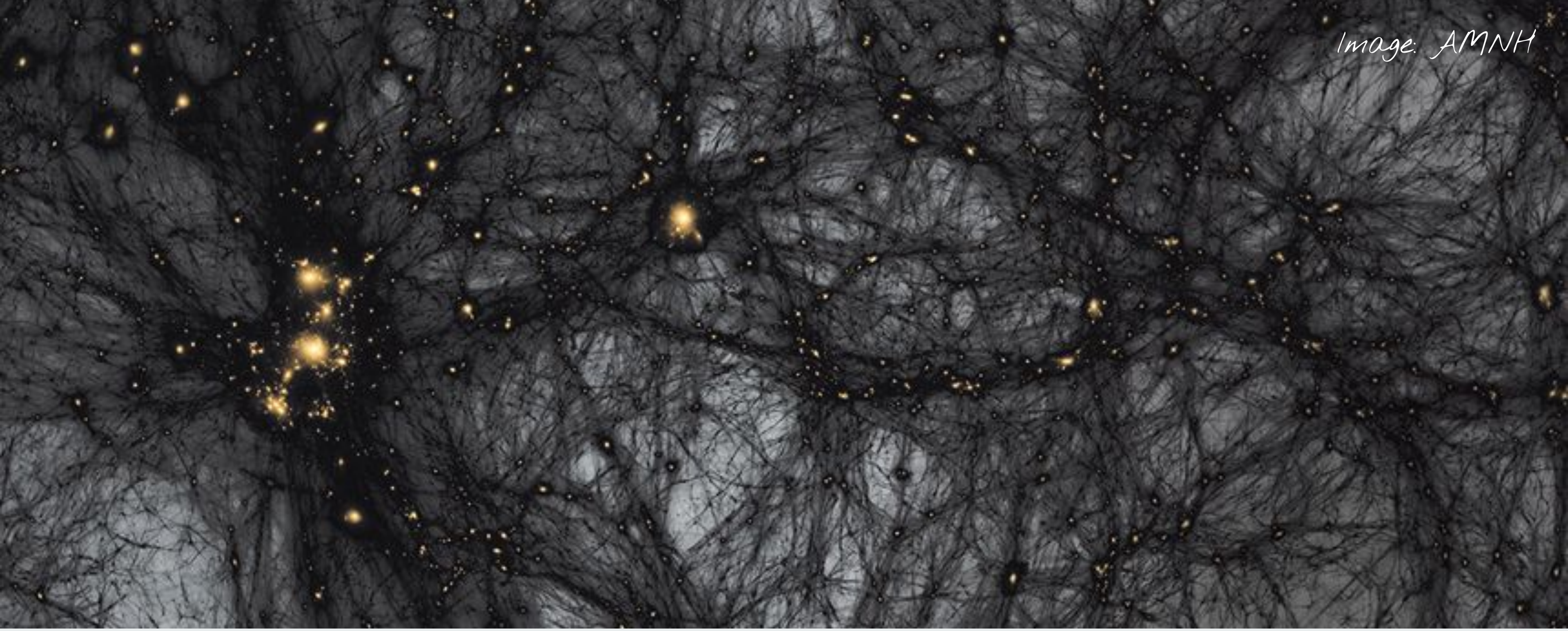
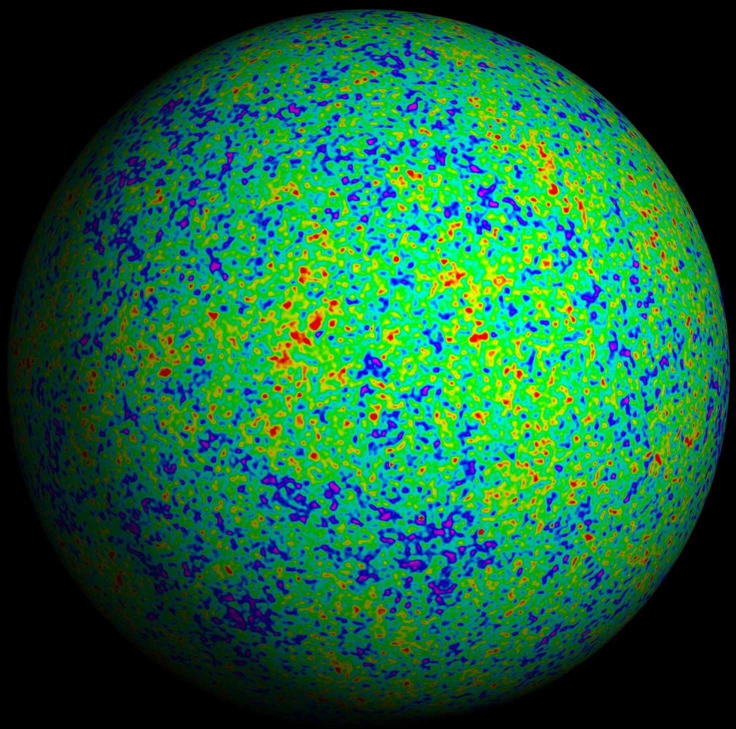


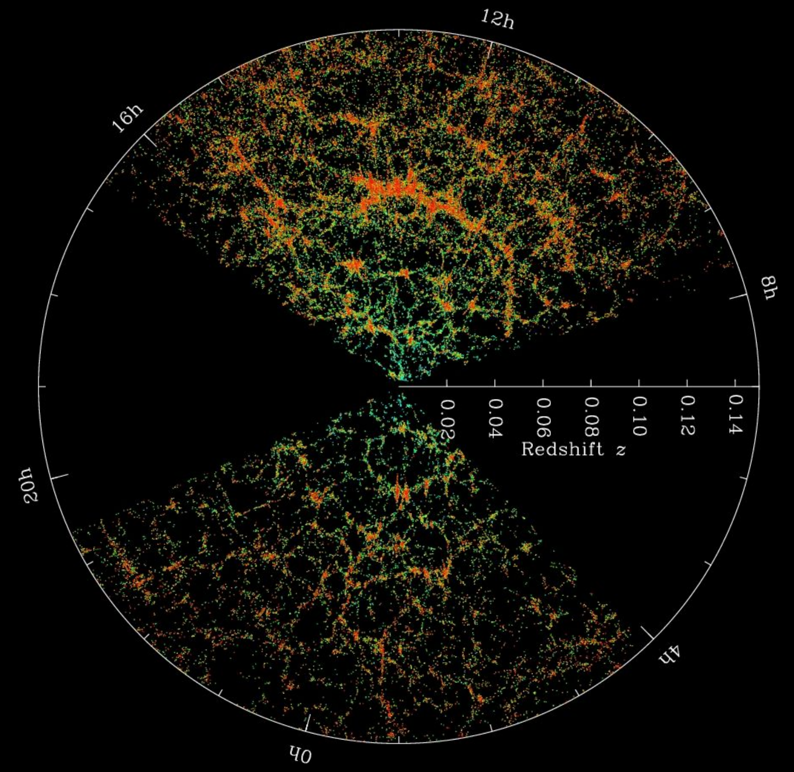
Large-Scale Structure Formation

Jia Liu (Princeton University)
Dark Matter Summer School, July 16, 2018
University at Albany

Goals

How Did This Happen?

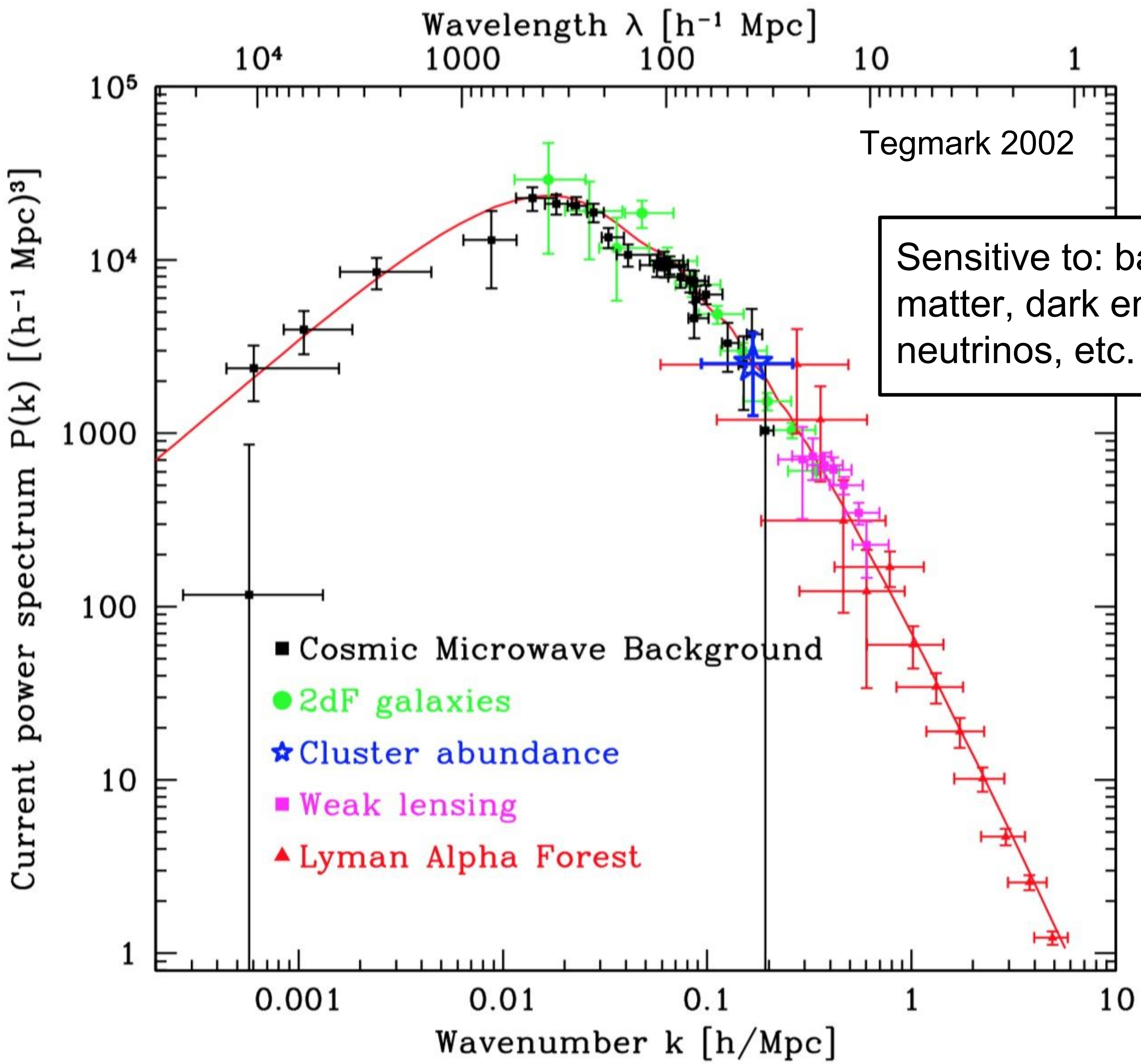




Extremely Gaussian Field
z=1100

Highly Nonlinear Structures
z<1

The Matter Power Spectrum



Sensitive to: baryons, dark matter, dark energy, neutrinos, etc.

Plan

1. Cosmological Observations 10’
2. Growth of Structure 25’
3. Gaussian Random Field 5’
4. The Matter Power Spectrum 20’
5. Large Scale Structure Probes 10’

Don’t be afraid of asking questions in class (and in life)!

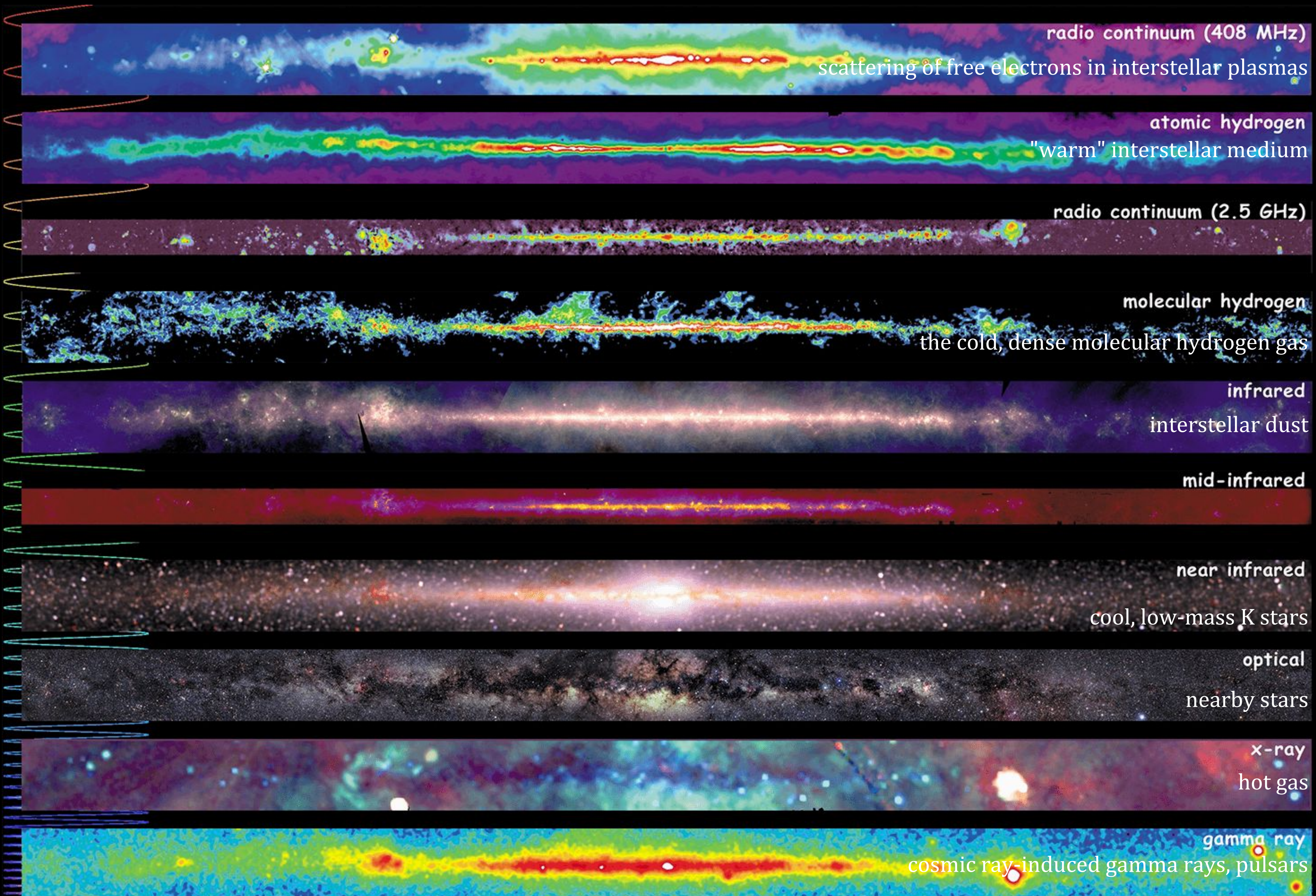
– “I’m the only one who doesn’t know this.”
You will be surprised how many people are just as puzzled..

– “I can’t frame this question clearly and eloquently.”
Start practicing now!

– “Maybe she said it already when I dozed off just now...”
While it doesn’t hurt for others to hear one thing twice, you may not understand the rest of the class if you miss this point!

(slide inspired by: Jae Woo Lee)

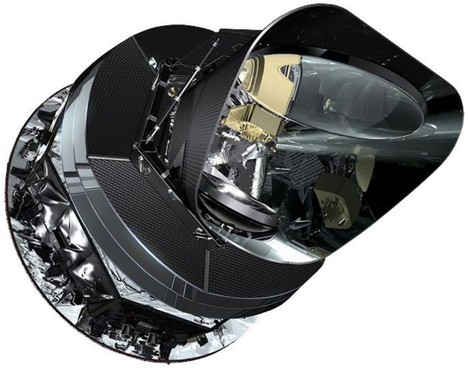
Cosmological Observations

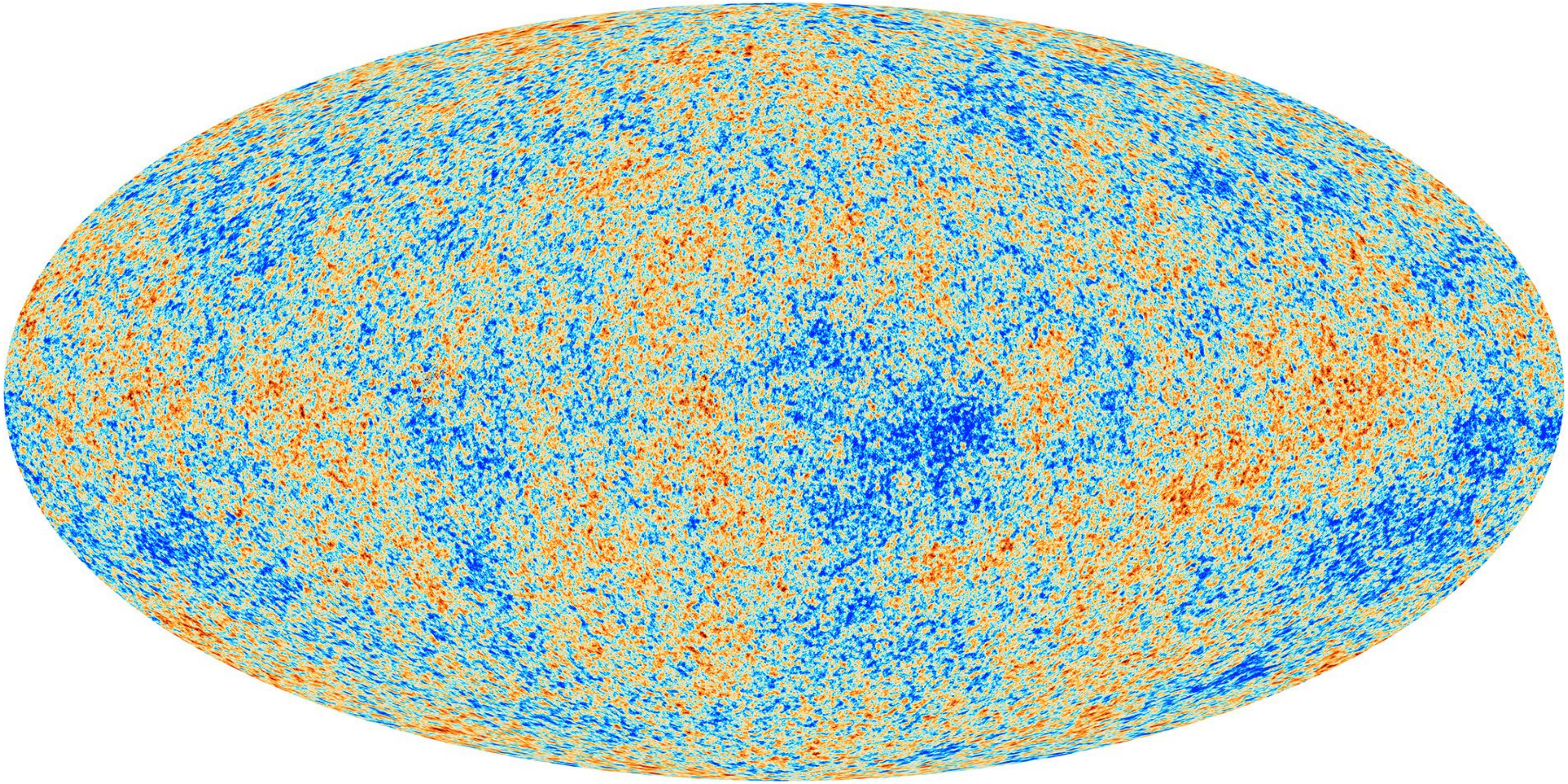


Multiwavelength Milky Way. Each wavelength gives us unique info. We can also apply the same technique to our Universe.

Cosmic Microwave Background
Planck Satellite

$z=1100$, Last scattering surface
380,000 years after the Big Bang



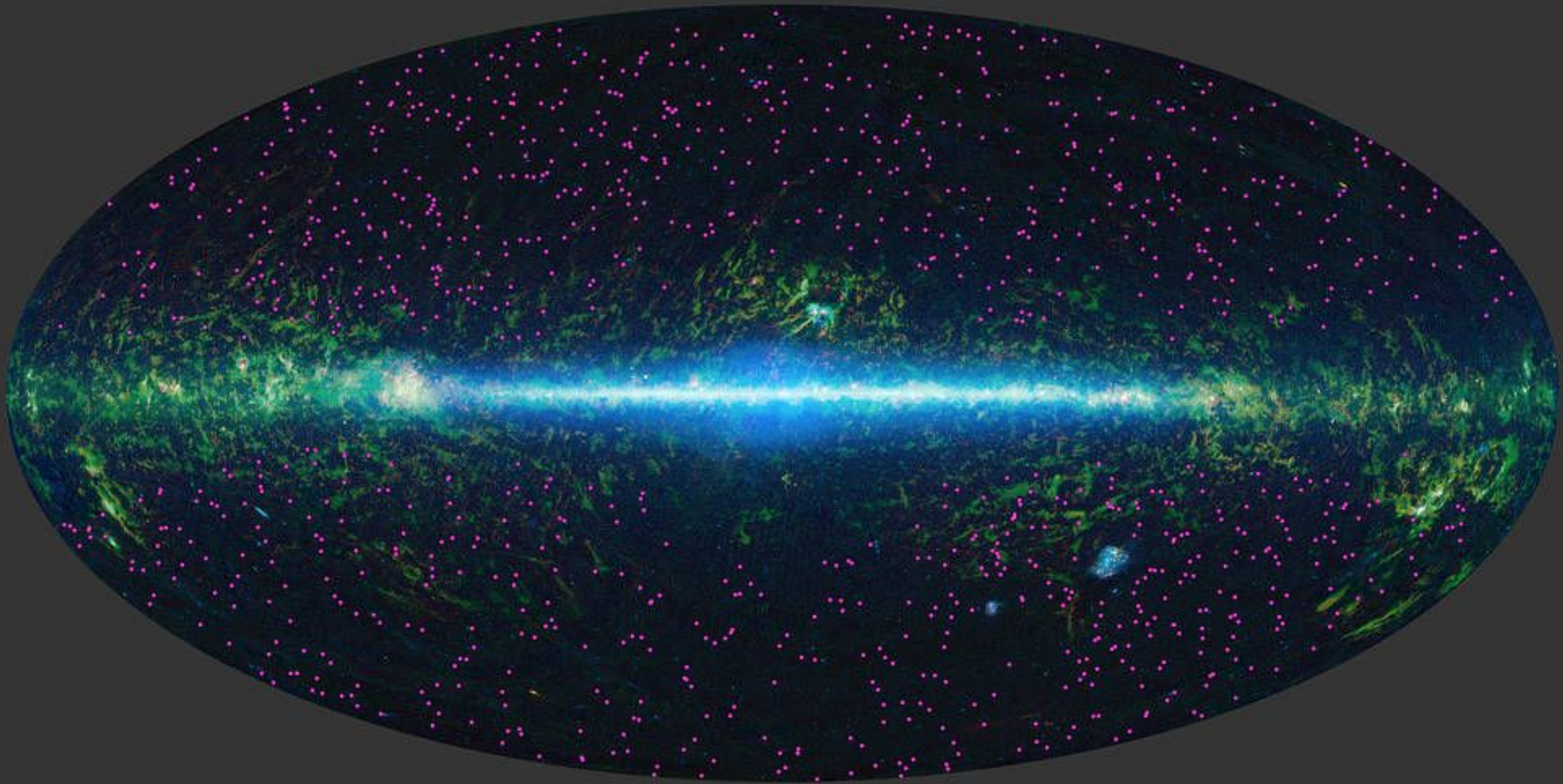


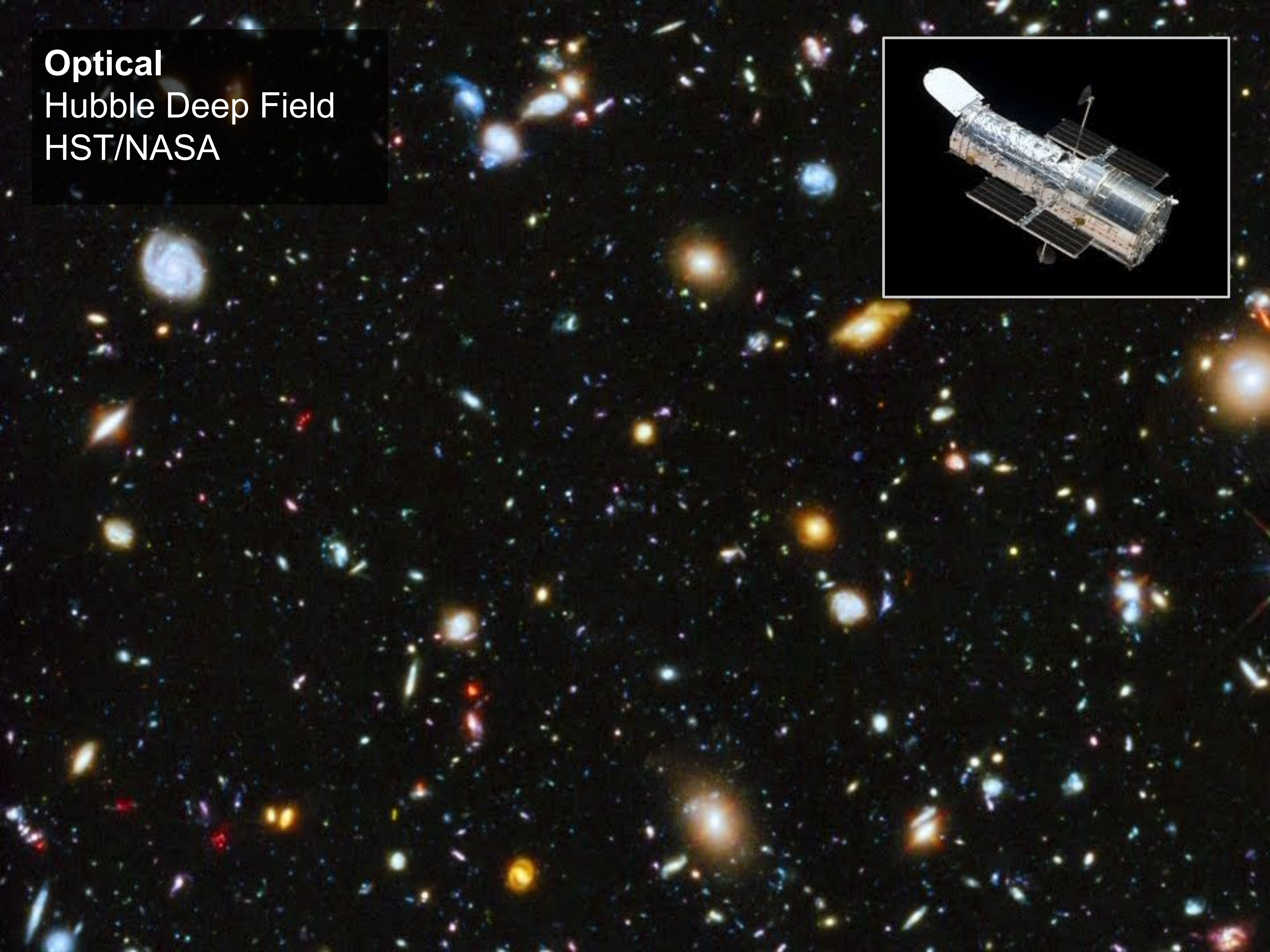
Infrared

Wide-field Infrared Survey Explorer (WISE)

