



Tune & Chromaticity measurements at GSI

What did we learn?

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**Workshop on ‘Extracting information from Electro-Magnetic Monitors
in Hadron Accelerators’**

Geneva

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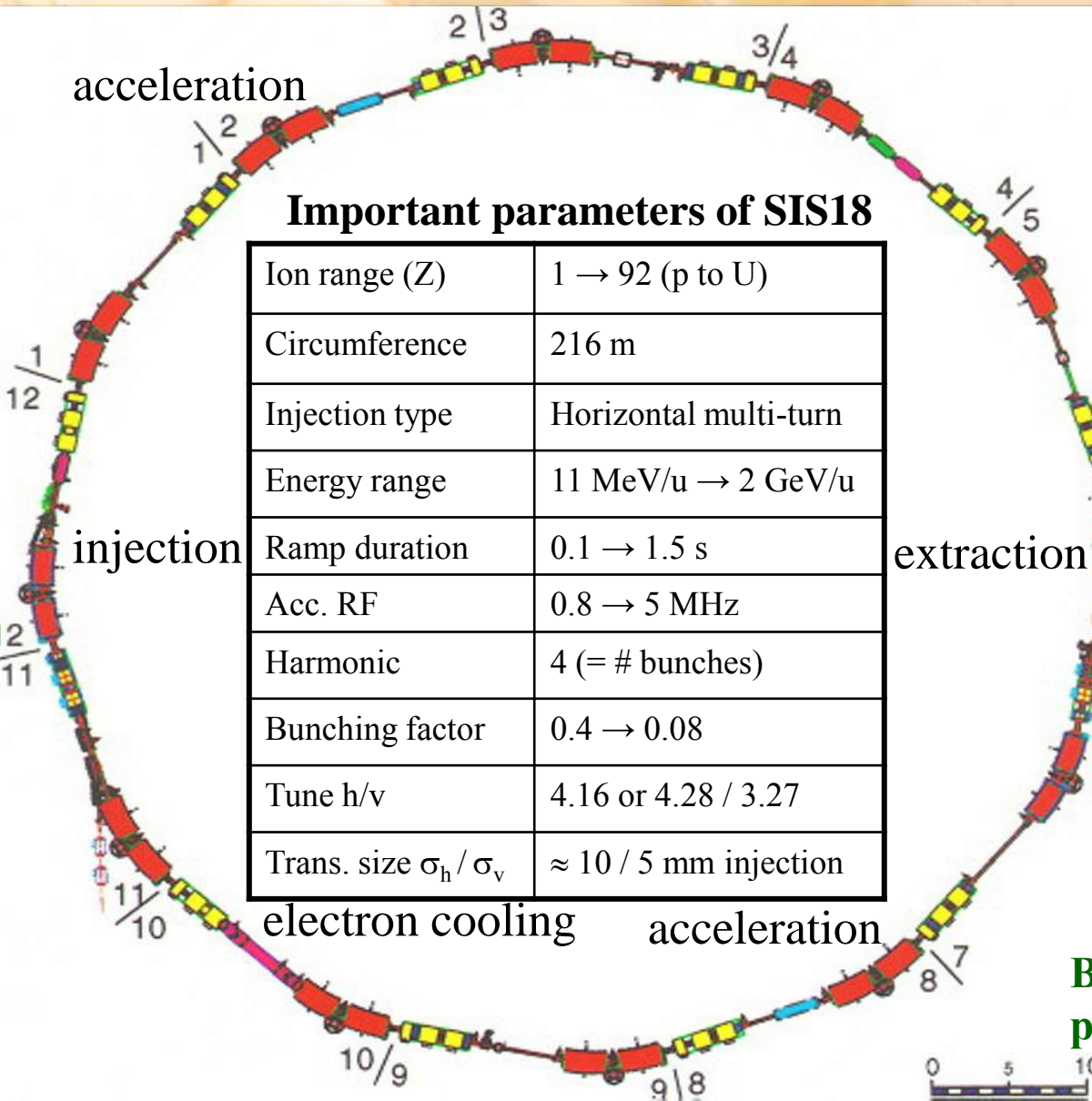


Outline



- Hardware details: BPM system(s) and tune measurement }
- Low intensity tune spectra : Bunched beam transverse Schottky }
- Head-tail modes : Head-tail mode excitation }
- High intensity effects: Direct space charge and Impedances }
- Measurements and interpretation: Tune spectra for high intensity beams }
- Chromaticity measurement: Head-tail phase shift }

GSI Heavy Ion Synchrotron SIS18 ($B\rho=18Tm$): Overview



Important parameters of SIS18

Ion range (Z)	1 \rightarrow 92 (p to U)
Circumference	216 m
Injection type	Horizontal multi-turn
Energy range	11 MeV/u \rightarrow 2 GeV/u
Ramp duration	0.1 \rightarrow 1.5 s
Acc. RF	0.8 \rightarrow 5 MHz
Harmonic	4 (= # bunches)
Bunching factor	0.4 \rightarrow 0.08
Tune h/v	4.16 or 4.28 / 3.27
Trans. size σ_h / σ_v	\approx 10 / 5 mm injection

Dipole, quadrupole, rf cavity $0.8 < f < 5$ MHz



Upcoming FAIR:

- SIS18 used as booster
- high intensities up to 'space charge limit'
- precise control of beam parameter for emittance conservation & low losses

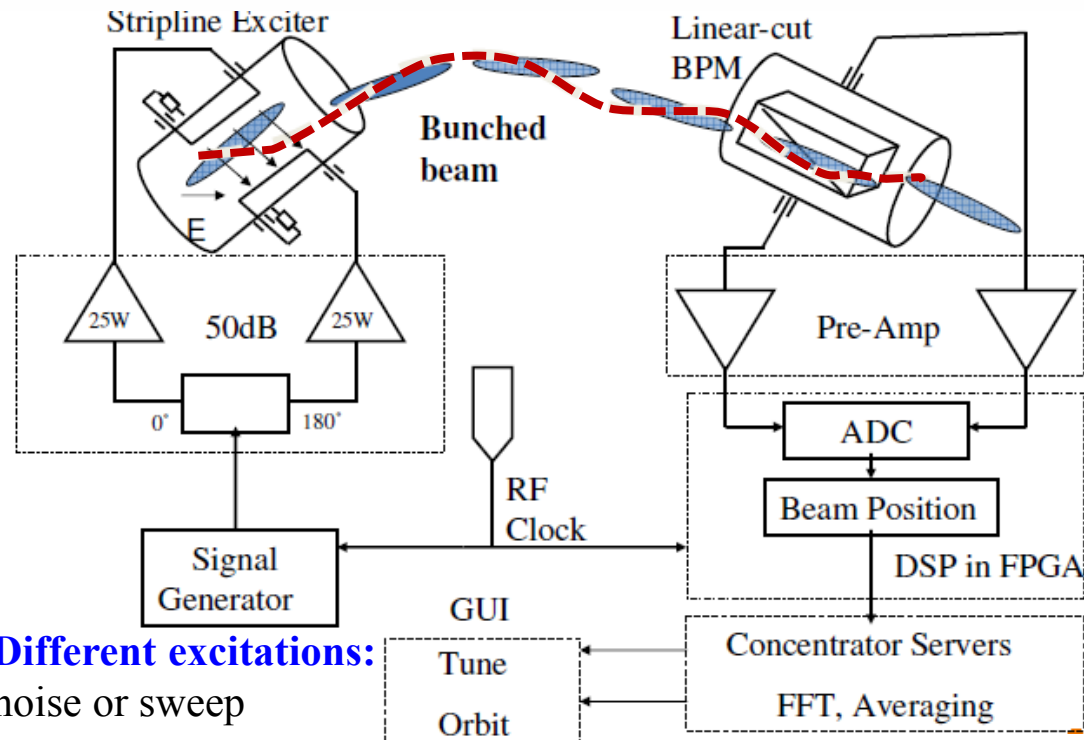
BPMs: 12 regular + 5 for special purpose stripline exciter

Tune , Orbit & PPosition System TOPOS → Oversampling



General functionality:

- The beam is excited by band-limited noise or sweep
- Broadband amplification & oversampling of bunches
- Position value for each bunch
- Fourier transformation gives the non-integer tune Q^f
- Mainly spectrum in baseband i.e. $Q^f < 1/2$



Different excitations:

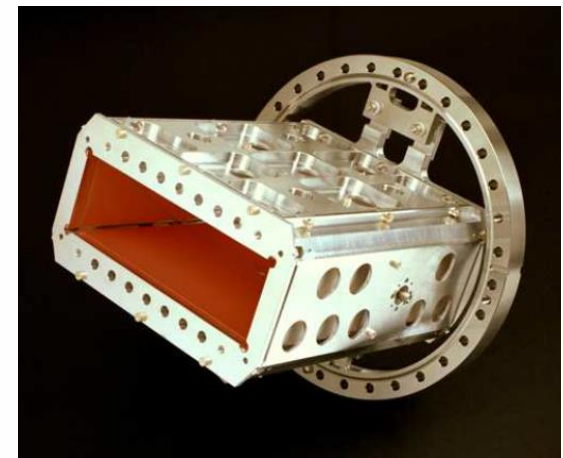
noise or sweep

Linear cut BPM:

Size: 200 x 70 mm², length 260 mm

Position sensitivity:

$$S_x = 0.44 \text{ \%/mm}, S_y = 1.6 \text{ \%/mm}$$



Digital Electronics

(LIBERA from I-Tech):

- ADC with 125 MSa/s
- ~9 effective bits
- FPGA: position evaluation etc.

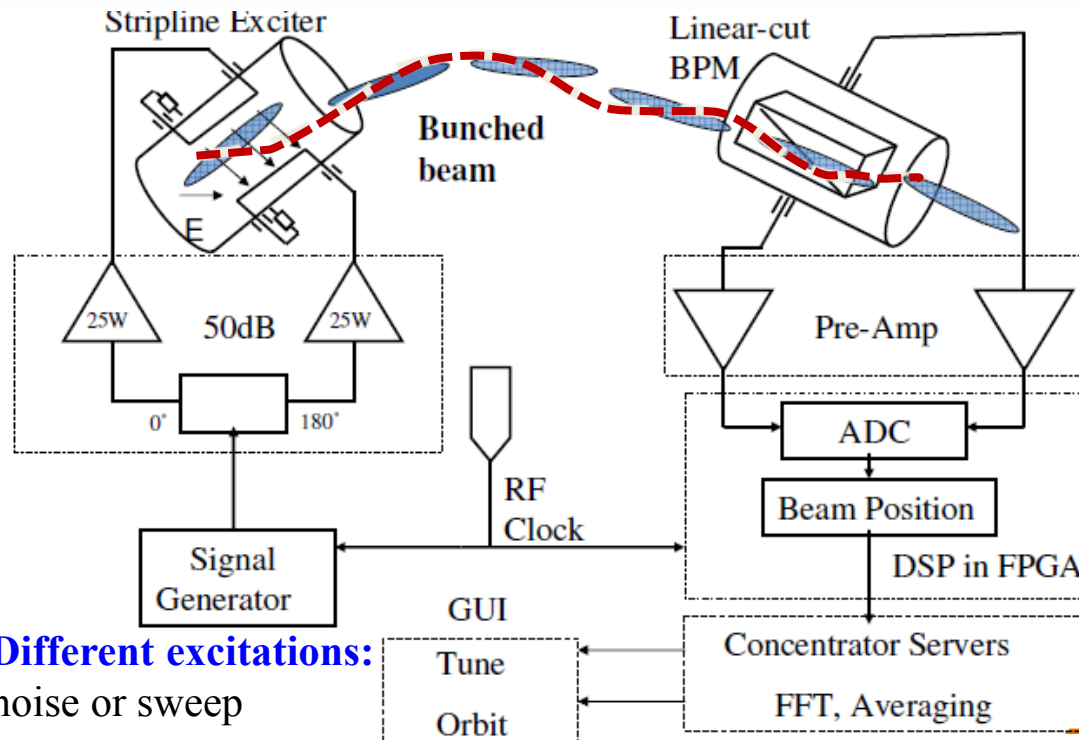
Remark: For FAIR-SIS100 12 eff. bits ADC

Tune , Orbit & PPosition System TOPOS → Oversampling

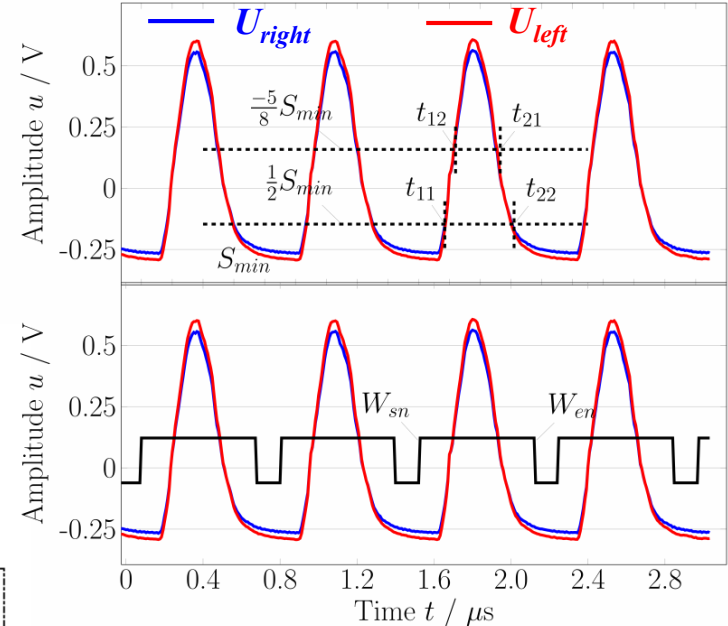


General functionality:

- The beam is excited by band-limited noise or sweep
- Broadband amplification & oversampling of bunches
- Position value for each bunch
- Fourier transformation gives the non-integer tune Q^f
- Mainly spectrum in baseband i.e. $Q^f < 1/2$



Example: One turn = 4 bunches @ 35 MeV/u



Steps for digital processing:

- Baseline restoration
- Integration of bunches
⇒ position for each bunch
- **Tune**: FFT on position of **same** bunch turn-by-turn i.e. 1 of 4 per turn
- From **raw data**: bunching factor, ω_{synch} & head-tail bunch shape

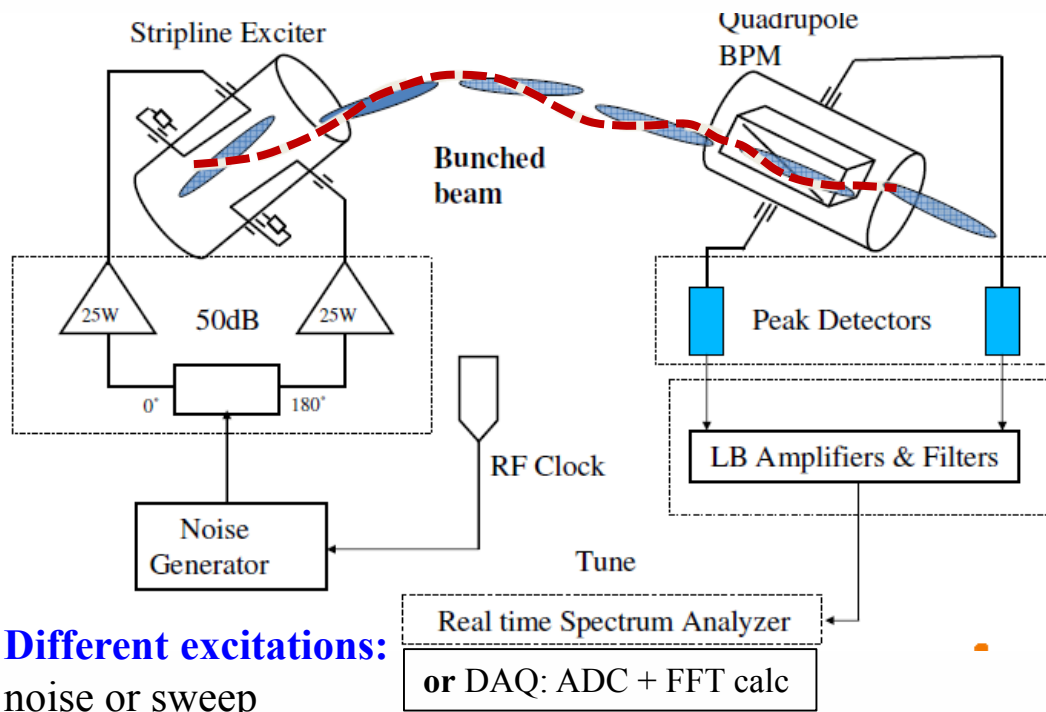
Base-band Tune system BBQ → analog Peak Detection



