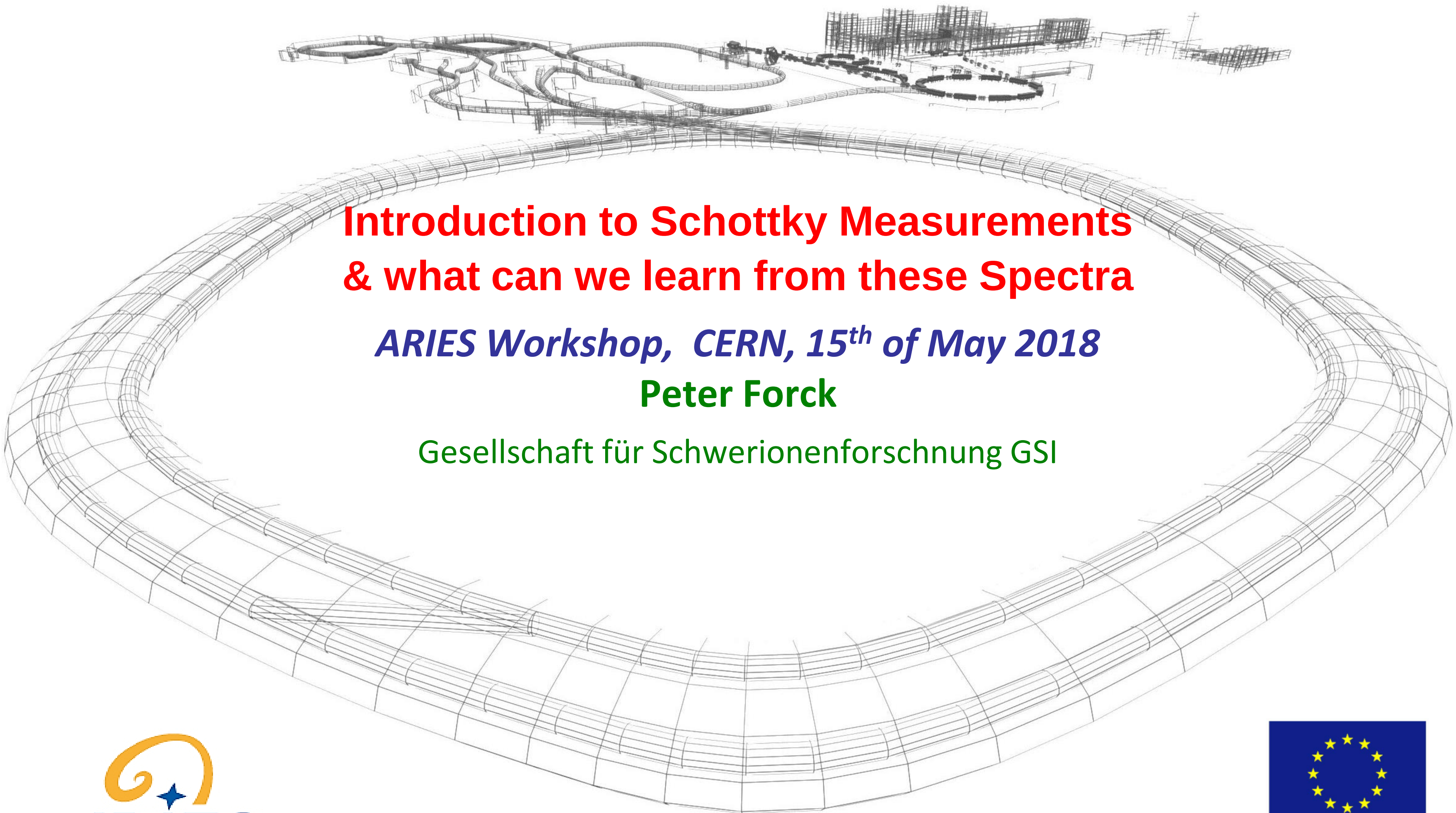


Introduction to Schottky Measurements & what can we learn from these Spectra

ARIES Workshop, CERN, 15th of May 2018

Peter Forck

Gesellschaft für Schwerionenforschung GSI

This project has received funding from the European Union’s Horizon 2020 Research and Innovation programme under Grant Agreement No 730871

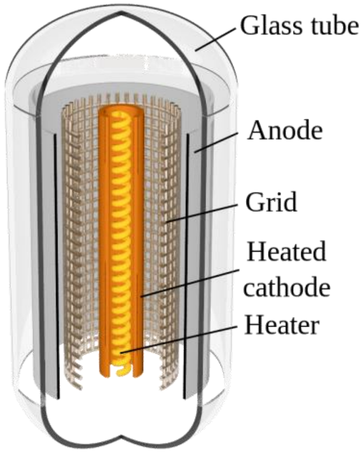
Shot Noise for free Charge Carriers (here Electrons)

Emission of electrons in a vacuum tube:

W. Schottky, ‘Spontaneous current fluctuations in various electrical conductors’, Ann. Phys. 57 (1918)
[original German title:‘Über spontane Stromschwankungen in verschiedenen Elektrizitätsleitern’]

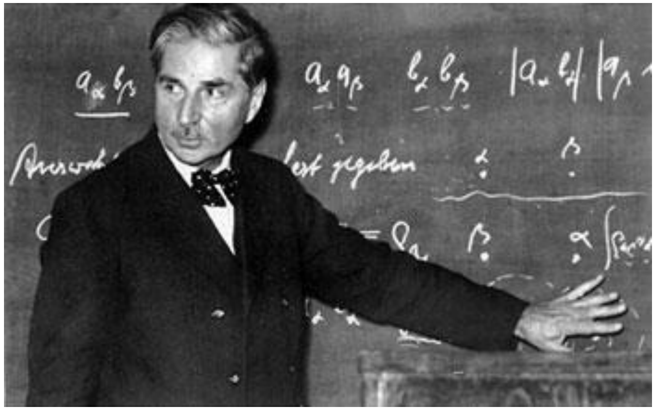
Result: Emission of electrons follows statistical law ⇒ white noise

Physical reason: Charge carrier of final mass and charge



Walter Schottky (1886 – 1976):

- German physicist at Universities Jena, Würzburg & Rostock and at company Siemens
- Investigated electron and ion emission from surfaces
- Design of vacuum tubes
- Super-heterodyne method i.e spectrum analyzer
- Solid state electronics e.g. metal-semiconductor interface called ‘Schottky diode’
- **No** connection to accelerators



Source: Wikipedia

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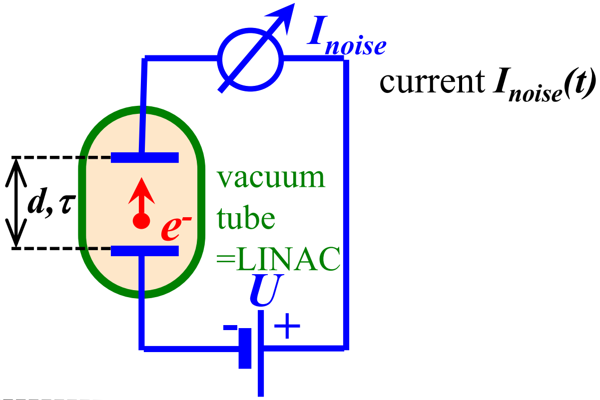
[original German title:'Über spontane Stromschwankungen in verschiedenen Elektrizitätsleitern']

Result: Emission of electrons follows statistical law ⇒ white noise

Physical reason: Charge carrier of final mass and charge for **single pass** arrangement
Assuming: charges of quantity e, N average charges per time interval and τ duration of travel

fluctuations as $I_{noise} = \sqrt{< I^2 >} = \sqrt{\frac{e^2 \cdot N}{\tau}} \propto \sqrt{N}$

$\Leftrightarrow \frac{I_{noise}}{I_{tot}} \propto \sqrt{1/N}$, I_{tot} is total current



This is white noise i.e. flat frequency spectrum

It is called **shot noise !**

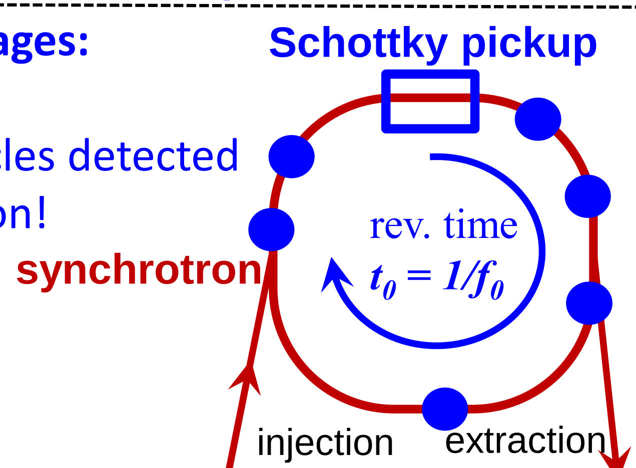
'Schottky signals' in circular accelerators of multiple passages:

This is **not** shot noise!

But the **fluctuations** caused by randomly distributed particles detected by the correlation of their **repeating** passage at one location!

⇒ The frequency spectrum has bands i.e. not flat

Schottky signal analysis: Developed at CERN ISR ≈ 1970th
for operation of stochastic cooling



Outline:

- Introduction to noise and fluctuations relevant for Schottky analysis
- Main part: Schottky signal generation and examples for:
  - **Longitudinal for coasting beams**
  - Transverse for coasting beams
  - Longitudinal for bunched beams
  - Transverse for bunched beams
- Some further examples for exotic beam parameters
- Conclusion

Remark:

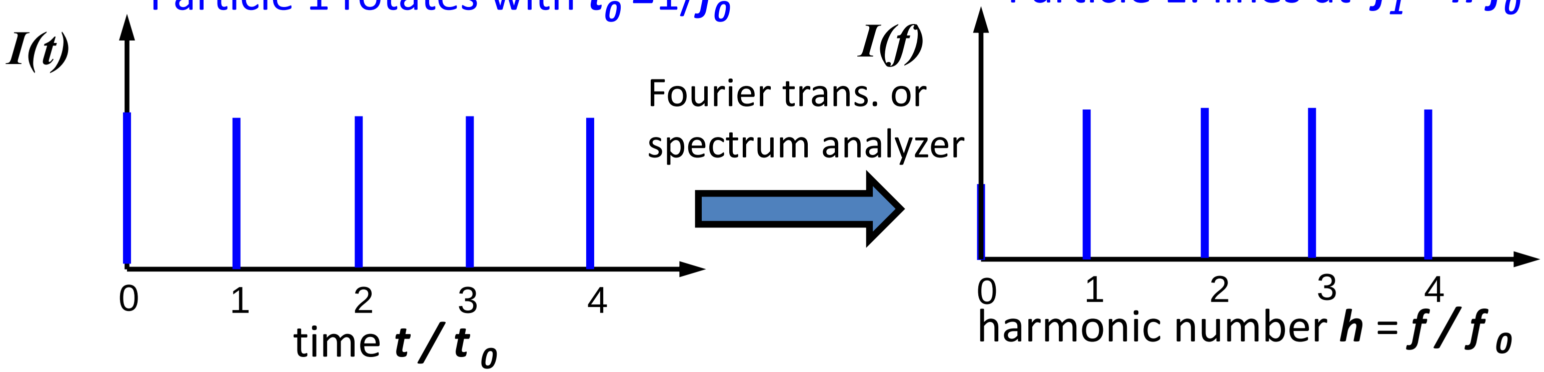
Assumption for the considered cases (if not stated otherwise):

- **Equal & constant synchrotron frequency for all particles $\Rightarrow \Delta f_{syn} = 0$**
- **No interaction between particles (e.g. space charge) $\Rightarrow$ no incoherent effect e.g. $\Delta Q_{incoh} = 0$**
- **No contributions by wake fields $\Rightarrow$ no coherent effects by impedances e.g. $\Delta Q_{coh} = 0$**

Longitudinal Schottky Analysis: 1st Step

Schottky noise analysis is based on the power spectrum
for consecutive passage of the **same** finite number of particles

Particle 1 rotates with $t_0 = 1/f_0$ Particle 1: lines at $f_1 = h \cdot f_0$



Particle 1 of charge e rotates with $t_1 = 1/f_0$:

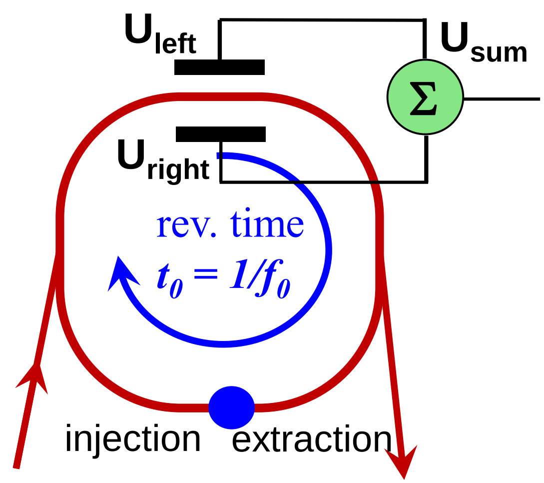
Current at pickup $I_1(t) = e f_0 \cdot \sum_{h=-\infty}^{\infty} \delta(t - h t_0)$

$$\Rightarrow I_1(f) = e f_0 + 2 e f_0 \cdot \sum_{h=1}^{\infty} \delta(f - h f_0)$$

i.e. frequency spectrum comprise of δ-functions at $h \cdot f_0$

This can be proven by **Fourier Series** for periodic signals
(and display of positive frequencies only)

Schottky pickup



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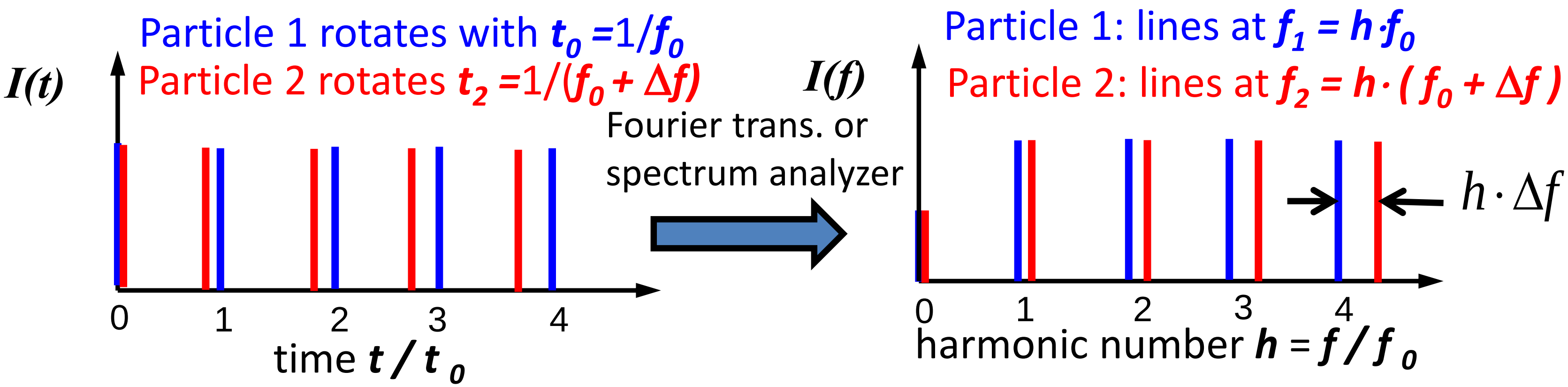


Particle 1 of charge ***e*** rotates with $t_1 = 1/f_0$ **:**

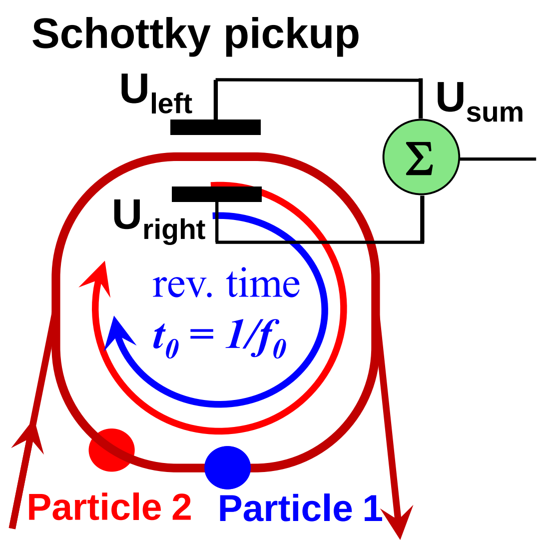
Current at pickup $I_1(t) = ef_0 \cdot \sum_{h=-\infty}^{\infty} \delta(t - ht_0)$

$$\Rightarrow I_1(f) = ef_0 + 2ef_0 \cdot \sum_{h=1}^{\infty} \delta(f - hf_0)$$

Particle 2 of charge ***e*** rotating with $t_2 = 1/(f_0 + \Delta f)$ **:**

Current at pickup $I_2(t) = ef_0 \cdot \sum_{h=-\infty}^{\infty} \delta(t - ht_2)$

$$\Rightarrow I_2(f) = ef_0 + 2ef_0 \cdot \sum_{h=1}^{\infty} \delta(f - h \cdot [\, f_0 + \Delta f \,])$$



Important result for 1st step:

- The **entire** information is available around all harmonics
- The distance in frequency domain scales with $h \cdot \Delta f$

Longitudinal Schottky Analysis: 2nd Step

Averaging over many particles for a coasting beam:

Assuming ***N* randomly** distributed particles characterized by phase θ_1, θ_2, θ_3 ... θ_N
with **same** revolution time $t_0 = 1/f_0 \Leftrightarrow$ same revolution frequency f_0

The total beam current is: $I(t) = ef_0 \sum_{n=1}^{N} \cos\theta_n + 2ef_0 \sum_{n=1}^{N} \sum_{h=1}^{\infty} \cos(2\pi f_0 h t + h\theta_n)$

