The FEEBLE INTERACTION

Or

WHY IS PARITY RESTORED?

Jean-Marie Frère
And Now?
Once upon a time, we had

- The Electromagnetic
- The Weak
- The Strong interactions
- Gravitation

Naïve question: where is the border between « weak » and « strong »?

Answer (ca 1970) ...didn’t you realize they are 10 orders of magnitude apart ??
<table>
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<tr>
<th>particule</th>
<th>Mass (GeV/c²)</th>
<th>« lifetime »</th>
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<tbody>
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<td>π (1800)</td>
<td>1.8</td>
<td>3.3 $10^{-24}$ s</td>
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<tr>
<td>lepton $\tau$</td>
<td>1.777</td>
<td>2.9 $10^{-13}$ s</td>
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<tr>
<td>muon</td>
<td>0.113</td>
<td>2.2 $10^{-6}$ s</td>
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« Dimensional analysis » expectation ....

\[ 1 \text{ GeV} \quad T = 6.58 \times 10^{-25} \text{ s} \]
Playing either on MW or g allows to fit lifetimes, but to fit all, a combination of M of order 100GeV and g of order of the electric charge is needed....

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The width of the Z boson is 2.5 GeV ...definitely NOT a weak interaction value!

So, there are no « weak interactions » ....

Still, the « old weak interactions » could be characterized differently, namely by the breaking of discrete symmetries (P, CP.....). Would this remain a valid characterization?
P violation is of course no problem for pure gauge interactions in 3+1 dim ....

P violation is in fact the EXPECTED situation in all gauge theories!

NOTHING to do with the presence or absence of right-handed neutrino!

In fact, we knew it all along, e.g. K to 3π vs K to 2π

Left Speaks to Left, Right to Right, and the couplings are in general different!
In fact, the mystery would rather be ....
Why is Parity respected around us?

Whether SU(5) unification is true or not... (or any SuSy approach) we can use it to classify particles.

All fermions are re-written in terms of the Left-Handed spinors

e.g; \((u^c_R)_L\) .... In 10 and \(\bar{5}\)
In fact, the mystery would rather be .... Why is Parity respected around us?
For so many centuries, we have perceived our universe as governed by LR symmetrical laws, with “accidental boundary conditions”: more R handed people than L, heart generally to the left, DNA with R helicity ...shells coiling, all seen as accidental, and probably uncorrelated (my thks to G. ‘t Hooft for a discussion)

This is why the experimental discovery of Parity violation was such a shock (contrary to general belief, CP violation was not unexpected afterwards ...se Lev B. Okun)!
In other terms, gauge interactions in 3+1 D are naturally P violating. Why has Nature in some way to make us believe that L and R were equivalent? In Gravitation, Electromagnetism and Strong interactions (which are all in some way long distance forces), Parity is the rule...

*Is it an accident that after breaking, the « long-distance » gauge interactions (in which I would include U(1)em but also the unbroken SU(3)color ) are parity invariant ???*

Why?? (I will bring no definitive answer, but explore a few hints).
A first (technical) answer:

Hint 1: **anomaly matching** probably forces it (as long as we only have a few small representations) ....but is there something more fundamental at play?

In a grand unified approach, and after breaking, only the “never anomalous” SU(2) remains P-violating, while the “unbroken” U(1) and SU(3) become P conserving...

Anomaly matching: no anomalies in minimal SU(5) to start with → none after breaking. For SU(3), only 3L and 3̅L available (equiv to 3R) → parity restored due to
- Anomaly matching
- Restricted choice of representations (nothing else than 3 and 3̅ in the end, would be different if 6 available?)
Another answer and an old conjecture: ...mass!

Long range forces (SU(3), U(1)em) mean unbroken symmetry.

Gauge invariance of mass term:

\[
\begin{align*}
E_e^- & \rightarrow e^{-i\alpha_L E_e^-} \\
& \Downarrow \\
& \text{Invariant under } U(1)_{\text{em}} \\
\Rightarrow \alpha_L = \alpha_R \Rightarrow P
\end{align*}
\]
Thus, if we have
MASSIVE particles coupled to an
EXACT gauge symmetry (SU(3), U(1)_{em})
Parity is automatically guaranteed.

Old conjecture: all charges particles are massive ....

(new formulation: all particles charged under a conserved gauge symmetry ....

Why? Technical problem with forward singularities for massless fermions ...???
Why does this not apply to weak interactions (for instance Z couplings of leptons?)

Broken symmetry!
What about weak interactions and CP (or T) violation?

- We know that CP (or T) violation is observed in many channels of mesons decays (K and B physics) in “weak interactions”
- More CP violation is expected to “explain” the “defeat of antimatter”, namely the survival of a non-zero Baryon number from an initially symmetrical (or made so by inflation) Universe.

