Non-SUSY dark matter

(by which I mean something other than MSSM neutralinos!)

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Perspective

- IMNSHO, A dark matter model isn't worth much if all it does is "explain" dark matter.
- It really should fit into a larger framework that solves some or all of the Standard Model's problems:
  - Baryon asymmetry problem
  - Gauge hierarchy problem
  - Strong CP problem (cf. Winslow's talk?)
  - Triviality problems
  - Unification problem
  - Charge quantization problem
  - Quantum gravity problem
  - Cosmological constant problem
  - Neutrino mass problem
  - Flavor problems
  - Too many couplings problem
  - Many anomalies at the 3 sigma and larger level

Perspective (cont.)

- There are also cosmological and astrophysical problems we would like to solve:
  - All the problems that motivate inflation (horizon problem, flatness problem, etc.)
  - Cuspy halo, missing satellites, too big to fail
  - Positron excess (PAMELA, AMS)
  - Weird things at the galactic center
- BTW, a reason to not believe in GIMP DM: inflation → reheating → production of DM. Gravity won't do it.
- At a minimum, DM has couplings to the inflaton.

Perspective (cont.)

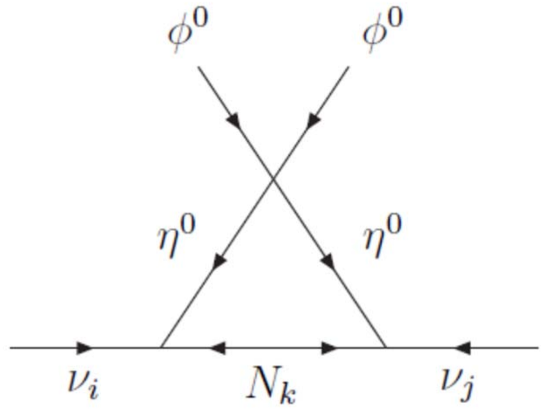
- “Stringy” models have the potential to solve all of these things, but they tend to come along with supersymmetry (SUSY).
- Thus the generic outcome is neutralino WIMP dark matter.
- But this is not necessarily so, and other interesting stringy models can be written down that don’t have low energy (~TeV) SUSY.
- As a string phenomenologist, I like to think of all models as somehow embedded into a string construction. But that’s just me.
- So even non-SUSY dark matter, which is the topic of this talk, is somehow for me SUSY at a high enough scale. But it is mostly irrelevant due to fact that phenomenology only cares about the low energy effective theory.

- We are more likely to “strike it rich” if we look at EFTs that also solve other problems, rather than some arbitrarily chosen EFT that only addresses dark matter.
- That is because nature want ALL the problems solved.

Scotogenic model [Ma, 2006]

- Extend usual see-saw by adding scalar doublet.
- Different from SM Higgs in that it is odd under a new Z(2) symmetry, as are the RH neutrinos.
- Assuming the Z(2) is not spontaneously broken, neutrinos get mass at one-loop and there is a LSP dark matter candidate (lightest state odd under Z(2)).
- So, there is an amusing connection between neutrino physics and dark matter physics.

Neutrino mass

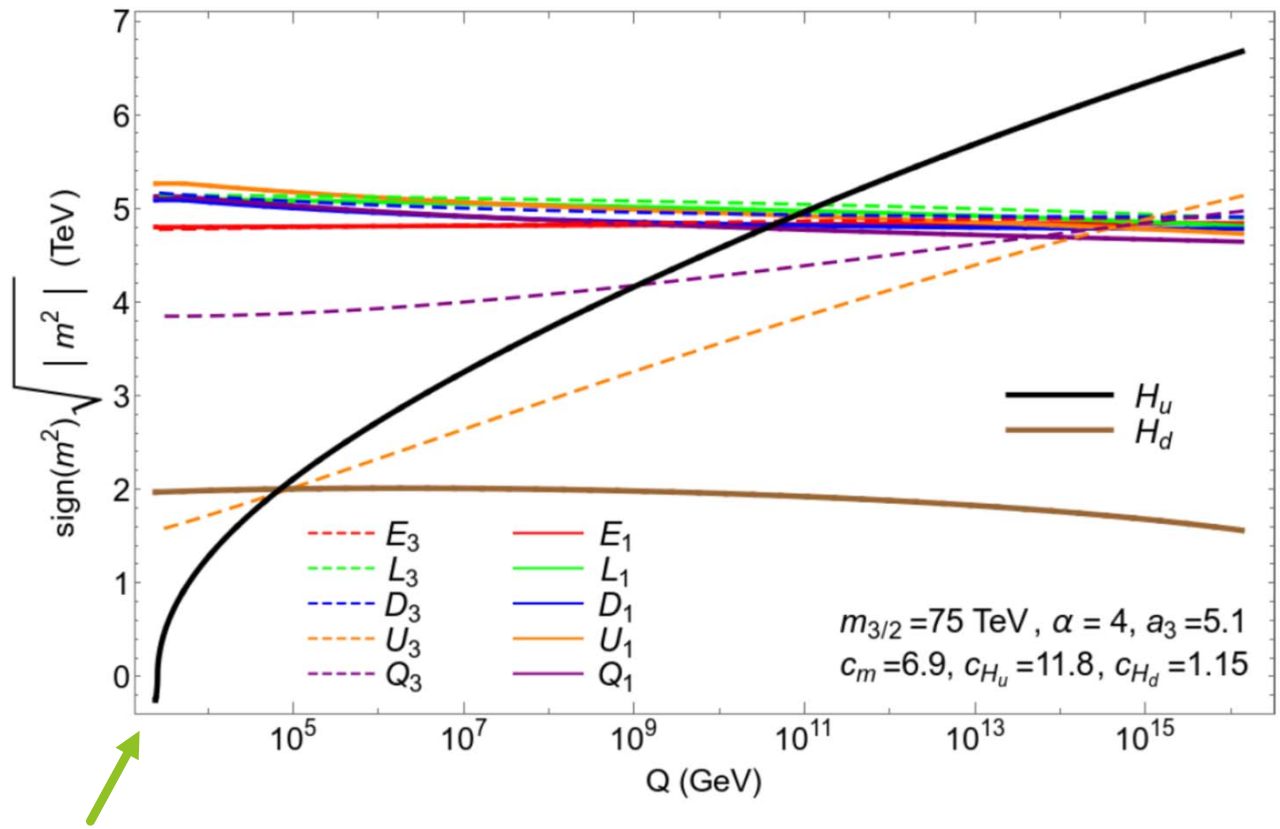


- The scalar potential, in complete generality, is:

$$V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2}\lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2}\lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta)$$
$$+\lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) + \frac{1}{2}\lambda_5 [(\Phi^\dagger \eta)^2 + \mathrm{h.c.}]$$

- You can see the complete proliferation of couplings. SUSY by contrast is much more economical; all the proliferation comes from SUSY breaking (soft SUSY breaking Lagrangian).

- Of course there are many questions:
  - Why is $m_1^2 < 0$ but $m_2^2 > 0$? Is this just put in by hand? SUSY does better.
  - How does this fit into a GUT, and what about the failure to unify couplings?
  - What about the hierarchy problem?
  - Are there other “flavor” problems, such as too large CPV, FCNCs, EDMs, etc.?



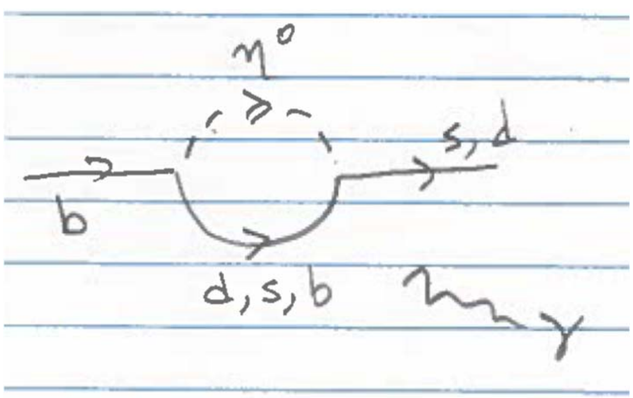
Baer et al., 1610.06205

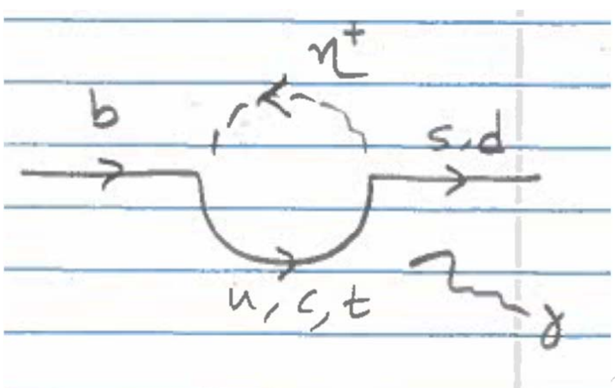
- Advantages include:
  - See-saw scale can be as low as 1 TeV, and thus directly accessible at LHC.
  - Dark matter and neutrino mass matrix are tied to each other.
    - Either RH neutrino (fermion) or
    - Re η^0 (scalar)