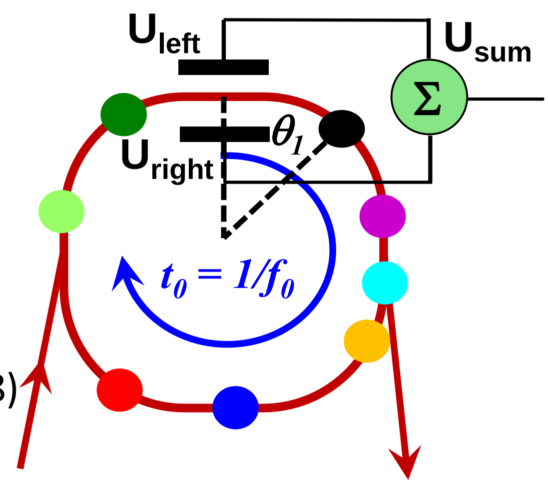
For observations much longer than one turn: average current $\langle I \rangle_h = 0$ for **each** harm. $h \neq 1$
but In a band around **each** harmonics ***h*** the *rms* current $I_{rms}(h) = \sqrt{\langle I^2 \rangle_h}$ remains:

$$\langle I^2 \rangle_h = \left(2ef_0 \sum_{n=1}^{N} \cos(h\theta_n) \right)^2 = (2ef_0)^2 \cdot (\cos h\theta_1 + \cos h\theta_2 + \ldots \cos h\theta_N)^2$$

$$\equiv (2ef_0)^2 \cdot N \langle \cos^2 h\theta_i \rangle = (2ef_0)^2 \cdot N \cdot \frac{1}{2} = 2\,e^2 f_0^2 \cdot N \quad \text{due to the random phases } \theta_n$$

The power at each harmonic ***h*** is: $P_h = Z_t \langle I^2 \rangle_h = 2\,Z_t\, e^2 f_0^2 \cdot N$

measured with a pickup of transfer impedance Z_t



Important result for 2nd step:

- The **integrated** power in each band is constant and $P_h \propto N$

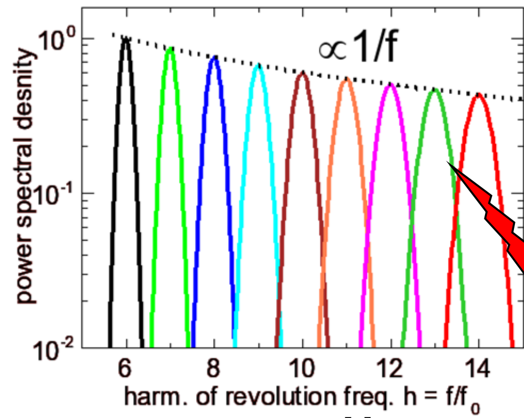
Remark: Random distribution is connected to shot noise & W. Schottky (1918)
Regular BPM processing for bunched beams: $P_h^{BPM} \propto N^2$

Introducing a frequencies distribution for many particles:

The dependence of the distribution per band is: $\frac{dP_h}{df} = Z_t \cdot \frac{d}{df}\langle I^2 \rangle_h = 2Z_t e^2 f_0^2 N \cdot \frac{dN}{df}$

Inserting the acc. quantity $\frac{df}{f_0} = h\,\eta \cdot \frac{dp}{p_0}$ leads to : $\frac{dP_h}{df} = 2Z_t e^2 p_0 N \cdot \frac{f_0}{h} \cdot \frac{1}{\eta} \cdot \frac{dN}{dp}$

Important results from 1st to 3rd step:



Example:
Gaussian
Δ*p*/*p* = 2%
η = 1

- The power spectral density $\frac{dP_h}{df}$ in **each** band

  reflects the particle's **momentum distribution**: $\frac{dP_h}{df} \propto \frac{dN}{dp}$

- The maxima of each band scales $\left.\frac{dP_h}{df}\right]_{max} \propto \frac{1}{h}$

  Measurement: **Low** ***f*** preferred for good signal-to-noise ratio

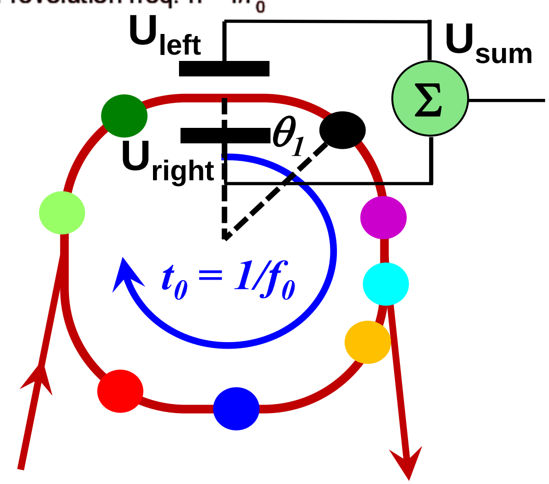
- The width increase for each band: $\frac{dP_h}{df} \propto h$

  Measurement: **High** ***f*** preferred for good frequency resolution

- The power scales only as $\frac{dP_h}{df} \propto N$ due to random phases of particles

  i.e. <u>**in**</u>coherent single particles' contribution

- For ions A^{q+} the power scales $\frac{dP_h}{df} \propto q^2 \Rightarrow$ larger signals for ions

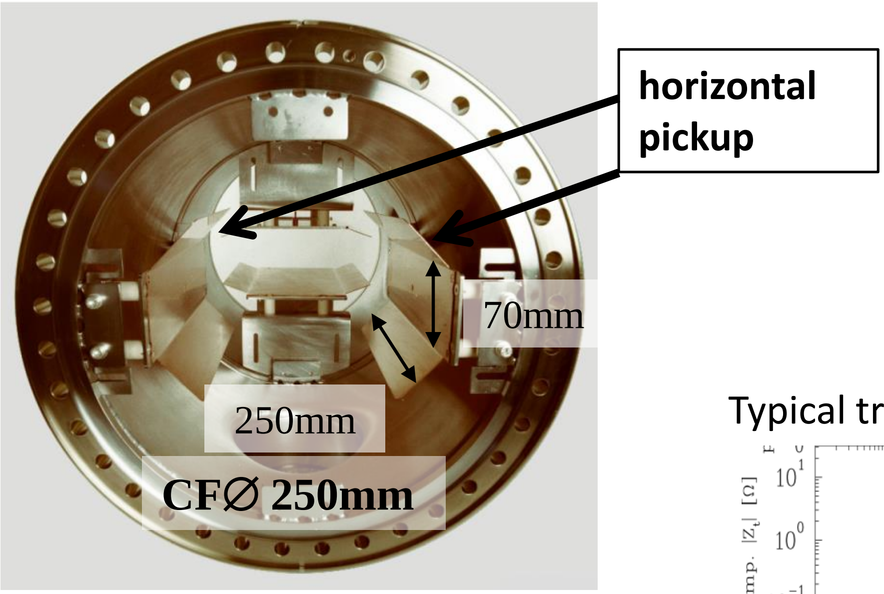


Remark: The 'power spectral density' $\frac{dP_h}{df}$ is called only 'power' P_h below

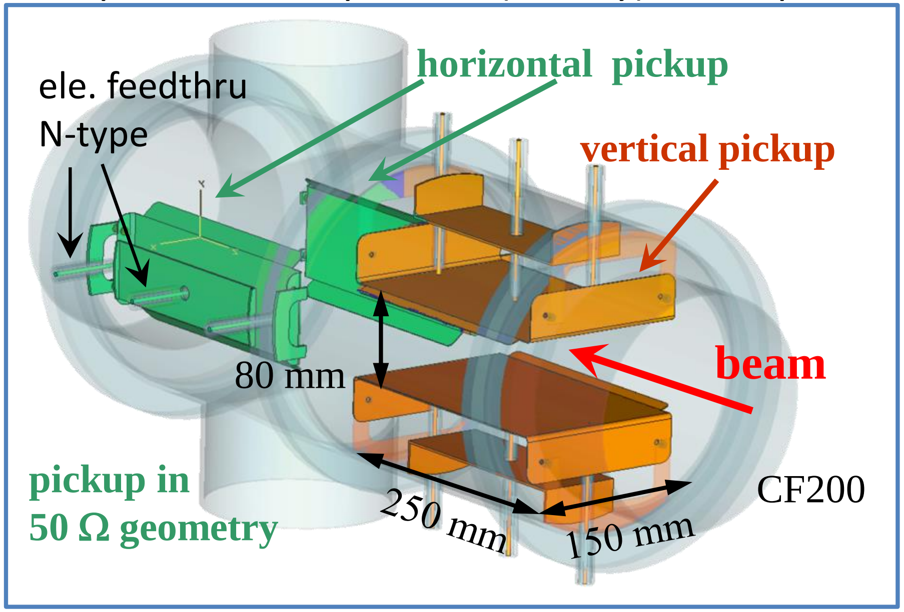
Pickup for Schottky Signals: Capacitive Pickup

A Schottky pickup can be like a capacitive BPM:
- Typ. 20...50 cm insertion length
- High position sensitivity for transverse Schottky
- Allows for broadband processing
- Linearity for position **not** important

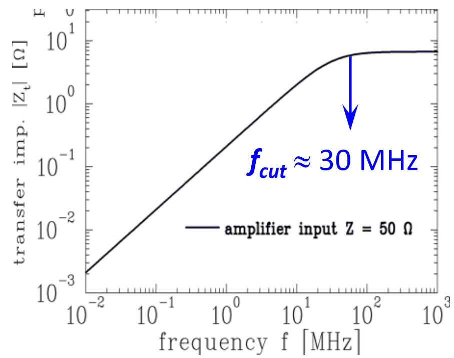
Example: Schottky pickup at GSI synhrotron



Example: 50 Ω Schottky for HIT, Heidelberg operated as capacitive (mostly) or strip-line



Typical transfer impedance



Challenge for electronics:
- Low noise amplifier
- Multi stage amplifier:
  prevent for signal saturation

Transfer impedance:

Coupling to beam $U_{signal} = Z_t \cdot I_{beam}$

Typically $Z_t \approx 1\ \Omega$, $R = 50\ \Omega$, $C \approx 100$ pF

$\Rightarrow f_{cut} = (RC)^{-1} \approx 30$ MHz

$\Rightarrow$ operation rang $f = 30 \dots 200$ MHzi.e. above f_{cut} & below signal distortion

Longitudinal Schottky for Momentum Spread $\Delta p/p_0$ Analysis

Momentum spread $\Delta p/p_0$ measurement after multi-turn injection & de-bunching of t < 1ms duration to stay within momentum acceptance during acceleration

Method: Variation of buncher voltage

i.e. rotation in longitudinal phase space

→ minimizing of momentum spread $\Delta p/p_0$

Example: 10^{10} U^{28+} at 11.4 MeV/u

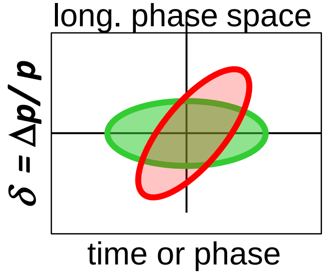
injection plateau 150 ms, η = 0.94

Longitudinal Schottky at harmonics h = 117

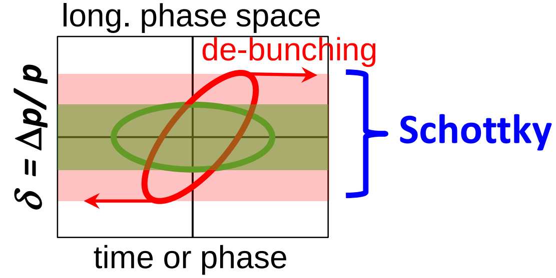
Momentum spread variation:

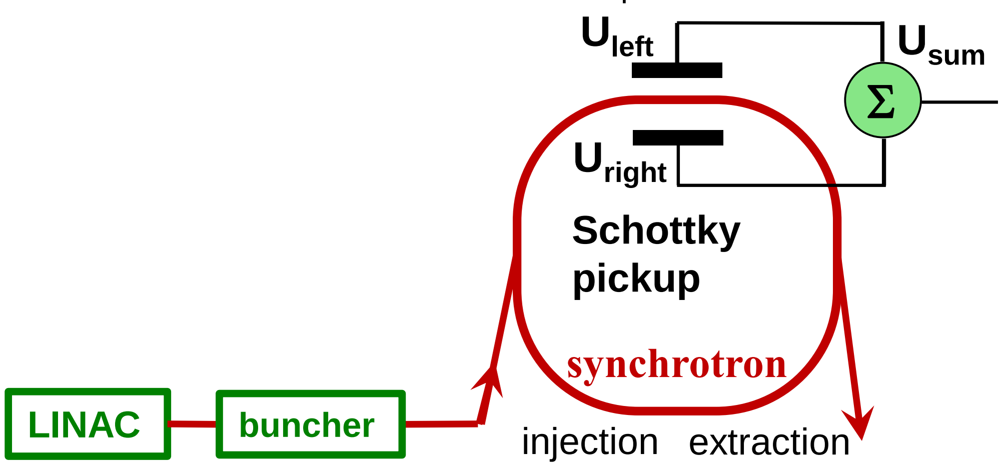
$\Delta p/p \approx (0.6 \ldots 2.5) \cdot 10^{-3}$ (1σ)

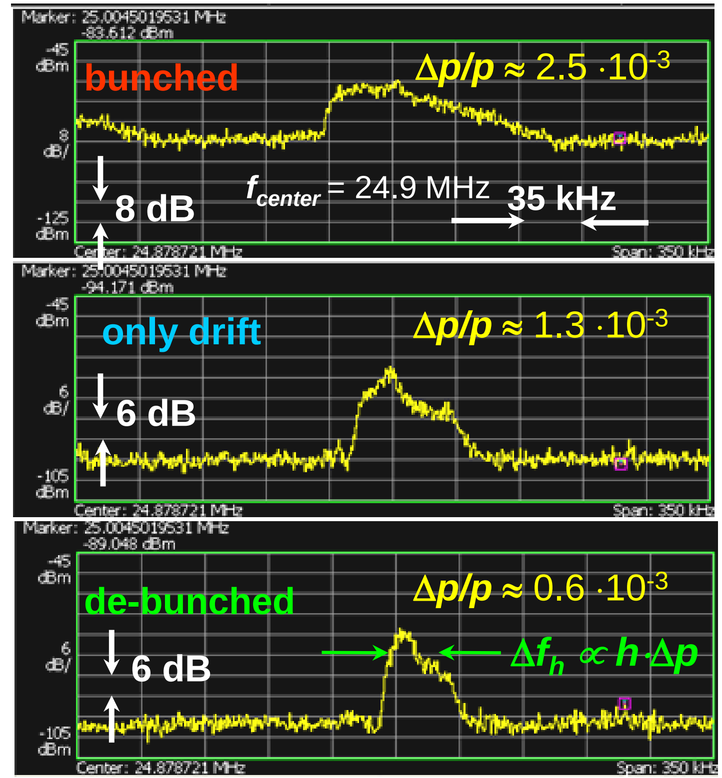
LINAC bunches at injection:



De-bunching after some ms:

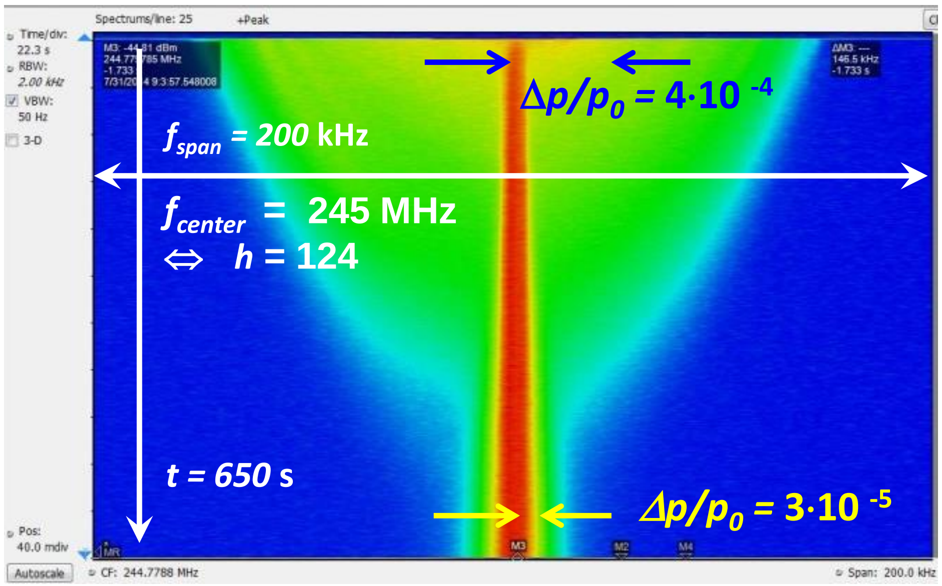




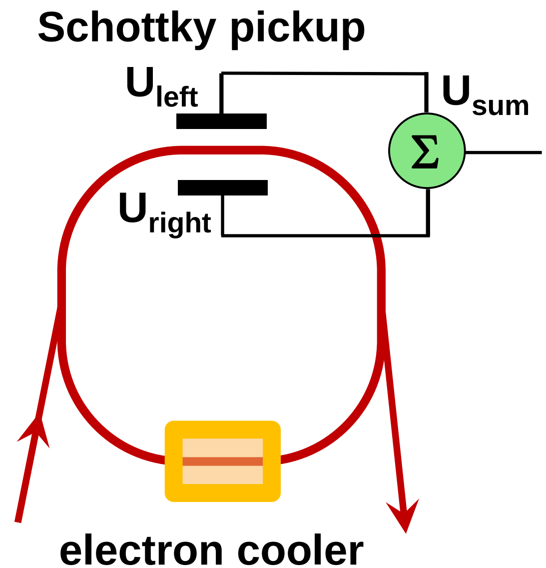


Electron Cooling: Monitoring of Cooling Process

Example: Observation of cooling process at GSI storage ring
Ion beam: 10^8 protons at 400 MeV
Electron beam $\boldsymbol{I_{ele}}$ = 250 mA
Momentum spread (1σ): $\Delta p/p = 4 \cdot 10^{-4} \rightarrow 3 \cdot 10^{-5}$ within 650 s