Yet, CP is THE emblematic symmetry of (pure) gauge interactions in 3+1 D

*(pure: no scalar terms, including fermion masses, allowed)*
An anti- $K^\circ$ is produced at the center together with other particles ($K^-$). It propagates upwards (invisible track) before disintegrating.

When it disintegrates, the presence of an $e^+$ (in red) shows that it has meanwhile transformed into a $K^\circ$.

Experiment proves that the anti-$K$ to $K$ transition is slightly more probable ($0.6\%$) than the opposite.

Microscopic physics is not invariant under Time reversal... And the same asymmetry exists between particles and antiparticles!

Credit: CP Lear
CP is the intrinsic symmetry of gauge interactions in 3+1 dimensions see Basic Building Bloc: chiral fermion and gauge boson
So... while one characteristic of “Weak Interactions”, namely P violation is easily accommodated by the gauge forces (NOTHING WEAK)

Something beyond Gauge in 3+1 D is needed to account for CP (or T violation)

ENTERS THE *feeble* FORCE!
The Scalar Boson (B-E-H) which was introduced to give mass to the gauge bosons ... finds another use. It is also responsible for the fermion masses!

A priori, 2 different roles, BUT

• SU(2) breaking is needed to split masses in a multiplet
• Such splitting necessarily contributes to the W and Z masses

\[ \lambda_e = \frac{g}{\sqrt{2}} \frac{m_e}{M_W} \approx 6 \times 10^{-6} \frac{g}{\sqrt{2}} \]
But CP violation is easily introduced by arbitrary, complex couplings.

\[ \Phi^- \rightarrow e^-_R \]

Hermitian Conj.

\[ \lambda \]

\[ \nu_L \]

\[ \Phi^+ \rightarrow e^-_R \]

CP

\[ \lambda^* \]

\[ \nu_L \]

\[ \Phi^+ \rightarrow e^{+}_L \]

\[ \lambda^* \]

\[ (\nu)_R \]

\[ \lambda \neq \lambda^* \] ALLOWS CP violation

(provided it cannot be rotated away → 3 generations in SM, but 1 sufficient in LR)
Indeed, the couplings to the first families are “feeble” (the only exception being the top quark).

Why the “joker”?

- At the difference of (non-abelian) gauge couplings, all parameters are completely arbitrary
- Complex couplings allow for CP violation

The resulting CKM (without forgetting GIM…) mixing matrix reproduces all the CP pheno, ...and it was a surprise that it accounted for direct CP in K, and the details of CP in B, which were complete predictions!

By now, mass is seen as an “interaction”, to some extent a dynamical quantity rather than a “static” parameter…. This also means we must understand their structure…
Moving from a « current » basis to a « mass basis », the Scalar (feeble) interaction accounts for the instability of the heavy flavours ..
This *feeble* coupling also explains the enormous time gap between the discovery at CERN of the W and Z bosons *(80 and 91 GeV)* in 1983 and the *H* *(125 GeV)* of barely higher mass in *(2012)*.

**Hint:** the LHC could be the machine for the discovery of *feeably interacting particles* *(but we have no longer any firm prediction for such particles)*
Must we settle for FUNDAMENTAL scalars?

- Obviously, Scalars are needed (and successful in predicting CP in the B sector)
- But the arbitrariness of the many couplings seems to defy the move to a fundamental understanding?
- Could they be composite?
- Could they result from another, deeper interaction (the width of the H hints at a much higher scale then)
- Could they be a hint for more than 4 dimensions?
- Could CP have a deeper origin than arbitrary complex numbers?

In the following slides, we will suggest or explore some of these directions
Let us first stay in 3+1 dimensions

Imagine the scalar sector comes from a more fundamental (gauge) theory ...

We would then expect CP to be conserved, and consequently also in the scalar potential and the Yukawa couplings.

*Can we break CP spontaneously, and agree with experiment?*
Spontaneous CP violation (3+1) dim within SM gauge content

In the Standard Model: SU(3) X SU(2) X U(1) attempts with n Scalar doublets ... n=3 avoids Flavour Changing Neutral Currents at tree level ... obtains CP violation, but with real CKM matrix ... now excluded!

See for instance: Spontaneous CP Nonconservation and Natural Flavor Conservation: A Minimal Model

(what happens if one relaxes absence of FCNC at tree level or introduces more particles? )

One possibility (this would deserve more work ...)

A Minimal model with natural suppression of strong CP violation
Moving from a « current » basis to a « mass basis », the Scalar (feeble) interaction accounts for the instability of the heavy flavours..

Flavour Changing Neutral Currents (interactions) are strongly suppressed in Nature ((10 orders of magnitude in the K system ... it is a major success of the minimal Standard Model (through the Glashow-Iliopoulos-Maiani mechanism) that they are absent at tree level.)
Spontaneous CP violation (3+1) dim

In the Left-Right symmetrical model $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$

A very nice model, where Parity symmetry can be restored at high energy, and an elegant structure!