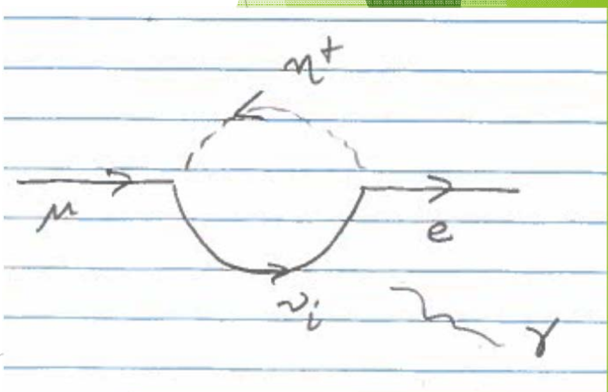
- Notice that b/c of Z(2) we don't have dangerous couplings like:

$$\lambda_{ij}^{u}\eta_u Q_{Li}u_{Lj}^{c} + \lambda_{ij}^{d}\eta_d Q_{Li}d_{Lj}^{c} + \lambda_{ij}^{e}\eta_d L_{Li}e_{Lj}^{c}$$

$$\eta_u = \eta, \quad \eta_{d\alpha} = \epsilon_{\alpha\beta}\eta_\beta^*$$



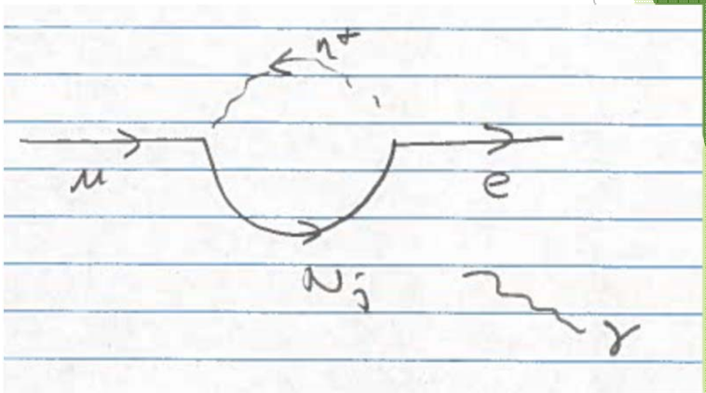




- We do however have this interesting process:
- We can do a quick back of the envelope:

$$\Gamma \sim \frac{h^2}{M_N^2 m_\eta^4} m_\mu^7 \sim 10^{-29}\text{GeV}$$

$$\text{BR}(\mu \to e\gamma) \sim 10^{-11}$$
$$\text{vs. expt. of BR} < 5.7 \times 10^{-13}$$



- So we have sensitivity to this rare process, and should do a careful check on any model of the Yukawa couplings and masses.

Leptogenesis from dark matter annihilations in scotogenic model

- Borah, Dasgupta & Kang, 1806.04689.
- Falls into class of models that try to explain $\Omega_{\rm DM} \approx 5\Omega_{\rm B}$ in a non-coincidental ("unified") way.
- Since it is the Ma model, in addition to RH neutrinos, we also have the SU(2) double scalar (like Higgs). But it and the RH neutrinos have a Z(2) parity, so we get a different sort of Dirac mass term-like Yukawa:

$$\Delta\mathcal{L} = -\lambda_{ij}\bar{L}_i\tilde{\eta}N_j$$

- However, it is assumed that Z(2) does not break spontaneously, so η does not get a vev, and this is not really a mass term.

- DM annihilations → baryon asymmetry == WIMPy leptogenesis
- Ties in with neutrinos b/c Ma model relates all three
- Want 3 things simultaneously:

1. DM relic density
2. Baryon asymmetry (leptogensis)
3. Neutrino masses

- Borah et al find it is not easy to have all 3.

- They have to introduce a singlet scalar.
- It has the interesting side-effect of making charged LFV a live possibility in the model, presumably because the leptogenesis scale is lowered to 5 TeV.

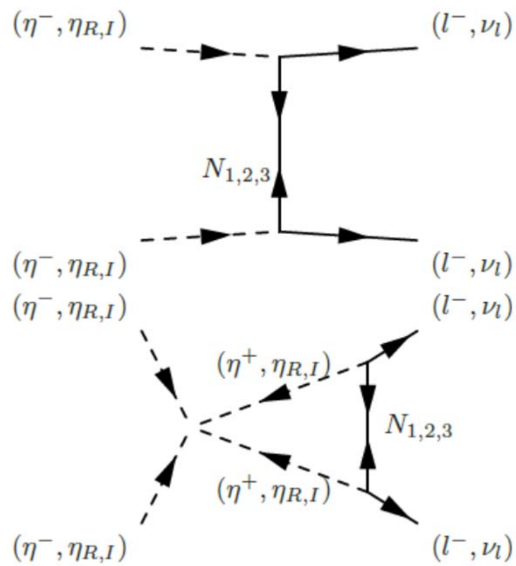
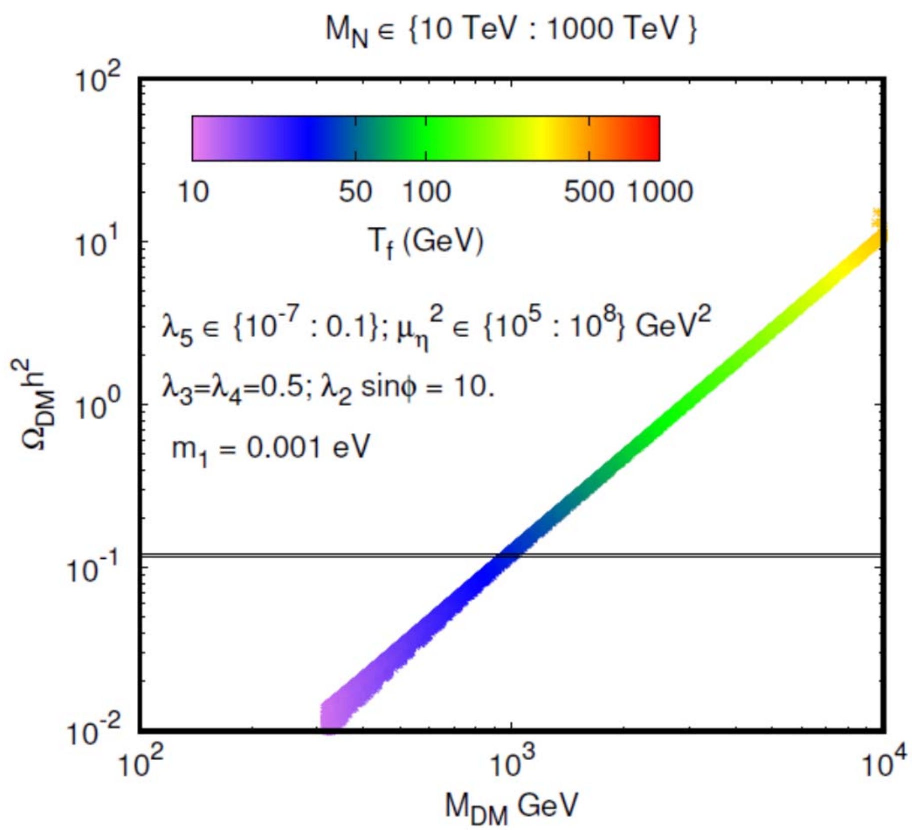


FIG. 1. Feynman diagrams contributing to $\langle \sigma v \rangle_{\mathrm{DMDM} \to LL}$ and ϵ.



T(f) > 150 GeV for the sphaleron processes to go → overproduction of DM

- Moreover Yukawa couplings must be O(1) for lepton asymmetry, forcing λ_5 to be very, very small in order to get small neutrino masses.
- But this then runs into direct detection limits (Z-mediated).
- Lower bound:

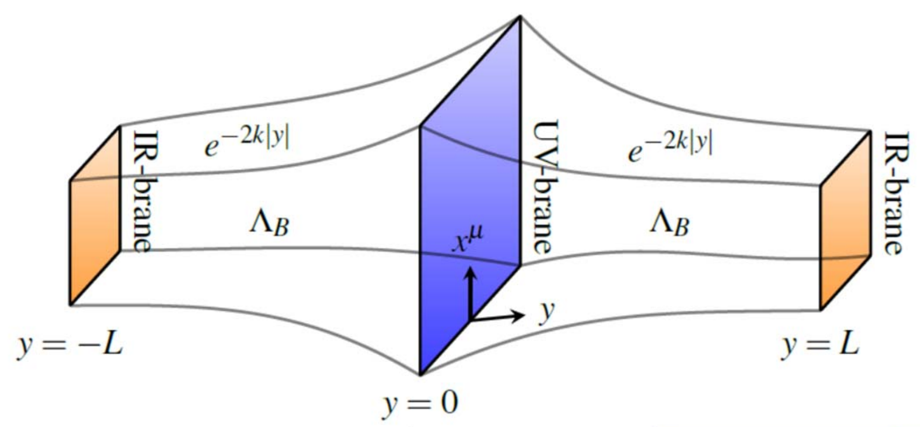
$$\lambda_5 \approx 1.65 \times 10^{-7} \left(\frac{\delta}{100\ \mathrm{keV}}\right)\left(\frac{M_{\mathrm{DM}}}{100\ \mathrm{GeV}}\right)$$

- For this reason “vanilla scotogenic” doesn’t work and they have to add a singlet scalar.