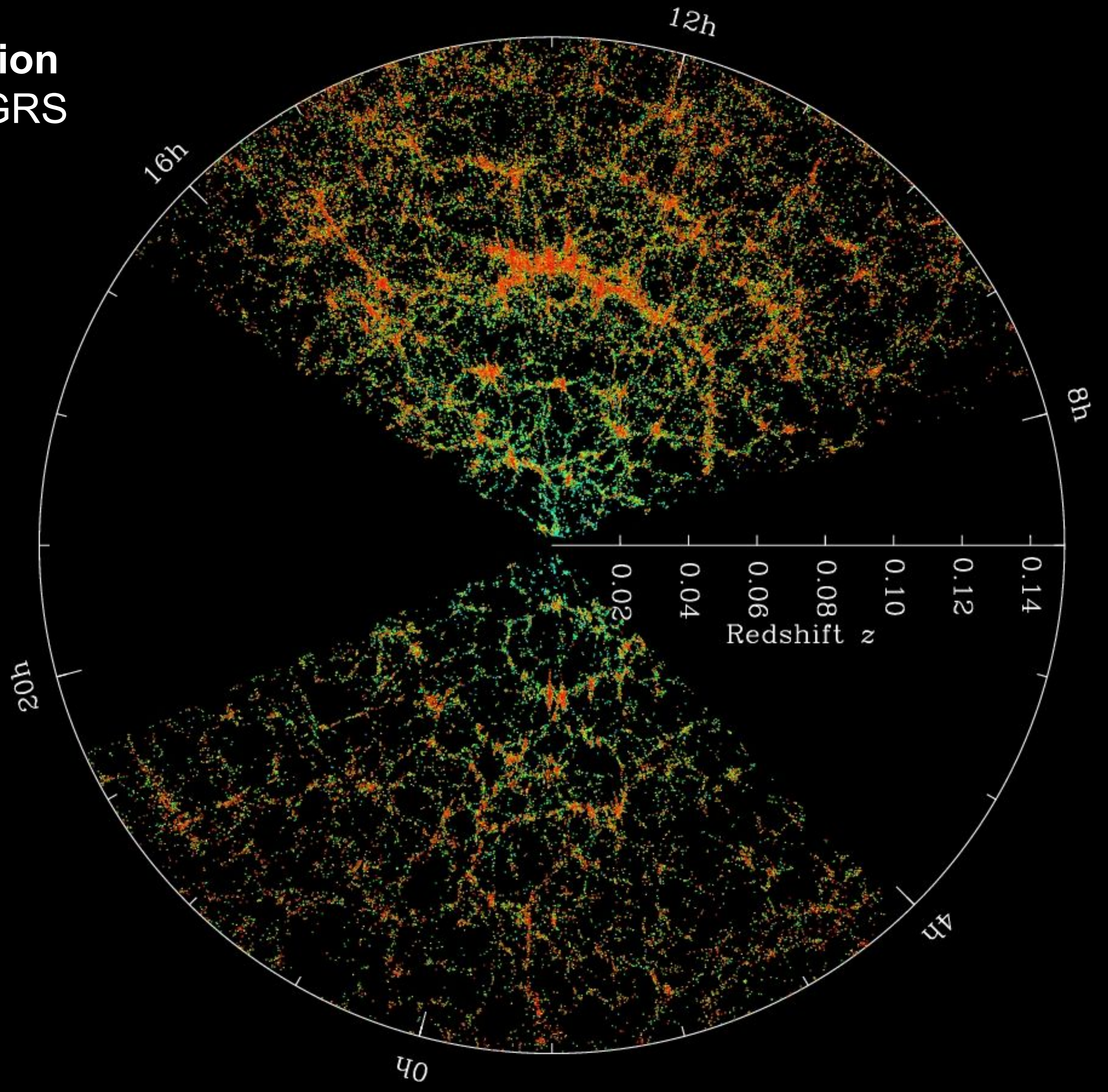
All Sky IR map of dusty/star forming galaxies

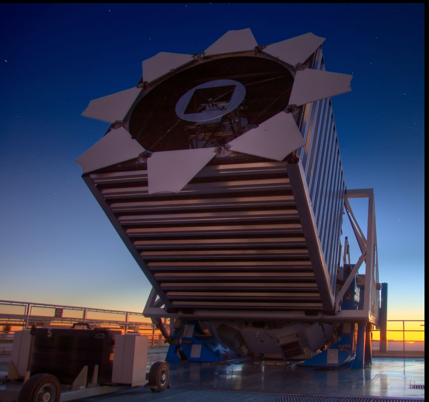
At 3.3, 4.7, 12, and 23 micrometers



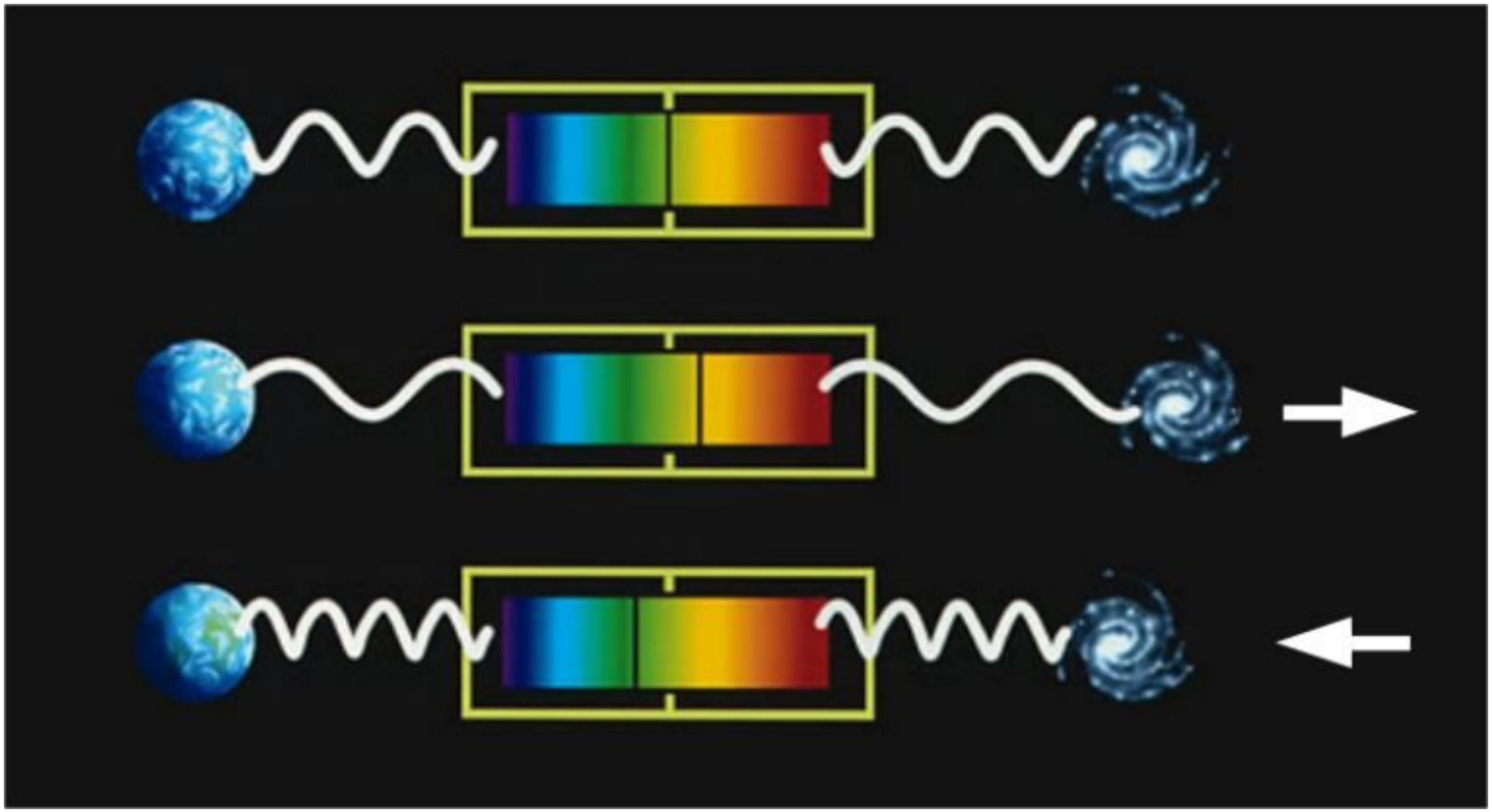

Optical
Hubble Deep Field
HST/NASA

Galaxy Distribution
SDSS/CfA2/2dFGRS



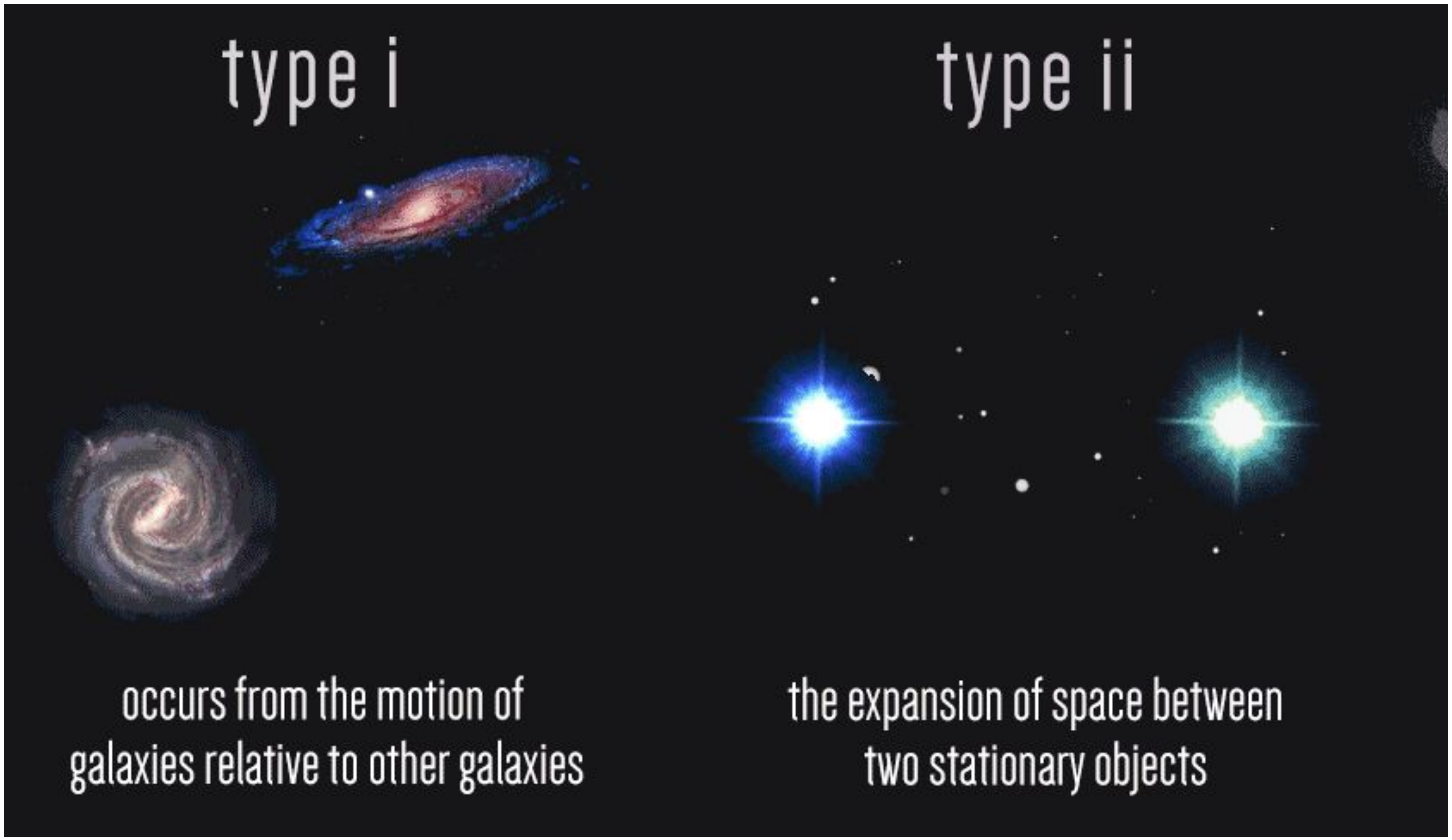


Redshift as Time and/or Distance



$$z = \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1$$

Redshift as Time and/or Distance



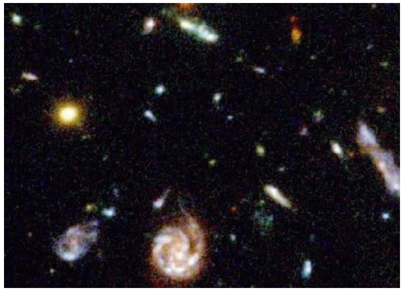
http://princetoninnovation.org/magazine/2015/12/09/redshift-universes-doppler-effect/

Redshift	v/c	Present Distance (Mpc)	Present Distance (10^6 light-years)	Look-Back Time (millions of years)
0.000	0.000	0	0	0
0.010	0.010	43	139	139
0.025	0.025	107	347	343
0.050	0.049	212	691	674
0.100	0.095	419	1370	1300
0.200	0.180	820	2670	2440
0.250	0.220	1010	3300	2950
0.500	0.385	1910	6210	5080
0.750	0.508	2680	8750	6650
1.000	0.600	3350	10,900	7820
1.500	0.724	4450	14,500	9420
2.000	0.800	5300	17,300	10,400
3.000	0.882	6520	21,300	11,600
4.000	0.923	7370	24,000	12,200
5.000	0.946	8000	26,100	12,600
6.000	0.960	8490	27,700	12,800
7.000	0.969	8890	29,000	13,000
8.000	0.976	9220	30,100	13,100
9.000	0.980	9500	31,000	13,200
10.000	0.984	9740	31,800	13,300
50.000	0.999	12,400	40,400	13,700

Distance units used in Cosmology:

Parsec (pc) = 3.26 years
Typical distance between 2 stars

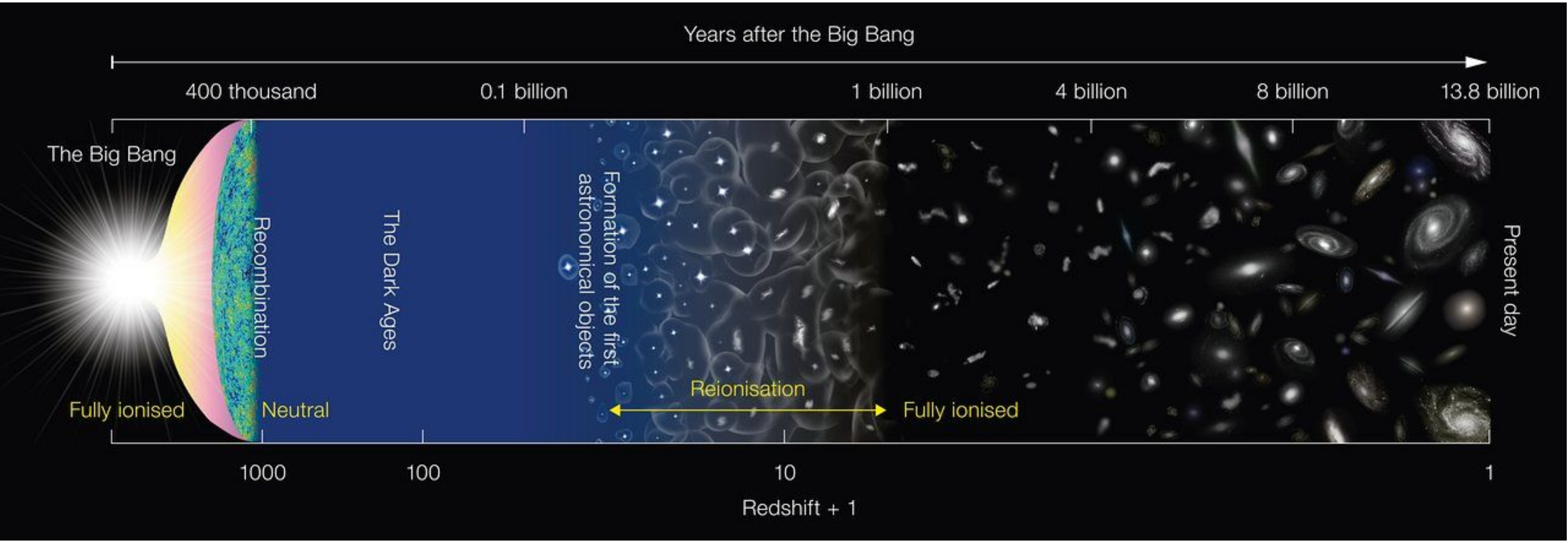
Megaparsec (Mpc) = 10^6 pc
Typical distance between 2 galaxies



Growth of Structure

Universe Timeline

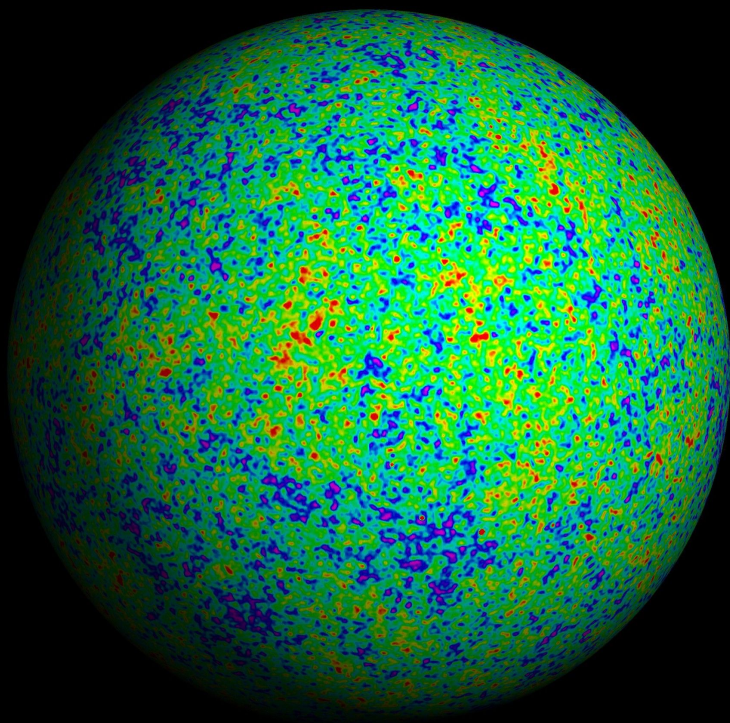
The Cosmic Microwave Background at z=1100



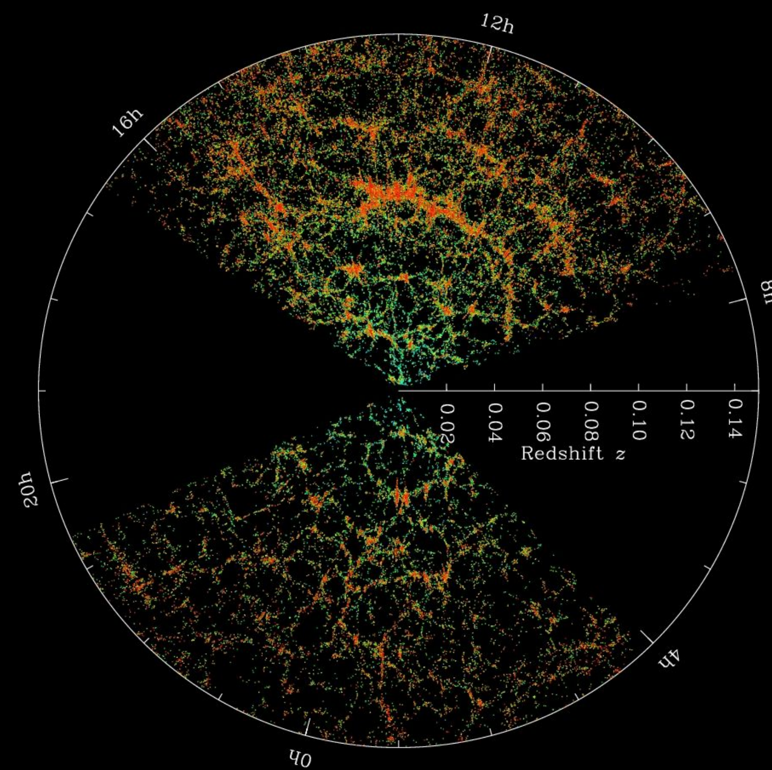
High redshift (distant Universe): Difficult to observe

More/better data at lower redshifts

How Did This Happen?



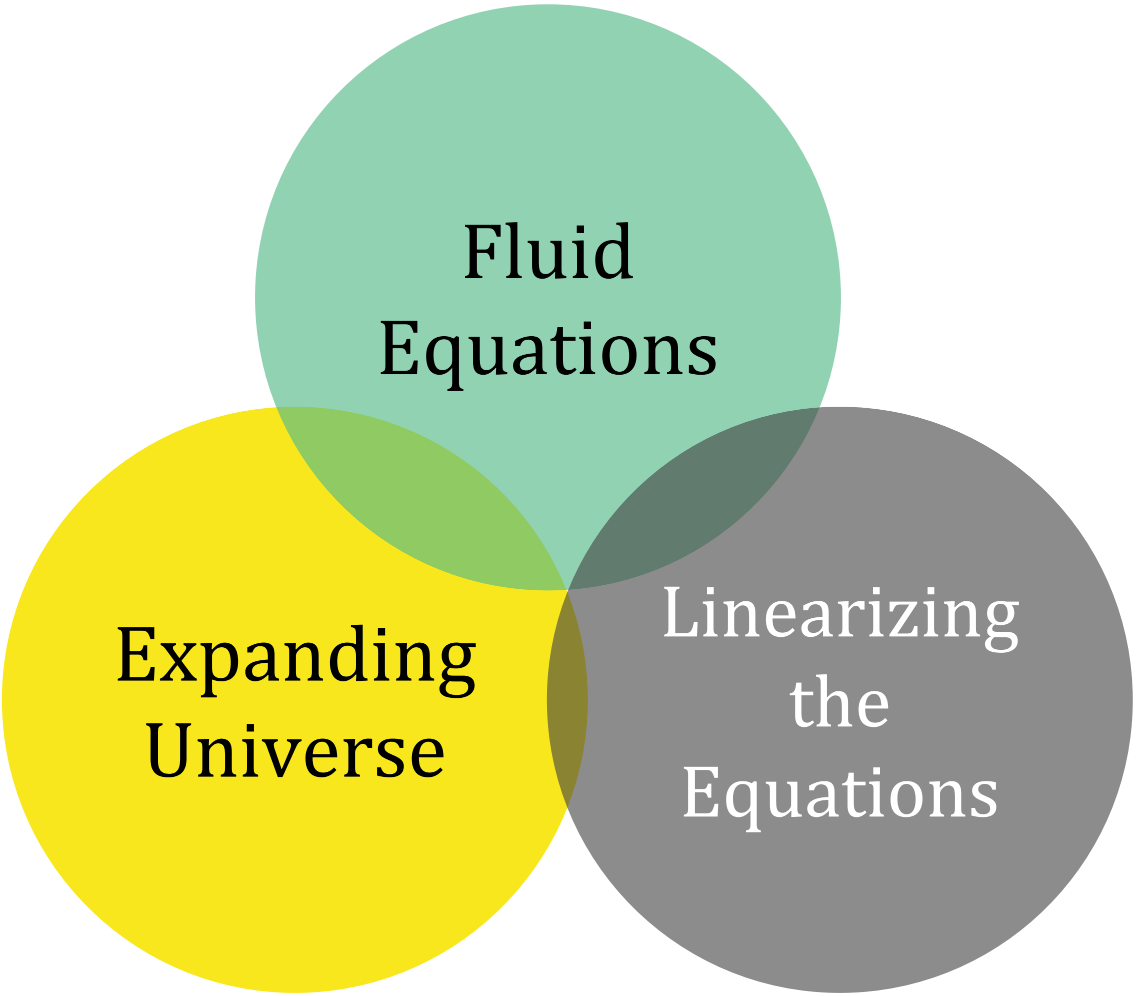
Extremely Gaussian Field



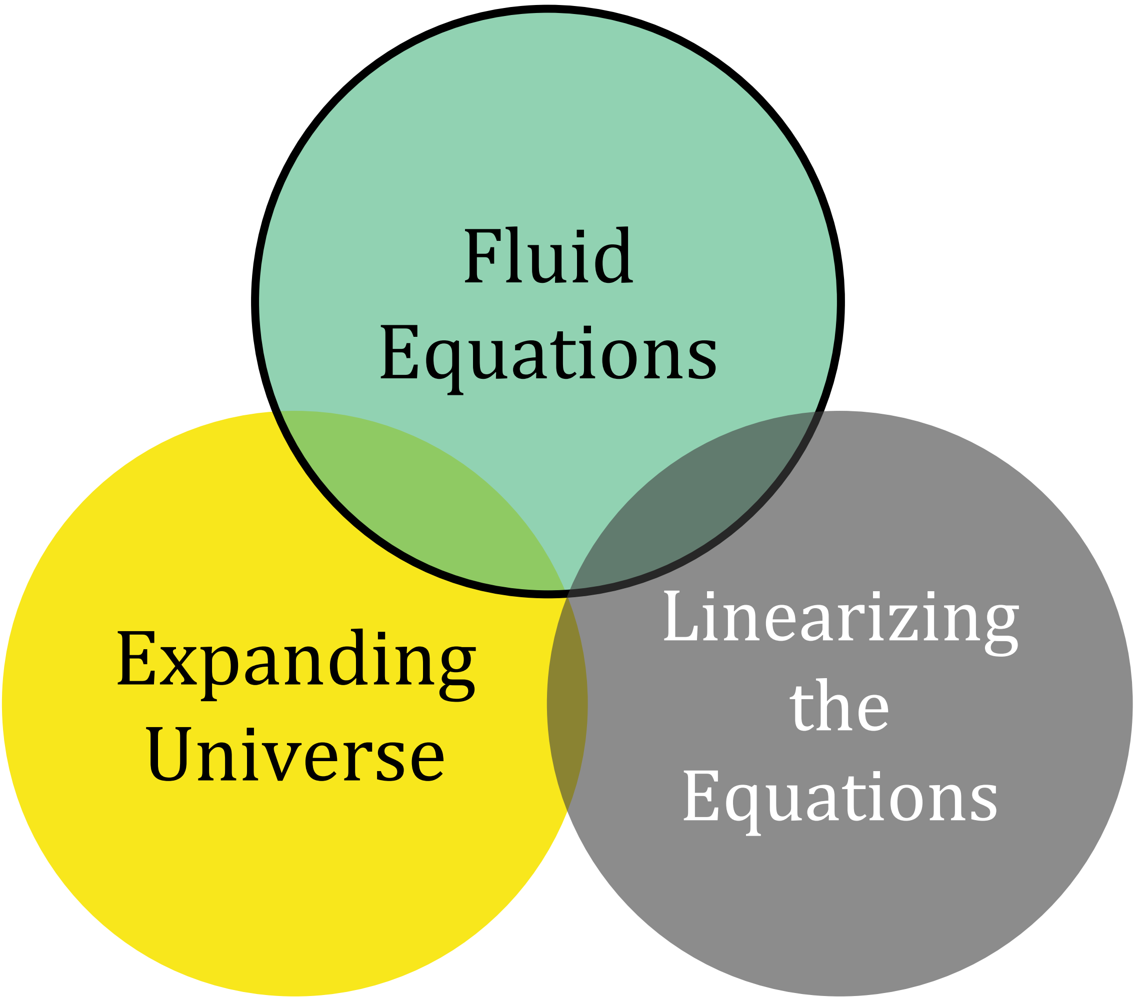
Highly Nonlinear Structures


z = 0.7
T = 7.22 Gyr
500 kpc

Ingredients For Structure Growth



Ingredients For Structure Growth

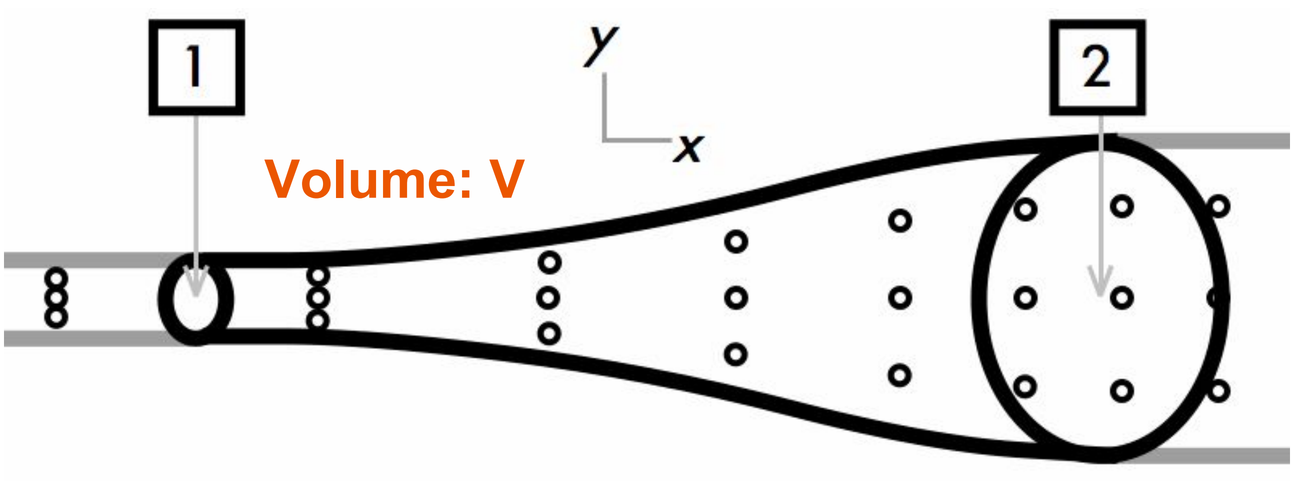


1. Continuity Equation (Mass Conservation)

Change in the density inside a fixed volume V

$$\partial_t \rho = -\nabla_{\boldsymbol{r}} \cdot (\rho \boldsymbol{u})$$

Total mass flow in/out of the boundary

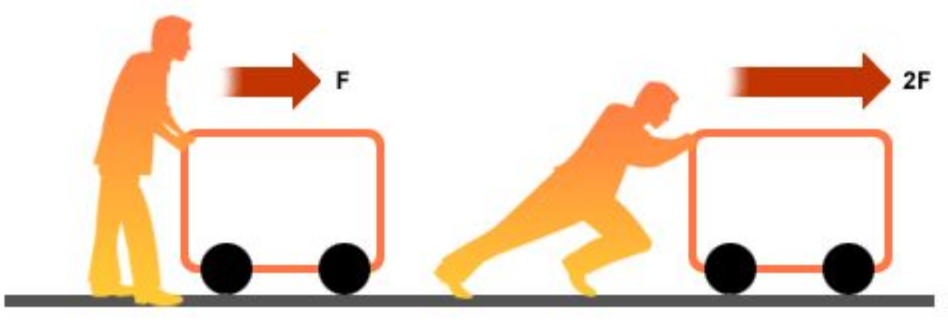


2. Euler Equation (Momentum Conservation)

$$(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla_r})\, \boldsymbol{u} = -\frac{\boldsymbol{\nabla_r} P}{\rho} - \boldsymbol{\nabla_r} \Phi$$

