PDF Uncertainties in VBS

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PDF sets and their representation

- PDFs contain a lot of uncertain parameters - all uncertainties and their correlations need to be represented in a compact way:
  - Monte-Carlo representation
    - Ensemble of PDFs (replicas)
    - Central value given by the average
    - Uncertainty given by the standard deviation
  - Hessian representation
    - Central value + eigenvectors of the covariance matrix in the parameter space
- To evaluate how the PDF uncertainty is propagating to some variable uncertainty (usually cross-section) this variable must be evaluated for each member of PDF set

Calculating PDF uncertainties [1]

1. Monte-Carlo representation:
   (a) Gaussian distribution:

$$\delta^{\text{pdf}}\sigma = \sqrt{\frac{1}{N_{\text{mem}}-1}\sum_{k=1}^{N_{\text{mem}}}(\sigma^{(k)} - \langle\sigma\rangle)^2}, \qquad \langle\sigma\rangle = \frac{1}{N_{\text{mem}}}\sum_{k=1}^{N_{\text{mem}}}\sigma^{(k)}$$

   (b) Non-Gaussian distribution:
   Reorder cross-sections: $\sigma^{(1)} \leq \sigma^{(2)} \leq \ldots \leq \sigma^{(N_{\text{mem}}-1)} \leq \sigma^{(N_{\text{mem}})}$
   Calculate uncertainty (100 members): $\delta^{\text{pdf}}\sigma = \frac{\sigma^{(84)} - \sigma^{(16)}}{2}$

2. Hessian representation:

$$\delta^{\text{pdf}}\sigma = \sqrt{\sum_{k=1}^{N_{\text{mem}}}(\sigma^{(k)} - \sigma^{(0)})^2}, \qquad \sigma^{(0)}\text{- central PDF}$$

- The N_{mem} is number of PDF error sets - central PDF and eventual parameter variations (α_S) are not included

[1] J. Butterworth, et al: PDF4LHC recommendations for LHC Run II, arXiv:1510.03865

PDF + α_S uncertainty [2]

- PDF variation calculated separately at central value of the α_S
- α_S variation calculated at the borders of the 68% CL interval (two additional members of the set)
  - Current PDG average: $\alpha_S(m_Z^2) = 0.1187 \pm 0.0007$
  - In LHAPDF sets (more conservative) : $\alpha_S(m_Z^2) = 0.118 \pm 0.0015$
- It follows that combined uncertainties can be evaluated as:

$$\delta^{\text{pdf}+\alpha_S}\sigma = \sqrt{(\delta^{\text{pdf}}\sigma)^2 + (\delta^{\alpha_S}\sigma)^2}$$

- All the methods for calculating the PDF and α_S uncertainty are implemented in the `LHAPDF::PDFUncertainty` structure [LHAPDF6, arXiv:1510.03865]

[2] J. Butterworth, et al: PDF4LHC recommendations for LHC Run II, arXiv:1510.03865