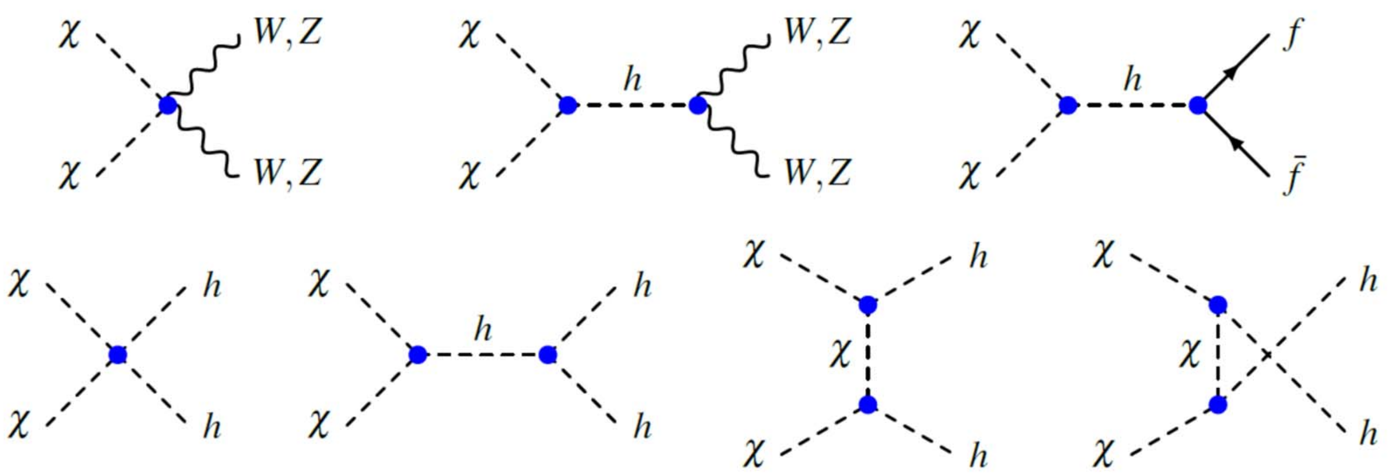
- By adding the singlet, they can decouple the mass splitting δ from the neutrino mass.
- I guess the original model was “too predictive.”

Ahmed et al., https://arxiv.org/abs/1510.05722

- “Dark Higgs” is Z(2) odd version of Higgs in a warped extra dimension (IR-UV-IR) set-up.
- “it is found that the dark-Higgs can provide only a small fraction of the observed dark matter abundance”
- KK parity Z(2) is allowed because they take the IR-UV-IR set-up rather than the original RS1 UV-IR set-up.



- They work out the low energy effective 4d theory and coannihilation diagrams



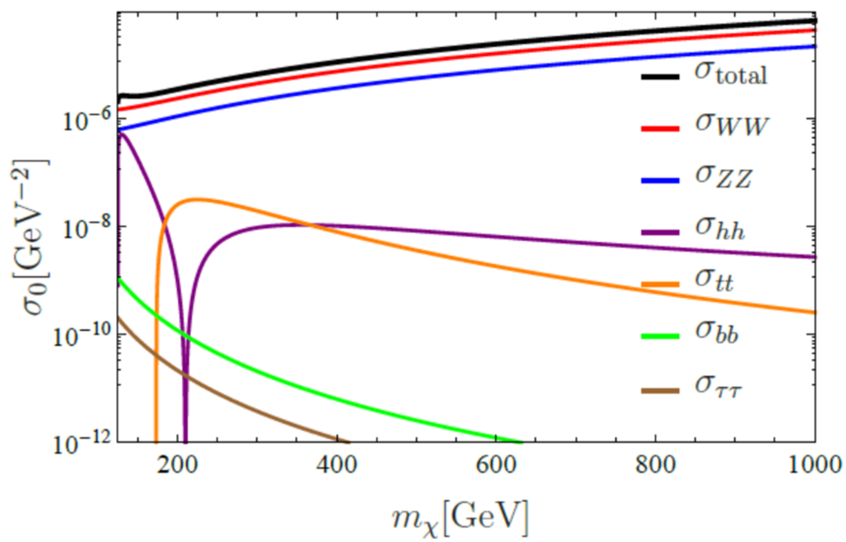
- From this they obtain the ingredients for calculating the cross section and hence relic density:

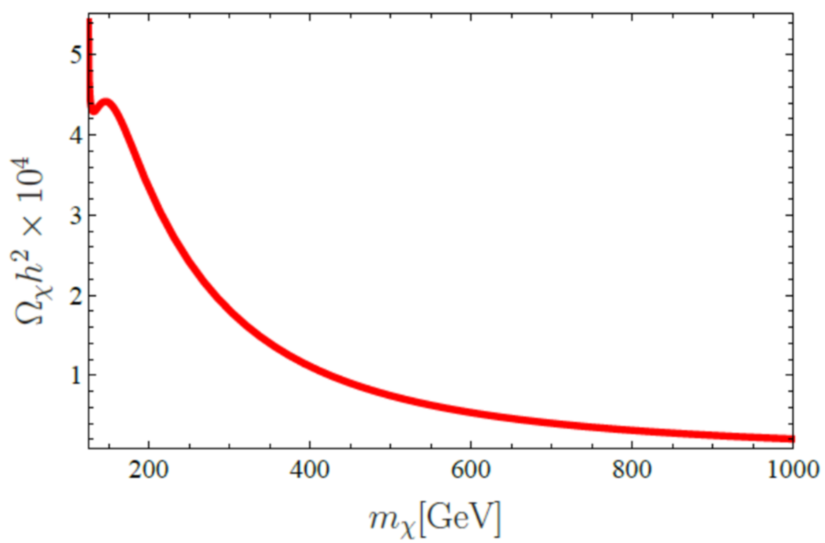
$$|\mathcal{M}(\chi\chi \to \tilde{V}\tilde{V})|^2 = \frac{4m_{\tilde{V}}^4}{S_{\tilde{V}} v^4}\left(1+\frac{3m_h^2}{s-m_h^2}\right)^2 \left[2+\left(1-\frac{s}{2m_{\tilde{V}}^2}\right)^2\right],$$

$$|\mathcal{M}(\chi\chi \to f\bar{f})|^2 = 18N_c \frac{m_f^2 m_h^4}{v^4} \frac{s-4m_f^2}{(s-m_h^2)^2},$$

$$|\mathcal{M}(\chi\chi \to hh)|^2 = \frac{9m_h^4}{2v^4}\left[1+3m_h^2\left(\frac{1}{s-m_h^2}+\frac{1}{t-m_\chi^2}+\frac{1}{u-m_\chi^2}\right)\right]^2$$

- You can see, it is very, very small:





- So it doesn’t work.

L. Vecchi, https://arxiv.org/abs/1310.7862

- Neutrino mass and proton decay from high scale physics is generic and of the form:

$$c_\nu \frac{\ell\ell HH}{\Lambda} \qquad\qquad c_p \frac{qqq\ell}{\Lambda^2}$$

- Experimental constraints and O(1) coefficients give:

$$\Lambda(\text{neutrino}) \geq 6 \times 10^{14}\ \text{GeV} \qquad\qquad \Lambda(\text{proton}) \geq 10^{16} GeV$$

- However, we can bring down the scale of new physics if the coefficients are very small for some reason.
- The best way to achieve this is through approximate conservation of L and B.
- So we see the connection to other physics.
- In fact, some aspect of this is necessary in order to preserve baryon asymmetry.

- If one has a neutral particle X, with B and L charges q(B), q(L), then the decay

$$X \to (pe)^m n^n \nu^p (\cdots)$$

is forbidden under the conditions (fermion)

$$q_B + q_L \neq \text{odd} \quad \text{or} \quad q_B - q_L \neq \text{odd}$$

or (boson)

$$q_B + q_L \neq \text{even} \quad \text{or} \quad q_B - q_L \neq \text{even}$$

- Thus we see that there is an accidental dark matter parity symmetry (preventing decay) that is a consequence of L and B conservation.

- In a gravitational theory, global symmetries are expected to be violated.
- So when we attempt something like warped extra dimensions, we have to make B and L gauge symmetries (local).
- Thus they are spontaneously broken in the UV, with some residual version that explains the small coefficients.

- In fact we are going to have to violate B and/or L at some level to get the baryon asymmetry.
- Then the stability of DM over cosmological time scales becomes a question that has to be investigated further.

- Vecchi introduces both Dirac and “Majorana” masses in the bulk (for fermions):

$$-M_D\bar{\Psi}\Psi - \left(\frac{M_M}{2}\bar{\Psi}\Psi^c + \mathrm{h.c.}\right)$$

