Calculating the Rotation Curve due to Matter in the Milky Way Galaxy

Dark Matter Summer School at Albany

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Outline

- Discuss components of the observed mass distribution of the visible matter in the Milky Way Galaxy.
- Use the Shell Theorem and 2-D analogy to calculate the gravitational force due to the visible matter.
- Calculate the rotational velocity of the visible matter in the Milky Way Galaxy.
- Infer the the amount of dark matter in the Milky Way Galaxy.
- Compare the total mass of visible matter and dark matter in the Milky Way Galaxy.

Visible Matter Distribution in the Milky Way Galaxy

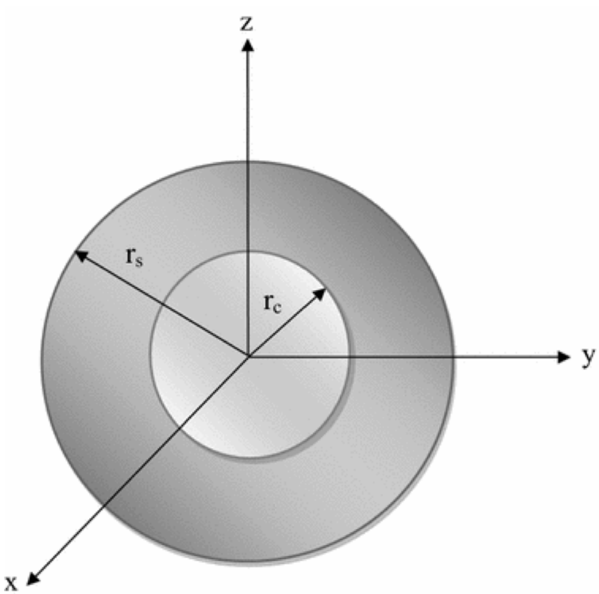
Mass Component	Total Mass ($M_\odot$)	Scale Radius(kpc)	Center Density ($M_\odot \cdot pc^3$)	Distribution
Black Hole	4.0E+06	——	——	Point
Inner Bulge	5.0E+07	0.0038	3.6E+04	Sphere
Main Bulge	8.4E+09	0.12	1.9E+02	Sphere
Galactic Disk	4.4E+10	3.0	15	Disk

"Rotation Curve and Mass Distribution in the Galactic Center—From Black Hole to Entire Galaxy—", Yoshiaki Sofue, 2013

1 $M_\odot$ = 2×10^{30} kg   1 pc = 3.0857×10^{16}m

Shell Theorem

- For 3-dimensional spherically symmetric distributions, the Shell Theorem describes the gravitational field.
- Is there a 2-dimensional analogy for a disk?



Black Hole, Inner Bulge, Main Bulge

- According to the Shell Theorem, only the enclosed mass must be taken into account for spherically symmetric mass distributions.
- The mass density of the bulges shows an exponential decay.

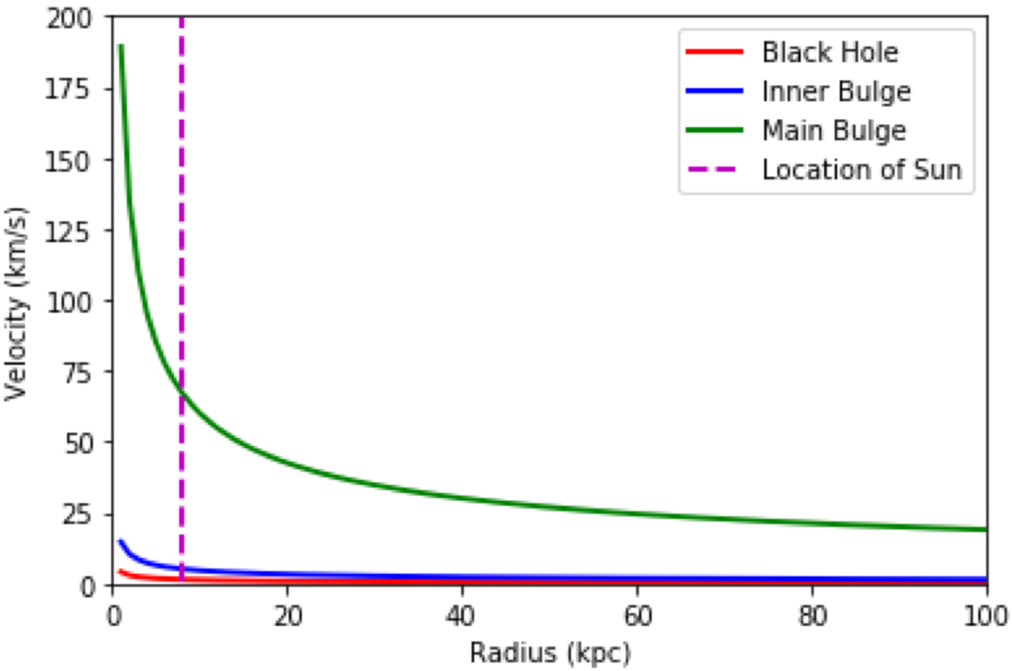
$$M(r)_{enclosed} = \int \rho_o \cdot e^{-\frac{r}{r_{scale}}} \cdot dV$$