The beam is excited to betatron oscillation by band-limited noise or chirp:

- The beam position is determined by analog manner via peak detector measured
- Filtering of base-band component deliver the non-integer tune Q^f

System designed by M. Gasior (CERN)



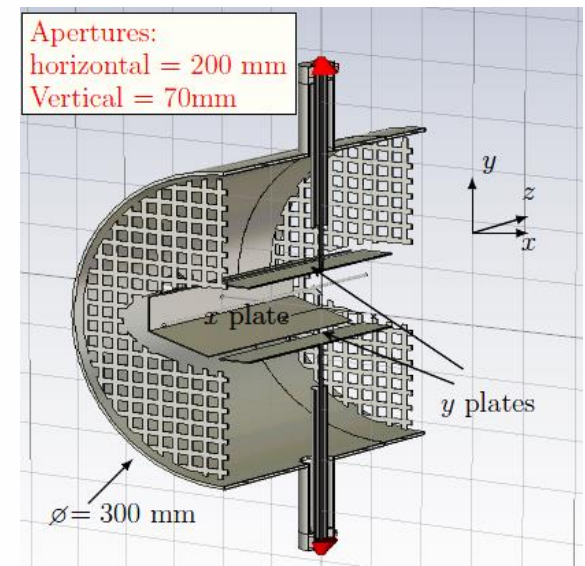
Different excitations:
noise or sweep

‘Quadrupolar’ BPM:

Size: 200 x 70 mm², length 210 mm

Position sensitivity:

$$S_x = 1.4 \text{ \%/mm}, S_y = 2.1 \text{ \%/mm}$$



BBQ design: M. Gasior BIW'12; GSI measurements; R. Singh (GSI) et al., Proc. HB'12 and DIPAC'13

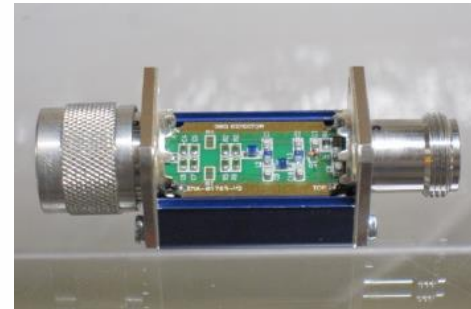
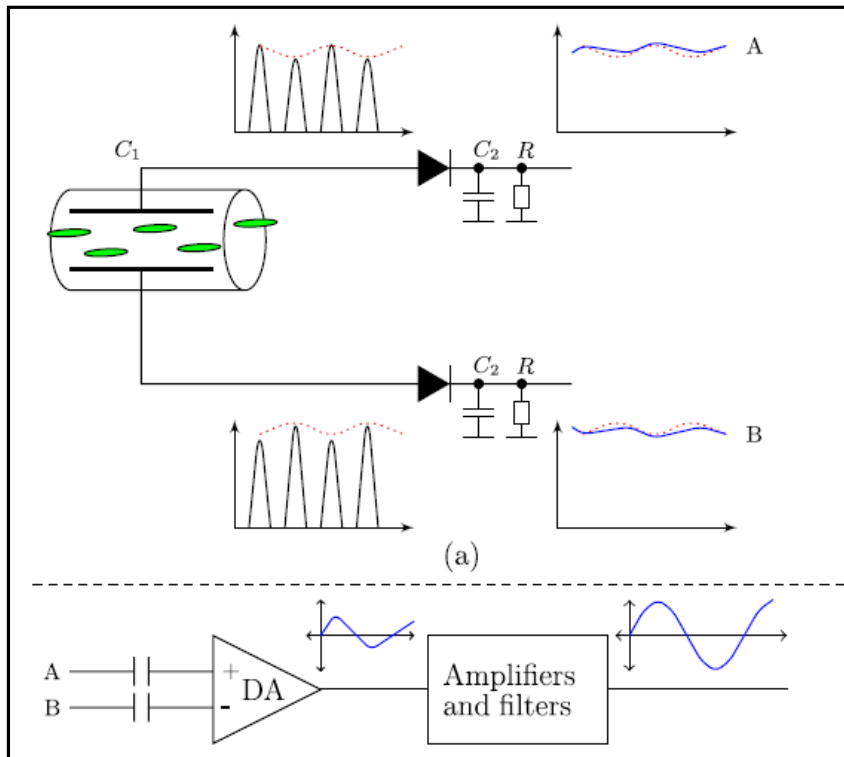
Base-band Tune system BBQ → analog Peak Detection



The beam is excited to betatron oscillation by band-limited noise or chirp:

- The beam position is followed using peak detectors i.e. time constants as parameters
- Filtering of base-band component deliver the non-integer tune Q^f

System designed by M. Gasior (CERN)



Steps of analog processing:

- Peak detection
- Amplification of the difference
- Filtering
- Feeding to spectrum analyzer or DAQ
- ⇒ weighted folding of spectrum to baseband

BBQ design: M. Gasior BIW'12; GSI measurements; R. Singh (GSI) et al., Proc. HB'12 and DIPAC'13

Comparison BBQ versus TOPOS



BBQ:

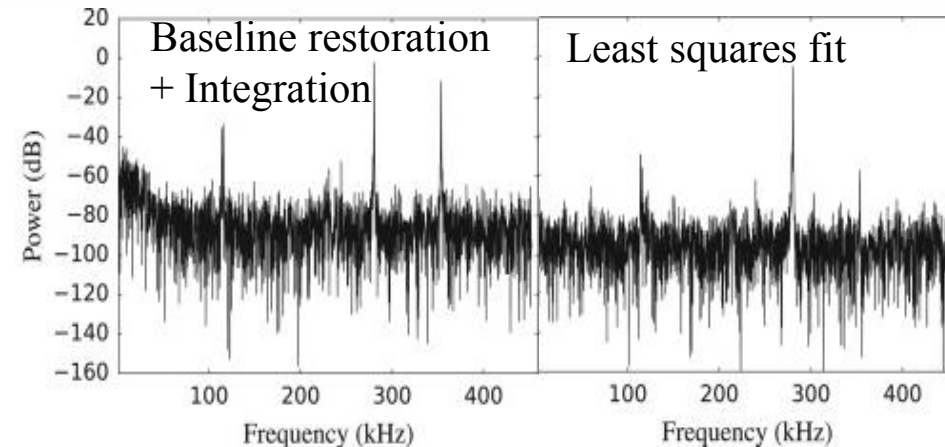
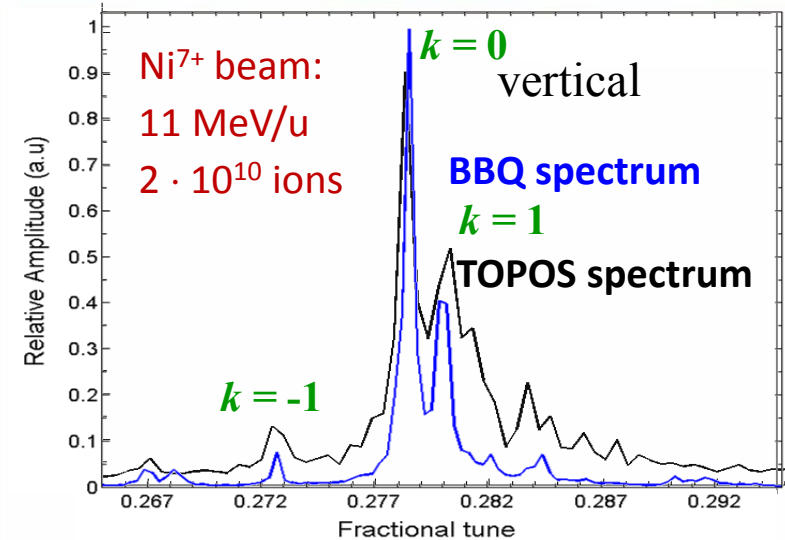
- Peak detection using analog circuit
i.e. no further treatment possible
 - High dynamic range
 - Result: tune with higher sensitivity
- ⇒ 'Easy-to-use' device

TOPOS:

- Oversampled digitization of the BPM signals
 - Full time domain information
 - Versatile data processing possible
e.g. picking 1 of 4 bunches, filtering ...
 - Results: Position, tune, longitudinal profile
synchrotron frequency ω_s
- ⇒ versatile due to full information stored

Remark: Improved position algorithm based on least squares fit brings the sensitivity close to BBQ at SIS-18

A. Reiter and R. Singh, NIM-A, 2018 [10.1016/j.nima.2018.02.046](https://doi.org/10.1016/j.nima.2018.02.046)



5 x higher signal-to-background + higher resolution
+ better common mode suppression

Tune Determination at SIS18: Online Display in Control Room

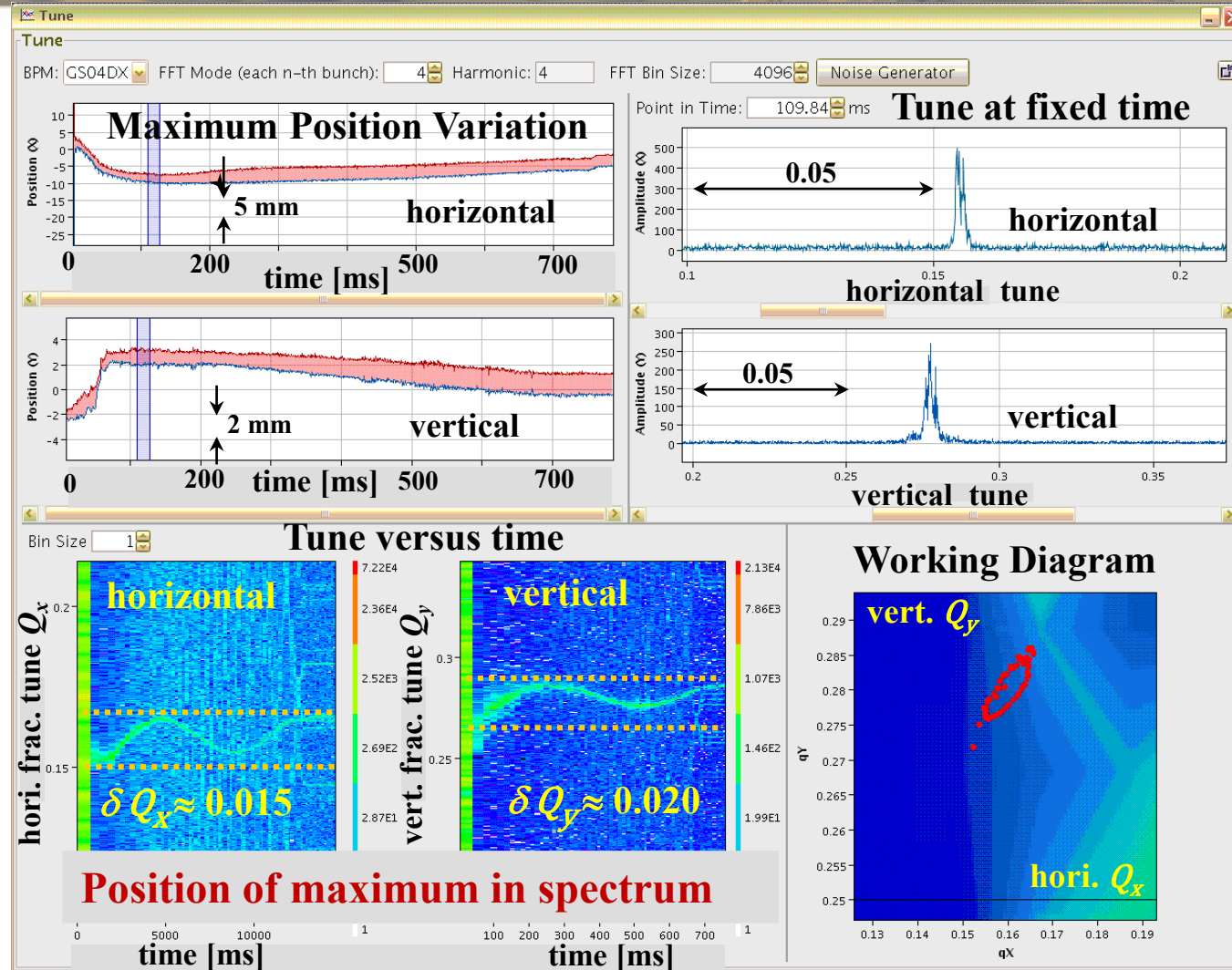


Online display for tune measurement during acceleration

Excitation with band-limited noise
 Time resolution: 4096 turns $\approx 20 - 4.5$ ms
 Variation during ramp: tripllett to duplett focusing

- Result:**
- Online display for user
 - Sufficient signal strength with moderate excitation
 - Minor emittance growth

Beam parameter:
 10^{10} Ar¹⁸⁺, 11 \rightarrow 300 MeV/u within 0.7 s
 \rightarrow tune variation by imperfect focusing ramp



P. Kowina et al., Proc. BIW'10; R. Haseitl et al., Proc. DIPAC'11, G. Jansa et al., Proc. ICALEPCS'09



Outline



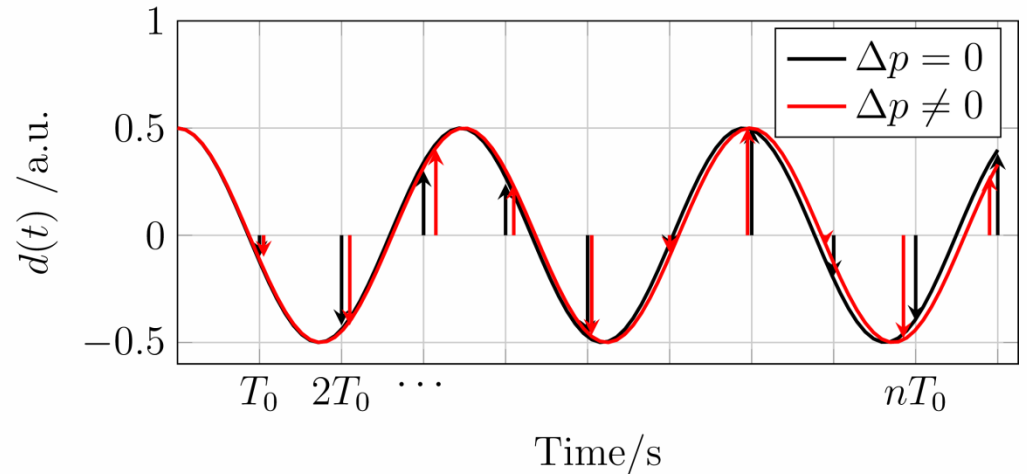
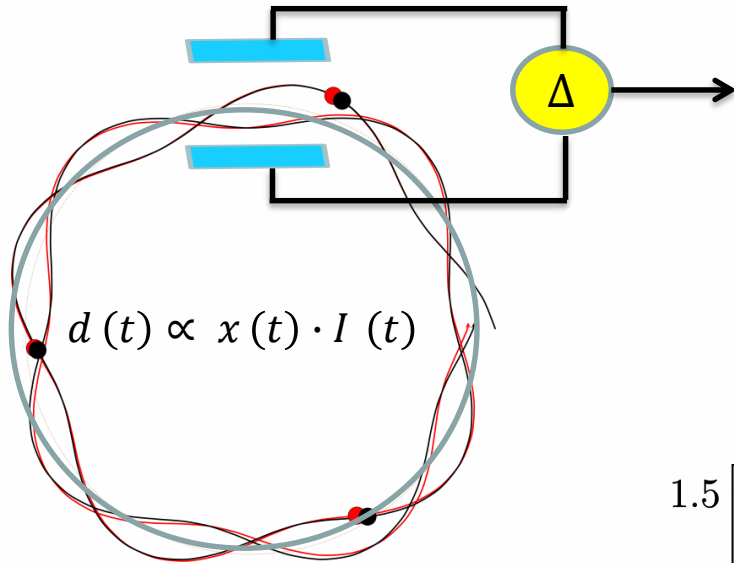
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Basic of Tune Spectrum: Non-interacting Particle Model