$$\Psi^c = C_5\Psi^*, \quad C_5 = -i\gamma^5\gamma^2$$

- To perform the KK decomp. it is good to break out chirality components

$$\Psi = \begin{pmatrix} \chi \\ \epsilon\psi^* \end{pmatrix}$$

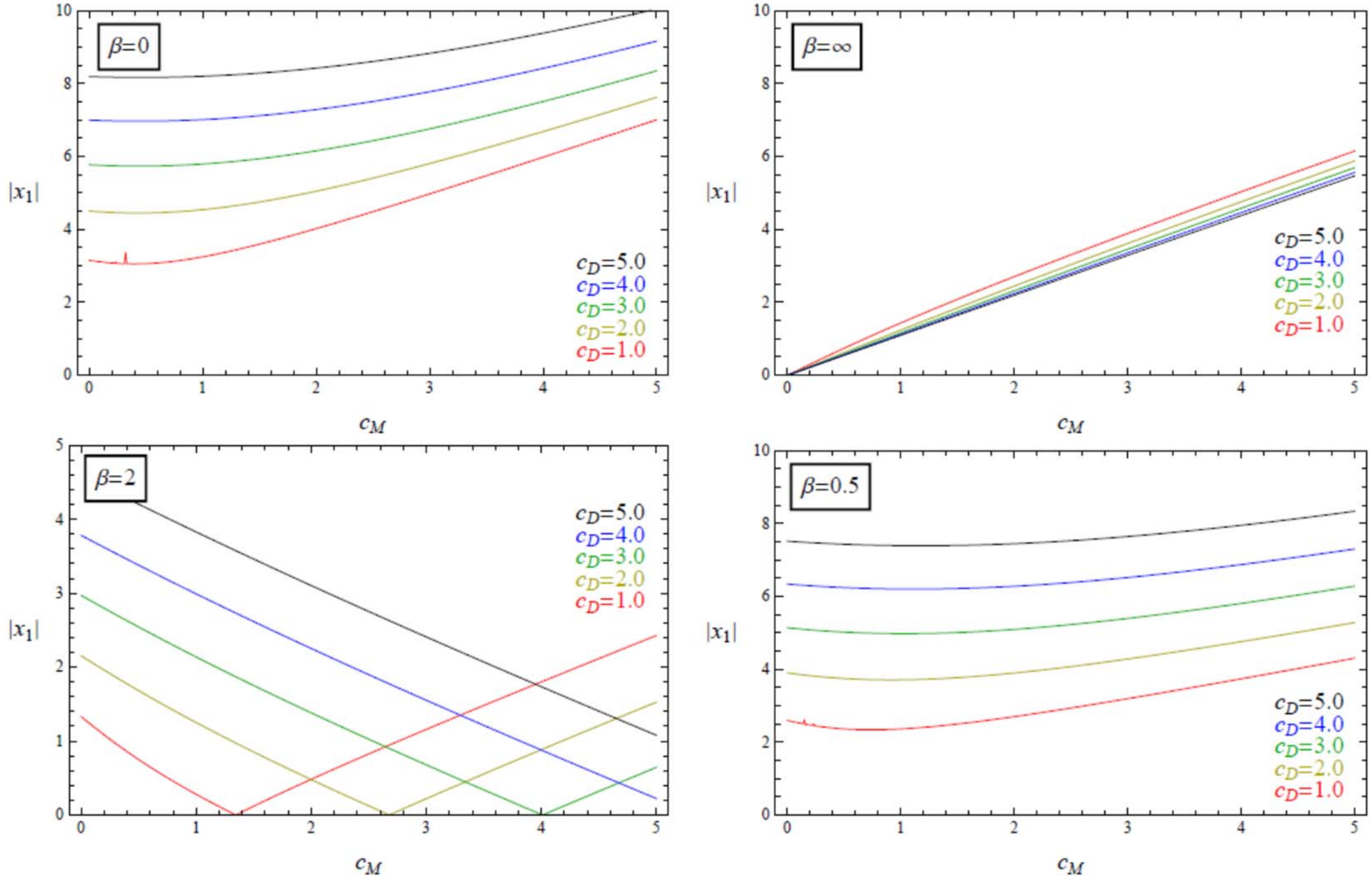
- The AdS(5) geometry is described by

$$ds^2 = a^2(z)(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \quad a(z) = \frac{L}{z}$$

$$\Delta = i\tau^2(\partial_z + 2a^{-1}\partial_z a) - a(\tau^1 M_D + \tau^3 M_M)$$

- Then the mode equation becomes:

$$\Delta \begin{pmatrix} \chi_n \\ \psi_n \end{pmatrix} = m_n \begin{pmatrix} \chi_n \\ \psi_n \end{pmatrix}$$



multi-TeV masses for Majorana DM candidate

Relic density

- Using standard formulae

$$\Omega_X h^2 \approx \frac{1.07 \times 10^9}{\mathrm{GeV}} \frac{x_f}{g_*^{1/2} M_{\mathrm{Pl}}\, \sigma v_{\mathrm{rel}}}$$

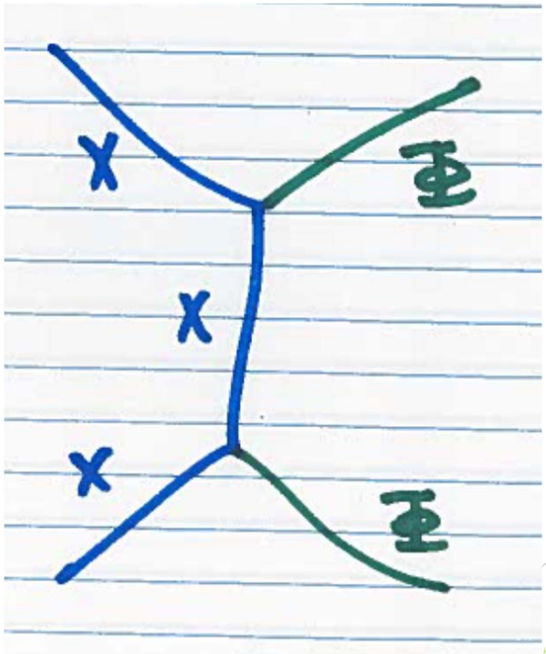
$$x_f^{1/2} e^{x_f} \approx \frac{5}{4\pi^3} \sqrt{\frac{45}{8}} \frac{M_{\mathrm{Pl}} m_X \sigma v_{\mathrm{rel}}}{g_*^{1/2}}$$

- In order to calculate the all important $\langle \sigma v \rangle$ we need to know what X interacts with!
- Certainly it interacts with the spin-2 KK gravitons.
- It also interacts with the radion σ
- But these are velocity suppressed and smaller than the mode that will now be discussed:
- The Goldberger-Wise field Φ
- This is the field associated with stabilizing the separation between the UV and IR branes.

- The interaction with X takes the form

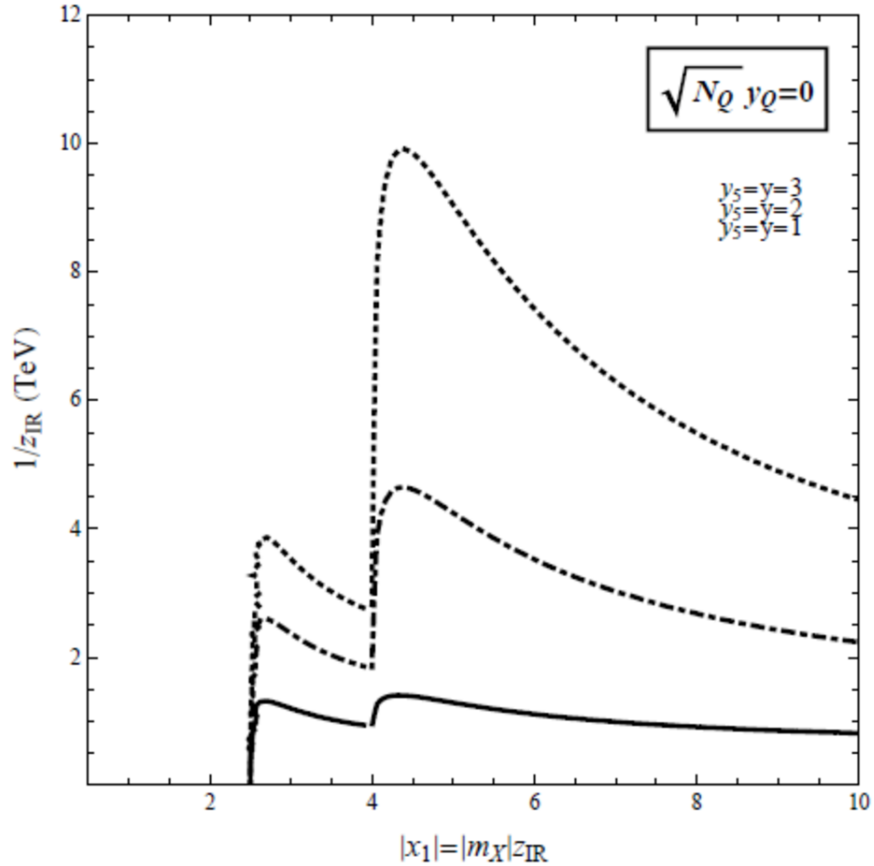
$$\frac{y}{2}\Phi\bar{X}X + i\frac{y_5}{2}\Phi\bar{X}\gamma^5 X$$

- Generically, the Yukawa couplings are O(1).
- So a typical thing you would get is t-channel exchange of X:



- This gives:

$$\sigma(XX \to \Phi\Phi)v_{\rm rel} = \frac{y^2 y_5^2}{8\pi m_X^2} \frac{\left(1 - \frac{m_\Phi^2}{m_X^2}\right)^{1/2}}{\left(1 - \frac{m_\Phi^2}{2m_X^2}\right)^2},$$



