Eikonal model analysis of elastic pp collisions at high energies

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Contemporary situation concerning description of el. scattering

- description and understanding of el. pp scattering is still not fully satisfactory
- contemporary situation summarized recently, e.g., in a strategical document A. Andreazza et al., "What Next: White Paper of the INFN-CSN1", Frascati Phys. Ser. **60**, 1–302 (2015); Section 7.5 - Total, elastic and diffractive cross sections:

  "Several theoretical models have been developed during the last decades to interpret the experimental results. Unfortunately, the perturbative QCD approach cannot be used in this context since most of the processes contributing to the total cross section are characterised by low momentum transfer. Some of the models are still based on Regge theory, while others prefer using optical or eikonal approaches. Moreover, so-called QCD-inspired models are trying to connect the concepts of Pomeron trajectories and proton opacity to the QCD description of elementary interactions between quarks and gluons. At the moment, no model manages to describe qualitatively and quantitatively the large amount of data available; they all have merits and shortcomings. Typically, they successfully describe the experimental results in a certain kinematic range but completely fail in other ones."
- at the moment only the *eikonal model approach* allows to take into account and study both Coulomb-hadronic interference *and* dependence of collisions on impact parameter - more fundamental description than other approaches discussed in the literature
  $\Rightarrow$ this talk is (therefore) devoted mainly to possibilities of the eikonal model approach

Contemporary descriptions of elastic collisions of charged hadrons

- measured elastic $\mathrm{d}\sigma/\mathrm{d}t$ of two charged hadrons given by

$$\frac{\mathrm{d}\sigma(s,t)}{\mathrm{d}t} = \frac{\pi}{sp^2}\left|F^{\mathrm{C+N}}(s,t)\right|^2 \tag{1}$$

  - $F^{\mathrm{C+N}}(s,t)$ - complete elastic scattering amplitude of Coulomb-hadronic interaction depending on both Coulomb $F^{\mathrm{C}}(s,t)$ and hadronic $F^{\mathrm{N}}(s,t)$ amplitudes
  - Coulomb interaction is usually assumed to be well known from QED (except electromagnetic form factors); Eq. (1) allows "separation" of Coulomb interaction from data and to study less known hadron (nuclear) scattering (QCD inapplicable)
  - Coulomb-hadronic interference used to constrain t-dependence of phase of $F^{\mathrm{N}}(s,t)$

- two approaches for description of elastic collisions of charged hadrons (amplitude $F^{\mathrm{C+N}}(s,t)$)
  - West and Yennie (Feynman diagram technique)
  - eikonal model

Coulomb-hadronic interference in the West and Yennie approach

- H. A. Bethe, "Scattering and polarization of protons by nuclei", Ann. Phys. **3**, 190–240 (1958)

$$F^{\mathrm{C+N}}(s,t) = F^{\mathrm{C}}(s,t)\, \mathrm{e}^{\mathrm{i}\alpha\phi(s,t)} + F^{\mathrm{N}}(s,t) \tag{2}$$

- G. B. West and D. R. Yennie, "Coulomb interference in high-energy scattering", Phys. Rev. **172**, 1413–1422 (1968)
  integral formula for relative phase (derived only for "small" values of $|t|$)

$$\alpha\phi(s,t) = \mp\alpha\left[\ln\left(\frac{-t}{s}\right) + \int_{-4p^2}^{0} \frac{\mathrm{d}t'}{|t-t'|}\left(1 - \frac{F^{\mathrm{N}}(s,t')}{F^{\mathrm{N}}(s,t)}\right)\right]. \tag{3}$$

- simplified interference formula of WY (1968)

$$F^{\mathrm{C+N}}_{\mathrm{WY}}(s,t) = \pm\frac{\alpha s}{t} G_1(t) G_2(t)\, \mathrm{e}^{\mathrm{i}\alpha\phi(s,t)} + \frac{\sigma^{\mathrm{tot,N}}(s)}{4\pi} p\sqrt{s}\,(\rho(s)+\mathrm{i})\, \mathrm{e}^{B(s)t/2} \tag{4}$$

  where (see also Locher 1967)

$$\alpha\phi(s,t) = \mp\alpha\left[\ln\left(\frac{-B(s)t}{2}\right) + \gamma\right] \tag{5}$$

  assuming for all kinematically allowed values of t
  - t-independence of phase of $F^{\mathrm{N}}(s,t)$, i.e., quantity $\rho(t) = \frac{\mathrm{Re}\, F^{\mathrm{N}}(t)}{\mathrm{Im}\, F^{\mathrm{N}}(t)}$ = const
  - purely exponential $\left|F^{\mathrm{N}}(s,t)\right|$ in t, i.e., diffractive slope $B(t) = \frac{2}{|F^{\mathrm{N}}(t)|}\frac{\mathrm{d}}{\mathrm{d}t}\left|F^{\mathrm{N}}(t)\right|$ = const
- used widely in the era of ISR for determination of $\sigma^{\mathrm{tot,N}}$, quantity $\rho(t=0)$ and $B(t=0)$

Problems and limitations involved in the WY approach

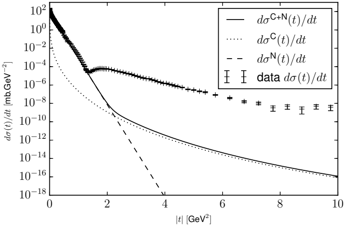
- relative phase $\phi(s,t)$ is real (defined as imaginary part of a complex function) $\Rightarrow$ the integral WY formula (3) consistent only with $\rho(t) = \text{const}$

  V. Kundrát, M. Lokajíček, and I. Vrkoč, "Limited validity of West and Yennie integral formula for elastic scattering of hadrons", Phys. Lett. **B656**, 182–185 (2007)

  J. Procházka and V. Kundrát, "Eikonal model analysis of elastic hadron collisions at high energies", arXiv:1606.09479 (2016)

- if $B(t)$ is t-independent $\Rightarrow$ contradiction to existence of observed dip-bump structure

  J. Kašpar, V. Kundrát, M. Lokajíček, and J. Procházka, "Phenomenological models of elastic nucleon scattering and predictions for LHC", Nucl.Phys. **B843**, 84–106 (2011)



- whole approach a priory limited and applied to data in region of only very small values of $|t|$ ($|t| \lesssim 0.01$ GeV^2 at 52.8 GeV)
- form factors $G_{1,2}(t)$ added by hand to the final interference formula(s)
- dependence of elastic hadronic collisions on impact parameter not considered
- ...