Dipole moment $\mathbf{d}(t)$ comprises betatron = amplitude $x(t)$ & synchrotron = phase $I(t)$ modulation



➤ Frequency of harmonics m & mode k

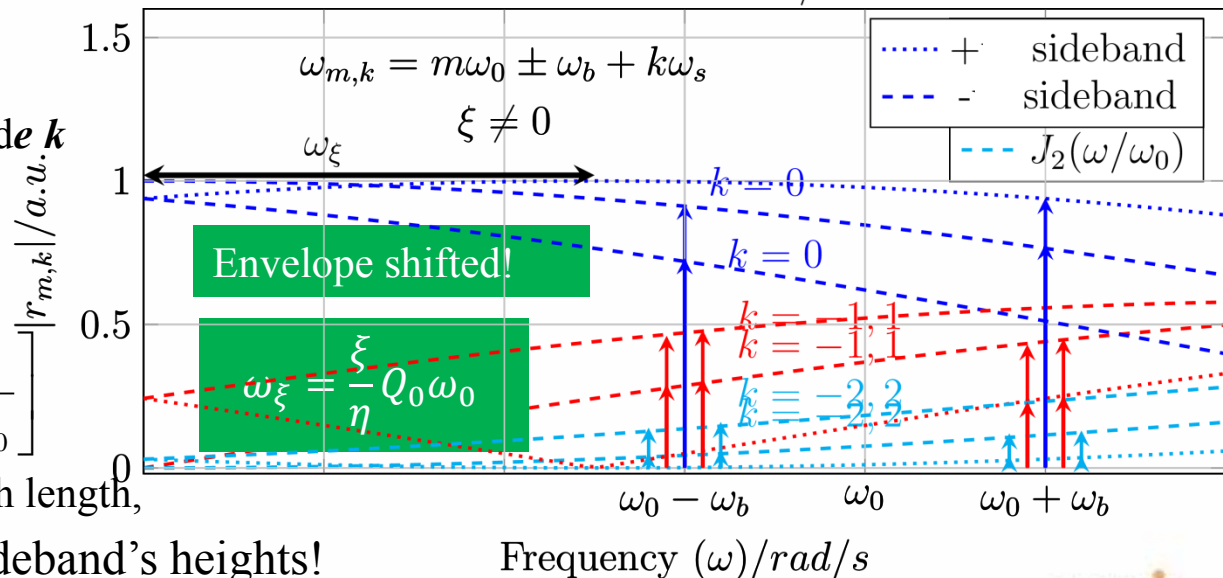
$$\omega_{m,k}^{\pm} = m\omega_0 \pm \omega_b \pm \omega_k$$

➤ Sideband amplitude given by Bessel-function J_k of order k

$$\text{Amplitude } r_{m,k}^{\pm} \propto J_k \left[\left(\omega_{m,k}^{\pm} - \omega_{\xi} \right) \frac{a}{\omega_0} \right]$$

ω_b betatron freq. ω_0 rev. freq. a bunch length,

Finite chromaticity ξ modifies sideband's heights!



Time Domain Mode Structure: Non-interacting Particle Model



Bessel's function as envelope represent unrealistic "hollow bunches". F. Sacherer (in 1978) found sinusoidal Eigen-functions good approximations for parabolic bunches

$$\bar{x}_k(\tau) = \cos(\pi(k+1)\tau/\tau_b) \cdot e^{-i\omega_\xi \tau}$$

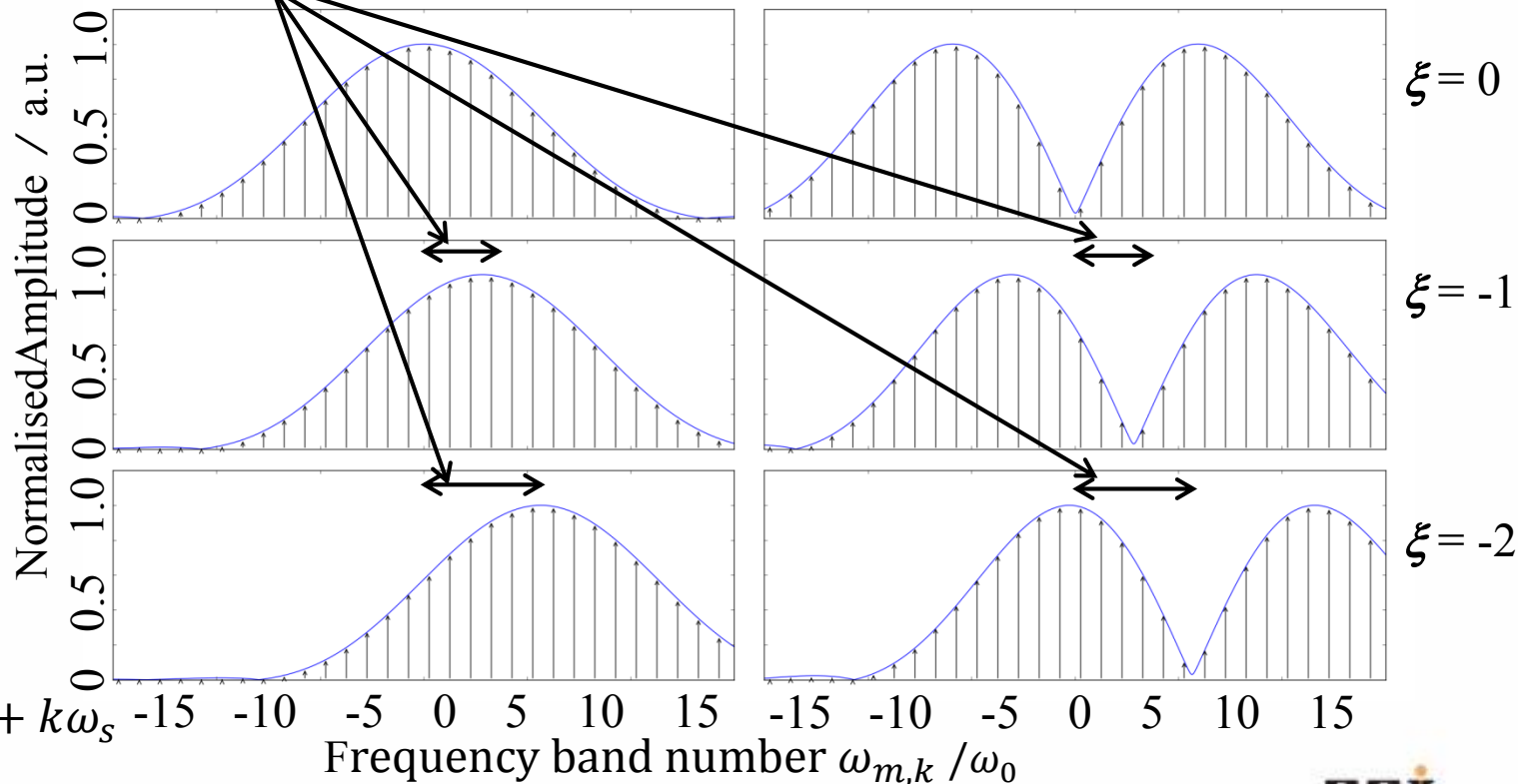
F. Sacherer: Transverse bunched beam instabilities : Theory

Chromatic tune $\omega_\xi/\omega_0 = \frac{\xi}{\eta} Q_0$

$k = 0$

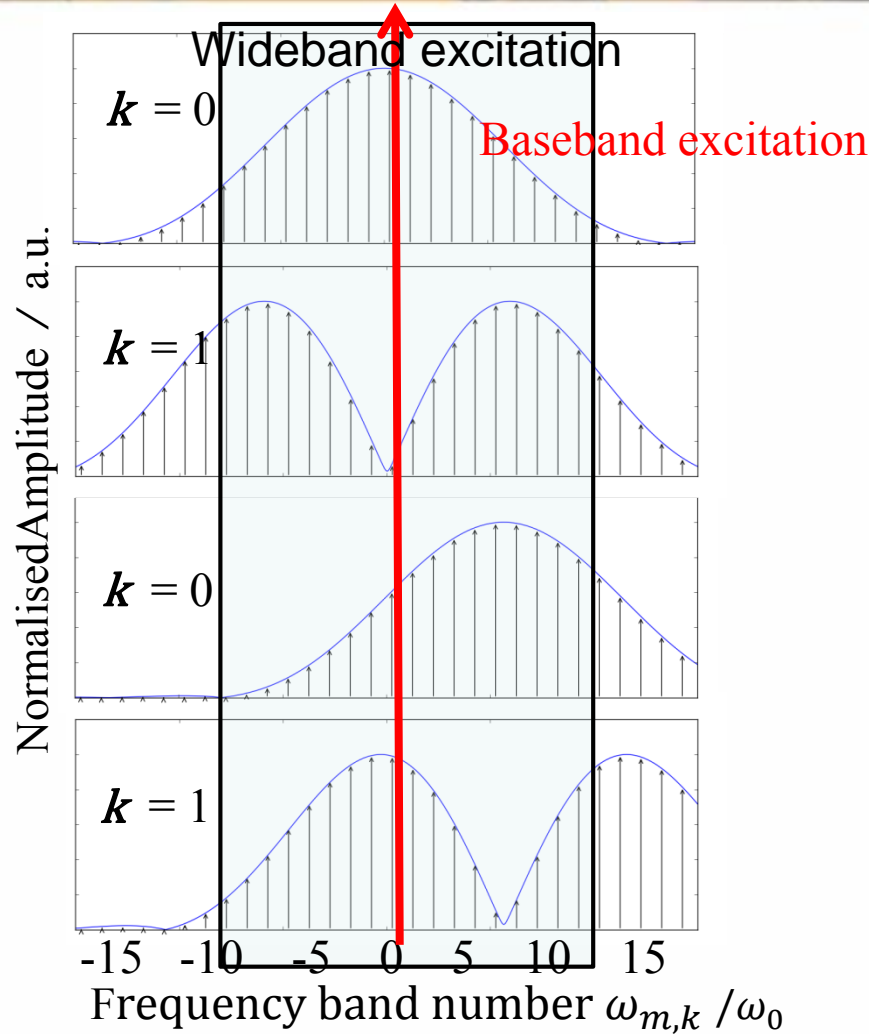
$k = 1$

SIS18 injection:
 $Q_0 = 3.27, \eta = -0.92$



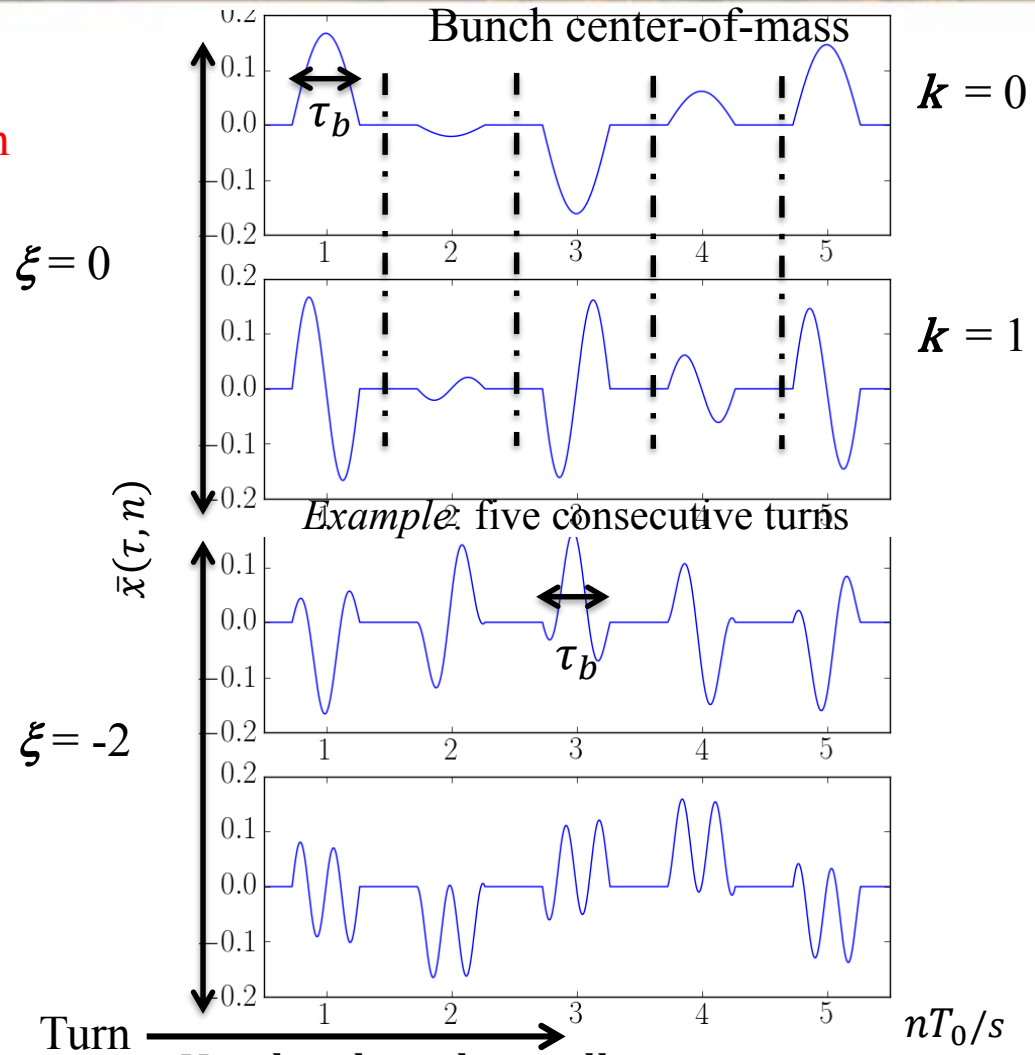
$$\omega_{m,k} = m\omega_0 + \omega_b + k\omega_s$$