Scenarios with the correct relic density

Dark QCD

- An old paper by Spergel & Steinhardt [PRL 84 (2000) 3760] shows that the problem of cuspy halos can be cured if dark matter has a self interaction that satisfies:

$$\frac{\sigma}{m} \sim 1\ \mathrm{cm}^2/\mathrm{g}$$

- If the dark sector is analogous to QCD, then there will be a single scale (the one generated by dimensional transmutation), and so we can estimate:

$$\sigma \sim \Lambda_{\mathrm{DQCD}}^{-2}, \quad m \sim \Lambda_{\mathrm{DQCD}}$$

$$\frac{\sigma}{m} \sim \Lambda_{\mathrm{DQCD}}^{-3}$$

- Using the astrophysical constraint (fixing cuspy halos), we find, quite amusingly:

$$\Lambda_{\rm DQCD} \sim 100 \text{ MeV}$$

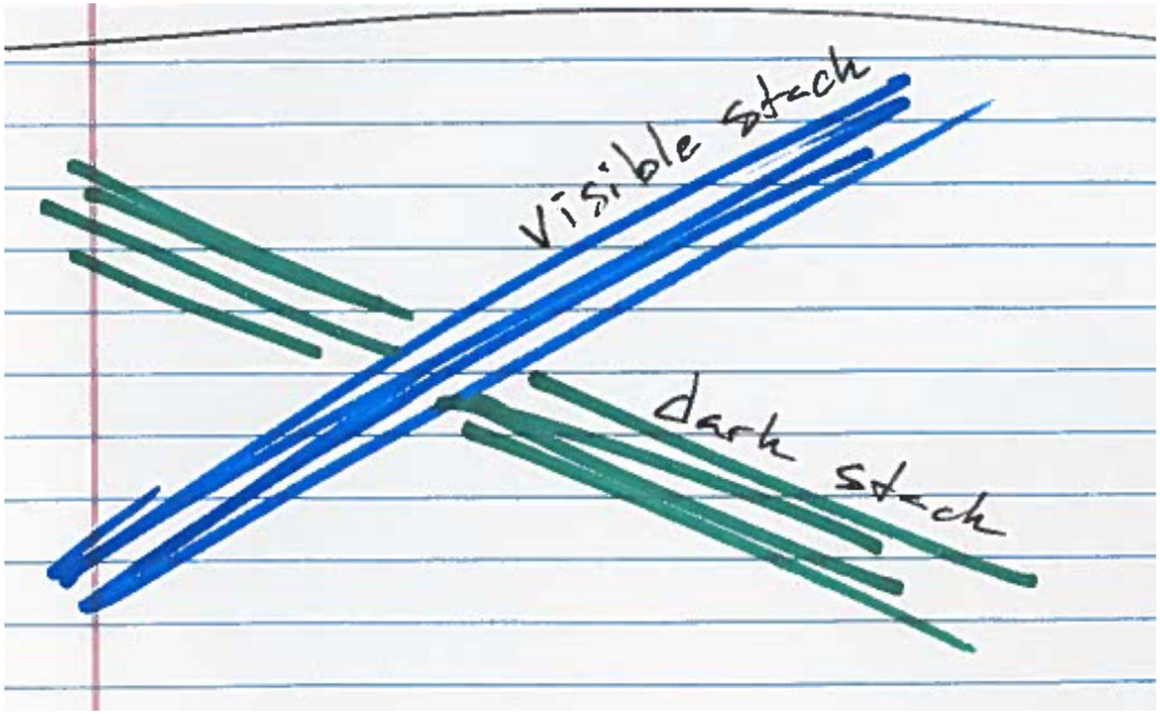
- This is amusing because it is the SAME as QCD.

- Note that these are short range interactions!
- There is no long-range dissipative force, so dark matter halos remain approximately spherical (they do not form disks).

- I like these models because:
  - I worked on string compactifications where we had strongly interacting matter in hidden sectors, driving gaugino condensation, hence SUGRA mediation.
  - Now the hidden sector can also be a dark sector.
  - I do (mostly) lattice gauge theory these days, so new strong interactions are kind of exciting and nonperturbative things we could in principle study on the lattice.
  - I have been promoting this idea for a long time and now lattice "BSM" groups are obtaining results on "composite dark matter."

- But I've actually been working on a somewhat different tack.
- One of the questions is how this sort of dark sector interacts with the visible sector, both for production of the relic density, and for detection.
- So I use holographic duality:
  - N D3-branes to produce a strongly interacting nonabelian gauge theory.
  - Klebanov-Witten background to get N=1 SUSY, or better yet Klebanov-Strassler to get N=1 with confinement.
  - N(f) D7-branes to get "quarks", i.e., flavor.

- So far that is straightforward and stuff we were doing 10 years ago.
- It has a warped extra dimension that can be mapped onto RS1 [Gherghetta & JG, 2006].
- SUSY breaking can also be addressed using a non-SUSY background deformation of KS [Gabella, Gherghetta & JG 2007].
- But now I am introducing two stacks of D7 branes at angles (two embeddings into KS).
- Interactions through the nonabelian extension of the DBI action allows us extract interactions between the visible sector (Stack A) and the dark sector (Stack B).



Strings stretching between the stacks mediate interactions:
Non-abelian Dirac-Born-Infeld tells us what these are.

- As this progresses we will be able to address the types of embeddings that allow for realisting dark matter relic density.

Stabilizing the hierarchy

- Hierarchies in warped compactifications can be supported by RR and NS fluxes [Giddings, Kachru & Polchinski 2001]
- This is a stringy realization of the Goldberger & Wise (1999) phenomenological mechanism for stabilizing the radion.
- In string theory this is a moduli field, and so it is not surprising that fluxes can stabilize it.

Dual picture of the hierarchy

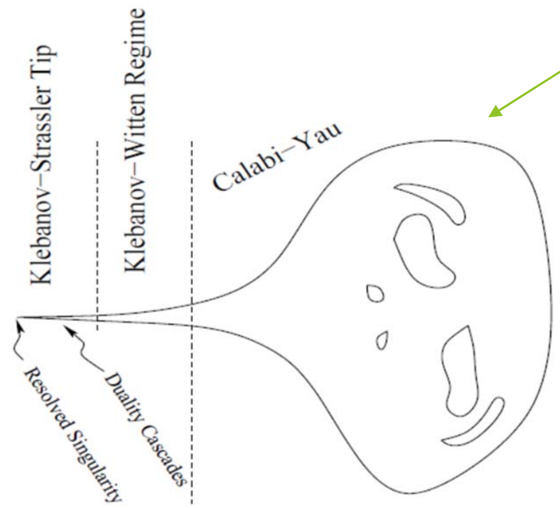
- In the dual gauge theory, this is the usual story of a hierarchy due to dimensional transmutation with a weak coupling at the high scale.

$$\Lambda = a^{-1} e^{-1/bg^2}$$

- Planck brane, negative tension brane, needed for RSI
- Stringy: O3 planes, wrapped D7 branes

Refinement of RS1:
Follow viewpoint of GKP

Replaces Planck (UV) brane



Replaces TeV (IR) brane

Gherghetta & JG 2006

- Conifold: present as embedding into 4-complex-dimensional space.

$$ds^2 = |dz_1|^2 + |dz_2|^2 + |dz_3|^3 + |dz_4|^2$$

$$z_1 z_2 - z_3 z_4 = 0$$

Satisfying the embedding condition

- The Gaussian coordinates (or at least one possible choice) are given by:

$$z_1 = r^{3/2} e^{\frac{i}{2}(\psi - \phi_1 - \phi_2)} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$z_2 = r^{3/2} e^{\frac{i}{2}(\psi + \phi_1 + \phi_2)} \cos\frac{\theta_1}{2} \cos\frac{\theta_2}{2}$$

$$z_3 = r^{3/2} e^{\frac{i}{2}(\psi + \phi_1 - \phi_2)} \cos\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$z_4 = r^{3/2} e^{\frac{i}{2}(\psi - \phi_1 + \phi_2)} \sin\frac{\theta_1}{2} \cos\frac{\theta_2}{2}$$

- Then the metric for the conifold becomes:

$$ds_6^2 = dr^2 + r^2 ds_{T^{1,1}}^2$$

$$ds_{T^{1,1}}^2 = \frac{1}{9}(d\psi + \sum_{i=1}^{2} \cos\theta_i d\phi_i)^2 + \frac{1}{6}\sum_{i=1}^{2}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2)$$

- The full 10d metric is given by:

$$ds_{10}^2 = H^{-1/2}(r)(-dt^2 + d\vec{x}^2) + H^{1/2}(r)ds_6^2$$

$$H(r) = 1 + \frac{L^4}{r^4}$$

$$L^4 = 4\pi g_s N(\alpha')^2$$

- N D3 branes at the conical singularity r=0.

- Here, the 5d base of the cone is the well-known quotient manifold $T^{1,1}$, which is among those classified by Romans as having reduced supersymmetry (fewer than the maximal number of Killing spinors).

$$T^{1,1} \simeq \frac{SU(2) \times SU(2)}{U(1)} \simeq \frac{S^3 \times S^3}{S^1}$$

$$Q = T_1^3 \oplus T_2^3$$

- Others in this class are $T^{p,q}$

$$Q = pT_1^3 \oplus qT_2^3$$

- The isometry group is easier to see with a change of coordinates (isn't that what GR is all about?):

$$z_1 = w_1 + iw_2, \quad z_2 = w_1 - iw_2$$

$$z_3 = w_3 + iw_4, \quad z_4 = -(w_3 - iw_4)$$

- Then the conifold is described by the more transparent equation

$$\sum_i w_i^2 = \det(w_4 \mathbb{I}_2 + i\sigma_a w_a) = 0$$

Symmetries and intersections

- This clearly has an $SU(2) \times SU(2) \times U(1)$ invariance:

$$w_4 \mathbb{I}_2 + i\sigma_a w_a \to e^{i\alpha} U (w_4 \mathbb{I}_2 + i\sigma_a w_a) V$$

- The $T^{1,1}$ base is the intersection of this with the 7-sphere:

$$2 \sum_i |w_i|^2 = \mathrm{Tr}(w_4 \mathbb{I}_2 + i\sigma_a w_a)(w_4 \mathbb{I}_2 + i\sigma_a w_a)^\dagger = 2r^3$$

- It also clearly has the $SU(2) \times SU(2) \times U(1)$ invariance.

