Extracting information from Electro-Magnetic Monitors in Hadron Accelerators

Peak detected Schottky - what does it provide us?

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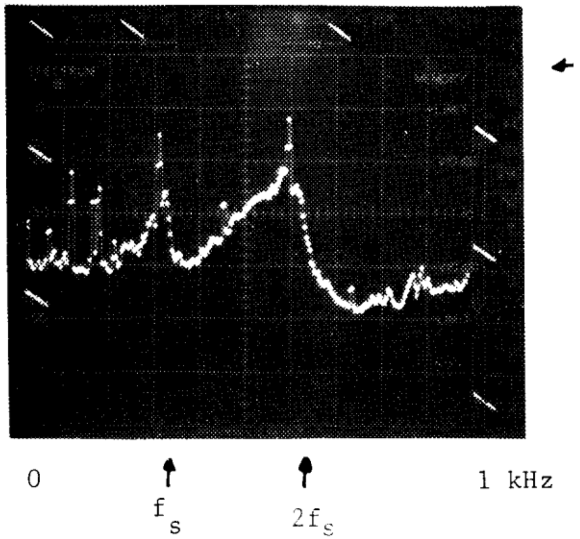
Acknowledgements: T. Bohl, I. Karpov, T. Linnecar, J. Esteban Muller, H. Timko

History

- The "peak detected (PD) Schottky" is a beam diagnostics tool developed by D. Boussard and T. Linnecar and extensively used in the SPS since the late 70s
- The quadrupole line was believed to represent well the particle distribution in synchrotron frequencies, similar to longitudinal Schottky spectrum of unbunched beam for revolution frequencies
- In the SPS " traditional" Schottky system (high sensitivity PU at ~1 GHz) was not used anymore (at least in longitudinal plane)
- PD Schottky diagnostics was also installed in the LHC

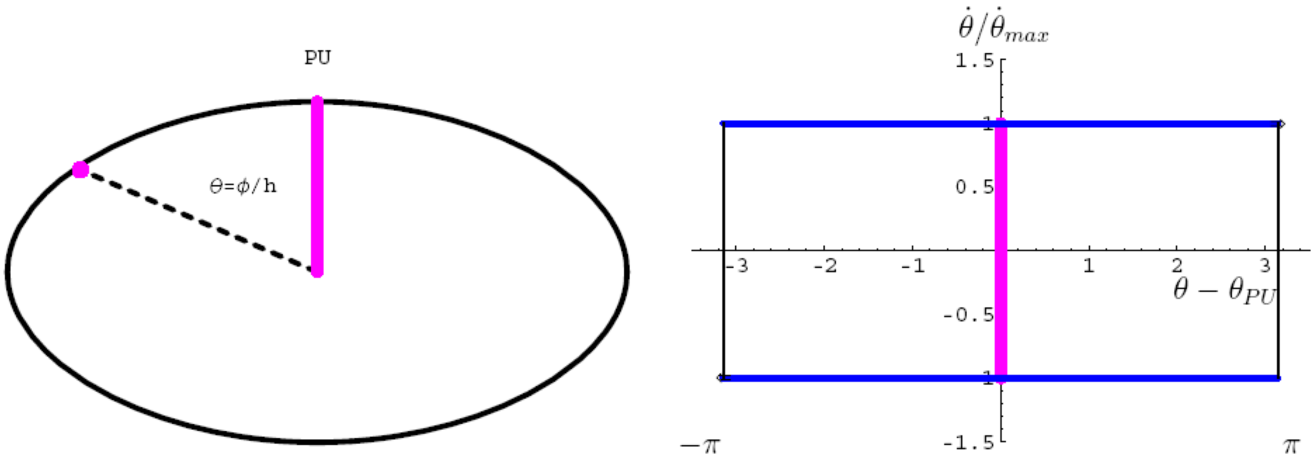
First measurements in the SPS,
coast at 120 GeV/c, 1978
SPS Improvement Reports 154, 162, 167



References for this work:

[1] E. S., CERN-BE-2009-010 RF, 2009
[2] E. S., T. Bohl, T. Linnecar, PAC'09
[3] E. S. HB2010

Longitudinal Schottky spectrum: unbunched beam (1/2)



Particles contributing to the Schottky signal at any given moment at the pick-up
Each particle contributes once per revolution period.

Longitudinal Schottky spectrum: unbunched beam (2/2)

For a single (n-th) particle circulating in the ring with a revolution frequency ω_n the time-domain signal at the pick-up

$$I_n(t) = e\omega_n \sum_{k=-\infty}^{\infty} \delta(\omega_n t + \theta_n - 2\pi k) = \frac{e\omega_n}{2\pi} \sum_{k=-\infty}^{\infty} \mathrm{e}^{-ik(\omega_n t + \theta_n)},$$

where θ_n is the particle azimuthal position at moment t=0 relative to the pick-up. The total beam current is a sum over all N particles

$$I(t) = \sum_{n=1}^{N} I_n(t) = \frac{e}{2\pi} \sum_{n=1}^{N} \sum_{k=-\infty}^{\infty} \omega_n \, \mathrm{e}^{-ik(\omega_n t + \theta_n)}.$$

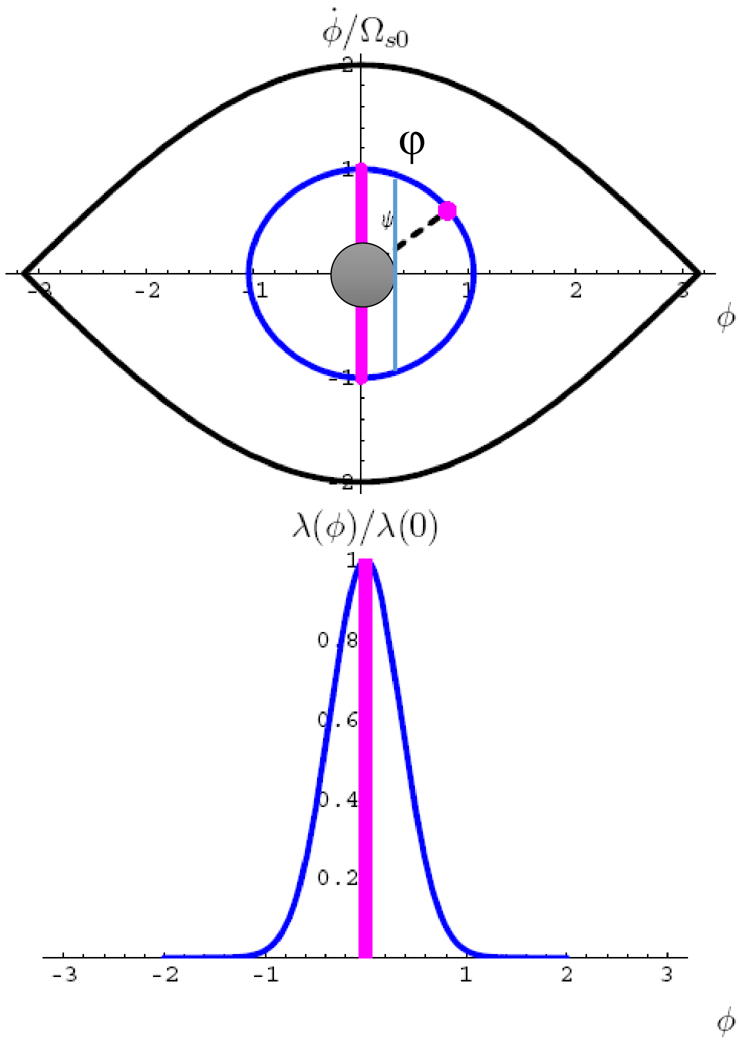
At positive frequencies

$$I(t) = \frac{e}{2\pi} \sum_{n=1}^{N} \omega_n + \frac{e}{\pi} \sum_{n=1}^{N} \sum_{k=1}^{\infty} \omega_n \cos k(\omega_n t + \theta_n).$$

The r.m.s. beam current per band (Schottky current) has lines at all harmonics k with increasing width $k\Delta\omega_0$. The power spectral density ~1/k.

References: J. Borer et al. (1974), D. Boussard (1987) …

Peak amplitude signal



- Power spectrum of *idealistic* Peak Amplitude (PA) signal [1]:

$$P(\omega) = \frac{e^2 N}{\pi} \sum_{m=2}^{\infty} \frac{\omega^2}{m^3} F(\omega/m)$$

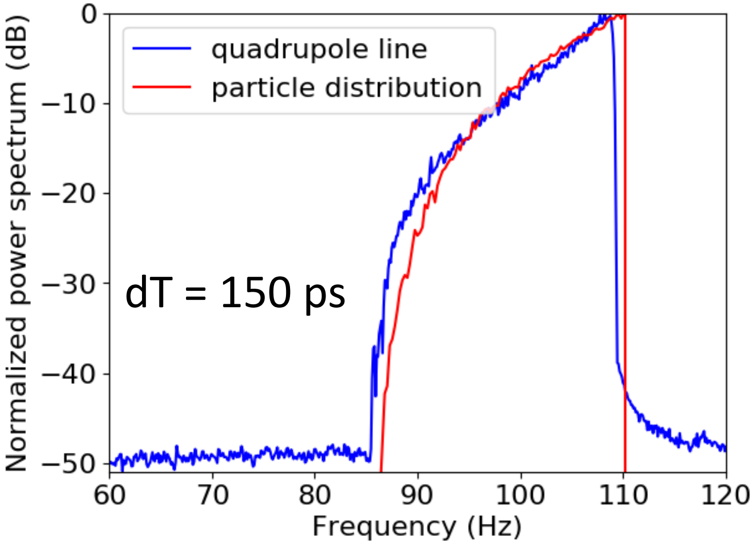
- Only particles with oscillation amplitude $\varphi_\alpha \geq \varphi$ contribute to the *realistic* PA signal at position φ twice per synchrotron period → core is under-represented

