Why the Maximal Odderon violates unitarity

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1. Unitarity

$$\mathbf{S}\mathbf{S}^\dagger = \mathrm{I} \quad (\text{let } \mathbf{S} = \mathrm{I} + \mathrm{i}\mathbf{A}) \quad \rightarrow \quad \underline{\mathrm{i}(\mathbf{A}^\dagger - \mathbf{A}) = \mathbf{A}^\dagger\mathbf{A}}$$

Unitarity equation

$$\underline{2\,\mathrm{Im}A_{\mathrm{el}}(b)} = \sum_n |A_{i\to n}(b)|^2 = \underline{|A_{\mathrm{el}}(b)|^2 + G_{\mathrm{inel}}(b)}$$

$$\text{where } G_{\mathrm{inel}}(b) = \sum_{n\neq i} |A_{i\to n}(b)|^2 < 1 = \underline{\text{probability}} \text{ of inelastic scatt.}$$

Solution of unitarity eq.

$$A(b) \equiv \underline{A_{\mathrm{el}}(b) = i(1 - e^{-\Omega(b)/2})} \quad \text{with } \mathrm{Re}\Omega(b) \geq 0$$

No solution of unitarity eq. if $G_{\mathrm{inel}}(b) > 1$. Let us calculate $G_{\mathrm{inel}}(b)$

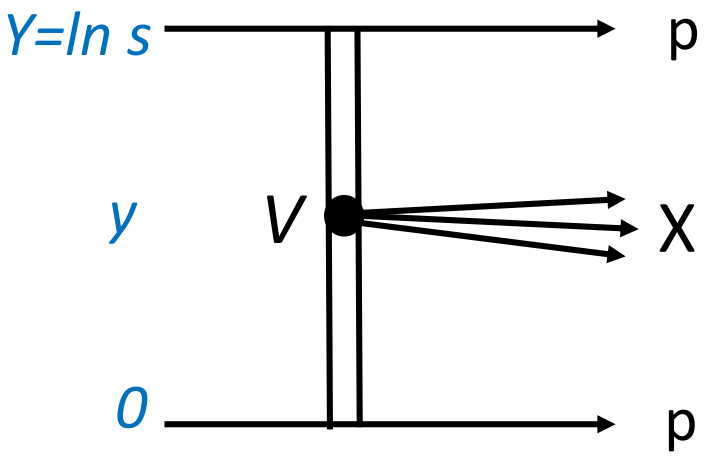
2. Finkelstein-Kajantie problem: σ(diff^{ve}) > σ(total) due to $\int_0^{\ln s} dy \ldots \sim \ln s$

Simple example: Central Exclusive Prod. pp → p+X+p

In the Froissart limit $\sigma_{\rm CEP} \sim ln^5 s$

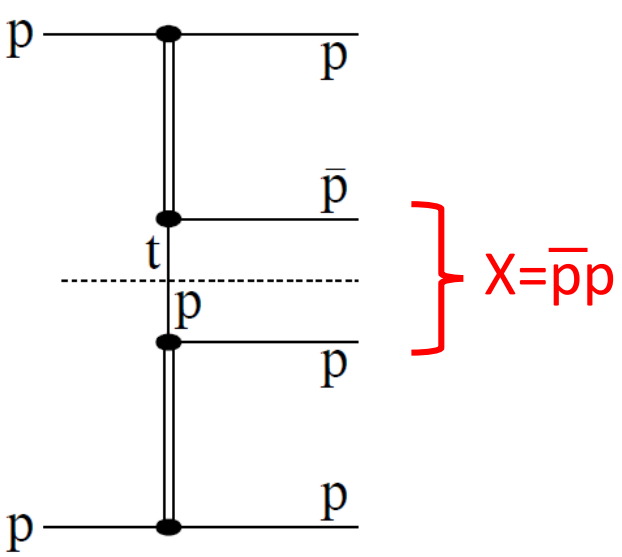
so $\sigma_{\rm CEP} > \sigma_{\rm tot} \sim ln^2 s$

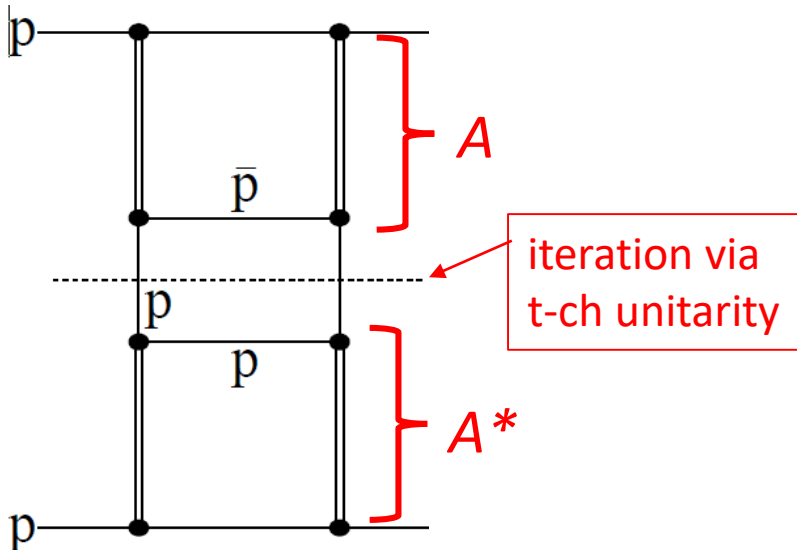
Could the explanation be that vertex $V = 0$? **No**



Can show, for example, that the $p\bar{p}$ component of X generated by t-channel unitarity has V ≠ 0, and cannot be compensated due to the singularity/pole at $t=m_p^2$.

