W Reconstruction in Vector Boson Scattering

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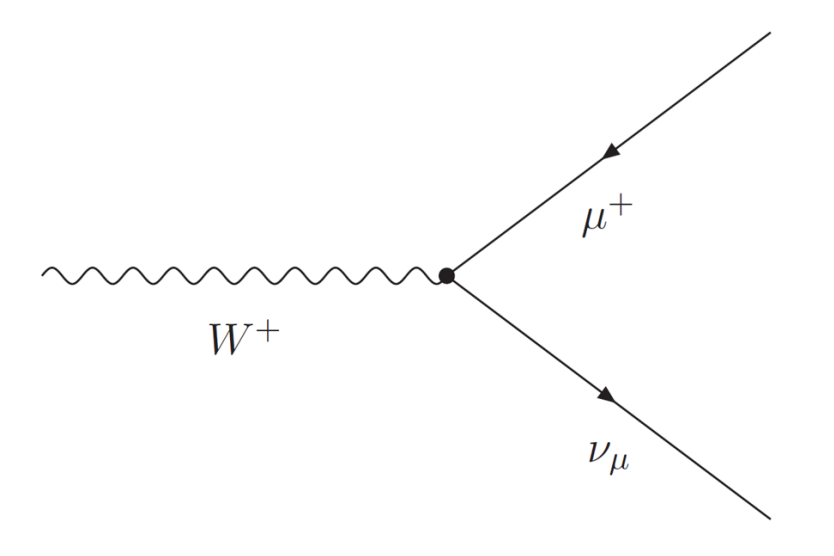
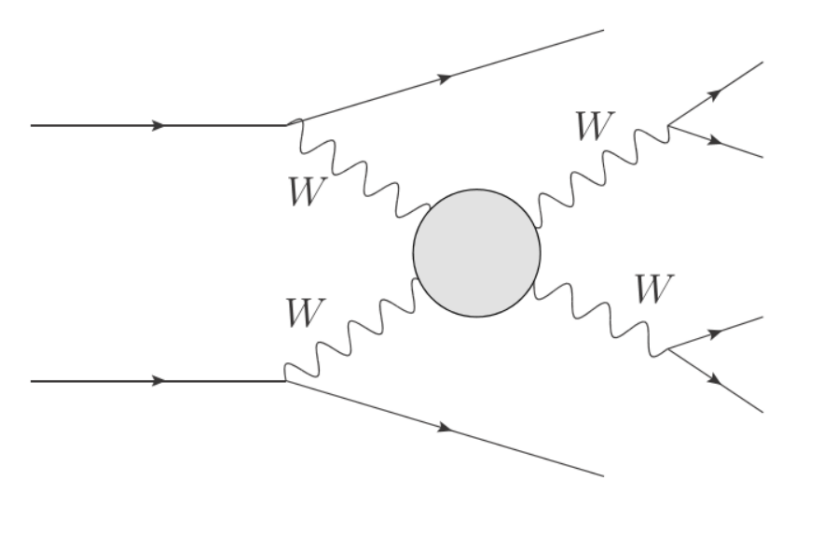
Motivation

From single W decay...

- The final goal of the exercise is to determine how well the full event kinematics of VBS can be reconstructed in order to measure the polarization of the participating W bosons.





....to VBS process

Formula Derivation for $W \to \mu\nu$

$$m_{\mathrm{W}}^2 = (p_\mu + p_\nu)^2 \quad \textit{ultra relativistic limit} \quad \overset{m\to 0}{\longrightarrow} \quad 2p_\mu p_\nu;$$

- Let's solve for the longitudinal component of the neutrino $p_{\nu L}$;

$$\underbrace{\left(p_{lL}^2 - E_l^2\right)}_{a} p_{\nu L}^2 +$$

$$\underbrace{\left(m_{\mathrm{W}}^2 p_{lL} + 2p_{lL}\vec{p}_{lT}\vec{p}_{\nu T}\right)}_{b} p_{\nu L} +$$

$$\underbrace{\frac{m_{\mathrm{W}}^4}{4} + (\vec{p}_{lT}\vec{p}_{\nu T})^2 + m_{\mathrm{W}}^2 \vec{p}_{lT}\vec{p}_{\nu T} - E_l^2\vec{p}_{\nu T}^2)}_{c} = 0;$$

$$\boxed{p_{\nu L_{1,2}} = \frac{-b \pm \sqrt{\Delta}}{2a}} \quad \text{where} \quad \Delta = b^2 - 4ac$$

As a second order parametric equation, Δ determines the number of solution and their nature.

$m_W \Rightarrow$ **fixed value** $(80.385$ GeV$)$

- if $\Delta > 0 \Rightarrow$ 2 solutions $(+/-)$
- if $\Delta < 0$, from the formula:

$$a > 0 \ \rightarrow \ \left(p_{lL}^2 - E_l^2\right) > 0 \Rightarrow \ |E_l| \leq p_{lL}$$

we have two working options:

$$\Delta(p_L) \begin{cases} \text{put } \Delta = 0 \\ m_W = m_W T \Rightarrow \quad \text{correct } m_W \text{ with transverse mass} \end{cases}$$