- We follow Levi & Ouyang (2005) and add probe D7 branes to introduce "quark" flavors.
- Builds on Klebanov-Witten: less SUSY
- D7 branes give rise to a "meson" spectrum that L&O analyze, including numerically.
- Tony & I wanted to see if we could relate this to Randall-Sundrum type phenomenology.
- To do this, we had to derive an effective 5d action from the L&O setup.

- The D7 embedding generally looks like

$$X^M(\xi), \quad \xi = (\xi^0, \ldots, \xi^7), \quad M = 0, \ldots, 9$$

- But we take

$$X^M = (x^\mu, r, \theta_1, \theta_2, \phi_1, \phi_2, \psi)$$

with each of these eight components depending on ξ

- Then we further take for the “background embedding” (i.e., ignoring fluctuations)

$$\xi^\mu \equiv x^\mu, \quad \mu = 0, \ldots, 3$$

$$\xi^4 = \theta_1, \quad \xi^5 = \theta_2, \quad \xi^6 = \phi_1, \quad \xi^7 = \phi_2$$

$$r = r_0(\theta_1, \theta_2) \quad \text{s.t.} \quad \mu = r_0^{3/2} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$\psi = \psi_0(\phi_1, \phi_2) \quad \text{s.t.} \quad \psi_0 = \phi_1 + \phi_2$$

- The action for the D7 probe brane is given by DBI:

$$S_{\mathrm{DBI}} = -\tau_7 \int d^4x \; d^2\theta \; d^2\phi \; \sqrt{\varphi^*(g) + \varphi^*(B) + 2\pi\alpha' F}$$

- The pullback of the metric (which we need for our scalar analysis) is

$$\varphi^*(g) = (g_0)_{ab} = \frac{\partial X^M}{\partial \xi^a}\frac{\partial X^N}{\partial \xi^b} g_{MN}$$

$$(g_0)_{ab} = \mathrm{diag}(r_0^2 \eta_{\mu\nu}, g_{\theta_i\theta_j}, g_{\phi_i\phi_j})$$

- …and in detail…

$$g_{\theta_i\theta_j} = \begin{pmatrix} \frac{1}{6} + \frac{1}{9}\cot^2\frac{\theta_1}{2} & \frac{1}{9}\cot\frac{\theta_1}{2}\cot\frac{\theta_2}{2} \\ \frac{1}{9}\cot\frac{\theta_1}{2}\cot\frac{\theta_2}{2} & \frac{1}{6} + \frac{1}{9}\cot^2\frac{\theta_2}{2} \end{pmatrix}$$

$$g_{\phi_i\phi_j} = \begin{pmatrix} \frac{1}{6}\sin^2\theta_1 + \frac{1}{9}(1+\cos\theta_1)^2 & \frac{1}{9}(1+\cos\theta_1)(1+\cos\theta_2) \\ \frac{1}{9}(1+\cos\theta_1)(1+\cos\theta_2) & \frac{1}{6}\sin^2\theta_2 + \frac{1}{9}(1+\cos\theta_2)^2 \end{pmatrix}$$

- We can carry out a geometric of analysis for the D7 embedding. Note that in terms of the original conifold coordinates it looks like (Y_4):

$$\mu z_2 = z_3 z_4$$

- This has a $U(1) \times U(1)$ invariance with charges $(2,1,1)$ and $(0,1,-1)$ for the three complex coordinates under each U(1).
- It also has a scaling symmetry $\Gamma: \quad z_2 \to \lambda^2 z_2, \quad z_{3,4} \to \lambda z_{3,4}$
- The base of the cone Y_4 is therefore given by $X_3 = Y_4/\Gamma$

- We can parameterize Y_4 by:

$$z_3 = \rho e^{i\alpha} \cos\frac{\gamma}{2}, \quad z_4 = \rho e^{i\beta} \sin\frac{\gamma}{2}$$

- The base just corresponds to the intersection with the S^3 of radius ρ, or the equation:

$$|z_3|^2 + |z_4|^2 = \rho^2$$

- Consider the homeomorphism

$$\mu z_2 = (1-s) z_3 z_4, \quad s \in [0,1]$$

- At s=1, we have $z_2 = 0; \quad z_3, z_4 \ \textbf{arbitrary} \quad \Rightarrow \quad \mathbb{R}_+ \times S^3$
- As s=0 we recover our D7 embedding.

- Thus our D7 embedding is homeomorphic to $\mathbb{R}_+ \times S^3$
- From this we conclude that the base X_3 is topologically equivalent to S^3

- On this basis we take the lowest mass states of the KK decomposition of D7 embedding fluctuations to be constant modes of angular coordinates; i.e., the lowest hyperspherical harmonic.

- It is of interest to relate Y^4 to the conifold geometry, particularly the coordinate r.

$$r^3 = \sum_{i=1}^{4} |z_i|^2 = \mu^2 + \rho^2 + \frac{1}{4\mu^2}\rho^4 \sin^2\gamma$$

- First, note that as $r \to \mu^{2/3}$, the $X_3 \simeq S^3$ radius ρ shrinks to zero. This shows in detail how the D7 branes “end” in the AdS_5 radial direction.

- Thus, since we have identified the eight worldvolume parameters with eight of the coordinates of the underlying 10d geometry, the fluctuations in the D7 embedding will correspond to changes in the two functions r_0 and ψ_0

$$r = r_0(1+\chi)$$

$$\psi = \psi_0 + 3\eta$$

- Then one substitutes these into the pullback of the metric and expands the DBI action:

$$\varphi^*(g) = g_0 + \delta g_0(\chi,\eta) \qquad \sqrt{\det(g_0+\delta g_0)} = \exp\frac{1}{2}\mathrm{Tr}(\ln g_0 + \ln(1+g_0^{-1}\delta g_0))$$

good stuff!

- Straightforward but tedious to get the quadratic action:

$$
\begin{aligned}
S &= -\tau_7 \int d^4x\ d^2\theta\ d^2\phi\ \Bigg\{ \sqrt{-g_0}\Bigg[\frac{g_0^{ab}}{2C}(\partial_a\chi\partial_b\chi + \partial_a\eta\partial_b\eta) \\
&\quad + \frac{4}{C}(\sin^2\frac{\theta_i}{2})^{-1}\chi\partial_{\phi_i}\eta - \frac{2}{C^2}(\sin^2\frac{\theta_i}{2})^{-1}\cot\frac{\theta_j}{2}\partial_{\theta_j}(\chi\partial_{\phi_i}\eta)\Bigg] \\
&\quad -\partial_{\theta_i}\left[\frac{\sqrt{-g_0}}{C}\cot\frac{\theta_i}{2}(3\chi^2 + 2\chi)\right]\Bigg\}
\end{aligned}
$$

$$
C = 1 + \frac{2}{3}\cot^2\frac{\theta_1}{2} + \frac{2}{3}\cot^2\frac{\theta_2}{2}
$$

- Embedding of K D7 branes:

$$z_1 = \mu = r^{3/2} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

- We make a change of variables

$$\theta_\pm = \frac{1}{2}(\theta_1 \pm \theta_2)$$

- This allows us to eliminate θ_+ in favor of $\hat{r} = r/\mu^{2/3}$

$$2\mu = r^{3/2}(\cos\theta_- - \cos\theta_+)$$

- Then we have integration over r instead of the angle θ_+ , which is what we want. I.e., the RS model has an integration over the 5th dimension r.
- We still have to deal with the integral over θ_- . The domain of integration depends on r.

$$\theta_- \in [-\frac{\pi}{2} + \sin^{-1}\frac{\mu}{r^{3/2}}, \frac{\pi}{2} - \sin^{-1}\frac{\mu}{r^{3/2}}]$$

- This introduces another dependence on r that must be kept track of.

- Thus reducing to the zeromodes of X3 we have

$$S = -24\pi^2 \mu^{8/3} \tau_7 \int d^4x \int_1^\infty d\hat{r} \Big\{ \frac{1}{2} \mu^{-4/3} \hat{r}^{-9/2} F_1(\hat{r}) \eta^{\mu\nu} (\partial_\mu \chi \partial_\nu \chi + \partial_\mu \eta \partial_\nu \eta)$$

$$+ \frac{1}{2} \hat{r}^{-1/2} \tilde{F}_1(\hat{r}) [(\partial_{\hat{r}} \chi)^2 + (\partial_{\hat{r}} \eta)^2] + \text{t.d.} \Big\}$$

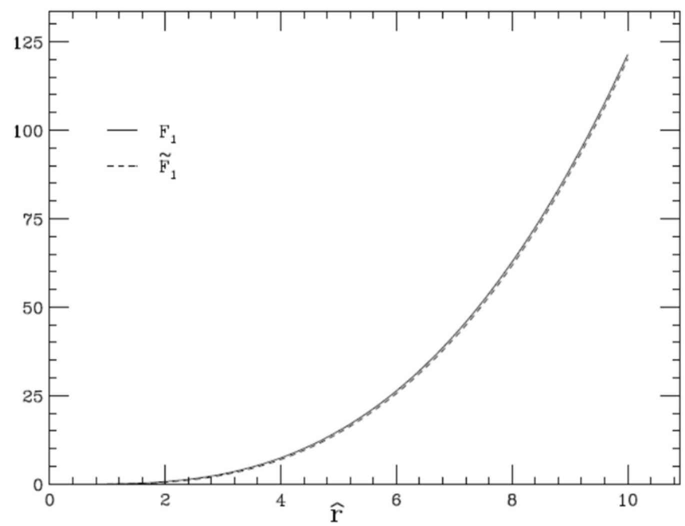
$$F_1(\hat{r}) = \int_{-\theta_0(\hat{r})}^{\theta_0(\hat{r})} d\theta_- \frac{\sqrt{-g}}{\sin \theta_+ C} (\hat{r}, \theta_-)$$

$$\tilde{F}_1(\hat{r}) = \int_{-\theta_0(\hat{r})}^{\theta_0(\hat{r})} d\theta_- \frac{\sqrt{-g}(C-1)}{\sin \theta_+ C^2} (\hat{r}, \theta_-)$$