$\Rightarrow$ WY approach inapplicable for reliable data analysis; not usable for studying t-dependence of hadronic amplitude; see detailed discussion in, e.g., J. Procházka and V. Kundrát, "Eikonal model analysis of elastic hadron collisions at high energies", arXiv:1606.09479 (2016)

Additional comments to the WY approach

many descriptions of elastic scattering negativelly influenced by the simplified approach of WY

- quantity $\rho(t=0)$ and diffractive slope $B(t=0)$
  - unclear physical meaning (only very indirect relation to particle characteristics/interactions)
  - importance of these quantities overestimated in many contemporary hadronic models mainly under the influence of the WY approach where they are determining $F^{\mathrm{N}}(s,t)$ at all values of t - both quantities assumed, without any reasoning, to be t-independent at *all* kinematically allowed values of t, see page 4
- measured $\mathrm{d}\sigma/\mathrm{d}t$ commonly divided into two parts
  1. region of very low values of $|t|$ (e.g., $|t| \lesssim 0.01$ GeV^2 at 52.8 GeV) analyzed with the help of the simplified WY interference formula assuming specific t-dependence of $F^{\mathrm{N}}(s,t)$ at *all* values of t
  2. region of higher values of $|t|$ (containing dip-bump structure) described with the help of elastic hadronic models having different t-dependence of hadronic amplitude than the one assumed in the WY approach

  $\Rightarrow$ inconsistent dual description of data

$\Rightarrow$ one should look for different and more general description of (Coulomb-)hadronic elastic scattering; one should transition from initial to final states, full physical picture

- t-dependence of $F^{\mathrm{N}}(s,t)$? I.e., t-dependences of hadronic modulus and phase? or, equivalently, t-dependences of quantities $\rho(t)$ and $B(t)$?
- b-dependent characteristics of collisions taking into account that initial states corresponding given value of b have different frequencies (weights)? (one should not mix characteristics of collisions at different impact parameter values)
- corresponding physical properties of colliding particles?

Eikonal model approach

- introduces dependence of elastic collisions on impact parameter
- several authors started from it or have been developing it (Glauber, van Hove, Miettinen, Islam, Cahn,...); results on various level of sophistication
- Coulomb-hadronic interference formula derived by Kundrát and Lokajíček (1994)

$$F^{C+N}_{eik}(s,t) = \pm \frac{\alpha s}{t} G_1(t) G_2(t) + F^N(s,t)[1 \mp i\alpha \bar{G}(s,t)] \tag{6}$$

where

$$\bar{G}(s,t) = \int\limits_{t_{min}}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}[G_1(t')G_2(t')] - \frac{1}{2\pi}\left[\frac{F^N(s,t')}{F^N(s,t)} - 1\right] I(t,t') \right\} \tag{7}$$

and

$$I(t,t') = \int\limits_{0}^{2\pi} d\Phi'' \frac{G_1(t'')G_2(t'')}{t''}. \tag{8}$$

- derived for *any* t and s (high energy) with the aim *not* to impose any restriction on t-dependence of $F^N(s,t)$
- allows description of data in the *whole* measured t-range (NB: to calculate $F^{C+N}(s,t)$ at given value of t needs to be known $F^N(s,t)$ at *all* values of t - even *outside* measured t-range)
  $\Rightarrow$ consistent description of data (no duality)

Hadronic quantities I

- modulus and phase of hadronic amplitude may be defined as

$$F^{\mathrm{N}}(s,t) = \mathrm{i}\left|F^{\mathrm{N}}(s,t)\right| \mathrm{e}^{-\mathrm{i}\zeta^{\mathrm{N}}(s,t)} \tag{9}$$

  it means

$$\tan \zeta^{\mathrm{N}}(s,t) = \rho(s,t) \tag{10}$$

- t-dependent quantities may be introduced

$$B(s,t) = \frac{\mathrm{d}}{\mathrm{d}t}\left[\ln \frac{\mathrm{d}\sigma^{\mathrm{N}}}{\mathrm{d}t}(s,t)\right] = \frac{2}{\left|F^{\mathrm{N}}(s,t)\right|}\frac{\mathrm{d}}{\mathrm{d}t}\left|F^{\mathrm{N}}(s,t)\right| \tag{11}$$

$$\rho(s,t) = \frac{\mathrm{Re}\, F^{\mathrm{N}}(s,t)}{\mathrm{Im}\, F^{\mathrm{N}}(s,t)} \tag{12}$$

Several physically interesting quantities derived from the hadronic amplitude $F^{\mathrm{N}}(s,t)$

- total cross section (optical theorem)

$$\sigma^{\mathrm{tot,N}}(s) = \frac{4\pi}{p\sqrt{s}}\, \mathrm{Im}\, F^{\mathrm{N}}(s,t=0) \tag{13}$$

- elastic and inelastic cross sections

$$\sigma^{\mathrm{el,N}} = \int \frac{\mathrm{d}\sigma^{\mathrm{el,N}}}{\mathrm{d}t} = \int \frac{\pi}{sp^2}\left|F^{\mathrm{N}}(s,t)\right|^2; \qquad \sigma^{\mathrm{inel}} = \sigma^{\mathrm{tot,N}} - \sigma^{\mathrm{el,N}} \tag{14}$$

Hadronic quantities II

- elastic hadronic amplitude in b-space - Fourier-Bessel transformation (Adachi, Kotani, Takeda, Islam, ...)

$$\begin{aligned} h_{\mathrm{el}}(s,b) &= h_1(s,b) + h_2(s,b) \\ &= \frac{1}{4p\sqrt{s}} \int_{t_{\mathrm{min}}}^{0} F^{\mathrm{N}}(s,t) J_0(b\sqrt{-t})\mathrm{d}t + \frac{1}{4p\sqrt{s}} \int_{-\infty}^{t_{\mathrm{min}}} \lambda(s,t) J_0(b\sqrt{-t})\mathrm{d}t \end{aligned} \tag{15}$$

- unitarity condition at *finite* energies

$$\mathrm{Im}\, h_1(s,b) + c(s,b) = |h_1(s,b)|^2 + g_1(s,b) + K(s,b) + c(s,b) \tag{16}$$

- profile functions
  - main b-dependent characteristics of collisions, introduced in analogy to description of some optics phenomena (light meeting an obstacle of a given profile which describes its absorptive properties)
  - sometimes interpreted as probabilities of total, elastic or inelastic collision at given value of impact parameter
  -

$$D^{\mathrm{el}}(s,b) \equiv 4\,|h_1(s,b)|^2, \tag{17}$$

$$D^{\mathrm{tot}}(s,b) \equiv 4\,(\mathrm{Im}\, h_1(s,b) + c(s,b)), \tag{18}$$

$$D^{\mathrm{inel}}(s,b) \equiv 4\,(g_1(s,b) + K(s,b) + c(s,b)) \tag{19}$$

- cross sections determined on the basis of the profile functions (X=tot, el, inel)

$$\sigma^{\mathrm{X}}(s) = 2\pi \int_0^{\infty} b\mathrm{d}b\; D^{\mathrm{X}}(s,b). \tag{20}$$

Hadronic quantities III

mean-square values of impact parameter b

- definition ($n = 2$ and $w(b) = 2\pi b$)

$$\langle b^n \rangle^{\mathrm{X}} = \frac{\int\limits_0^\infty b^n \; w(b) D^{\mathrm{X}}(s,b) \mathrm{d}b}{\int\limits_0^\infty w(b) D^{\mathrm{X}}(s,b) \mathrm{d}b} \tag{21}$$