J. Roßbach et al., Cool 2015, p. 136 (2015)



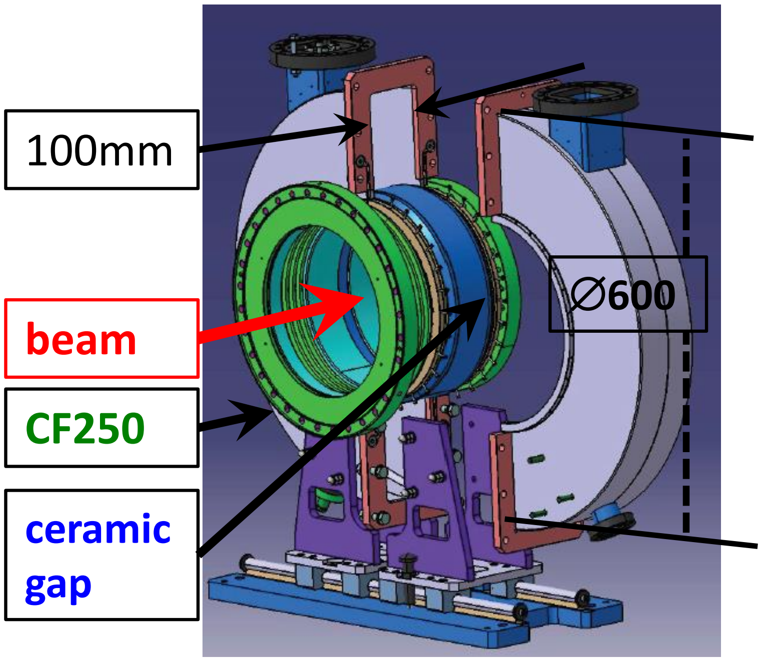
Application:

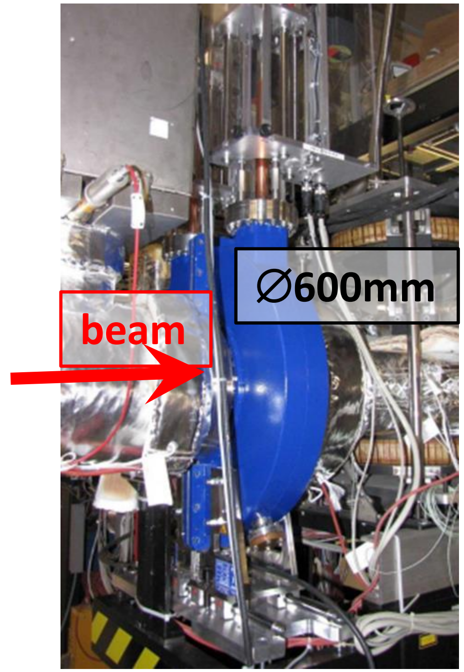
- Alignment of cooler parameter and electron-ion overlap
- Cooling force & intra-beam scattering measurement

Pillbox Cavity for vey low Detection Threshold

Enhancement of signal strength by a cavity

Example: Pillbox cavity at GSI and Lanzhou storage ring for with variable frequency

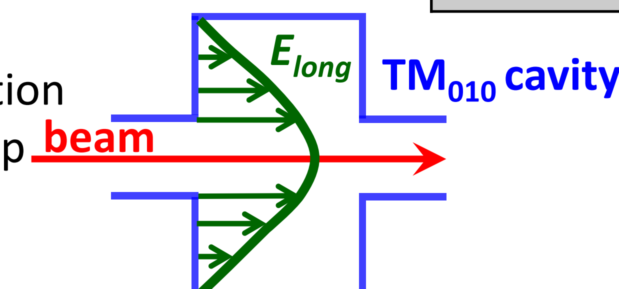




Outer $\varnothing_{out}$	600 mm
Beam pipe $\varnothing_{in}$	250 mm
Mode (monopole)	TM_{010}
Res. freq. f_{res} Variable by plunger	≈ 244 MHz ± 2 MHz
Quality factor Q_0	≈ 1100
Loaded Q_l	≈ 550
R/Q_0	≈ 30 Ω
Coupling	Inductive loop

Advantage:

- Sensitive down to single ion observation
- Part of cavity in air due to ceramic gap
- Can be sort-circuited to prevent for wake-field excitation

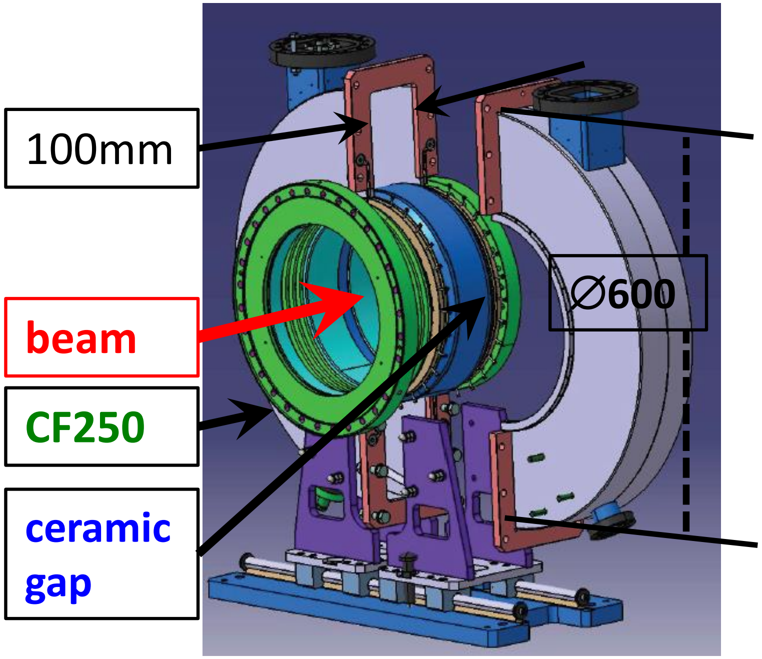


F. Nolden et al., NIM A 659, p.69 (2011), F. Nolden et al., DIPAC'11, p.107 (2011), F. Suzaki et al., HIAT'15, p.98 (2015)
For RHIC design: W. Barry et al., EPAC'98, p. 1514 (1998), K.A. Brown et al., Phys. Rev. AB, 12, 012801 (2009)

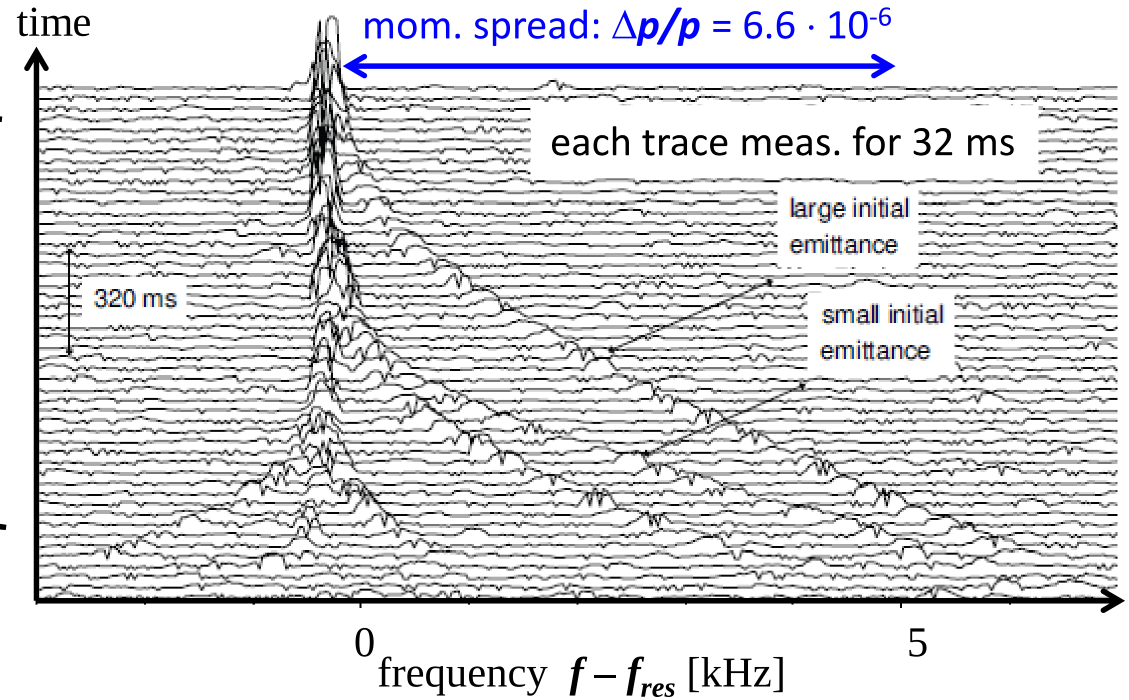
Pillbox Cavity for single Ion Detection

Observation of *single* ions is possible:

Example: Storage of **six** $^{142}Pm^{59+}$ at 400 MeV/u during electron cooling



f_{res} = 244.965 MHz



Application:

- Single ion observation for basic accelerator research
- Observation of radio-active nuclei for life time and mass measurements

F. Nolden et al., NIM A 659, p.69 (2011), F. Nolden et al., DIPAC'11, p.107 (2011), F. Suzaki et al., HIAT'15, p.98 (2015)

Outline of the tutorial:

- Introduction to noise and fluctuations relevant for Schottky analysis
- Main part: Schottky signal generation and examples for the following case:
  - Longitudinal for coasting beams
  - **Transverse for coasting beams**
  - Longitudinal for bunched beams
  - Transverse for bunched beams
- Some further examples for exotic beam parameters
- Conclusion

Observation of the difference signal of two pickup electrodes:

Betatron motion by a single particle 1 at Schottky pickup:

Displacement: $\boldsymbol{x_1(t)} = A_1 \cdot \cos(2\pi q f_0 t)$

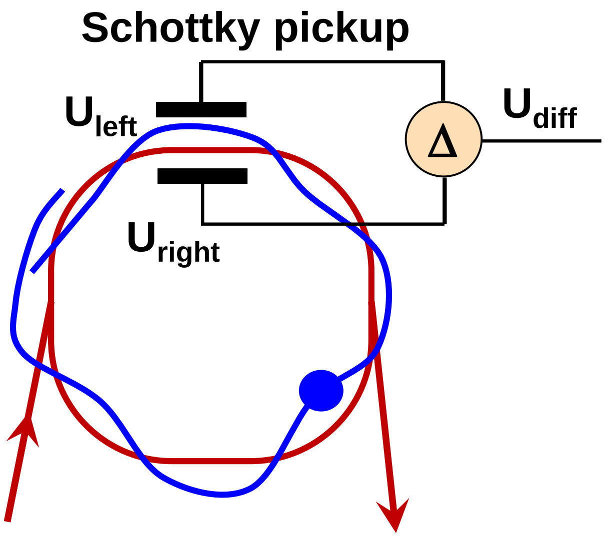
A₁: single particle trans. amplitude

q: non-integer part of tune

Dipole moment: $\boldsymbol{d_1(t)} = x_1(t) \cdot \; I(t)$

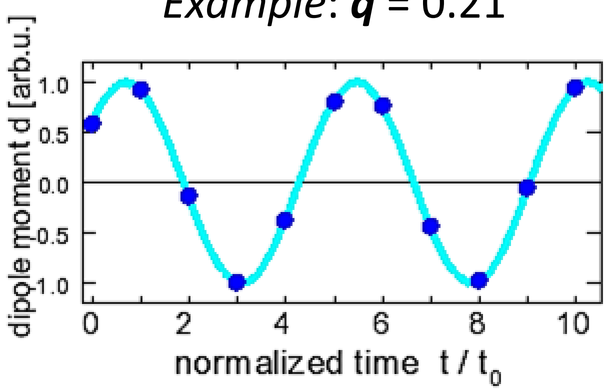
transverse part equals 'signal'

longitudinal part equals 'carrier'



Pickup voltage: $\boldsymbol{U_1(t)} = Z_\perp \cdot \; d_1(t)$

Example: ***q*** = 0.21



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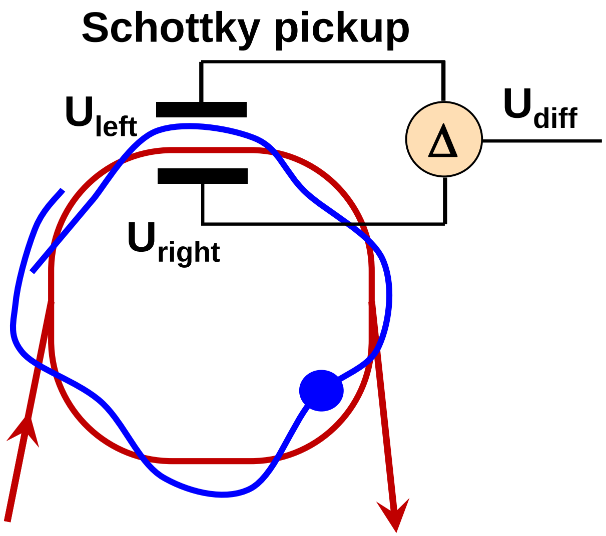


Dipole moment: $\boldsymbol{d_1(t)} = x_1(t) \cdot I(t)$

transverse part equals 'signal'

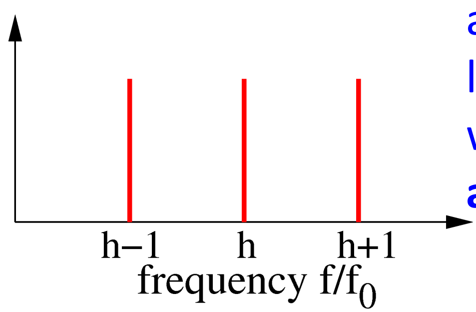
longitudinal part equals 'carrier'

Inserting longitudinal Fourier series: $\boldsymbol{d_1(f)} =$

$$e f_0 \cdot A_1 + 2 e f_0 A_1 \cdot \sum_{h=1}^{\infty} \cos(2\pi q f_0 t) \cdot \cos(2\pi h f_0 t)$$

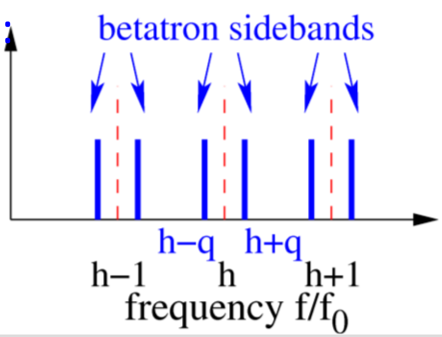
$$= e f_0 \cdot A_1 + e f_0 A_1 \cdot \sum_{h=1}^{\infty} \cos(2\pi[h-q] f_0 t) \cdot \cos(2\pi[h+q] f_0 t)$$

longitudinal Schottky



amplitude modulation:
left & right sideband
with distance ***q***
at each harmonics

transverse Schottky



Principle of Amplitude Modulation

Composition of two waves:

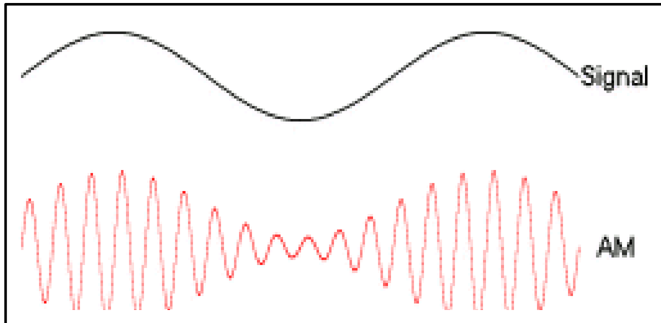
- **Carrier:** For synchrotron→ revolution freq. $f_0 = 1/t_0$

  $U_c(t) = \hat{U}_C \cdot \cos(2\pi f_0 t)$

- **Signal:** For synchrotron→ betatron frequency $f_\beta = q \cdot f_0$

  $q < 1$ non-integer part of tune $Q = n + q$

  $U_\beta(t) = \hat{U}_\beta \cdot \cos(2\pi q f_0 t)$



Source: wikipedia

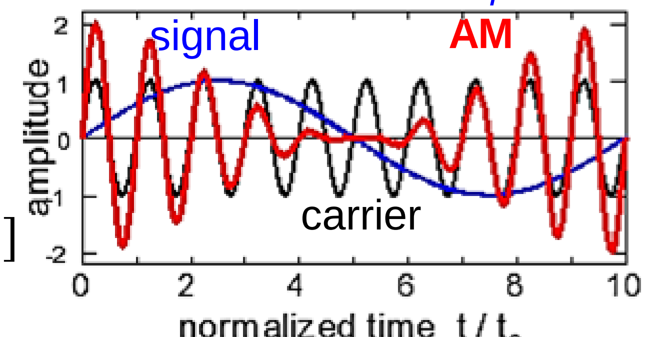
Amplitude multiplication of both signals $m_\beta = \frac{\hat{U}_\beta}{\hat{U}_C} = 1$

$$\Rightarrow U_{tot}(t) = \left[\hat{U}_C + \hat{U}_\beta \cdot \cos(2\pi q f_0 t)\right] \cdot \cos(2\pi f_0 t)$$

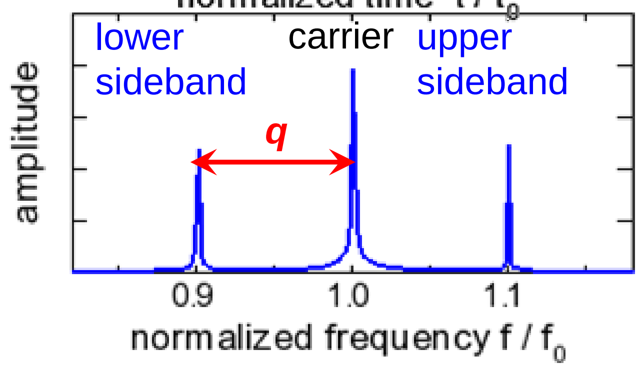
$$= \hat{U}_C \cdot \cos(2\pi f_0 t)$$

$$+ {}^1/_2\, \hat{U}_\beta \cdot [\, \cos(2\pi[1-q] f_0 t) + \cos(2\pi[1+q] f_0 t)\,]$$

Example: $q = 0.1$, $\hat{U}_\beta = \hat{U}_C$



Using: $\cos(x) \cdot \cos(y) = {}^1/_2 [\cos(x-y) + \cos(x+y)]$



Remark:

Pickup difference signal $\Rightarrow$ central carrier peak vanish

if beam well centered in pickup

Transverse Spectrum for a coasting Beam: Many Particles

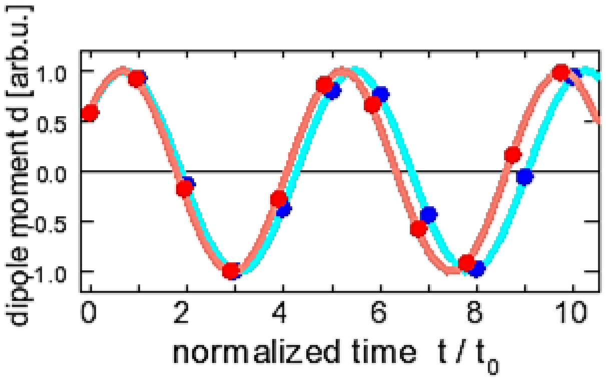
Observation of the difference signal of two pickup electrodes:

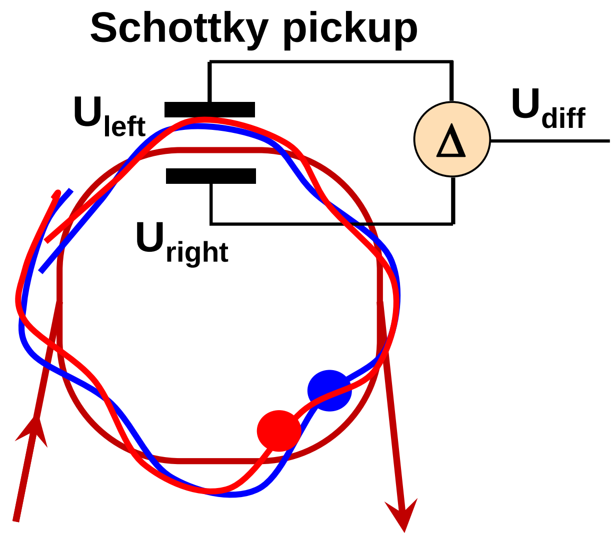
Betatron motion by two particles at pickup:

Displacements: $x_1(t) = A_1 \cdot \cos(2\pi q_1 f_0 t)$

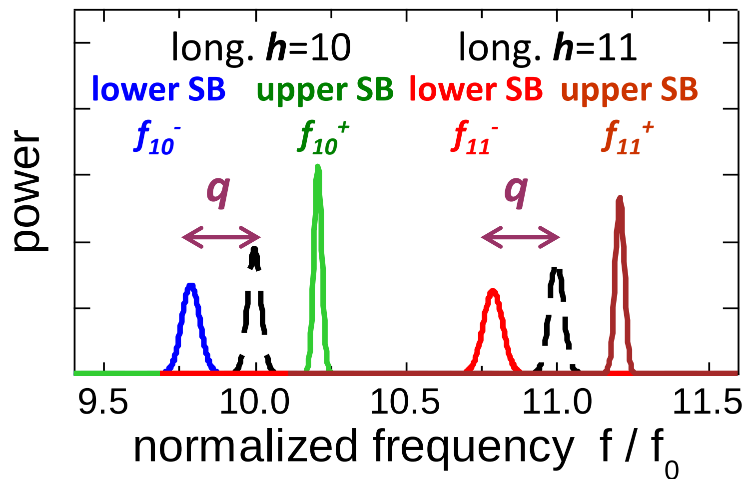
: $x_2(t) = A_2 \cdot \cos(2\pi q_2 f_0 t + \varphi_2)$

Example: $q_1 = 0.21$ & : $q_2 = 0.26$





Example: $Q = 4.21$, $\Delta p/p_0 = 2\cdot 10^{-3}$, $\eta = 1$, $\xi = -1$



Transverse Schottky band for a distribution:

- Amplitude modulation of longitudinal signal (i.e. 'spread of carrier')
- Two sideband centered at $f_h^{\pm} = (h \pm q)\cdot f_0$
  $\Rightarrow$ tune measurement
- The width is unequal for both sidebands (see below)
- The integrated power is constant (see below)

Example for Tune Measurement using transverse Schottky

Example of a transverse Schottky spectrum:

- Wide scan with lower and upper sideband
- Tune from central position of both sidebands

$$q = h \cdot \frac{f_h^+ - f_h^-}{f_h^+ + f_h^-}$$

- Sidebands have different shape
- Tune measurement without beam influence

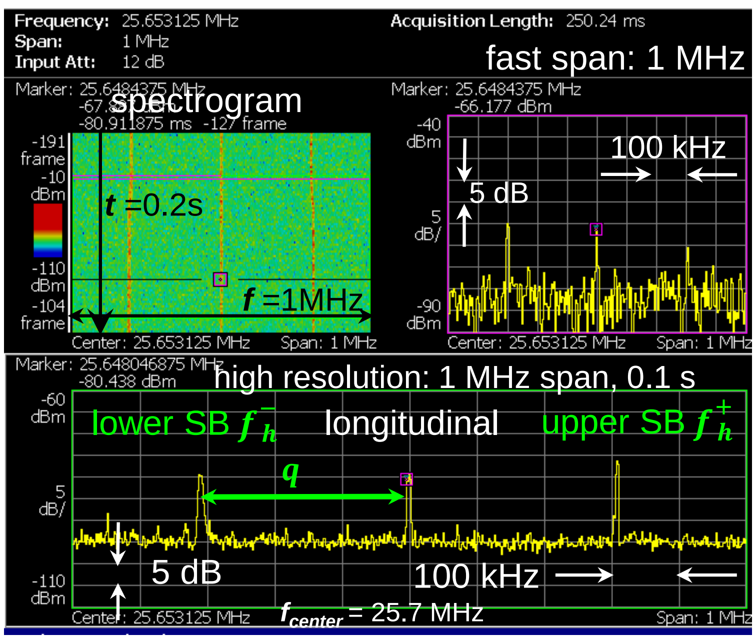
⇒ usage during regular operation

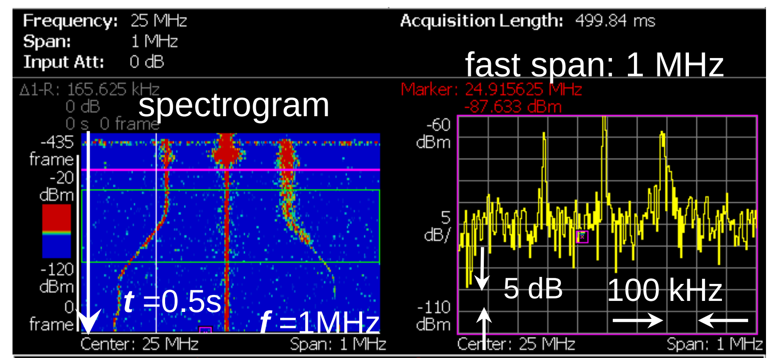
Example: Horizontal tune Q_h = 4.161 → 4.305 within 0.3 s for preparation of slow extraction

Beam Kr^{33+} at 700 MeV/u,

f_0 = 1.136 MHz ⇔ h = 22

Characteristic movements of sidebands visible





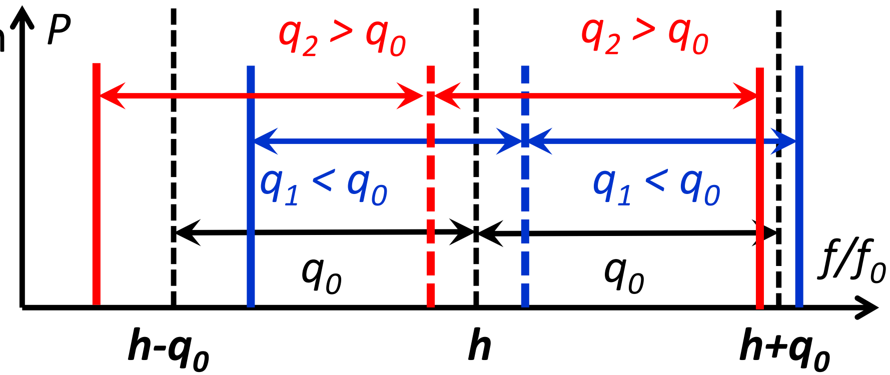
Sideband Width for a coasting Beam

Calculation of the sideband width:

The sidebands at $f_h^{\pm} = (h \pm q) \cdot f_0$ comprises of

- Longitudinal spread expressed via momentum
  $$\frac{\Delta f}{f_0} = \eta \cdot \frac{\Delta p}{p_0}$$ (η: freq. dispersion)
- Transverse tune spread $\Delta Q = \Delta q$
  for low current dominated by chromaticity
  $$\frac{\Delta q}{Q_0} = \xi \cdot \frac{\Delta p}{p_0} = \frac{\xi}{\eta} \cdot \frac{\Delta f}{f_0}$$

Depictive Example: $\eta = 1$, $\xi = -1$



Reference particle: tune q_0

Particle 1 with $p_1 > p_0 \Rightarrow q_1 = q_0 - |\xi \cdot \Delta p_1/p_0| < q_0$

Particle 2 with $p_2 < p_0 \Rightarrow q_2 = q_0 + |\xi \cdot \Delta p_2/p_0| > q_0$

Sideband Width for a coasting Beam

Calculation of the sideband width:

The sidebands at $f_h^{\pm} = (h \pm q) \cdot f_0$ comprises of

- Longitudinal spread expressed via momentum
  $$\frac{\Delta f}{f_0} = \eta \cdot \frac{\Delta p}{p_0} \quad (\eta\text{: freq. dispersion})$$
- Transverse tune spread $\Delta Q = \Delta q$
  for low current dominated by chromaticity
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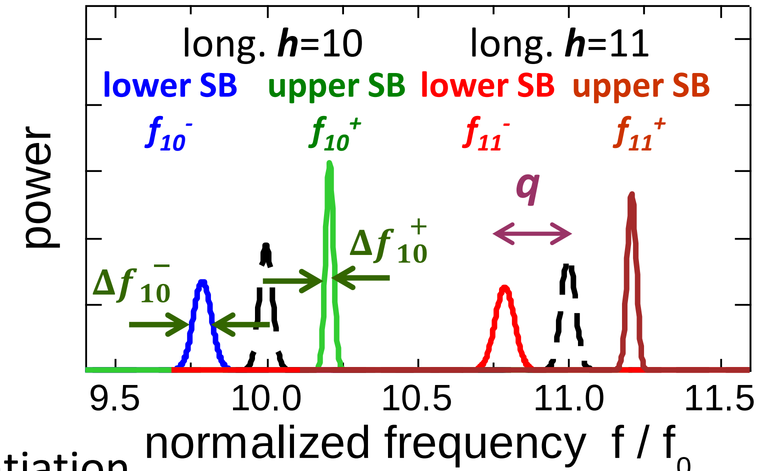
Example: $Q = 4.21$, $\Delta p/p_0 = 2 \cdot 10^{-3}$, $\eta = 1$, $\xi = -1$



Using $f_h^{\pm} = (h \pm q) \cdot f_0$ & product rule for differentiation

$\Rightarrow$ lower sideband : $$\Delta f_h^- = (h-q) \cdot \Delta f_h - \Delta q \cdot f_0 = \eta \frac{\Delta p}{p_0} \cdot f_0 \left(h - q - \frac{\xi}{\eta} Q_0\right)$$

$\Rightarrow$ upper sideband: $$\Delta f_h^+ = (h+q) \cdot \Delta f_h + \Delta q \cdot f_0 = \eta \frac{\Delta p}{p_0} \cdot f_0 \left(h + q + \frac{\xi}{\eta} Q_0\right)$$

long. part — trans. — chromatic coupling

Results:

- Sidebands have different width in dependence of Q_0, η and ξ
  i.e. 'longitudinal ± transverse ± coupling' $\Rightarrow$ 'chromatic tune'
- The width measurement can be used for chromaticity ξ measurements

Power per Band for a coasting Beam & transverse *rms* Emittance

Dipole moment for a harmonics ***h*** for a particle with betatron amplitude $\boldsymbol{A_n}$:

$$d_n(hf) = 2ef_0A_n \cdot \cos(2\pi q f_0 t + \theta_n) \cdot \cos(2\pi h f_0 t + \varphi_n)$$

Averaging over betatron phase θ_n and spatial distribution for the ***n = 1…N*** particles:

$$\Rightarrow \langle d^2 \rangle = e^2 {f_0}^2 \cdot N/2 \cdot \langle A^2 \rangle \cdot N/2$$

with $\langle A^2 \rangle \equiv {x_{rms}}^2 = \varepsilon_{rms}\beta$ square of average transverse amplitudes

$$\Rightarrow {P_h}^{\pm} \propto \langle d^2 \rangle = e^2 {f_0}^2 \cdot \frac{N}{2} \cdot \varepsilon_{rms}\beta$$ with ε_{rms} transvers emittance and β-function at pickup

Results:

- Power ${P_h}^{\pm}$ is the same at each harmonics ***h***
- Power decreases for lower emittance beams (due to decreasing modulation power)

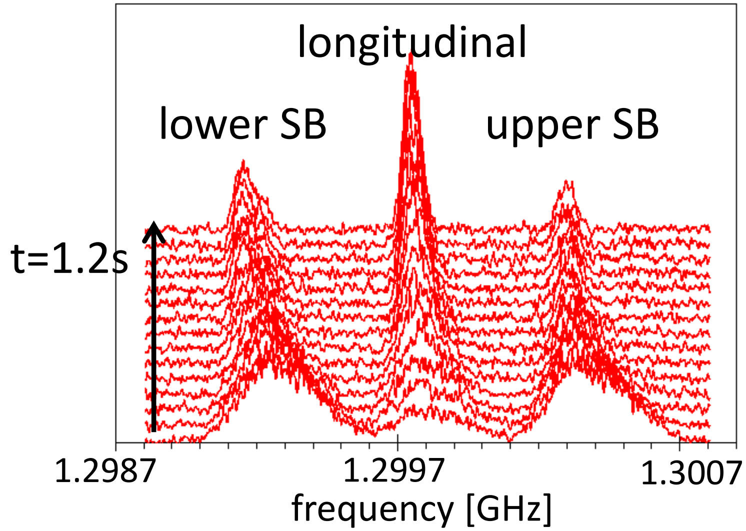
$\Rightarrow$ measurement of rms emittance is possible.

*Example for s*ideband behavior*:*

Emittance shrinkage during stochastic cooling at GSI

- Width: smaller due to longitudinal cooling
- Height: ≈ constant due to transverse cooling
- Power ${P_h}^{\pm}$ decreases $\Rightarrow$ emittance determination,

  but requires normalization by profile monitor



F. Nolden , DIPAC'01, p. 6 (2001)

Movable Schottky cavity at RHIC $\Rightarrow$ absolute calibration for $\boldsymbol{\varepsilon}$
see K.A. Brown et al., Phys. Rev. AB, 12, 012801 (2009),
W. Barry et al., EPAC'98, p. 1514 (1998)

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Frequency modulation by composition of two waves:

- **Carrier:** For synchrotron → revolution freq. $f_0 = 1/t_0$

  $U_c(t) = \hat{U}_C \cdot \cos(2\pi f_0 t)$

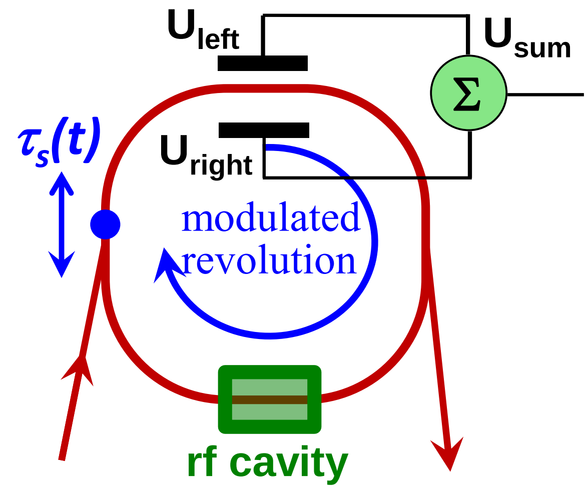
- **Signal:** For synchrotron → synchrotron freq. $f_s = Q_s \cdot f_0$

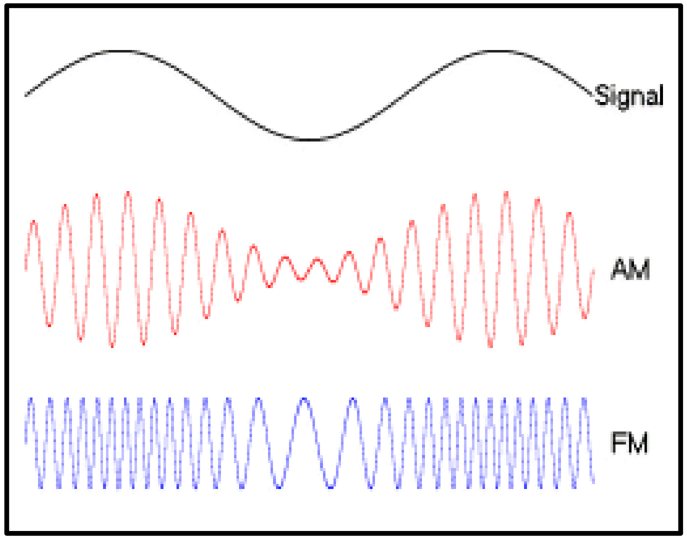
  $Q_s << 1$ synchrotron tune i.e. long. oscillations per turn

  $\tau_s(t) = \hat{\tau}_s \cdot \cos(2\pi f_s t)$

Frequency modulation is: $U_{tot}(t) = \hat{U}_C \cdot \cos\left(2\pi f_0 t + m_s \cdot \int_0^t \tau_s(t')dt'\right)$

$$= \hat{U}_C \cdot \cos\left(2\pi f_0 t + \frac{m_s \hat{\tau}_s}{2\pi f_s} \cdot \sin(2\pi f_s t)\right)$$