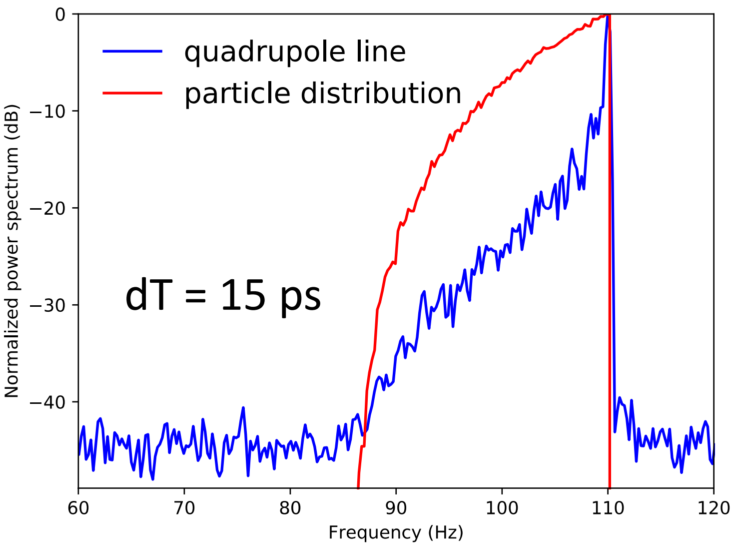
- Measured peak amplitude signal:

$$I_{av}^{pd}(t_k) = \frac{1}{2\Phi} \int_{-\Phi}^{\Phi} I(t_k, \phi) d\phi$$

Peak Amplitude (PA) signal - simulations

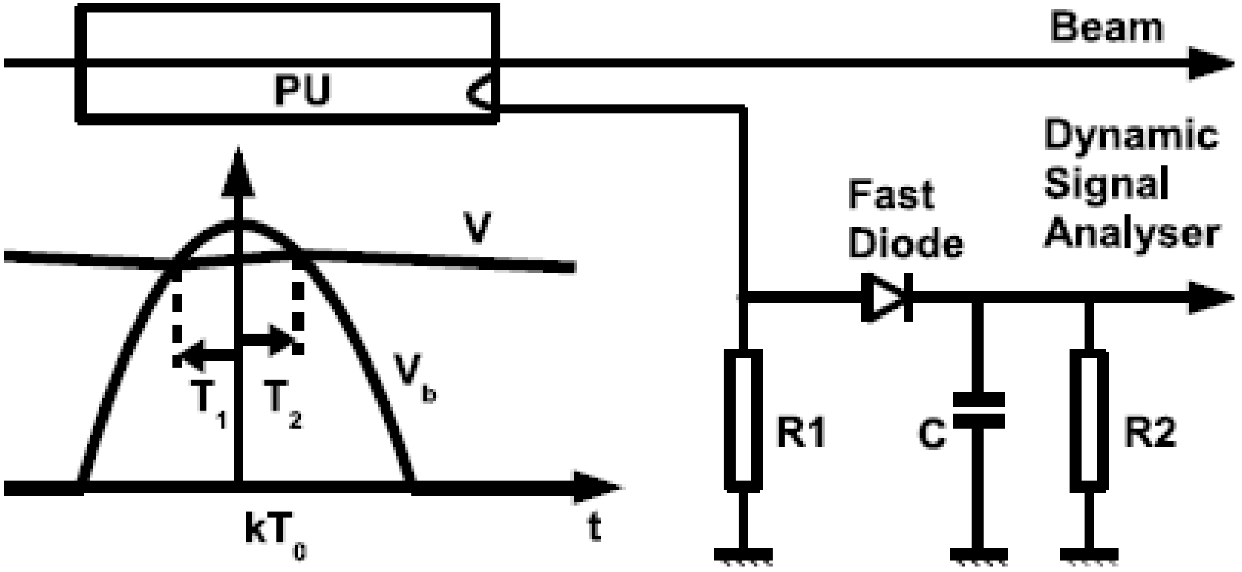
- Tracking of 10^5 macro-particles at 450 GeV in LHC by I. Karpov: 1.4 ns bunch, fs = 55 Hz
- Fourier transform of the PA signal acquired over 8.9 s (10^5 turns) and averaged over 100 “acquisitions”





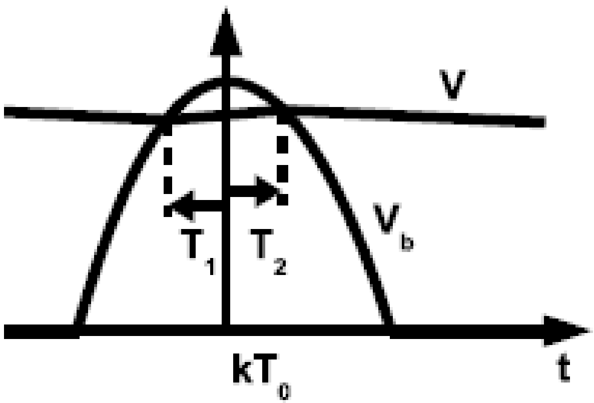
Peak Detected (PD) signal (1/5)

The simplified PD scheme used for longitudinal Schottky measurements in the SPS and LHC



50 Ohm 1 MOhm

PD signal (2/5)



The fast diode is open during the bunch passage (current I_b), when $V_b = I_b R > V$.

The voltage V measured at resistance R_2 during time interval $(-T_1, T_2)$ can be found from the equation (valid for $R_2 >> R_1$)

$$\frac{dV}{dt} = \alpha(V_b - V),$$

where $\alpha = 1/(R_1 C) + 1/(R_2 C)$

The solution

$$V(t) = \alpha \int_{-T_1}^{t} V_b(t')\, e^{-\alpha(t-t')}\, dt' + V(-T_1) e^{-\alpha(t+T_1)}$$

The diode is off for the rest of the revolution period and we have

$$\frac{dV}{dt} = -\mu V, \qquad V(t) = V(T_2)\, e^{-\mu(t-T_2)}$$, where $\mu = 1/(R_2 C)$.

PD signal (3/5)

Typical SPS and LHC parameters relevant to the PD Schottky measurements

Parameter			SPS	LHC
revol. period	T_0	μs	23.0	88.9
RF harmonic	h		4620	35640
resistance	R_1	Ω	50	50
resistance	R_2	$M\Omega$	1.0	1.0
capacitance	C	pF	240	920
PD decay time	$1/\mu$	μs	240	920
PD growth time	$1/\alpha$	ns	12	12
acquisition time	T_a	s	1.6	3.2

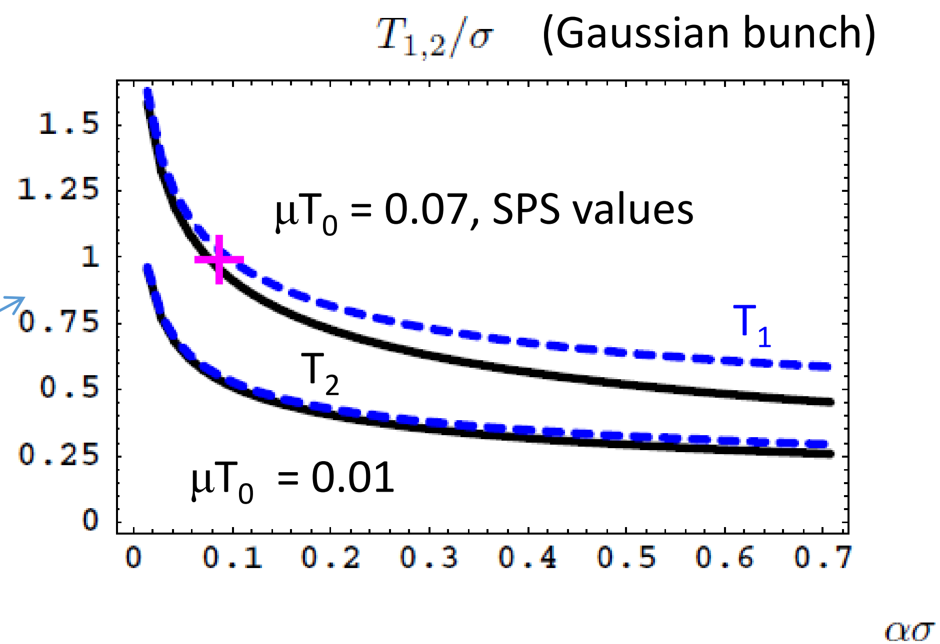
PD signal (4/5)

After a transient period variations of T_1 and T_2 from turn to turn are small and defined only by statistical fluctuations (Schottky noise). Then

$$V(-T_1) = V(T_2)\, e^{-\mu T0}.$$

The stationary values of $T_{1,2}$ can be found from conditions

$$V(-T_1) = V_b(-T_1), \quad V(T_2) = V_b(T_2)$$



→ For the SPS max $\alpha\sigma = 1/12$ and $\mathbf{T_1 \approx T_2 \approx \sigma}$

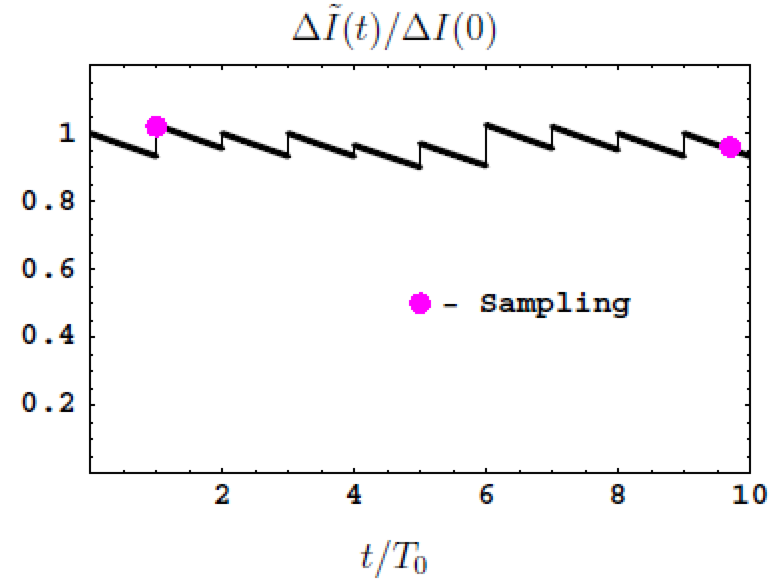
PD signal (5/5)