Reweighting

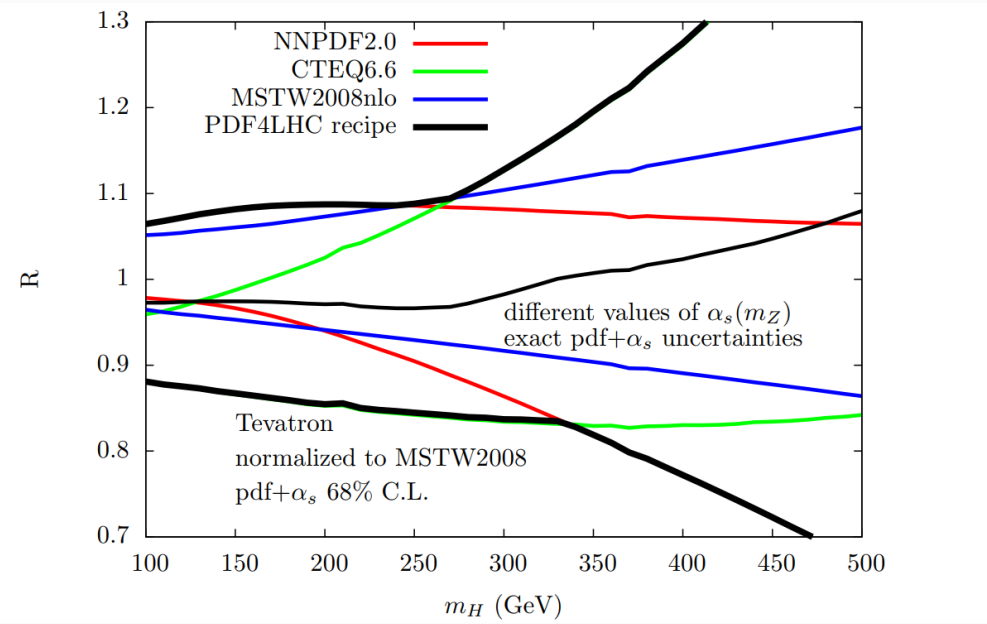
- Less computationally intense, then generating sample for each member in the PDF set
- All sets are 100% correlated, which means that statistical error does not affect the uncertainty calculation as it would in case of statistically independent samples
- Statistics of the sample used still contributes to the uncertainty (10 M events in this case)
- LO reweighting from PDF set A to PDF set B:

$$w_{A \to B} = \frac{f_1^B(x_1, Q) f_2^B(x_2, Q)}{f_1^A(x_1, Q) f_2^A(x_2, Q)}$$

- Many event generators have possibility of (N)NLO reweighting: `MadGraph5_aMC@NLO`, `POWHEG`, `Sherpa`, `FEWZ`, `RESBOS`

Statistical combinations of PDF sets

- Differences among different sets should be accounted for in PDF uncertainties
- In many cases it is better to take into account more than just one PDF set
- PDF uncertainty deduced from the envelope of bands of different PDF sets can be overestimated. $\Rightarrow$ Statistical combinations are a better choice [PDF4LHC, arXiv:1510.03865]



Plot taken from M. Bitje, el al.: The PDF4LHC Working Group Interim Recommendations, arXiv:1101.0538

PDF4LHC15 set

- Statistical combination of the **CT14** (Hessian), **MMHT2014** (Hessian) and **NNPDF3.0** (MC) sets
- Available in LHAPDF in three different delivery options: [PDF4LHC, arXiv:1510.03865]
  - Monte-Carlo (CMC-PDFs)
  - Hessian, with 30 eigenvectors (META-PDFs)
  - Hessian, with 100 eigenvector (MCH-PDFs)

PDF set	Pert. order	Error Type	N_{mem}	α_S var.
`PDF4LHC15_nlo_100`	NLO	`symhessian`	100	No
`PDF4LHC15_nlo_30`	NLO	`symhessian`	30	No
`PDF4LHC15_nlo_mc`	NLO	`replicas`	100	No
`PDF4LHC15_nlo_30_pdfas`	NLO	`symhessian+as`	32	Yes
`PDF4LHC15_nlo_mc_pdfas`	NLO	`replicas+as`	102	Yes

Full-leptonic VBS - Samples

- Process: $pp \to jje^+\nu_e\mu^+\nu_\mu$ (LO)
- Parton level events
- MC generator: `PHANTOM`
- Three samples were generated, with `PDF4LHC15_nlo_mc_pdfas`, `PDF4LHC15_nlo_30_pdfas`, and `PDF4LHC15_nlo_100` (10M events each)
- Cross-section variation has been achieved by reweighting from the central value of each set to a corresponding error set

Kinematical cuts:

- $p_{\mathsf{T}}^{\ell} > 20\text{GeV}$
- $|\eta^{\ell}| < 2.5$
- $p_{\mathsf{T}}^{j} > 30\text{GeV}$
- $|\eta^{\ell}| < 4.5$
- $p_{\mathsf{T}}^{\text{miss}} > 40\text{GeV}$
- $m_{jj} > 500\text{GeV}$
- $\Delta R_{j\ell} > 0.3$
- $\Delta R_{\ell\ell} > 0.3$