- expressions for the mean-square values as functions of $F^{\mathrm{N}}(s,t)$

  V. Kundrát, M. V. Lokajíček, and D. Krupa, "Impact parameter structure derived from elastic collisions", Phys. Lett. B **544**, 132–138 (2002)

$$\begin{aligned}
\langle b^2 \rangle^{\mathrm{el}} =& \langle b^2 \rangle^{\mathrm{mod}} + \langle b^2 \rangle^{\mathrm{ph}} \\
=& \frac{4 \int\limits_{t_{\min}}^{0} \mathrm{d}t |t| \left(\frac{\mathrm{d}}{\mathrm{d}t} \left| F^{\mathrm{N}}(s,t) \right| \right)^2}{\int\limits_{t_{\min}}^{0} \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2} + \frac{4 \int\limits_{t_{\min}}^{0} \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2 |t| \left(\frac{\mathrm{d}}{\mathrm{d}t} \zeta^{\mathrm{N}}(s,t) \right)^2}{\int\limits_{t_{\min}}^{0} \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2} \\
\langle b^2 \rangle^{\mathrm{tot}} =& 4 \left(\frac{\frac{\mathrm{d}}{\mathrm{d}t} \left| F^{\mathrm{N}}(s,t) \right|}{\left| F^{\mathrm{N}}(s,t) \right|} - \tan \zeta^{\mathrm{N}}(s,t) \frac{\mathrm{d}}{\mathrm{d}t} \zeta^{\mathrm{N}}(s,t) \right) \Bigg|_{t=0} \\
\langle b^2 \rangle^{\mathrm{inel}} =& \frac{\sigma^{\mathrm{tot,N}}(s) \langle b^2 \rangle^{\mathrm{tot}} - \sigma^{\mathrm{el,N}}(s) \langle b^2 \rangle^{\mathrm{el}}}{\sigma^{\mathrm{inel}}(s)}
\end{aligned} \tag{22}$$

Definition: central vs. peripheral behaviour of elastic collisions

- Definition: two basic types of behaviour of elastic hadron collisions (models) in dependence on impact parameter may be distinguished
  1. peripheral: $\sqrt{\langle b^2 \rangle^{\text{el}}} > \sqrt{\langle b^2 \rangle^{\text{inel}}}$
     i.e., if elastic collisions correspond in average to higher impact parameter b then the inelastic ones; corresponds to usual ideas of collisions of two matter objects
  2. central: $\sqrt{\langle b^2 \rangle^{\text{el}}} < \sqrt{\langle b^2 \rangle^{\text{inel}}}$
     the opposite; anti-ontological behaviour; some kind of transparency of colliding particles; never sufficiently explained in literature

Comparison of proton electromagnetic form factors

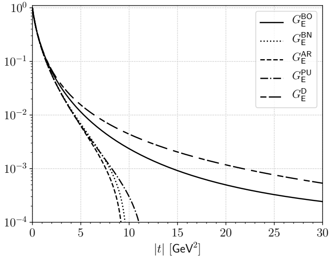


Figure: electric form factors

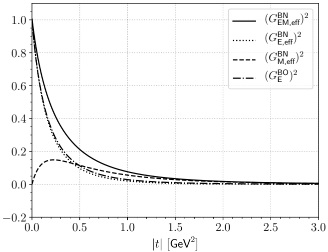


Figure: effective form factors

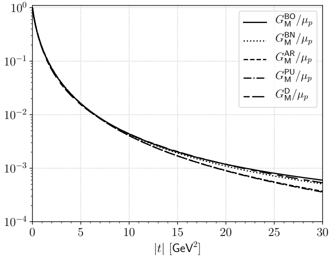


Figure: magnetic form factors

- different authors/analyses
  - different shapes of form factors
- electric form factors
  - differences visible at $|t| > 2.5$ GeV^2
- electric vs. effective electromagnetic form factors
  - very significant differences already at lower $|t|$ values
- impact on calculations in the (eikonal) interference formula?

Hadronic amplitude $F^{\mathrm{N}}(s,t)$

1. hadronic amplitude in many contemporary hadronic models is a priory strongly constrained (without sufficient reasoning) by requiring:
   1.1 dominance of the imaginary part of $F^{\mathrm{N}}(s,t)$ in quite broad interval of t in forward direction
   1.2 vanishing of the imaginary part of $F^{\mathrm{N}}(s,t)$ at (or around) $t = t_{\mathrm{dip}}$
   1.3 change of sign of the real part of $F^{\mathrm{N}}(s,t)$ at "low" value of $|t|$ (required by Martin's theorem [8] derived under certain conditions)

   the corresponding t-dependence of $F^{\mathrm{N}}(s,t)$ (its phase) is strongly constrained by these requirements and it may be shown that mainly the first requirement leads to central behaviour of elastic collisions
2. one may ask if it is possible to obtain description of data which would lead to peripheral behaviour of elastic collisions (without imposing the previous constrains);
   1981 [9] - peripheral solution of the scattering problem may be obtained if hadronic phase has specific t-dependence

$\Rightarrow$ one may try to determine $F^{\mathrm{N}}(s,t)$ on the basis of experimental data under given set of assumptions (constrains)

Fitting procedure I

- eikonal Coulomb-hadronic interference formula used for determination of hadronic amplitude from data

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2}\left|F^{C+N}_{eik}(s,t)\right|^2 \tag{23}$$

- hadronic amplitude may be parameterized

$$F^{N}(s,t) = i\left|F^{N}(s,t)\right| e^{-i\zeta^{N}(s,t)} \tag{24}$$

- modulus (very general parameterization)

$$\left|F^{N}(s,t)\right| = (a_1 + a_2 t)\, e^{b_1 t + b_2 t^2 + b_3 t^3} + (c_1 + c_2 t)\, e^{d_1 t + d_2 t^2 + d_3 t^3} \tag{25}$$

- phase (also very general parameterization, analytic if κ is positive integer)

$$\zeta^{N}(s,t) = \zeta_0 + \zeta_1 \left|\frac{t}{t_0}\right|^{\kappa} e^{\nu t} \tag{26}$$

  it may reproduce various t shapes in dependence on values of free parameters
- another phase parameterization strongly limiting allowed t-dependences (used in the past)

$$\zeta^{N}(s,t) = \arctan \frac{\rho_0}{1 - \left|\frac{t}{t_{dip}}\right|} \tag{27}$$

  - leads to the dominance the imaginary part of $F^{N}(s,t)$ in quite broad interval of t in forward direction
  - vanishing of the imaginary part of $F^{N}(s,t)$ at $t = t_{dip}$
  - no change of sign of the real part of $F^{N}(s,t)$ at "low" value of $|t|$
  - non-analytic