$$v(r) = \sqrt{\frac{GM}{r}}$$

- Where r is the distance from the center of the objects and r_{scale} is the scale radius of the objects

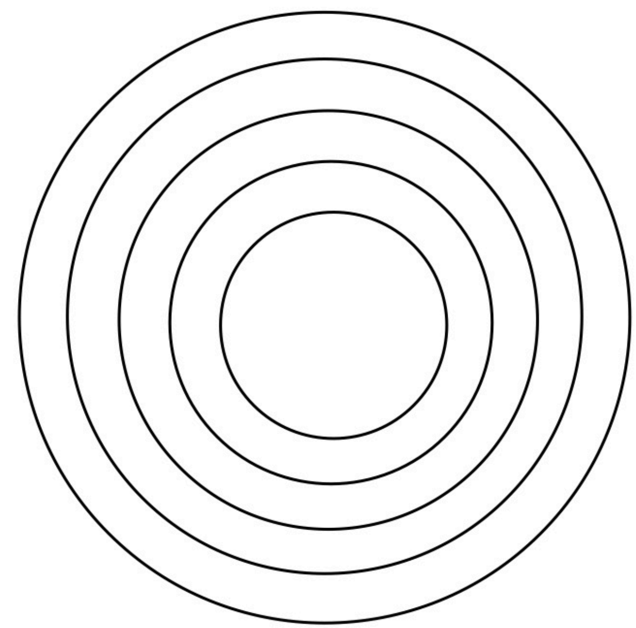
Rotational Velocity due to the Black Hole and the Bulges

- Summary of velocity curves using the 3-D Shell Theorem.
- The rotational velocity is dominated by the Main Bulge.
- The rotational curve of the sun is around 220 km/s.
- These three parts contributes less to the rotational curve of the sun. So, we need to calculate the rotational velocity due to the Galactic Disk.



Ring Method to Calculate the Gravitational Force from a Disk

- Ring method provides a way to calculate the gravitational force from a **Non-Uniform** disk.
- Break the disk into numerous rings
- Calculate the gravitational force from each ring
- Sum the gravitational force from each ring up
- Verified by applying to uniform disk.



Theorems to Calculate Gravitational Force from a Disk

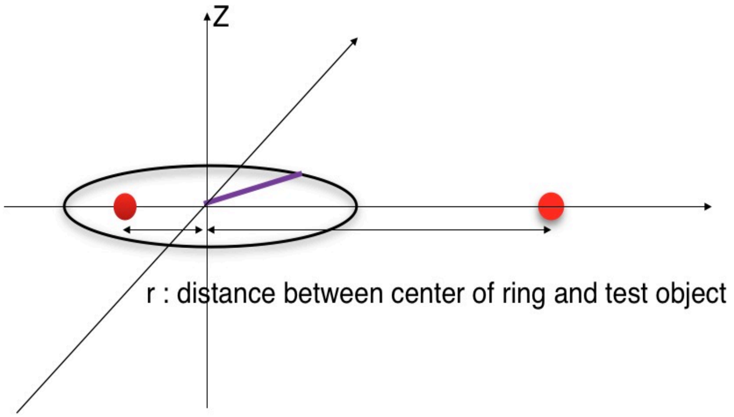
- Uniform Disk model

(*"THE GRAVITATIONAL FIELD OF A DISK", Fred T. Krogh, Edward W. NG, and William V. Snyder*)