The signal detected at the moment t, after the k-th bunch passage, is

$V_k e^{-\mu(t-t_k)}$, where $t_k = kT_0$

with voltage

$$V_k = \Delta V_k + V_{k-1} e^{-\delta} = \sum_{q=0}^{k} \Delta V_{k-q}\ \mathrm{e}^{-q\delta}$$



Here $\delta = \alpha(T_2 + T_1) + \mu T_0.$

$\delta \simeq 0.25$ for $T_2 = T_1 = 1$ ns

The change in voltage at each revolution turn is proportional to the average bunch peak amplitude:

$$\Delta V_k = R_1 \alpha \int_{t_k - T_1}^{t_k + T_2} I_b(t_k - t')\ \mathrm{e}^{-\alpha(t_k + T_2 - t')} dt'.$$

PD Schottky spectrum (1/5)

A particle will be **at position φ** when its synchrotron angle ψ_n is equal to

$$\begin{aligned} \psi_n &= \Omega_n t_\phi + 2\pi m, \\ \psi_n &= \pi - \Omega_n t_\phi + 2\pi m, \end{aligned} \qquad t_\phi = t_\phi(\mathcal{E}_n, \phi) = \int_0^\phi \frac{d\phi'}{\sqrt{2[\mathcal{E}_n - W(\phi')]}}.$$

This will happen at times t_1 and t_2. Then similar to the "ideal" (PA) case a single particle contribution to a bunch current is

$$I_n(t,\phi) = \frac{e}{2}\sum_m [\delta(t-t_1) + \delta(t-t_2)] = \frac{e\Omega_n}{4\pi}\sum_m [\mathrm{e}^{im\Omega_n t_\phi} + \mathrm{e}^{im(\pi-\Omega_n t_\phi)}]\,\mathrm{e}^{-im(\Omega_n t + \psi_{n0})}.$$

Collecting possible contributions at φ from all particles

$$I(t,\phi) = \sum_n I_n(t,\phi).$$

PD Schottky spectrum (2/5)

The increase in voltage during bunch passage ~ average bunch peak amplitude

$$\Delta V_k = R_1 \alpha_\phi \int_{-\Phi}^{\Phi} I(t_k, \phi) \mathrm{e}^{-\alpha_\phi(\Phi-\phi)} d\phi.$$

$$\alpha_\phi = \alpha/(h\omega_0)$$

or

$$\Delta V_k = \frac{e}{2\pi} B \sum_n \sum_m \Omega_n A_m(\mathcal{E}_n) \, \mathrm{e}^{-im(\Omega_n t_k + \psi_{n0})},$$

$$B = 2R_1 \alpha_\phi \Phi \, \mathrm{e}^{-\alpha_\phi \Phi}.$$

where formfactor A

$$A_m(\mathcal{E}_n) = \frac{1}{4\Phi} \int_{-\Phi_{max}}^{\Phi_{max}} \mathrm{e}^{\alpha_\phi \phi} [\mathrm{e}^{im\Omega_n t_\phi} + \mathrm{e}^{im(\pi - \Omega_n t_\phi)}] d\phi.$$

$$\begin{aligned} \Phi_{max} &= \Phi & \text{for } \mathcal{E}_n > W(\Phi), \\ \Phi_{max} &= \phi_a(\mathcal{E}_n) & \text{for } \mathcal{E}_n < W(\Phi), \end{aligned}$$

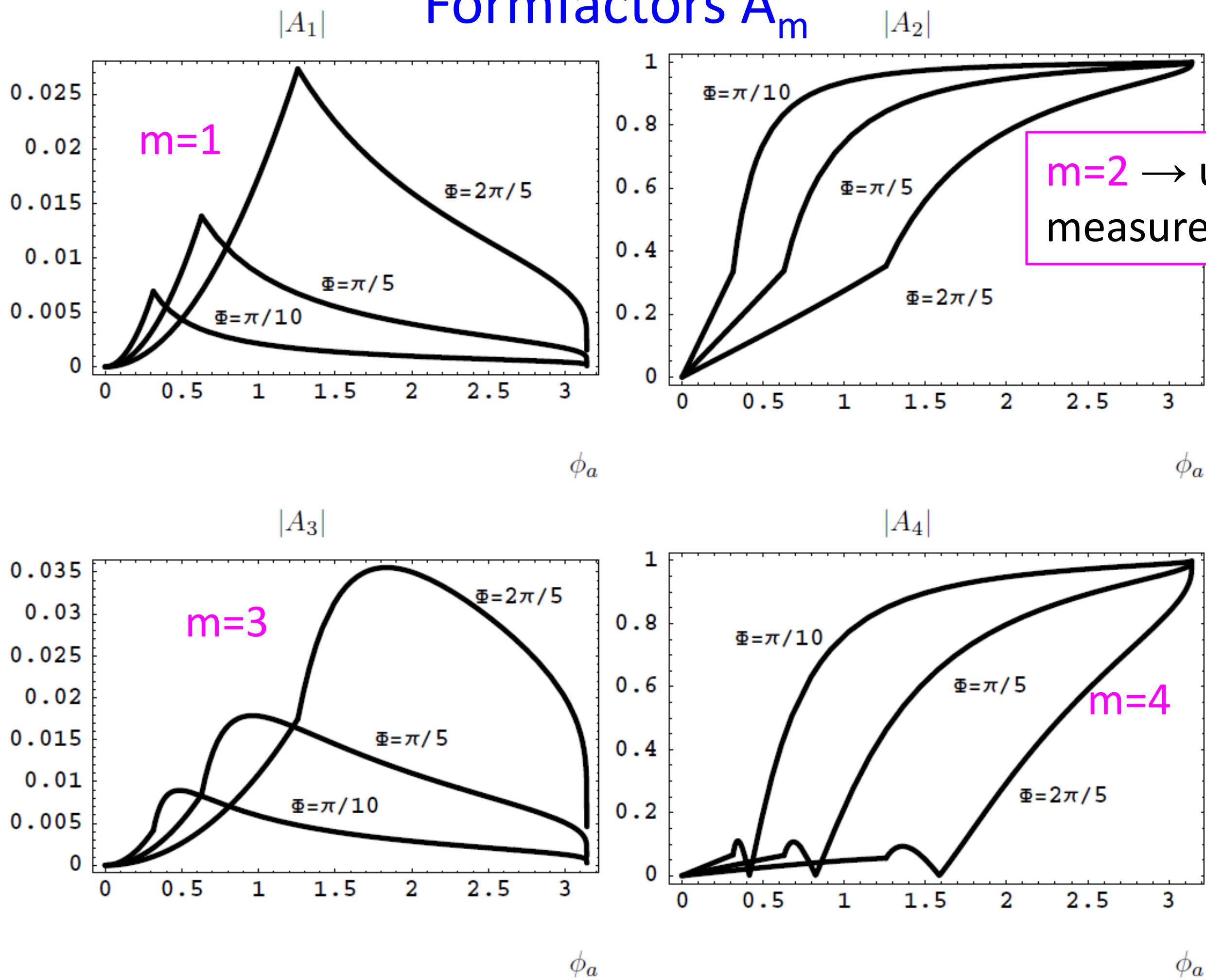
$$\mathcal{E}_n = W(\phi_a)$$

Terms with odd m are suppressed as ~ $\alpha_\varphi \Phi$

$$\begin{aligned} A_1 &= \frac{\alpha_\phi \phi_a}{3} \left(\frac{\phi_a}{\Phi}\right) & \text{for } \phi_a \le \Phi \\ A_1 &= \frac{\alpha_\phi \Phi}{3} \left(\frac{\Phi}{\phi_a}\right) & \text{for } \phi_a \ge \Phi \\ A_2 &= \frac{1}{3} \left(\frac{\phi_a}{\phi}\right) & \text{for } \phi_a \le \Phi \\ A_2 &= 1 - \frac{2}{3} \left(\frac{\Phi}{\phi_a}\right)^2 & \text{for } \phi_a \ge \Phi \end{aligned}$$

PD Schottky spectrum (3/5)

Formfactors A_m



m=2 → used in measurements!