VBS Semileptonic channel

VBS Semileptonic W boson reconstruction

PHANTOM PARAMETER for the production

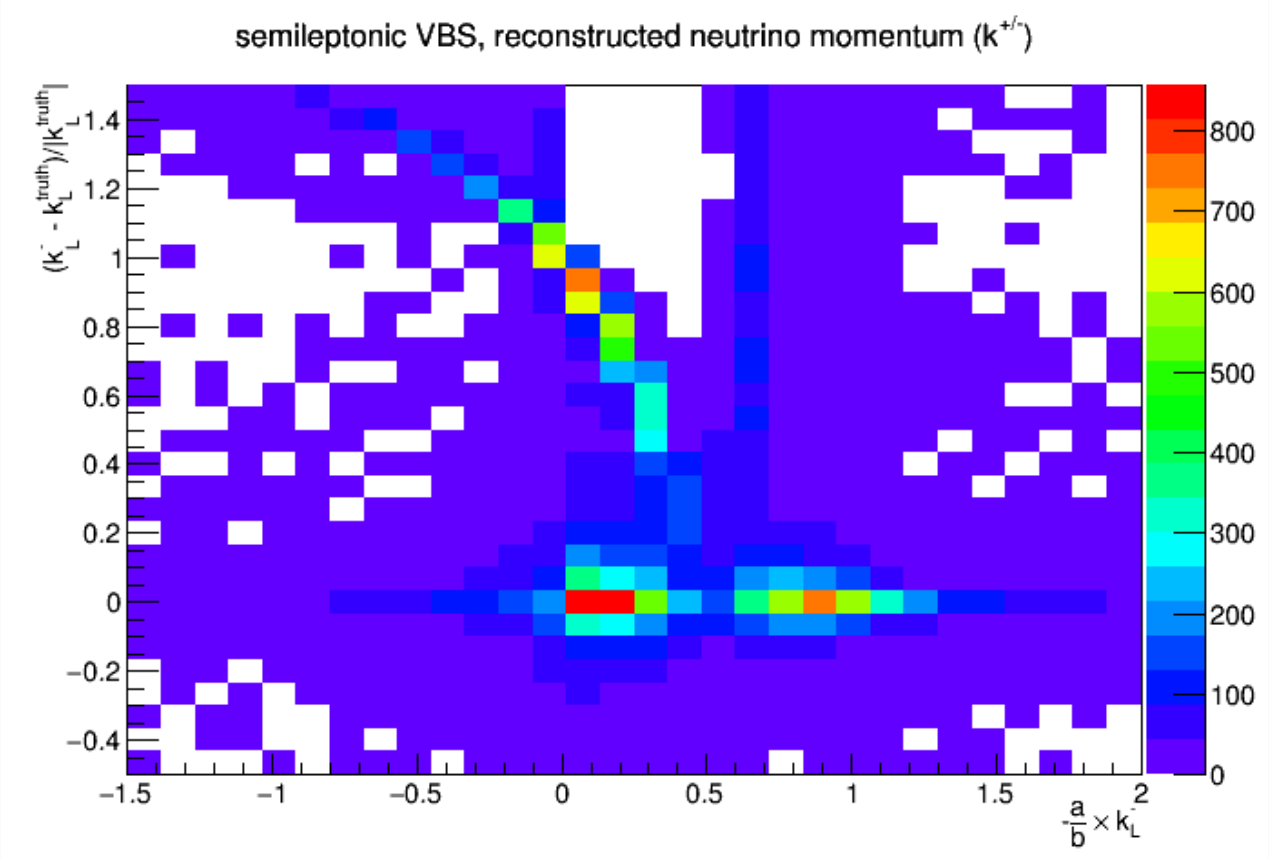
- Process: $pp \to jj\mu^+\nu_\mu$
- Parton level events
- MC generator: `Phantom`
- 1M samples generated, with `NNPDF30_nnlo_as_0118`
- CALCULATION TYPE: α_e^6
- SCALE CHOICE: (invariant mass of the 2 central jets and of 2 leptons)$/\sqrt{2}$

Kinematical cuts:

- $p_{\mathrm{T}}^{\ell} > 20$ GeV
- $|\eta^{\ell}| < 3$
- $p_{\mathrm{T}}^{min} > 30$ GeV
- $|\eta_j| < 5.4$
- $p_{\mathrm{T}}^{\mathrm{miss}} > 20$ GeV
- $m_{jj} > 500$ GeV
- $\Delta R_{j\ell} > 0.3$

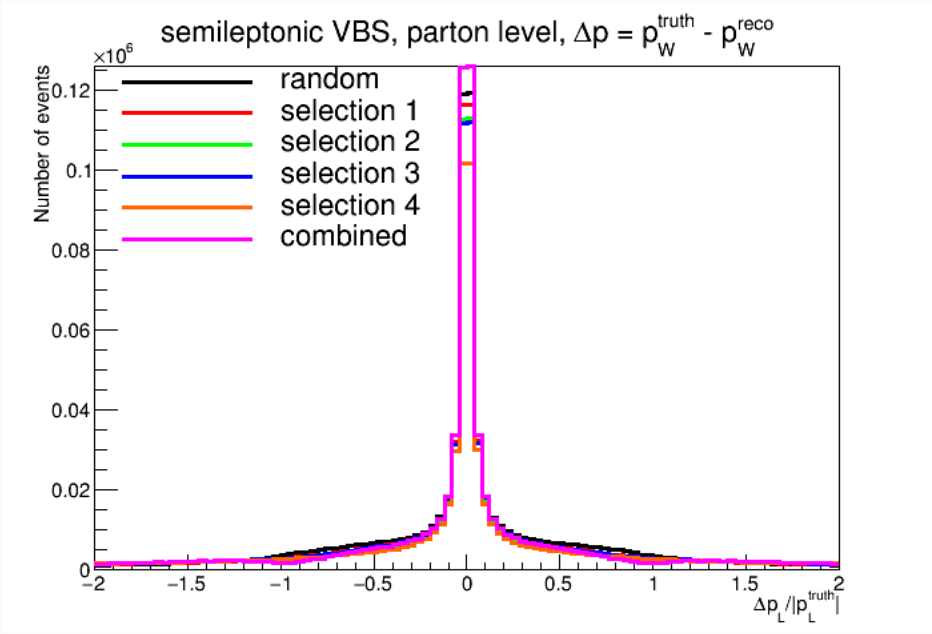
Criteria Selection at Parton Level

Selection 4 algorithm discards all the solutions for which $-K_L(minus) * a/b < 0.5$ and viceversa for $plus$ solution. If both solutions pass or don't pass this criterium, random solution is chosen.