- Lass' Ring model

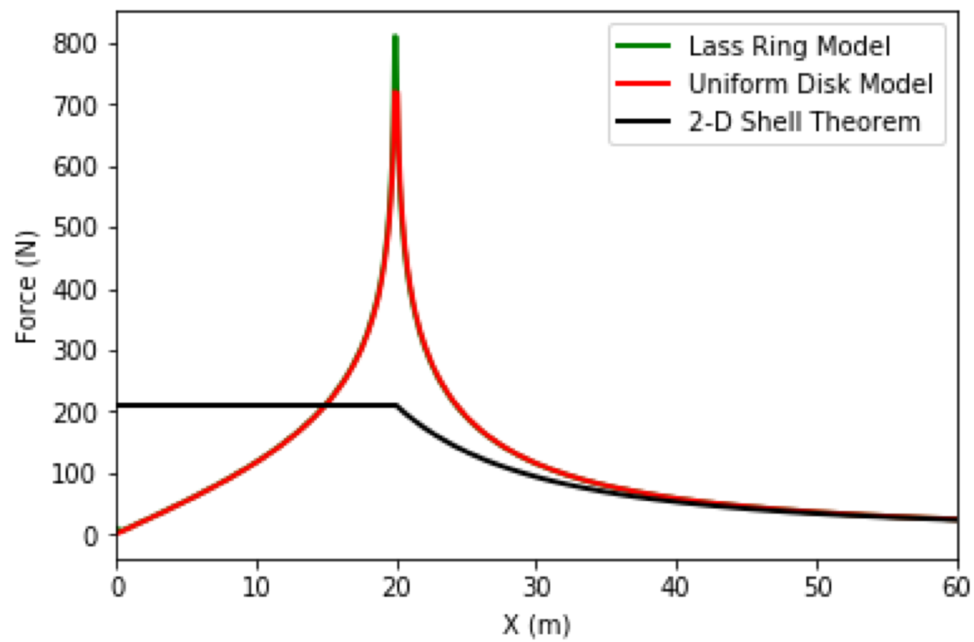
(*"THE GRAVITATIONAL POTENTIAL DUE TO UNIFORM DISKS AND RINGS"* , HARRY LASS)

- Direct Derivation Ring model



Comparison of Different Models

- The radius of the uniform disk is 20m.
- The mass of the test object is 1.0E12kg.
- All the three models give consistent results, so the ring method works.
- According to the figure, the 2-D Shell Theorem does not accurately model the gravitational force from a disk.
- Around 2~3 radii, there is no difference.