Here, some flavour changing currents remain, mediated by some Scalar Bosons, but they can be controlled if the relevant scalars are kept heavy.

*Spontaneous CP violation can be achieved by a phase between vacuum expectation values*

\[
\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix}
\]
In the spontaneously broken CP LR model, 
Assuming the FCNC scalars to be heavy (and thus neglected), 
we came up with a clear prediction:

\[
\text{and is excluded if } \sin 2\beta > 0.1.
\]

Shortly after the paper, the value of \( \sin 2\beta \) was measured...(this is the amount of CP violation in the B meson decay to J/\( \Psi \) and K mesons)

**Current value : 0.63 (0.06) !**

Here also, **minimal structure excluded... worth re-visiting if we discover, say a WR boson at a few TeV**
Another path ...

**CP violation from extra dimensions?**

CP is the symmetry of gauge interactions in 3+1 D, but not in for other dimensions: for instance, in 5+1, the relevant symmetry is C ...

Can CP violation occur in “pure gauge” (no scalars) through dimensional reduction?

Indeed, if CP is the “natural” symmetry of pure gauge in 3+1 Dim, this is not general for larger dimensions, For 5+1 dim for instance, C is the automatic symmetry, while P occurs only for specific particle choices ... hence CP is generally broken!
Before going to more than 3+1 dimensions, 
Must apologize for being WASTEFUL ...

In 4+1 dim, 
Minimal spinor has 
4 components,

\[
\begin{pmatrix}
\Psi_L \\
\Psi_R
\end{pmatrix}
\]

Reduction to
3+1

In 5+1 dim, 
Minimal (chiral) spinor
has 
4 components, 
(Dirac has 8)

\[
\begin{pmatrix}
\Psi_L \\
\Psi_R \\
\chi_L \\
\chi_R
\end{pmatrix}
\]

\[
\begin{pmatrix}
\Psi_L \\
0 \\
0 \\
0
\end{pmatrix}
\]
A schematic example in 4+1 dimensions

\[ L_{4+1} = m \overline{\Psi} \Psi + \lambda \overline{\Psi} \Phi \Psi + g \overline{\Psi} \gamma_A W^A \Psi \]

Since \( \gamma^4 = i \gamma_5 \), the last term corresponds in 3+1 to a pseudoscalar term,

Hosotani loop: \[ \oint W_4 dx^4 = B \]

\[ m \overline{\Psi} \Psi + \lambda \overline{\Psi} < \Phi > \Psi + g \overline{\Psi} i \gamma_5 B \Psi \]

We can generate CP violation, starting from purely real parameters, and even breaking of the group; Here however, we have « cheated » in 2 ways:
We have still introduced a scalar interaction (the mass \( m \)), without it the CP violation would be eliminated by a chiral rotation (in fact, the full example is formulated with an SU(2) group).
We have also not checked the stability of the compactification.
Things are better in 5+1 dim:

The components W4 and W5 of the gauge field play respectively as pseudoscalar and scalar fields after reduction to 3+1 dim ...

We obtained a coherent model using only one fermion coupled to gauge in 6D and yielding a massive fermion with electric dipole moment (CP and T violating) in 3+1 dim ..... This is however mainly a “proof of concept”, and a realistic model would be quite difficult to build.

JMF, M. Libanov, S. Mollet JHEP 1406 (2014) 103

Figure 8: One-loop contributions to EDM for $\psi_{3\bar{3}0}$. Contributions with $\psi_{-i\bar{n}-m}$ in the loop must be included as well.
Living with scalars, but trying to reduce the number of parameters through dimensional reduction ... some interesting results

Not a new idea ... assuming 4+1 dim, with $x_1, x_2, x_3, t$ and extra dimension $y$

Can “multilocalize” the fermions at different locations in the extra dimension $y$
(with flat or curved metric $\rightarrow$ Randal-Sudrum )

In principle, NOT very PREDICTIVE:

just map mass patterns into cartography of the extra dimension, with strong sensitivity due to the « gaussian tails » $\rightarrow$ EASY HIERARCHIES
Things get more interesting in 3+1 +2 dim

$$\Phi = e^{in\phi}$$

The 3 fermion modes have different shapes in r, and different winding properties in the extra dimension variable $\phi$

1 family in 6D $\rightarrow$ 3 families in 4D

Vortex with winding number n localizes n chiral massless fermion modes in 3+1
The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable $\phi$.

The 4D mass matrices are obtained by integrating $r$ and $\phi$, and are the convolution of these curves.