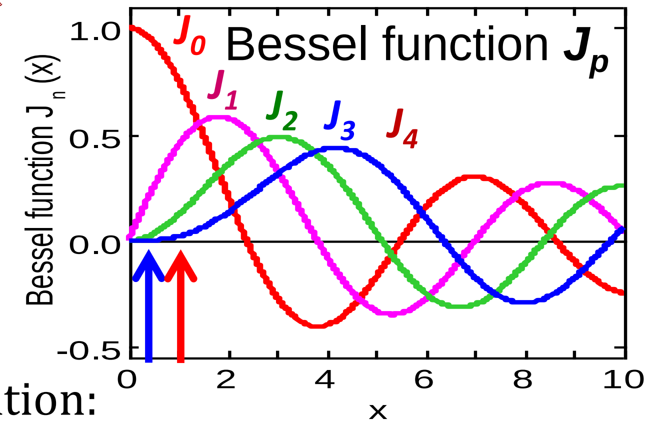


Source: wikipedia

Bunched Beam: Longitudinal Schottky Spectrum for a single Particle

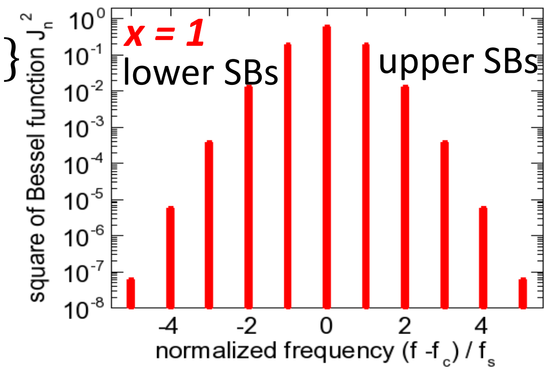
Frequency modulation by composition of two waves:

- **Carrier:** For synchrotron → revolution freq. $f_0 = 1/t_0$
  $U_c(t) = \hat{U}_C \cdot \cos(2\pi f_0 t)$
- **Signal:** For synchrotron → synchrotron freq. $f_s = Q_s \cdot f_0$
  $Q_s << 1$ synchrotron tune i.e. long. oscillations per turn
  $\tau_s(t) = \hat{\tau}_s \cdot \cos(2\pi f_s t)$



Modification of coasting beam case by synchrotron oscillation:

$$I_1(t) = ef_0 + 2ef_0 \sum_{h=0}^{\infty} \cos\{2\pi h f_0 [t + \hat{\tau}_s \cos(2\pi f_s t + \psi)]\}$$

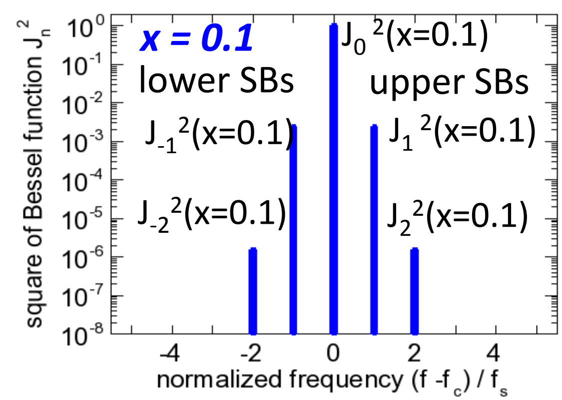


Each harmonics h comprises of lower and upper sidebands:

$$\sum_{p=-\infty}^{\infty} J_p(2\pi h f_0 \hat{\tau}_s) \cdot \cos(2\pi h f_0 t + 2\pi p f_s t + p\psi)$$

For ***each*** revolution harmonics ***h*** the longitudinal is split

- Central peak at hf_0 with height $J_0(2\pi \cdot hf_0 \cdot \hat{\tau}_s)$
- Sidebands at $hf_0 \pm pf_s$ with height $J_p(2\pi \cdot hf_0 \cdot \hat{\tau}_s)$

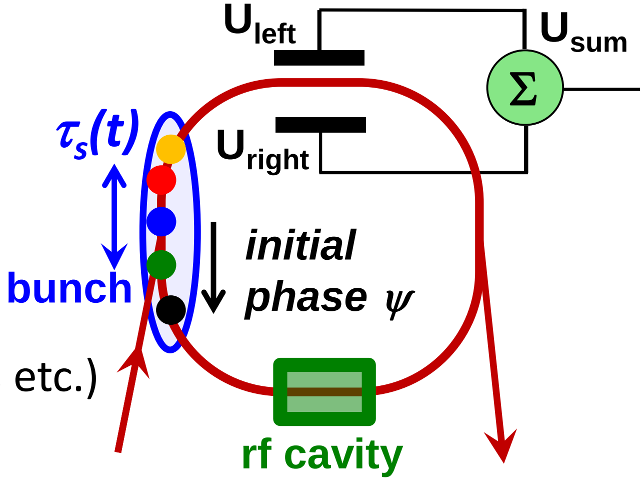


Note:

- The argument of Bessel functions contains amplitude of synchrotron oscillation $\hat{\tau}_s$ & harmonics h
- Distance of sidebands are independent on harmonics h

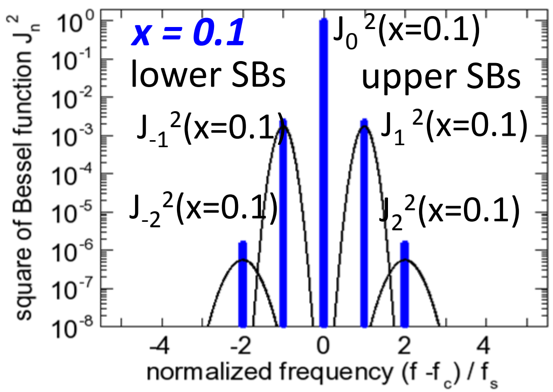
Bunched Beam: Longitudinal Schottky Spectrum for many Particles

Particles have different amplitudes $\hat{\tau}_s$ and initial phases ψ
$\Rightarrow$ averaging over initial parameters for *n = 1...N* particles:



Results:

- **Central peak *p = 0*:** No initial phase for single particles
  $U_0(t) \propto J_0(2\pi \cdot hf_0 \cdot \hat{\tau}_s) \cdot \cos(2\pi hf_0 t)$
  $\Rightarrow$ **Total power $P_{tot}(p=0) \propto N^2$**
  i.e. contribution from 1...***N*** particles add up **coherently**
  $\Rightarrow$ **Width: $\sigma_{p=0}$ = 0** (ideally without power supplier ripples etc.)
  Remark: This signal part is used in regular BPMs
  $\Rightarrow$ this is **not** a Schottky line in a **stringent** definition
- **Side bands *p ≠ 0*:** initial phases ψ appearing
  $U_p(t) \propto J_p(2\pi \cdot hf_0 \cdot \hat{\tau}_s) \cdot \cos(2\pi hf_0 t + 2\pi p f_s t + p\psi)$
  $\Rightarrow$ **Total power $P_{tot}(p \neq 0) \propto N$**
  i.e. contribution from 1...***N*** particles add up **incoherently**
  $\Rightarrow$ **Width: $\sigma_{p\neq 0} \propto p \cdot \Delta f_s$** lines getting wider
  due to momentum spread **$\Delta p / p_0$** &
  possible spread of synchrotron frequency **Δf_s**

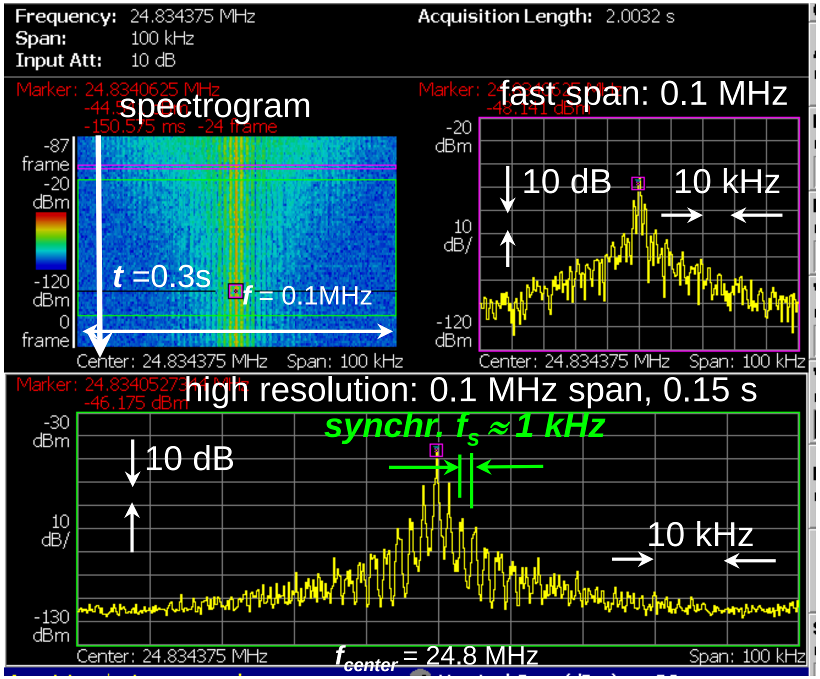


Example for scaling of power:

If ***N*** = *10^{10} then* $P_{tot}(p=0) \approx 100\text{dB} \cdot P_{tot}(p \neq 0)$

Example of longitudinal Schottky Analysis for a bunched Beam

Example: **Bunched** beam at GSI synchrotron
Beam: Injection E_{kin} = 11. 4 MeV/u harm. h = 120



Power spectrum with $P \propto J_p^2$

Application for 'regular' beams:

- Determination of synchrotron frequency f_s
- Determination of momentum spread:
  - envelope does **not** represent directly coasting beam
    $\Rightarrow$ **not** directly usable for daily operation
  - but can be extracted with detailed analysis due to the theorem $\sum_{p=-\infty}^{\infty} J_p^2(x) = 1$ for all x
    $\sum_{p=-\infty}^{\infty} J_p(x) = 1$ and $J_{-p}(x) = (-1)^p J_p(x)$
    $\Rightarrow$ for each band h: $\int P_{bunch} df = \int P_{coasting} df$

Application for intense beams:

- The sidebands reflect the distribution $P(f_s)$ of the synchrotron freq. due to their incoherent nature
  see e.g. E. Shaposhnikova et al., HB'10, p. 363 (2010) & PAC'09, p. 3531 (2009), V. Balbecov et al., EPAC'04, p. 791 (2004)
- However, the spectrum is significantly deformed amplitude $\hat{\tau}_s$ dependent synchrotron freq. $f_s(\hat{\tau}_s)$
  see e.g. O. Boine-Frankenheim, V. Kornilov., Phys. Rev. AB 12. 114201 (2009)

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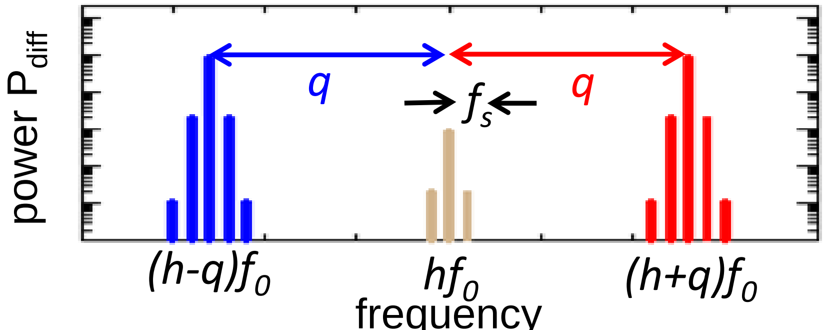
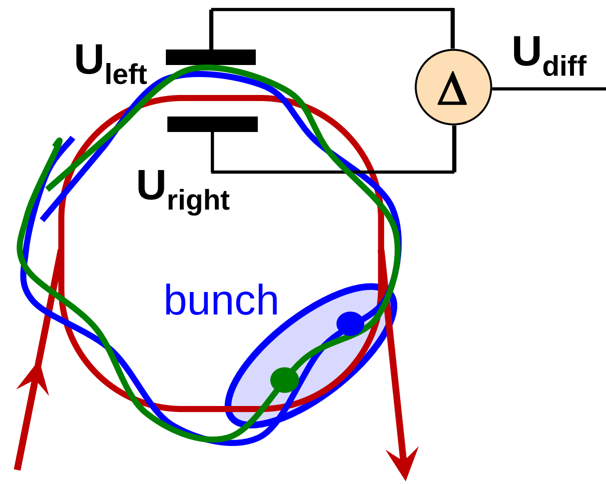
Transverse Schottky Analysis for bunched Beams

Transverse Schottky signals are understood as

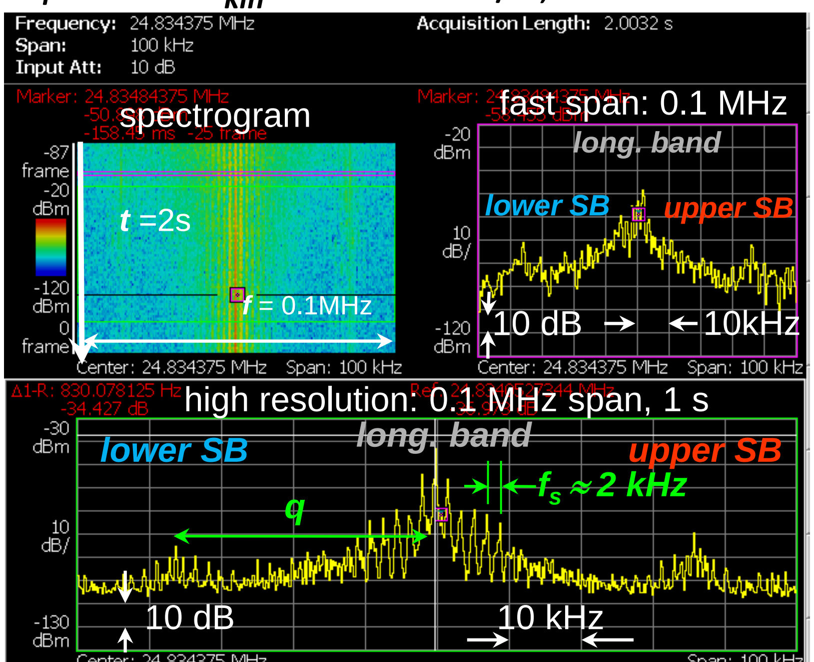
- **amplitude modulation of the longitudinal signal**
- **convolution by transverse sideband**



Schottky pickup



Example: GSI $\boldsymbol{E_{kin}}$ = 11.4 MeV/u, harmonics ***h*** = 119



Structure of spectrum:

- **Longitudinal** peak with synchrotron SB
  - central peak $P_0 \propto N^2$ called coherent
  - sidebands $P_p \propto N$ called incoherent
- **Transverse** peaks comprises of
  - replication of coherent long. structure
  - incoherent base might be visible

Remark: Spectrum can be described by lengthy formula
see e.g. S. Chattopadhay, CERN 84-11 (1984)

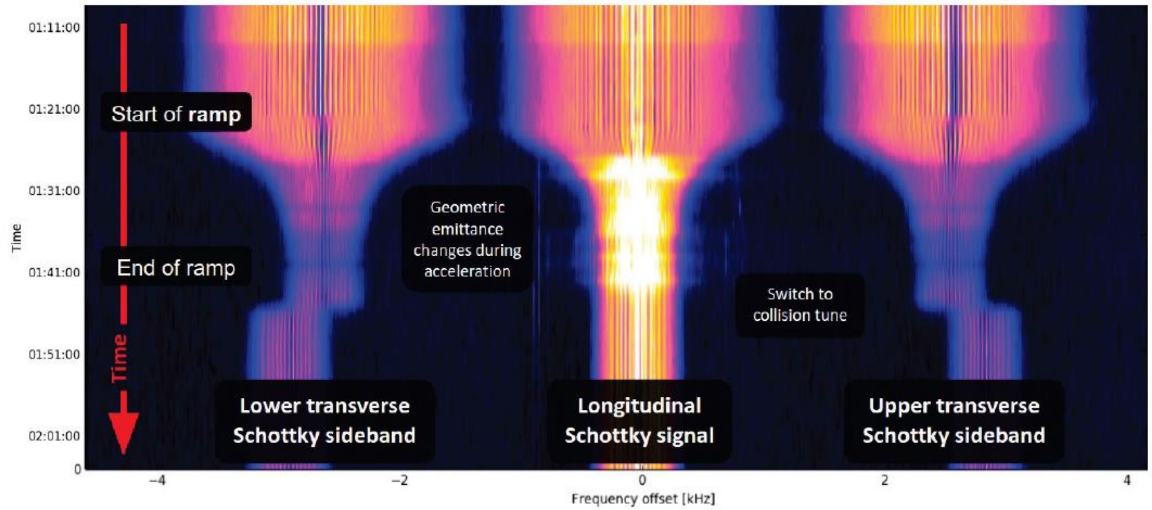
Remark: Height of long. band depends
center of the beam in the pickup

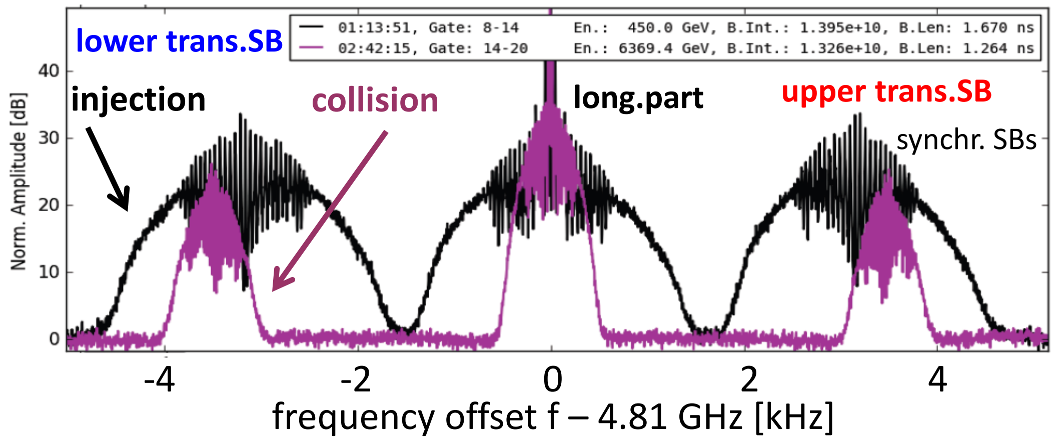
Schottky spectrogram during LHC ramp and collision:

The interesting information is in the ***in****coherent part of the spectrum (i.e. like for coasting beams)*

- **Longitudinal part**
  - ***Width:*** → momentum spread
- **Transverse part**
  - **Center:** → tune
  - **Width**: → chromaticity difference of lower & upper SB
  - **Integral** : → emittance

Example: LHC nominal filling with Pb^{82+} ´,harm. $h \approx 4\cdot10^5$
→ acceleration & collisional optics within ≈ 50 min





FNAL realization and measurement:
A. Jansson et al., EPAC'04, p. 2777 (2004) &
R. Pasquinelli, A. Jansson, Phys. Rev AB **14**, 072803 (2011)

CERN: M. Betz et al. IPAC'16, p. 226 (2016),
M. Betz et al., NIM A 874, p. 113 (2017)

LHC 4.8 GHz Schottky: Tune and Chromaticity Measurement

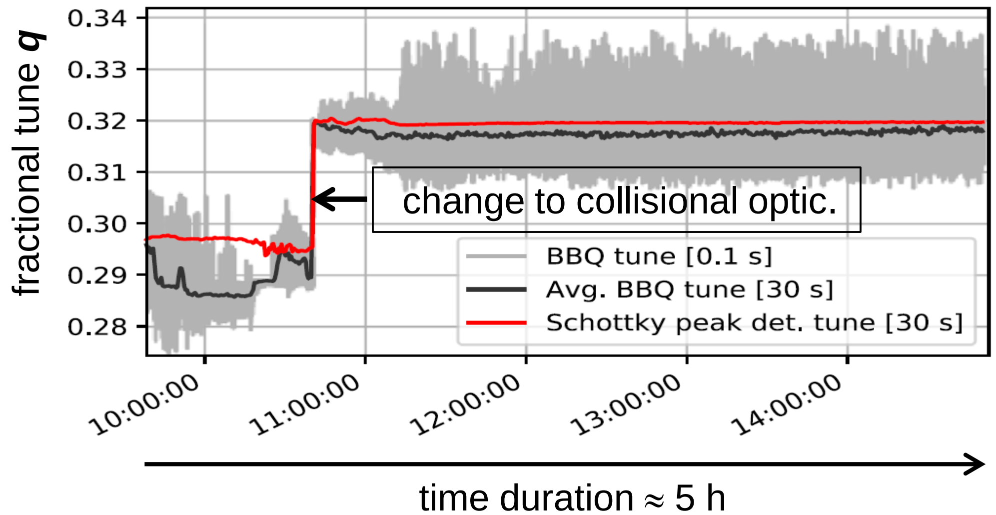
Tune from position of sideband:

Permanent monitoring of tune

- Without excitation
- High accuracy down to 10^{-4} possible
- Time resolution here 30 s

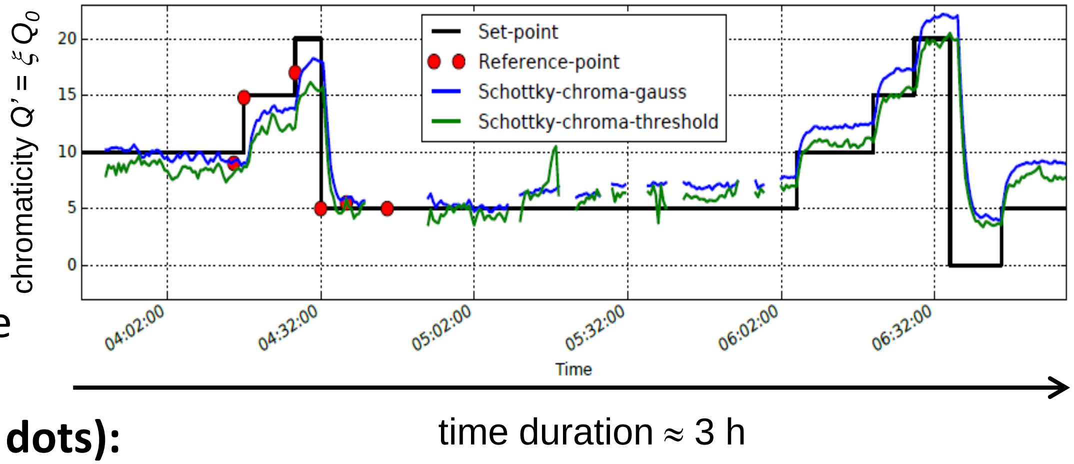
Comparison to BBQ system based on:

- Transverse (gentle) excitation
- Bunch center detection
- Time resolution here 1 s



Chromaticity from width of sidebands of incoherent part:

- Two different offline algorithms
- Satisfactory accuracy
- Time resolution here 30 s
- Performed at MD time, breaks are due to experimental realignments



Comparison to traditional method (red dots):

- Change of bunching frequency ⇒ $\delta p = p_{actual} - p_0$
- Tune measurement and fit $\Delta Q / Q_0 = \xi \cdot \delta p / p_0$

M. Betz et al. IPAC'16, p. 226 (2016),
M. Betz et al., NIM A 874, p. 113 (2017)

Challenge for bunched beam Schottky:
Suppression of broadband sum signal to prevent for saturation of electronics

Design consideration:

Remember scaling: width $\Delta f \propto h$, power $P \propto 1/h$

- Low sum signal i.e. outside of bunch spectrum (LHC: acceleration by f_{acc} = 25 MHz)
- Avoiding overlapping Schottky bands
- Sufficient bandwidth to allow switching

Technical choice:

- Narrow band pickup by two wave guide for TE_{10} mode, cut-off at 3.2 GHz
- Coupling slots for beam's TEM mode

$\Rightarrow$ center f_c =4.8 GHz $\Leftrightarrow$ harm. $h \approx 4 \cdot 10^5$ & $BW \approx$0.2 GHz

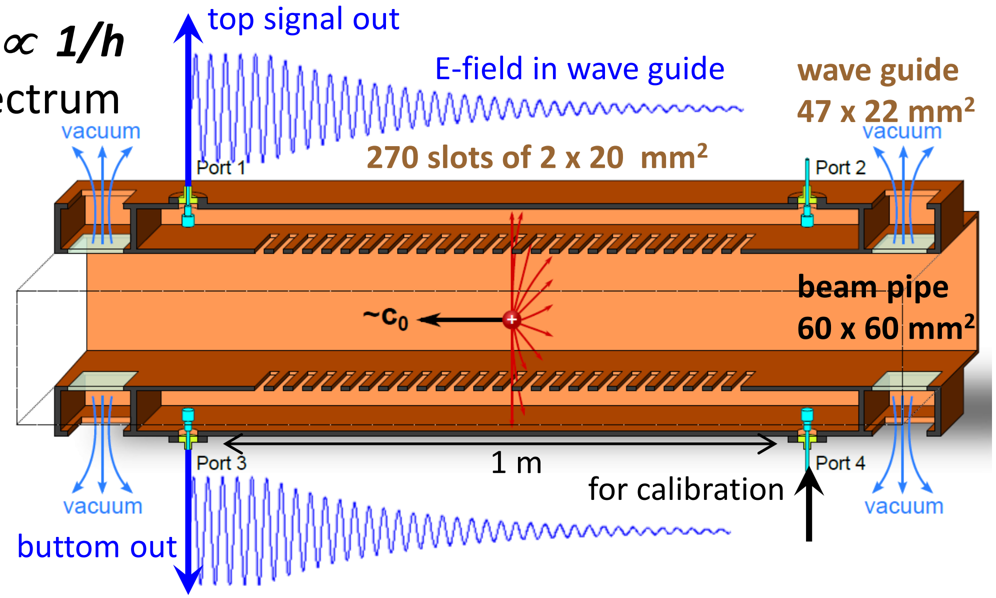
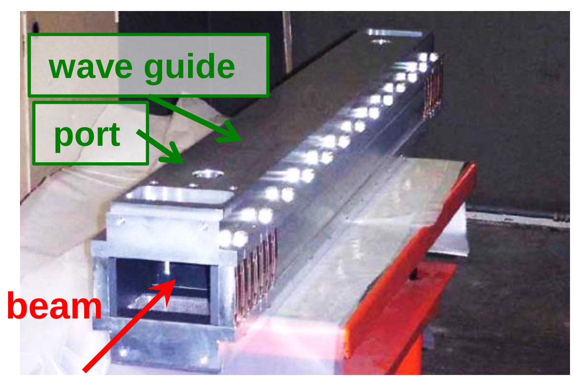
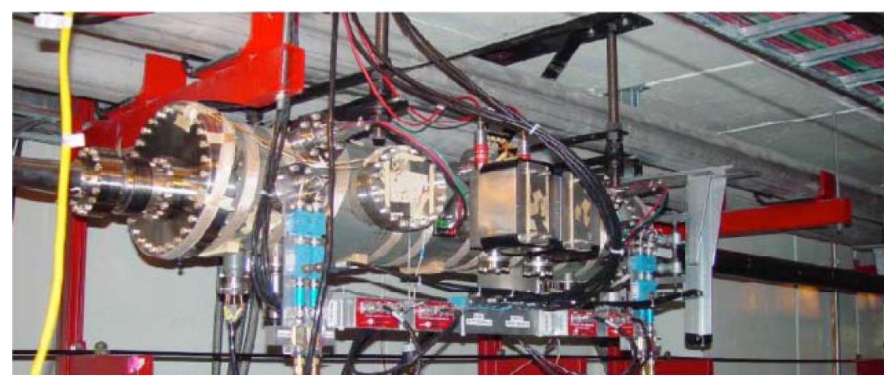


Photo of 1.8 GHz Schottky pickup at FNAL recycler





CERN: M. Wendt et al. IBIC'16, p. 453 (2016), M. Betz, NIM A 874, p. 113 (2017)
FNAL: R. Pasquinelli et al., PAC'03, p. 3068 (2003) & R. Pasquinelli, A. Jansson, Phys. Rev AB **14**, 072803 (2011).

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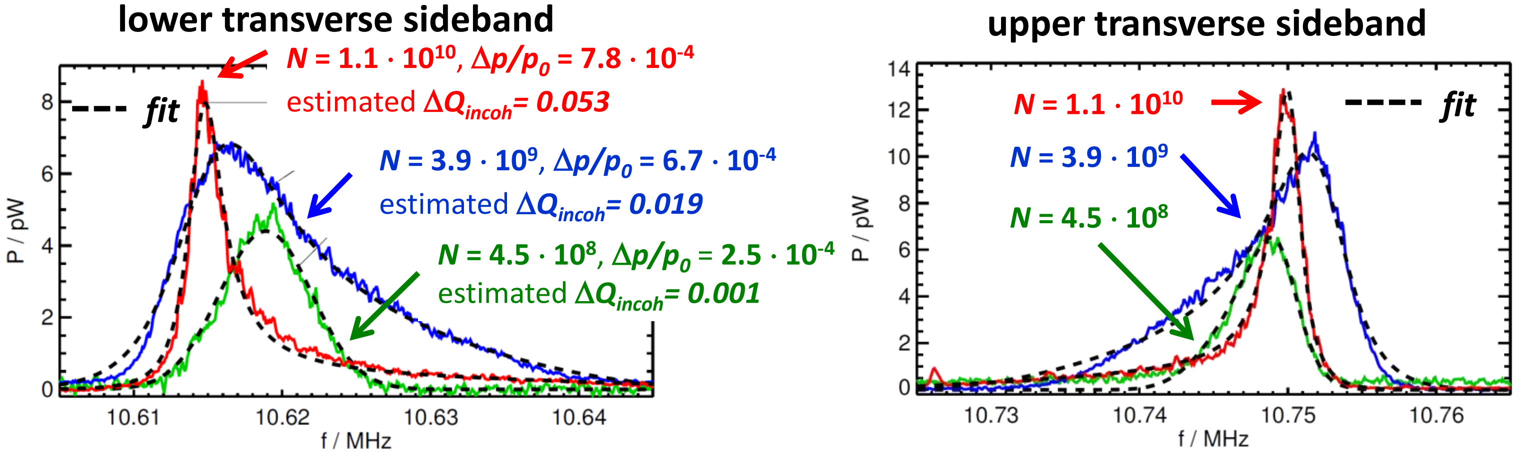
Deformed Schottky Spectra for high Intensity coasting Beams

Transverse spectra can be deformed even at 'moderate' intensities for lower energies

Remember: Transverse sidebands were introduced as **coherent** amplitude modulation

Goal: Modeling of a possible deformation leading to correct interpretation of spectra
Extracting parameters like tune spread ΔQ_{incoh} by comparison to detailed simulations

Example: Coasting beam GSI synchrotron Ar^{18+} at 11.4 MeV/u, harm. h = 40, coherent $\Delta Q_{coh} \approx 0$



Method:

- Calculation of space charge & impedance modification
- Calculation of beam's frequency spectrum
- Comparison to the experimental results

⇒ Model delivers reliable beam parameters, spectra can be explained

Schottky diagnostics:

- Spectra do not necessarily represents the distribution, but parameter can be extracted

O. Boine-Frankenheim et al., Phys. Rev. AB 12, 114201 (2009) , S. Paret et al., Phys. Rev. AB 13, 022802 (2010)

Longitudinal Schottky: Modification for very cold Beams

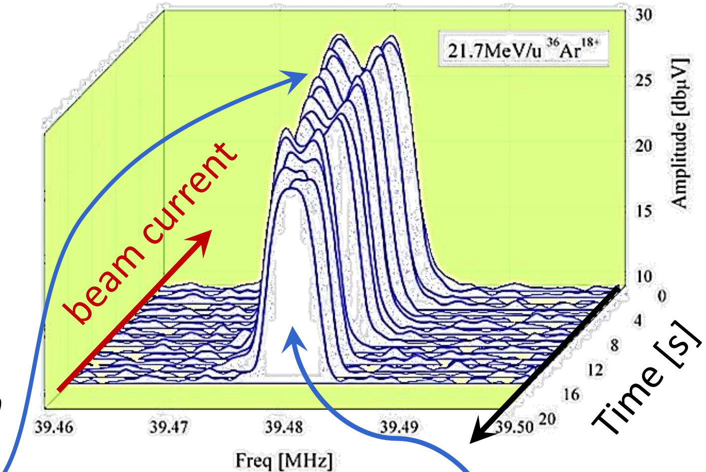
Very high phase space density leads to modification of the longitudinal Schottky spectrum

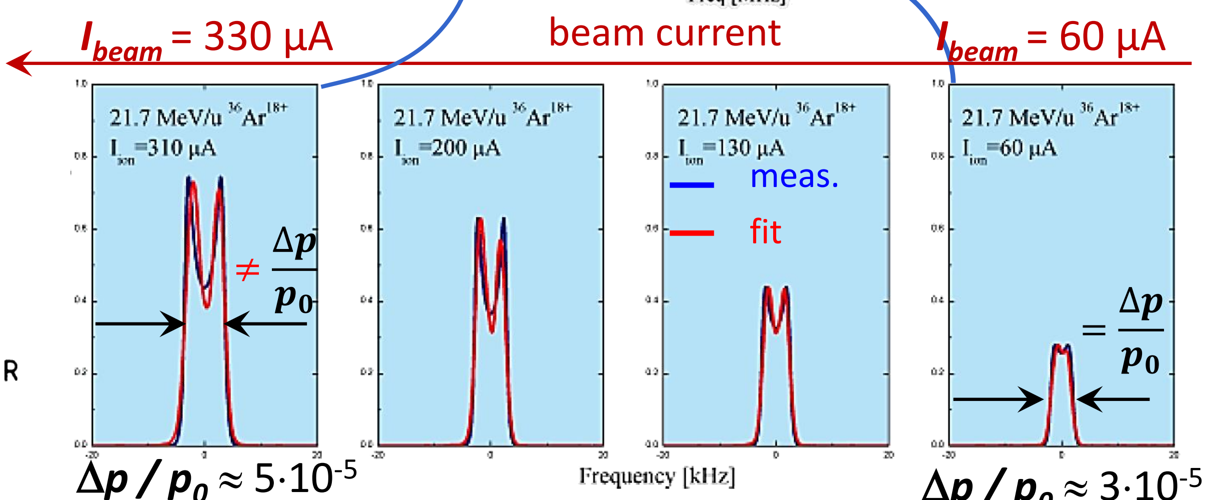
Low energy electron cooler ring:
High long. & trans. phase space density
$\Rightarrow$ Strong coupling between the ions
$\Rightarrow$ Excitation of co-&counter propagation plasma waves by wake-fields (beam impedance)

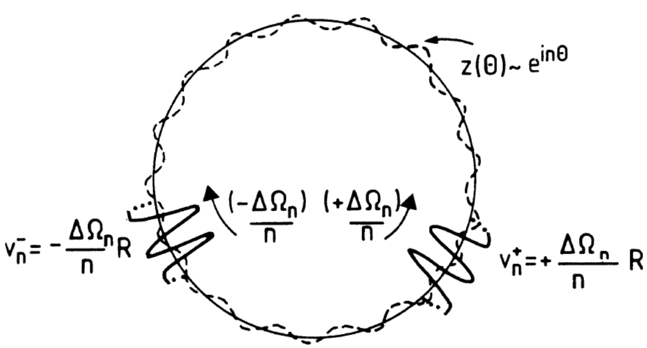
This collective density modulation is a coherent effect!