Excitation of Head-tail Modes



Choose any/many frequency bands to excite

$$\omega_{m,k} = m\omega_0 + \omega_b + k\omega_s$$

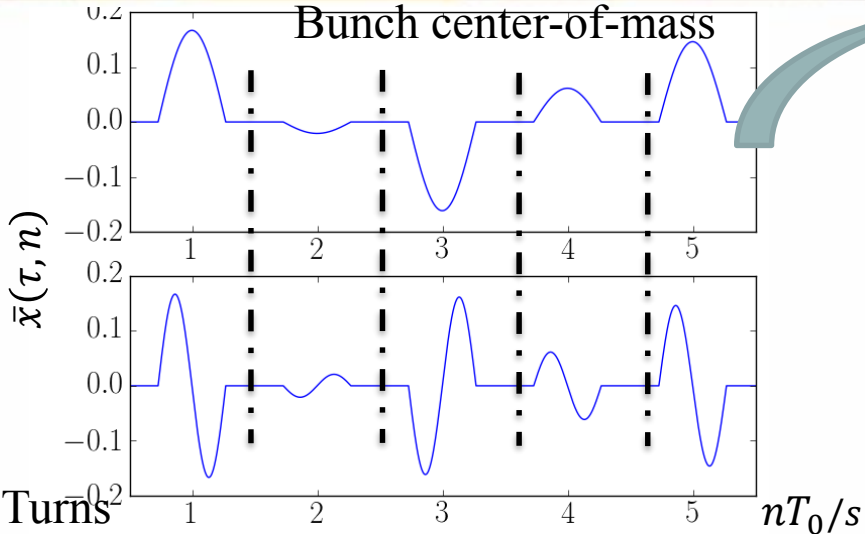


Head-tail mode oscillations:

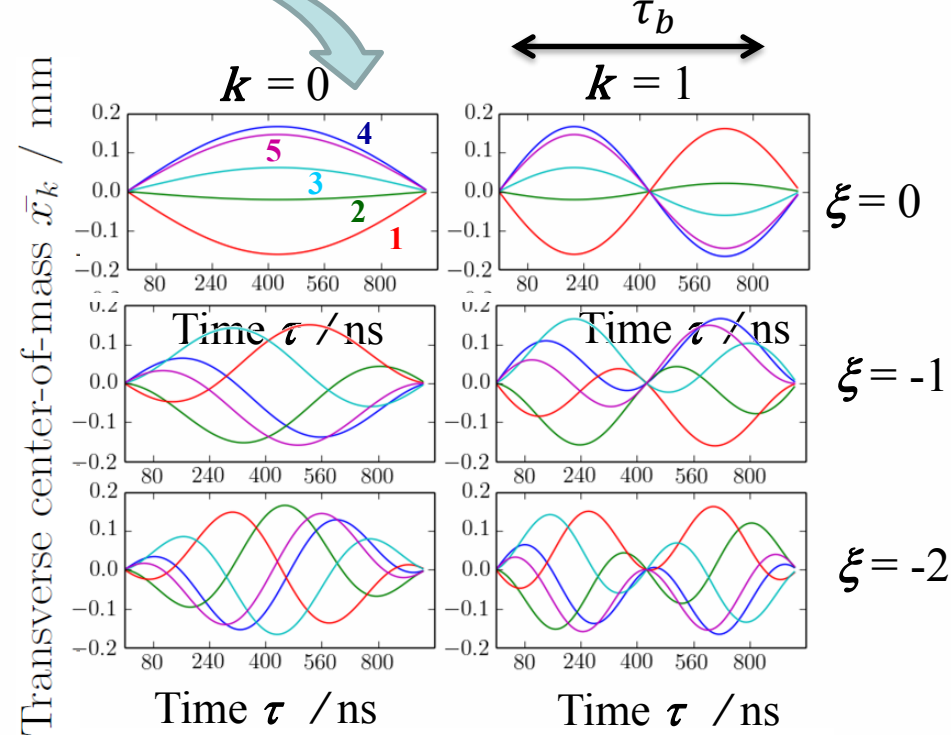
$$\bar{x}_k(\tau, n) = \text{Re}(\bar{x}_k(\tau) \cdot e^{-i(\omega_b + k\omega_s)T_0 n})$$



Time Domain Mode Structure: Non-interacting Particle Model



Example: five consecutive turns



$$\bar{x}_k(\tau, t) = \bar{x}_k(\tau) \cdot \cos[(\omega_b + k\omega_s)t + \omega_\xi \tau]$$

Non-zero chromaticity \leftrightarrow intra-bunch movement

Phase shift depending on the separation $\Delta\tau$

For $k = 0$, $\Delta\varphi = \omega_\xi \Delta\tau = \frac{\xi}{\eta} Q_0 \omega_0 \Delta\tau$

$$\xi = \frac{\Delta\varphi \eta}{Q_0 \omega_0 \Delta\tau}$$

SIS18 injection:
 $Q_0 = 3.27, \eta = -0.92$

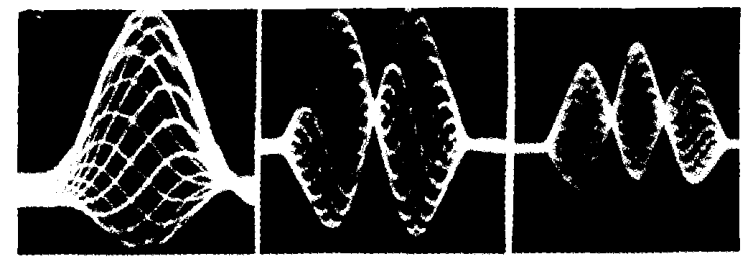
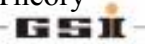


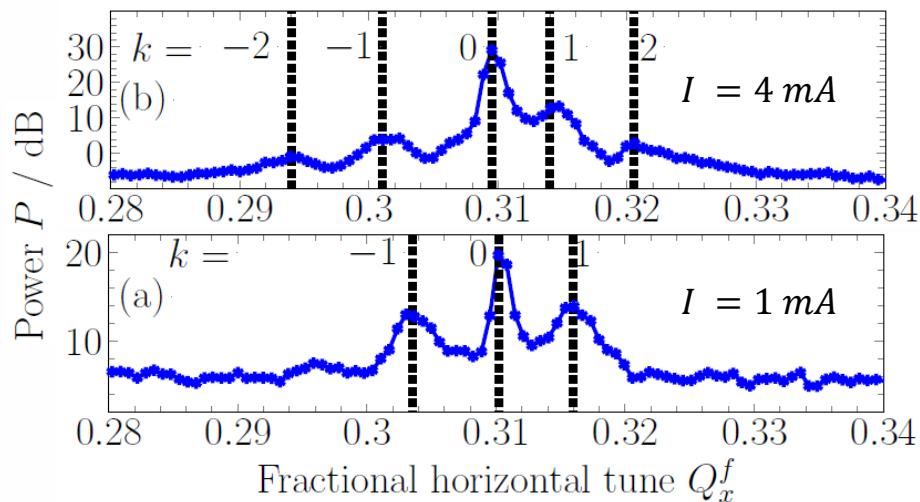
Fig. 2 Head-tail modes observed in the CERN Booster⁶.
 F. Sacherer: Transverse bunched beam instabilities : Theory



Tune Spectra at SIS18: Modification with intensity



Higher current: The „global“ peak moves to the left. The symmetry of the spectrum is broken.



Low current: Tune spectra has symmetric sidebands due to synchrotron motion.

Relevant GSI SIS18 parameters

Parameter	Typical Value
Circumference	216 m
Beam current	$10^7 - 10^{13}$ charges
Injection energy	11.4 MeV/u ($\beta = 0.15$)
Betatron tune Q_x & Q_y	4.31 or 4.17 & 3.28
Synchrotron tune Q_s	0.007 (~ 1.4 kHz)
Trans. size σ_x & σ_y	6...10 & 3...5 mm

Most measurement and tests at prolonged injection flat-top with highest ΔQ performed.

General questions:

- Why is the tune spectrum modified at increasing beam current ?
- Very practical: Which peak is the coherently shifted tune ?
- What can we learn from this modification for further beam parameters ?

Not high enough for instability

Outline



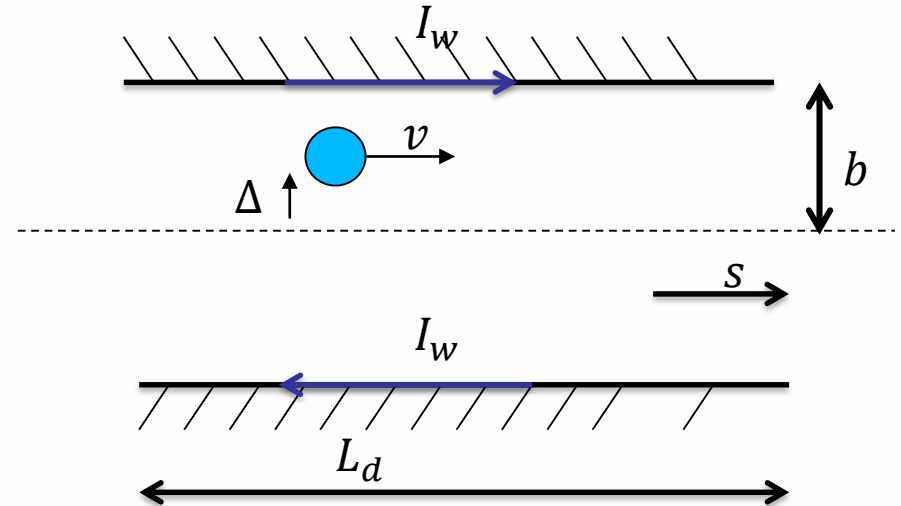
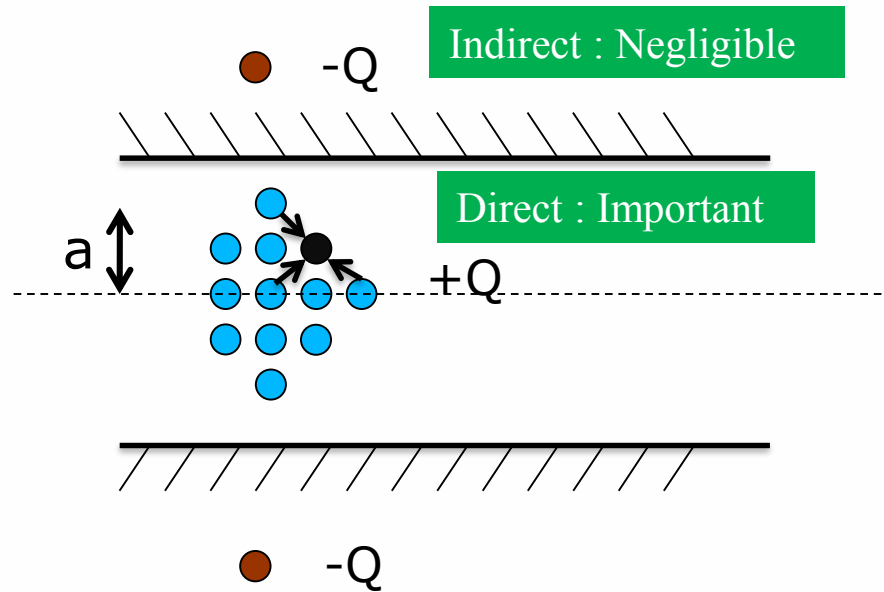
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Space charge and impedances for transverse motion



Model assumptions: round constant beam & pipe radius, constant or KV distribution,



Interaction of individual particles with each other and the boundaries \rightarrow Incoherent effect

Interaction of center-of-mass motion with boundaries \rightarrow Coherent effect

Non-linear force with strong dependence on beam distribution \rightarrow Will lead to a negative tune spread

$$Z_{\perp}(\omega) = -j \frac{\int_0^{L_d} (E(s, \omega) + v \times B(s, \omega))_{\perp} ds}{\beta I \Delta}$$

Formulas: coherent Tune Shift and incoh. Tune Spread



Model assumptions: round constant beam & and perfectly conducting pipe radius, KV distribution, constant synchrotron tune ...

$$\text{Transverse impedance } Z_{\perp} = i \frac{Z_0}{2\pi(\beta_0\gamma_0 b)^2}$$

$$\text{Coherent tune shift } \Delta Q_c = i \frac{qI_p R^2 Z_{\perp}}{2Q_0\beta_0 W_0}$$

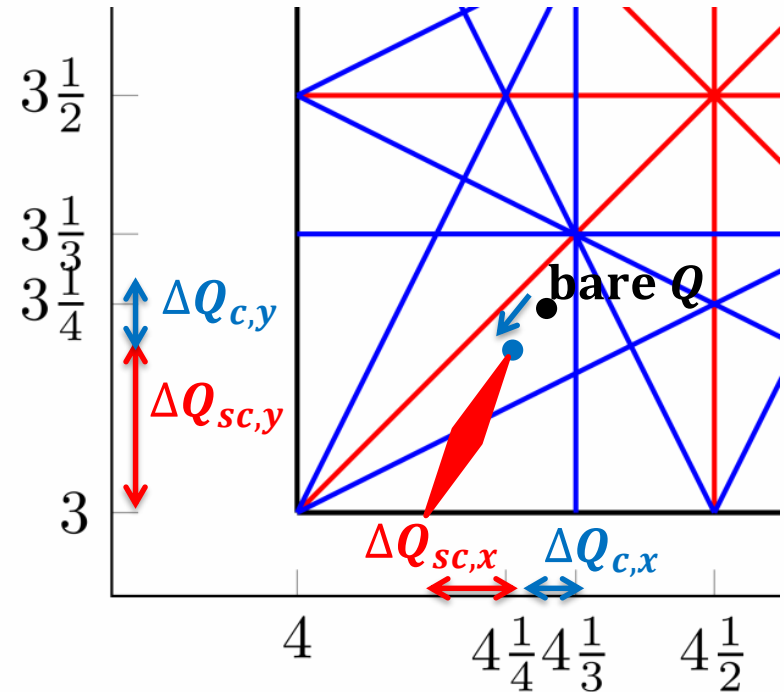
$$\text{Incoh. tune spread } \Delta Q_{sc} = \frac{qI_p R}{4\pi\epsilon_0 c W_0 \beta_0^3 \gamma_0^2 \epsilon}$$

$$\text{Relation } \Delta Q_c = \Delta Q_{sc} \cdot \left(\frac{a}{b}\right)^2$$

← beam radius
← pipe radius

$$\text{Space charge parameter } q_{sc} = \frac{|\Delta Q_{sc}|}{Q_s}$$

q_{sc} includes effect of longitudinal oscillations



b pipe radius, R synchrotron radius, $Z_0 = \sqrt{\mu_0/\epsilon_0}$
 q ion charge, I_p peak current, Q_0 bare tune
 W_0 total energy, β_0 velocity, γ_0 Lorentz factor
 a beam radius, ϵ transverse emittance

Analytical Model: Space Charge Modification of Tune Spectra



Assumption of analytical description of head-tail modes by M. Blaskiewicz (1998):

- Transverse phase space: KV-distribution
- Longitudinal ‘airbag’ phase space: phase \rightarrow constant momentum only two velocities $\mathbf{v}_s = \pm \mathbf{v}_{s0}$
 \Rightarrow synchrotron tune: $Q_s = \pi v_{s0} / (R \omega_0 a)$

Low intensity: spacing of sidebands: $\Delta Q_k = k Q_s$

High intensity: spacing of sidebands:

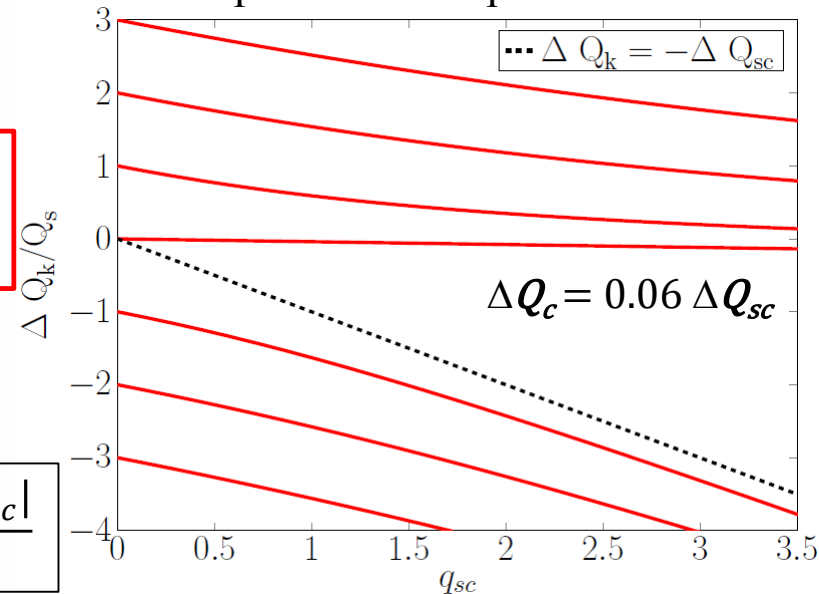
$$\Delta Q_{\pm k} = -\frac{\Delta Q_{sc} + \Delta Q_c}{2} \pm \sqrt{\frac{(\Delta Q_{sc} - \Delta Q_c)^2}{4} + (k Q_s)^2}$$