Full-leptonic VBS - Total cross-section uncertainties

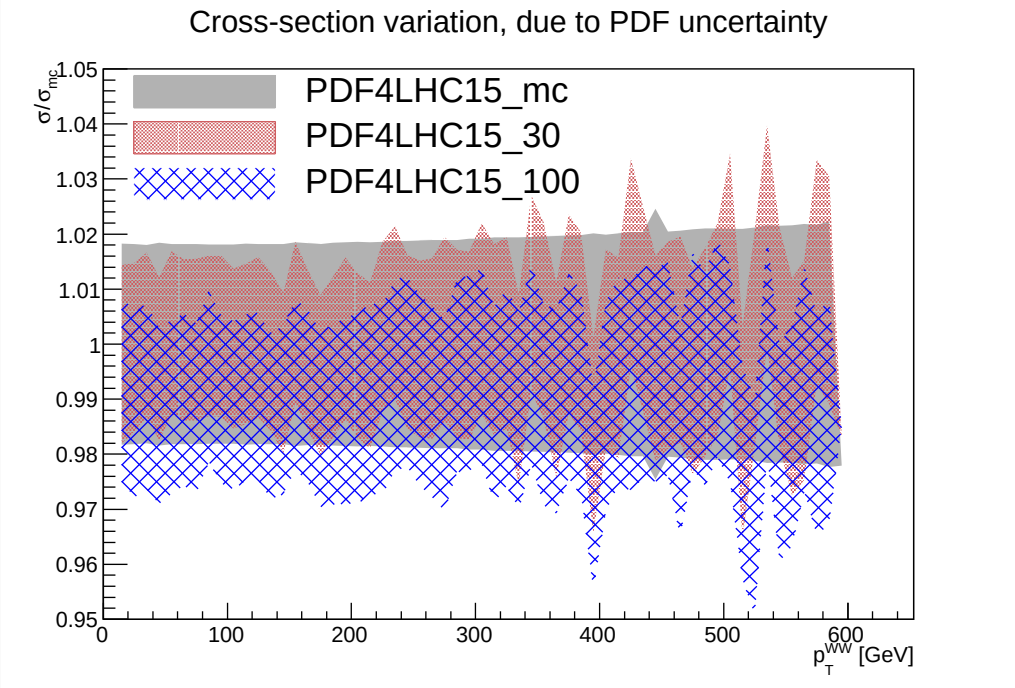
PDF type	Total LO xsection [fb]	Statistical error [%]
PDF4LHC15_nlo_100	2.15271	3.17778×10^{-2}
PDF4LHC15_nlo_30_pdfas	2.15298	3.16176×10^{-2}
PDF4LHC15_nlo_mc_pdfas	2.15333	3.16156×10^{-2}

PDF unc MC (a) [%]	PDF unc MC (b) [%]	PDF unc Hess [%]	α_S unc [%]	Combined unc [%]
-	-	**1.76815**	-	-
-	-	**1.6248**	$\mathbf{1.3131 \times 10^{-2}}$	**1.6249**
1.8329	1.92104	-	$\mathbf{1.1046 \times 10^{-2}}$	**1.8329**

- Gaussianity test (Shapiro-Wilk):
  PDF4LHC15_nlo_mc_pdfas: $W = 0.9891, \quad p\text{-value} = 0.5928$

