1. Unitarity

\[ SS^\dagger = I \quad (\text{let} \quad S = I + iA) \quad \rightarrow \quad i(A^\dagger - A) = A^\dagger A \]

Unitarity equation

\[ 2 \text{Im} A_{el}(b) = \sum_n |A_{i\rightarrow n}(b)|^2 = |A_{el}(b)|^2 + G_{inel}(b) \]

where \( G_{inel}(b) = \sum_{n \neq i} |A_{i\rightarrow n}(b)|^2 < 1 \) = probability of inelastic scatt.

Solution of unitarity eq.

\[ A(b) \equiv A_{el}(b) = i(1 - e^{-\Omega(b)/2}) \quad \text{with} \quad \text{Re} \Omega(b) \geq 0 \]

No solution of unitarity eq. if \( G_{inel}(b) > 1. \) Let us calculate \( G_{inel}(b) \)
2. Finkelstein-Kajantie problem: $\sigma(\text{diff}^\text{ve}) > \sigma(\text{total})$ due to $\int_0^{\ln s} dy \ldots \sim \ln s$

Simple example: Central Exclusive Prod. $pp \rightarrow p + X + p$

In the Froissart limit $\sigma_{\text{CEP}} \sim \ln^5 s$ so $\sigma_{\text{CEP}} > \sigma_{\text{tot}} \sim \ln^2 s$

Could the explanation be that vertex $V = 0$? No

Can show, for example, that the $p\bar{p}$ component of $X$ generated by t-channel unitarity has $V \neq 0$, and cannot be compensated due to the singularity/pole at $t = m_p^2$.

So starting from $A_{\text{el}}$ we see t-ch unitarity gives a component of $G_{\text{inel}}(b)$ increasing faster than $\int_0^{\ln s} dy \ldots \sim \ln s$

Figs: amplitude (left) and cross section (right) of $\bar{p}p$ Central Exclusive Prod. generated by t-ch unitarity
3. Solution to the Finkelstein-Kajantie problem

Complete CEP must include rescattering $S_{el}$ (that is the survival probability $S^2 = |S_{el}|^2$ of the rapidity gaps)

$$A_{CEP}(b) = S_{el}(b) \ A_{bare}(b)$$

where

$$|S_{el}(b)|^2 = |1 + iA_{el}(b)|^2 = e^{-\text{Re}\Omega(b)}$$

Black disc asymptotics: $\text{Re}\Omega \rightarrow \infty$, $A_{el}(b) \rightarrow i$, $S^2(b) \rightarrow 0$ for $b < R$

If $\sigma_{tot}$ increases, Black disc is the only known solution to the FK problem

To repeat, if at least one component of $G_{inel}$ increases (as $\int dy \sim \ln s$) as $s \rightarrow \infty$, violating unitarity, the only way to cancel it is to have $S(b) \rightarrow 0$

4. Maximal Odderon contradicts unitarity as $s \rightarrow \infty$

Asymptotically MO means $\text{Re}A/\text{Im}A \rightarrow \text{constant} \neq 0$

In this case $S^2(b) = |1 + iA(b)|^2 \geq |\text{Re}A(b)|^2 
eq 0$

so there is no possibility to compensate the growth of $\sigma_{CEP}$. 
The Odderon in QCD

1. Estimate of Odderon contribution

QCD lowest $\alpha_s$ order

Ryskin (1987)

\[ A(b) = i \left( 1 - e^{-\Omega(b)/2} \right) \]

with \( \Omega = \Omega_{\text{even}} + \Omega_{\text{odd}} \)

\[ \rho = \frac{\text{Re}A}{\text{Im}A} \]
2. Maximal Odderon excluded

As \( s \to \infty \) \( \text{Re} \ A/\text{Im} \ A \to 0 \) maximal Odderon excluded

3. Life above the black disc limit

\[
\text{Im} A(b) = \int \sqrt{\frac{d\sigma_{el}}{dt} \frac{16\pi}{1 + \rho^2 J_0(b q_t) \frac{q_t dq_t}{4\pi}}}
\]

\[
\text{Im} A(0) = \frac{\sigma_{tot}}{4\pi B_{el}}
\]

<table>
<thead>
<tr>
<th>( \sqrt{s} ) TeV</th>
<th>UA4: 0.541</th>
<th>LHC</th>
<th>2.76</th>
<th>7</th>
<th>8</th>
<th>13</th>
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</thead>
<tbody>
<tr>
<td>ImA(0)</td>
<td>0.84± 0.025</td>
<td>TOTEM</td>
<td>1.01 ± 0.043</td>
<td>1.01 ± 0.03</td>
<td>1.045±0.032</td>
<td>1.11±0.032</td>
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<tr>
<td>ImA(0)</td>
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<td>ATLAS</td>
<td>0.988±0.02</td>
<td>0.996±0.015</td>
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</table>

TOTEM exceed the black disc limit (\( \text{Im} A(t=0) \leq 1 \)) by \( > 3\sigma \) at 13 TeV

If confirmed, this is a most important result