ΔQ_c coherent tune shift

ΔQ_{sc} incoherent tune spread

$$q_{sc} = \frac{|\Delta Q_{sc}|}{Q_s}$$

Example for SIS18 parameter



Findings:

- $k = 0$ it is $\Delta Q_0 = -\Delta Q_c$ i.e. value of shifted bare tune
- $k > 0$, head-tail modes come closer as $q_{sc} = \Delta Q_{sc} / Q_s$ increases
- $k < 0$ almost constant slope (but larger Landau damping \Leftrightarrow broader and lower spectral lines)

M. Blaskiewicz: *Fast head-tail instabilities with space charge*, Phys. Rev. Acc. Beams 1, 044201, (1998)

A. Burov, *Head-tail modes for strong space charge*, Phys. Rev. Acc. Beams 12, 044202, (2009)

O. Boine-Frankenheim et al., *Transverse Schottky noise spectrum for bunches with space charge*, Phys. Rev. Acc. Beams 12, 114201, (2009)

Measurements: Overview of Beam Parameters



Parameter	Symbols	Value	Value	
Beam	$A \text{ Ion } q^+$	$^{238} \text{U } ^{73+}$	$^{14} \text{N } ^{7+}$	
Energy	W_{kin}	11.4 MeV/u	11.56 MeV/u	Longitudinal Schottky
No. of particles	N_p	$(1 \dots 12) \cdot 10^8$	$(1 \dots 15) \cdot 10^9$	Current Transformer
Emittance	$\varepsilon_x \ \& \ \varepsilon_y (2\sigma)$	45 & 22 mm-mrad	33 & 12 mm-mrad	Profile Monitor
Tune	$Q_{x0} \ \& \ Q_{y0}$	4.31 & 3.27	4.16 & 3.27	TOPOS or BBQ
Bunching factor	B_f	0.4	0.37	TOPOS
Synchrotron tune	$Q_{s0} \ \& \ Q_{s,meas}$	0.007 & 0.0065	0.006 & 0.0057	
Chromaticity	$\xi_x \ \& \ \xi_y$	-0.94 & -1.85 set	-1.7 & -2.1 meas.	

Measurement of all relevant beam parameters

to calculate q_{sc} for horizontal and vertical direction, respectively:

$$q_{sc,x} = \frac{|\Delta Q_{sc}|}{Q_{s,meas}} = \frac{1}{2\varepsilon_0} \cdot \frac{q^2 N_p / B_f}{Q_{s,meas} W_0 \beta_0^2 \gamma_0^2 \varepsilon_{eff,x}}$$

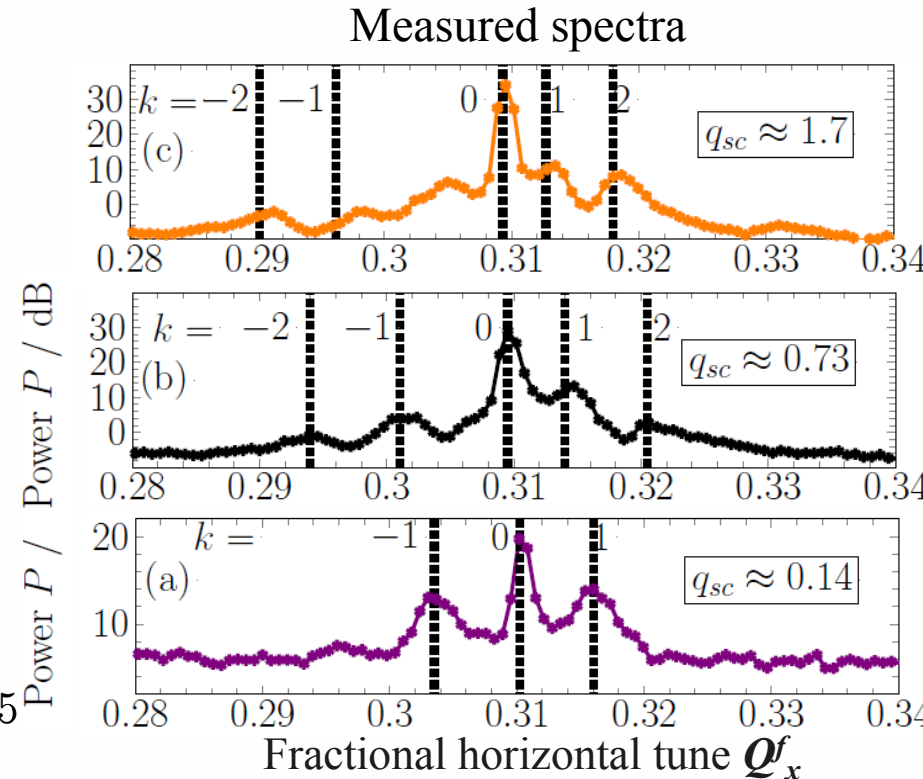
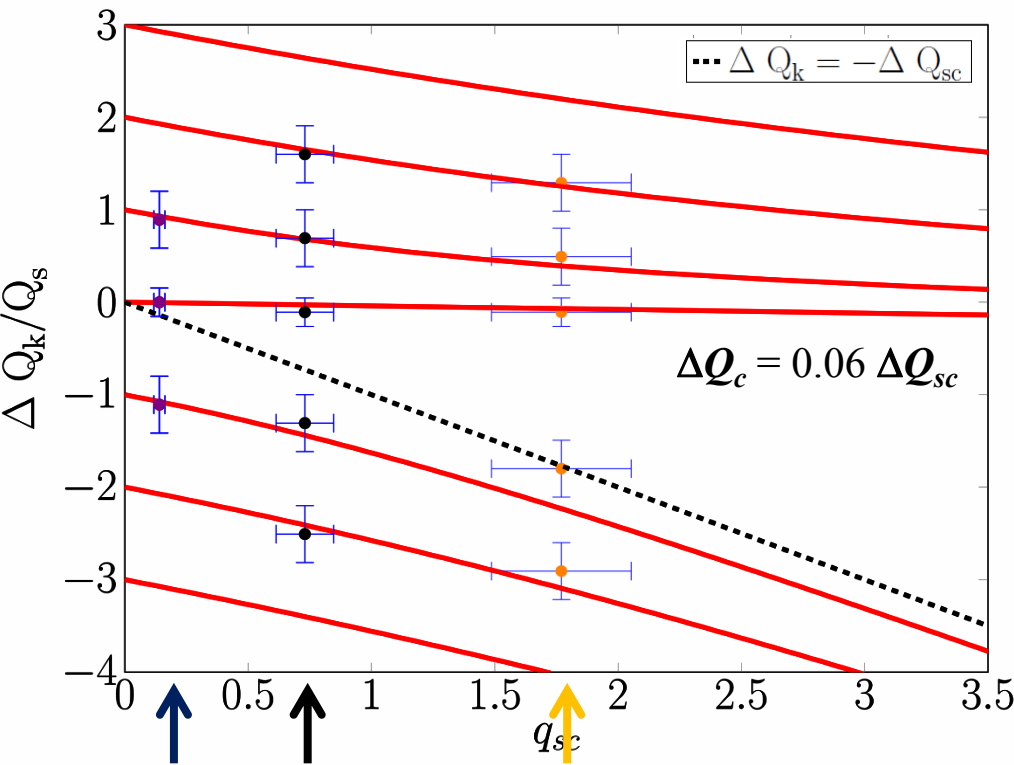
Presented tune spectra are measured by BBQ or TOPOS

effective emittance $\varepsilon_{eff,x} = 1/2 (\varepsilon_x + \sqrt{\varepsilon_x \varepsilon_y Q_{x0}/Q_{y0}})$

Measurements: Moderate Space Charge Parameter



Beam: U^{73+} , 11.4 MeV/u, $(1 \dots 12) \cdot 10^8$ ions, $Q_s = 0.007 \Leftrightarrow f_s = \frac{Q_s \omega_0}{2\pi} = 1.4$ kHz



Red lines: predicted lines using **measured** beam parameters:

$$\Delta Q_{\pm k} = -\frac{\Delta Q_{sc} + \Delta Q_c}{2} \pm \sqrt{\frac{(\Delta Q_{sc} - \Delta Q_c)^2}{4} + (kQ_{s,meas})^2}$$

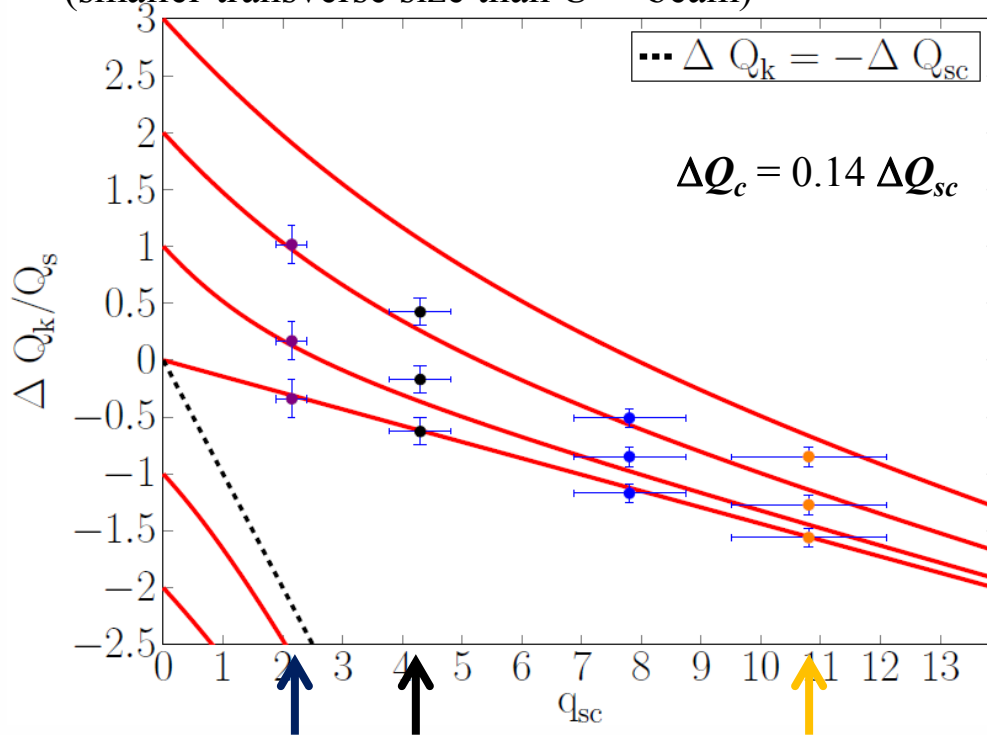
$$q_{sc} = \frac{|\Delta Q_{sc}|}{Q_{s,meas}}$$

Measurements: Strong Space Charge Parameter



Beam: N^{7+} , 11.56 MeV/u, $(3 \dots 15) \cdot 10^9$ ion, $Q_s = 0.006 \Leftrightarrow f_s = \frac{Q_s \omega_0}{2\pi} = 1.2$ kHz

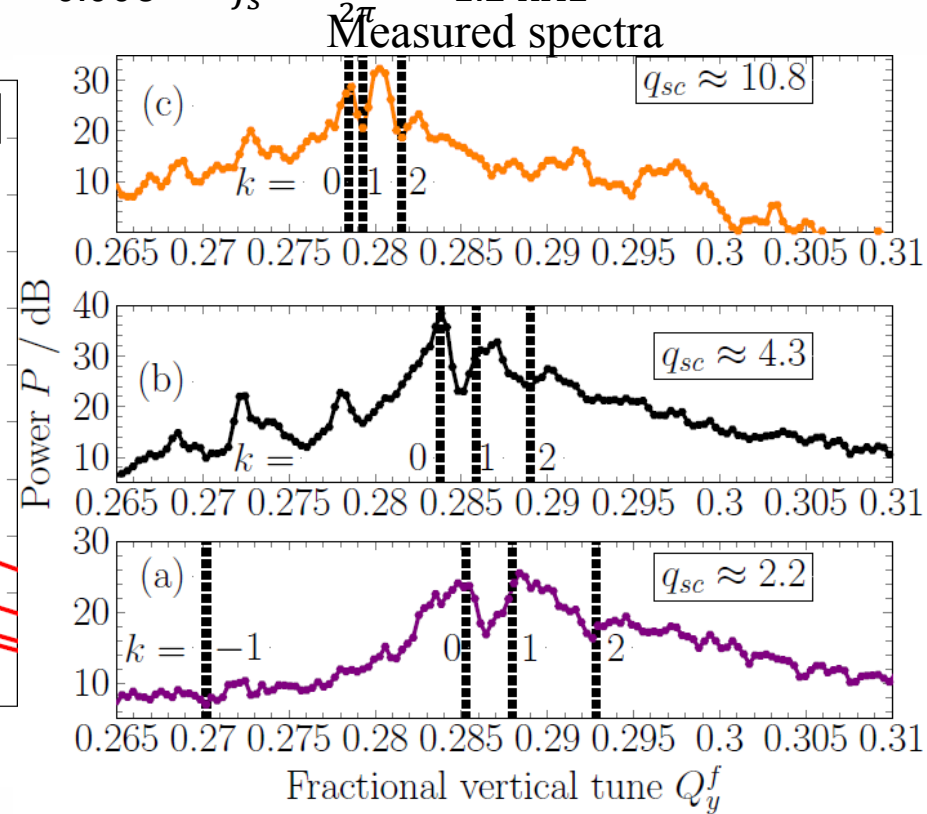
(smaller transverse size than U^{73+} beam)



