Muons are identified by combining information from the Inner Detector (ID) and Muon Spectrometer (MS) detectors. About 96% of muons are reconstructed by fitting hits from ID and MS tracks. In order to precisely measure the momentum of muons, two types of corrections are applied:

- Scale correction and smearing in simulation, based on template fitting on the invariant mass distribution of $\mu^-\mu^+$ pairs at the $Z$ and $\psi$ poles.
- Correction of local charge-dependent momentum scale biases in data, only corrects $p_T$ vs $p_T^*$, leaving the di-muon scale invariant.

**Charge-dependent Corrections**

Certain systematic misalignment modes can cause sagitta changes, for example, the curl and the twist changes, biasing the measured $p_T$ of positive and negative muons in opposite directions. This effect is dominant in data of prompt reconstruction and that after inclusion of the effect in the alignment procedure the bias is greatly reduced. Sagitta biases cause a mass shift for charge asymmetric final states.

The sagitta bias correction is applied in data:

$$ p_T \rightarrow \frac{p_T}{1 + \sqrt{q^2 p_T \delta_s}} $$

These misalignment modes leave the track chi-square unchanged.

**Scale Correction and Smearing**

$$ p_T \rightarrow \frac{\Delta p_T}{\sqrt{1 + (\Delta_1 + \Delta_2) \cdot p_T}} $$

$$ G(\Delta_2, \Delta_1, \Delta_3) = p_T \cdot (1 + \Delta_0) $$

- $\Delta_0$: Energy loss in calorimeter and other materials, MS only
- $\Delta_1$: Magnetic field integral and radial distortions of the detector
- $\Delta_2$: Energy loss fluctuations in the material, MS only
- $\Delta_3$: Multiple scattering, local radial distortions and local distortions of magnetic field

**Scale and Resolution**

The plots show the momentum scale and resolution of $\mu^-\mu^+$ pairs as functions of pseudo-rapidity of the leading muon obtained from reconstructed $Z \rightarrow \mu^-\mu^+$ candidates after all correction applied. Systematic uncertainties (filled area), from energy loss modelling and imperfect modelling of second order effects in the momentum parametrization, are derived from $\pm 1\sigma$ variations of the parameters used to derive the correction. The smallest uncertainty on scale is about 0.5% in central and up to 2% in forward region, on resolution is 5% in central and up to 20% in forward region.

**Invariant Mass Distribution**

Left two plots show the comparison of momentum scale and resolution between data and MC from 2016 33.3 fb$^{-1}$ full year data, and right two from 2017 15.4 fb$^{-1}$ data, after all corrections.

Invariant mass distribution of $\mu^-\mu^+$ pairs at the Z pole from 2017 data and simulation sample before and after correction.