Brout-Englert-Higgs field $H$

Vortex Profile $e^{i3\phi}$
### Fermion-vortex couplings

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### “Yukawa” fermion-scalars couplings

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Parity Restored? JMF SPLIT 2018
For Quarks and Dirac fermions, we get a mass matrix like:

\[
\begin{pmatrix}
\text{small} & \varepsilon \\
\text{medium} & \varepsilon \\
\text{large} & \\
\end{pmatrix}
\]

Additional couplings involving the vortex field, with winding \( e^{i\phi} \) can give the small \( \varepsilon \) leading to Cabibbo mixings.

The scheme is very constrained, as the profiles are dictated by the equations, instead of being imposed by hand, like in multilocalisation.
Neutrinos ARE different

In the same context (0th order in Cabibbo mixing), we will get indeed (see later):

\[ M_\nu \sim \begin{pmatrix} \cdot & \cdot & m \\ \cdot & \mu & \cdot \\ m & \cdot & \cdot \end{pmatrix} \]

Where \( m \gg \mu \)

After 45° 1-3 rotation and 23 permutation, this leads to an inverted hierarchy, (minute solar mass difference found between the heavier neutrinos)

\[ M_\nu \sim \begin{pmatrix} m & \cdot & \cdot \\ \cdot & -m & \cdot \\ \cdot & \cdot & \mu \end{pmatrix} \]

The – sign may be absorbed in the mixing matrix, but contributes destructively to the effective mass for neutrinoless double beta decay (Pseudo-Dirac structure when full Cabibbo-like mixing is introduced)
Generic prediction: large mixings, inverted hierarchy suppressed neutrinoless double beta decay

**NEUTRINOS MASSES**

- Consequences of this structure

  - $0
\nu\beta\beta$ decay

  partial suppression

\[
|\langle m_{\beta\beta}\rangle| \simeq \frac{1}{3} \sqrt{\Delta m^2_{\odot}}
\]

\[ M_\nu \sim \begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix} \]

Automatically get

Inverted Hierarchy

Mass scale
**Neutrino masses**

| \( m_i \) | \( 5.46 \cdot 10^{-2} \text{ eV} \) | — |
| \( m_2 \) | \( 5.53 \cdot 10^{-2} \text{ eV} \) | — |
| \( m_3 \) | \( 4.17 \cdot 10^{-5} \text{ eV} \) | — |
| \( \Delta m_{21}^2 \) | \( 7.96 \cdot 10^{-5} \text{ eV}^2 \) | \( (7.50 \pm 0.185) \cdot 10^{-5} \text{ eV}^2 \) |
| \( \Delta m_{13}^2 \) | \( 2.98 \cdot 10^{-3} \text{ eV}^2 \) | \( (2.47^{+0.069}_{-0.067}) \cdot 10^{-3} \text{ eV}^2 \) |

**Lepton mixing matrix**

\[
|U_{PMNS}| = \begin{pmatrix} 0.76 & 0.63 & 0.13 \\ 0.39 & 0.58 & 0.72 \\ 0.52 & 0.52 & 0.68 \end{pmatrix}
\]

\[
\langle m_{\beta\beta} \rangle = 0.013 \text{ eV}
\]

\[
\begin{align*} J & = 0.019 \\ \theta_{12} & = 39.7^\circ \\ \theta_{23} & = 46.5^\circ \\ \theta_{13} & = 7.2^\circ \end{align*}
\]

\[
\simeq \begin{pmatrix} 0.795 & -0.846 & 0.513 & -0.585 & 0.126 & -0.178 \\ 0.205 & -0.543 & 0.416 & -0.730 & 0.579 & -0.808 \\ 0.215 & -0.548 & 0.409 & -0.725 & 0.567 & -0.800 \end{pmatrix}
\]

\[
\lesssim 0.3 \text{ eV} \ [31]
\]

\[
\lesssim 0.036
\]

\[
\simeq (31.09^\circ - 35.89^\circ)
\]

\[
\simeq (35.8^\circ - 54.8^\circ)
\]

\[
\simeq (7.19^\circ - 9.96^\circ)
\]

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*JMF,M Libanov, FS Ling, S Mollet, S Troitsky*

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**Note**: a non-vanishing \( \theta_{13} \) was predicted *before its observation*
Partial conclusions:

• The Brout-Englert-Higgs mechanism has opened the way to unifying the “weak interactions” with electromagnetism ....
  There are in fact NO WEAK INTERACTIONS in the “old” sense!
• The mass of fermions is now seen as a type of interaction, and the Scalar boson (composite or fundamental) is the vector of new interactions. For its very small coupling to the first generations, we can call it the feeble force.
  (and LHC is a machine for detecting feebly interacting particles)

• The nature of Scalars remains mysterious, and also its relation to CP violation
  • Various ideas have been presented to investigate its nature (spontaneous CP violation, CP violation from dimensional reduction, mass spectrum from extradimensions) and much work remains!
Difficulties to unify gravity and other fundamental forces?

...at 10^{28} eV ...