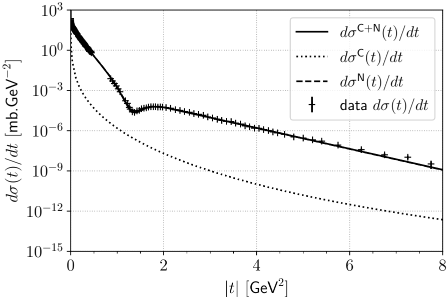
Fitting procedure II

several fits of measured elastic differential cross section at two very different energies 52.8 GeV and 8 TeV performed under different assumptions:

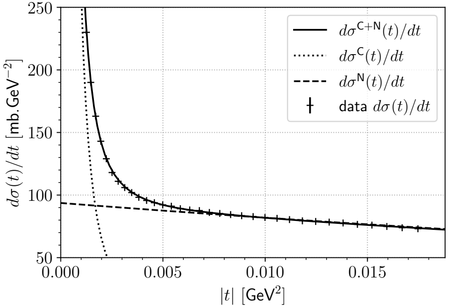
1. different choices of form factors (effective electric vs. effective electromagnetic)
2. different constrains of hadronic amplitude $F^{\mathrm{N}}(s,t)$
   - 2.1 widely used constrains imposed on $F^{\mathrm{N}}(s,t)$ strongly limit possible t-dependences of hadronic phase - mainly the required dominance of the imaginary part of $F^{\mathrm{N}}(s,t)$ leads to centrality of elastic collisions
     - 52.8 GeV - used general phase parameterization (26) and adding the constrains
     - 8 TeV - used already restricted phase parameterization (27)
   - 2.2 *peripheral* case
     - no unique solution (parameterization of the used phase (26) allows very different t-dependences)
       $\Rightarrow$ added additional constrain on required value of $\sqrt{\langle b^2\rangle^{\mathrm{el}}}$ and 3 different alternatives shown
     - needed to solve complicated problem of bounded extremes

- 8 different fits/models of data showed in the following at each energy (1 central, 3 different peripheral; each with different form factor)
  - effective *electric* form factors - Fit 1a central, Fits. 2a-4a peripheral
  - effective *electromagnetic* form factors - Fit 1b central, Fits. 2b-4b peripheral

Results at 52.8 GeV - fitted $d\sigma/dt$



(a) full available $|t|$-range of measured data



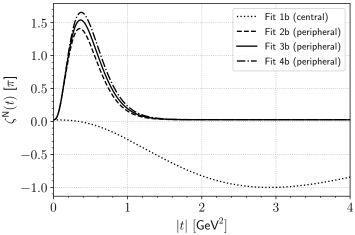
(b) region of very low values of $|t|$

- fits in the very broad interval $|t| \in \langle 0.00126, 7.75 \rangle$ GeV^2 including *both* peak at the lowest measured values of $|t|$ and dip-bump structure at higher values of $|t|$
- all the performed fits at 52.8 GeV lead to similar t-dependences of hadronic $d\sigma/dt$

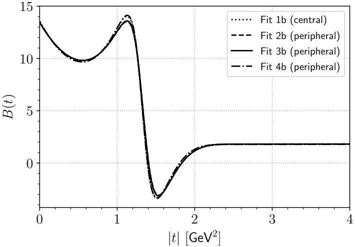
$\sqrt{s}$	[GeV]	52.8	52.8	52.8	52.8
Fit		1a	2a	3a	4a
Case		central	peripheral	peripheral	peripheral
Form factor		effective electric	effective electric	effective electric	effective electric
ζ_0		0.0758 $\pm$ 0.0017	0.0817 $\pm$ 0.0018	0.0816 $\pm$ 0.0017	0.0815 $\pm$ 0.0017
ζ_1		-2.604	1975 $\pm$ 37	1979 $\pm$ 38	1991 $\pm$ 38
κ		3	3	3	3
ν	[GeV^{-2}]	1.028	8.49 $\pm$ 0.12	8.26 $\pm$ 0.14	8.02 $\pm$ 0.14
a_1		12147.3 $\pm$ 9.2	12198 $\pm$ 12	12197.7 $\pm$ 9.6	12197.0 $\pm$ 9.9
a_2	[GeV^{-2}]	10664 $\pm$ 29	10762 $\pm$ 73	10771.3 $\pm$ 40.7	10780 $\pm$ 43
b_1	[GeV^{-2}]	5.911 $\pm$ 0.018	5.877 $\pm$ 0.018	5.876 $\pm$ 0.019	5.875 $\pm$ 0.018
b_2	[GeV^{-4}]	3.693 $\pm$ 0.063	3.487 $\pm$ 0.063	3.488 $\pm$ 0.068	3.492 $\pm$ 0.062
b_3	[GeV^{-6}]	1.692 $\pm$ 0.042	1.550 $\pm$ 0.040	1.551 $\pm$ 0.045	1.555 $\pm$ 0.041
c_1		58.9 $\pm$ 1.4	60 $\pm$ 10	59.8 $\pm$ 3.6	59.6 $\pm$ 3.8
c_2	[GeV^{-2}]	-3.0e-6 $\pm$ 2.7	-3.2e-8 $\pm$ 2.1	9.9e-7 $\pm$ 2.3	-6.0e-8 $\pm$ 1.7
d_1	[GeV^{-2}]	0.901 $\pm$ 0.047	0.91 $\pm$ 0.15	0.90 $\pm$ 0.031	0.90 $\pm$ 0.39
χ^2/ndf		344/206	306/204	305/204	311/204
$\Delta\chi^2$		0	6.8	4.0	7.7
$\rho(t=0)$		0.0760 $\pm$ 0.0017	0.082 $\pm$ 0.0018	0.0818 $\pm$ 0.0017	0.0817 $\pm$ 0.0017
$B(t=0)$	[GeV^{-2}]	13.522 $\pm$ 0.035	13.461 $\pm$ 0.040	13.462 $\pm$ 0.039	13.461 $\pm$ 0.036
σ^{tot}	[mb]	42.686 $\pm$ 0.033	42.849 $\pm$ 0.055	42.846 $\pm$ 0.036	42.844 $\pm$ 0.037
σ^{el}	[mb]	7.466	7.529	7.528	7.528
σ^{inel}	[mb]	35.22	35.32	35.32	35.32
$\sigma^{\text{el}}/\sigma^{\text{tot}}$		0.1749	0.1757	0.1757	0.1757
$\mathrm{d}\sigma^{\text{N}}/\mathrm{d}t(t=0)$	[mb.GeV^{-2}]	93.63	94.43	94.42	94.41
$\sqrt{\langle b^2\rangle^{\text{tot}}}$	[fm]	1.026	1.024	1.024	1.024
$\sqrt{\langle b^2\rangle^{\text{el}}}$	[fm]	0.6777	1.853	1.950	2.069
$\sqrt{\langle b^2\rangle^{\text{inel}}}$	[fm]	1.086	0.7348	0.6789	0.5992
$D^{\text{tot}}(b=0)$		1.29	1.30	1.30	1.30
$D^{\text{el}}(b=0)$		0.530	0.0316	0.0331	0.0490
$D^{\text{inel}}(b=0)$		0.761	1.27	1.27	1.25