Equivalent to

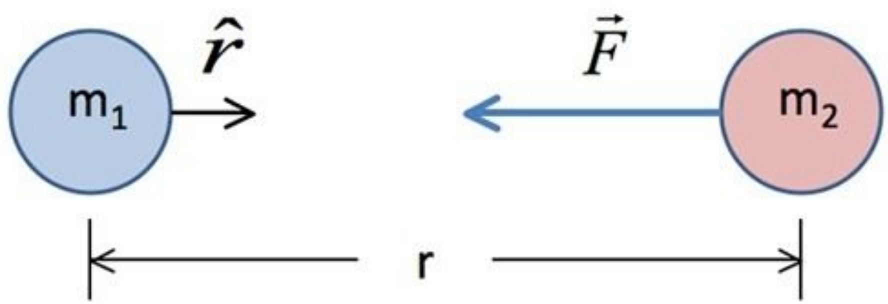
a = F/m



3. Poisson Equation

$$\nabla_r^2 \Phi = 4\pi G \rho$$

Equivalent to
Newton's Law of Gravity



$$\vec{F} = -\frac{G m_1 m_2}{r^2} \hat{r}$$

Fluid Equations

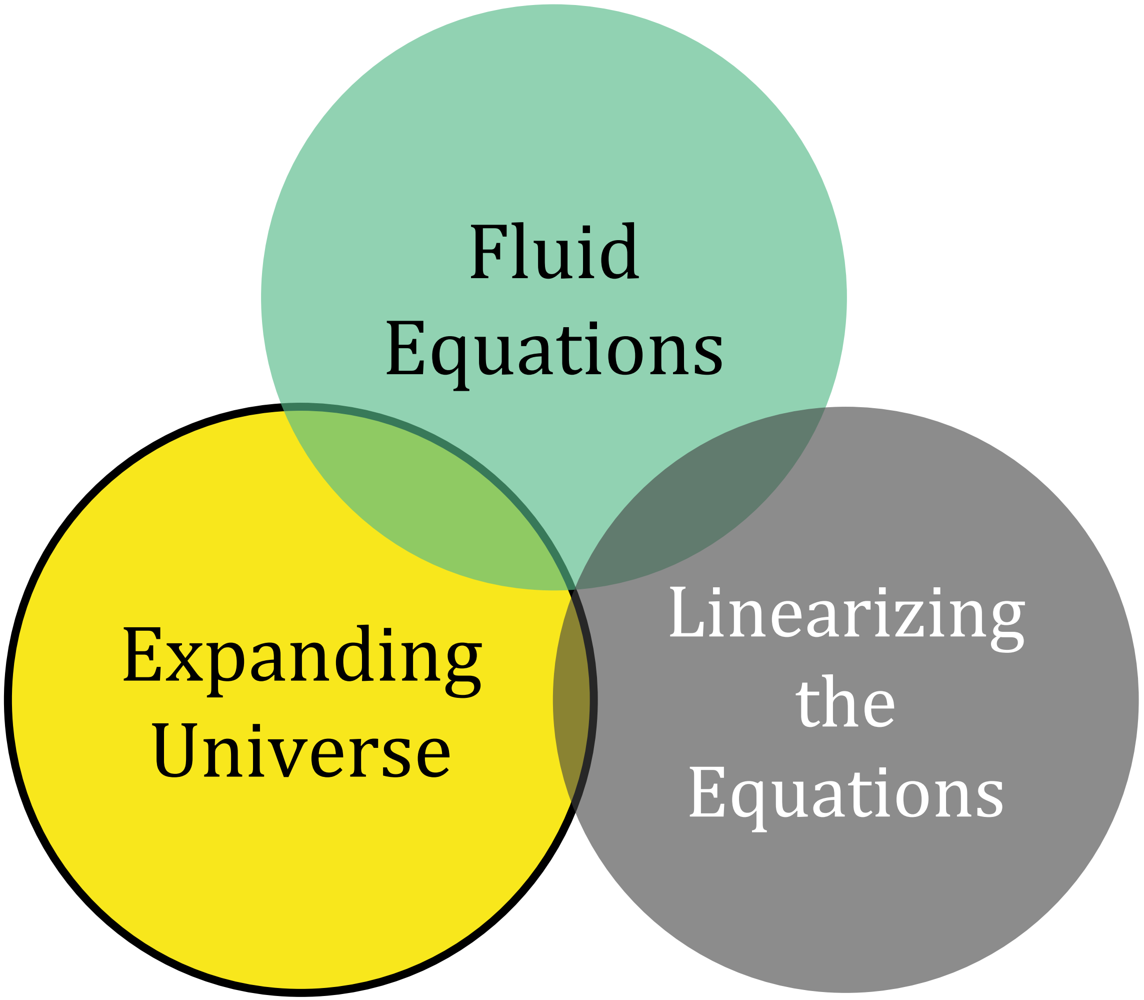
$\partial_t \rho = -\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot (\rho \boldsymbol{u})$	Continuity Equation (Mass Conservation)
$(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}})\, \boldsymbol{u} = -\frac{\boldsymbol{\nabla}_{\boldsymbol{r}} P}{\rho} - \boldsymbol{\nabla}_{\boldsymbol{r}} \Phi$	Euler Equation (Momentum Conservation)
$\nabla_{\boldsymbol{r}}^2 \Phi = 4\pi G \rho$	Poisson Equation

ϱ: density Φ: grav. potential P: pressure u: velocity

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \qquad \partial_t = \partial/\partial t$$

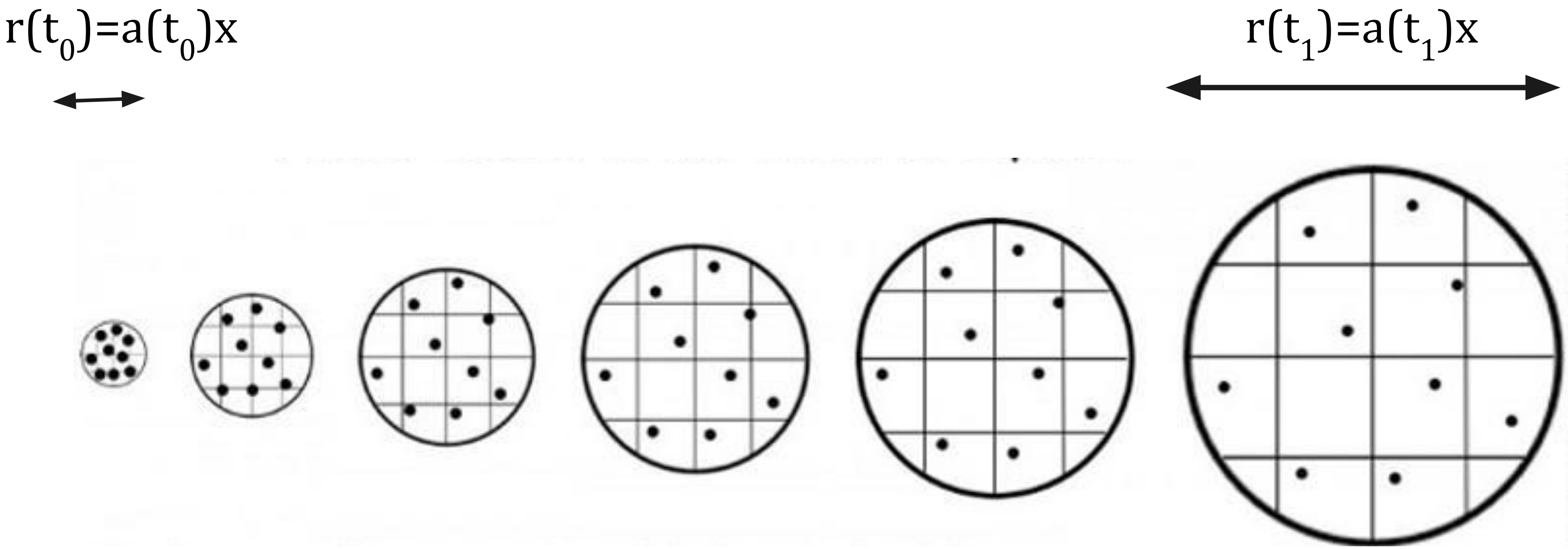
Detailed derivation see Daniel Baumann's online notes:
http://www.damtp.cam.ac.uk/user/db275/Cosmology.pdf

Ingredients For Structure Growth



Expanding Universe

r: physical, x: comoving



$r(t)=a(t)x$: the comoving coordinate x of all galaxies remain the same, while physical distances r are scaled by the scale factor a.

Friedmann Equation

(governs the expansion of the universe)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

G = gravitation constant

ρ = density

k = curvature parameter

Sometimes expressed as:

$$H(a) = H_0\sqrt{\Omega_k a^{-2} + \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{\mathrm{DE}} a^{-3(1+w)}}$$

$$\rho_c = \frac{3H^2}{8\pi G} \qquad \Omega = \frac{\rho}{\rho_c}$$

Origin: 00 component of Einstein's Field Equation (links energy and space curvature)

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Friedmann Equation

(governs the expansion of the universe)

Expansion

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

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Friedmann Equation

(governs the expansion of the universe)

Expansion — Density

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Friedmann Equation

(governs the expansion of the universe)

Expansion | Density | Curvature

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Friedmann Equation

(governs the expansion of the universe)

Expansion | Density | Curvature | Dark Energy

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

G = gravitation constant

ρ = density

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Sometimes expressed as:

$$H(a) = H_0\sqrt{\Omega_k a^{-2} + \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{\mathrm{DE}} a^{-3(1+w)}}$$

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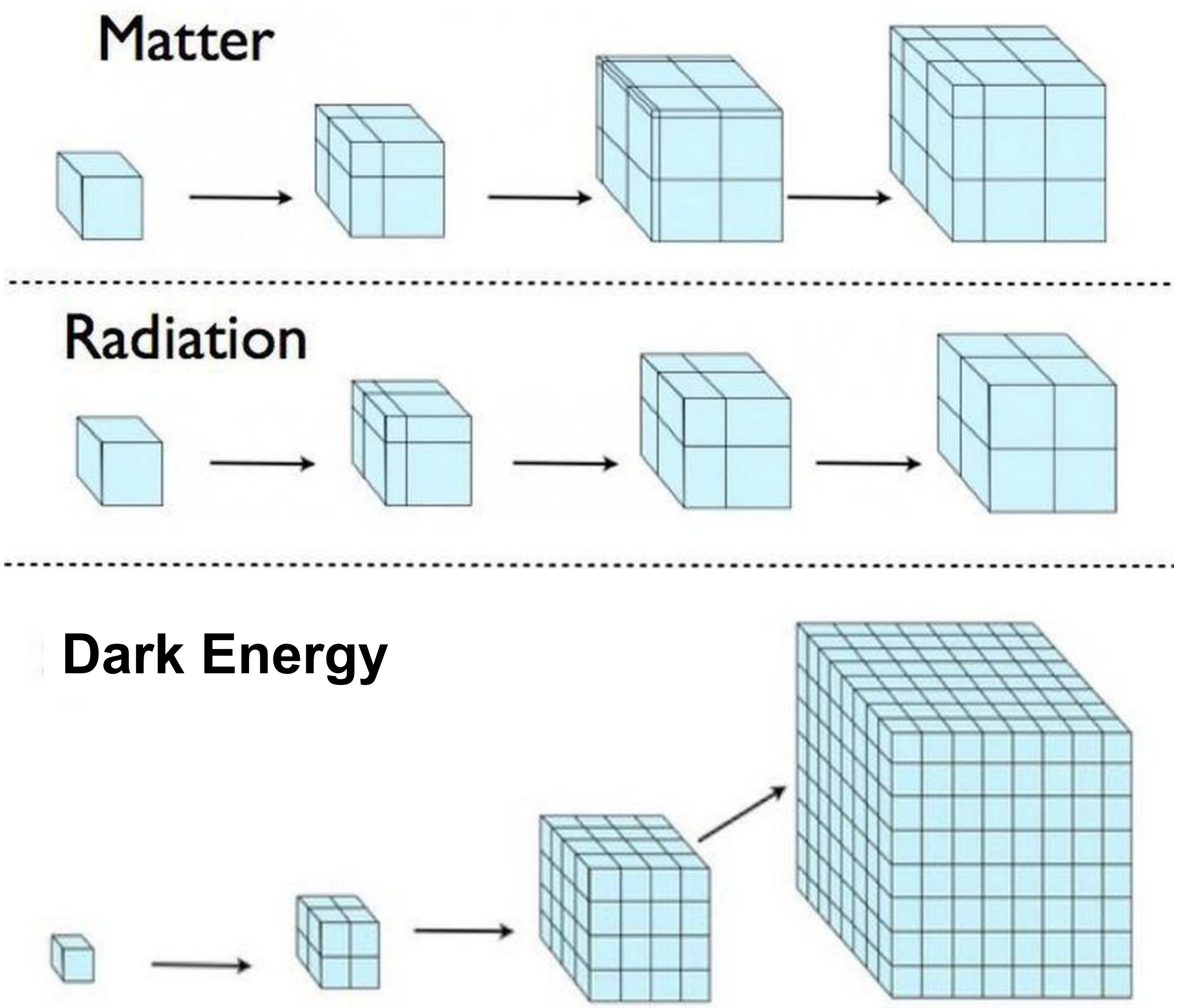
Origin: 00 component of Einstein's Field Equation (links energy and space curvature)

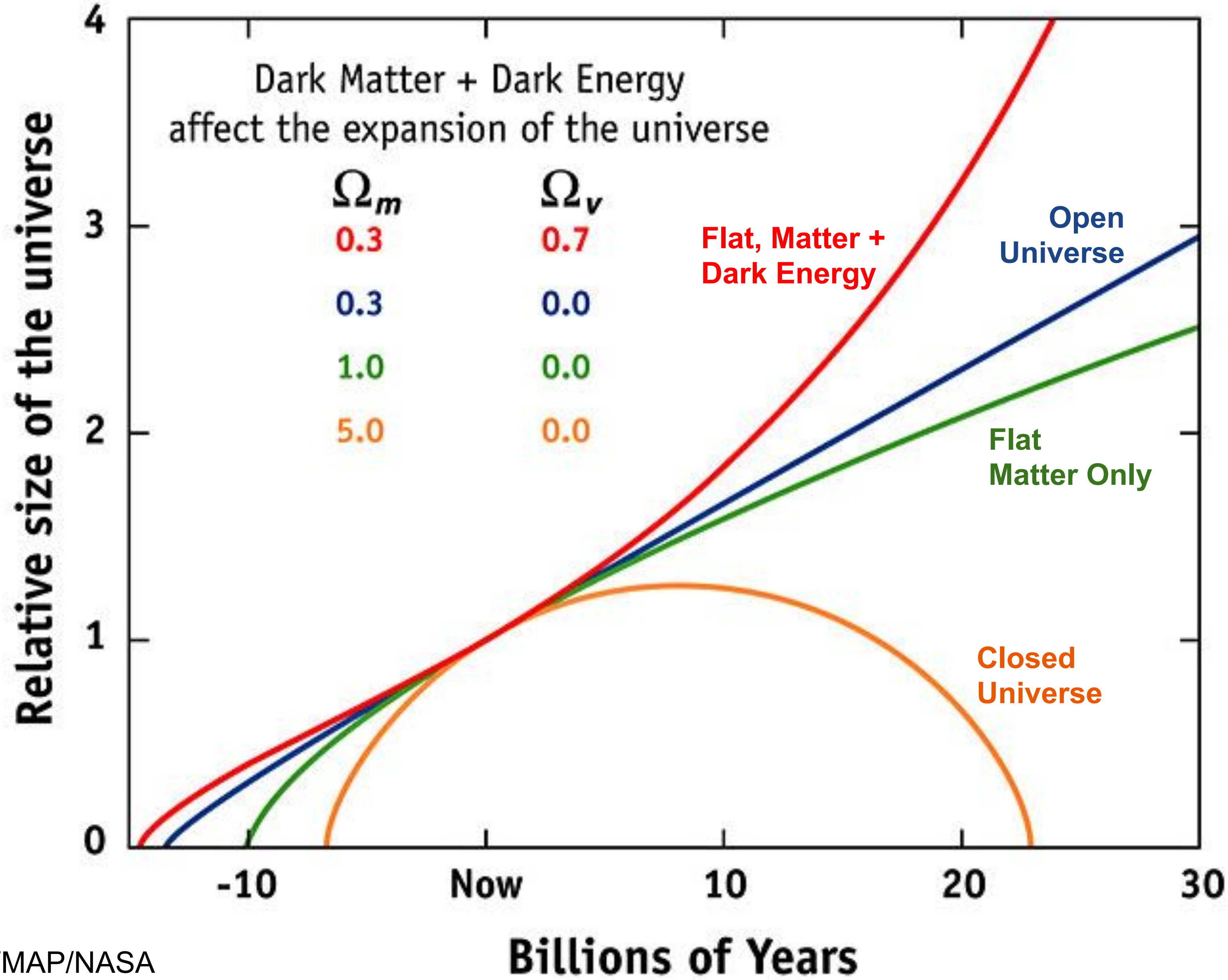
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Background (scale factor a) Expansion



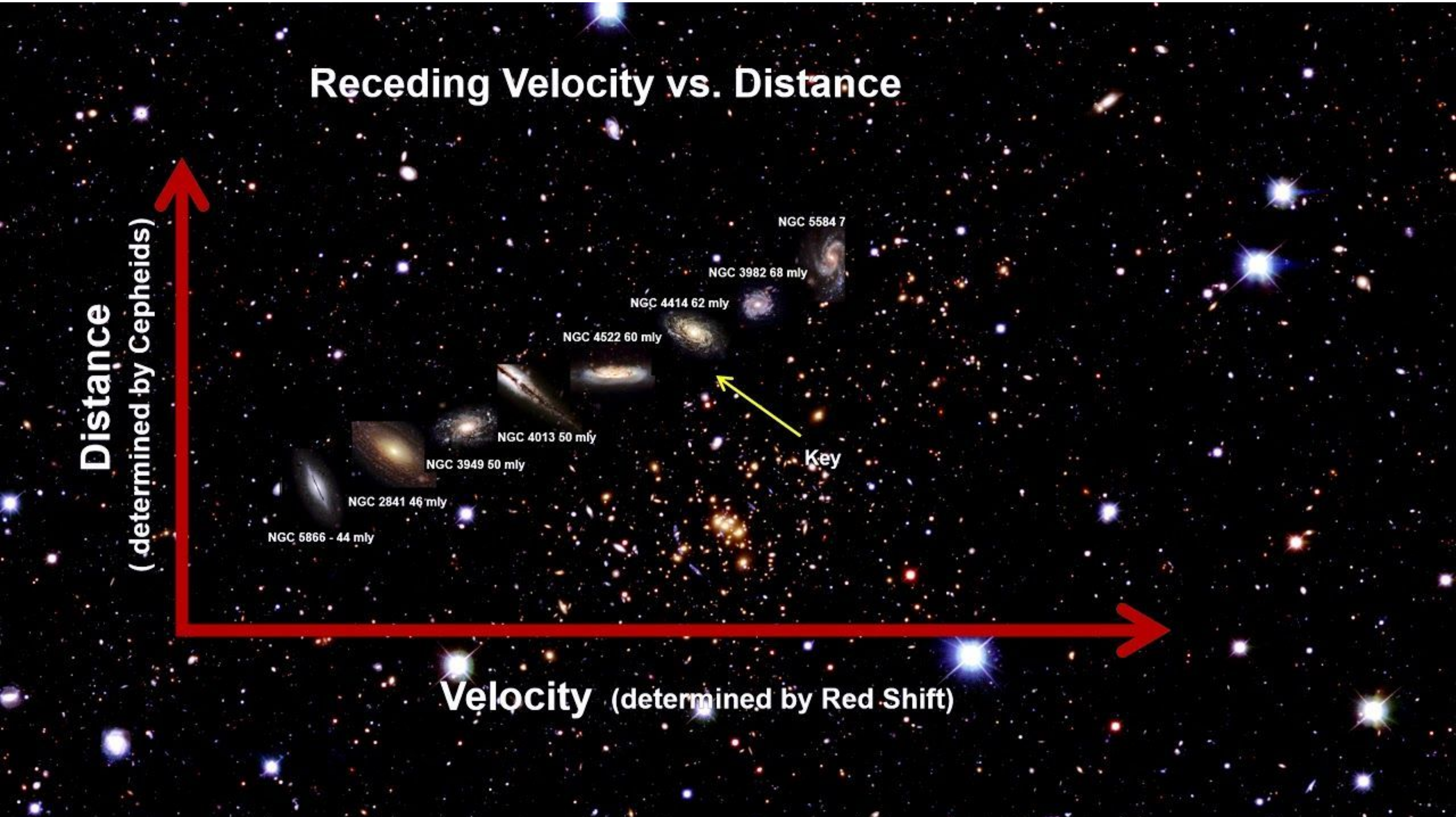
An illustration of how spacetime expands when it's dominated by Matter, Radiation or Dark Energy. All three of these solutions are derivable from the Friedmann equations. (Credit: E. Siegel)

Expansion of the Universe

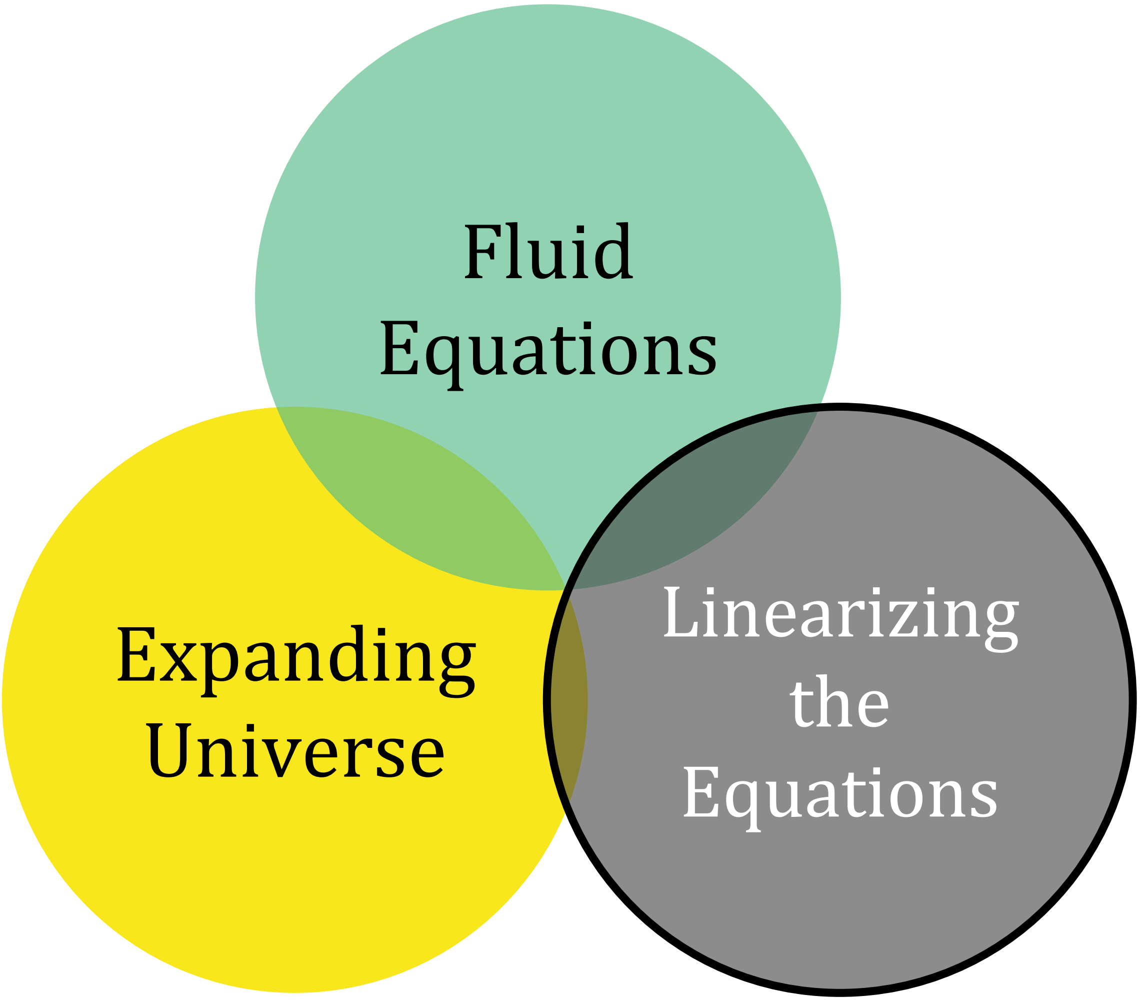


Credit: WMAP/NASA

Measuring the Hubble Parameter H: Distance vs Velocity (Redshift)



Ingredients For Structure Growth



Linearized Fluid Equations

(replace $x \rightarrow x_0 + \delta x$ and keep only the first order quantities)

1. Continuity Equation (Mass Conservation)

$$\partial_t \rho = -\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot (\rho \boldsymbol{u}) \quad \Longrightarrow \quad \dot{\delta} = -\frac{1}{a} \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

2. Euler Equation (Momentum Conservation)

$$(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}})\, \boldsymbol{u} = -\frac{\boldsymbol{\nabla}_{\boldsymbol{r}} P}{\rho} - \boldsymbol{\nabla}_{\boldsymbol{r}} \Phi \quad \Longrightarrow \quad \dot{\boldsymbol{v}} + H \boldsymbol{v} = -\frac{1}{a \bar{\rho}} \boldsymbol{\nabla} \delta P - \frac{1}{a} \boldsymbol{\nabla} \delta \Phi$$

3. Poisson Equation

$$\nabla_{\boldsymbol{r}}^2 \Phi = 4\pi G \rho \quad \Longrightarrow \quad \boldsymbol{\nabla}^2 \delta \Phi = 4\pi G a^2 \bar{\rho} \delta$$

Structure Evolution (Linear Theory)

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta = 4\pi G\bar{\rho}\delta$$

$$H(a) = H_0\sqrt{\Omega_k a^{-2} + \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{\rm DE} a^{-3(1+w)}}$$