PD Schottky spectrum (4/5)

Finally the PD signal

$$V_k = \frac{eB}{2\pi}\sum_n \sum_m \Omega_n\, A_m(\mathcal{E}_n)\, Q_m(\Omega_n)\, e^{-im(\Omega_n t_k + \psi_{n0})}$$

The sum Q_m is due to the previous turns

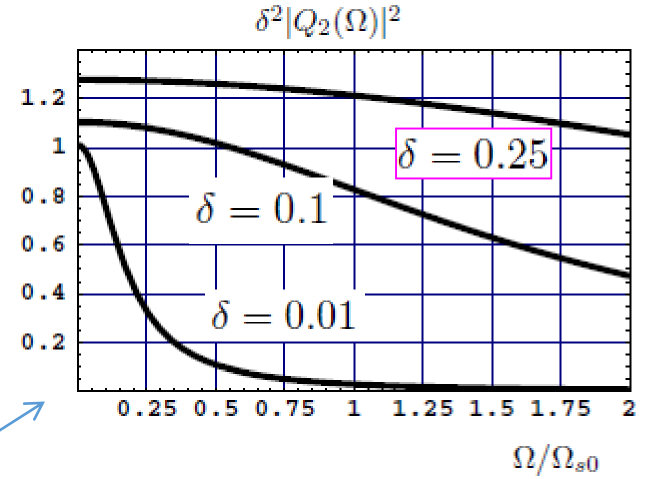
$$Q_m(\Omega_n) = \sum_{q=0}^{k} e^{im\Omega_n T_0 q - q\delta} \simeq \frac{1}{1 - e^{im\Omega_n T_0 - \delta}}$$

The power spectral density of the PD signal

$$P(\omega) = \frac{P_0}{\Omega_{s0}^2}\sum_{m=1}^{\infty}\int \Omega^2 F(\Omega)\, |A_m(\Omega)|^2 |Q_m(\Omega)|^2\, S^2 d\Omega,$$

where $P_0=e^2Nf_{so}B^2$ and for acquisition time T_a

$$S^2 = |S(\omega - m\Omega)|^2 = \frac{2T_a}{T_{s0}}\,\frac{\sin^2[(\omega - m\Omega)T_a/2]}{[(\omega - m\Omega)T_a/2]^2}.$$

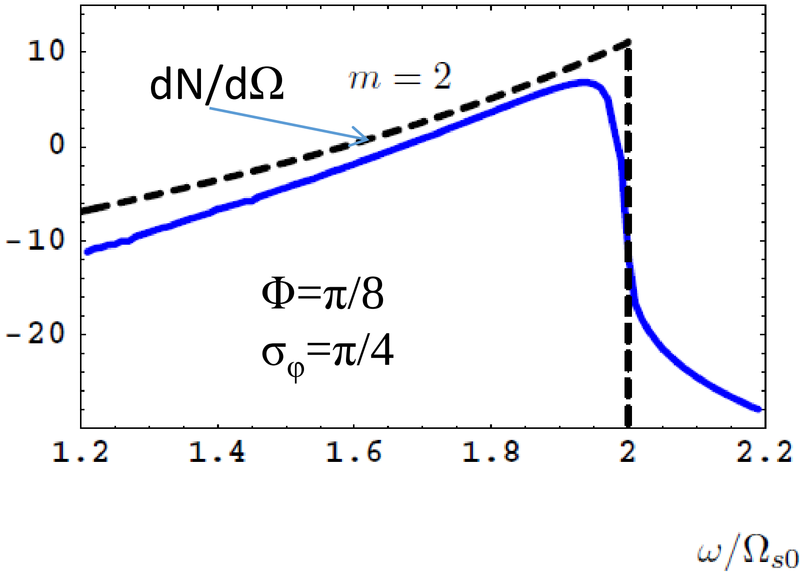


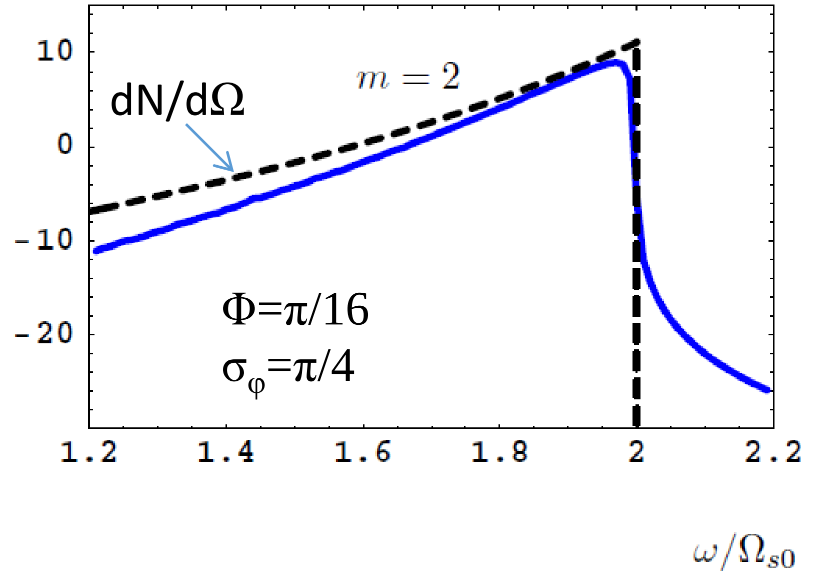
- Q_m is a farly flat function (for not too small δ, in SPS δ=0.25)

Here $\delta = \alpha(T_2 - T_1) + \mu T_0$.

PD Schottky spectrum (5/5)

Peak Detected Schottky: quadrupole bands P_2/P_0

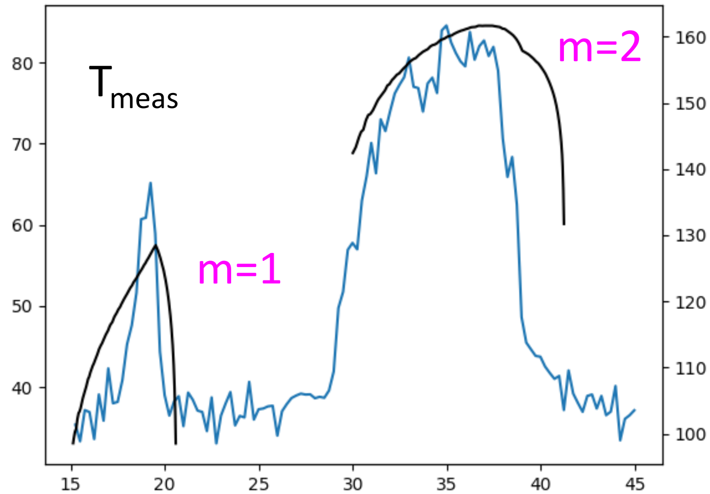


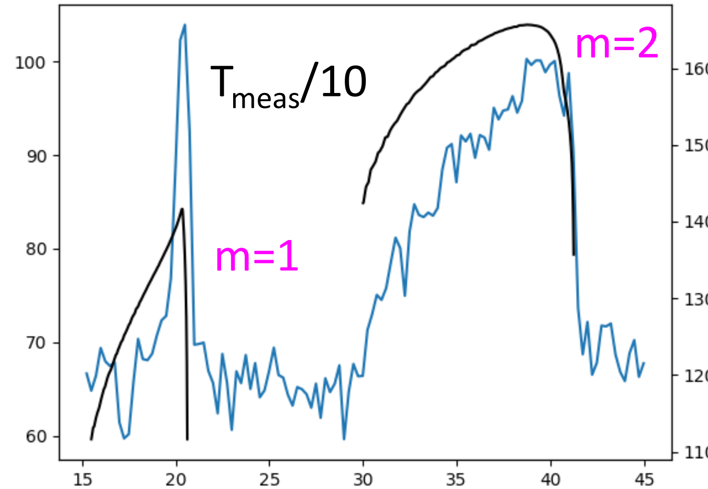


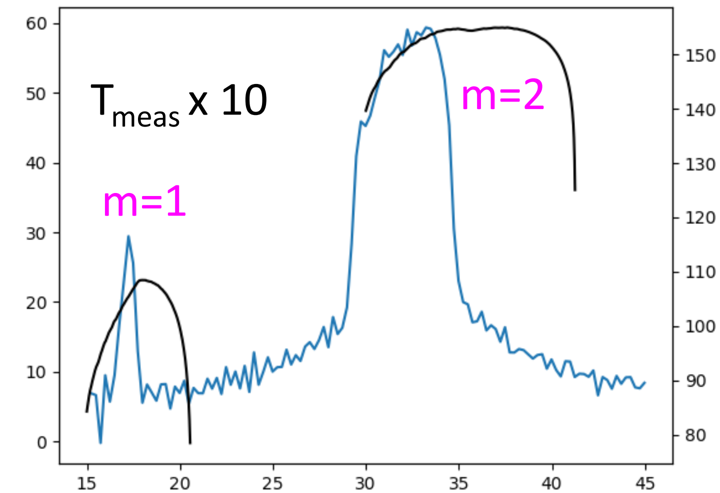
⇒ The measured PD Schottky spectrum deviates from distribution function dN/dΩ mainly due to $|A(\Omega)|^2$

⇒ The distortion is smaller for smaller averaging distance Φ

Numerical simulations of PD Schottky



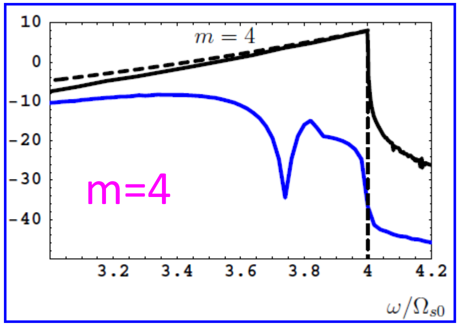




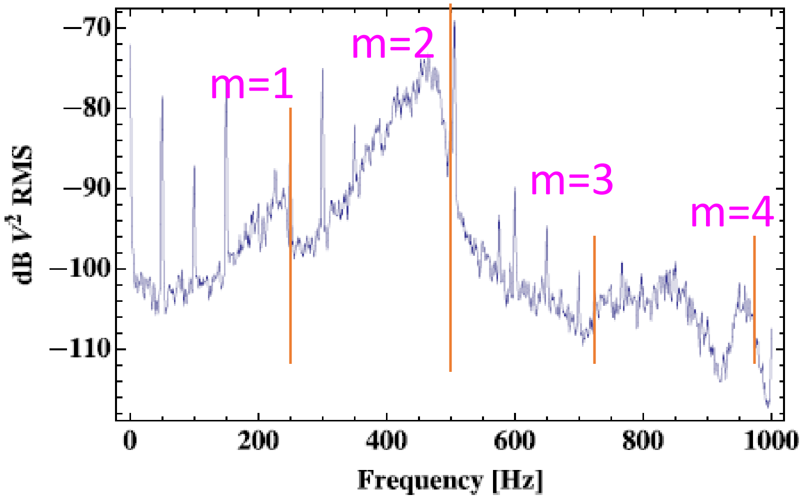
First simulations of LHC PD Schottky
by J. Esteban Muller (ESS), 2018