$\sqrt{s}$	[GeV]	52.8	52.8	52.8	52.8
Fit		1b	2b	3b	4b
Case		central	peripheral	peripheral	peripheral
Form factor		effective electromagnetic	effective electromagnetic	effective electromagnetic	effective electromagnetic
ζ_0		0.0762 ± 0.0017	0.0824 ± 0.0017	0.0825 ± 0.0017	0.0849 ± 0.0017
ζ_1		-2.605	1970 ± 37	1974 ± 37	1964 ± 38
κ		3	3	3	3
ν	[GeV^{-2}]	1.028	8.48 ± 0.13	8.23 ± 0.14	8.02 ± 0.15
a_1		12149.8 ± 9.2	12203 ± 10	12202.3 ± 9.3	12225 ± 30
a_2	[GeV^{-2}]	10705 ± 29	10760 ± 39	10767 ± 33	10760 ± 150
b_1	[GeV^{-2}]	5.905 ± 0.017	5.867 ± 0.018	5.868 ± 0.017	5.934 ± 0.021
b_2	[GeV^{-4}]	3.677 ± 0.063	3.440 ± 0.062	3.445 ± 0.060	3.722 ± 0.080
b_3	[GeV^{-6}]	1.678 ± 0.041	1.518 ± 0.040	1.520 ± 0.038	1.698 ± 0.062
c_1		58.8 ± 1.4	60.6 ± 3.0	60.4 ± 1.9	57.4 ± 26
c_2	[GeV^{-2}]	-5.4e-6 $\pm$ 2.9	-1.8e-5 $\pm$ 2.4	-6.3e-8 $\pm$ 2.3	-1.4 ± 4.4
d_1	[GeV^{-2}]	0.901 ± 0.050	0.908 ± 0.042	0.907 ± 0.041	0.918 ± 0.039
χ^2/ndf		345/206	305/204	303/204	322/204
$\Delta\chi^2$		0	6.9	4.0	1.9
$\rho(t=0)$		0.0763 ± 0.0017	0.0826 ± 0.0017	0.0827 ± 0.0016	0.0851 ± 0.017
$B(t=0)$	[GeV^{-2}]	13.515 ± 0.035	13.439 ± 0.037	13.444 ± 0.036	13.573 ± 0.055
σ^{tot}	[mb]	42.694 ± 0.033	42.864 ± 0.037	42.861 ± 0.034	42.917 ± 0.14
σ^{el}	[mb]	7.469	7.542	7.539	7.532
σ^{inel}	[mb]	35.22	35.32	35.32	35.39
$\sigma^{\mathrm{el}}/\sigma^{\mathrm{tot}}$		0.1750	0.1759	0.1759	0.1755
$\mathrm{d}\sigma^{\mathrm{N}}/\mathrm{d}t(t=0)$	[mb.GeV^{-2}]	93.67	94.52	94.51	94.80
$\sqrt{\langle b^2\rangle^{\mathrm{tot}}}$	[fm]	1.026	1.023	1.023	1.028
$\sqrt{\langle b^2\rangle^{\mathrm{el}}}$	[fm]	0.6778	1.854	1.959	2.045
$\sqrt{\langle b^2\rangle^{\mathrm{inel}}}$	[fm]	1.085	0.7322	0.671	0.6261
$D^{\mathrm{tot}}(b=0)$		1.29	1.30	1.30	1.29
$D^{\mathrm{el}}(b=0)$		0.530	0.0317	0.0342	0.0466
$D^{\mathrm{inel}}(b=0)$		0.762	1.27	1.27	1.25

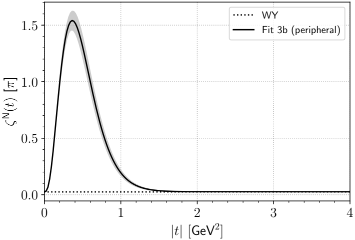
Results at 52.8 GeV - hadronic phase and diffractive slope



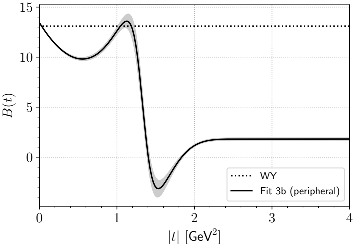
(a) hadronic phases



(b) diffractive slopes

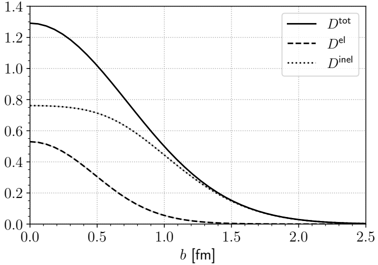


(c) hadronic phases - Fit 3b vs. WY

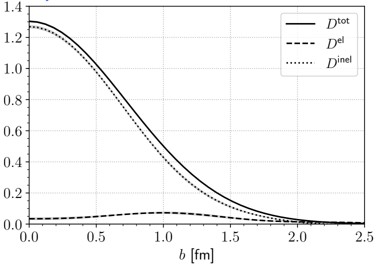


(d) diffractive slopes - Fit 3b vs. WY

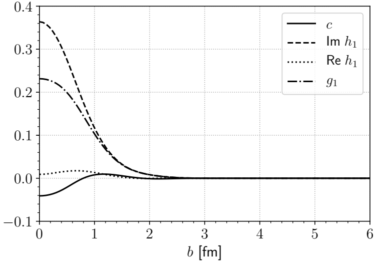
Results at 52.8 GeV - b-dependent functions



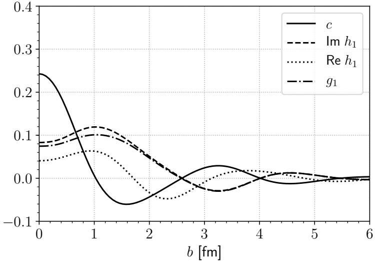
(a) profile functions: central case, Fit 1b