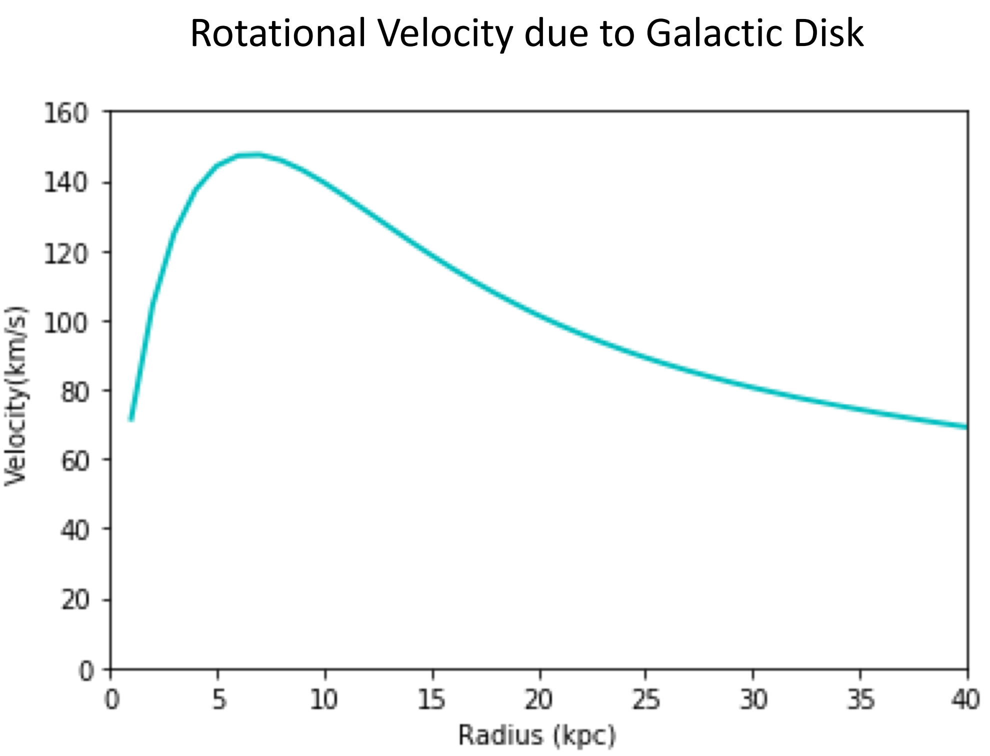


Galactic Disk

- The mass density of the galactic disk is not uniform.
- In using the ring method, each ring has a different mass density.

$$Density = \rho_o \cdot e^{-\frac{r}{r_{scale}}}$$

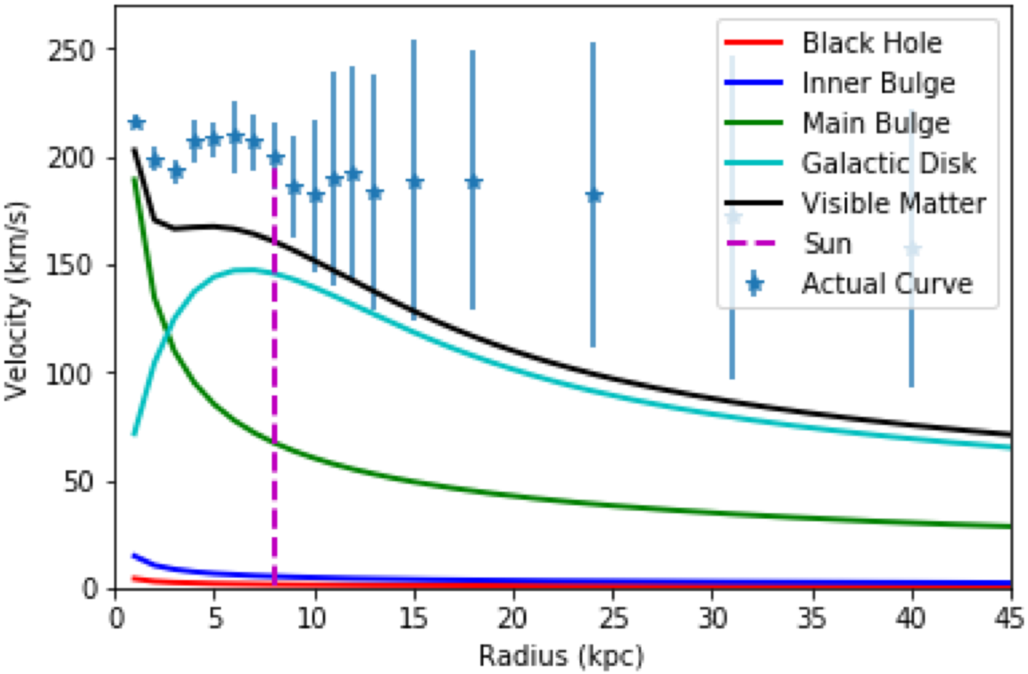
"Rotation Curve and Mass Distribution in the Galactic Center—From Black Hole to Entire Galaxy—", Yoshiaki Sofue, 2013



The Rotational Velocity due to Visible Matter

- The rotational velocity due to all the visible matter do not match the actual rotational velocity.