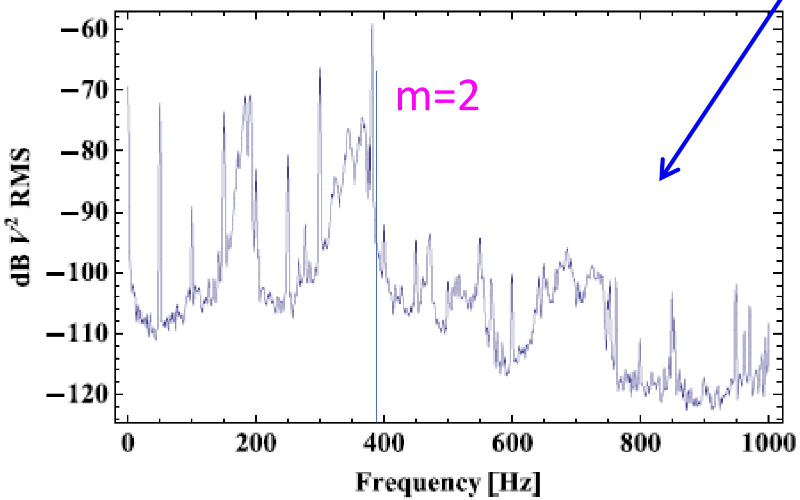
6.5 TeV, 12 MV, 1.15 ns bunch,
(no intensity effects)
fs= 20.64 Hz, 2fs=41.28 Hz
1/α=10.4 ns and 1/μ=970 μs

Measured PD Schottky spectrum in SPS



→ Only quadrupole line is really usable





Low intensity single bunch at 26 GeV/c, f_{s0}=240 Hz, $\sigma_\varphi=\pi/4$

High intensity bunch in coast at 270 GeV/c, f_{s0}=192 Hz, $\sigma_\varphi=\pi/12$

Courtesy T. Bohl

Comparison with bunched-beam ("traditional") longitudinal Schottky spectrum

The spectral density of current fluctuations for bunched beam.
(e.g. S. Chattopadhyay, Some fundamental aspects of fluctuations and coherence in charged-particle beams in storage rings, CERN-84-11, 1984)

$$P_L(\omega) = \frac{e^2 N \omega_0^2}{2\pi} \sum_{p=-\infty}^{\infty} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \frac{1}{m} F\left(\frac{\omega - p\omega_0}{m}\right) |I_{mp}(\mathcal{E})|^2,$$

where $\mathcal{E}$ corresponds to the synchrotron frequency $\Omega = \omega - p\omega_0$
and the "distortion" factor is

$$I_{mp}(\mathcal{E}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{p\phi(\mathcal{E},\psi)/h - im\psi} \, d\psi.$$

For short bunches in a single RF system

$$I_{mp}(\mathcal{E}) \simeq i^m J_m(p\phi_a/h_1)$$

Bunched-beam ("traditional") longitudinal Schottky spectrum

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Traditional Schottky spectrum: dipole sidebands

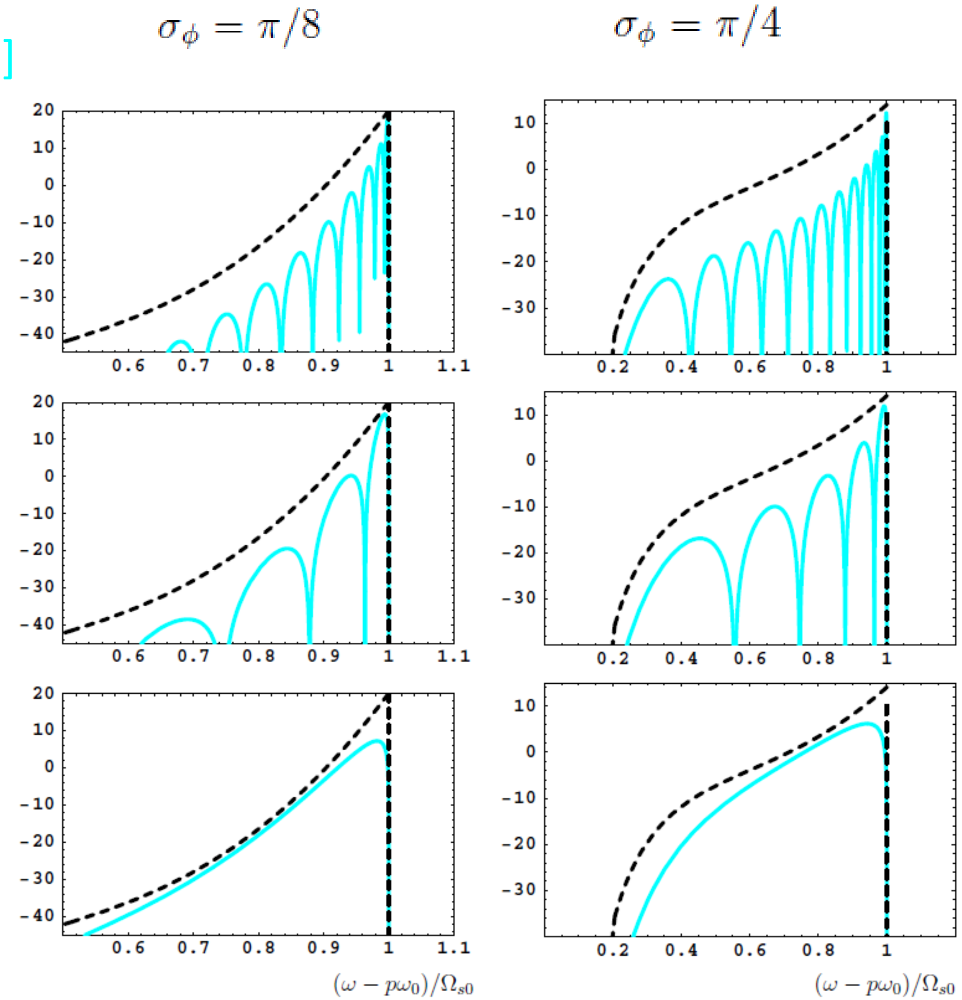
P_{pm}/P_0 [dB]