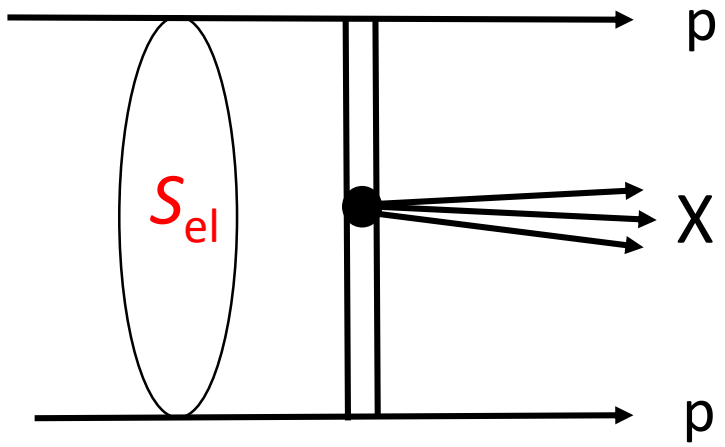
So starting from A_{el} we see t-ch unitarity gives a component of $G_{inel}(b)$ increasing faster than $\int_0^{\ln s} dy.. \sim \ln s$





Figs: amplitude (left) and cross section (right) of $\bar{p}p$ Central Exclusive Prod. generated by t-ch unitarity

3. Solution to the Finkelstein-Kajantie problem



Complete CEP must include rescattering S_{el} (that is the survival probability $S^2 = |S_{el}|^2$ of the rapidity gaps)

$$A_{CEP}(b) = S_{el}(b)\, A_{bare}(b)$$

where $\quad |S_{el}(b)|^2 \;=\; |1 + iA_{el}(b)|^2 \;=\; e^{-\mathrm{Re}\Omega(b)}$

Black disc asymptotics: $\quad \mathrm{Re}\Omega \to \infty, \;\; A_{el}(b) \to i, \quad \mathbf{S^2(b) \to 0 \text{ for } b < R}$

If σ_{tot} increases, Black disc is the only known solution to the FK problem

To repeat, if at least one component of G_{inel} increases (as $\int dy \sim \ln s$) as $s \to \infty$, violating unitarity, the only way to cancel it is to have $S(b) \to 0$

4. Maximal Odderon contradicts unitarity as s →∞

MO

Asymptotially MO means $\mathbf{Re}A/\mathbf{Im}A \to$ constant $\neq 0$

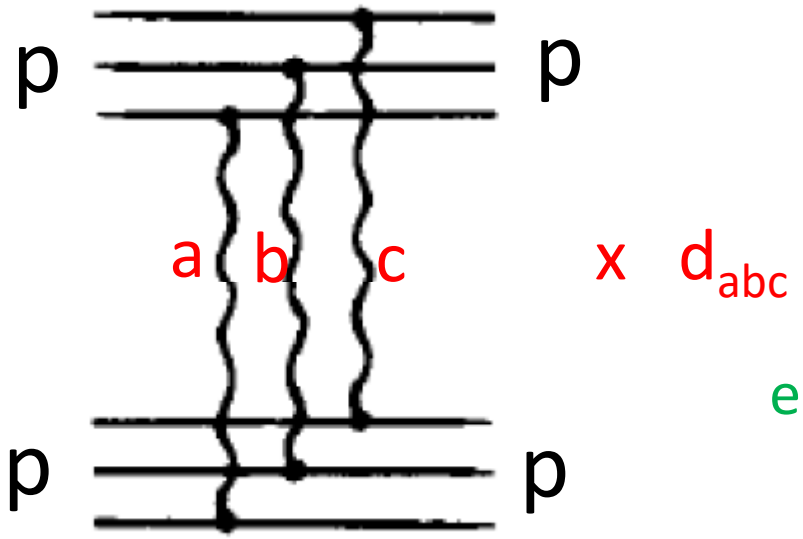
In this case $\quad S^2(b) \;=\; |1 + iA(b)|^2 \;\geq\; |\mathrm{Re}A(b)|^2 \;\neq 0$

so there is no possibility to compensate the growth of σ_{CEP}.