(b) profile functions: peripheral case, Fit 3b



(c) central case - Fit 1b



(d) peripheral case - Fit 3b

Real and imaginary parts of $F^{\mathrm{N}}(s,t)$ at 52.8 GeV

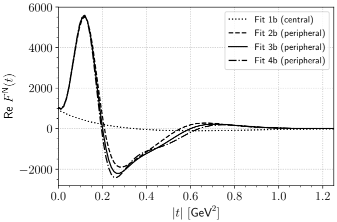


Figure: real parts - Fits. 1b-4b

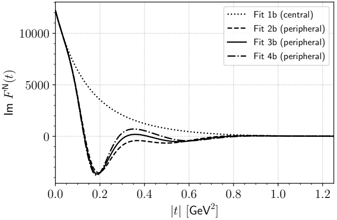


Figure: imaginary parts - Fits. 1b-4b

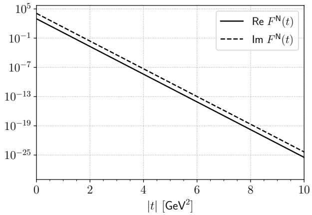
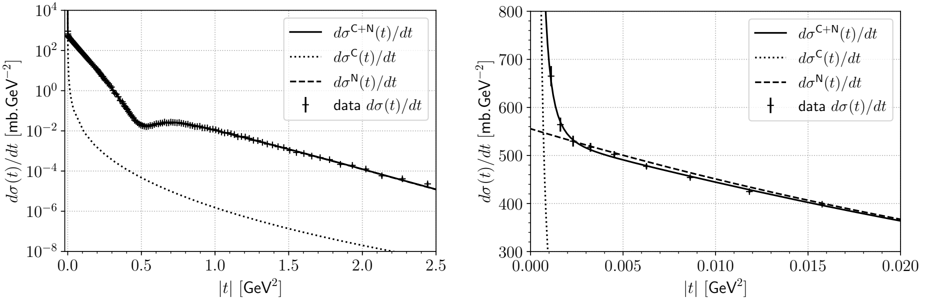


Figure: WY - real and imaginary parts (corresponding free parameters of $F^{\mathrm{N}}(s,t)$ taken from [10])

- Eikonal model
  - Fit 1b (central) - real part of $F^{\mathrm{N}}(s,t)$ changes sign at $|t| \approx 0.35$ GeV^2
  - Fits. 2b-4b (peripheral) - real part of $F^{\mathrm{N}}(s,t)$ changes sign at $|t| \approx 0.2$ GeV^2
    $\Rightarrow$ conclusion of the Martin theorem *fulfilled*
- WY
  - real part of $F^{\mathrm{N}}(s,t)$ does not change sign at any t value (hadronic phase assumed to be t-independent)
    $\Rightarrow$ conclusion of the Martin theorem *not fulfilled*

Results at 8 TeV - fitted $d\sigma/dt$



(a) full available $|t|$-range of measured data

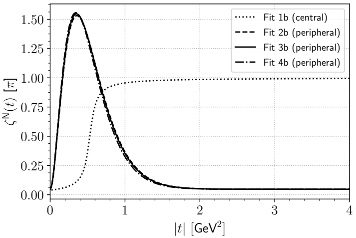
(b) region of very low values of $|t|$

- TOTEM 8 TeV data (1000m and 90m optics data up to $|t| = 0.2$ GeV^2, see [11, 12]); extended by renormalized TOTEM 7 TeV data [13] up to 2.5 GeV^2 to obtain data in wider t-region ($|t| \in \langle 6 \times 10^{-4}, 2.5\rangle$ GeV^2) including both region of peak at very low values of $|t|$ and dip-bump structure region $\Rightarrow$ approximate data denoted as "8 TeV data" (only statistical errors taken into account)
- all the performed fits at 8 TeV leads to similar t-dependences of hadronic $d\sigma/dt$

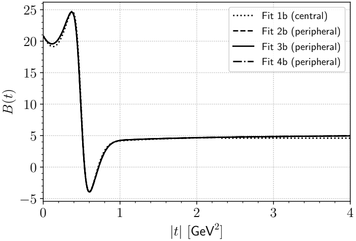
$\sqrt{s}$	[GeV]	8000	8000	8000	8000
Fit		1a	2a	3a	4a
Case		central	peripheral	peripheral	peripheral
Form factor		effective electric	effective electric	effective electric	effective electric
t_{dip}	[GeV^2]	-0.53	-	-	-
ρ_0		0.131 $\pm$ 0.015	-	-	-
ζ_0		-	0.152 $\pm$ 0.019	0.154 $\pm$ 0.016	0.156 $\pm$ 0.017
ζ_1		-	277.0 $\pm$ 8.6	292.3 $\pm$ 8.9	309.0 $\pm$ 8.9
κ		-	2	2	2
ν	[GeV^{-2}]	-	5.75 $\pm$ 0.15	5.85 $\pm$ 0.15	5.98 $\pm$ 0.14
a_1	[10^{-7}]	66.47 $\pm$ 0.28	66.70 $\pm$ 0.18	66.77 $\pm$ 0.14	66.84 $\pm$ 0.10
a_2	[10^{-7}GeV^{-2}]	164.52 $\pm$ 0.83	170.08 $\pm$ 0.39	170.10 $\pm$ 0.39	170.11 $\pm$ 0.35
b_1	[GeV^{-2}]	8.265 $\pm$ 0.026	8.224 $\pm$ 0.048	8.241 $\pm$ 0.029	8.261 $\pm$ 0.035
b_2	[GeV^{-4}]	9.19 $\pm$ 0.17	8.00 $\pm$ 0.28	8.06 $\pm$ 0.17	8.12 $\pm$ 0.20
b_3	[GeV^{-6}]	14.60 $\pm$ 0.29	12.65 $\pm$ 0.40	12.71 $\pm$ 0.25	12.79 $\pm$ 0.29
c_1	[10^{-7}]	1.74 $\pm$ 0.26	2.215 $\pm$ 0.098	2.202 $\pm$ 0.085	2.188 $\pm$ 0.058
c_2	[10^{-7}GeV^{-2}]	-2.70 $\pm$ 0.57	-1.91 $\pm$ 0.27	-1.94 $\pm$ 0.20	-1.969 $\pm$ 0.17
d_1	[GeV^{-2}]	2.70 $\pm$ 0.071	2.609 $\pm$ 0.043	2.614 $\pm$ 0.031	2.618 $\pm$ 0.032
χ^2/ndf		242 / 131	357 / 129	367 / 129	384 / 129
$\Delta\chi^2$		0	11	10	4.8
$\rho(t=0)$		0.131 $\pm$ 0.015	0.154 $\pm$ 0.019	0.155 $\pm$ 0.017	0.148 $\pm$ 0.017
$B(t=0)$	[GeV^{-2}]	20.992 $\pm$ 0.085	20.967 $\pm$ 0.098	20.999 $\pm$ 0.062	21.035 $\pm$ 0.071
$\sigma^{\mathrm{tot,N}}$	[mb]	103.42 $\pm$ 0.61	104.16 $\pm$ 0.44	104.22 $\pm$ 0.36	104.2 $\pm$ 0.33
$\sigma^{\mathrm{el,N}}$	[mb]	27.6	28.0	28.0	28.0
σ^{inel}	[mb]	75.8	76.2	76.2	76.3
$\sigma^{\mathrm{el,N}}/\sigma^{\mathrm{tot,N}}$		0.267	0.269	0.269	0.269
$\mathrm{d}\sigma^{\mathrm{N}}/\mathrm{d}t(t=0)$	[mb.GeV^{-2}]	556	567	568	569
$\sqrt{\langle b^2\rangle^{\mathrm{tot}}}$	[fm]	1.28	1.28	1.28	1.28
$\sqrt{\langle b^2\rangle^{\mathrm{el}}}$	[fm]	0.879	1.83	1.88	1.93
$\sqrt{\langle b^2\rangle^{\mathrm{inel}}}$	[fm]	1.40	1.00	0.971	0.937
$D^{\mathrm{tot}}(b=0)$		2.01	2.03	2.03	2.03
$D^{\mathrm{el}}(b=0)$		1.02	0.213	0.197	0.182
$D^{\mathrm{inel}}(b=0)$		0.985	1.82	1.83	1.84