Predicted spectra : red lines

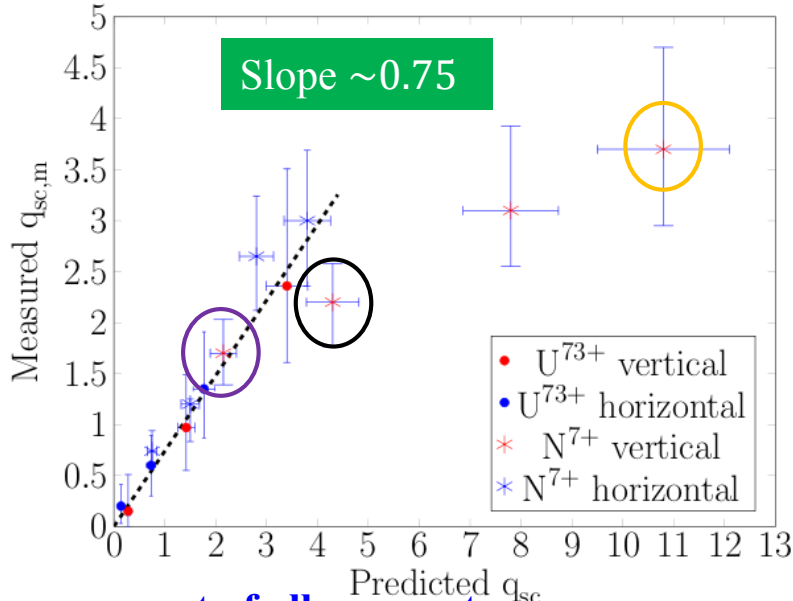
$$\Delta Q_{\pm k} = -\frac{\Delta Q_{sc} + \Delta Q_c}{2} \pm \sqrt{\frac{(\Delta Q_{sc} - \Delta Q_c)^2}{4} + (kQ_{s,meas})^2}$$

$$q_{sc} = \frac{|\Delta Q_{sc}|}{Q_{s,meas}}$$



Comparison of Predicted and Measured Space Charge Parameter

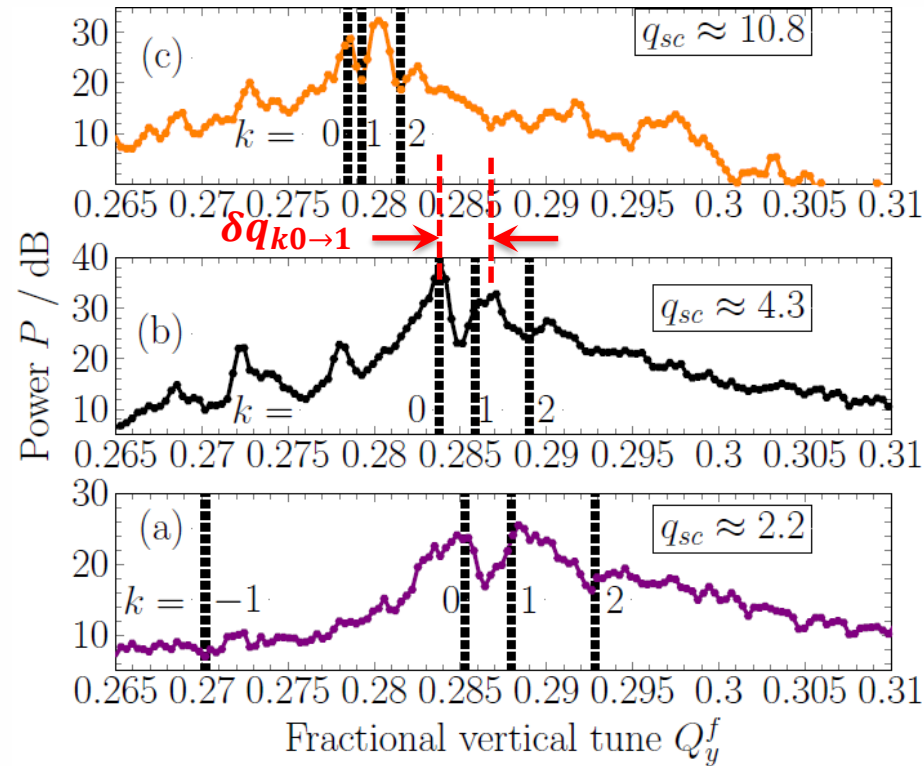
The values q_{sc} from the depicted measurements with U^{73+} and N^{7+}



Measurement of all parameters:

- **Predicted** $q_{sc,x} = \frac{1}{2\epsilon_0} \cdot \frac{q^2 N_p / B_f}{Q_{s,meas} W_0 \beta_0^2 \gamma_0^2 \epsilon_{eff,x}}$
- **Measured** q_{sc} based on the distance between lines of modes $k = 0$ and 1 : $\delta q_{k0 \rightarrow 1}$

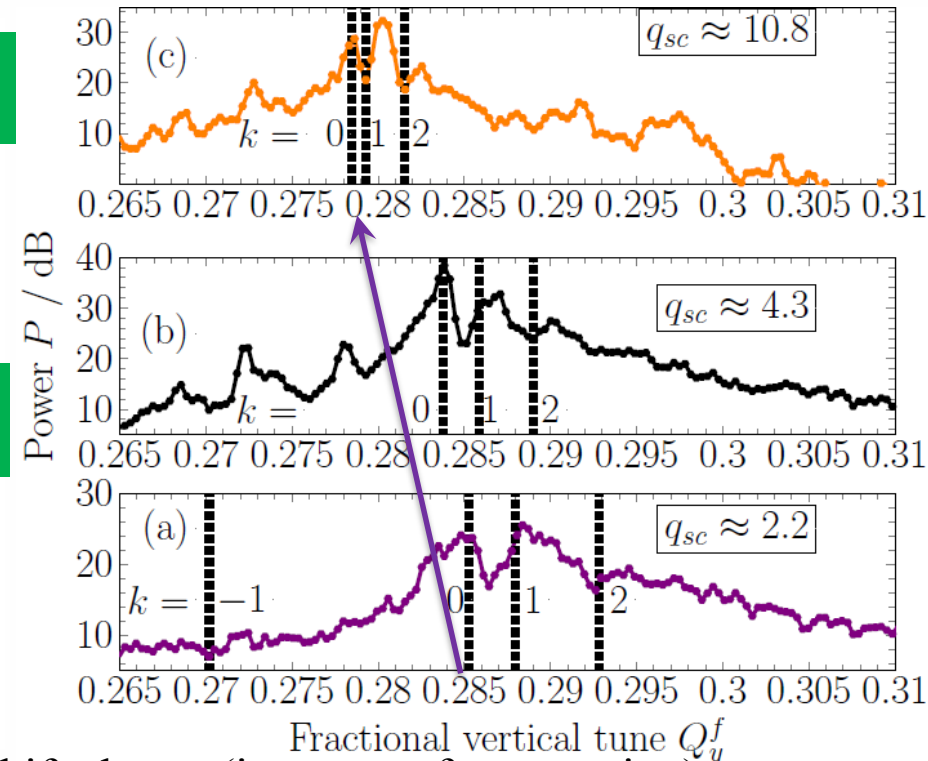
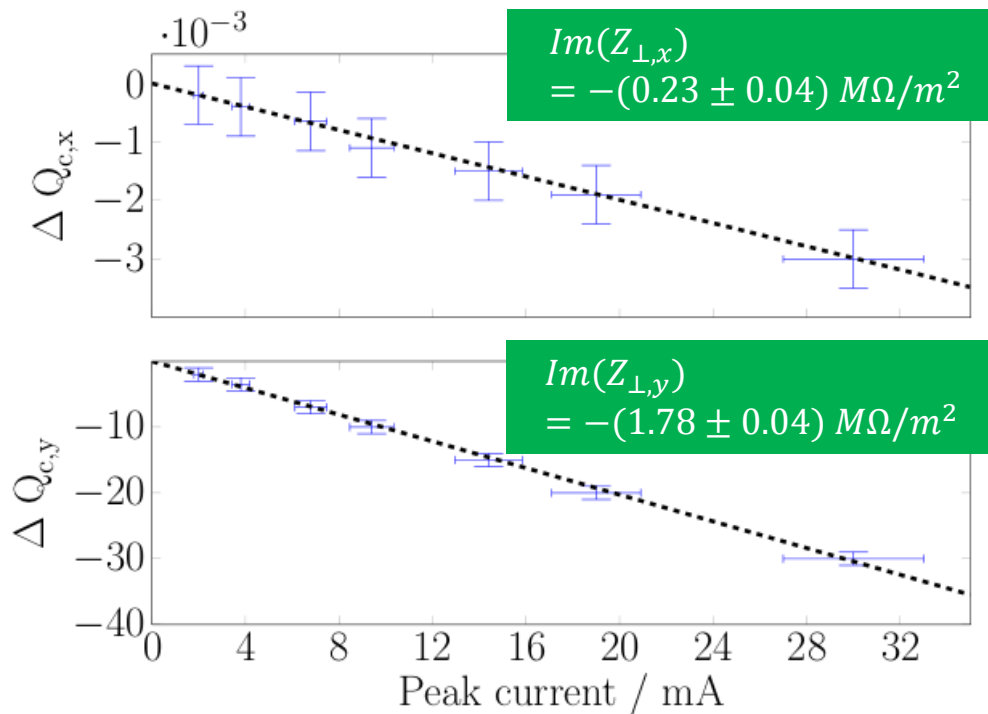
- Findings:**
- General trend well reproduced i.e. reason for modified tune spectra was found
 - Significant deviation for higher values of q_{sc} (as expected from analytic model)
 - Does a better model with high prediction power exists?
 - Can such model be applied to SIS18 parameter providing a plots of tune spectra?



Measurements: Determination of coherent Tune Shift ΔQ_c



The coherent tune shift ΔQ_c from the depicted N^{7+} spectra and further spectra with Ar^{18+}



For $k = 0$: $\Delta Q_c = \Delta Q_{k=0}$ i.e. value of coherently shifted tune (important for operating)

Results:

- Expected linear scaling coherent tune shifts versus peak beam current I_p
- For value of slope the resistive, effective impedance Z_{\perp} can be determined
 \Rightarrow estimation of effective beam pipe radius $b_x \sim 115 \pm 5.5$ mm & $b_y = 35 \pm 1$ mm

R. Singh et al., *Interpretation of transverse tune spectrum in a heavy-ion synchrotron*, Phys. Rev. Acc. Beams 13, 034201 (2013)



Time Domain Identification of Head-tail Modes

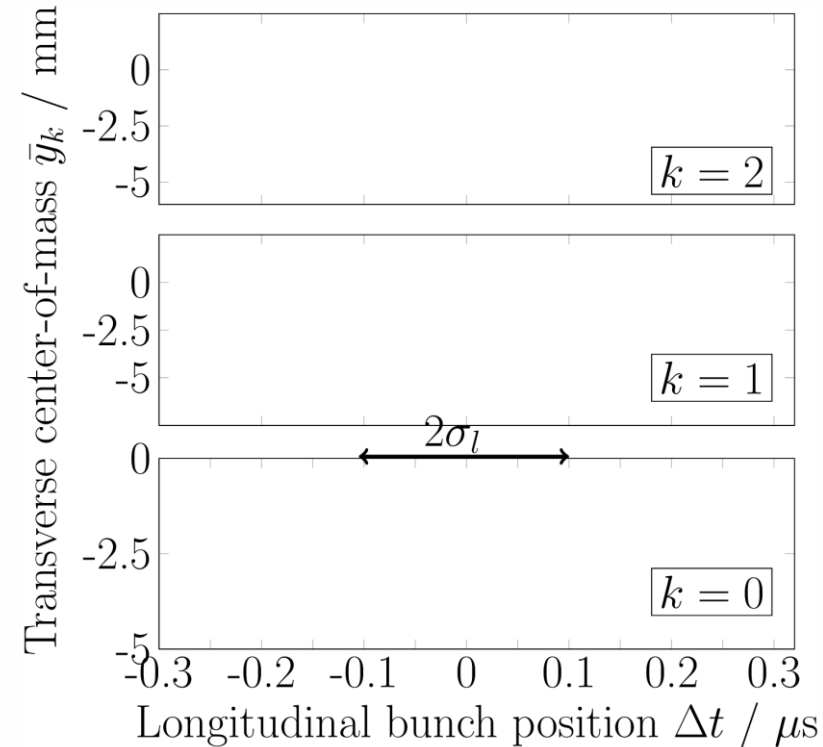
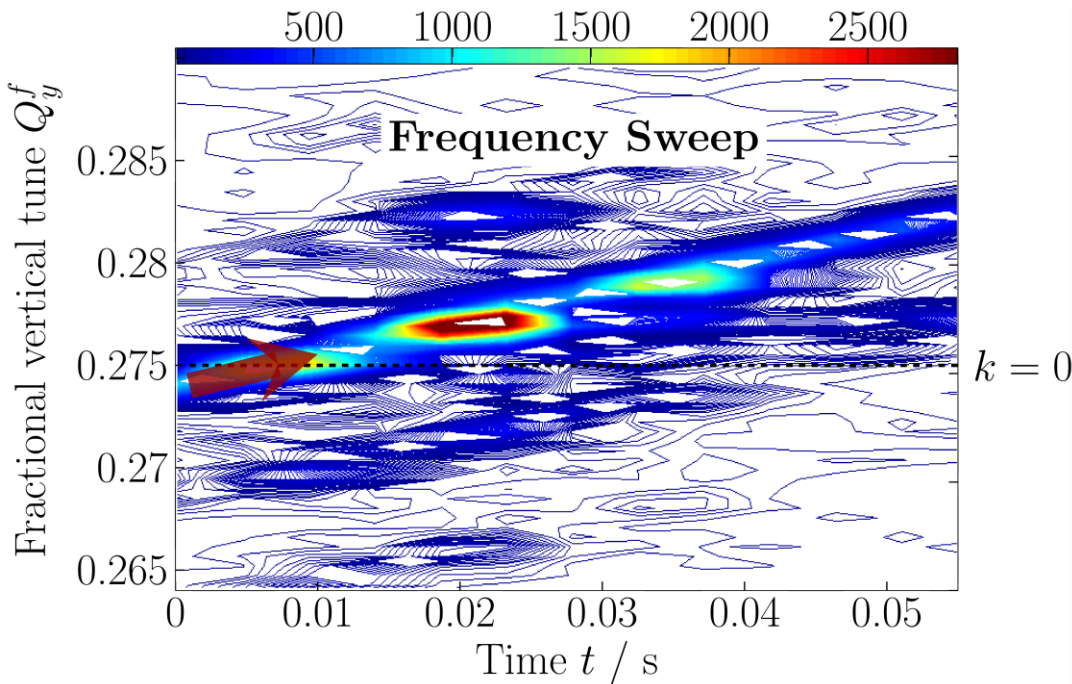


Frequency sweep allows the observation of bunch center oscillations i.e. head-tail modes

Beam parameter: $15 \cdot 10^9 \text{ N}^{7+}$ at 11.5 MeV/u, $Q_y = 3.75$, $\xi = -2.1$

Tune spectrogram during sweep:

11 consecutive turn-by-turn center-of-mass recordings:



Results:

- Sweep excitation allows excitation of individual head-tail modes
- The mode-structure verifies the spectra interpretation

Remark: Eigen-functions for high intensities could be determined

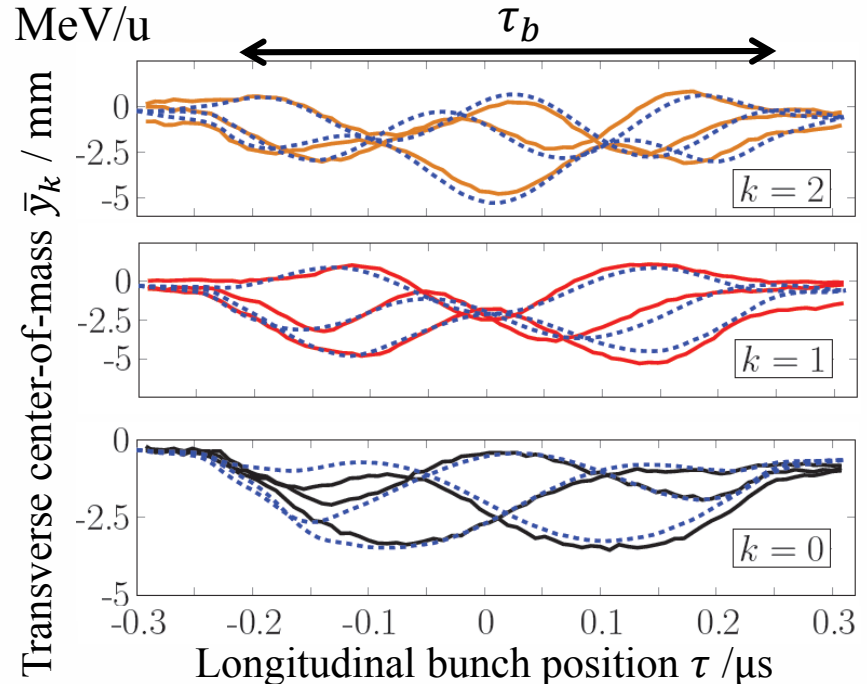
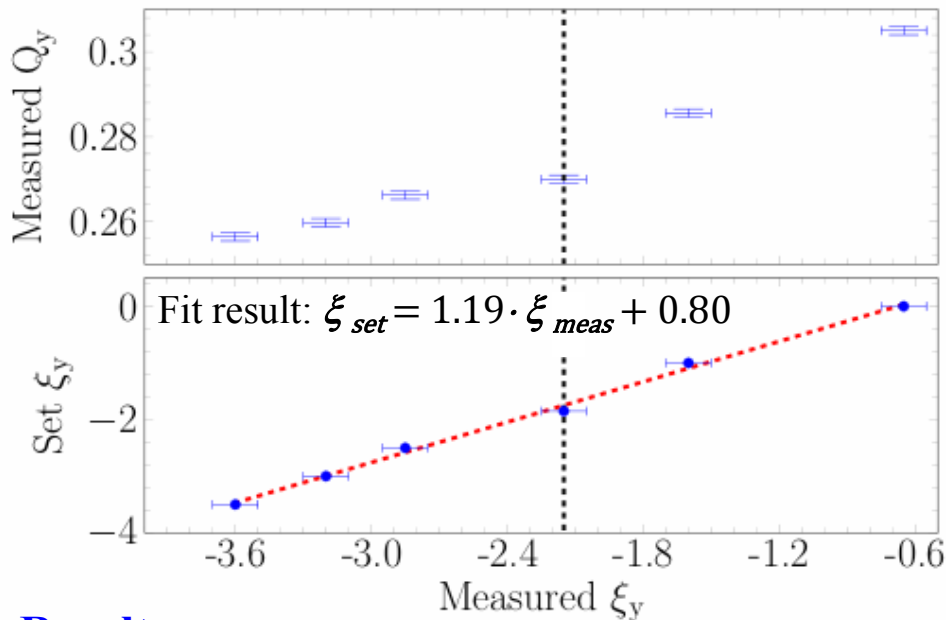
Determination of Chromaticity from Head-tail Oscillations