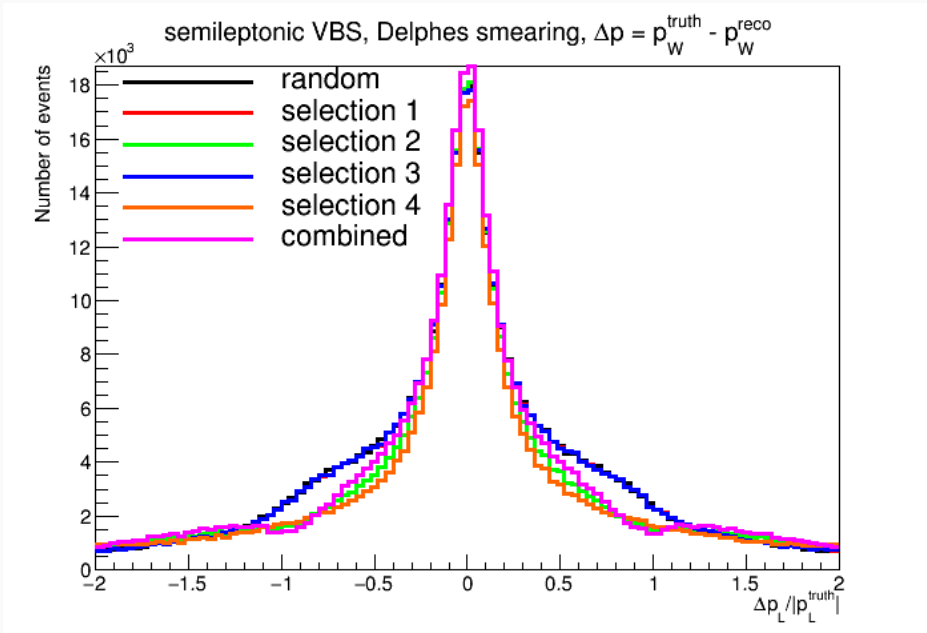


Criteria Application Outcome - Parton and Detector level

Reconstruction of W momentum at parton level according to different criteria.



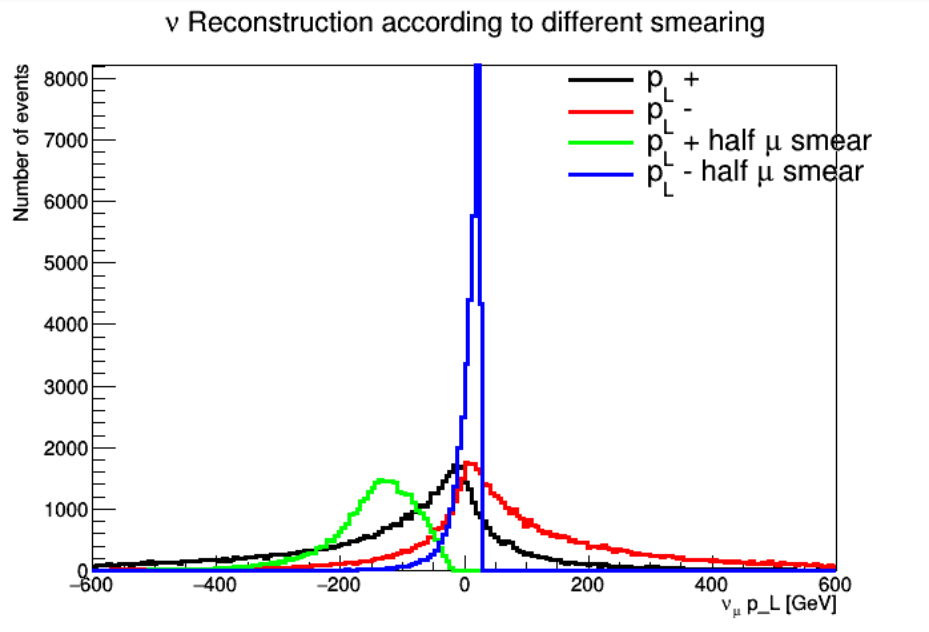
Reconstruction of W momentum after detector interaction[1] according to different criteria.



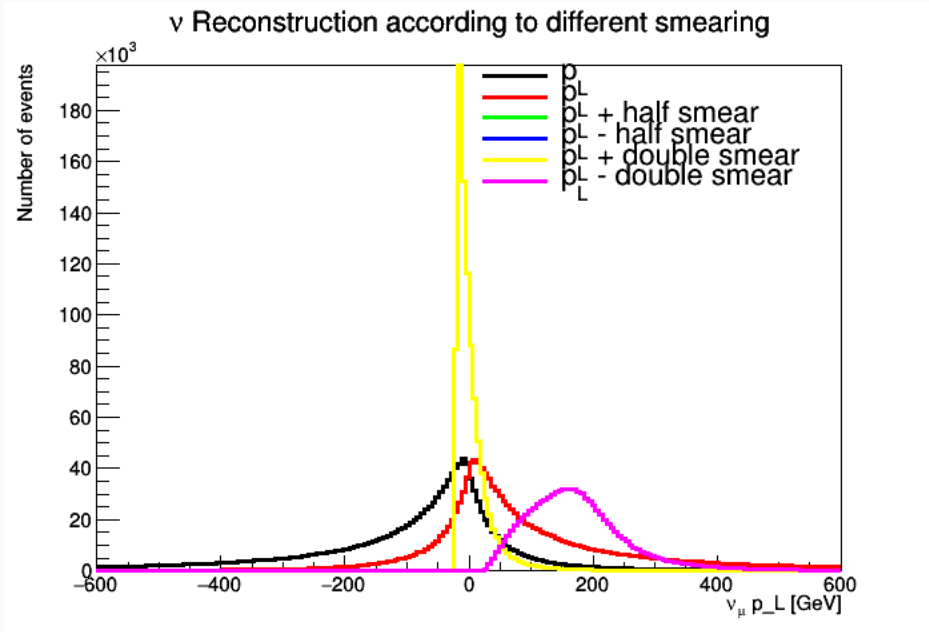
[1] `Delphes` simulation obtained using ATLAS card set up simulation

VBS Semileptonic - different muon smearing

Plot of $p_{\nu L}$ from `Delphes`[2] data according to plus and minus solution using different μ smearing



Plot of $p_{\nu L}$ from `Delphes`[3] data according to plus and minus solution using different MET smearing



[3]We used standard `Delphes` ATLAS card changing one parameter at a time.

