBA has to be checked with complete calculations

Recap: extraction of $sin^2\theta_{eff}$ at LEP

Form factors implemented in ZFITTER: [M. Awramik, M. Czakon, A. Freitas '06]

$$\begin{split} \mathcal{A}[e^{+}e^{-} \to f\bar{f}] &= 4\pi i \, \alpha \, \frac{Q_{e}Q_{f}}{s} \, \gamma_{\mu} \otimes \gamma^{\mu} \\ &+ i \frac{\sqrt{2}G_{\mu}M_{Z}^{2}}{1 + i\Gamma_{Z}/M_{Z}} \, I_{e}^{(3)} \, I_{f}^{(3)} \, \frac{1}{s - \overline{M}_{Z}^{2} + i\overline{M}_{Z}\overline{\Gamma}_{Z}} \\ &\times \, \rho_{ef} \left[\gamma_{\mu}(1 + \gamma_{5}) \otimes \gamma^{\mu}(1 + \gamma_{5}) \\ &- 4|Q_{e}|s_{W}^{2} \, \kappa_{e} \, \gamma_{\mu} \otimes \gamma^{\mu}(1 + \gamma_{5}) \\ &- 4|Q_{f}|s_{W}^{2} \, \kappa_{f} \, \gamma_{\mu}(1 + \gamma_{5}) \otimes \gamma^{\mu} \\ &+ 16|Q_{e}Q_{f}|s_{W}^{4} \, \kappa_{ef} \, \gamma_{\mu} \otimes \gamma^{\mu} \right] \end{split} \qquad \kappa_{ef}(s) = \kappa_{e}(s)\kappa_{f}(s) - \frac{M_{Z}^{2} - s}{s} \frac{1}{(a_{e}^{(0)} - v_{e}^{(0)})(a_{f}^{(0)} - v_{f}^{(0)})} \\ &\times \left[q_{e}^{(1)}q_{f}^{(0)} + q_{e}^{(1)}q_{e}^{(0)} \frac{v_{e}^{(0)}}{a_{f}^{(0)}} - p_{e}^{(1)}q_{f}^{(0)} \frac{v_{e}^{(0)}}{a_{e}^{(0)}} - q_{e}^{(0)}q_{f}^{(0)} \frac{\Sigma_{\gamma\gamma}}{\gamma} + \text{boxes} \right] \\ &\times \left[q_{e}^{(1)}q_{f}^{(0)} + q_{e}^{(1)}q_{e}^{(0)} - p_{f}^{(1)}q_{e}^{(0)} \frac{v_{e}^{(0)}}{a_{f}^{(0)}} - p_{e}^{(1)}q_{f}^{(0)} \frac{v_{e}^{(0)}}{a_{e}^{(0)}} - q_{e}^{(0)}q_{f}^{(0)} \frac{\Sigma_{\gamma\gamma}}{\gamma} + \text{boxes} \right] \\ &\times \left[q_{e}^{(1)}q_{f}^{(0)} + q_{e}^{(1)}q_{e}^{(0)} - p_{f}^{(1)}q_{e}^{(0)} \frac{v_{e}^{(0)}}{a_{f}^{(0)}} - p_{e}^{(1)}q_{f}^{(0)} \frac{v_{e}^{(0)}}{a_{e}^{(0)}} - q_{e}^{(0)}q_{f}^{(0)} \frac{\Sigma_{\gamma\gamma}}{\gamma} + \text{boxes} \right] \\ &+ 16|Q_{e}Q_{f}|s_{W}^{4} \, \kappa_{ef} \, \gamma_{\mu} \otimes \gamma^{\mu} \right]$$

Relation between $\sin^2\theta_{eff}$ determined from expansion around the complex pole and the one defined in *ZFITTER*:

$$\sin^{2} \theta_{\text{eff,pole}}^{f} = \overline{s}_{W}^{2} \operatorname{Re} \left\{ \overline{\kappa}_{Z}^{f}(M_{Z}^{2}) \right\} = \sin^{2} \theta_{\text{eff,ZFITTER}}^{f} - \frac{\Gamma_{Z}}{M_{Z}} \frac{q_{f}^{(0)}}{a_{e}^{(0)}(a_{f}^{(0)} - v_{f}^{(0)})} \operatorname{Im} \left\{ p_{e}^{(1)} \right\}$$

$$\operatorname{numerically}_{small, but}$$

$$\overline{s}_{W}^{2} = \left(1 - \frac{\overline{M}_{W}^{2}}{\overline{M}_{Z}^{2}} \right) = s_{W}^{2} \left[1 + \frac{c_{W}^{2}}{s_{W}^{2}} \left(\frac{\Gamma_{W}^{2}}{M_{W}^{2}} - \frac{\Gamma_{Z}^{2}}{M_{Z}^{2}} \right) \right]^{-1} \cdot \operatorname{required}_{small, but}$$

$$\operatorname{required}_{this order}_{Eectroweak Theory, Georg Weiglein, LHC Days in Split, Split, 09/2018} \xrightarrow{28}$$