* Example on black board: matter dominated era

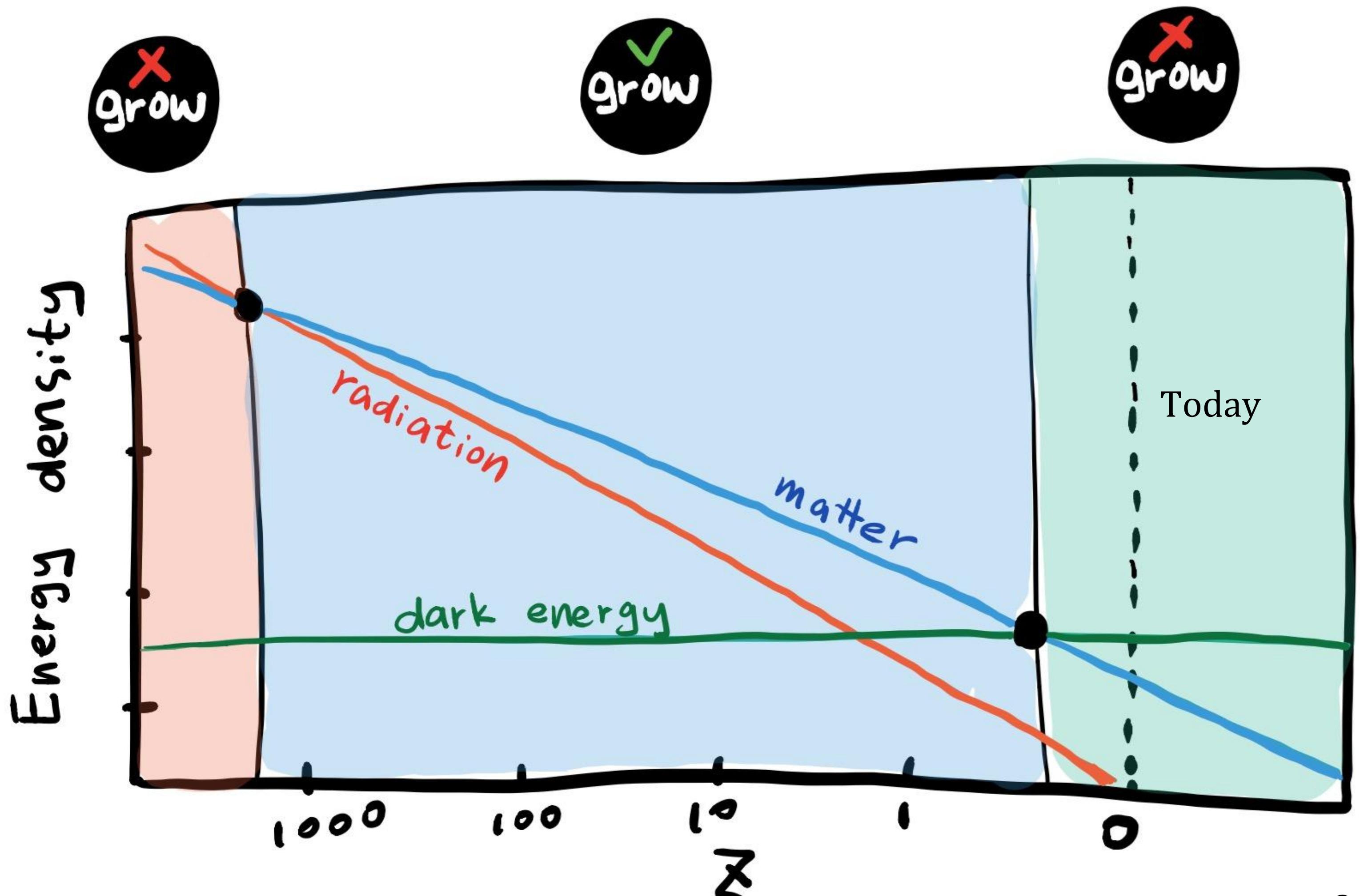
Structure Evolution (Linear Theory)

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta = 4\pi G\bar{\rho}\delta$$

$$H(a) = H_0\sqrt{\Omega_k a^{-2} + \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{\mathrm{DE}} a^{-3(1+w)}}$$

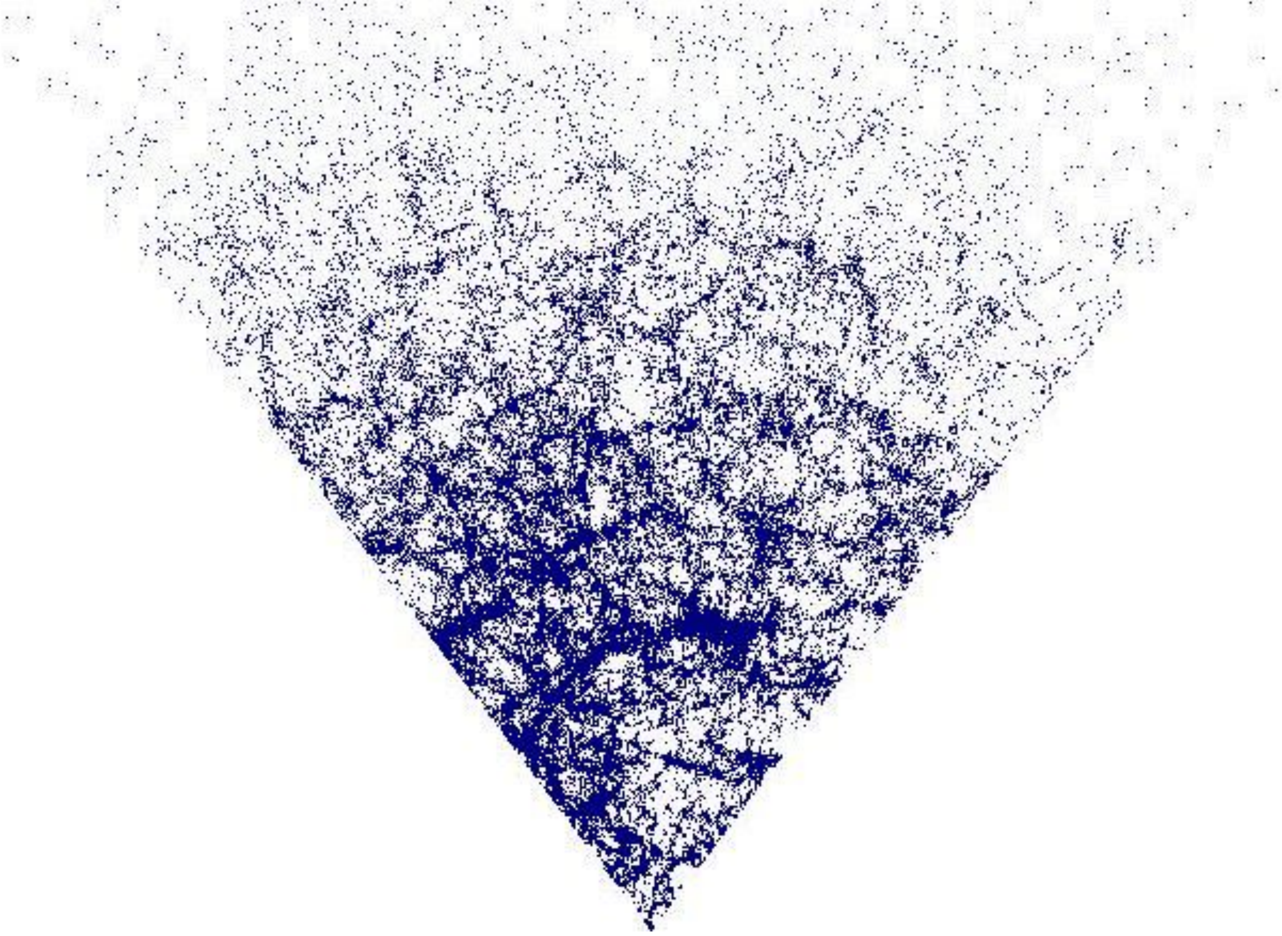
Radiation dominated era:	δ ~ ln a
Matter dominated era:	δ ~ a
Dark energy dominated era:	δ ~ constant

Key Result: Structure Grows Mainly in the Matter Dominated Epoch



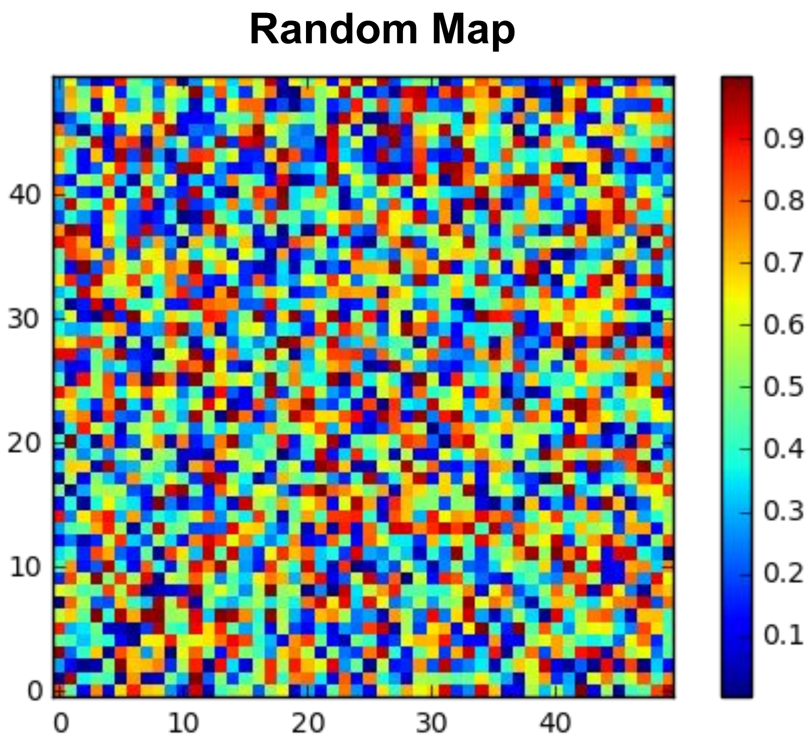
Credit: Jia Liu
Inspired by: Frieman, Turner, Huterer, 2008

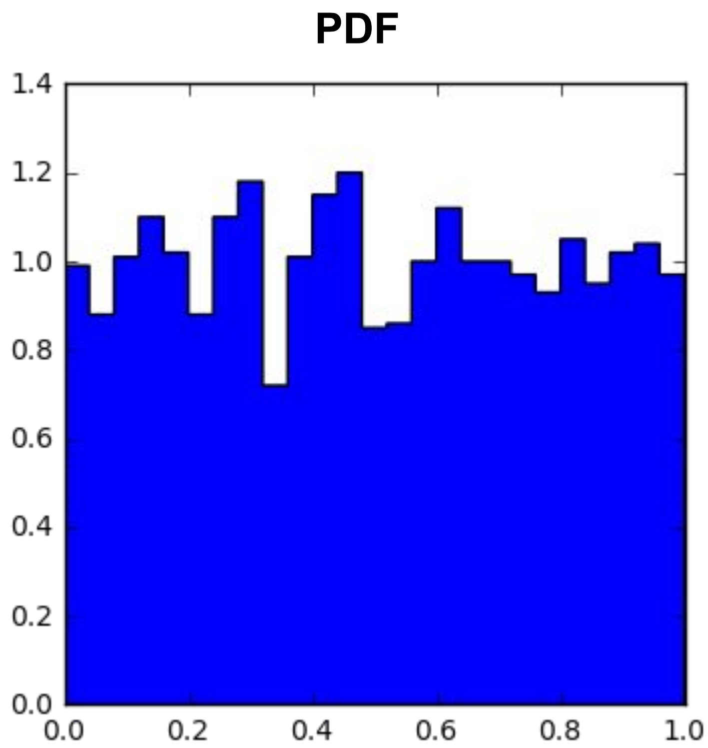
Gaussian Random Field



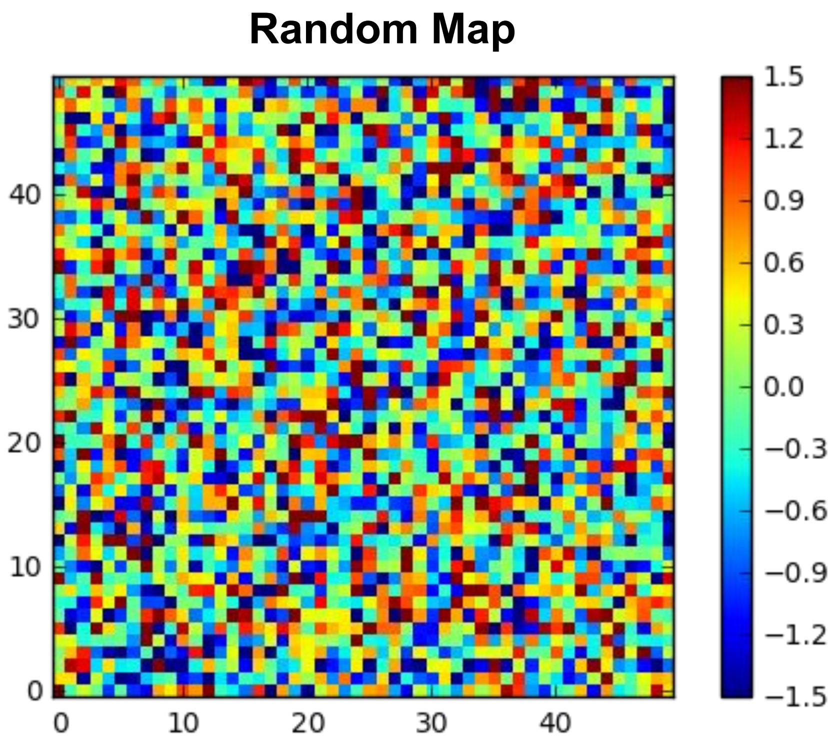
What do we do with all the beautiful cosmology data ?

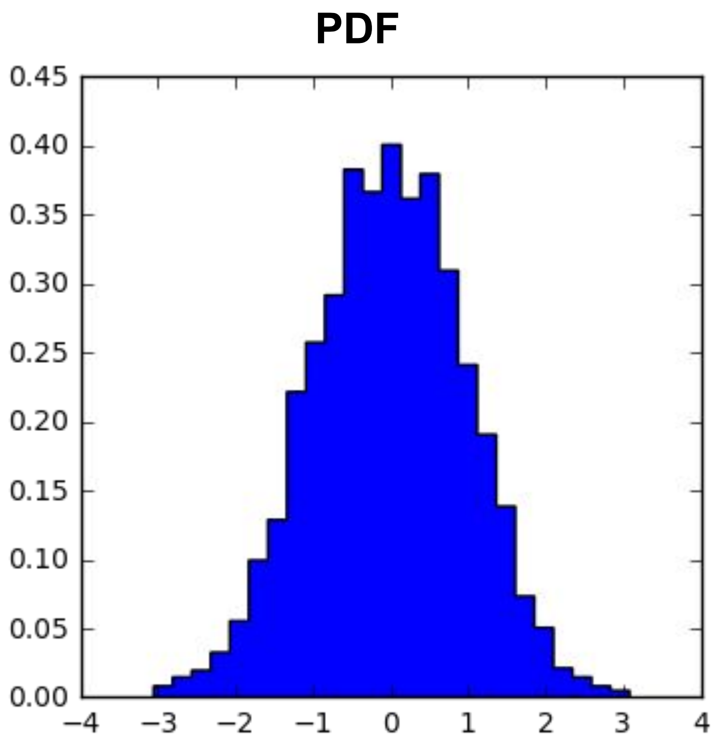
A Random Field with Flat Probability



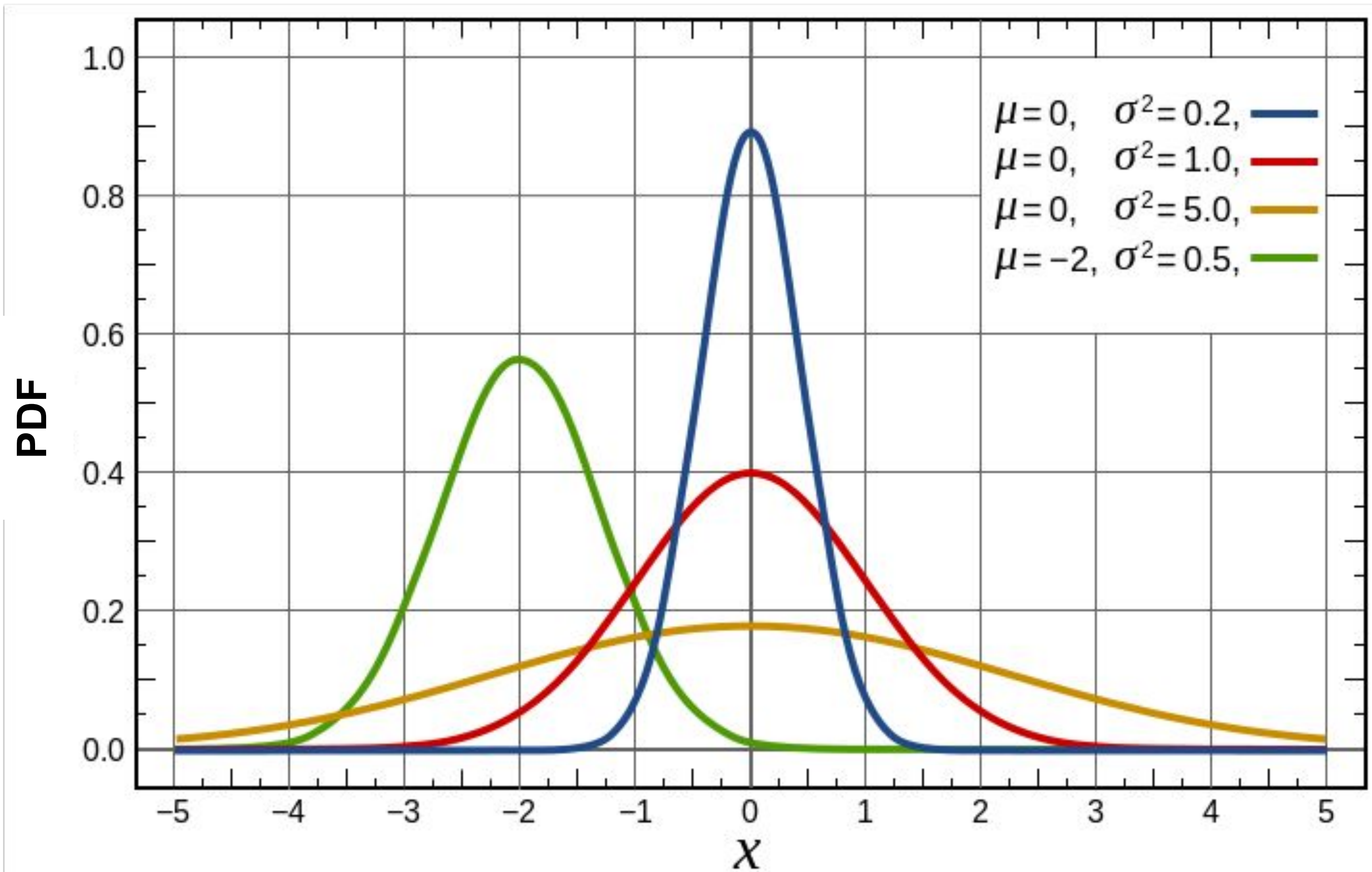


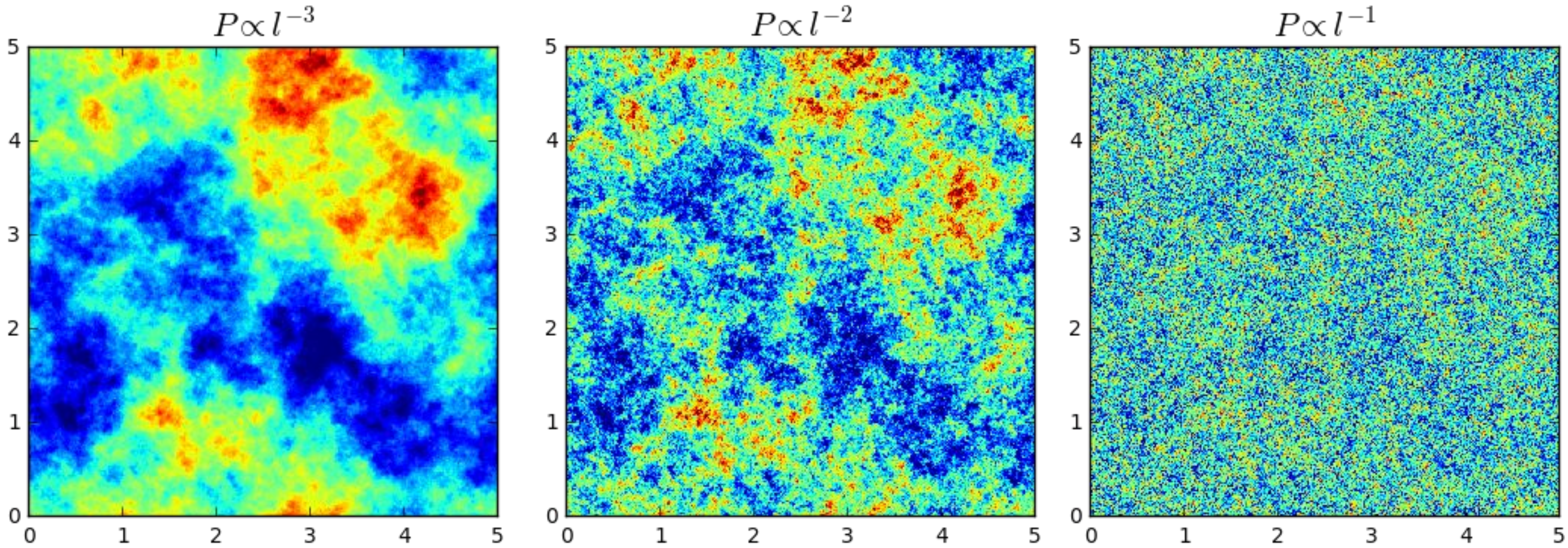
A Random Field with Gaussian Probability

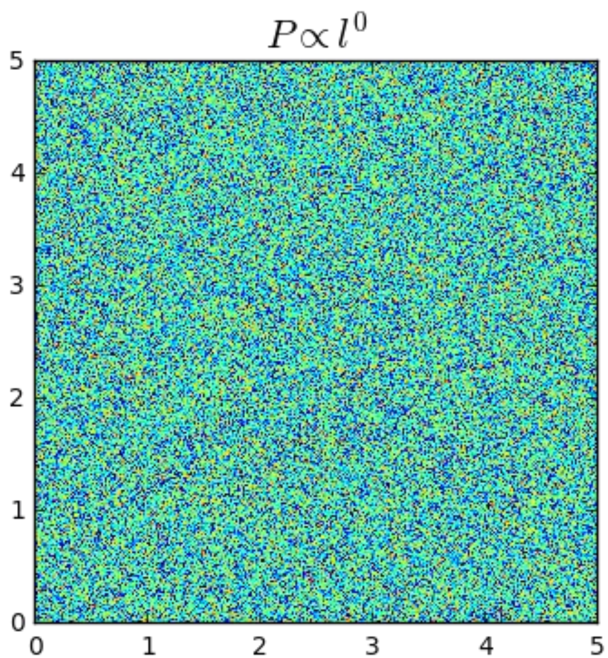




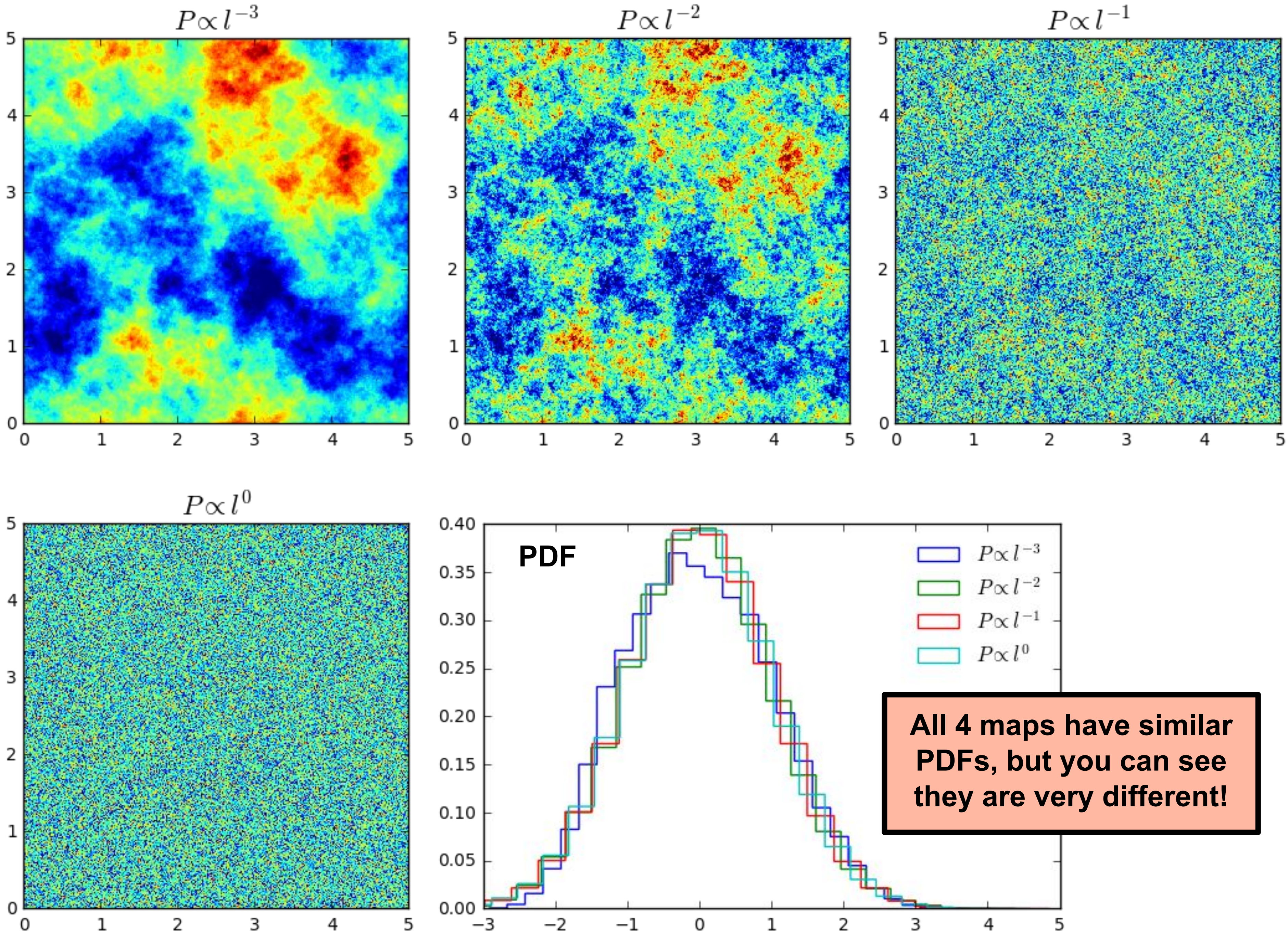
A Gaussian distribution is fully described by 2 numbers: Mean, Variance





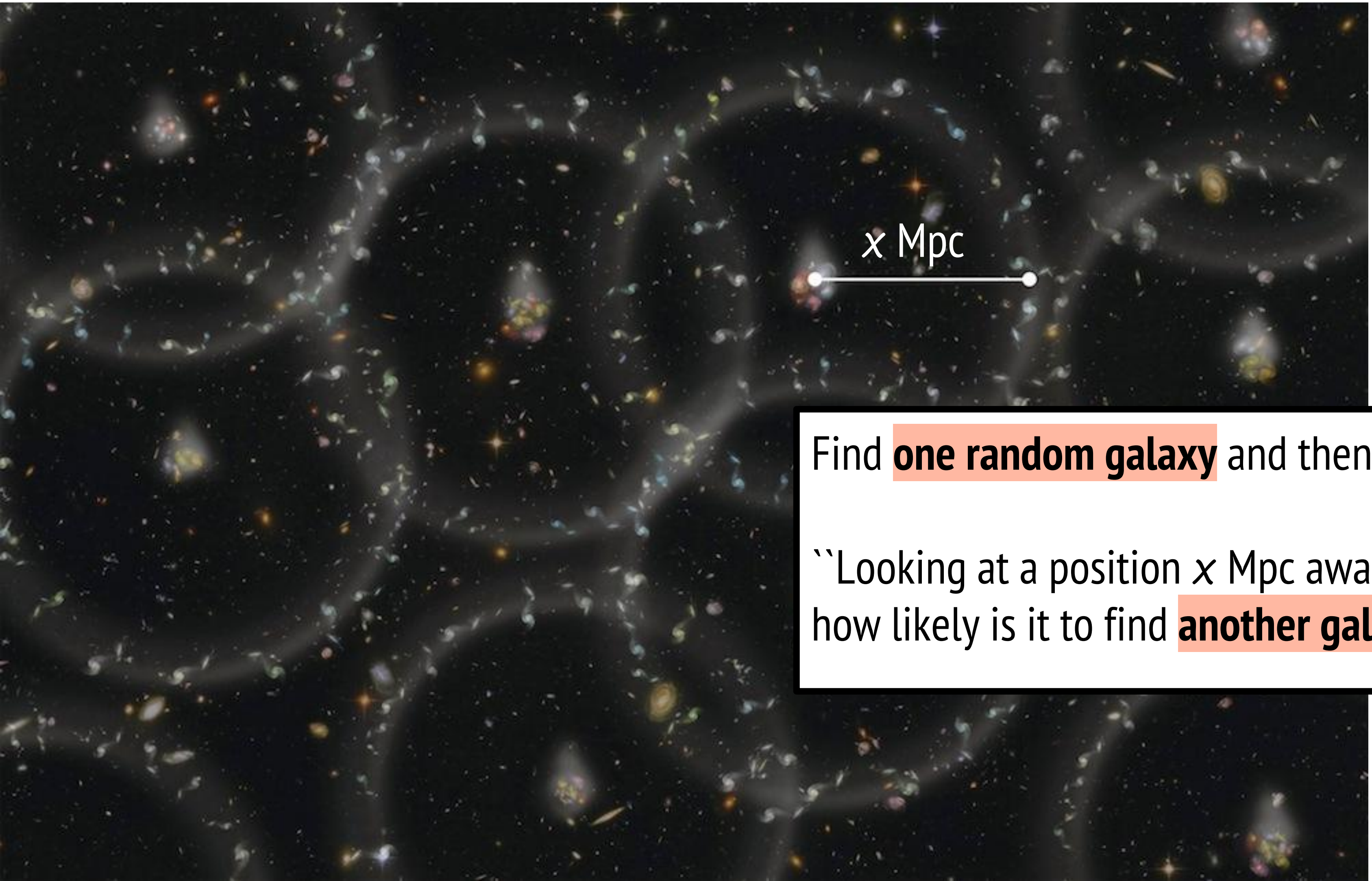


What is the distribution of these 4 maps? (same color scale)



$P \propto l^{-3}$
$P \propto l^{-2}$
$P \propto l^{-1}$
$P \propto l^{0}$
PDF
$P \propto l^{-3}$
$P \propto l^{-2}$
$P \propto l^{-1}$
$P \propto l^{0}$
All 4 maps have similar PDFs, but you can see they are very different!