VBS Fully leptonic channel

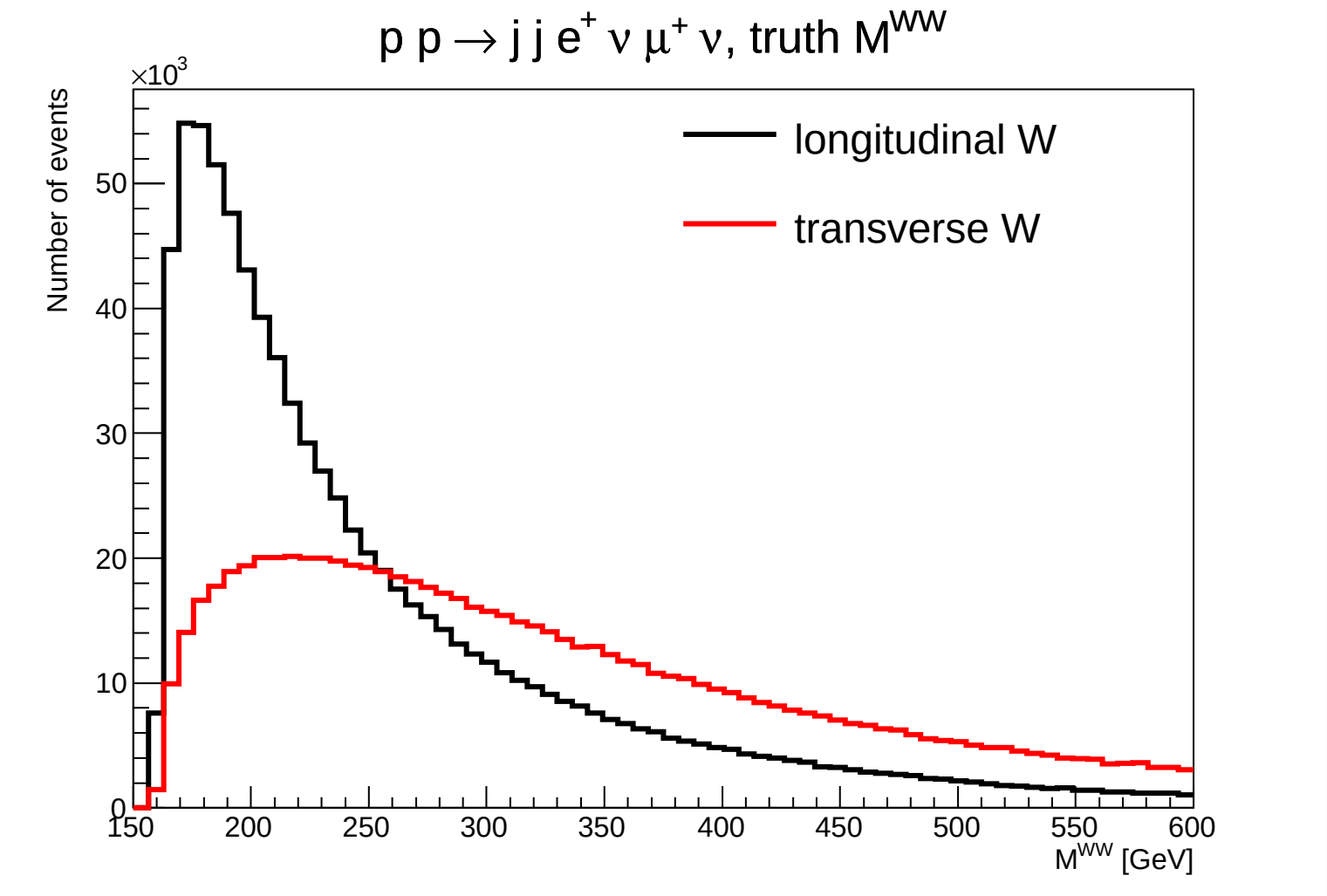
Reco of neutrino momentum in $pp \to jje^+\nu_e\mu^+\nu_\mu$

- 8 unknown parameters (2 x neutrino four momentum)
- 6 equations:
  - $\vec{p}_{\mathrm{T}}^{\,\nu_\mu} + \vec{p}_{\mathrm{T}}^{\,\nu_e} = \vec{p}_{\mathrm{T}}^{\,\mathrm{miss}}$ (2x)
  - $(p^\ell + p^\nu)^2 = m_W^2$ (2x)
  - $p_\nu^2 = 0$ (2x)
- Remaining 2 equations:
  1. Setting some parameters to fixed values - for example: $M_{WW}^2 = (p_e + p_{\nu_e} + p_\mu + p_{\nu_\mu})^2$ and $M_{\nu\nu}^2 = (p_{\nu_e} + p_{\nu_\mu})^2$, M_{WW} and $M_{\nu\nu}$ are fixed numbers.[1]
  2. Using of $MT2$-Assisted On-Shell (MAOS) quantites, i.e. minimization of the transverse masses of the lepton-neutrino pairs.[4]
  3. Other ideas ??

[1] arXiv:hep-ph/0603011, Higgs spin analysis in Collins-Soper frame using opening angles of different-flavour final state leptons