$\Omega_{s0}F_{dist}(\omega)$

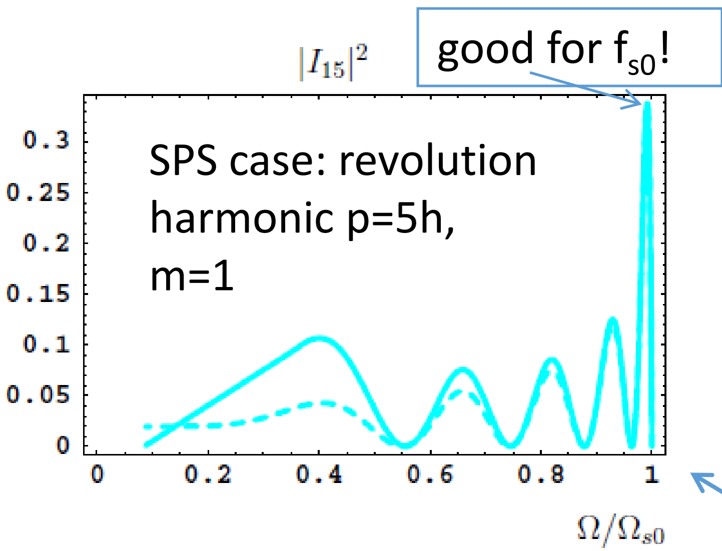
LHC: 4.8 GHz, p=12h →

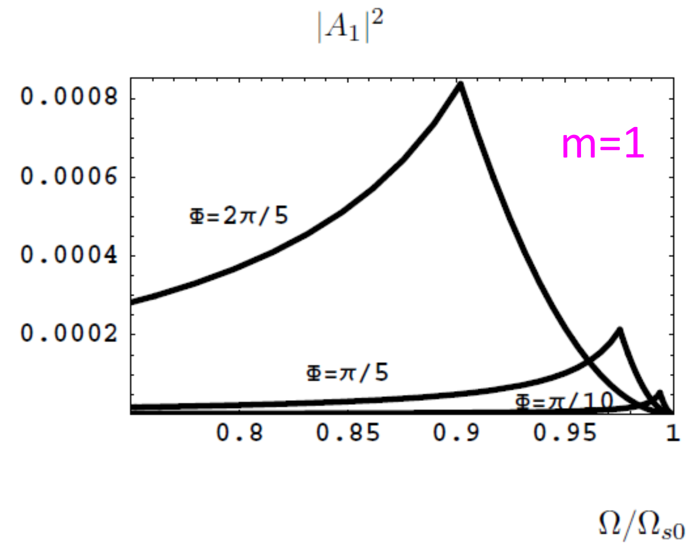
SPS: 1.0 GHz, p=5h →

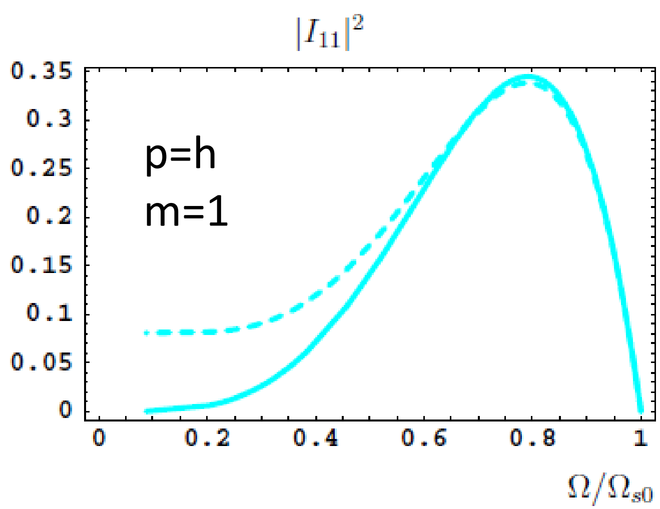
p=h →

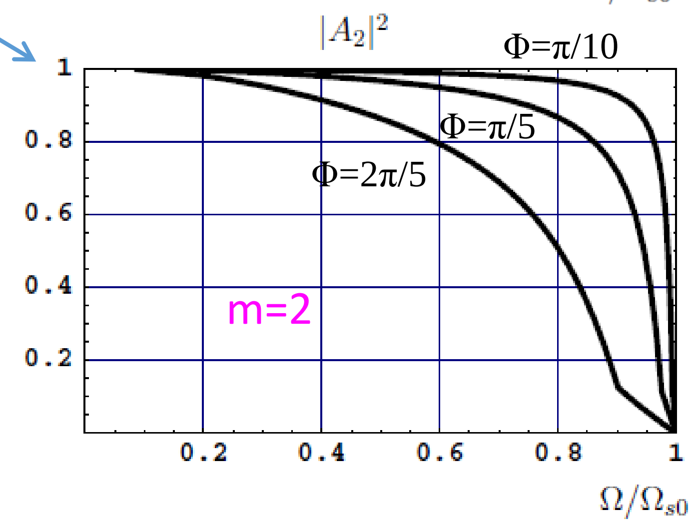


Traditional versus PD Schottky signals



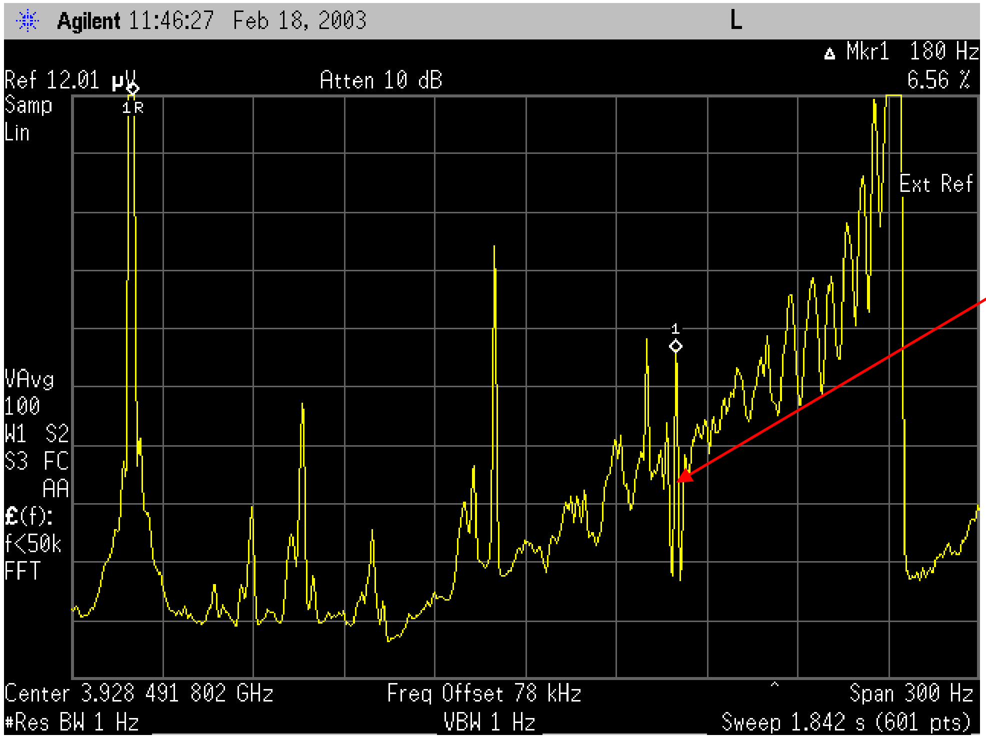






Application of traditional Schottky spectrum Losses along the injection plateau due to noise

- Due to noise particles are redistributed within the bunch or lost completely (resonant islands with bridges due to white noise?)
- These effects also seen in ppbar and in RHIC



RHIC

Traditional Schottky at 4 GHz.

Hole appears during coast.

In this case, particles are lost from the bunch, not redistributed.

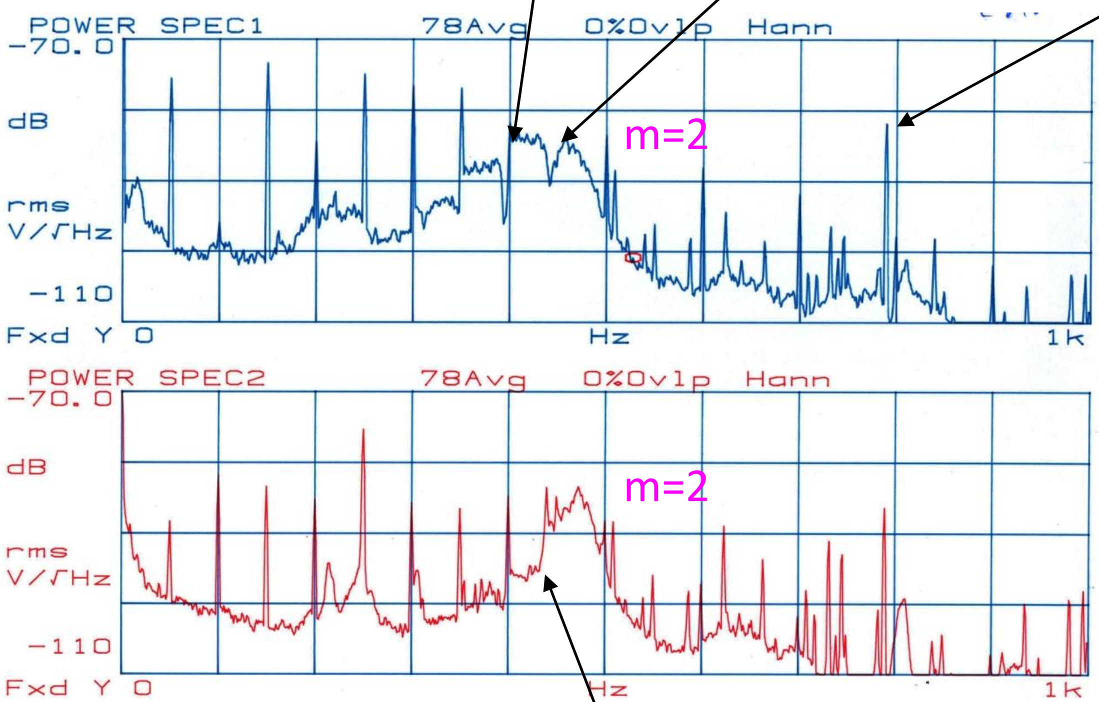
Cured by removing source in RF.

M. Brennan (BNL)

Application of PD Schottky:
Losses along the SPS injection plateau (noise)

Peak detected Schottky after 17 min store

395 Hz hole caused by coherent line at 790 Hz. The other?



Head of batch

Tail of batch

Depopulated area – impedance, RF, electron cloud …?