Two-Point Correlation Function



Find **one random galaxy** and then ask:

``Looking at a position *x* Mpc away,
how likely is it to find **another galaxy**?''

Two-Point Correlation Function

Two-point function measured the **lumpiness** at a certain scale:

Imagine randomly dropping rulers of various length. Then record the fraction of trails where you find galaxies at both end of the ruler.

Two-Point Correlation Function

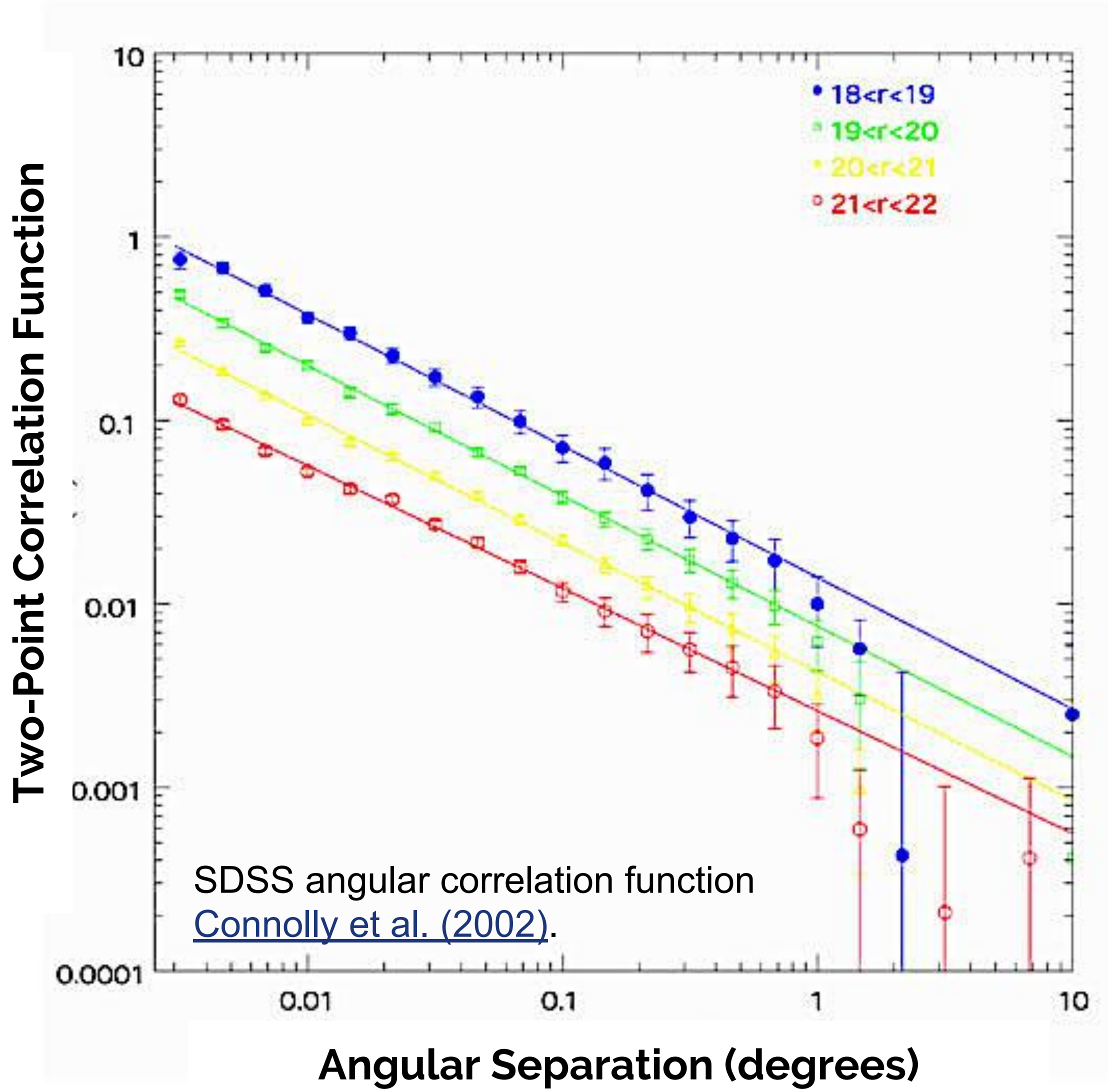
Assuming ISOTROPY (i.e. ξ depends on distance r, but not direction).

$$\xi(r) = \frac{\langle\, [\,\rho(\mathbf{x}+\mathbf{r}) - \langle\rho\rangle\,]\,[\,\rho(\mathbf{x}) - \langle\rho\rangle\,]\,\rangle_{\mathbf{x}}}{\langle\rho\rangle^2} = \langle\, \delta(\mathbf{x}+\mathbf{r})\,\delta(\mathbf{x})\,\rangle_{\mathbf{x}}$$

To a good approximation, ξ(r) can be described by a power law, with γ ~ -1.8 and r_0 depends on the specific object.

$$\xi(r) = \left(\frac{r}{r_0}\right)^{\gamma}$$

Two-Point Correlation Function



SDSS angular correlation function
[Connolly et al. (2002)](#).

* [Sample python code to compute two-point function](#)

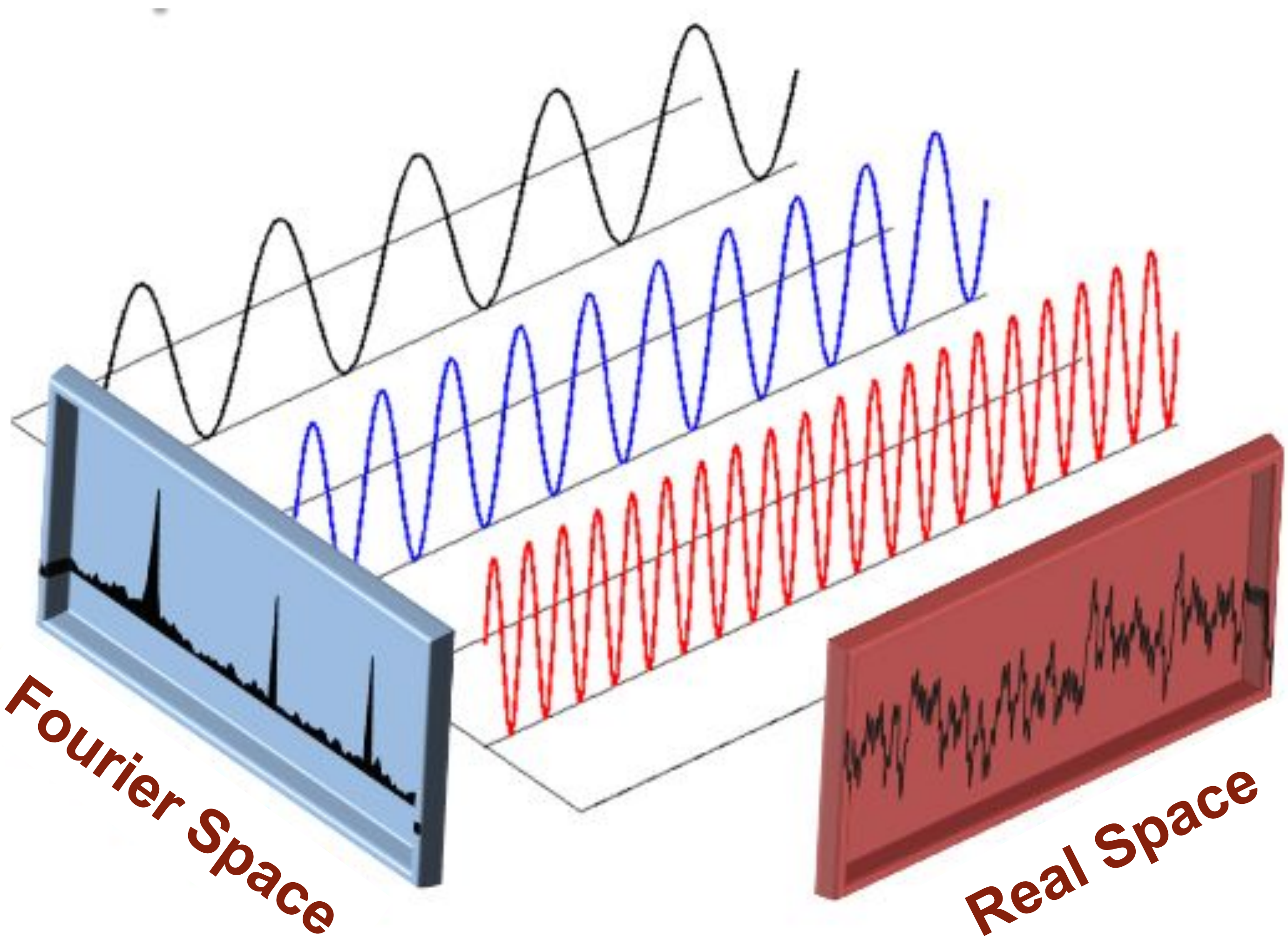
Power Spectrum

(Fourier Transformation of the Two-Point Correlation Function)

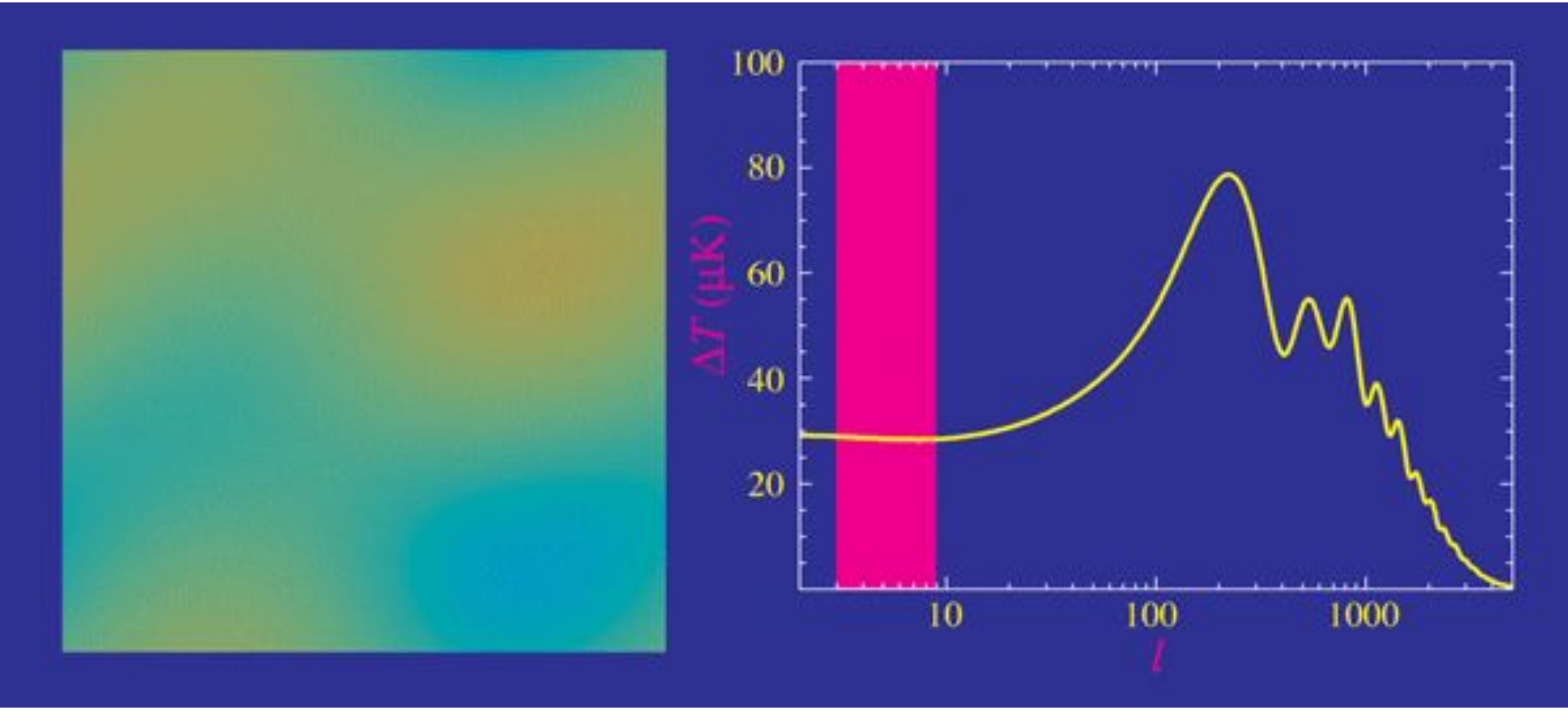
$$P(k) = \int \xi(r) e^{-i\vec{k}\vec{r}} d^3\vec{r}$$

Each Fourier k mode evolves independently

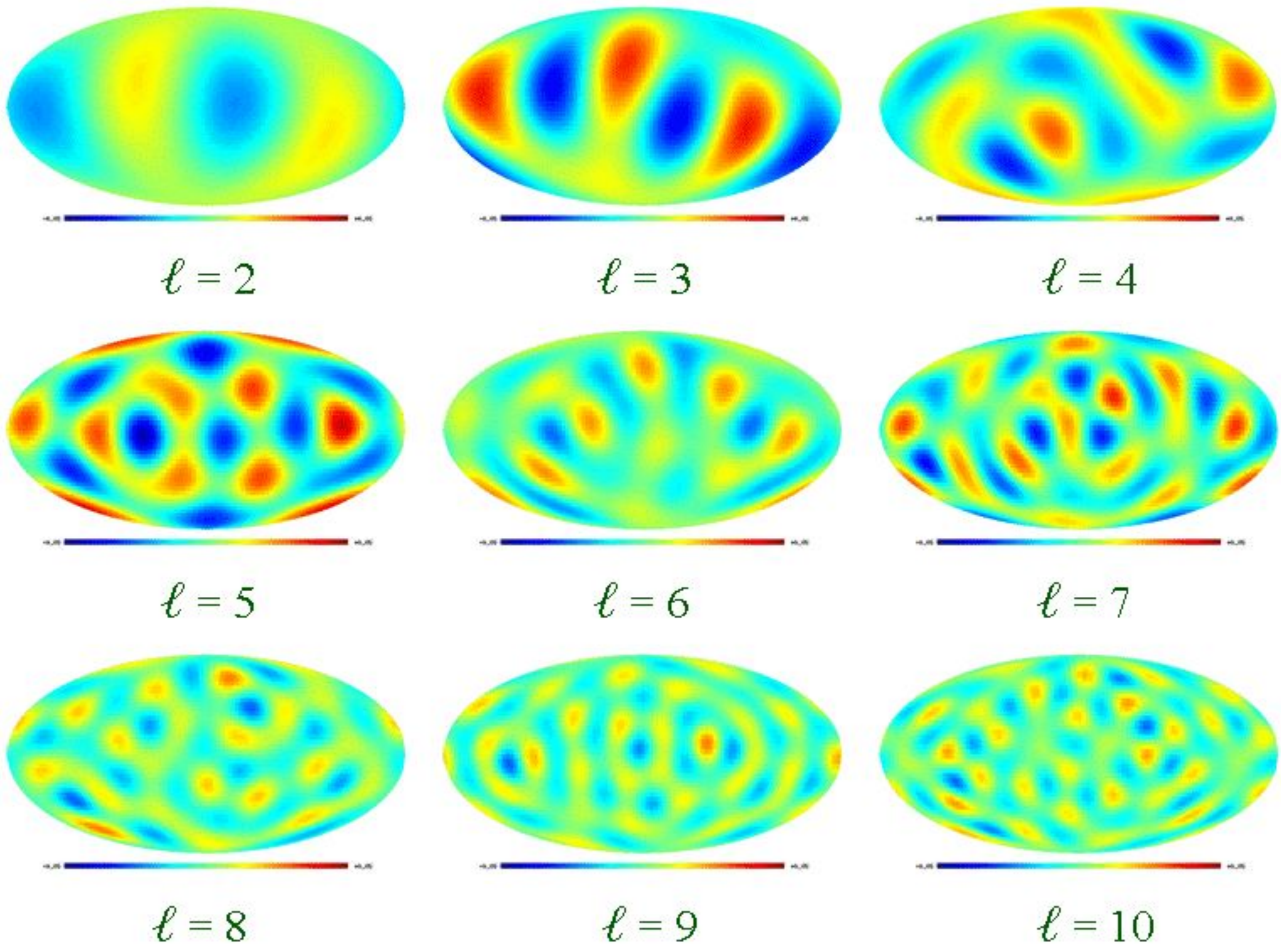
Power Spectrum



Example: Power Spectrum of The Cosmic Microwave Background



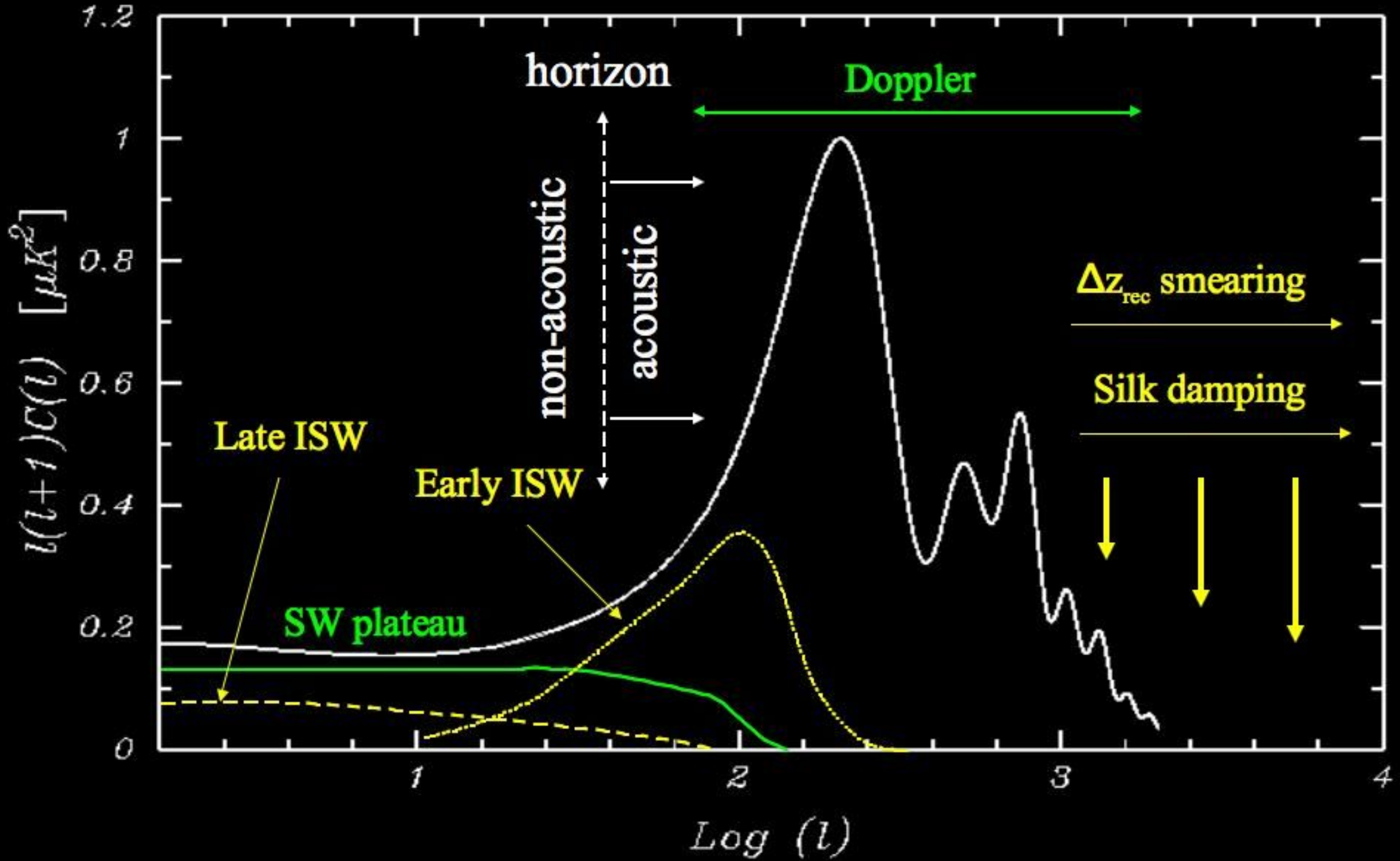
Wayne Hu's CMB tutorial



WMAP 3-year data filtered at various multipoles.
Chiang Lung-Yih

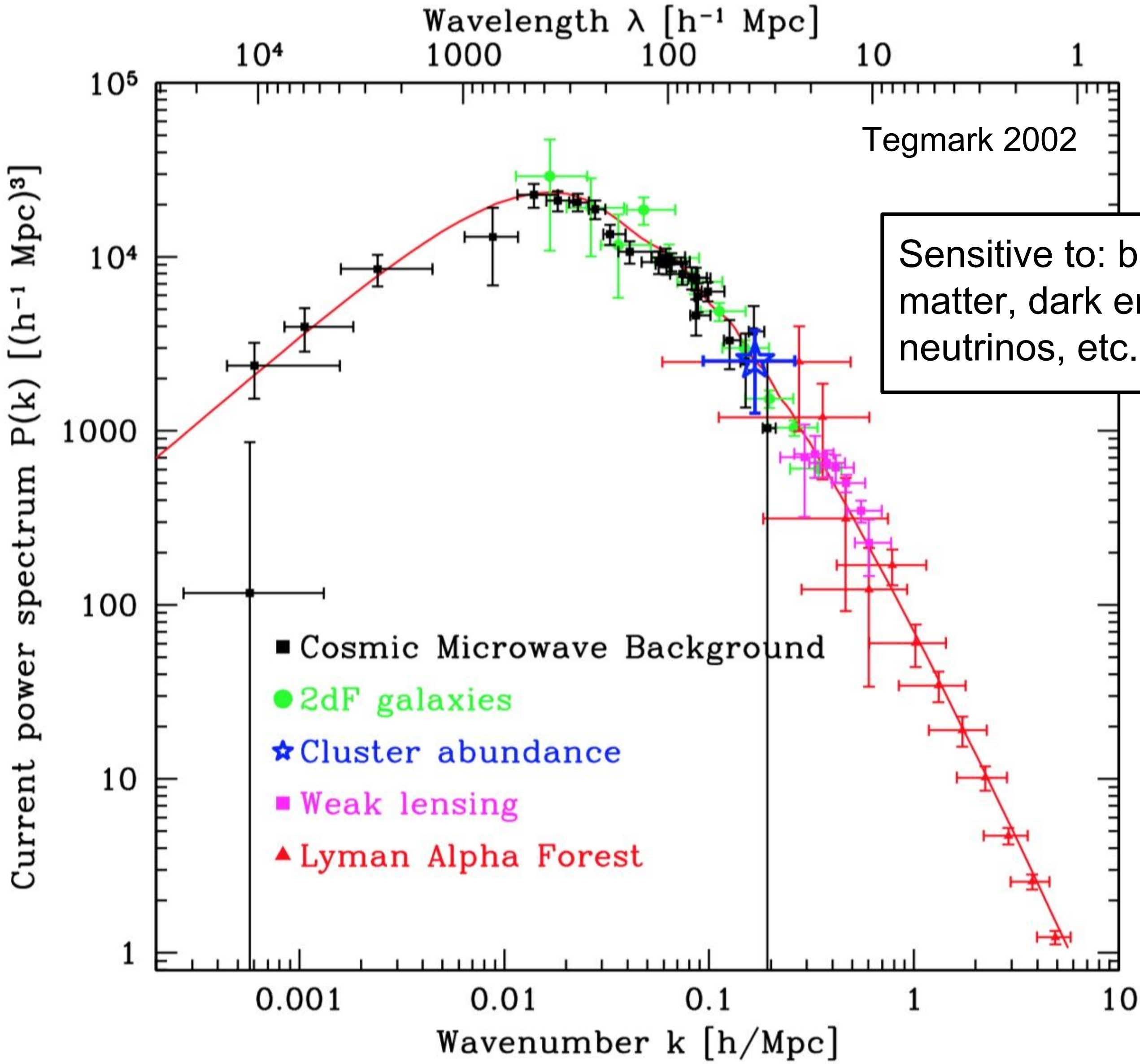
Fourier Space: Physics More Transparent

(see lecture by Mike Zemcov!)