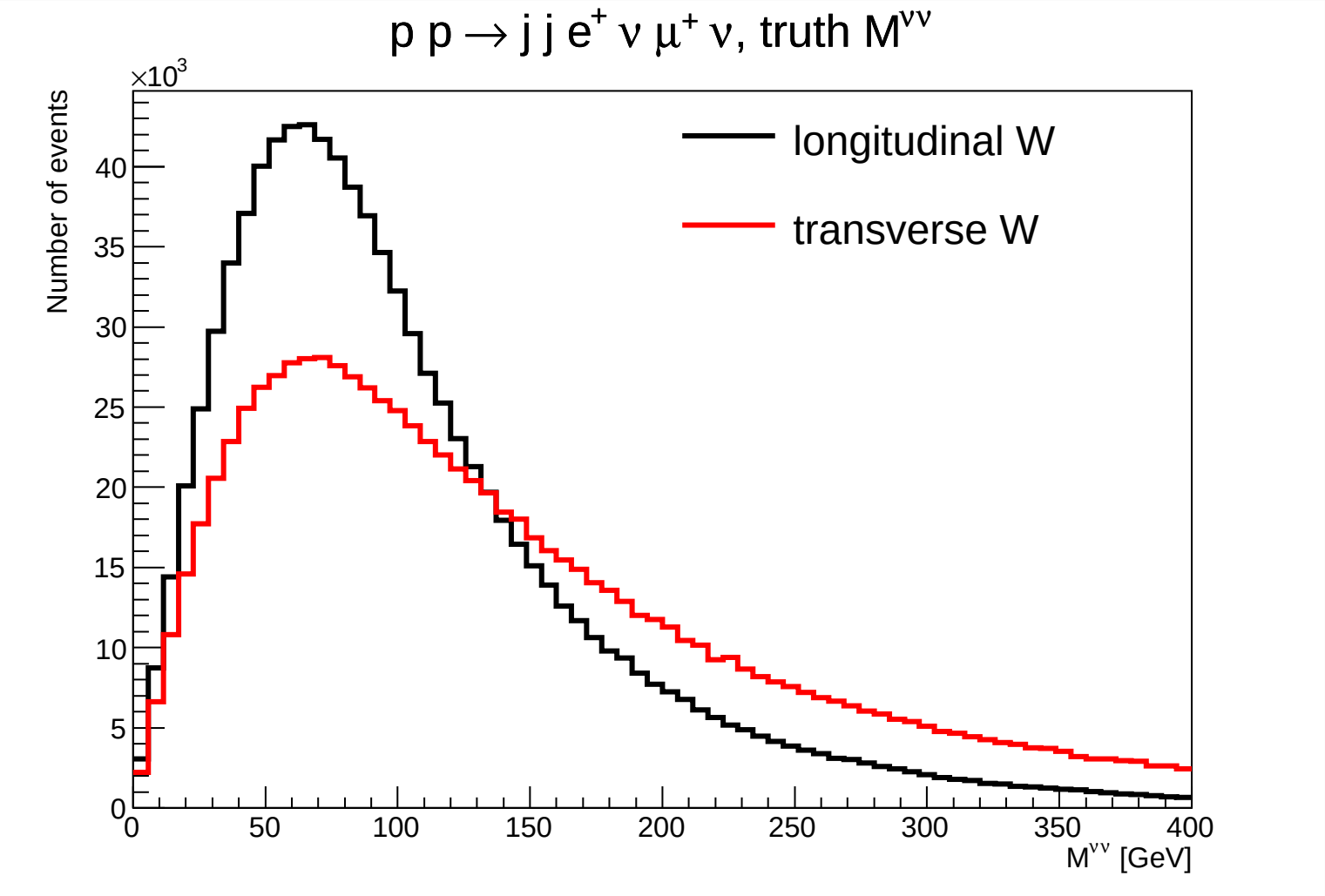
[4] arXiv:0908.0079

1. Reco with fixed parameters - Choice of M_{WW} and $M_{\nu\nu}$



$\Rightarrow M_{WW} \sim 250$ GeV

1. Reco with fixed parameters - Choice of M_{WW} and $M_{\nu\nu}$



$\Rightarrow M_{\nu\nu} \sim 100$ GeV

1. Reco with fixed parameters - Equations

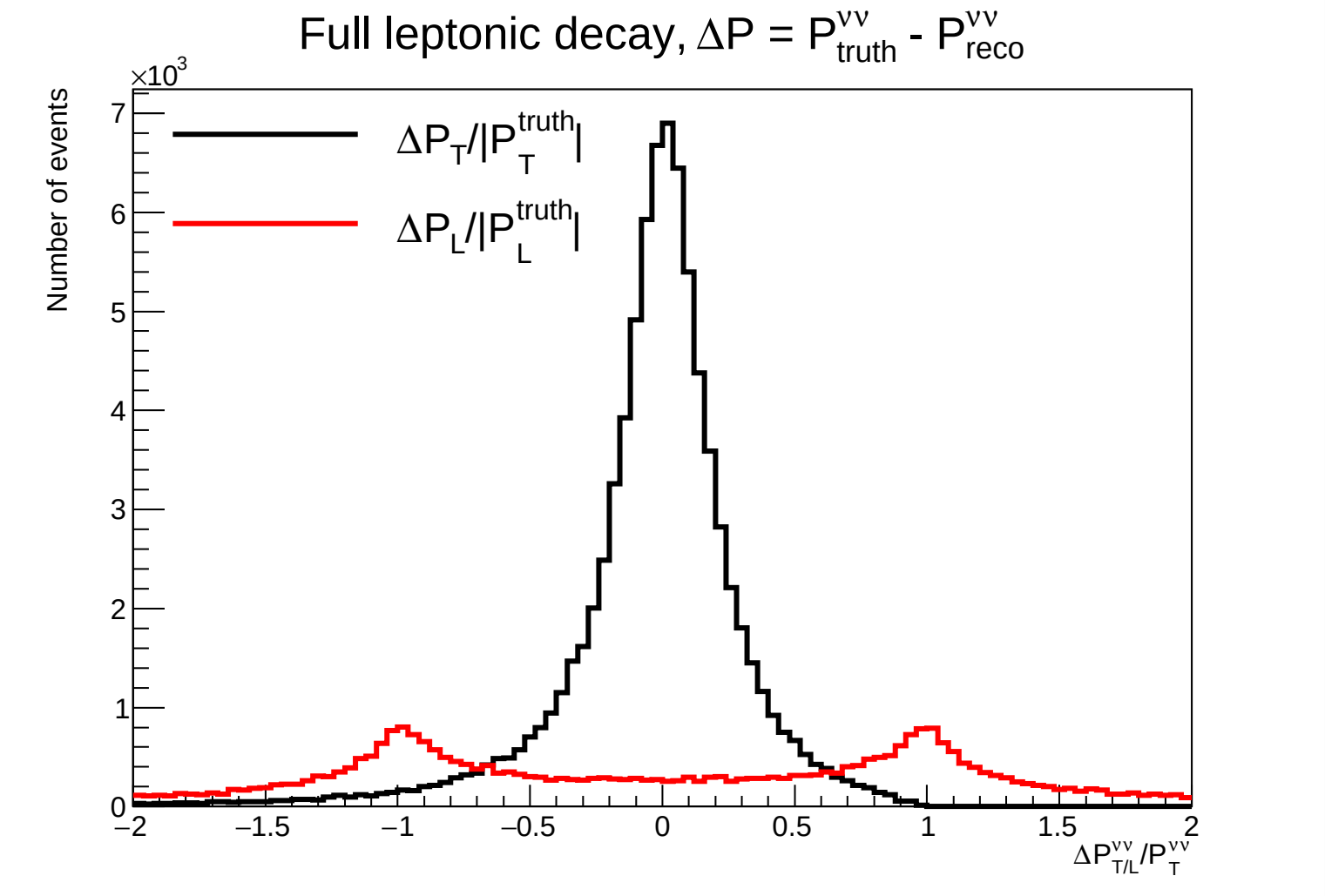
$$P_{WW} = P_{\nu\nu} + P_{\ell\ell}$$
$$M^2_{WW} = M^2_{\nu\nu} + M^2_{\ell\ell} + 2P_{\nu\nu} \cdot P_{\ell\ell}$$