 $\Rightarrow Precision measurements could rule out the SM and the MSSM!$ Electroweak Theory, Georg Weiglein, LHC Days in Split, Split, 09 / 2018

Sensitivity to new force carrier: present and future



Conclusions

Rich spectrum of electroweak physics at the LHC; effects can be much larger than naively expected. I could mention only a few aspects in this talk.

A very good understanding of both QCD and electroweak higher-order contributions is required for a discrimination between physics within and beyond the Standard Model.

Joint effort between experiment and theory is needed for the extraction of electroweak precision observables.

⇒ Electroweak physics at the LHC provides sensitivity to effects of new physics!



$\Delta r^{(N)MSSM(h.o.)} = \Delta r^{SM(h.o.)} + \Delta r^{SUSY(h.o.)}$ *M*_W prediction in the Standard Model

Contributions beyond one-loop order:

$$\Delta r^{\text{SM(h.o.)}} = \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r_{\text{ferm}}^{(\alpha^2)} + \Delta r_{\text{bos}}^{(\alpha^2)} + \Delta r^{(G_{\mu}^2\alpha_s m_t^4)} + \Delta r^{(G_{\mu}^3m_t^6)} + \Delta r^{(G_{\mu}m_t^2\alpha_s^3)}$$

Chetyrkin, Kuhn, Steinhauser, Djouadi, Verzegnassi, Awramik, Czakon, Freitas, Weiglein, Faisst, Seidensticker, Veretin, Boughezal, Kniehl, Sirlin, Halzen, Strong, ...

Impact of different contributions to Δr (x 10⁴) for fixed $M_W = 80.385$ GeV and $M_H^{SM} = 125.09$ GeV:

[O. Stål, G. W., L. Zeune '15]

$$\frac{\Delta r^{(\alpha)}}{297.17} \frac{\Delta r^{(\alpha\alpha_s)}}{36.28} \frac{\Delta r^{(\alpha\alpha_s^2)}}{7.03} \frac{\Delta r^{(\alpha^2)}}{29.14} + \Delta r^{(\alpha^2)}}{\Delta r^{(G_{\mu}^2 \alpha_s m_t^4)} + \Delta r^{(G_{\mu}^3 m_t^6)}} \frac{\Delta r^{(G_{\mu} m_t^2 \alpha_s^3)}}{1.23}$$

Methods for estimating theoretical uncertainties Mobosphermass highbe Sder corrections



Renormalisation scheme dependence

a) Uncertainty of $\mathcal{O}(\alpha^2)$ corrections beyond leading $\alpha^2 m_t^4$ and $\alpha^2 m_t^2$ from comparison of $\overline{\text{MS}}$ and OS schemes: Degrassi, Gambino, Sirlin '96 $\delta M_W \sim 2 \text{ MeV}$ (for $M_H \sim 100 \text{ GeV}$) Actual remaining $\mathcal{O}(\alpha^2)$ corrections: Freitas, Hollik, Walter, Weiglein '00

 $\delta M_{\rm W} \sim 3 \,{\rm MeV}$ (for $M_{\rm H} \sim 100 \,{\rm GeV}$)

b) Estimate of missing $O(\alpha^3)$ corrections from comparison of $\overline{\text{MS}}$ and OS results: Awramik, Czakon, Freitas, Weiglein '03 Degrassi, Gambino, Giardino '14

 $\delta M_{\rm W} \sim 4...5 \,\,{\rm MeV}$ (after accounting for $\mathcal{O}(\alpha_{\rm t} \alpha_{\rm s}^3)$ corrections)

 \rightarrow Saturates previous δM_W estimate!

Note: Differences in (implicitly) resummed higher-order contributions

[A. Freitas '15]

M_W prediction in the NMSSM

Higg Mass M shift to the tree-level hoggs, mass L. Zeune '15]

- In the NMSSM: Additional tree-level Higgs mass contribution Can reduce the size of the radiative corrections needed to 'push' the lightest Higgs mass up to the experimental value
- Here the NMSSM Higgs sector contribution to M_W is predominately SM like with $M_{h^{\rm SM}}=M_{h_1^{\rm NMSSM}}$