The Odderon in QCD

1. Estimate of Odderon contrib^n

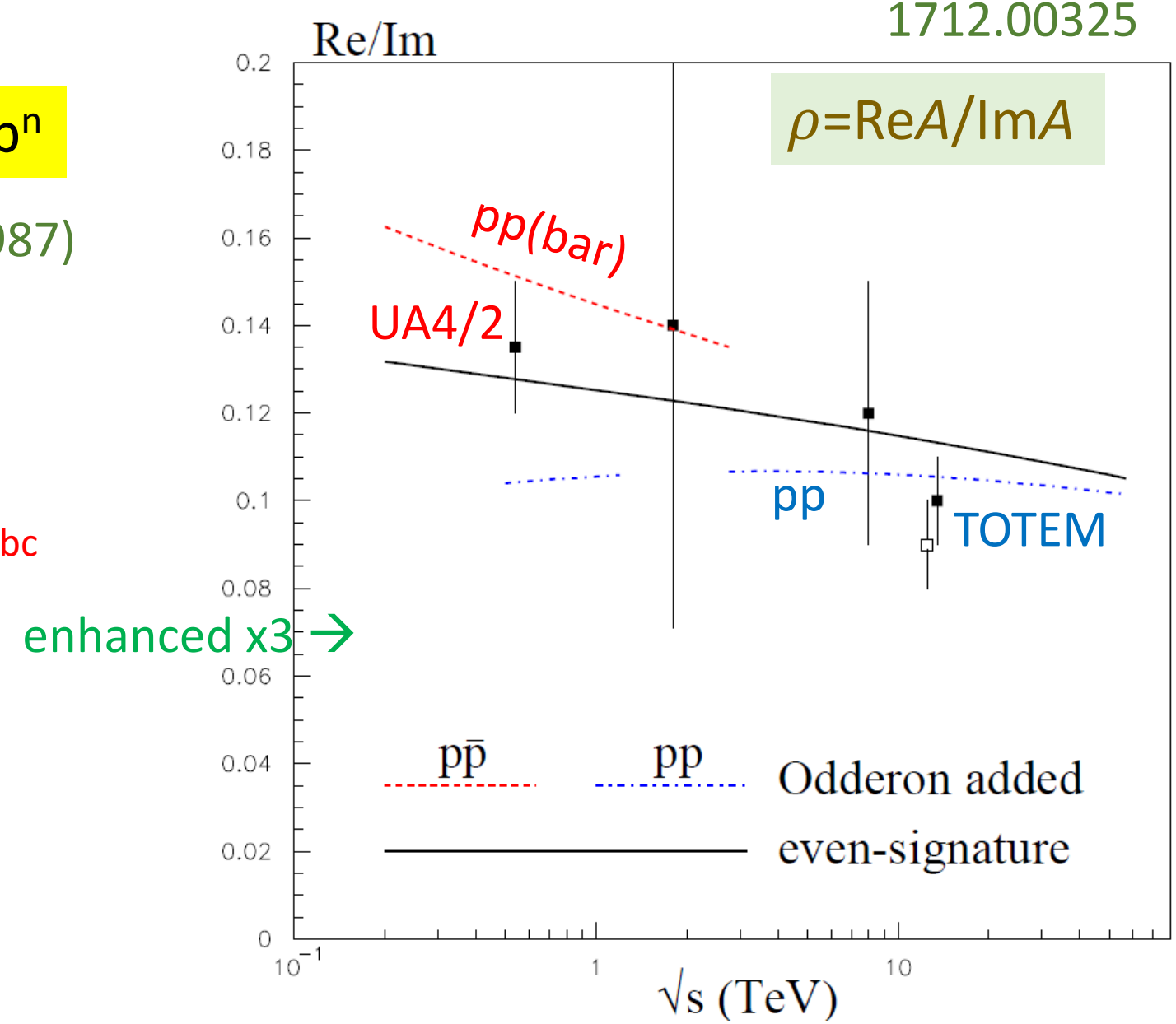
QCD lowest α_s order Ryskin (1987)



$$A(b) = i\left(1 - e^{-\Omega(b)/2}\right)$$

with $\Omega = \Omega_{\text{even}} + \Omega_{\text{odd}}$

1712.00325



2. Maximal Odderon excluded

As s → ∞ Re A/ImA → 0 maximal Odderon excluded

3. Life above the black disc limit

$$\mathrm{Im}A(b) = \int \sqrt{\frac{d\sigma_{\mathrm{el}}}{dt}\frac{16\pi}{1+\rho^2}} J_0(bq_t)\frac{q_t dq_t}{4\pi}$$

→

central collisions (b=0)

$$\boxed{\mathrm{Im}A(0) = \frac{\sigma_{\mathrm{tot}}}{4\pi B_{\mathrm{el}}}}$$

data

$\sqrt{s}$ TeV	UA4: 0.541	LHC	2.76	7	8	13
Im$A(0)$	0.84± 0.025	TOTEM	1.01± 0.043	1.01 ± 0.03	1.045±0.032	1.11±0.032
Im$A(0)$		ATLAS		0.988±0.02	0.996±0.015	

TOTEM exceed the black disc limit (ImA(t=0) ≤ 1) by > 3σ at 13 TeV

If confirmed, this is a most important result