Define: $M^2 \equiv M^2_{WW} - M^2_{\nu\nu} - M^2_{\ell\ell}$

$$M^2 = 2P_{\nu\nu} \cdot P_{\ell\ell};$$

- For extraction of the $P_{\mathrm{L}}^{\nu\nu}$ the same equations as for semileptonic channel apply.

1. Reco with fixed parameters - Performance



- Doesn't work

2. Reco with MAOS quantities - Equations

- MAOS estimations $\vec{p}_{\mathsf{T}}^{\,\nu_e\prime}$ and $\vec{p}_{\mathsf{T}}^{\,\nu_\mu\prime}$ for neutrinos transverse momentums can be obtained by minimizing the function $f(\vec{p}_1, \vec{p}_2) = \max\{M_{\mathsf{T}}^{W_1}, M_{\mathsf{T}}^{W_2}\}$, constrained by a bond $\vec{p}_1 + \vec{p}_2 = \vec{p}_{\mathsf{T}}^{\,\text{miss}}$, where

$$M_{\mathsf{T}}^{W_1} = 2(|\vec{p}_{\mathsf{T}}^{\,\mu}||\vec{p}_1| - \vec{p}_{\mathsf{T}}^{\,\mu} \cdot \vec{p}_1), \qquad M_{\mathsf{T}}^{W_2} = 2(|\vec{p}_{\mathsf{T}}^{\,e}||\vec{p}_2| - \vec{p}_{\mathsf{T}}^{\,e} \cdot \vec{p}_2)$$

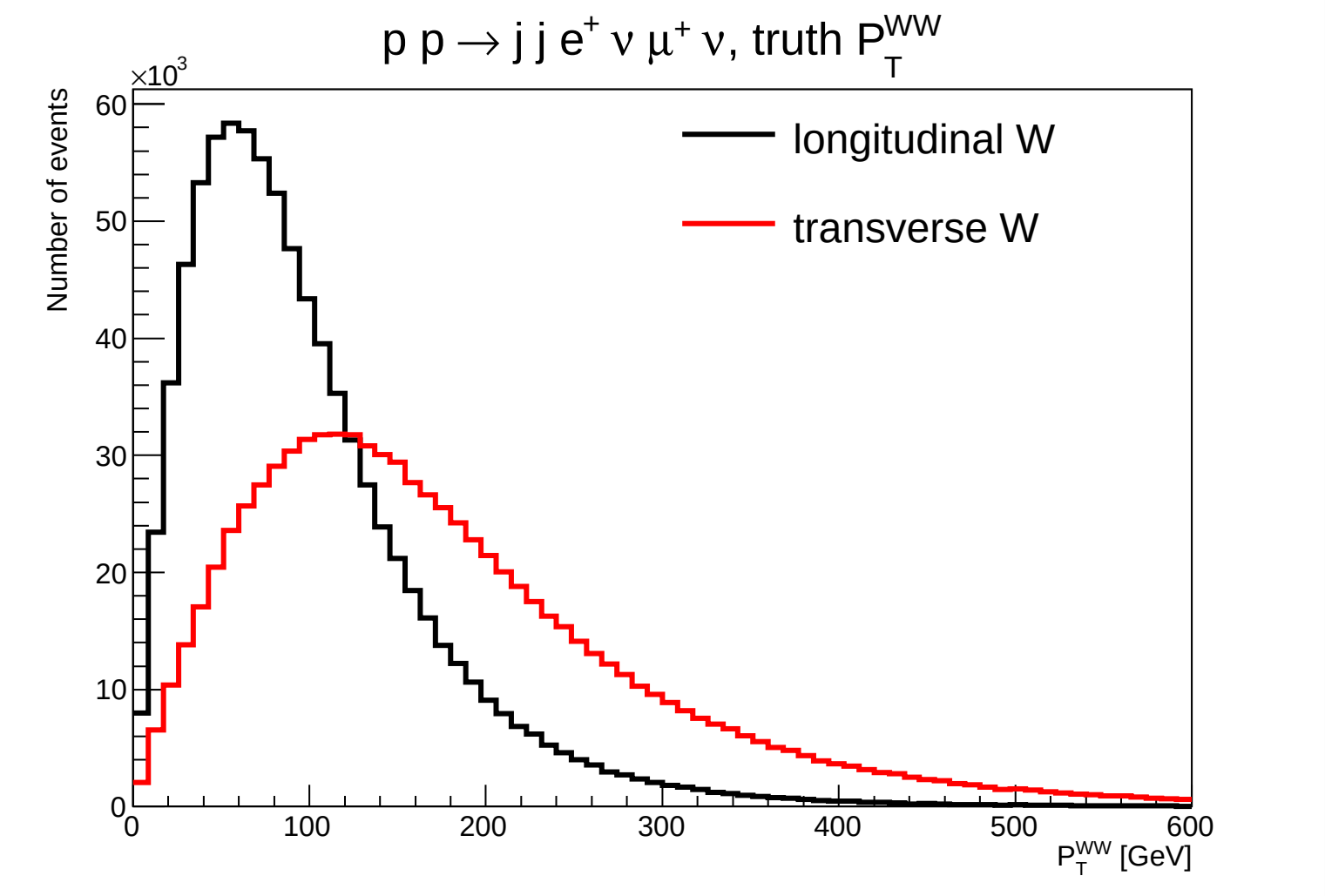
- Minimum of the function f defines quantity $M_{\mathsf{T}2}$:

$$M_{\mathsf{T}2} \equiv \min_{\vec{p}_1+\vec{p}_2=\vec{p}_{\mathsf{T}}^{\,\text{miss}}} f(\vec{p}_1, \vec{p}_2) = f|_{\vec{p}_{\mathsf{T}}^{\,\nu_e\prime}, \vec{p}_{\mathsf{T}}^{\,\nu_\mu\prime}} \tag{1}$$