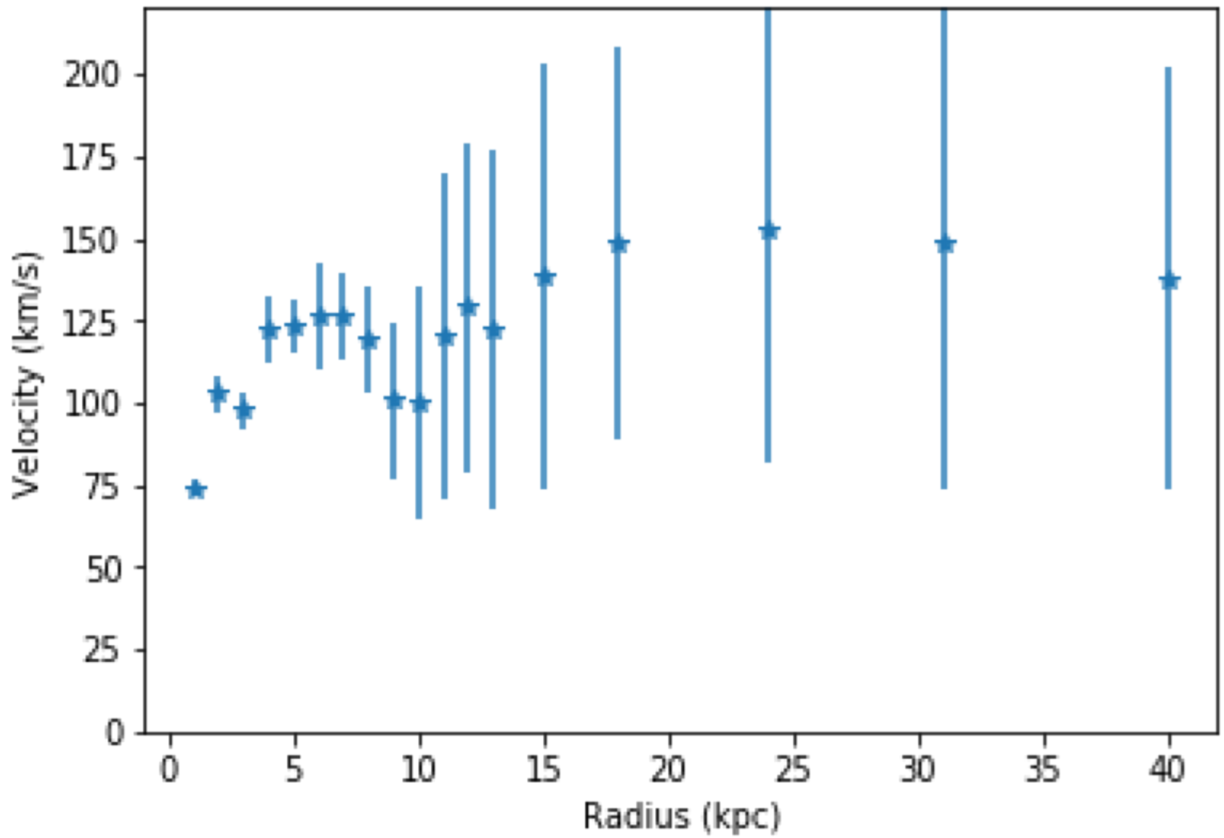
- There must be some other mass contributes to the total rotational velocity.



"Rotation Curve of the Milky Way out to ~200 kpc", Bhattacharjee, Pijushpani and Chaudhury, Soumini and Kundu, Susmita, 2014