$\Rightarrow$ Schottky spectrum comprises then
coherent part with power scaling $P \propto N^2$
+ the regular **incoherent** part with $P \propto N$
$\Leftrightarrow$ Schottky **doesn't** represent distribution e.g. $\sigma \neq \Delta p/p_0$
but $\Delta p / p_0$ can be gained from model fit

Example: at CSRe cooler ring in Lanzhou, China
Beam: Ar^{18+} at E_{kin} = 21 MeV/u, harm. $h \approx 100$





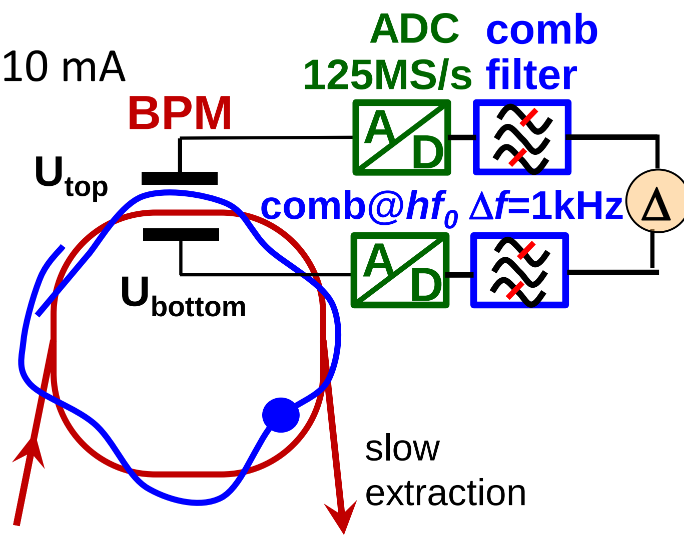


S. Chattopadhay, CERN 84-11 (1984)

L.J. Mao et al. IPAC'10, p. 1946 (2010)

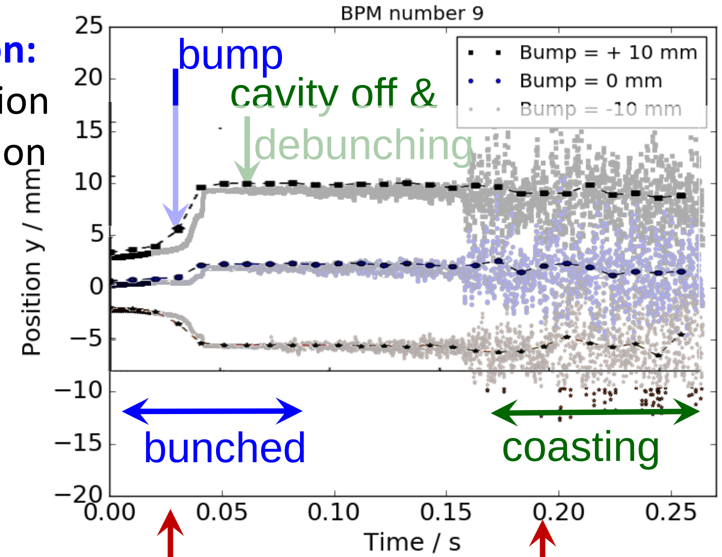
Position measurement with BPMs for a coasting beam

Beam: E_{kin} = 800 MeV/u, f_0 = 0.99 MHz, f_{rf} = 4.92 MHz, I_{beam} = 10 mA



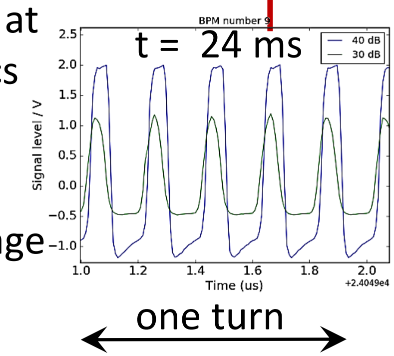
Steps of beam manipulation:

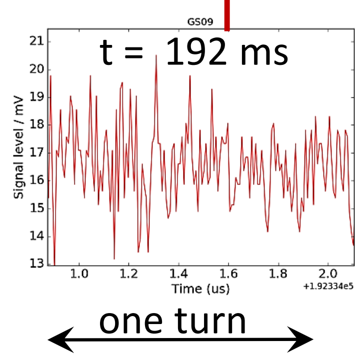
1. Bunched beam acceleration
2. Closed bump in one section
   → regular closed orbit measurement with with 80 µs time steps
3. Cavity switch off & frequency detuning
   → beam de-bunches



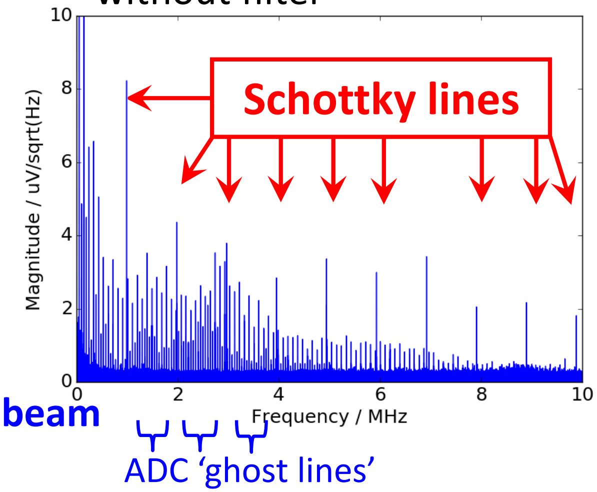
BPM data treatment:

1. Digital **comp filter** at Schottky harmonics ***f(h)*** for ***h*** = 1 ... 8 width Δf = 1 kHz
2. Time binning average with 8 ms steps





Coasting beam spectrum without filter

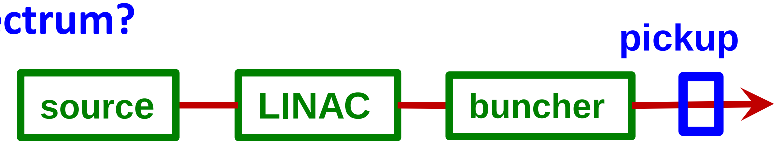


⇒ **Position resolution $\Delta x \approx$ 1 mm at $\Delta t \approx$ 10 ms time steps for coasting beam e.g. useful for slow extraction or cooling observation**

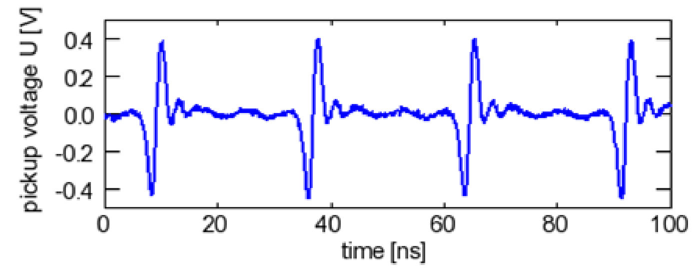
Longitudinal Schottky at a LINAC ??? → Result: Probably not possible

Is it possible to measure the momentum spread at a single pass accelerator i.e. is there an incoherent contribution to the bunch spectrum?

Experiment at GSI: broadband pickup & oscilloscope
(Schottky in synchrotron: Incoherent width $\Delta f_h \propto h$)

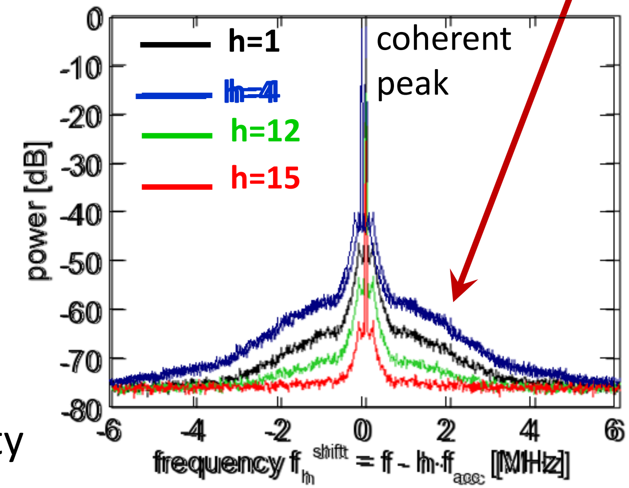


Beam: U^{28+} at 11.4 MeV/u, f_{acc} = 36 MHz

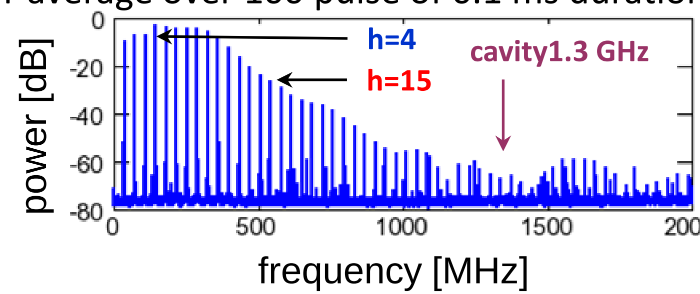


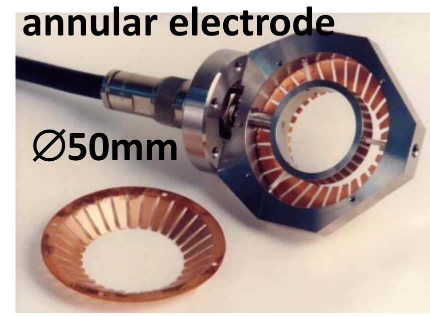
Result:
Peak structure does **not** change for different 'harmonics' ***h***:
⇒ no incoherent Schottky part!
Supported by spectra recorded with a cavity @ 1.3 GHz of high ***h*** and sensitivity

Is this the incoherent frequency spread $\propto \Delta p / p_0$?
No, but bunchs' ampl. & phase variation!



FFT average over 100 pulse of 0.1 ms duration





Interpretation:
Schottky signals require the periodic passage of the **same** particle to ensure the correlation to build up.

P. Kowina et al., HB'12, p. 538 (2012)

Summary

Schottky signals are based on modulations and fluctuations:

Modulation ⇔ coherent quantities:

- Measurement of f_0, Q_0 & f_s from peak center → frequent usage by GSI operators

Fluctuation ⇔ incoherent quantities:

- Measurement of $\Delta p/p_0$ & ξ from peak width → frequent usage for $\Delta p/p_0$ by GSI operators
- signature of Δf_s & ΔQ from peak shape → for machine development only at GSI

General scaling: incoherent signal power $P(h) \propto q^2 N / h$ and width $\Delta f(h) \propto h$

q: ion charge state, ***N***: number of ions, ***h***: harmonics

Signal spectrum: Partly complex, but computable for 'regular' cases

High intensity beams: Characteristic modifications, important for model verification

Detection:
- Recordable with wide range of pickups, measurement possible in each harmonics
- Electronics for very weak signals must be matched to the application

For valuable discussion I like to thank:

- P. Kowina GSI, R. Singh GSI, M. Wendt CERN for very intense discussion
- M. Betz LBL (formally CERN), O. Boine-Frankenheim GSI, O. Chorniy GSI, P. Hülsmann GSI , A. Jansson ESS (formally FNAL), A.S. Müller KIT, M. Steck GSI, J. Steinmann KIT and many others

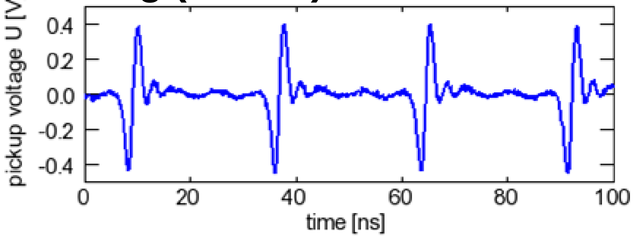
Thank you for your attention!

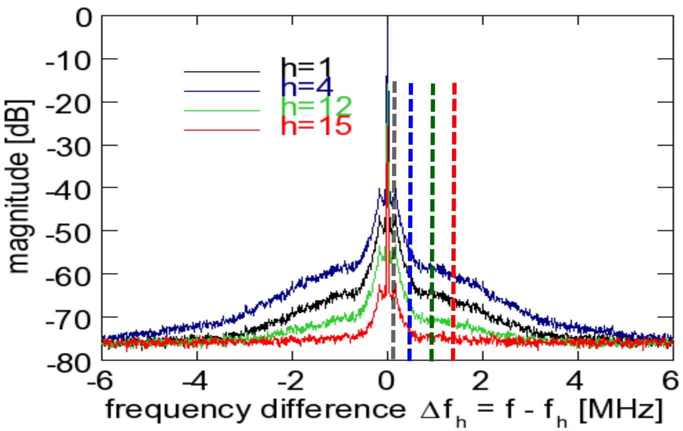
Spare slides

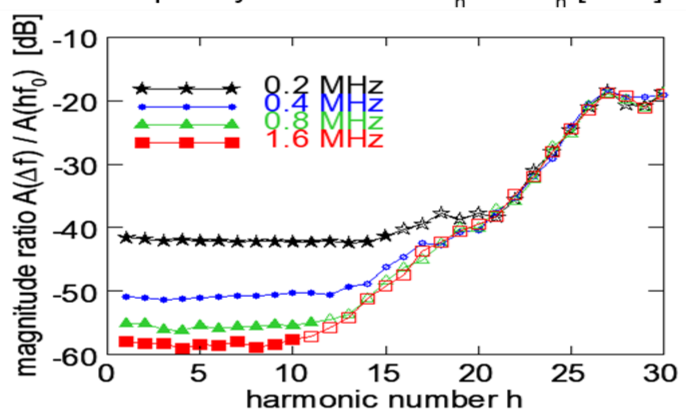
Longitudinal Schottky at a LINAC ??? ⇒ No !!!

Beam: U^{28+} at 11.4 MeV/u, $f_{acc} \equiv f_0$ = 36 MHz, I_{beram} =0.2 mA, average of 100 pulse with 0.1 ms duration

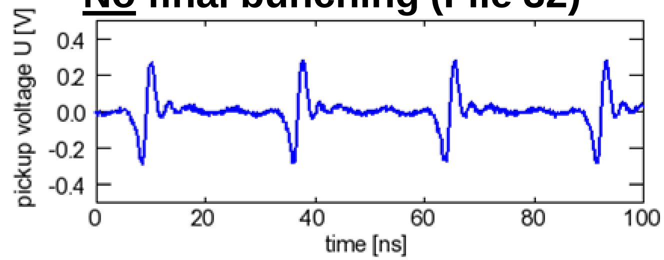
Final bunching (File 34)

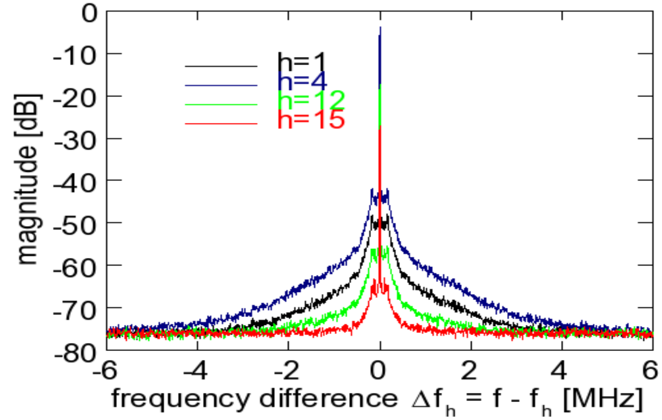


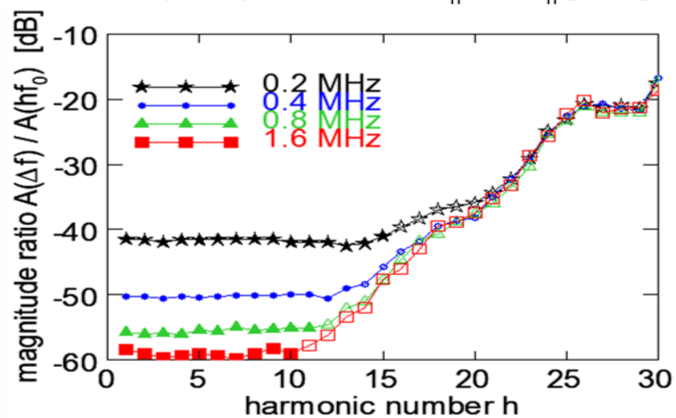




No final bunching (File 32)







Schottky Spectrum at Synchrotron Light Sources

Hadron synchrotron: most beams non-relativistic or $\gamma < 10$ (exp. LHC) $\Rightarrow$ **no** synch. light emission

$\Leftrightarrow$ stationary particle movement $\Rightarrow$ turn-by-turn correlation

Electron synchrotrons relativistic $\gamma \approx 5000$ $\Rightarrow$ synchrotron light emission

$\Leftrightarrow$ break-up of turn-by-turn correlation **?**

Test of longitudinal Schottky at ANKA (Germany):

Goal: determination of momentum spread $\Delta p / p_0$

Ring shaped electrode as broadband detector

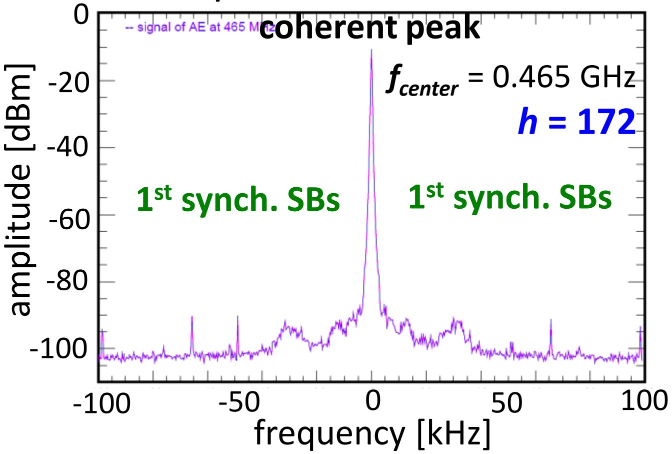
Results:

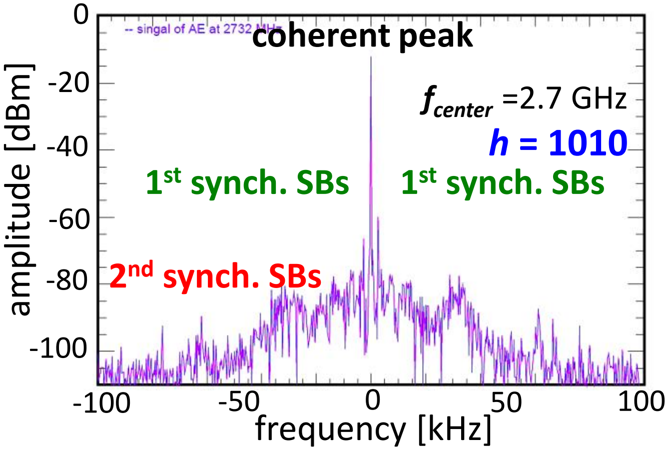
- Narrow coherent central peak
- Synchrotron sidebands clearly observed
- Sideband wider as central peak
  $\Rightarrow$ incoherent cntribution
- Ratio of power $P_{central} / P_{SB}$ as expected

$\Rightarrow$ Attempt started, feasibility shown!