$\sqrt{s}$	[GeV]	8000	8000	8000	8000
Fit		1b	2b	3b	4b
Case		central	peripheral	peripheral	peripheral
Form factor		effective electromagnetic	effective electromagnetic	effective electromagnetic	effective electromagnetic
t_{dip}	[GeV^2]	-0.53	-	-	-
ρ_0		0.123 ± 0.015	-	-	-
ζ_0		-	0.147 ± 0.017	0.148 ± 0.016	0.147 ± 0.017
ζ_1		-	270 ± 11	281 ± 11	298 ± 11
κ		-	2	2	2
ν	[GeV^{-2}]	-	5.60 ± 0.20	5.68 ± 0.20	5.83 ± 0.19
a_1	[10^{-7}]	66.48 ± 0.33	66.74 ± 0.15	66.79 ± 0.11	66.87 ± 0.21
a_2	[10^{-7}GeV^{-2}]	164.14 ± 0.96	170.35 ± 0.49	170.39 ± 0.39	170.44 ± 0.53
b_1	[GeV^{-2}]	8.261 ± 0.027	8.13 ± 0.034	8.137 ± 0.026	8.153 ± 0.029
b_2	[GeV^{-4}]	9.15 ± 0.17	7.57 ± 0.20	7.58 ± 0.16	7.62 ± 0.17
b_3	[GeV^{-6}]	14.59 ± 0.31	12.13 ± 0.30	12.15 ± 0.25	12.20 ± 0.27
c_1	[10^{-7}]	1.72 ± 0.30	2.06 ± 0.12	2.047 ± 0.067	2.03 ± 0.17
c_2	[10^{-7}GeV^{-2}]	-2.75 ± 0.67	-2.43 ± 0.32	-2.46 ± 0.14	-2.51 ± 0.44
d_1	[GeV^{-2}]	2.71 ± 0.081	2.683 ± 0.045	2.688 ± 0.019	2.700 ± 0.060
χ^2/ndf		236 / 131	358 / 129	368 / 129	377 / 129
$\Delta\chi^2$		0	18	16	4.8
$\rho(t=0)$		0.123 ± 0.015	0.148 ± 0.019	0.149 ± 0.016	0.148 ± 0.017
$B(t=0)$	[GeV^{-2}]	20.976 ± 0.095	20.811 ± 0.017	20.829 ± 0.055	20.859 ± 0.073
$\sigma^{\text{tot,N}}$	[mb]	103.50 ± 0.70	104.07 ± 0.38	104.12 ± 0.31	104.2 ± 0.48
$\sigma^{\text{el,N}}$	[mb]	27.6	28.0	28.0	28.0
σ^{inel}	[mb]	75.8	76.1	76.1	76.2
$\sigma^{\text{el,N}}/\sigma^{\text{tot,N}}$		0.267	0.269	0.269	0.269
$\mathrm{d}\sigma^{\text{N}}/\mathrm{d}t(t=0)$	[mb.GeV^{-2}]	556	566	566	567
$\sqrt{\langle b^2\rangle^{\text{tot}}}$	[fm]	1.28	1.27	1.27	1.27
$\sqrt{\langle b^2\rangle^{\text{el}}}$	[fm]	0.879	1.83	1.86	1.91
$\sqrt{\langle b^2\rangle^{\text{inel}}}$	[fm]	1.40	0.992	0.970	0.937
$D^{\text{tot}}(b=0)$		2.01	2.04	2.04	2.04
$D^{\text{el}}(b=0)$		1.02	0.216	0.205	0.190
$D^{\text{inel}}(b=0)$		0.989	1.83	1.84	1.85

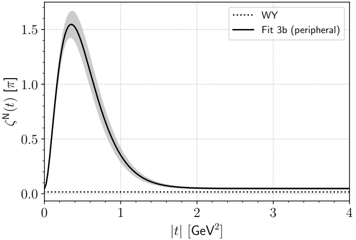
Results at 8 TeV - hadronic phase and diffractive slope



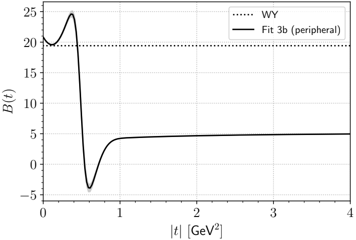
(a) hadronic phases



(b) diffractive slopes

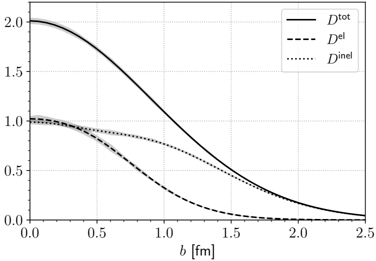


(c) hadronic phases - Fit 3b vs. WY

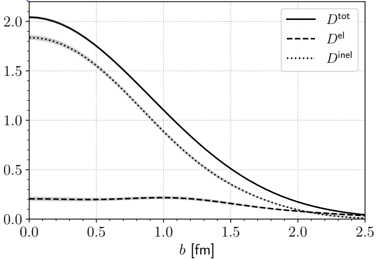


(d) diffractive slopes - Fit 3b vs. WY

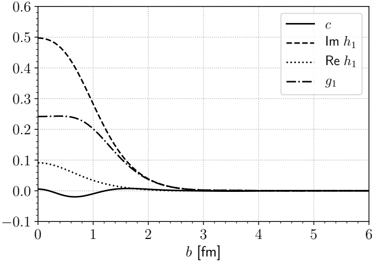
Results at 8 TeV - *b*-dependent functions



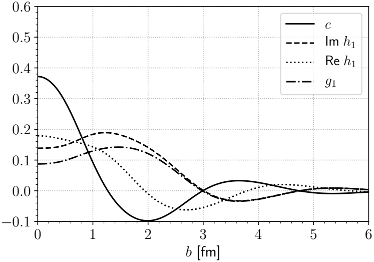
(a) profile functions: central case, Fit 1b



(b) profile functions: peripheral case, Fit 3b



(c) central case - Fit 1b



(d) peripheral case - Fit 3b

Real and imaginary parts of $F^{\mathrm{N}}(s,t)$ at 8 TeV

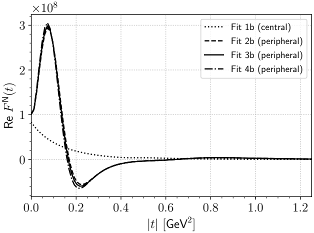


Figure: real parts - Fits. 1b-4b

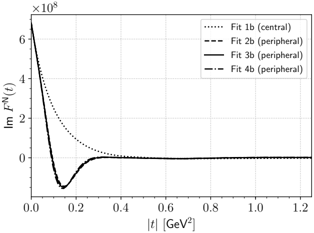


Figure: imaginary parts - Fits. 1b-4b

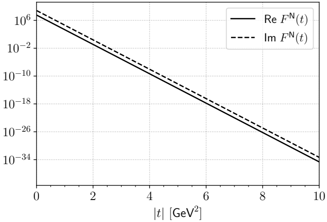


Figure: WY - real and imaginary parts (corresponding free parameters of $F^{\mathrm{N}}(s,t)$ taken from [11])