V_{RF} = 2MV, f_s = 257 Hz

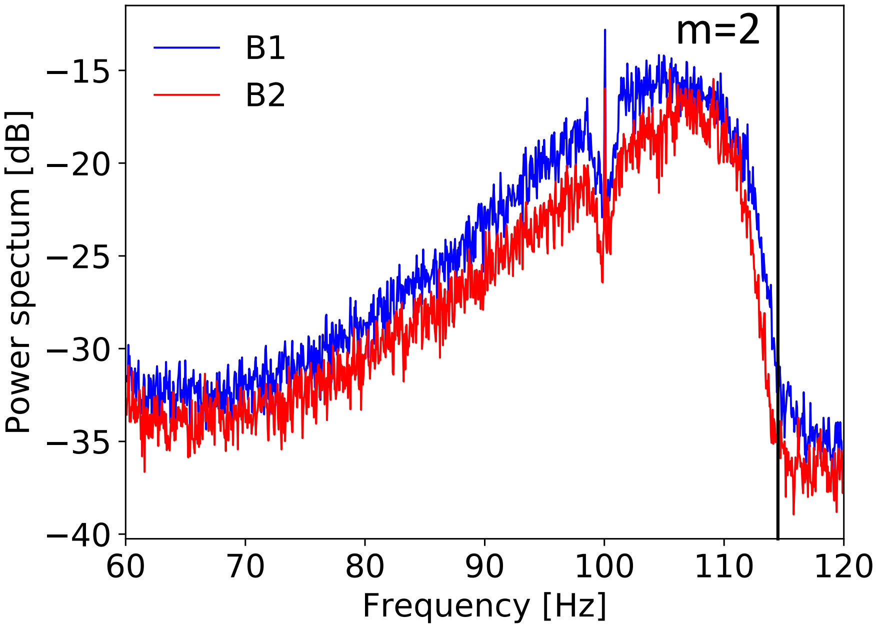
T. Linnecar, 2005

PD Schottky spectrum on LHC flat bottom

- LHC MD in 2017
- fs0= 57.5 Hz for 6 MV

→ Hole in particle distribution due to strong effect of n x 50 Hz lines

Not visible on bunch profiles

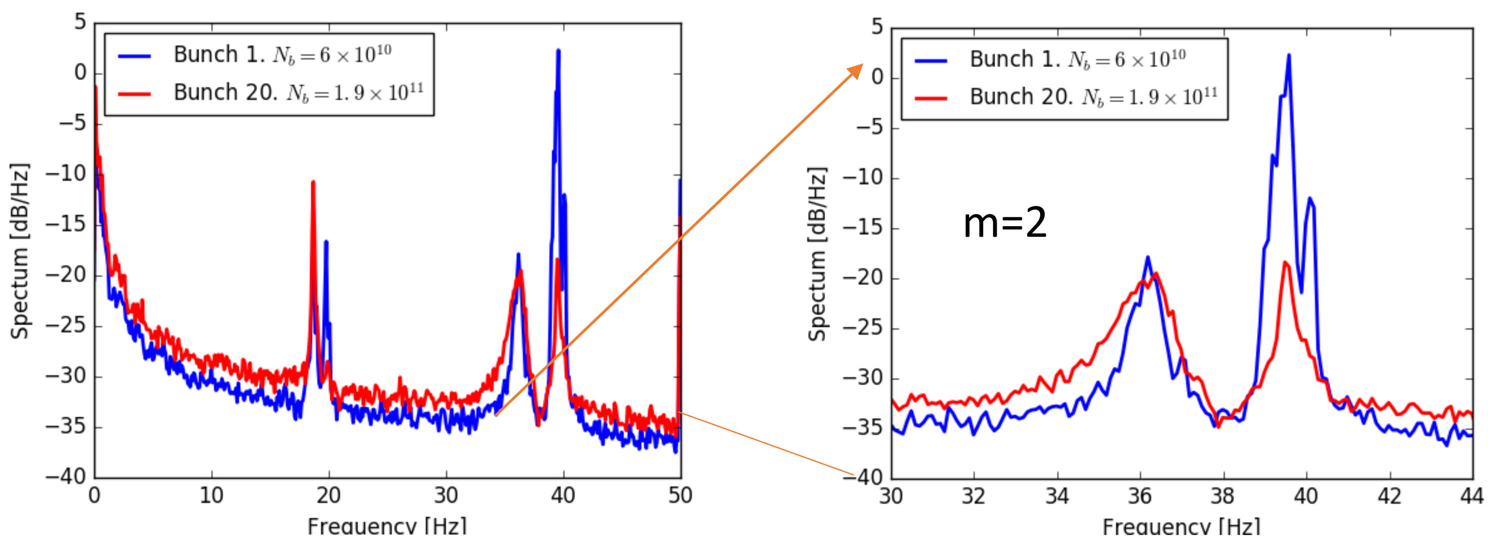


Courtesy I. Karpov

PD Schottky measurements in LHC: 4.5 TeV flat top

Particle distribution after controlled emittance blow-up (BUP),

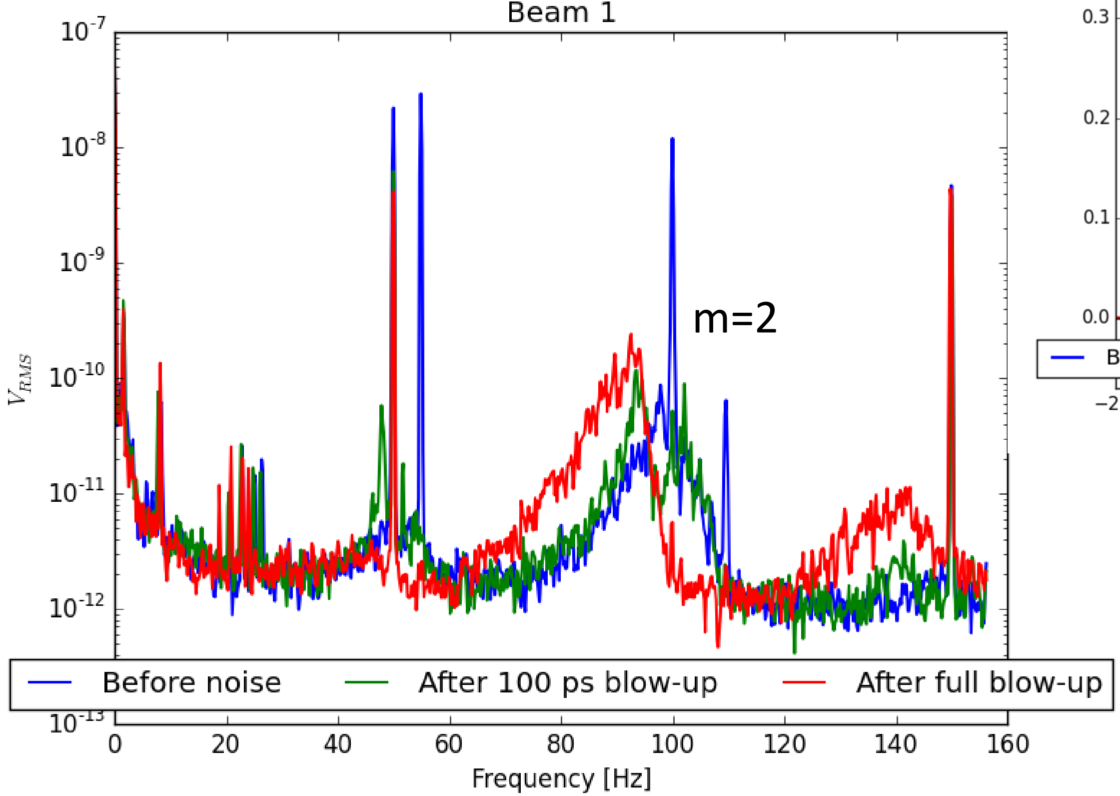
Huge hole due to action of phase loop (?)