The Matter Power Spectrum

The Matter Power Spectrum

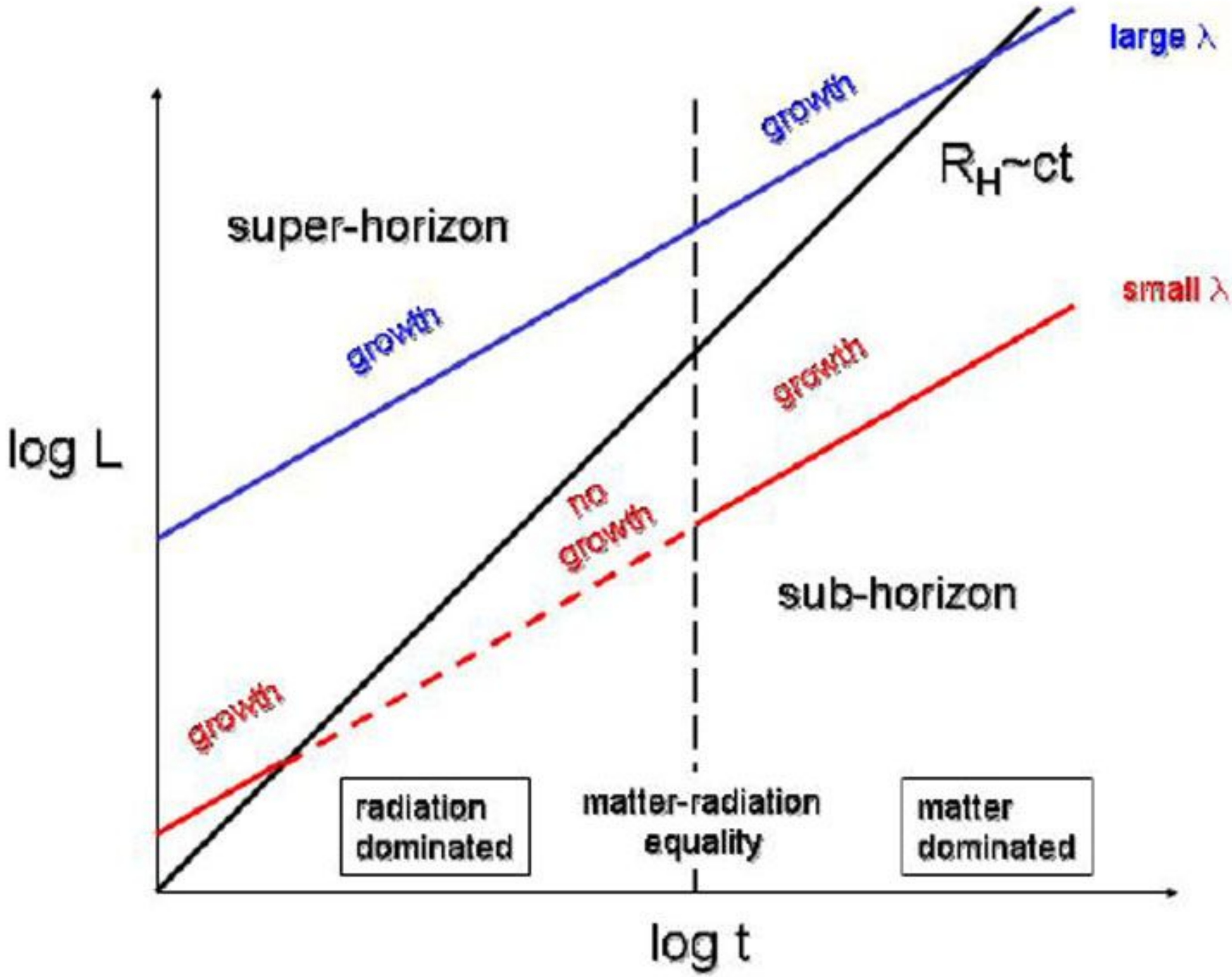


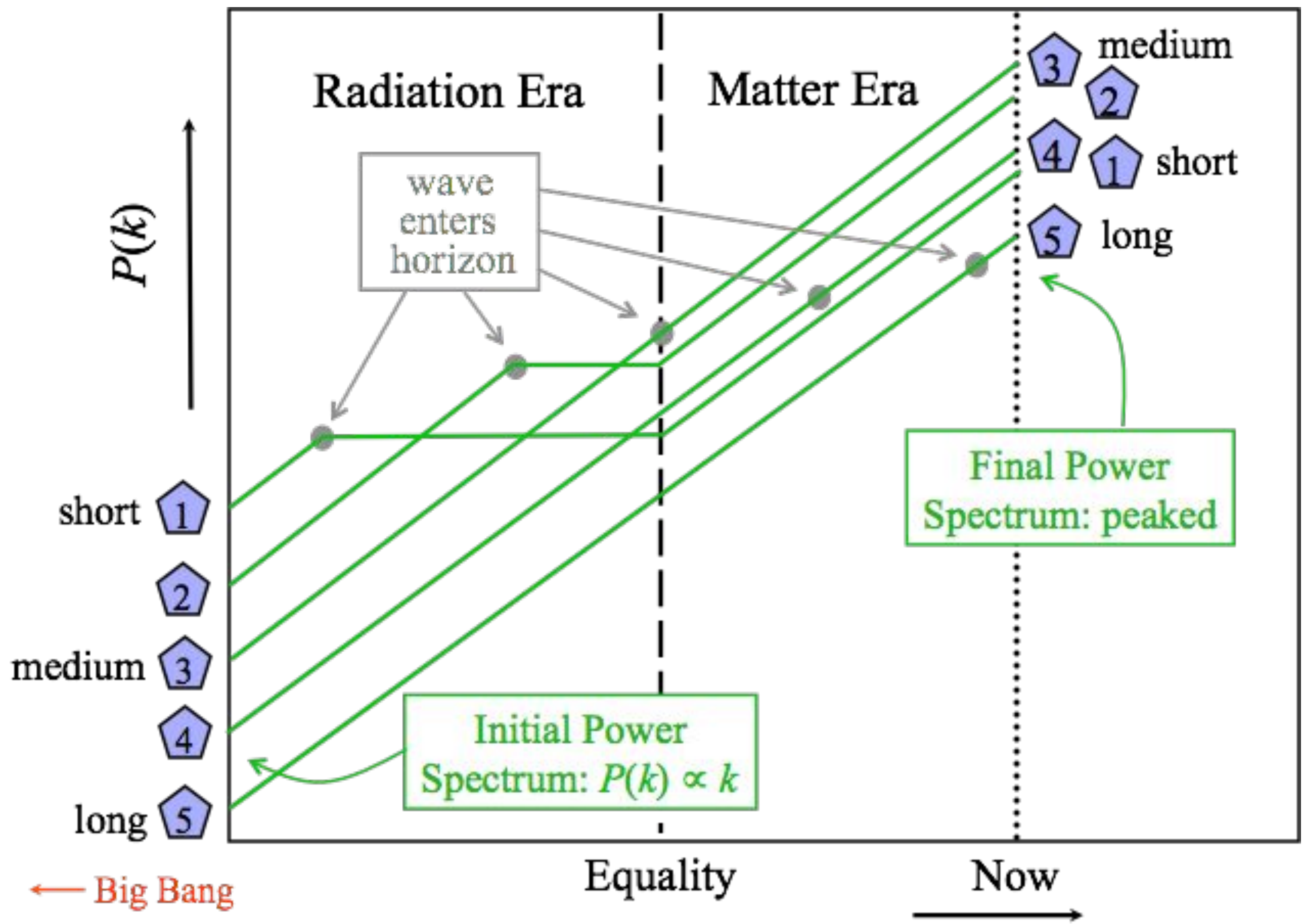
Sensitive to: baryons, dark matter, dark energy, neutrinos, etc.

Evolution of P(k)

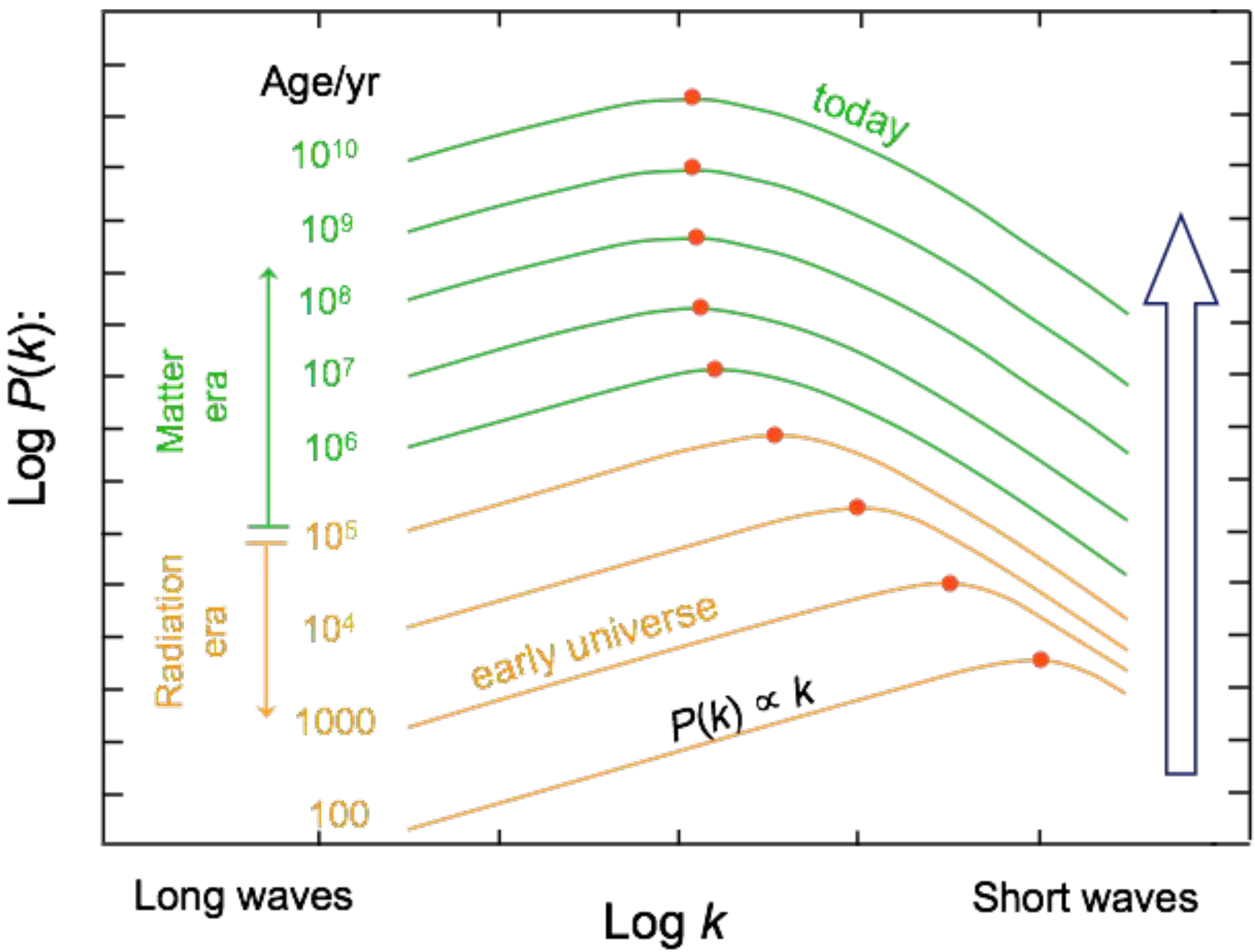
	Radiation Era	Matter Era
Super-horizon	□ $\sim a^2 \sim t$	□ $\sim a \sim t^{2/3}$
Sub-horizon	□ $\sim \ln a$ (frozen)	□ $\sim a \sim t^{2/3}$

Evolution of P(k)

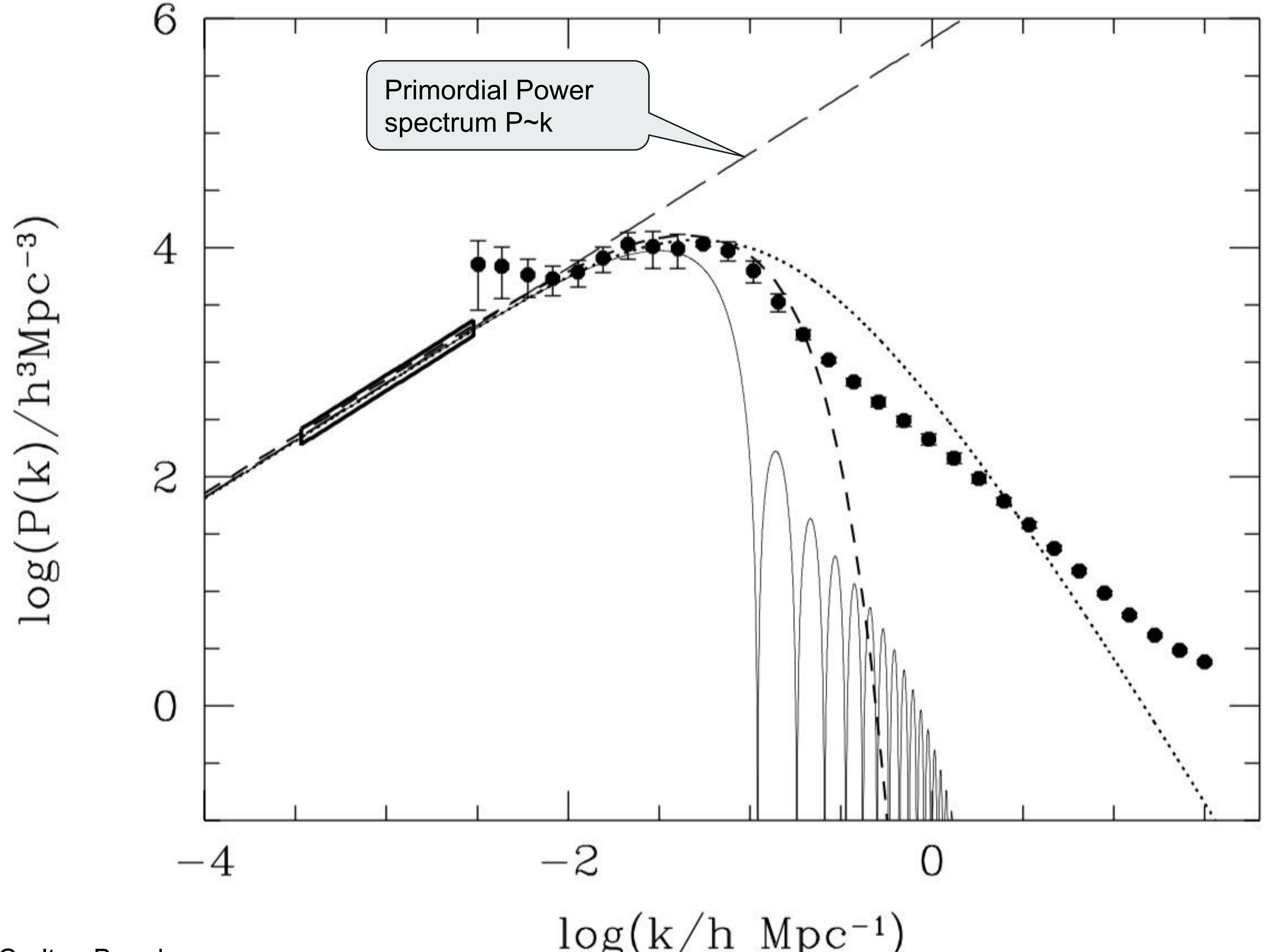




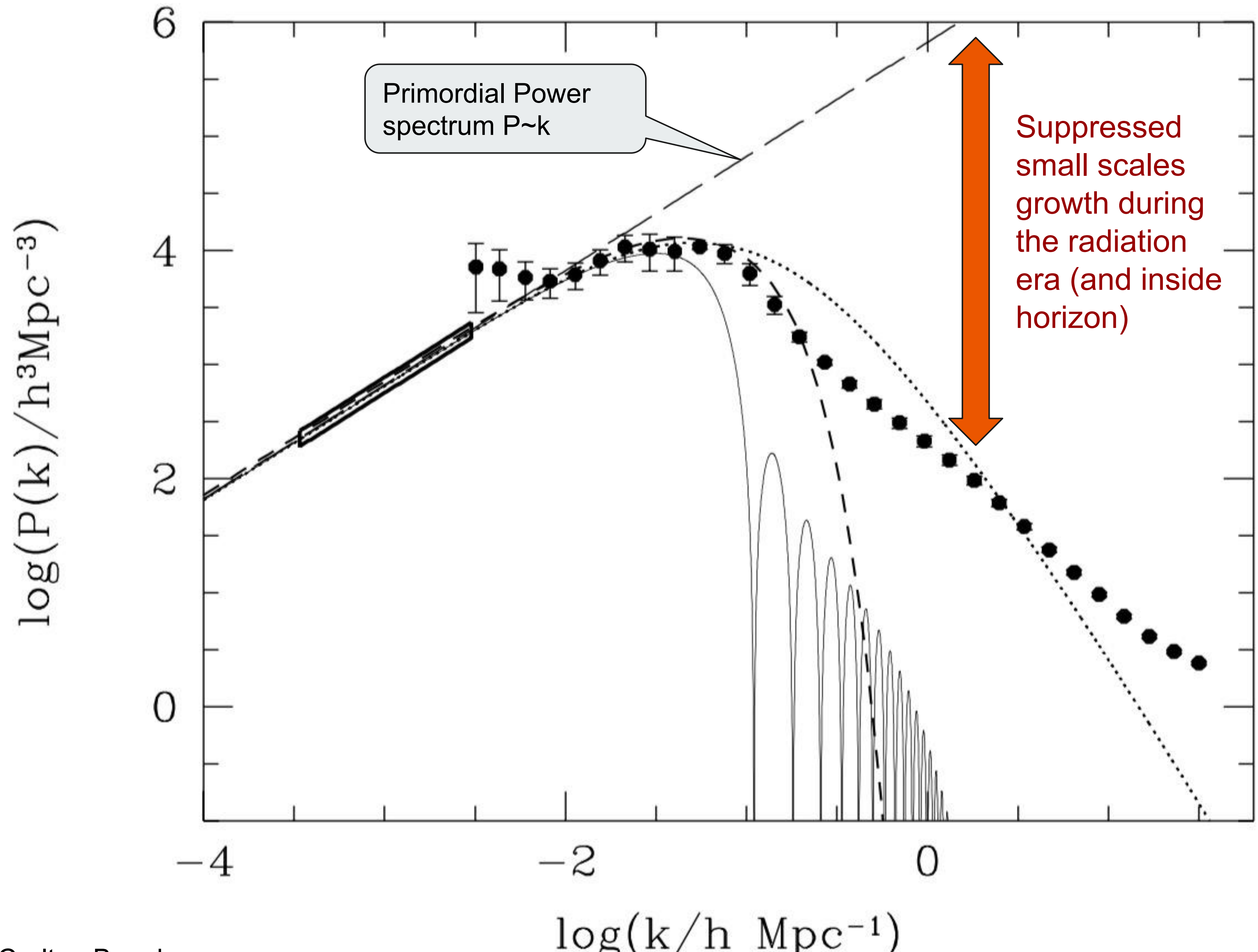
Radiation Era
Matter Era
wave enters horizon
P(k)
short
medium
long
1
2
3
4
5
medium
short
long
Final Power Spectrum: peaked
Initial Power Spectrum: P(k) ∝ k
Equality
Now
← Big Bang



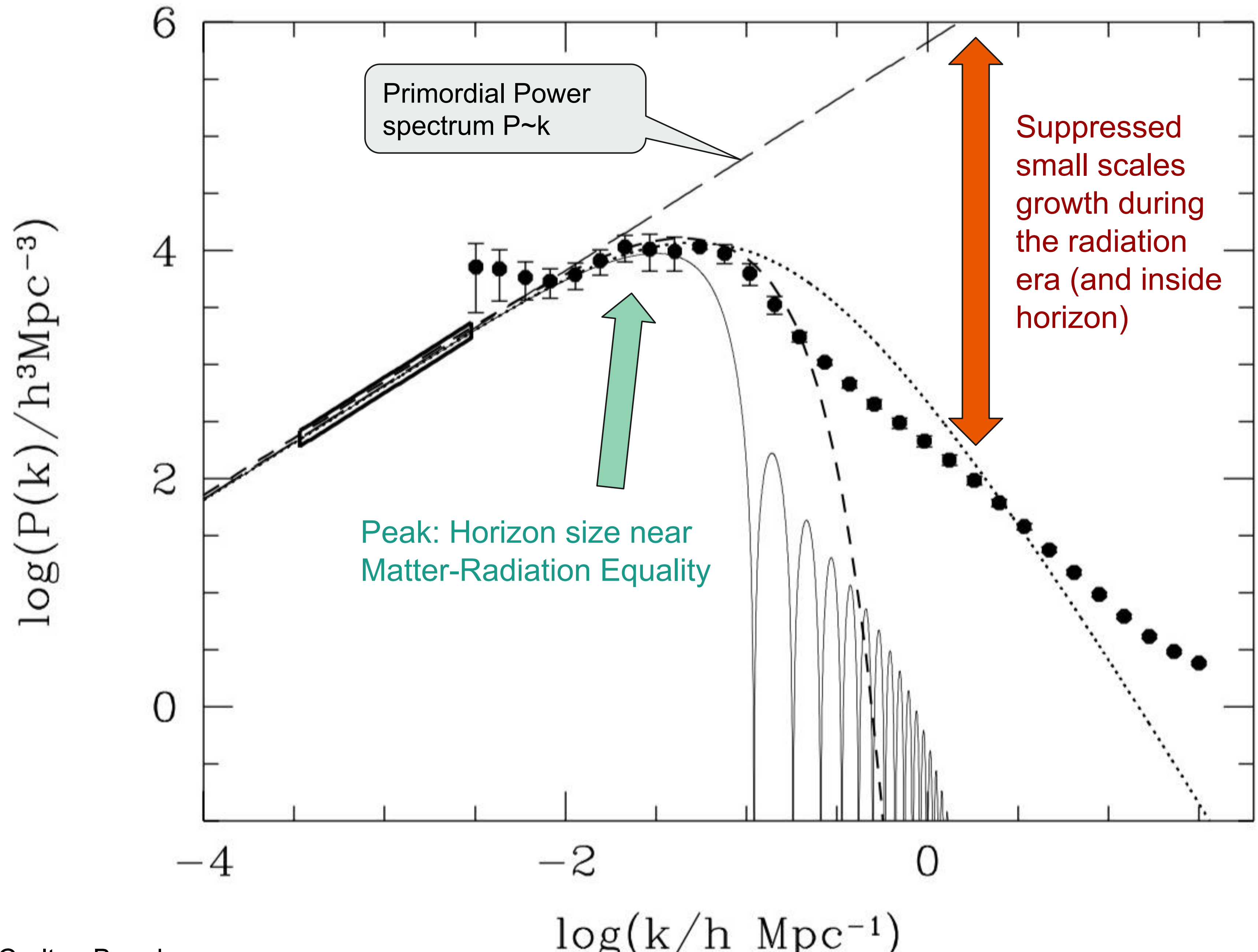
Log $P(k)$:
Age/yr
today
10^{10}
10^9
10^8
10^7
10^6
10^5
10^4
1000
100
Matter era
Radiation era
early universe
$P(k) \propto k$
Long waves
Log k
Short waves