- Eikonal model
  - Fit 1b (central) - real part of $F^{\mathrm{N}}(s,t)$ does not change sign at any $|t|$ value
    $\Rightarrow$ conclusion of the Martin theorem *not fulfilled*
  - Fits. 2b-4b (peripheral) - real part of $F^{\mathrm{N}}(s,t)$ changes sign at $|t| \approx 0.165$ GeV^2
    $\Rightarrow$ conclusion of the Martin theorem *fulfilled*
- WY
  - real part of $F^{\mathrm{N}}(s,t)$ does not change sign at any $|t|$ value (hadronic phase assumed to be t-independent)
    $\Rightarrow$ conclusion of the Martin theorem *not fulfilled*

Eikonal model - summary of the obtained results

1. measured $d\sigma/dt$ at two very different energies 52.8 GeV and 8 TeV analyzed in broad interval of $|t|$ values (including both peak at low values of $|t|$ and also region of dip-bump structure at higher $|t|$ values) under different assumptions (constrains) to determine their impact on hadronic quantities (and overall relevance of the given description)
2. for each description the corresponding t-dependence of $F^{\mathrm{N}}(s,t)$ at *all* values of t has been determined from data and several quantities characterizing the collisions process in t and b space have been calculated - to study the whole physical picture at given energy under the given set of assumptions
3. our results show:
    - choice of form factor (effective electric vs. effective electromagnetic) - *small or negligible* impact on determination of hadronic amplitude
    - choice of t-dependence of hadronic phase - may *completely* change behaviour of collisions in b-space; the phase is only weakly constrained by the eikonal interference formula
    - t-dependence of $\left|F^{\mathrm{N}}(s,t)\right|$ - *strongly* constrained by measured $d\sigma/dt$
4. hadronic amplitude in many models of elastic hadronic collisions is strongly a priory constrained without sufficient reasoning - our results show that these models then leads to *central* behaviour of elastic collisions (mainly due to required dominance of the imaginary part of $F^{\mathrm{N}}(s,t)$ in forward region), corresponding particle structure has never been sufficiently explained in the literature
5. we have shown that elastic collisions may be described as *peripheral* (in agreement with usual ideas corresponding to collisions of two matter objects) just by allowing hadronic phase to be strongly t-dependent already at low values of $|t|$; corresponding hadronic amplitudes showed in this talk are analytic, satisfy unitarity and conclusion of Martin's theorem

Conclusion I

1. WY approach
   - used widely at ISR for "measurement" of $\sigma^{\mathrm{tot,N}}$, $B(t=0)$ and $\rho(t=0)$
   - many problems and limitations identified later (several papers exist), see [5]
     ⇒ WY approach should be abandoned in the era of LHC as it may lead to wrong physical conclusions; it should not be used for constraining hadronic models based on assumptions inconsistent with the simplified model of WY
     ⇒ one should look for other description of el. scattering of (charged) hadrons
2. eikonal model approach
   - more general and relevant for analysis of el. data in the era of LHC than the WY simplified approach
   - more fundamental then other contemporary models of el. scattering as it may be used for description of Coulomb-hadronic interference *and* take into account also dependence of collisions on impact parameter (in order not to mix collisions corresponding to different values of impact parameter)
   - it allows to *study* full physical picture of el. scattering process (transition from initial to final states and characteristics of colliding particles)
   - recent results
     - 52.8 GeV: J. Procházka and V. Kundrát, "Eikonal model analysis of elastic hadron collisions at high energies", arXiv:1606.09479 (2016) (see also appendix B concerning discussion of the relative phase in the WY approach; new revision of the paper under preparation)
     - 8 TeV: TOTEM Collaboration, "Measurement of elastic pp scattering at $\sqrt{s} = 8$ TeV in the Coulomb-nuclear interference region – determination of the ρ-parameter and the total cross-section", Eur. Phys. J. C **76**, 661 (2016), see also CERN-PH-EP-2015-325, arXiv:1610.00603 (see also this talk for further details)
   - similar analysis of elastic data with the help of the eikonal model approach may be performed at any (other) high energy

Conclusion II

- several open questions and problems may be identified in all contemporary descriptions (models) of el. scattering [14]:
  1. divergence of Coulomb amplitude at $t=0$ corresponding to infinite value of impact parameter while in experiment the values of impact parameters are very limited
  2. extrapolation of $F^{\mathrm{N}}(s,t)$ to $t=0$ ($\mathrm{d}\sigma^{\mathrm{N}}/\mathrm{d}t$ at $t=0$ in all approaches required to have maximal value)
  3. relation of b-dependent profile functions to probabilities of corresponding collisions in dependence on b
  4. increase of $\sigma^{\mathrm{tot,N}}$, $\sigma^{\mathrm{el,N}}/\sigma^{\mathrm{inel}}$ and $\sigma^{\mathrm{el,N}}/\sigma^{\mathrm{tot,N}}$ with energy (never sufficiently explained)
  5. ...
- Proper analysis of elastic collisions in dependence on impact parameter may provide important insight concerning shapes and dimensions of collided particles which can be hardly obtained in a different way. One should carefully study the assumptions involved in any collision model, test the consequences and solve all the open problems and questions before making far-reaching conclusions concerning structure and properties of collided particles [5, 14–16].

Backup

Values of mean impact parameters predicted by some models

Model	$\sqrt{\langle b^2 \rangle^{\text{tot}}}$	$\sqrt{\langle b^2 \rangle^{\text{el}}}$	$\sqrt{\langle b^2 \rangle^{\text{inel}}}$
Bourelly et al.	1.249	0.876	1.399
Petrov et al. (2P)	1.227	0.875	1.324
Petrov et al. (3P)	1.263	0.901	1.375
Block et al.	1.223	0.883	1.336
Islam et al.	1.552	1.048	1.659

Table: Values of root-mean-squares of impact parameter (in femtometers) predicted by several contemporary phenomenological models of pp collisions at collision energy of 14 TeV[6, 17].

- all the models predict $\sqrt{\langle b^2 \rangle^{\text{el}}} < \sqrt{\langle b^2 \rangle^{\text{inel}}}$, i.e., *central* behaviour of elastic hadronic scattering
- J. Kašpar, V. Kundrát, M. Lokajíček, and J. Procházka, "Phenomenological models of elastic nucleon scattering and predictions for LHC", Nucl.Phys. **B843**, 84–106 (2011)

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