Fit of measured head-tail modes to the classical (i.e. without space charge) eigen-functions

$$\bar{y}_k(\tau, n) = \bar{y}_k(\tau) \cdot \cos[(\omega_b + k\omega_s)nT_0 + \omega_\xi\tau + \varphi_0] + y_{offset}$$

Beam parameter: low current $15 \cdot 10^9 \text{ N}^{7+}$ at 11.5 MeV/u



Results:

- Precise determination of chromaticity ξ
- SIS18: Deviation between set & actual value & coupling to tune due to uncorrected closed orbit
- ⇒ Reliable method but only for offline analysis

- Bold lines: measured bunch center
- Dotted lines: least squares fit with chromaticity ξ as fit parameter

R. Singh et al., *Interpretation of transverse tune spectrum in a heavy-ion synchrotron*, Phys. Rev. Acc. Beams 13, 034201 (2013)



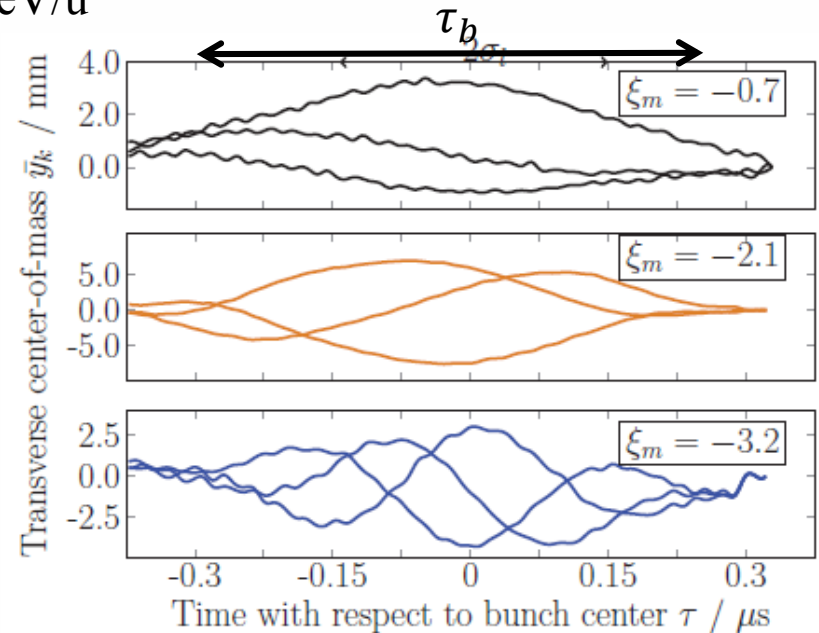
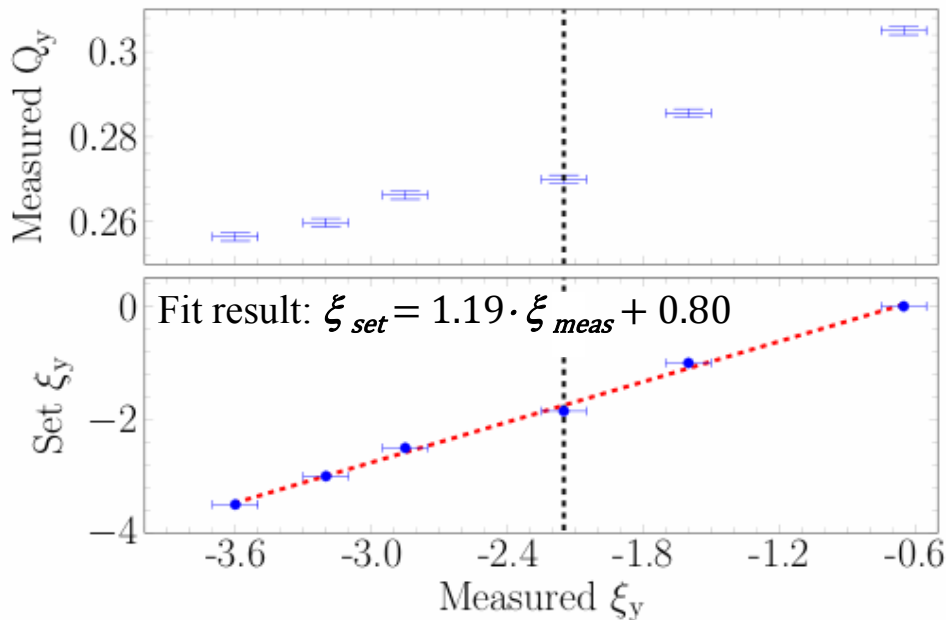
Determination of Chromaticity from Head-tail Oscillations



Fit of measured head-tail modes to the classical (i.e. without space charge) eigen-functions

$$\text{For } k = 0 \rightarrow \bar{y}_0(\tau, n) = \cos(\pi\tau/\tau_b) \cdot \cos[\omega_b n T_0 + \omega_\xi \tau + \varphi_0] + y_{offset}$$

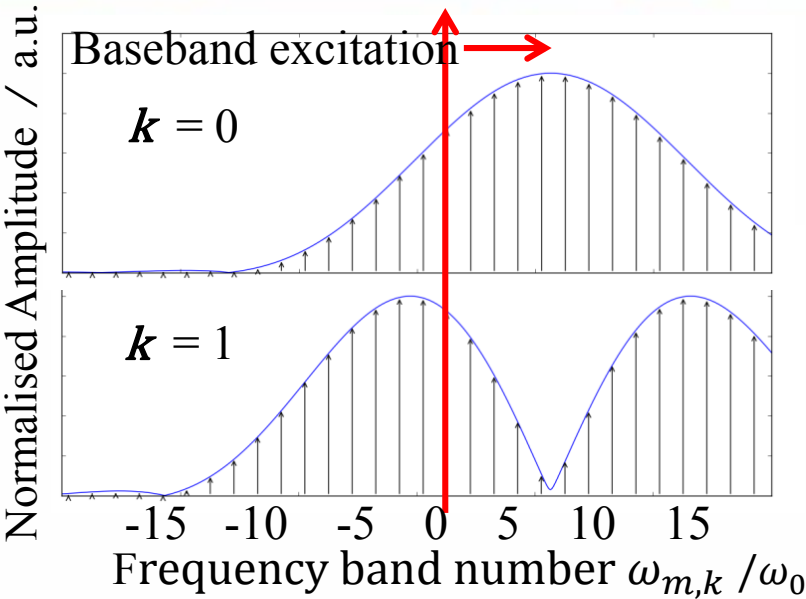
Beam parameter: low current $15 \cdot 10^9 \text{ N}^{7+}$ at 11.5 MeV/u



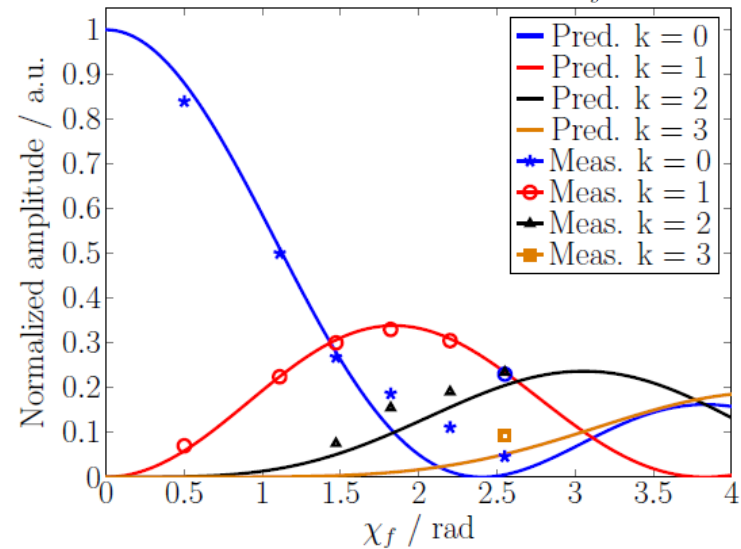
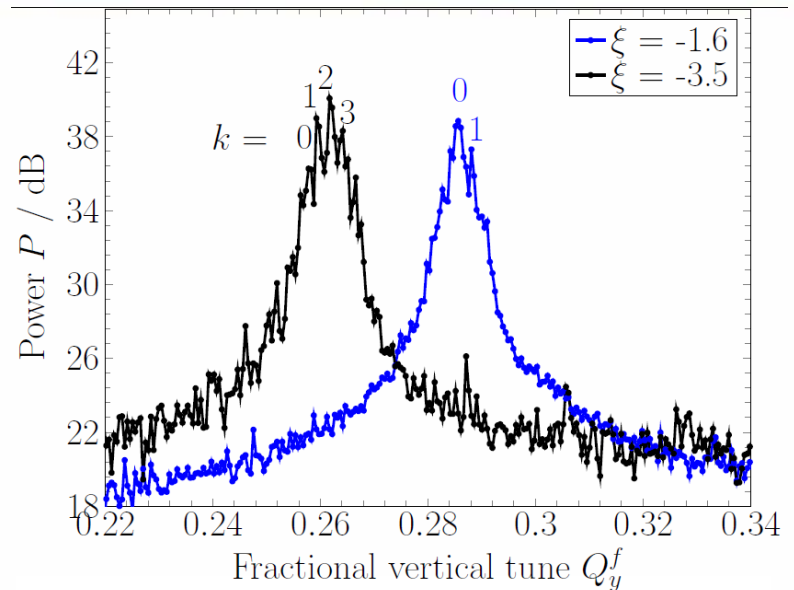
Results:

- SIS18: Deviation between set & actual value & coupling to tune due to uncorrected closed orbit
- Position measurement system not appropriate → a dedicated head tail monitor with hybrids better
- Beam losses at changed chromaticity
- The phase between two locations $\Delta\tau$ is a function of chromaticity as discussed earlier ($\omega_\xi \Delta\tau$)
- Phase shift constant between turns

Tune spectra vs Chromaticity



- Relative amplitudes of excited modes after a frequency sweep as a function of chromaticity
- Their excitability depends on the exact band/frequency of excitation
- To always have $k = 0$ mode as the dominant peak, excitation band should be close to ω_ξ



Tune Spectra at Higher Energies

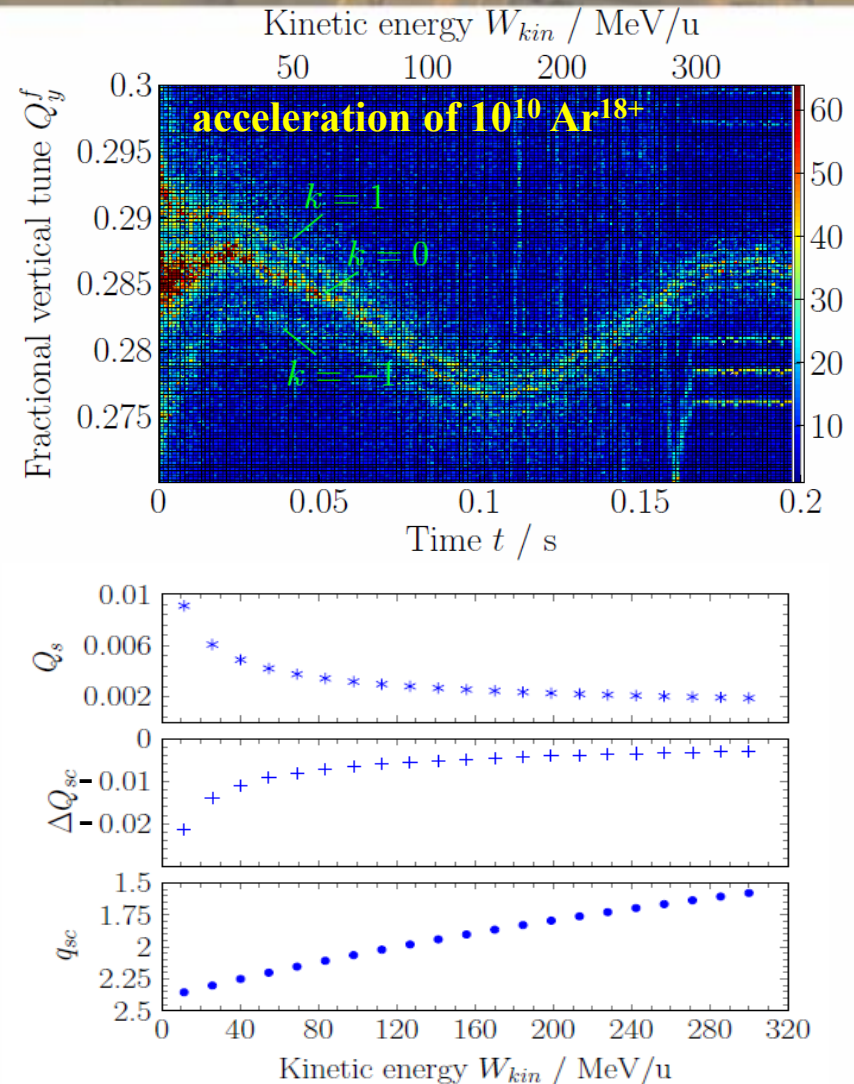
During acceleration:

- transverse emittance decreases
 - synchrotron tune Q_s decreases
- ⇒ $q_{sc} > 1$ i.e. relevant modification of spectrum

Typical values at other synchrotrons at injection:

Parameter	SIS18	SPS	RHIC	ANKA
I_{peak} / mA	10	1400	500	12
$\varepsilon / \text{mm}\cdot\text{mrad}$	22	0.2	10	0.15
Lorentz fac. γ	1	27	4	100
Tune spr. ΔQ_{sc}	-0.05	-0.1	-0.02	-10^{-4}
Synch. tune Q_s	0.007	0.015	0.0015	0.008
SC para. q_{sc}	~ 7	~ 7	~ 13	~ 0

Space charge modification of tune spectra is also relevant for other facilities !