- Careful study yields an approximate answer:

$$F_1(\hat{r}) \approx \frac{1}{6}\hat{r}^{5/2}\ln\hat{r}$$

- It’s good to about five decimal places.
- Numerically exact results are shown in the accompanying plot (next page):


125
100
75
50
25
0
0
2
4
6
8
10
$\hat{r}$
F_1
$\tilde{F}_1$

- It can be seen that the tilde cousin is quite close in absolute value:

$$\tilde{F}_1(\hat{r}) \approx F_1(\hat{r})$$

- However, the approximation becomes *relatively* poor for $\hat{r} = \mathcal{O}(1)$

- I.e., the relative error $\dfrac{\tilde{F}_1(\hat{r}) - F_1(\hat{r})}{F_1(\hat{r})}$ is not small.

- To obtain a 5d action with the usual normalizations, we have to rescale the scalar fluctuation fields:

$$\chi = \hat{r}^{3/2}(\ln\hat{r})^{-1/2}\chi'$$

$$\eta = \hat{r}^{3/2}(\ln\hat{r})^{-1/2}\eta'$$

- Then we obtain, after some manipulations:

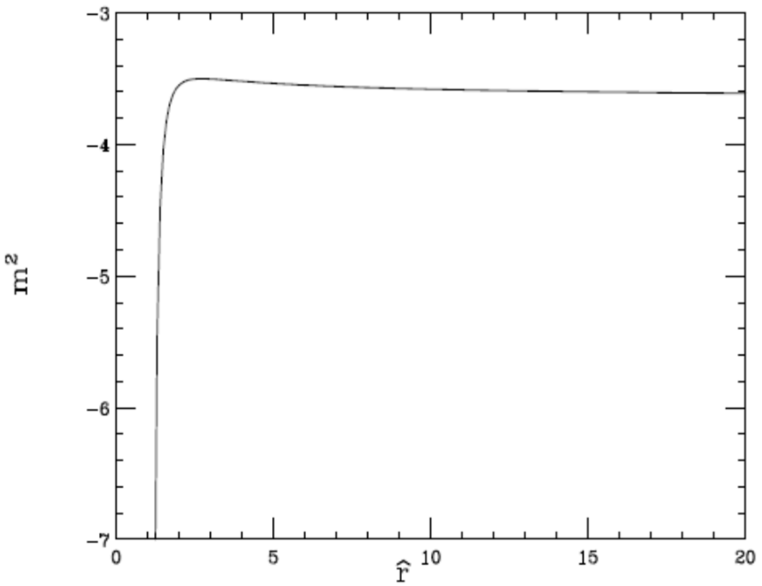
$$S(\chi') \approx -2\pi^2\mu^{8/3}\tau_7 \int d^4x \int_{\hat{R}}^{\infty} d\hat{r}\left\{\frac{\hat{r}}{\mu^{4/3}}\eta^{\mu\nu}\partial_\mu\chi'\partial_\nu\chi'\right.$$
$$\left. f(\hat{r})[\hat{r}^5(\partial_{\hat{r}}\chi')^2 + \hat{r}^3 m^2(\hat{r})\chi'^2]\right\}$$

$$f(\hat{r}) = \frac{\tilde{F}_1(\hat{r})}{F_1(\hat{r})} \approx 1 \quad \text{for} \quad \hat{r} \gg 1$$

- The $\hat{r}$ dependent mass is

$$m^2(\hat{r}) = -\frac{15}{4} + \frac{1}{2\ln\hat{r}} - \frac{1}{4(\ln\hat{r})^2}$$

- It looks like this:



- The large negative mass-squared implies Dirichlet BCs

$$\lim_{\hat{r}\to 1} \chi'(\hat{r}) = \lim_{\hat{r}\to 1} \eta'(\hat{r}) = 0$$

Ignoring logs, we get conformally coupled scalar

- Reintroducing the AdS radius L

$$S(\chi') \approx -2\pi^2 L^{-5}\tau_7 \int d^4x \int_R^\infty dr \left\{ \frac{r}{L}\eta^{\mu\nu}\partial_\mu\chi'\partial_\nu\chi' + \frac{r^5}{L^5}(\partial_r\chi')^2 - \frac{15}{4L^2}\frac{r^3}{L^3}\chi'^2 \right\}$$

Dual gauge theory

Klebanov & Witten 1998

- Superpotential of KW dual theory

$$W = \lambda \epsilon_{ik} \epsilon_{jl} \mathrm{Tr}(A_i B_j A_k B_l)$$

- $SU(N) \times SU(N)$ gauge theory with bifundamentals A_1, A_2, B_1, B_2

- NSVZ beta function: *N=1 SCFT* at IRFP with known anomalous dimensions.

- F-term condition $\partial W/\partial \phi_i = 0$ implies conifold condition if we take

$$z_1 = A_1 B_1, \quad z_2 = A_2 B_2$$
$$z_3 = A_1 B_2, \quad z_4 = A_2 B_1$$

$$\Rightarrow \quad z_1 z_2 - z_3 z_4 = 0$$

- So the geometry of Y_6 is the moduli space. Similar to *N=4* case but more interesting.

Adding fundamental flavors

Ouyang 2003; Levi & Ouyang 2005

- Add to the superpotential

$$\Delta W = h\tilde{q}A_1 Q + gqB_1\tilde{Q} + \mu_1 q\tilde{q} + \mu_2 Q\tilde{Q}$$

- Then demanding a massless mode gives:

$$A_1 B_1 = \frac{\mu_1\mu_2}{gh} \quad \Leftrightarrow \quad z_1 = \mu$$

- Levi & Ouyang go on to determine the dimensions of operators from the “meson” spectra (coming from the D7 brane embedding).
- It has the interpretation:

$$q(AB)\cdots(AB)\tilde{q}$$

n insertions of (AB)

$$\Delta \sim \frac{3}{2}n$$

- Dimensions follow from KW superpotential being marginal (dim=3).

SUSY breaking from 5d geometry

- Supersymmetry breaking is introduced by a Type IIB supergravity motivated deformation [Kuperstein & Sonnenschein, hep-th/0309011] of the Klebanov-Strassler background:

$$ds^2 = A^2(z)(-dt^2 + d\vec{x}^2 + dz^2)$$

$$A^2(z) = \frac{1}{(kz)^2}\left[1 - \epsilon\left(\frac{z}{z_1}\right)^4\right]$$

- According to holographic duality, this is supposed to be dual to soft SUSY breaking gaugino masses in the strongly coupled gauge theory.

- This is the 5d deformation that we extracted from the solution of KuSo, following techniques similar to those in the first part of this talk (KW).
- The KuSo background is 10d, and is a non-SUSY background that solved Type IIB SUGRA EOM following techniques of Borokhov & Gubser (2002).
- In that work, SUSY breaking deformations of KS were worked out.

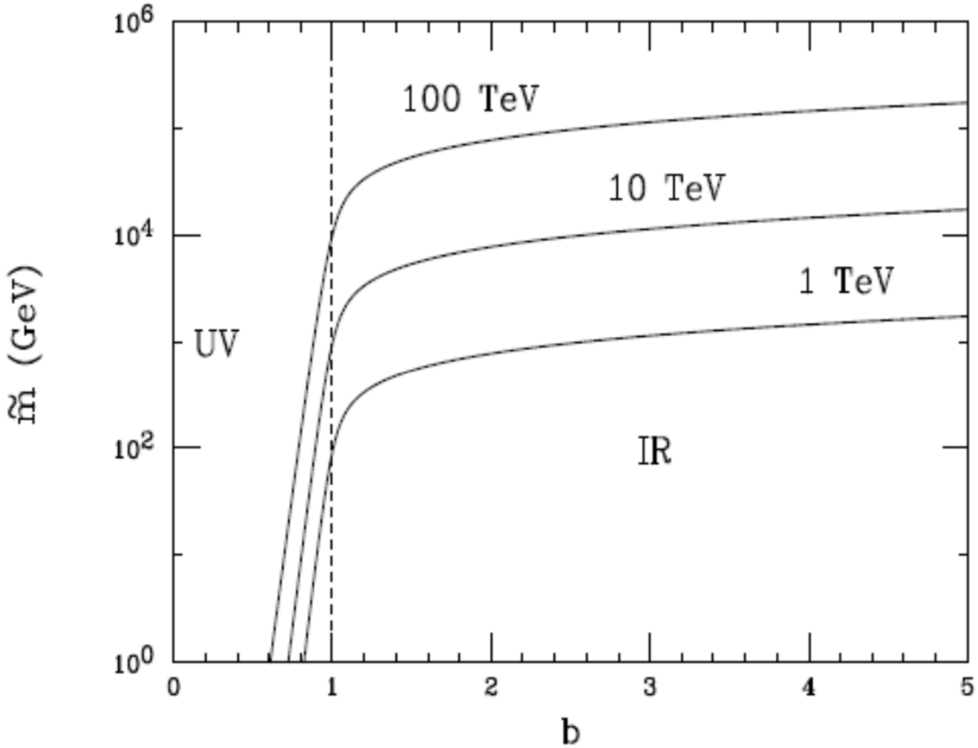
- Because of partial compositeness of fields in the bulk, this is a single-sector SUSY breaking model.