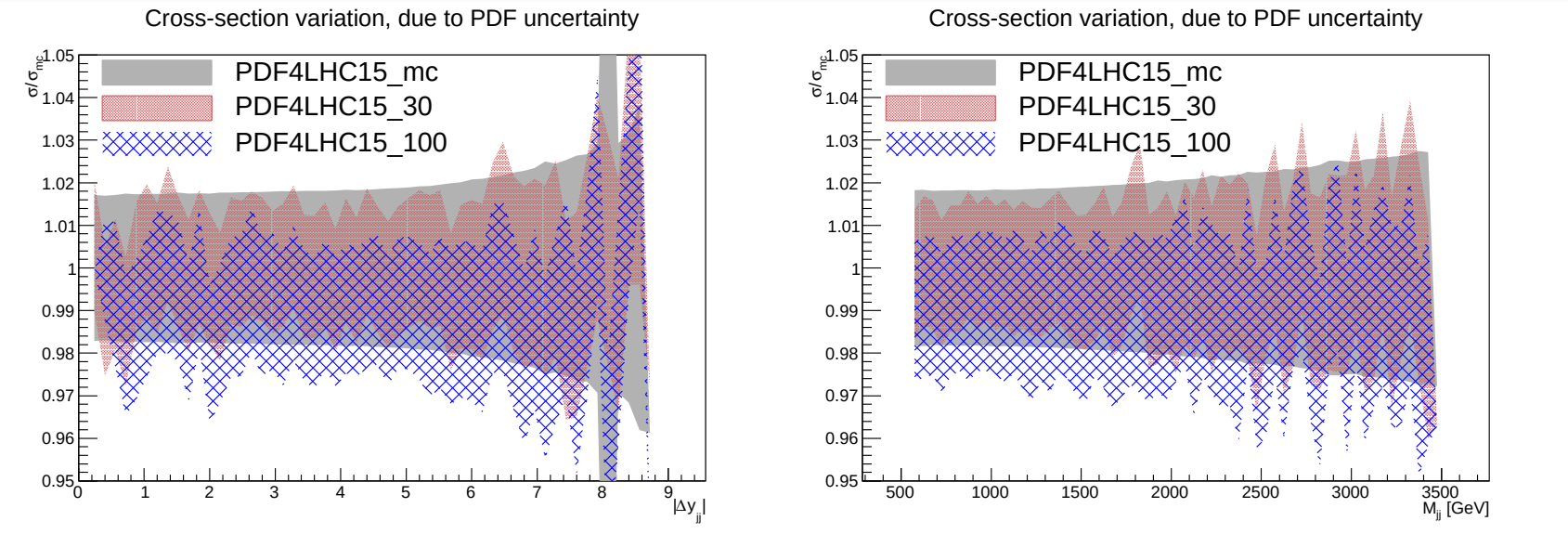
Full-leptonic VBS - Differential cross-section uncertainties

- 68% CL differential cross-section uncertainty band, due to PDF variation, along p_{T}^{WW}
- Cross-sections normalized to the MC set central value
- Central values differ among the PDF sets, but bands are compatible



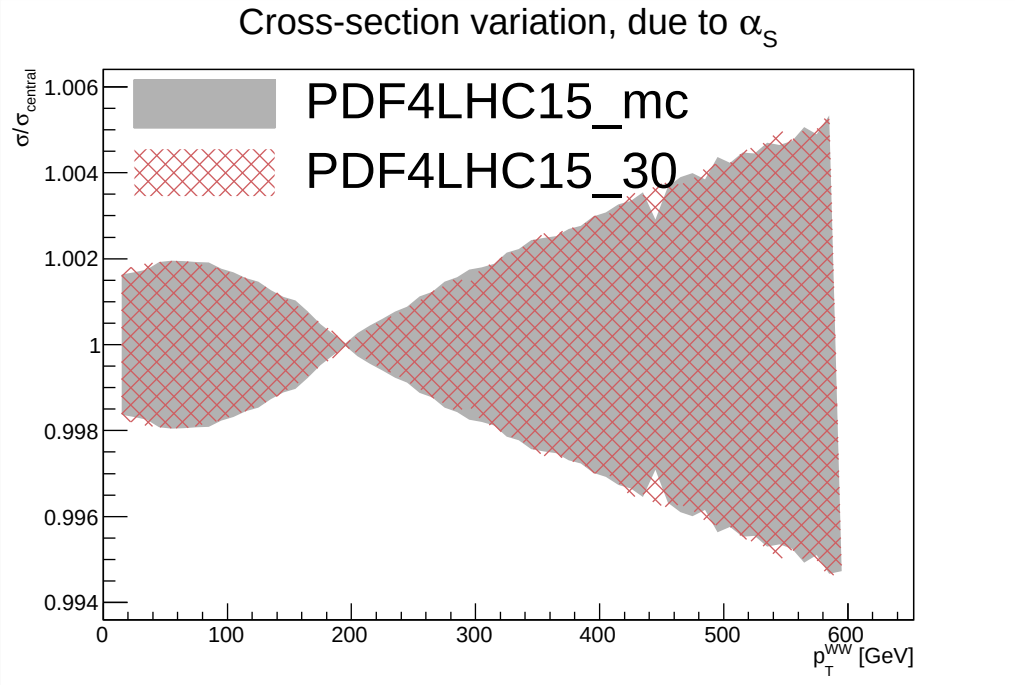
Full-leptonic VBS - Differential cross-section uncertainties