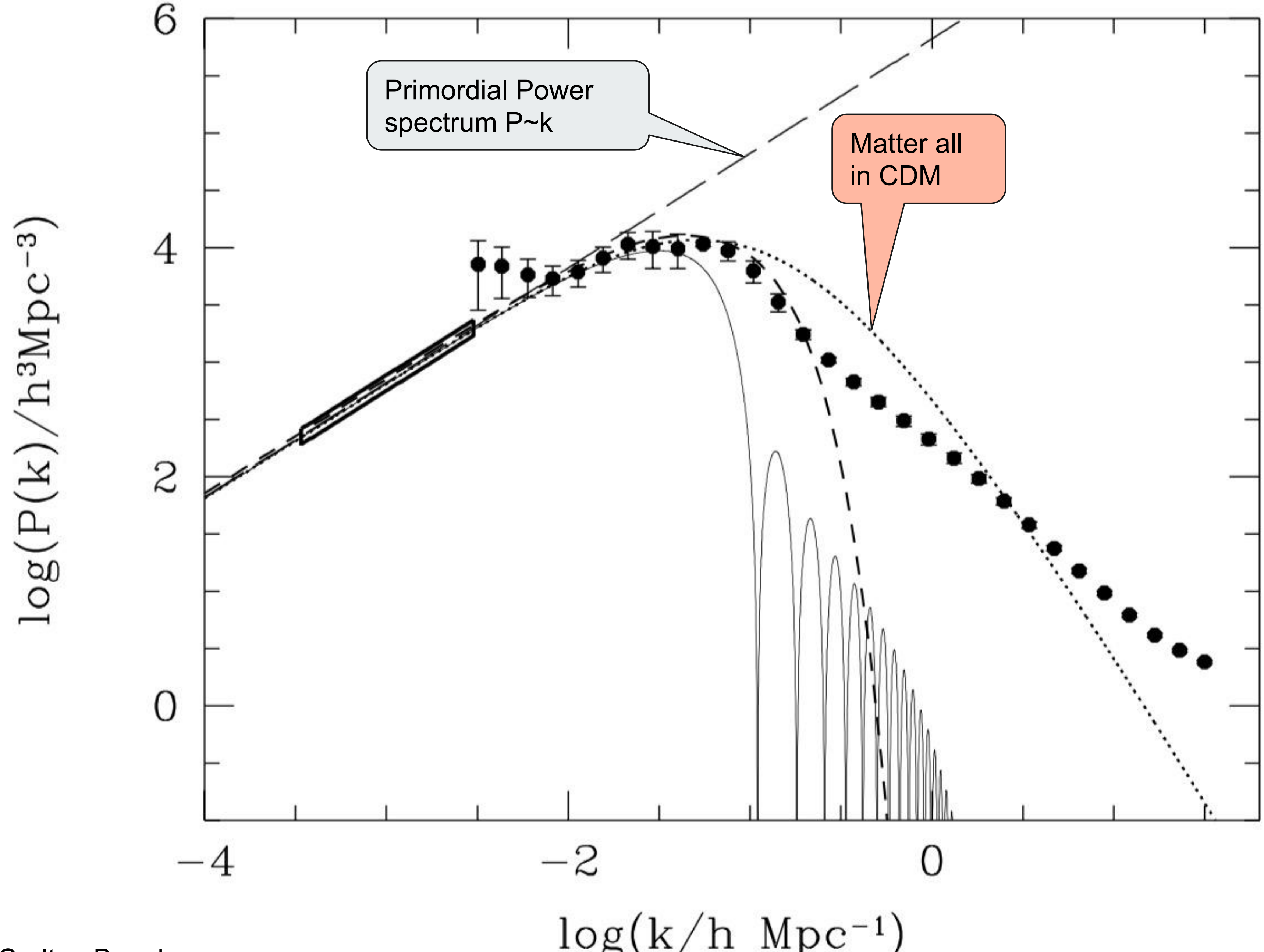
Carlton Baugh



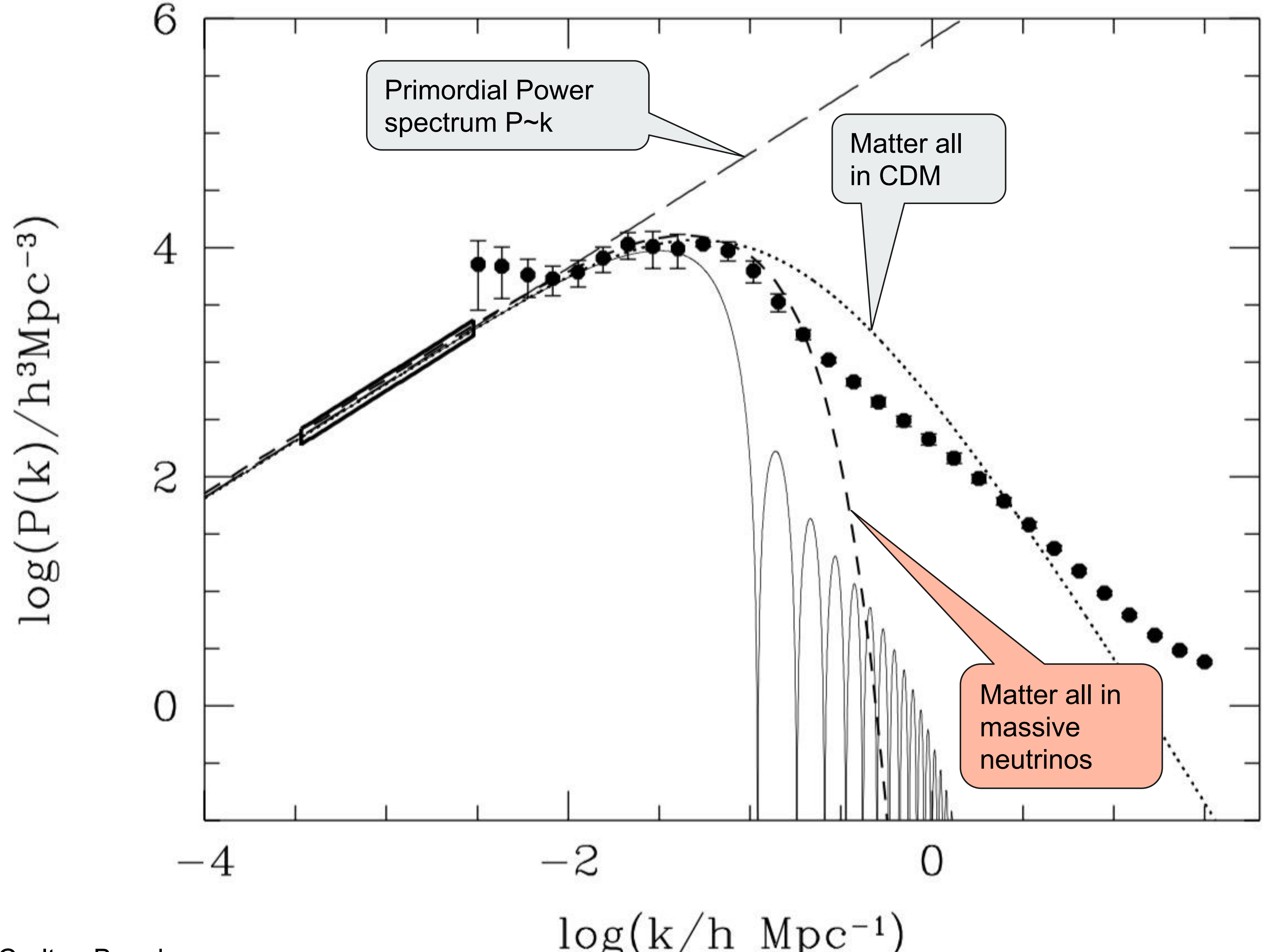
Carlton Baugh



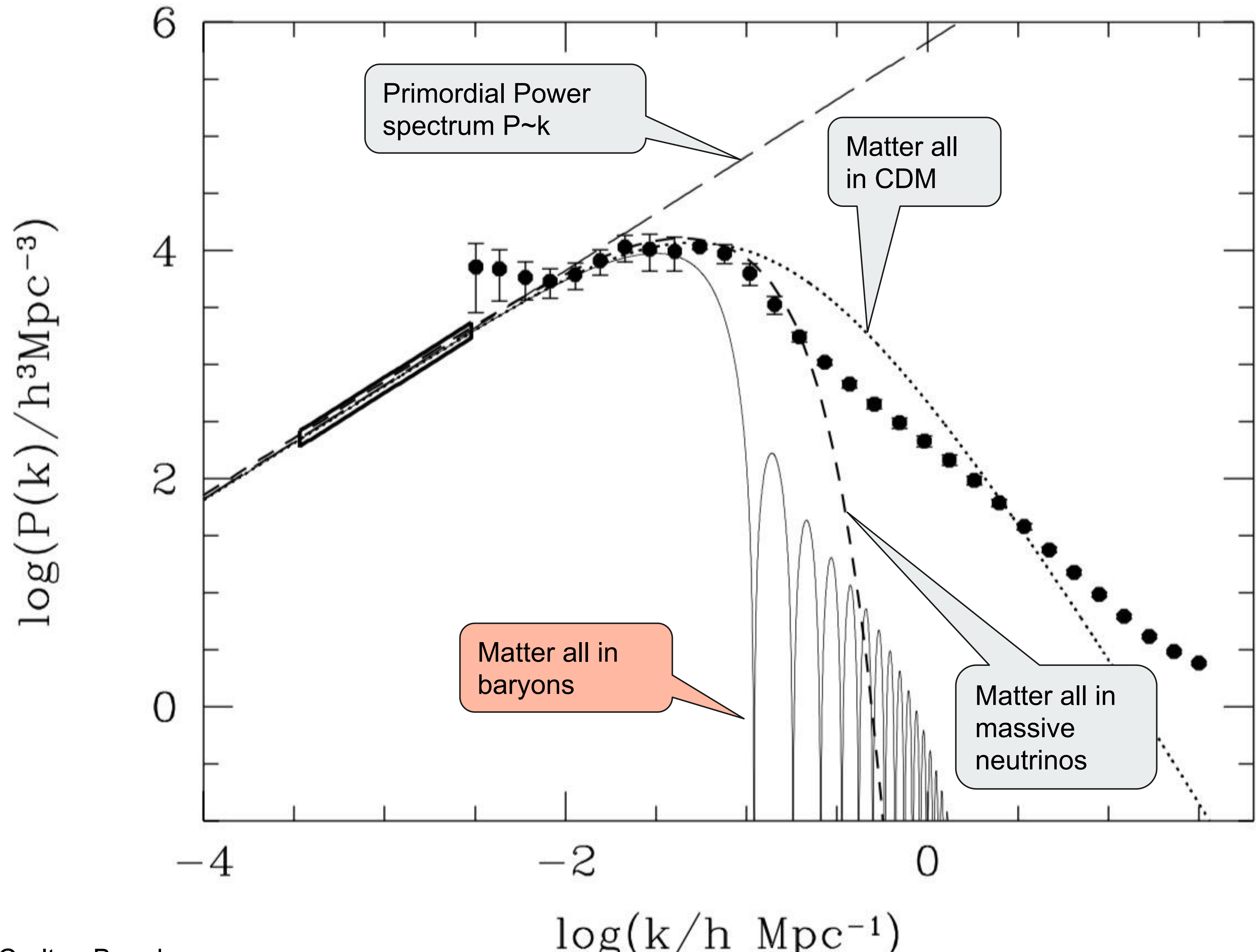
Carlton Baugh



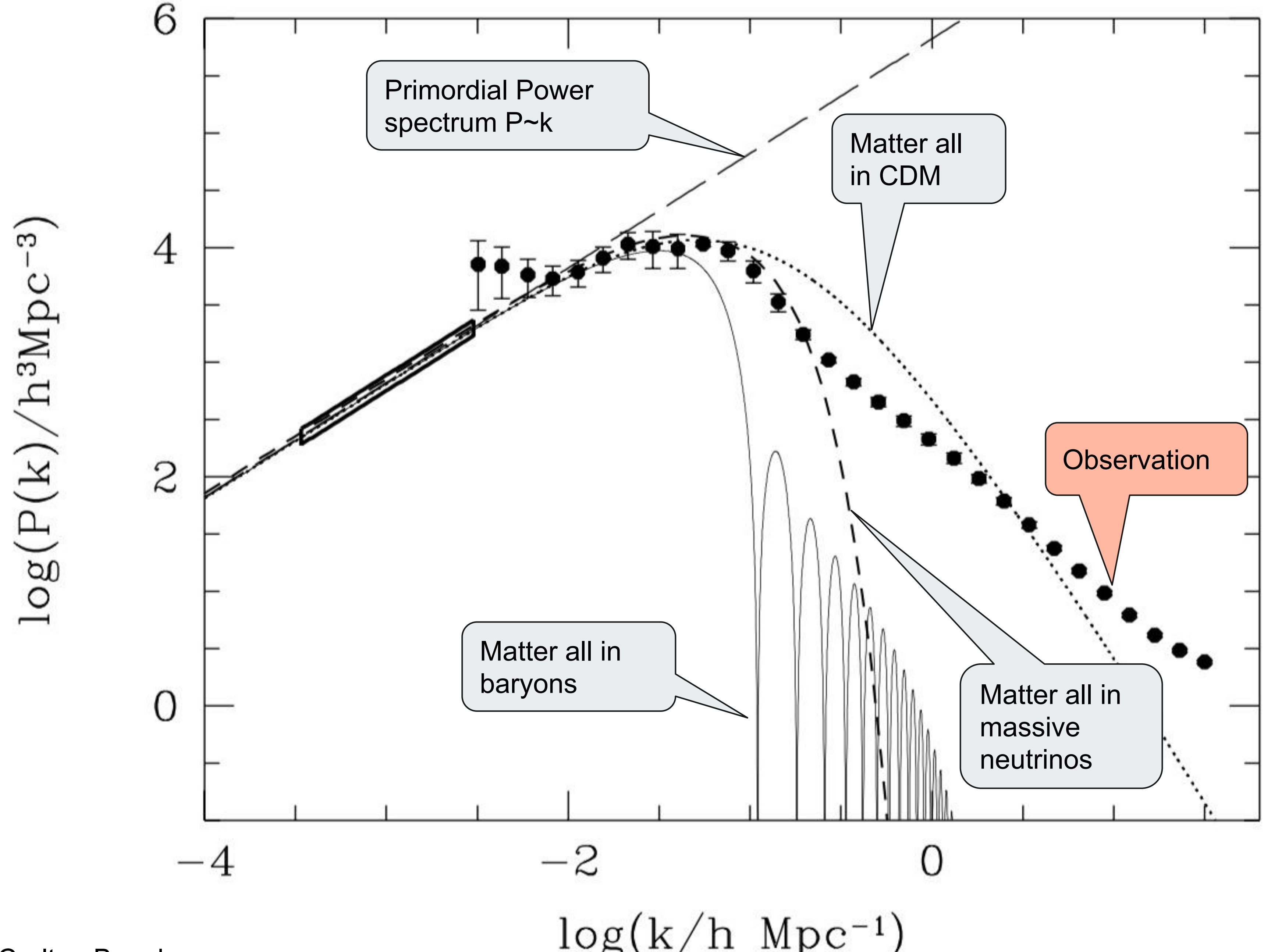
Carlton Baugh



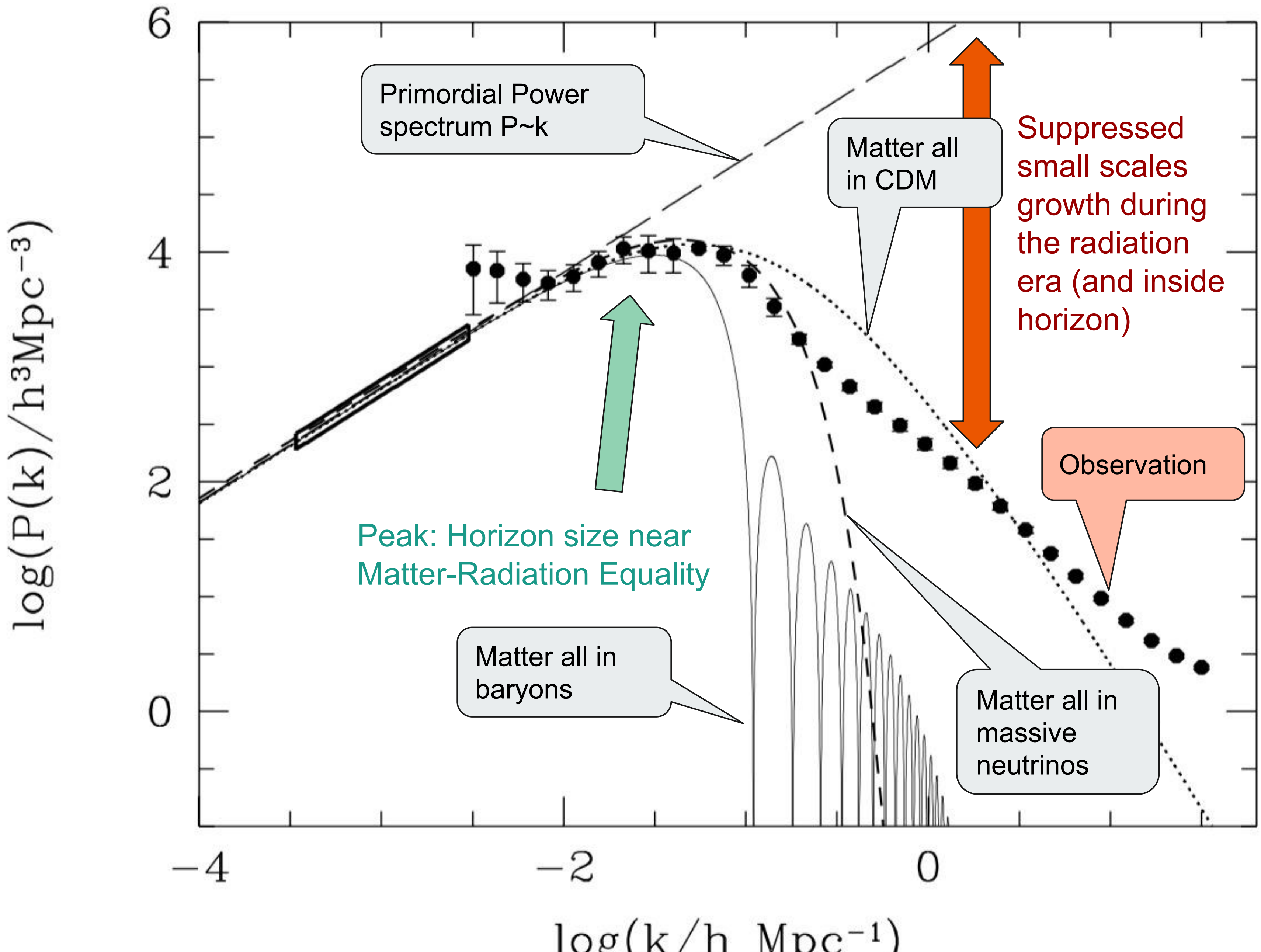
Carlton Baugh



Carlton Baugh

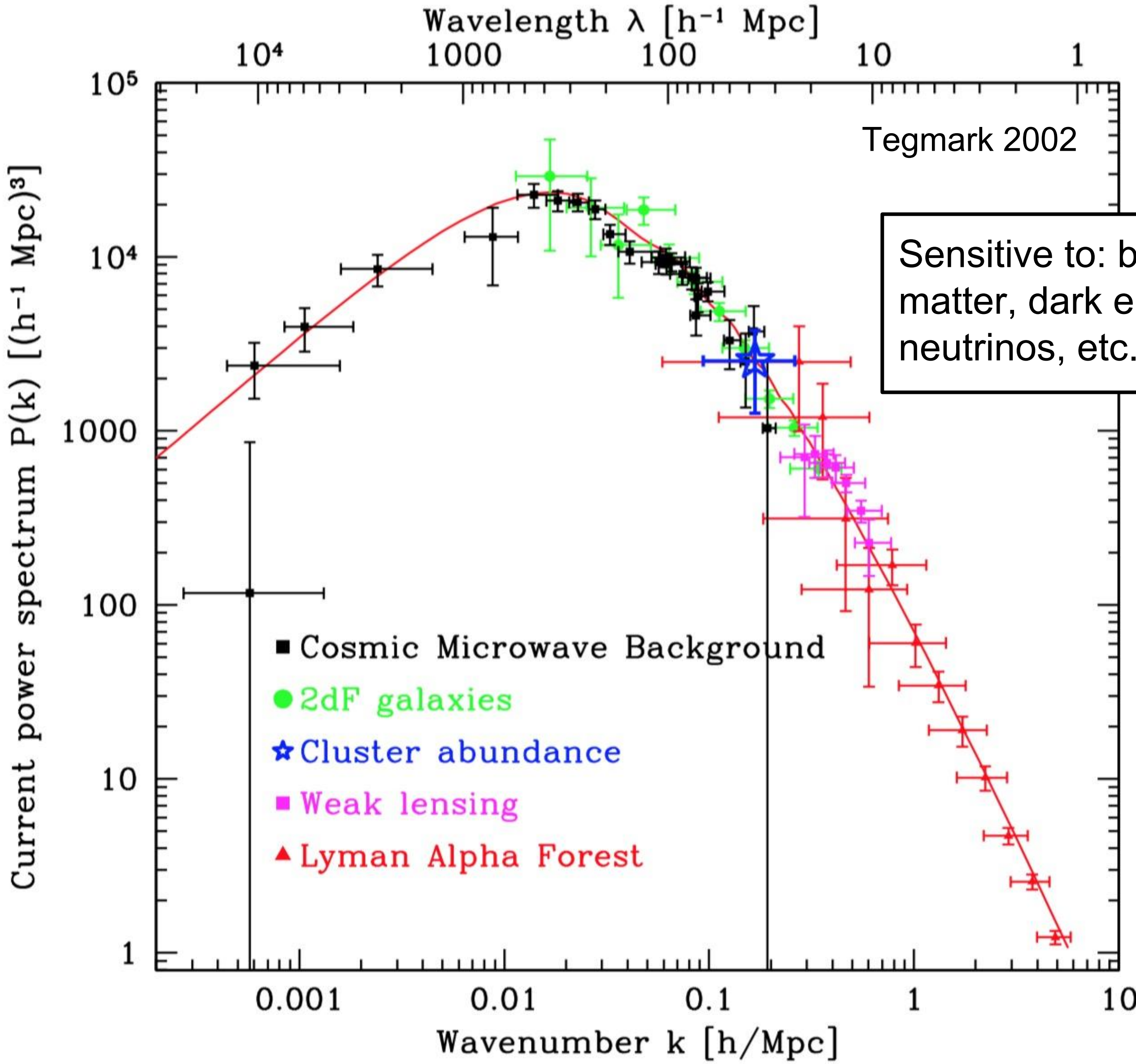


Carlton Baugh



Carlton Baugh

The Matter Power Spectrum



Sensitive to: baryons, dark matter, dark energy, neutrinos, etc.

Matter Power Spectrum

So far we have only discussed the **linear** growth of structure. At late times, structure growth **nonlinearly**, numerical simulations are needed to calibrate the matter power spectrum.

Popular tools including:

- Camb: http://camb.info/
- Class: http://class-code.net/
- Nicaea: http://www.cosmostat.org/software/nicaea

Large-Scale Structure Probes

Two Types of LSS Probes

GEOMETRY: H(z)

Supernova

Baryon Acoustic Oscillations

Strong Lensing

...

GROWTH: δ(z)

Weak Lensing

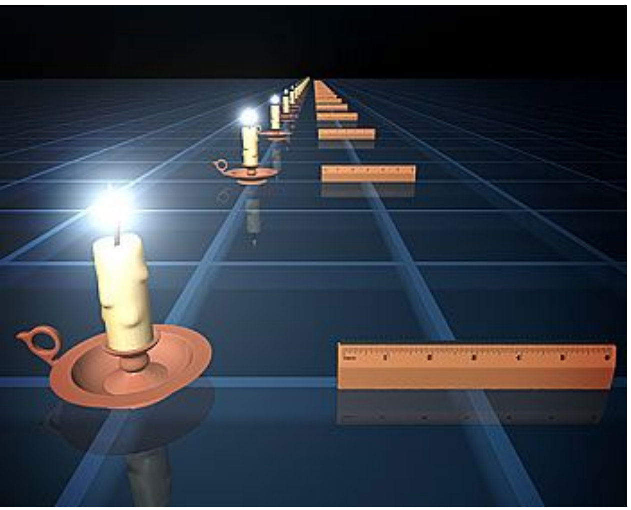
Clusters

Redshift Space Distortion

...

LSS Probes: GEOMETRY

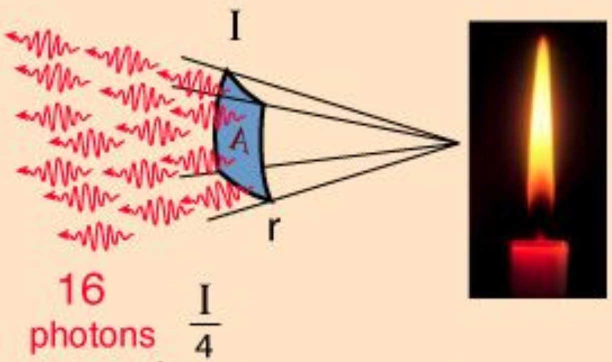
$$H(a) = H_0\sqrt{\Omega_k a^{-2} + \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{\mathrm{DE}} a^{-3(1+w)}}$$



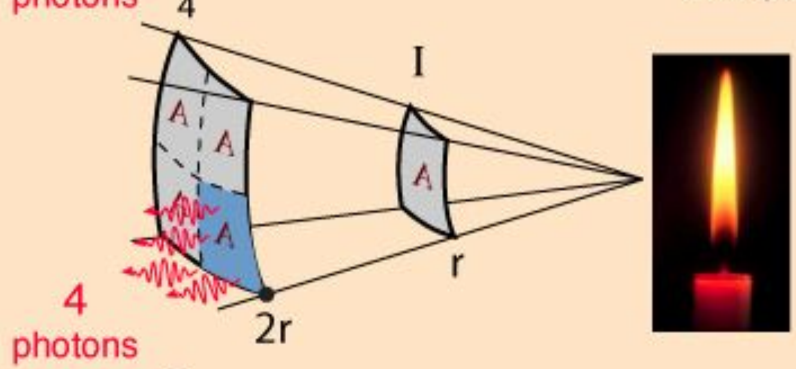
measurable	Definition
proper distance	$D(z) = \int_0^z \frac{dz'}{H(z')} = \begin{cases} \lvert k\rvert^{-1/2}\sin^{-1}\left[\lvert k\rvert^{1/2} r(z)\right] & k>0 \\ r(z) & k=0 \\ \lvert k\rvert^{-1/2}\sinh^{-1}\left[\lvert k\rvert^{1/2} r(z)\right] & k<0 \end{cases}$
luminosity distance (Standard Candle)	$d_L(z) = r(z)(1+z)$
angular diameter distance (Standard Ruler)	$d_A(z) = r(z)/(1+z)$
volume element	$dV = \frac{r^2(z)}{\sqrt{1-kr^2(z)}}\,dr\,d\Omega$

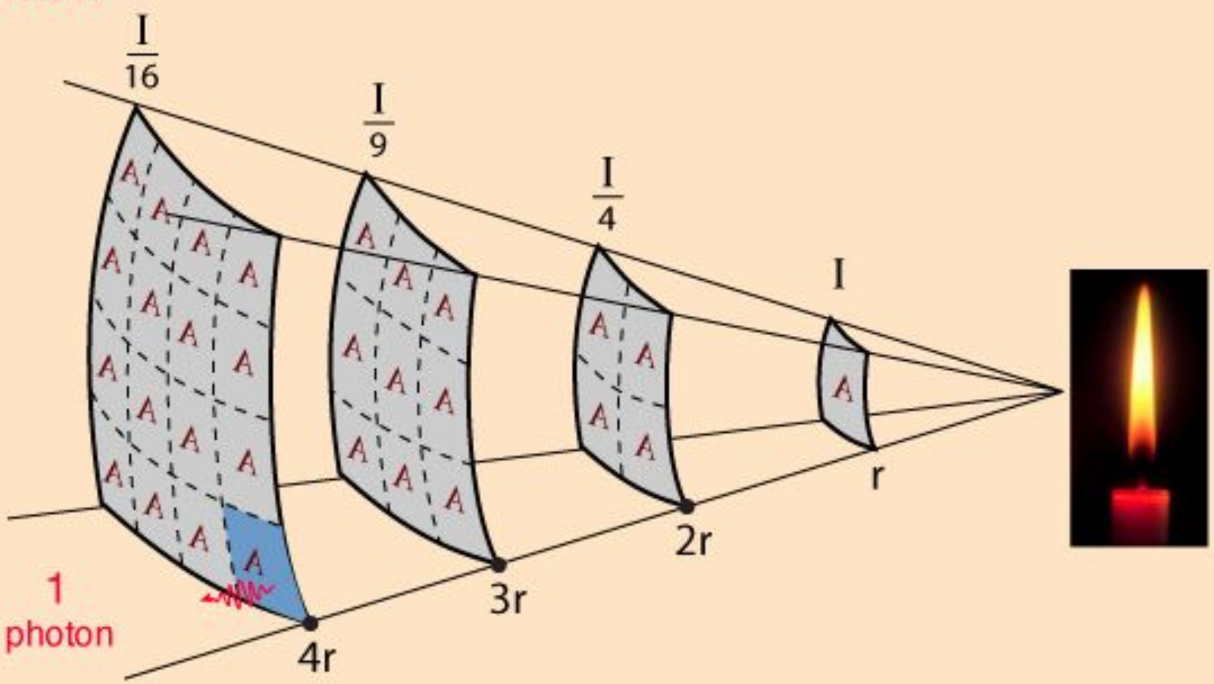
The "standard candle" approach to distance measurement.

If you know you have the same source strength of light, then counting the number of photons through a standard area detector tells you the distance to the source.



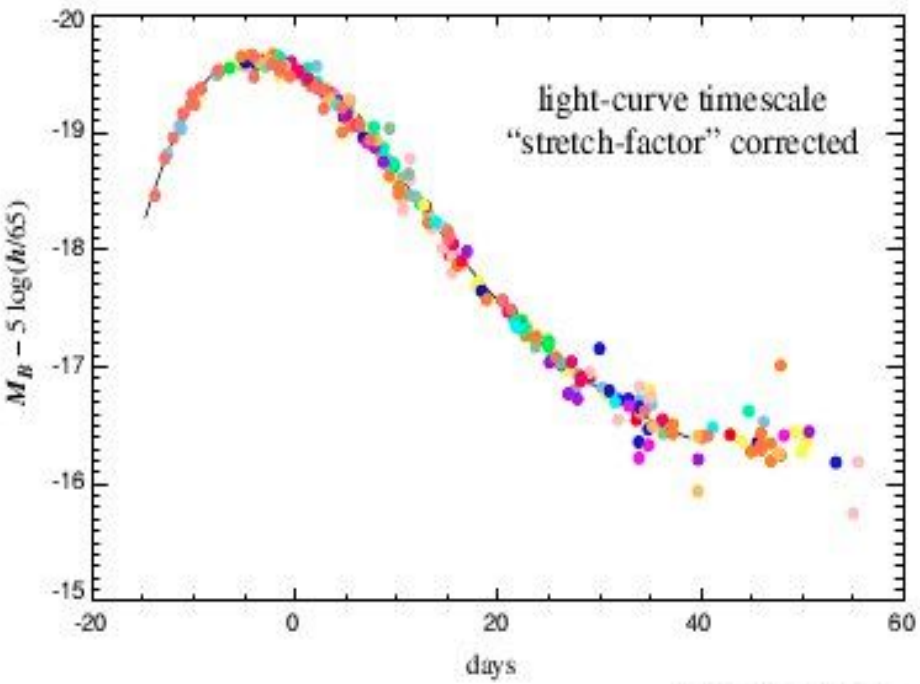
Light from a point source drops off according to the inverse square law, a strictly geometrical relationship.



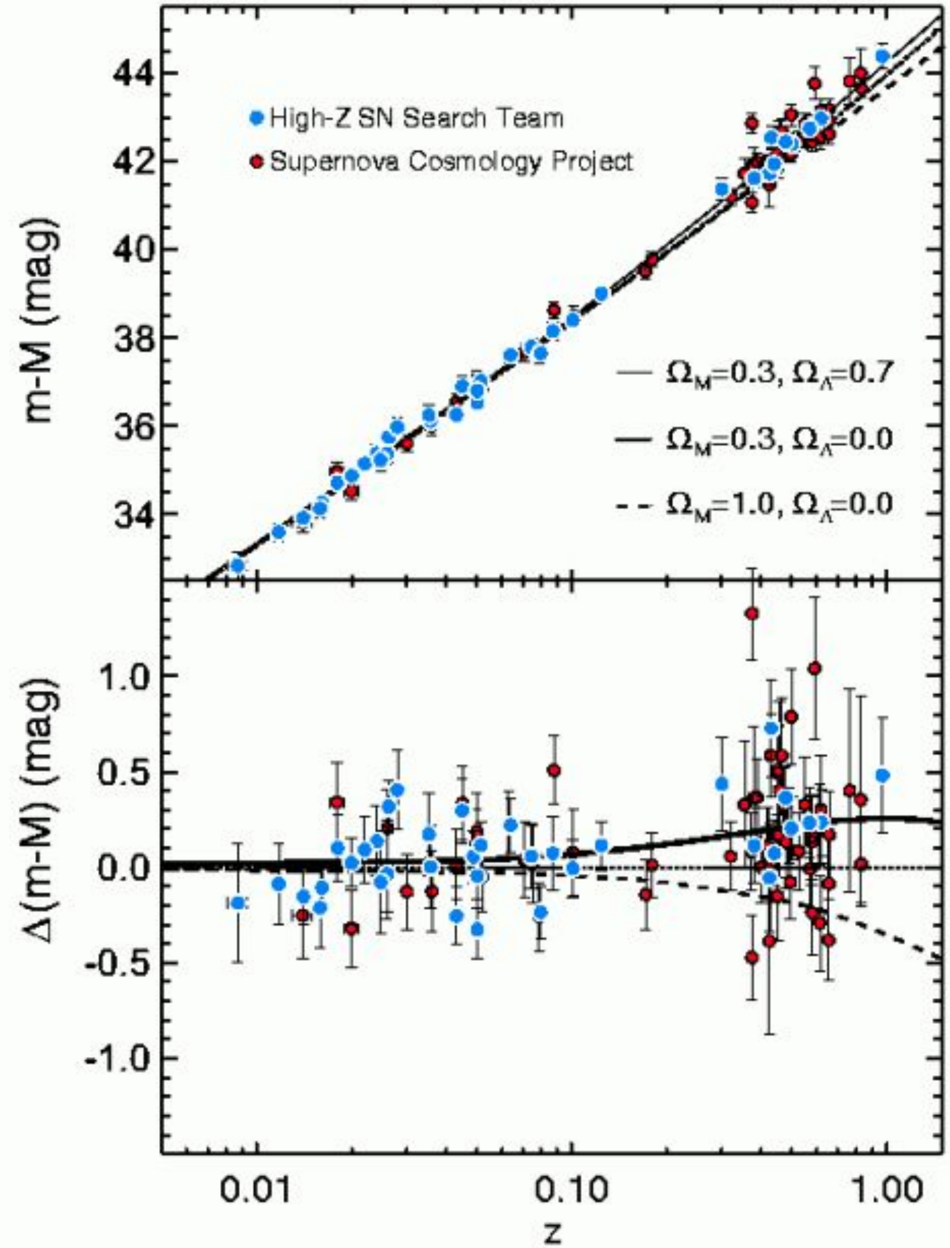


Standard Candle: Type Ia SN

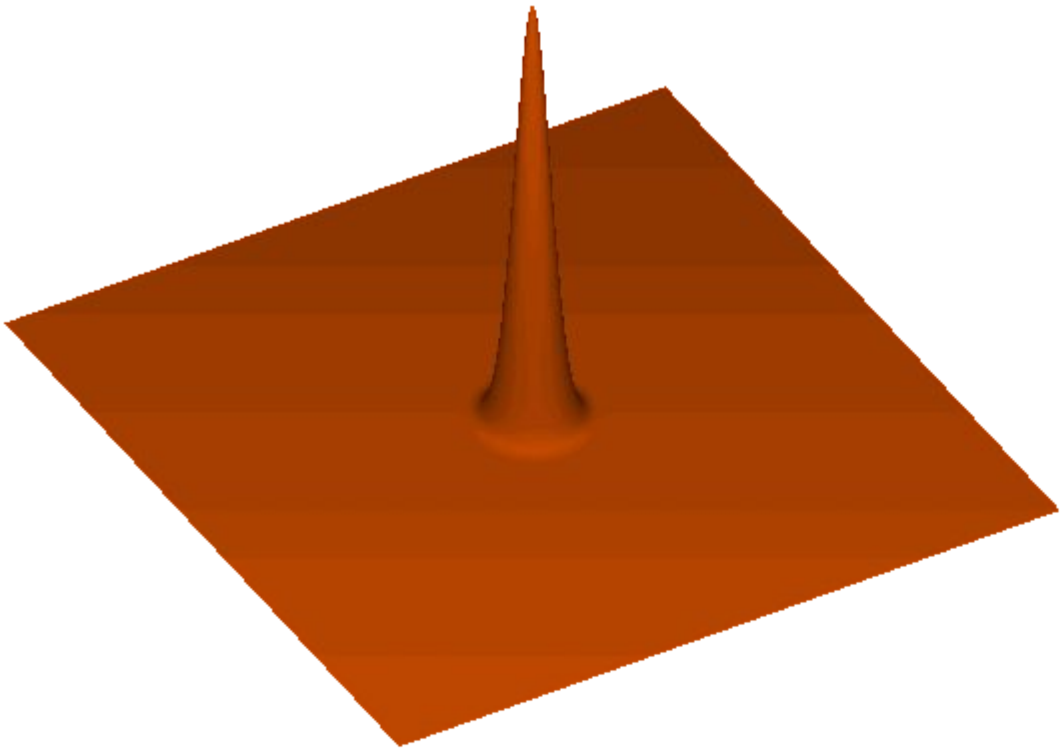
(exploding white dwarfs)