R. Singh et al., PRST-AB 2013 and IBIC'13

Comparison to Theoretical Predictions

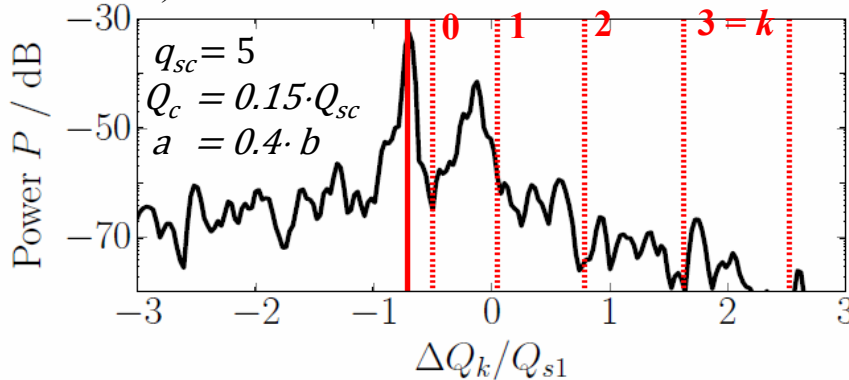


Theoretical investigations by other groups

as seen by a beam diagnostics person:

- Head-tail modes discussed for instabilities
- Eigen-frequencies predicted as a function of q_{sc}
- Landau damping stronger for negative modes
- Bunch length influences $Q_s(\sigma_{bunch}) \Rightarrow$ changes in spectrum
- Chromaticity influences the peak height and width
- **But:** Seldom plotted in terms of beam observables
 \Rightarrow Prediction missing for height & width for SIS18

Example: PIC Schottky spectrum by code PATRIC (Boine-Frankenheim et al.)



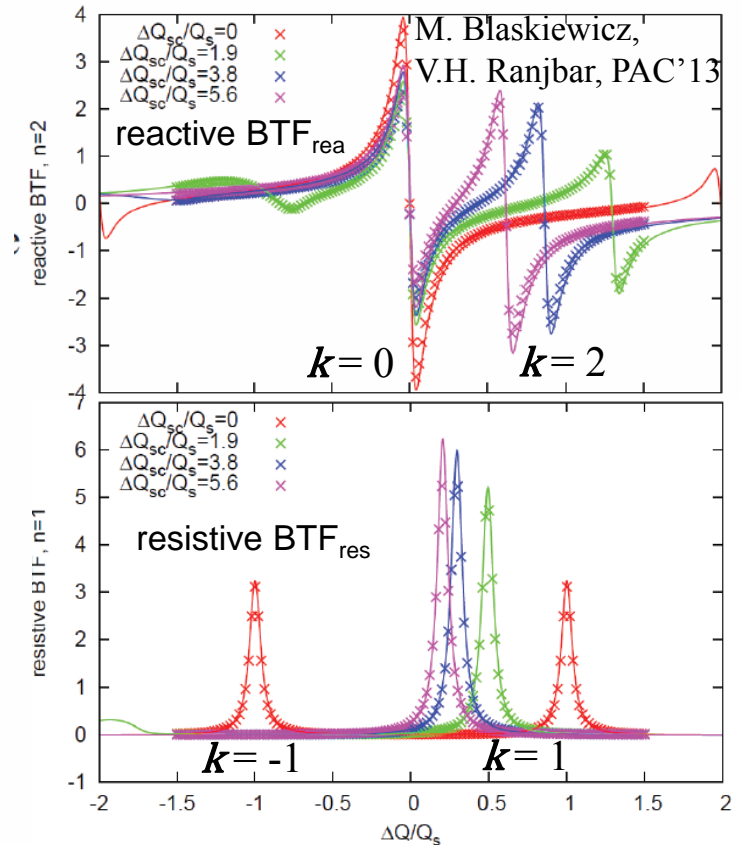
M. Blaskiewicz: *Fast head-tail instabilities with space charge*, Phys. Rev. Acc. Beams 1, 044201, (1998)

A. Burov, *Head-tail modes for strong space charge*, Phys. Rev. Acc. Beams 12, 044202, (2009)

O. Boine-Frankenheim et al., *Transverse Schottky noise spectrum for bunches with space charge*, Phys. Rev. Acc. Beams 12, 114201, (2009)

M. Blaskiewicz, V.H. Ranjbar *Transverse beam transfer functions via Vlasov equation*, Proc., PAC2013, Pasadena p. 1427 (2013).

Example: Plot of calculated $BTF(q_{sc})$
 $\xi=0, \Delta Q_i/Q_s = 0.04$



Measured tune spectrum:

$$P(\omega) = BTF_{rea}^2(\omega) + BTF_{res}^2(\omega)$$

Summary and Outlook



Experimental findings:

- System for **online** tune measurement realized, excitation with acceptable emittance growth
- Measurement of all beam parameters required for correct interpretation of tune spectrum
- Tune spectra: Eigen-frequencies significantly shifted for $q_{sc} \gtrsim 0.5$ by head-tail modes
- Coherently tune shift ΔQ_c measured by shift of $k=0$ mode \Rightarrow **However**, $k=0$ mode is **not** always the highest peak
 - \Rightarrow spectrum must be interpreted e.g. by recording the mode structure of bunches !
- Estimation of tune spread ΔQ_{sc} available \Rightarrow usage for operation and MDs?
- Chromaticity measurement using head-tail phase shift

Lessons and outlook:

- Do not blame the hardware if the spectrum looks ugly
- Excite individual head-tail mode by harmonic excitation to measure chromaticity \rightarrow Still to compare head tail chromaticity with rf modulation method
- Implications for tune feedback systems \rightarrow Excite approximately at chromatic frequency to obtain $k = 0$ mode as dominant peak

Thank you for your attention !



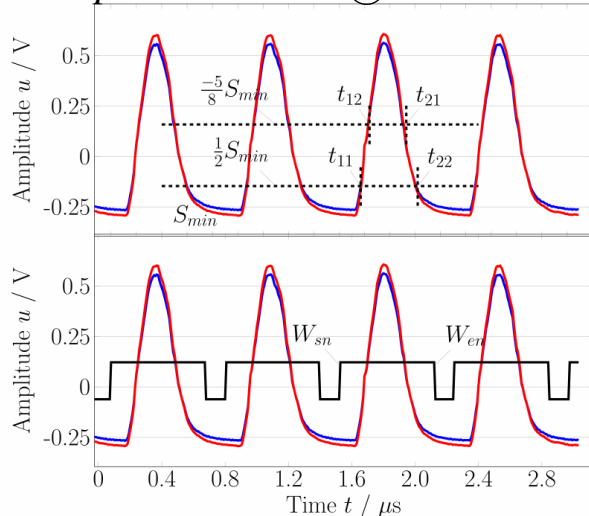
Extra slides

Position Determination: Integration versus least square Fit



Traditional: Integration

Example: One turn @ 35 MeV/u



1. Creation of window for one bunch of N samples
2. Determine baseline values $B_{l/r}$
3. Subtraction $U_{i,l/r} - B_{l/r}$ for $i = 1...N$
4. Integration of bunch signal

$$I_{l/r} = \sum_{i=1}^N (U_{i,l/r} - B_{l/r})$$
5. Position:
$$x = \frac{1}{S} \cdot \frac{I_l - I_r}{I_l + I_r}$$

S is the position sensitivity

Novel: Ordinary least square fit algorithm:

1. Creation of window of a single bunch with N samples
2. Calculation of sum and difference for each sample:

$$\Sigma_i = U_{i,l} + U_{i,r}$$

$$\Delta_i = U_{i,l} - U_{i,r}$$
 for $i = 1...N$
 'Plotting' Δ_i versus Σ_i
3. Assumption: From $x = \frac{1}{S} \cdot \frac{\Delta}{\Sigma} \Rightarrow \Delta_i = a \cdot \Sigma_i + b$
 with $a = x \cdot S$ and b as fit parameter
 \Rightarrow Least square fit of Δ_i as a function of Σ_i

General solution for fit parameter a :

$$a = \frac{\sum_{i=1}^N (\Sigma_i - \bar{\Sigma})(\Delta_i - \bar{\Delta})}{\sum_{i=1}^N (\Sigma_i - \bar{\Sigma})^2} = \frac{\text{cov}(\Sigma, \Delta)}{\text{var}(\Sigma)} \quad \& \quad b = \bar{\Delta} - a\bar{\Sigma}$$

with average value $\bar{\Sigma} = \frac{1}{N} \sum_{i=1}^N \Sigma_i$ & $\bar{\Delta} = \frac{1}{N} \sum_{i=1}^N \Delta_i$

4. Position $x = a/S$ i.e. from fit value from diff-over-sum

Advantage:

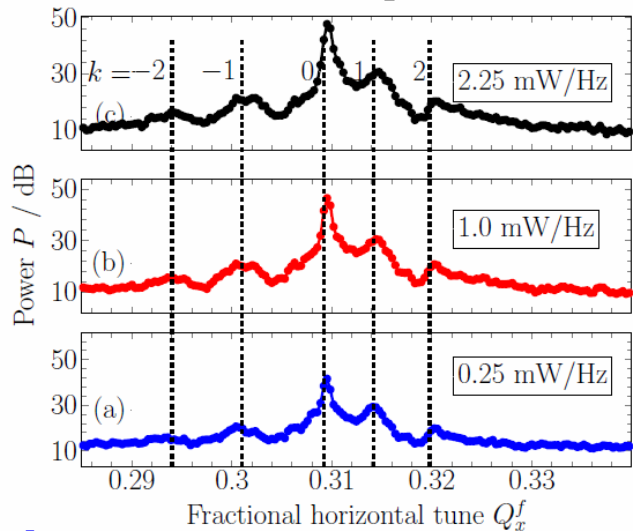
- No baseline reconstruction required
- Robust against offset etc. (here fit parameter b)
i.e. high common and differential mode suppression
- Easy to implement
- For closed orbit: window over many bunches



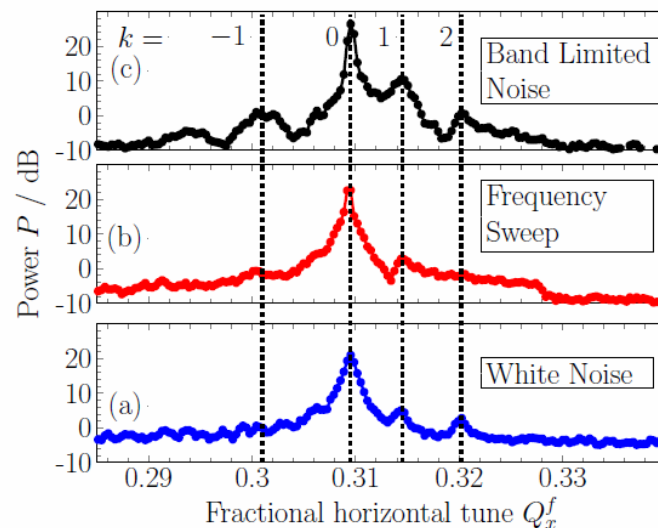
Tune Spectra for different experimental Parameter



Variation of excitation power:



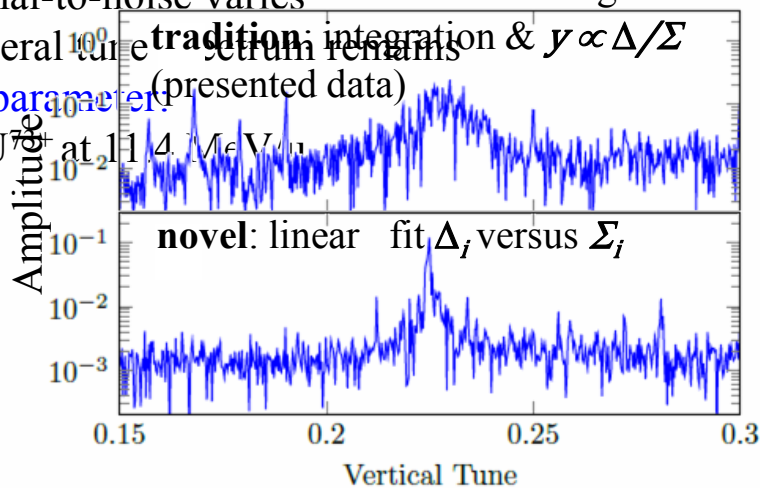
Variation of excitation type:



Result:

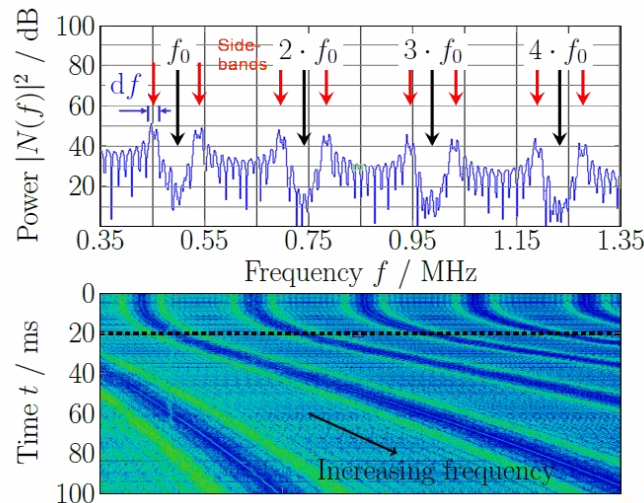
- signal-to-noise values
 - general tune spectrum remains
- Same raw data but different algorithms
 tradition: integration & $y \propto \Delta/\Sigma$
 (presented data)

Beam parameter
 $5 \cdot 10^8 U_{7+}$ at $11.4 MeV_{U_{11}}$

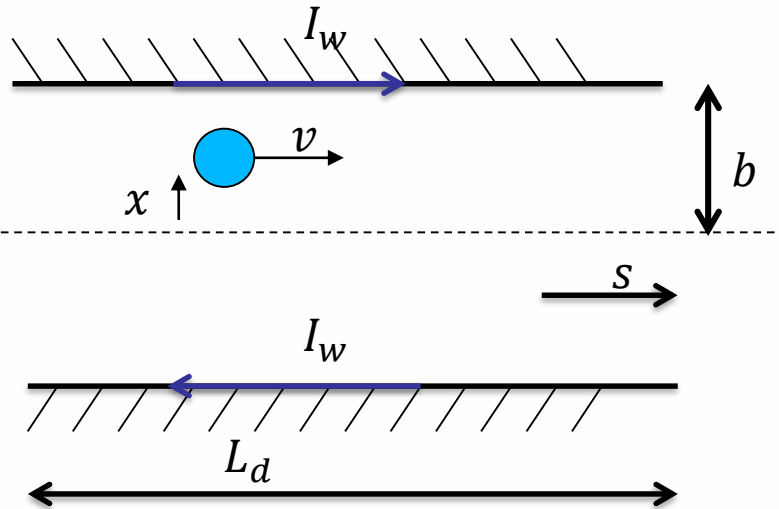


Result: No significant difference

Spectrum of band limited noise:



Transverse coupling impedances



- Effect of beam on itself through its environment
- $Im(Z_{\perp})$ defines the frequency shift of the coherent modes. (Reactive)
- $Re(Z_{\perp})$ defines the growth rate of these modes. (Resistive)

$$Z_{\perp}(\omega) = -j \frac{\int_0^{L_d} (E(s, \omega) + v \times B(s, \omega))_{\perp} ds}{\beta I x}$$

For a perfectly conducting beam pipe, it reduces to

Coherent tune shift

$$\Delta Q_c = -j \frac{q I_p R^2 Z_{\perp}}{2 Q_0 \beta_0 W_0} \propto \frac{\Delta Q_{sc} a^2}{b^2}$$

$$Z_{\perp} = -j \frac{Z_0}{2\pi \beta_0^2 \gamma_0^2 b^2}$$

Characteristic impedance