Provides gravity dual of older pheno models:
Arkani-Hamed, Luty & Terning 1997
Luty & Terning 1998
Cohen, Kaplan & Nelson 1996

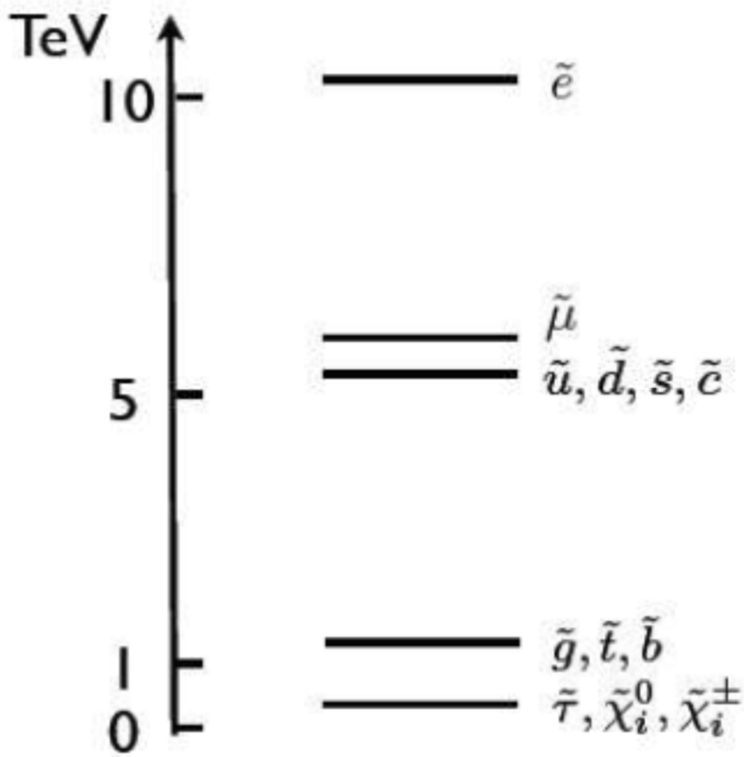
- Soft scalar masses are determined in terms of the deformation parameter and the localization parameter b:

$$\tilde{m}^2 \approx \epsilon k^2 \frac{(1-b)(b+10)}{(kz_1)^4} \frac{(kz_1)^{1+b} - (kz_1)^{1-b}}{(kz_1)^{1-b} - (kz_1)^{b-1}}$$

- Also have predictions for gaugino soft masses, etc.
- [Recall that susy pheno is determined by soft lagrangian.]
- In contrast to usual WED models, Higgs is confined to UV brane. Mass protected by SUSY in UV.

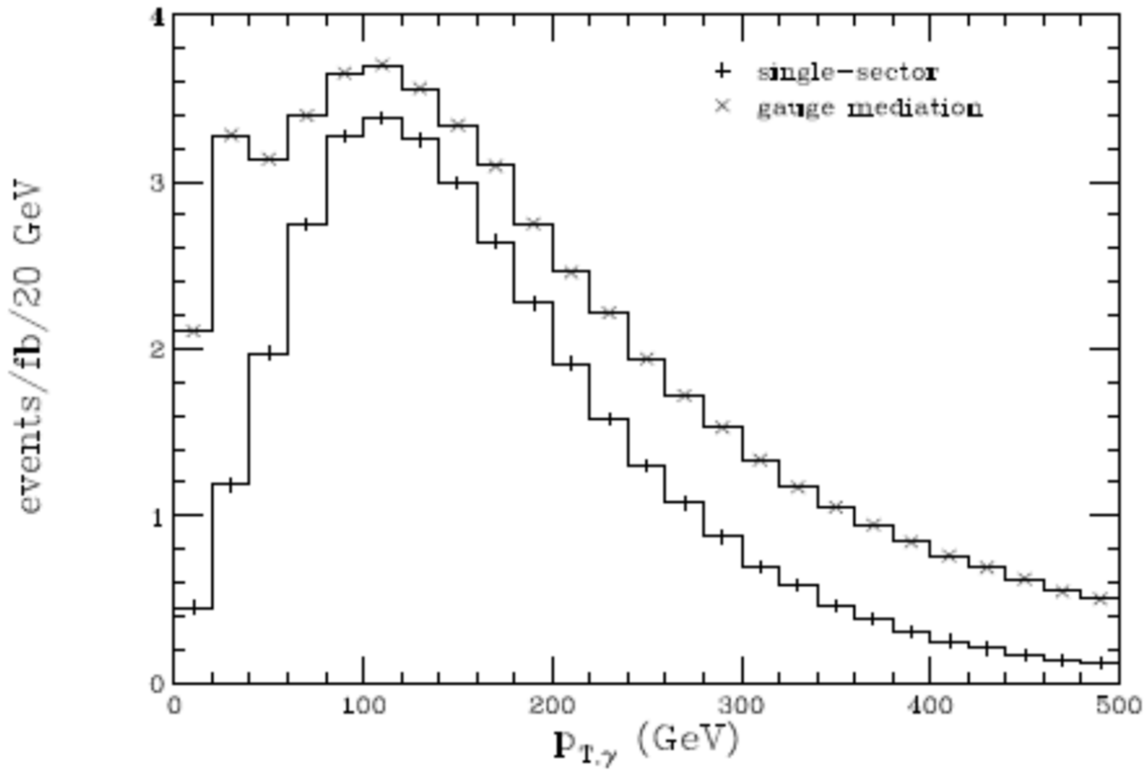


$z_1^{-1} = 1, 10, 100$ TeV


TeV
10
5
1
0
$\tilde{e}$
$\tilde{\mu}$
$\tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}$
$\tilde{g}, \tilde{t}, \tilde{b}$
$\tilde{\tau}, \tilde{\chi}_i^0, \tilde{\chi}_i^\pm$

$\tilde{e}_L, \tilde{e}_R, \tilde{\nu}_{eL}$	10160, 10150, 10160 GeV
$\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\nu}_{\mu L}$	5145, 5130, 5145 GeV
$\tilde{d}_L, \tilde{d}_R, \tilde{u}_L, \tilde{u}_R$	5905, 5885, 5970, 5890 GeV
$\tilde{s}_L, \tilde{s}_R, \tilde{c}_L, \tilde{c}_R$	5905, 5885, 5970, 5890 GeV
$\tilde{g}$	1615 GeV
$\tilde{b}_1, \tilde{b}_2, \tilde{t}_1, \tilde{t}_2$	1354, 1369, 1253, 1369 GeV
$\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\nu}_{\tau L}$	511, 630, 633 GeV
$\tilde{\chi}_1^{\pm}, \tilde{\chi}_2^{\pm}$	478, 593 GeV
$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$	288, 480, 511, 598 GeV
$h^0, A^0, H^0, H^{\pm}$	115, 646, 646, 651 GeV
$\tilde{G}$	2.35 eV

Diphoton signal (ruled out in 1st month of LHC running)



Conclusions

- Warped geometries, a hierarchy of scales, and dynamical SUSY breaking can be accommodated in a string/D-brane construction.
- Randall-Sundrum type set-up, with fields in the bulk (compositeness) can be extracted from probe D7 branes in the Klebanov-Witten and Klebanov-Strassler geometries.
- This provides a calculable approach to theories that are strongly coupled in the gauge theory dual.
- Realistic phenomenology can be derived in this set-up, but with stringy origins and modifications of the usual pheno assumptions.

2 stacks

- Now if we have 2 stacks, we have 2 embeddings:

$$r = \begin{pmatrix} r_1(\theta_1, \theta_2) & 0 \\ 0 & r_2(\theta_1, \theta_2) \end{pmatrix}$$

- This is a block diagonal form: N(1) D7 branes at r(1); N(2) D7 branes at r(2). But then we have to allow for fluctuations that are completely general, an [N(1)+N(2)] x [N(1) + N(2)] matrix:

$$r = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} + \begin{pmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{pmatrix}$$

- Here:

$$\chi_{11} \quad \text{is} \quad N_1 \times N_1, \quad \chi_{12} \quad \text{is} \quad N_1 \times N_2, \quad \ldots$$

- The non-abelian extension of DBI can be found in Johnson's lectures, eq. (209):

$$\begin{aligned} S_p &= -T_p \int d^{p+1}\xi\, e^{-\Phi}\mathcal{L}\,, \quad \text{where} \\ \mathcal{L} &= \mathrm{STr}\left\{\det{}^{1/2}[E_{ab} + E_{ai}(Q^{-1}-\delta)^{ij}E_{jb} + 2\pi\alpha' F_{ab}]\det{}^{1/2}[Q^i{}_j]\right\} \end{aligned} \tag{209}$$

where $Q^i{}_j = \delta^i{}_j + i2\pi\alpha'[\Phi^i, \Phi^k]E_{kj}$, and we have raised indices with E^{ij}.

$$E_{MN} = g_{MN} + B_{MN}$$

$\Phi_i \sim$ **embedding fluctuation orthogonal to brane, like χ and η**