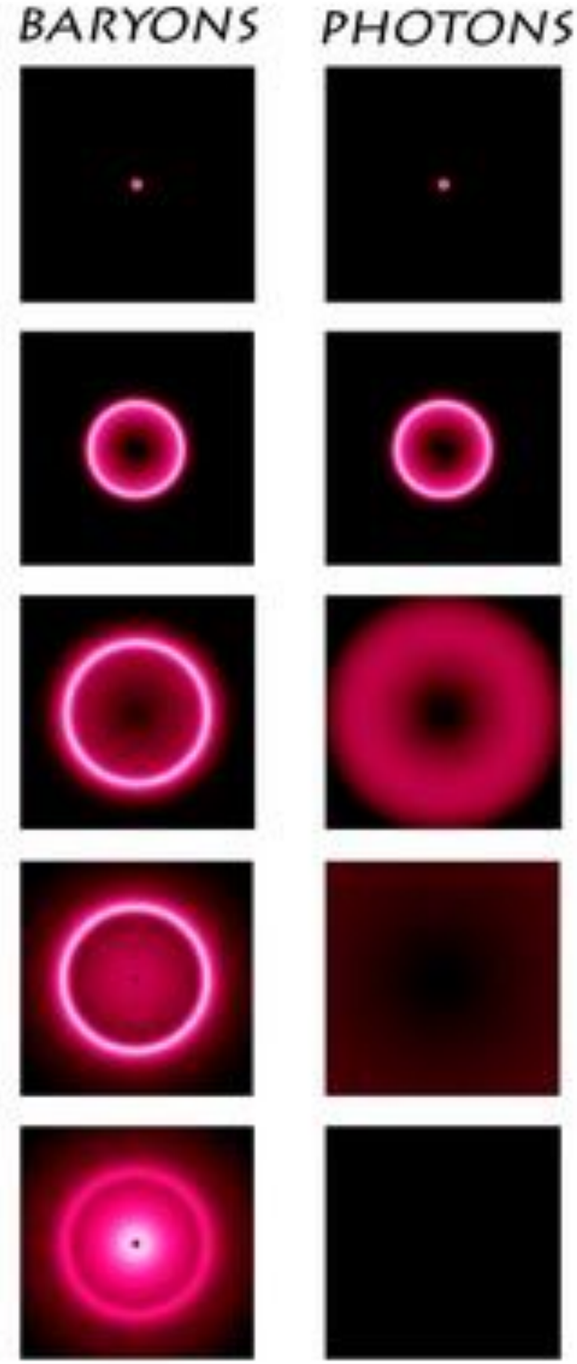


Kim, *et al.* (1997)



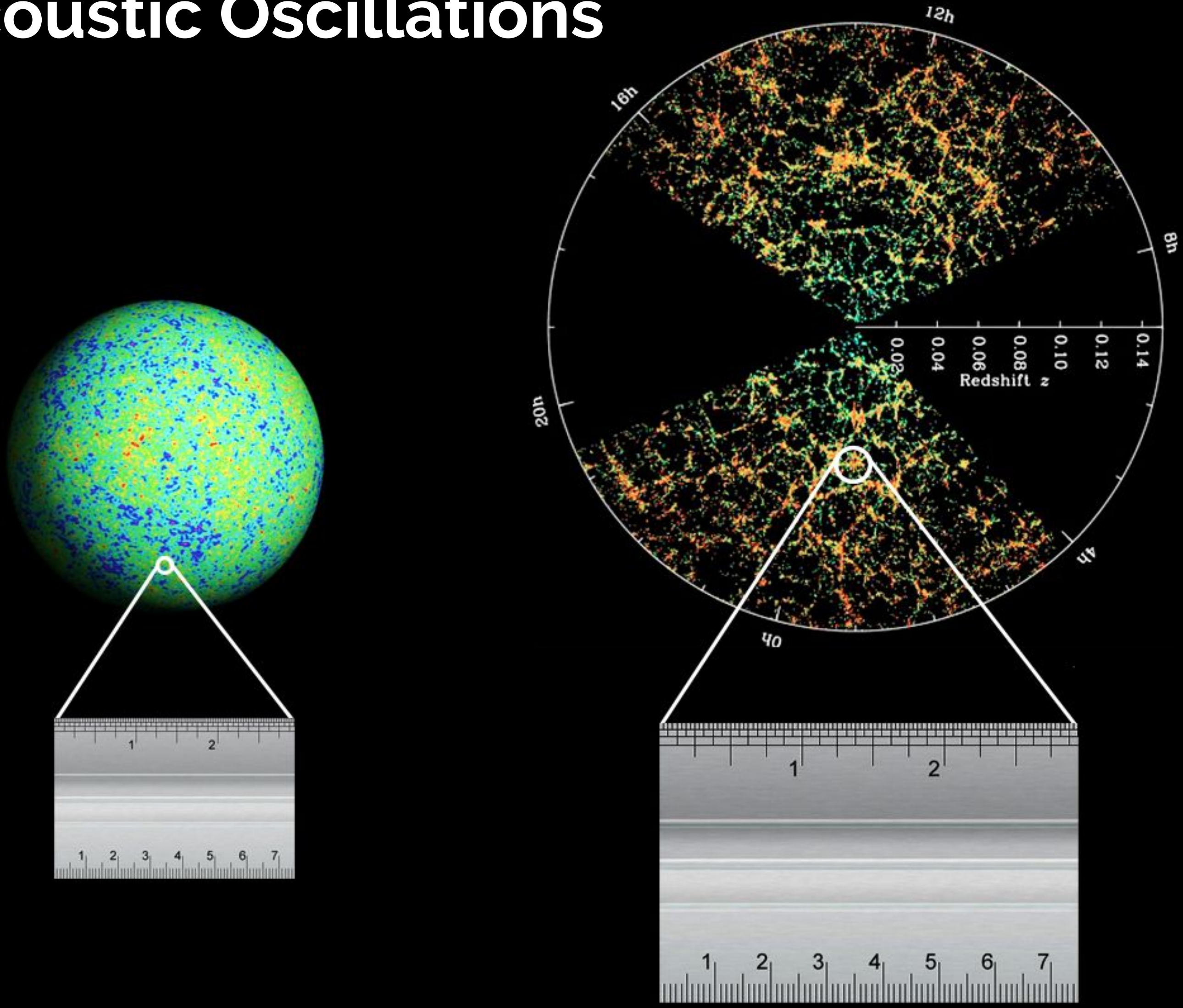
Standard Ruler: Baryon Acoustic Oscillations

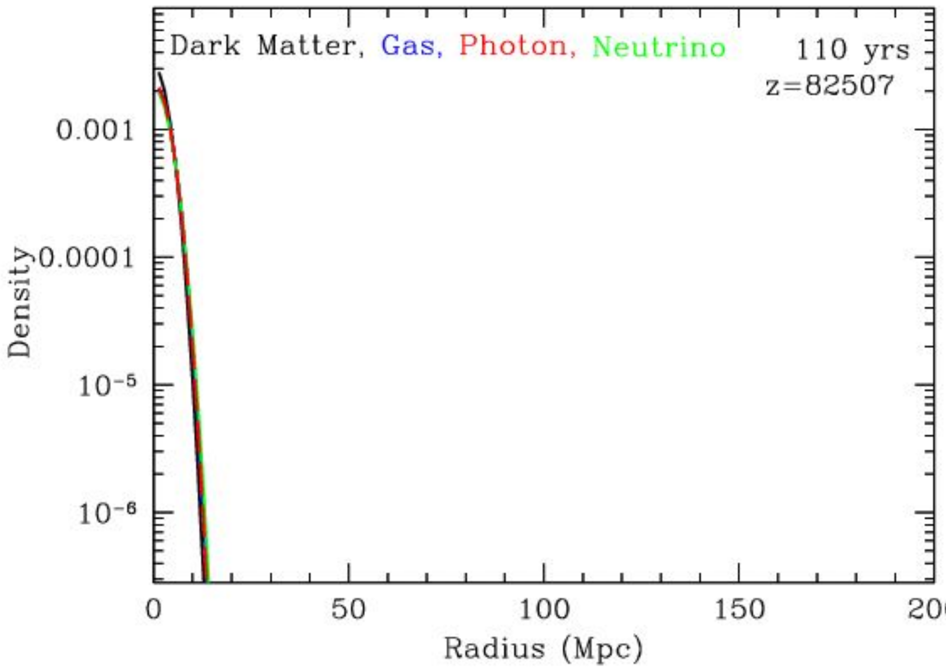


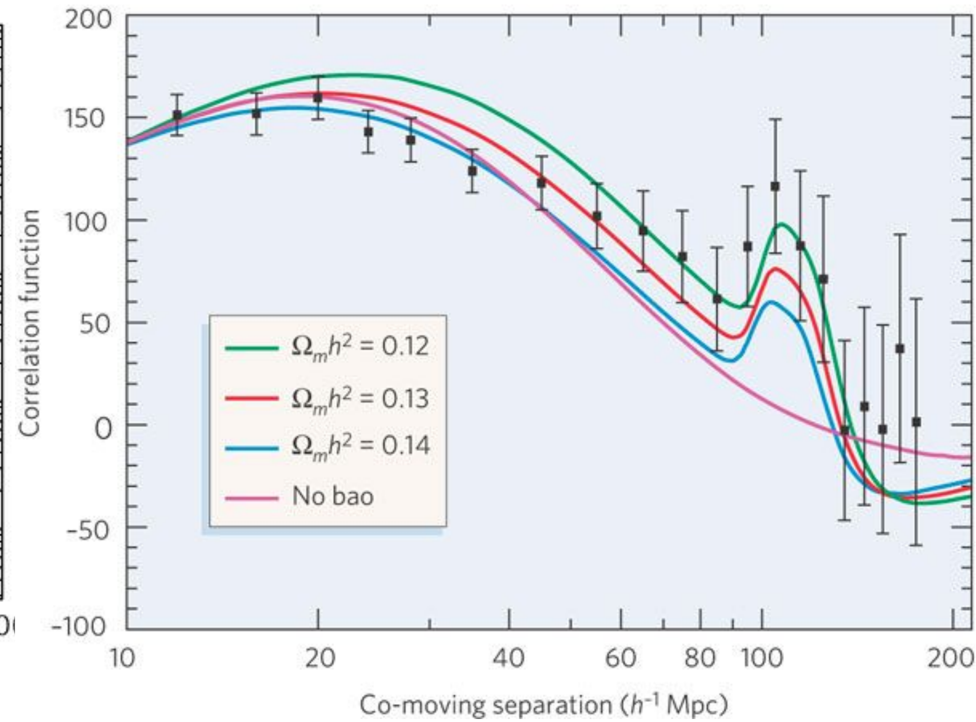


Martin White

Standard Ruler:
Baryon Acoustic Oscillations







Eisenstein & Bennett

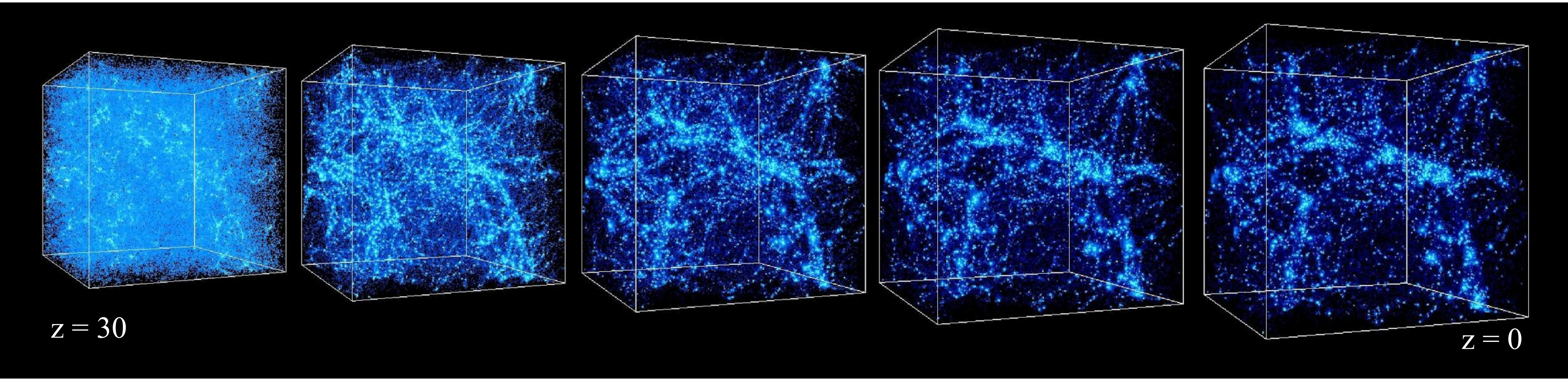
LSS Probes: GROWTH

Density fluctuation at a certain epoch is the result of competition between matter and dark energy.

Gravity/mass: pulls matter together → dense regions grow denser.

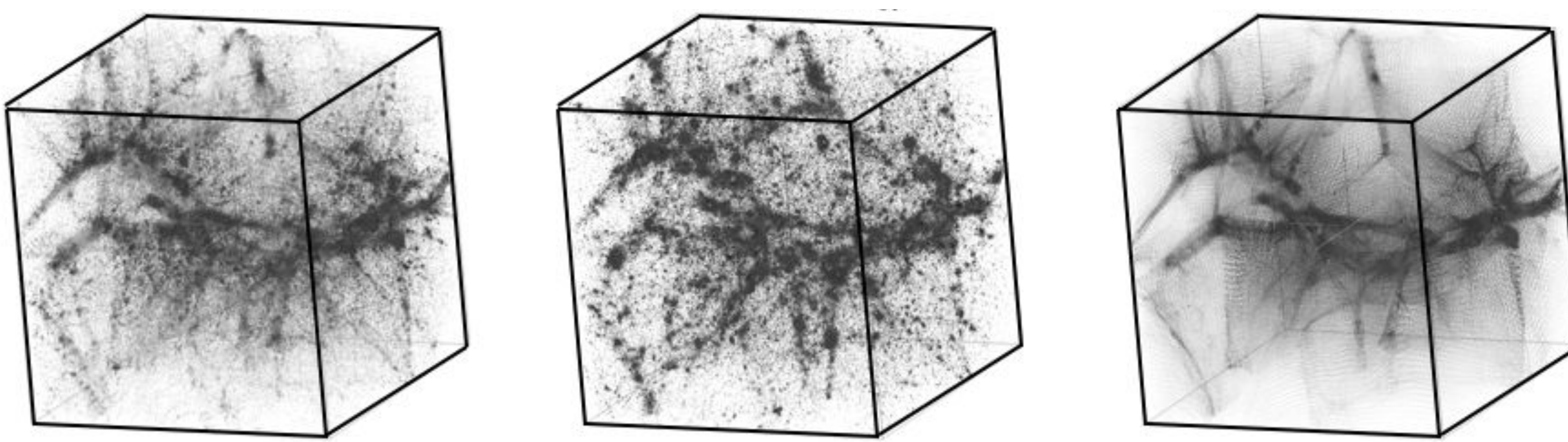
Dark energy: pushes matter apart → slows down the growth of structure.

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta = 4\pi G\bar{\rho}\delta$$



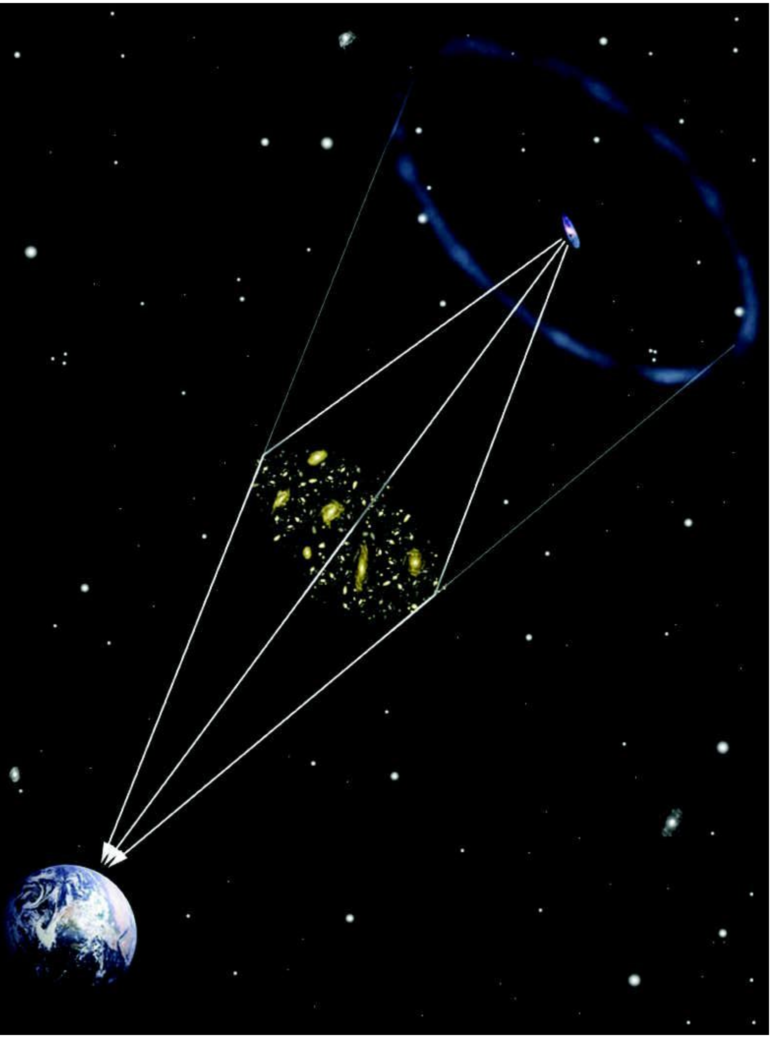
LSS Probes: GROWTH

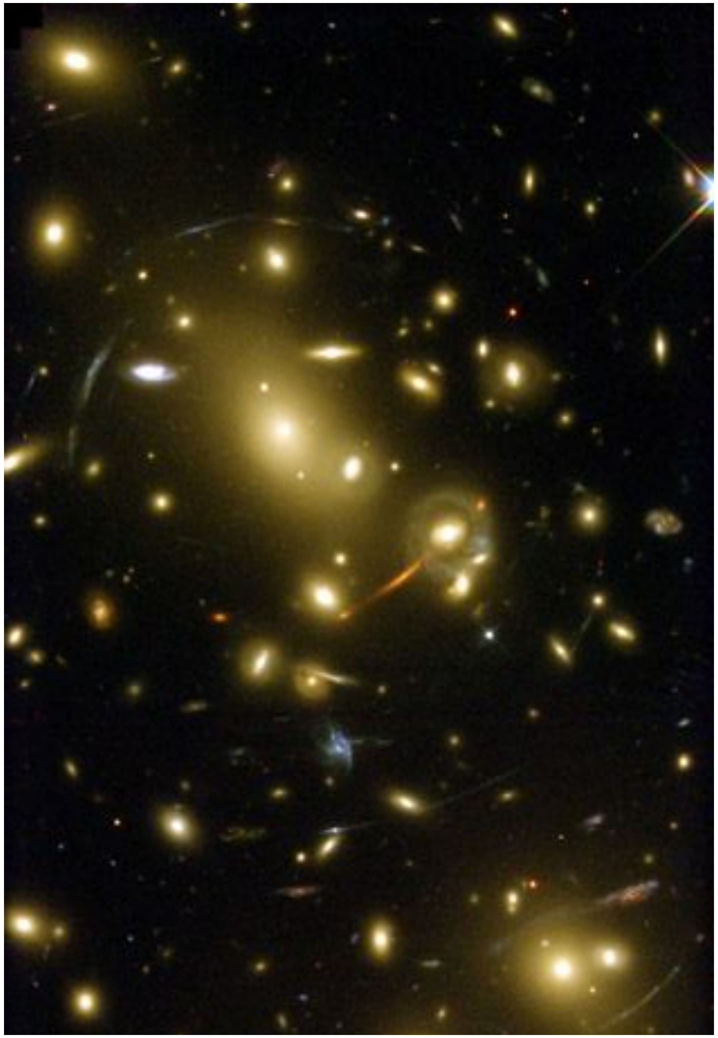
Standard Model *Dark Matter* *Massive Neutrinos*



Credit: Katrin Heitmann

LSS Probes: Weak Lensing

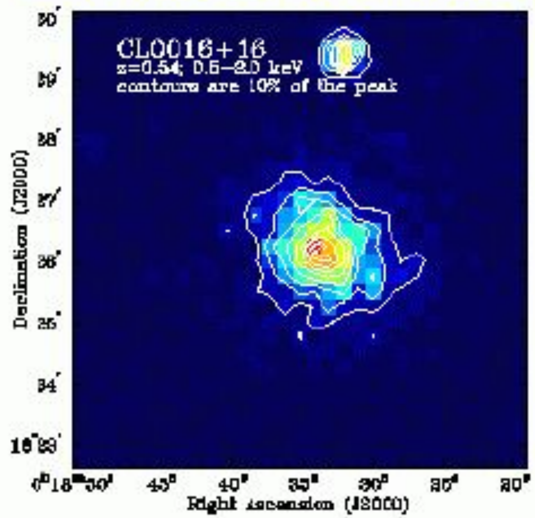




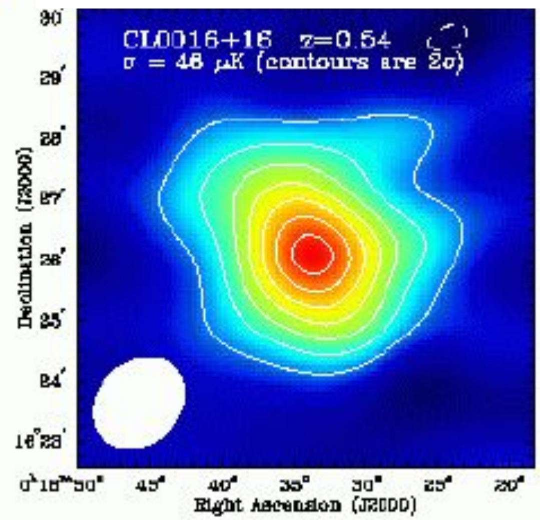
Credit: CFHTLenS

LSS Probes: Clusters

X-ray

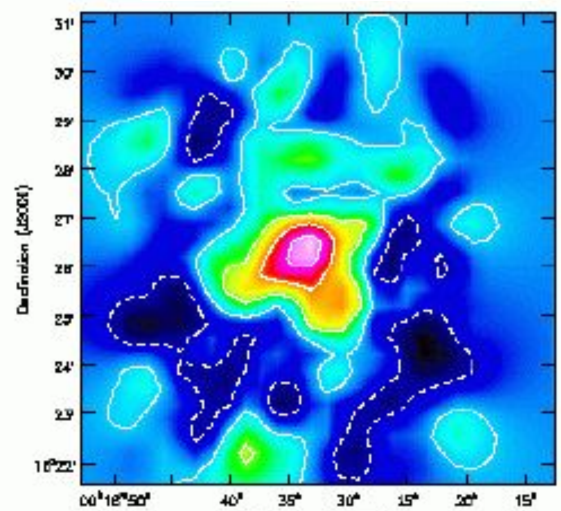


Sunyaev-Zel'dovich



Optical galaxy concentration

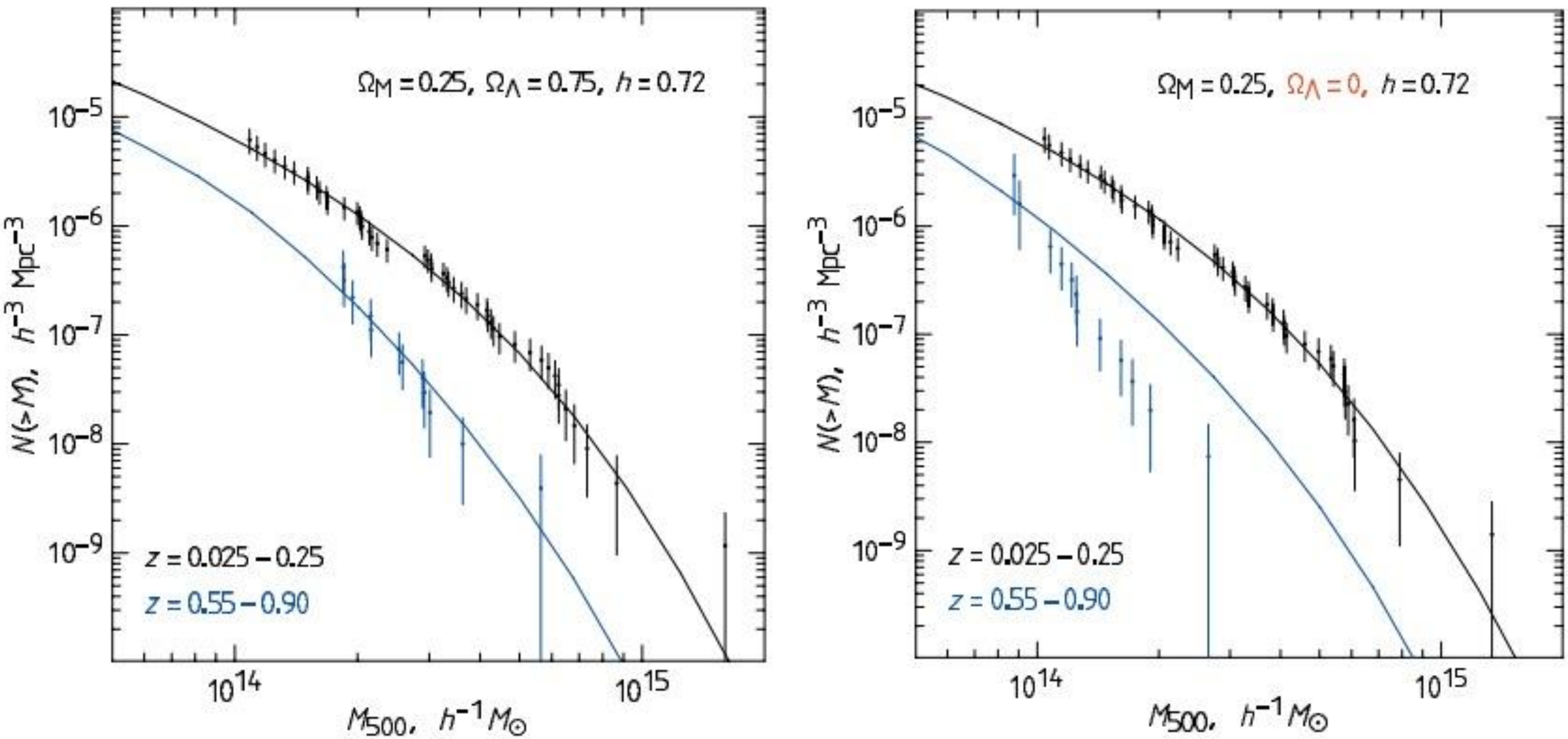
Weak Lensing



G. Holder

LSS Probes: Clusters

VIKHLININ ET AL.



Summary

1. Cosmological Observations: different wavelength gives us information about matter distribution at different time (redshift)
2. Growth of Structure: growths happens mainly during matter dominated era
3. Gaussian Random Field: the power spectrum
4. The Matter Power Spectrum: sensitive to dark matter, dark energy, massive neutrinos, etc.
5. Large Scale Structure Probes: SN Ia, BAO, weak lensing, clusters, etc.


Artificial satellites
MOLNIYA 3-45