- Solution of the problem (1), under assumption $p_{\mathsf{T}}^{WW} \sim 0$:

$$\vec{p}_{\mathsf{T}}^{\,\nu_e\prime} = -\vec{p}_{\mathsf{T}}^{\,\mu} \qquad \vec{p}_{\mathsf{T}}^{\,\nu_\mu\prime} = -\vec{p}_{\mathsf{T}}^{\,e}$$

2. Reco with MAOS quantities - Equations



- Assumption $p_{\mathrm{T}}^{WW} \sim 0$ obviously not valid in our case.

2. Reco with MAOS quantities - Equations

Exact solution:

- $\min\left[\max\{M_{\mathrm{T}}^{W_1}, M_{\mathrm{T}}^{W_2}\}\right]$ can always lie only on the intersection of $M_{\mathrm{T}}^{W_1}$ and $M_{\mathrm{T}}^{W_2}$. $\Rightarrow$ Additional bond: $M_{\mathrm{T}}^{W_1} = M_{\mathrm{T}}^{W_2}$
  - It follows that:

$$\Rightarrow \quad 2(|\vec{p}_{\mathrm{T}}^{\,\mu}||\vec{p}_1| - \vec{p}_{\mathrm{T}}^{\,\mu} \cdot \vec{p}_1) = 2(|\vec{p}_{\mathrm{T}}^{\,e}||\vec{p}_2| - \vec{p}_{\mathrm{T}}^{\,e} \cdot (\vec{p}_{\mathrm{T}}^{\,\mathrm{miss}} - \vec{p}_1))$$

$$|\vec{p}_{\mathrm{T}}^{\,\mu}||\vec{p}_1| - \vec{p}_{\mathrm{T}}^{\,\ell\ell} \cdot \vec{p}_1 + \vec{p}_{\mathrm{T}}^{\,e} \cdot \vec{p}_{\mathrm{T}}^{\,\mathrm{miss}} = |\vec{p}_{\mathrm{T}}^{\,e}|\sqrt{|\vec{p}_{\mathrm{T}}^{\,\mathrm{miss}}|^2 - 2\vec{p}_{\mathrm{T}}^{\,\mathrm{miss}} \cdot \vec{p}_1 + |\vec{p}_1|^2}$$

$$(|\vec{p}_{\mathrm{T}}^{\,\mu}||\vec{p}_1| - |\vec{p}_{\mathrm{T}}^{\,\ell\ell}||\vec{p}_1|\cos\varphi + \vec{p}_{\mathrm{T}}^{\,e} \cdot \vec{p}_{\mathrm{T}}^{\,\mathrm{miss}})^2 =$$

$$|\vec{p}_{\mathrm{T}}^{\,e}|^2|\vec{p}_{\mathrm{T}}^{\,\mathrm{miss}}|^2 - 2|\vec{p}_{\mathrm{T}}^{\,e}|^2|\vec{p}_{\mathrm{T}}^{\,\mathrm{miss}}||\vec{p}_1|\cos(\varphi + \varphi_0) + |\vec{p}_{\mathrm{T}}^{\,e}|^2|\vec{p}_1|^2$$

  - φ_0 - angle between $\vec{p}_{\mathrm{T}}^{\,\mathrm{miss}}$ and $\vec{p}_{\mathrm{T}}^{\,\ell\ell}$

    $\rightarrow$ Parameter of the equation: $\varphi_0 = \arccos\left(\dfrac{\vec{p}_{\mathrm{T}}^{\,\ell\ell} \cdot \vec{p}_{\mathrm{T}}^{\,\mathrm{miss}}}{|\vec{p}_{\mathrm{T}}^{\,\ell\ell}||\vec{p}_{\mathrm{T}}^{\,\mathrm{miss}}|}\right)$

  - φ - angle between $\vec{p}_{\mathrm{T}}^{\,\ell\ell}$ and $\vec{p}_1$;

2. Reco with MAOS quantities - Equations

$$\underbrace{\left(|\vec{p}_{\mathsf{T}}^{\,\mu}|^2 + |\vec{p}_{\mathsf{T}}^{\,\ell\ell}|^2\cos^2\varphi - 2|\vec{p}_{\mathsf{T}}^{\,\mu}||\vec{p}_{\mathsf{T}}^{\,\ell\ell}|\cos\varphi - |\vec{p}_{\mathsf{T}}^{\,e}|^2\right)}_{f(\varphi)}|\vec{p}_1|^2 \quad +$$

$$\underbrace{\left(2(|\vec{p}_{\mathsf{T}}^{\,\mu}| - |\vec{p}_{\mathsf{T}}^{\,\ell\ell}|\cos\varphi)\vec{p}_{\mathsf{T}}^{\,e}\cdot\vec{p}_{\mathsf{T}}^{\,\mathsf{miss}} + 2|\vec{p}_{\mathsf{T}}^{\,e}|^2|\vec{p}_{\mathsf{T}}^{\,\mathsf{miss}}|\cos(\varphi+\varphi_0)\right)}_{g(\varphi)}|\vec{p}_1| \quad +$$