- 68% CL PDF uncertainty bands along $|\Delta y|$ and m_{jj}



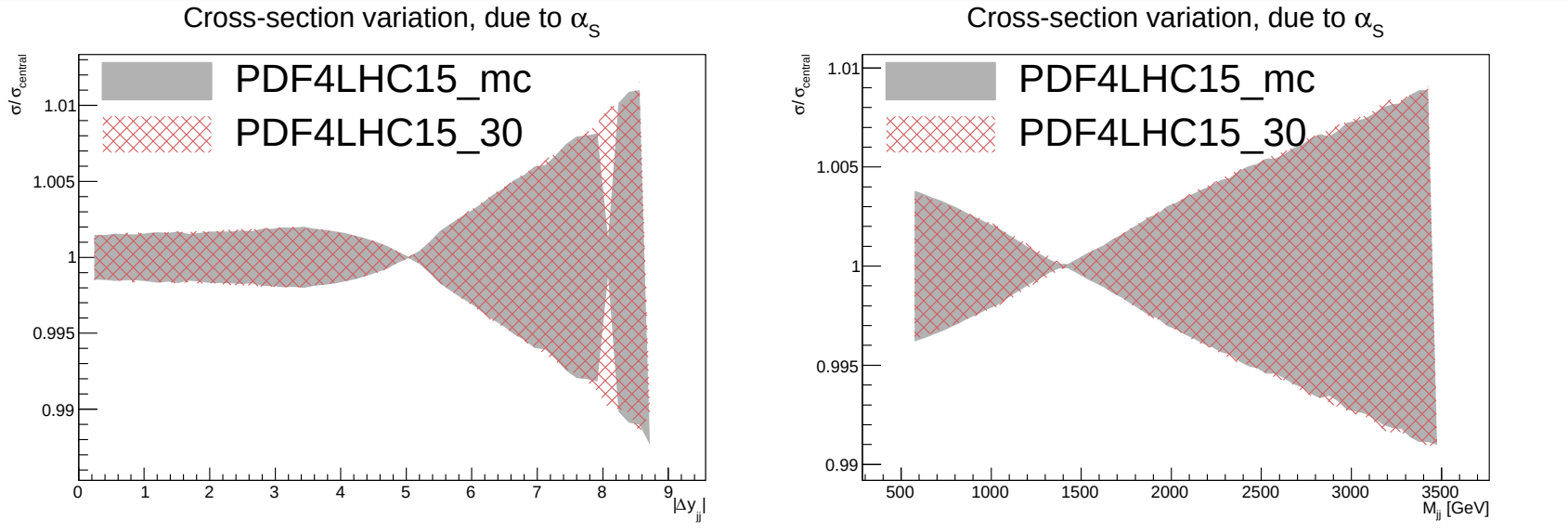
Full-leptonic VBS - Differential cross-section uncertainties

- 68% CL differential cross-section uncertainty band, due to α_S variation, along p_{T}^{WW}
- Error cross-sections normalized to the central value from the same PDF set



Full-leptonic VBS - Differential cross-section uncertainties

- 68% CL α_S uncertainty bands along $|\Delta y|$ and m_{jj}
- Cross-section uncertainty due to α_S variation much smaller than the uncertainty due to PDF variation



Conclusions

- The use of reweighting significantly reduces computational time
- Statistical combinations are in many cases better choice than the PDF envelope
- Currently available statistical combination in LHAPDF library is PDF4LHC15
- All three delivery options of PDF4LHC15 give compatible results
- α_S uncertainty significantly smaller than the PDF uncertainty
- We plan to perform scale variation at NLO QCD (and EWK if possible) and cross-check this procedure at LO