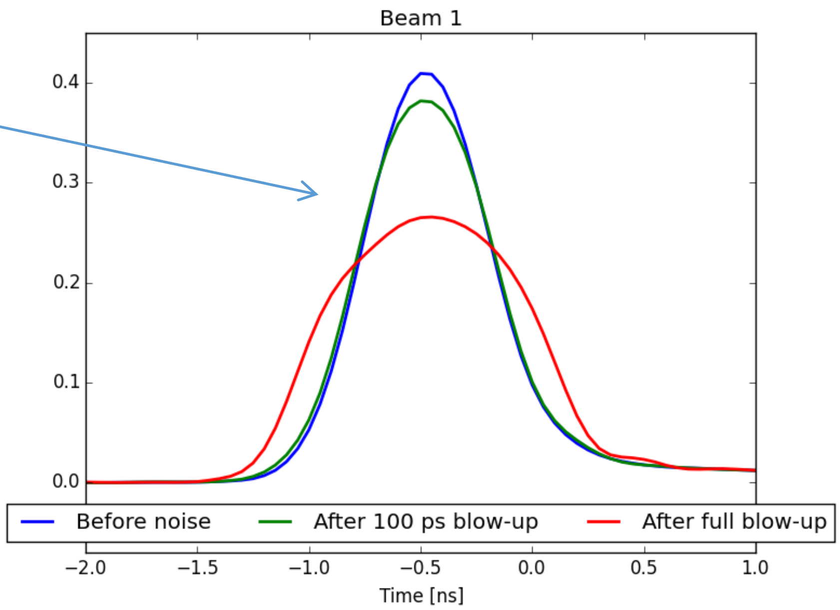


Courtesy J. E. Muller

Bunch flattening on the LHC flat bottom

PD Schottky spectrum of 3 bunches





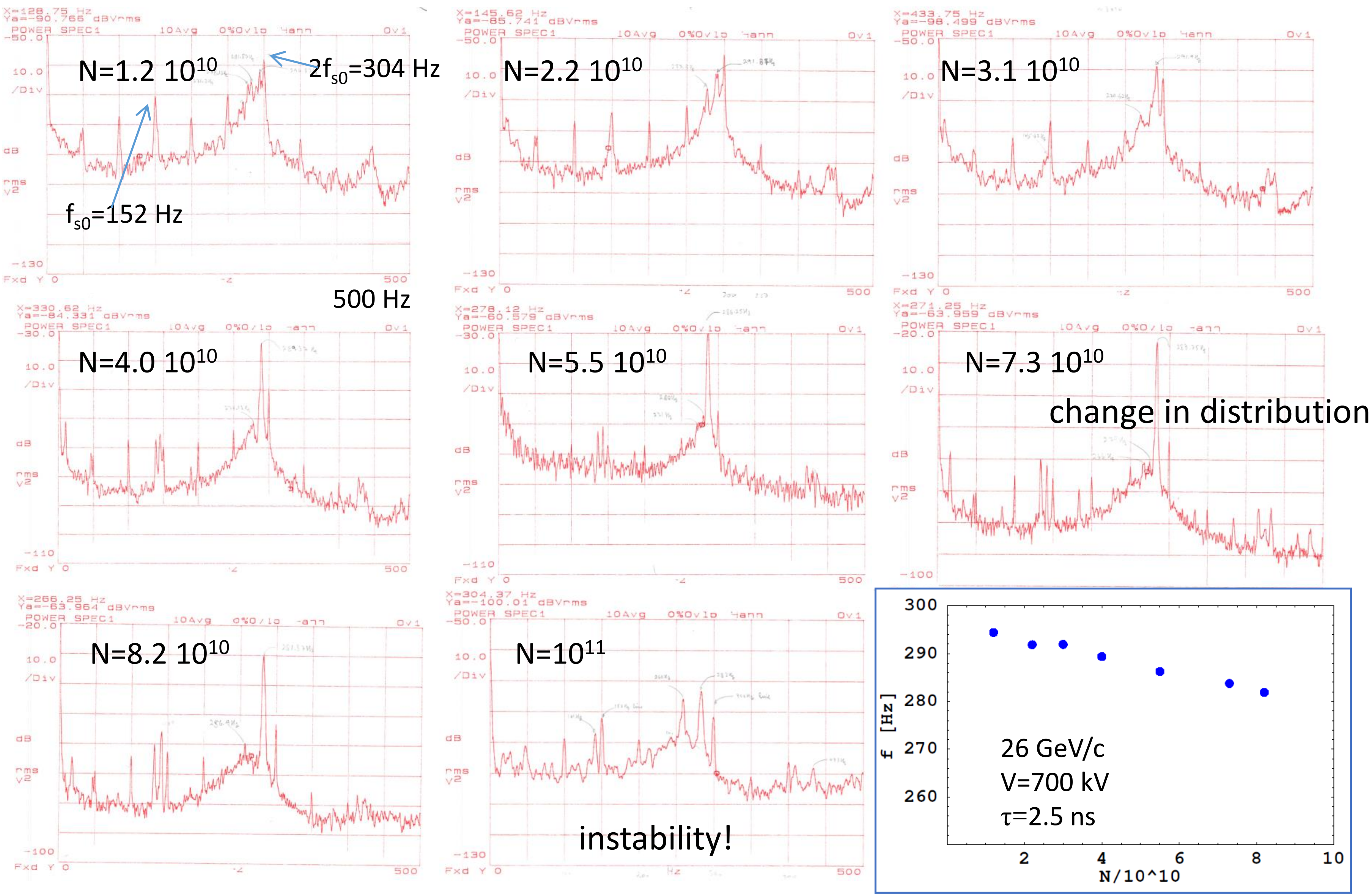
450 GeV, RF voltage 6 MV

fs = 55.09 Hz

Test on 29.04.2015

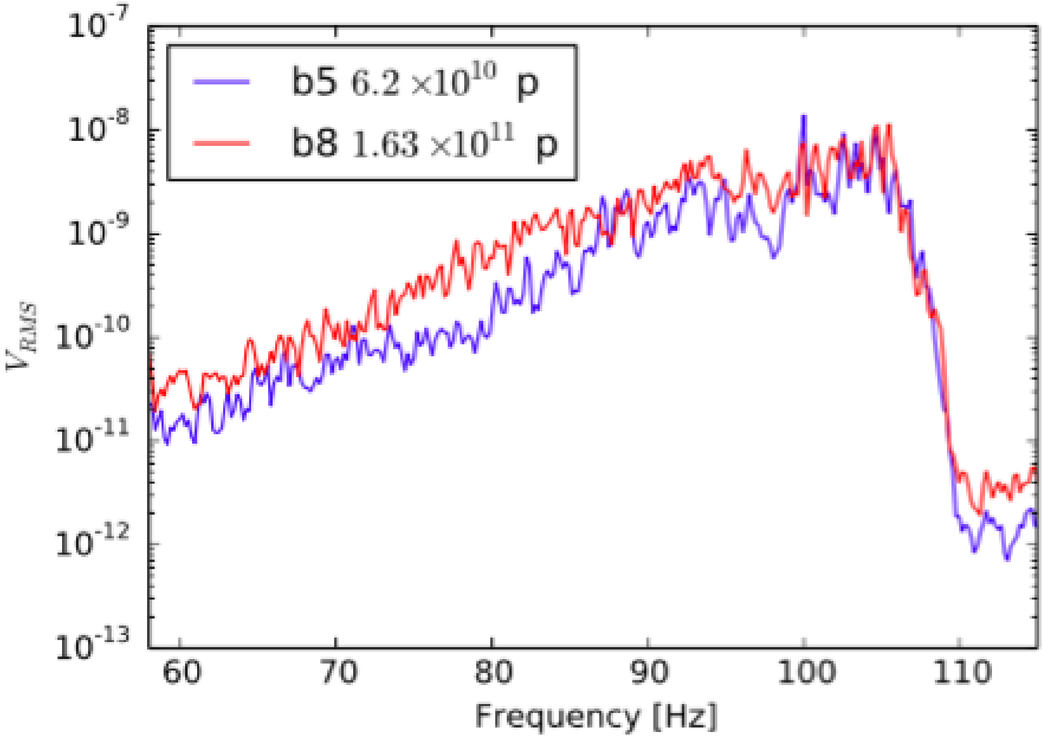
Courtesy J. E. Muller

Quadrupole frequency shift with intensity in SPS



Synchrotron frequency shift in LHC

Quadrupole line of PD Schottky spectrum of 2 bunches



For ImZ=0.1 Ohm
expected Δf_s = − 0.35 Hz

Frequency resolution ~ 0.2 Hz

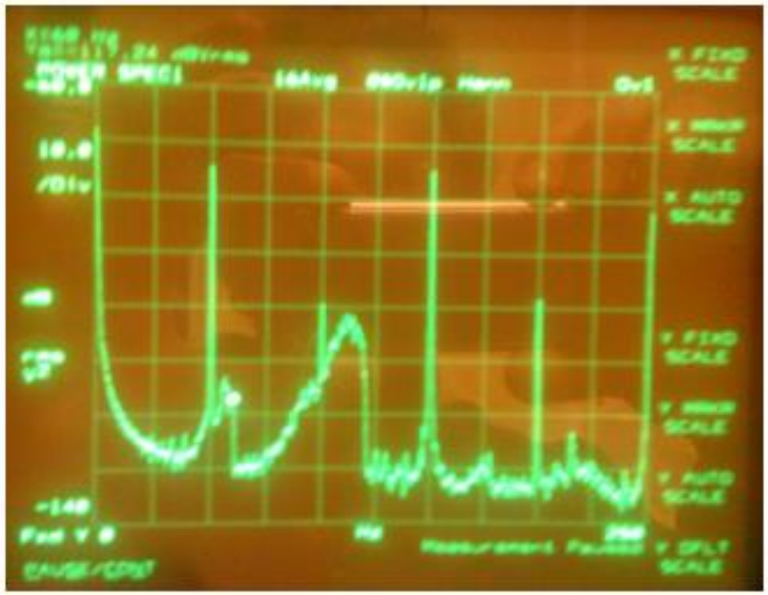
τ = 1.4 ns, f_{so} = 55.1 Hz for 450 GeV and V_{rf} = 6 MV.
MD on Nov. 28, 2012 (J. E. Muller et al.)

Summary

- The PD Schottky spectrum have been used at CERN for a long time as an additional diagnostic tool in longitudinal plane
- The quadrupole line of the PD Schottky spectrum represents the particle distribution in synchrotron frequency modified by the synchrotron frequency nonlinearity (Ω^2) and experimental set-up, function $(A_2Q_2)^2$
- The measured PD signal is proportional to an average over the bunch current around its peak value. The distortion mainly depends on the distance Φ over which the averaging is performed.
- For optimised experimental set-up, in the PD Schottky m=2 line, the deviation from synchrotron frequency distribution function is much less than for the traditional Schottky (which nevertheless can give better measurement of a zero-amplitude synchrotron frequency f_{s0})

Spare slides

First measured PD Schottky spectrum in LHC

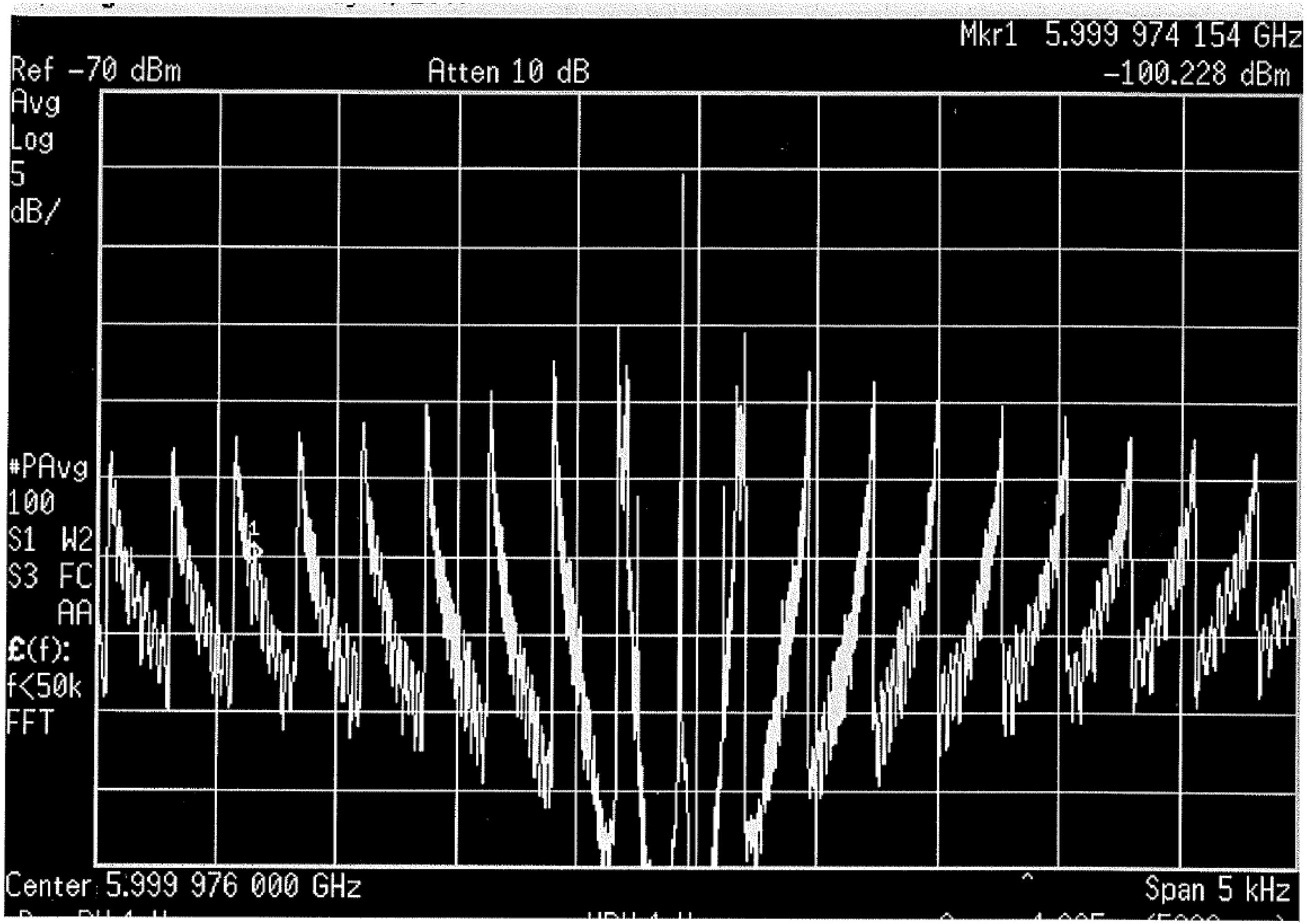


LHC coast at 450 GeV/c, September 2008,

f_{s0}=66 Hz, $\sigma_{\phi} = \pi/7$. Courtesy T. Bohl.

Scales: vertical 10dBV^{2}_{rms}/div, horizontal 25 Hz/div

Traditional longitudinal Schottky spectrum



measurements in RHIC, M. Brennan