Further investigations are ongoing

Example: ANKA at 2.5 GeV



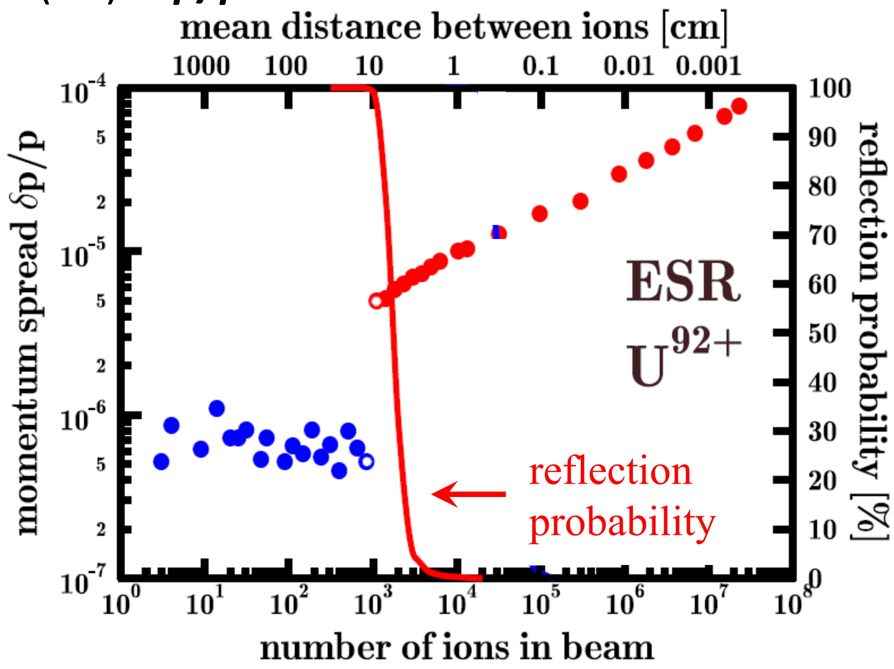


K.G. Sonnad et al., PAC'09, p. 3880 (2009)

Electron Cooling: Linear Chain by Minimal Momentum Spread

Example: Observation of longitudinal momentum at GSI storage ring
- Ion beam: U^{92+} at 360 MeV/u applied to electron cooling with I_{ele} = 250 mA
- Variation of stored ions by lifetime of $\tau \approx$ 10 min i.e. total store of several hours
- Longitudinal Schottky spectrum with 30 s integration every 10 min

$\Rightarrow$ Momentum spread (1σ): $\Delta p/p = 10^{-4}$ $\rightarrow$ below 10^{-6} when reaching an intensity threshold



Interpretation:
- Intra beam scattering as a heating mechanism is suppressed below the threshold
- Ions can't overtake each other, but building a 'linear chain' (transverse size σ_x < 30 µm)
- Momentum spread is basically given by stability of power suppliers

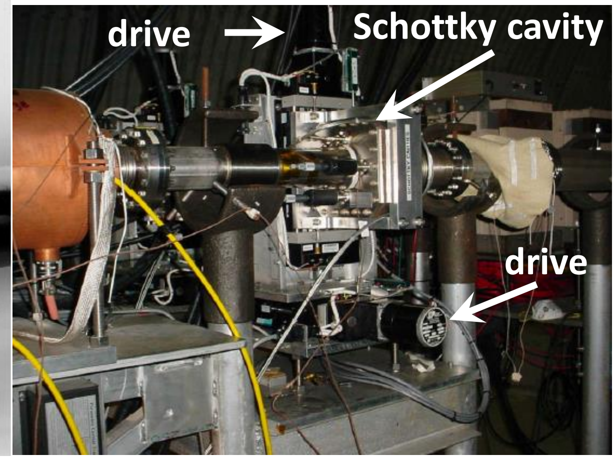
M. Steck et al., Phys. Rev. Lett 77, 3803 (1996), R.W. Hasse, EPAC 00, p. 1241 (2000)

Transverse rms Emittance Determination by Schottky Analysis at RHIC

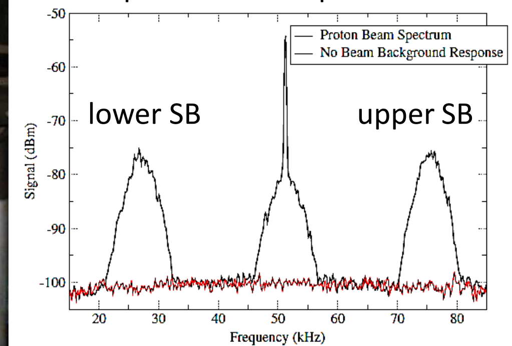
The integrated power in a sideband delivers the rms emittance $P_h^{\pm} \propto \langle d^2 \rangle \propto \varepsilon_{rms} \cdot \beta$

Example: Schottky cavity operated at dipole mode TM_{120} @ 2.071 GHz & TM_{210} @ 2.067 GHz i.e. a beam with offset excites the mode (like in cavity BPMs)

Peculiarity: The entire cavity is movable $\Rightarrow$ the stored power delivers a calibration ***P(x)***



power in long band depend on beam position ***x***



Result: rms emittances coincide with IPM measurement within the 20 % error bars

TABLE II. Results of Schottky emittance scan and comparison to RHIC IPM. Emittance values are normalized.

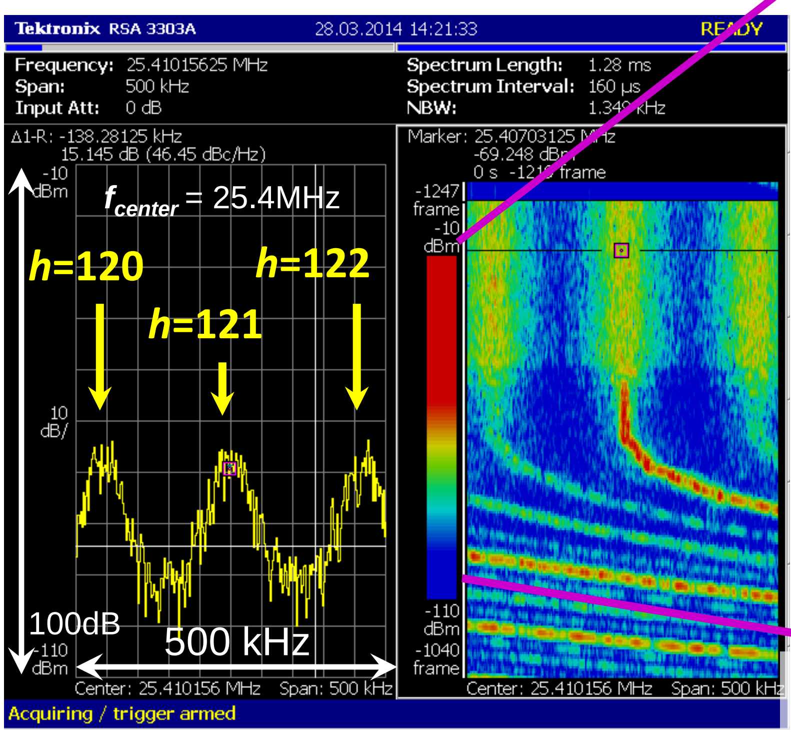
Ring and plane	Schottky β function (m)	Schottky rms beam size (mm)	Schottky emittance (π μm, 95%)	IPM emittance (π μm, 95%)
Blue horizontal	28 ± 4	1.04 ± 0.1	23 ± 5	24 ± 5
Blue vertical	27 ± 4	0.95 ± 0.1	20 ± 4	23 ± 3
Yellow horizontal	27 ± 4	0.99 ± 0.1	22 ± 4	19 ± 4
Yellow vertical	30 ± 5	1.15 ± 0.1	26 ± 5	28 ± 4

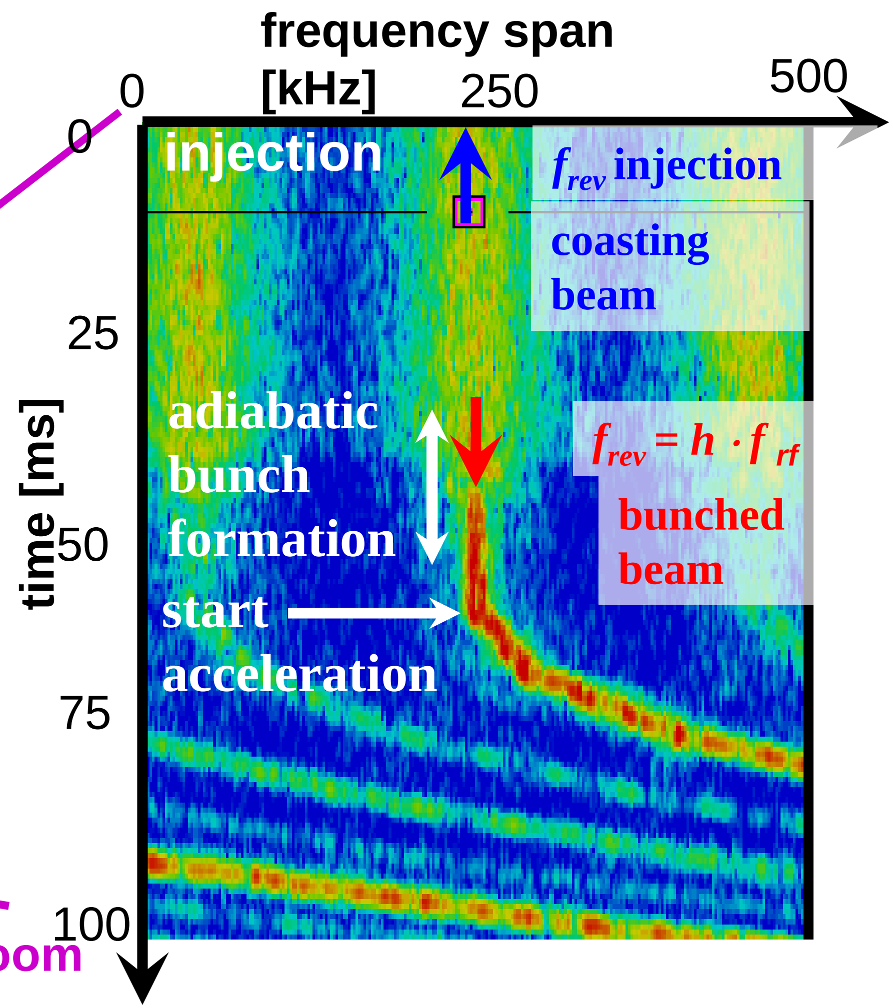
K.A. Brown et al., Phys. Rev. AB, 12, 012801 (2009), W. Barry et al., EPAC'98, p. 1514 (1998)

Longitudinal Schottky Noise Analysis for acceleration Ramp Operation

Example for longitudinal Schottky spectrum to check proper acceleration frequency:

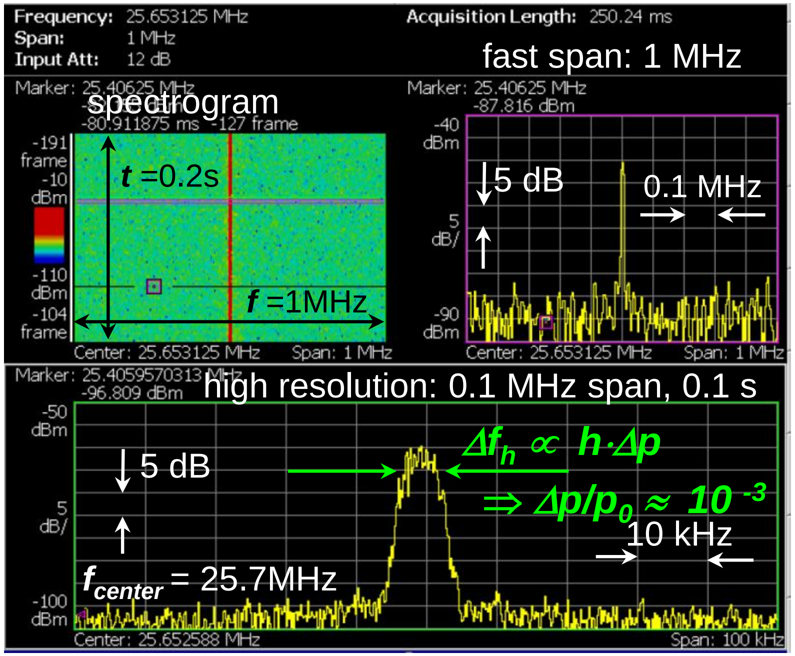
- Injection energy given by LINAC settings, here E_{kin} =11.4 MeV/u ⇔ β = 15.5 %, $\Delta p/p \approx 10^{-3}$ (1σ)
- multi-turn injection & **de-bunching within ≈ ms**
- adiabatic bunch formation & acceleration
- Measurement of revolution frequency f_{rev}
- Alignment of acc. f_{rf} to have $f_{rev} = h \cdot f_{rf}$
  i.e. no frequency jump !





Example: **Coasting** beam at GSI synchrotron at injection

E_{kin} = 11.4 MeV/u ⇔ β = 15.5 %, harmonic number h = 119



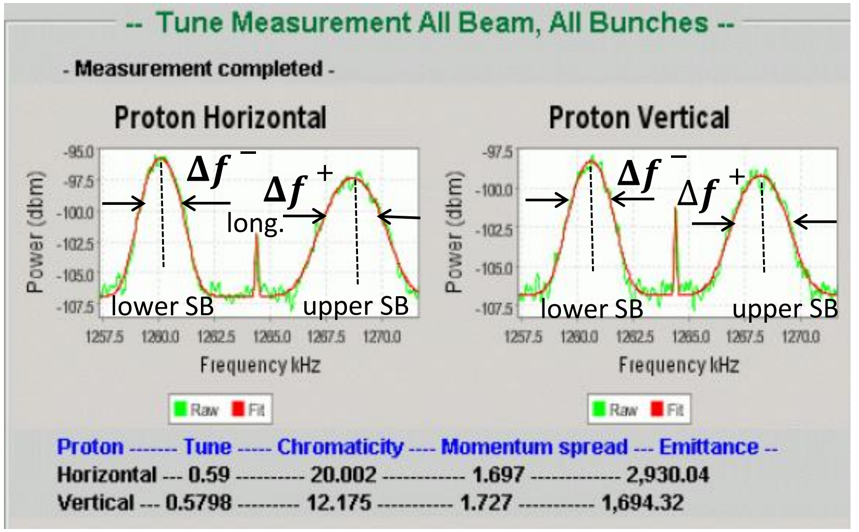
Application for coasting beam diagnostics:

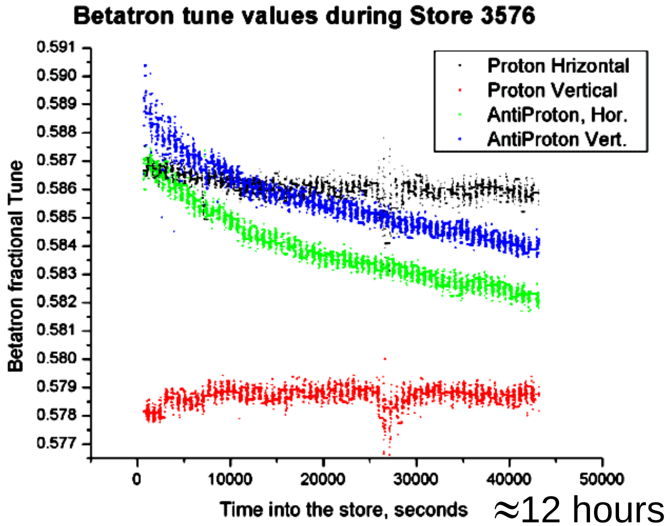
- Injection: momentum spread via $\frac{\Delta p}{p_0} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{h\, f_0}$ as influenced by re-buncher at LINAC
- Injection: matching i.e. f_{center} stable at begin of ramp
- Dynamics during beam manipulation e.g. cooling
- Relative current measurement for low current below the dc-transformer threshold of ≈ 1µA

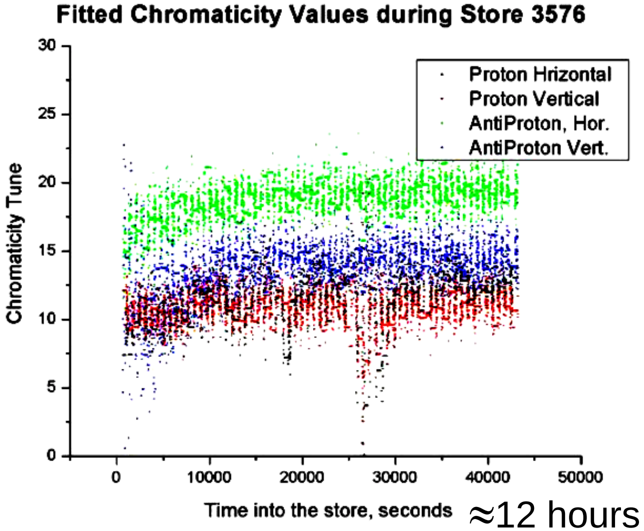
Example of Chromaticity Measurement at Tevatron

Permanent chromaticity monitoring at Tevatron:

- Sidebands around 1.7 GHz i.e. $h \approx 36{,}000$
  with slotted waveguide, see below for CERN type
- Gated, down-mixing & filtered
  by analog electronics
- Gaussian fit of sidebands
  Center $\rightarrow$ tune q
  Width $\rightarrow$ chromaticity ξ via $\Delta f^{+} - \Delta f^{-}$
  $\rightarrow$ momentum spread $\Delta p/p$ via $\Delta f^{+} + \Delta f^{-}$







Remark: Spectrum measured with bunched beam and gated signal path, see below

A. Jansson et al., EPAC'04, p. 2777 (2004) & R. Pasquinelli, A. Jansson, Phys. Rev AB **14**, 072803 (2011)