The Rotational Velocity due to the Dark Matter

- $V_{DM} = \sqrt{{V_{total}}^2 - {V_{BH}}^2 - {V_{Bulges}}^2 - {V_{disk}}^2}$

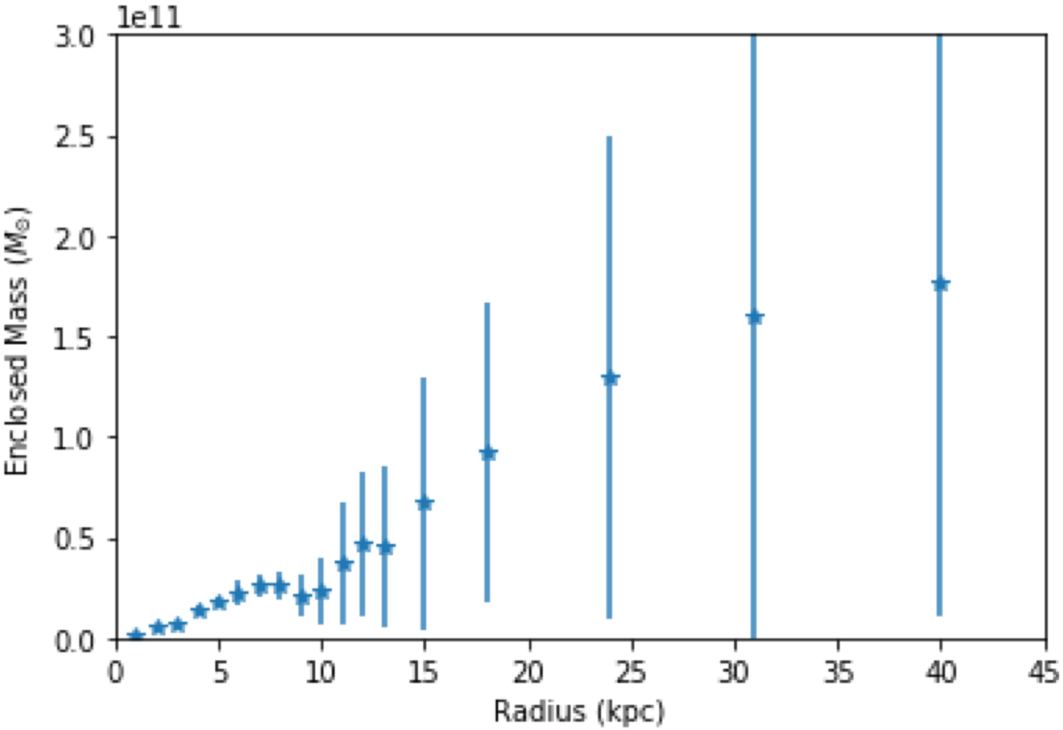


Mass distribution of the Dark Matter

- We assume the dark matter is a sphere and the Shell Theorem can be used.
- The mass distribution of the dark matter is calculated from its rotational velocity.

$$M_{enclosed} = \frac{{v_{rot}}^2 \cdot r}{G}$$

- Within 40 kpc, the enclosed mass of the dark matter is around 1.8E+11 $M_\odot$.



Summary

- The 2-D Shell Theorem can not be used to calculate the gravitational force due to the disk.
- Calculating the force due to the 2-dimensional galactic disk is more complicated than calculating the force due to the 3-dimensional bulges.
- The rotational velocity due to the galactic disk dominates at distances beyond 3 kpc.
- The calculated velocity distributions can be used to extract the velocity distribution due to the dark matter.
- Within 40 kpc, we estimate that the total mass due to the dark matter is 1.8E+11 $M_{\odot}$ and the mass due to the black hole, the bulges, and the disk is 0.54E+11 $M_{\odot}$. Within 40 kpc, the dark matter accounts for 75% of the total matter.

Equations for Different Models

- Uniform Disk model

(“*THE GRAVITATIONAL FIELD OF A DISK*”, *Fred T. Krogh, Edward W. NG, and William V. Snyder*)

  - $k^2 = \frac{4r\rho}{R^2+\rho^2+2r\rho}$ a=$\left(\frac{-4Ge\rho^{\frac{3}{2}}}{kr^{\frac{3}{2}}}\right)\{(1-\frac{1}{2}k^2)K(k)-E(k)\}$ $F = ma$

- Lass’ Ring model

(“*THE GRAVITATIONAL POTENTIAL DUE TO UNIFORM DISKS AND RINGS*” , HARRY LASS)

  - $k^2 = \frac{4r_{scale}\cdot \mathrm{r}}{z^2+(r_{scale}+r)^2}$
  - U(r,0)=-2mGρ$[(r_{scale} + r)E(k) + (r_{scale} - r)K(k)]$
  - $F = -\nabla U$

- Direct Derivation Ring model
  - $U = \frac{Gm\cdot dM}{dR} = \frac{GmM_{ring}}{\pi}\int_0^{\pi}(r^2 + r_{\mathrm{scale}}{}^2 - 2r \cdot r_{scale}\cos(\theta))^{-\frac{1}{2}} \cdot d\theta$
  - $F = -\nabla U$

Equation to Calculate Error Bar in Mass Distribution of the Dark Matter

- $m_{enclosed} = \frac{\mathrm{v}_{rot}{}^{2} \cdot r}{G}$
- $dm^2 = (\frac{2v_{rot} \cdot r}{G})^2 \cdot dv^2$

$$= 4 \cdot \left(\frac{\frac{v^2 \cdot r}{G}}{v}\right)^2 \cdot dv^2$$

$$= 4 \cdot \frac{m_{en}{}^2}{v_{rot}{}^2} \cdot dv^2$$

$$(\frac{dm}{m})^2 = 4 \cdot (\frac{dv}{v})^2$$

- $\frac{dm}{m} = 2 \cdot \frac{dv}{v}$