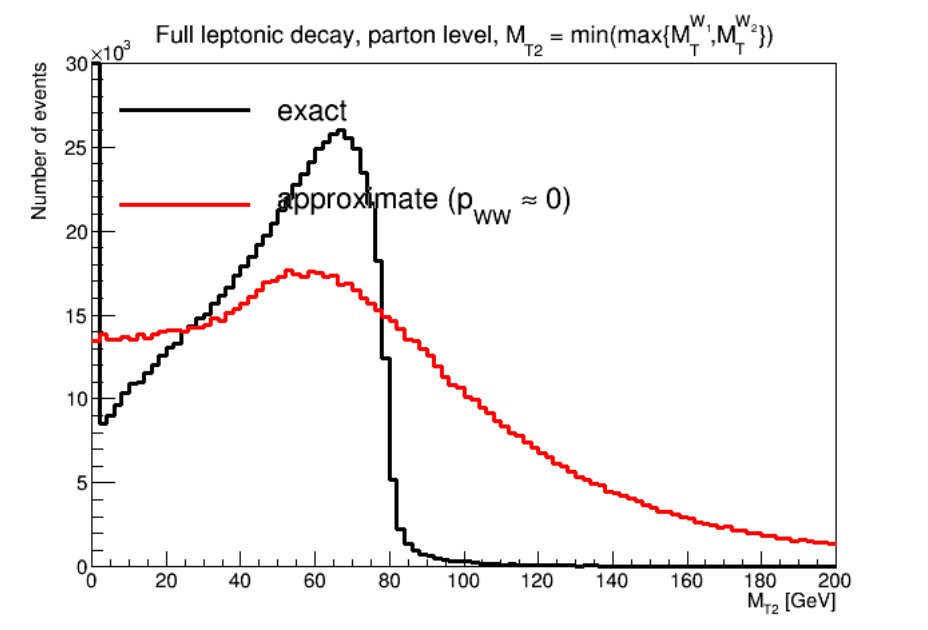
$$\underbrace{(\vec{p}_{\mathsf{T}}^{\,e}\cdot\vec{p}_{\mathsf{T}}^{\,\mathsf{miss}})^2 - |\vec{p}_{\mathsf{T}}^{\,\mathsf{miss}}|^2|\vec{p}_{\mathsf{T}}^{\,e}|^2}_{c} \quad = \quad 0$$

- Equation of the intersection curve in parametric form - x-axis of coordinate system coinciding with the $\vec{p}_{\mathsf{T}}^{\,\ell\ell}$ direction;

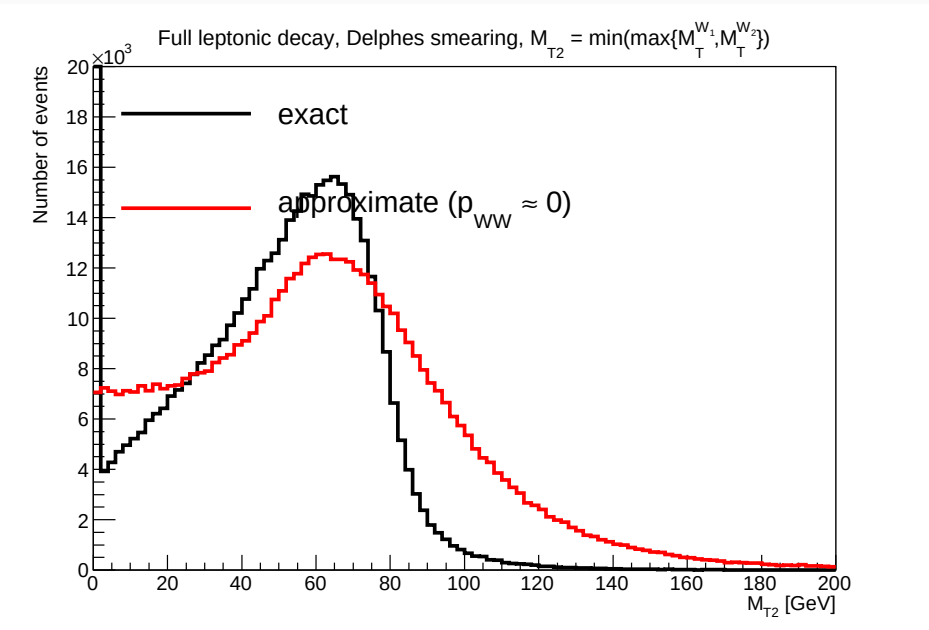
$$|\vec{p}_1| = \frac{-g(\varphi) \pm \sqrt{g(\varphi)^2 - 4cf(\varphi)}}{2f(\varphi)}, \qquad \vec{p}_2 = \vec{p}_{\mathsf{T}}^{\,\mathsf{miss}} - \vec{p}_1$$

- Minimum of $M_{\mathsf{T}2}$ on the intersection curve can be found numerically;
- The following plots are produced by evaluating $M_{\mathsf{T}2}$ in 2000 points;

2. Reco with MAOS quantities - Results

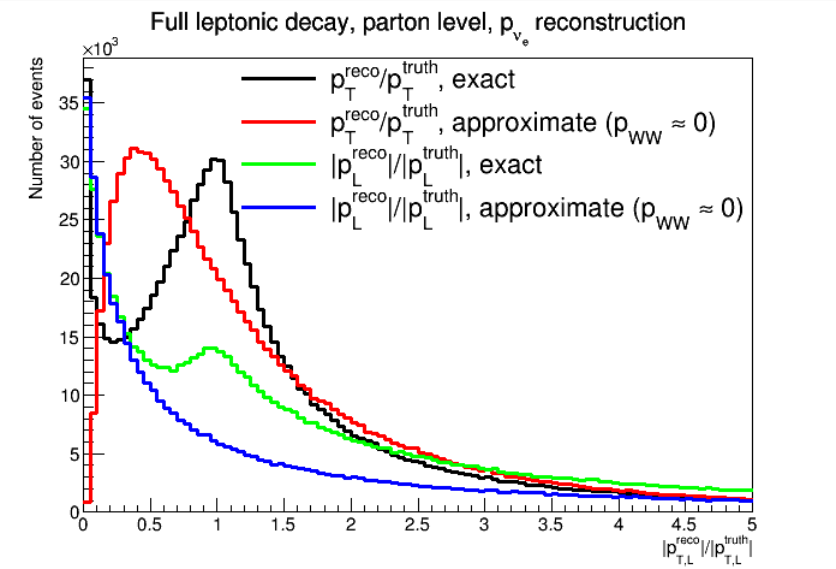


- evaluation of M_{T2} at parton level.

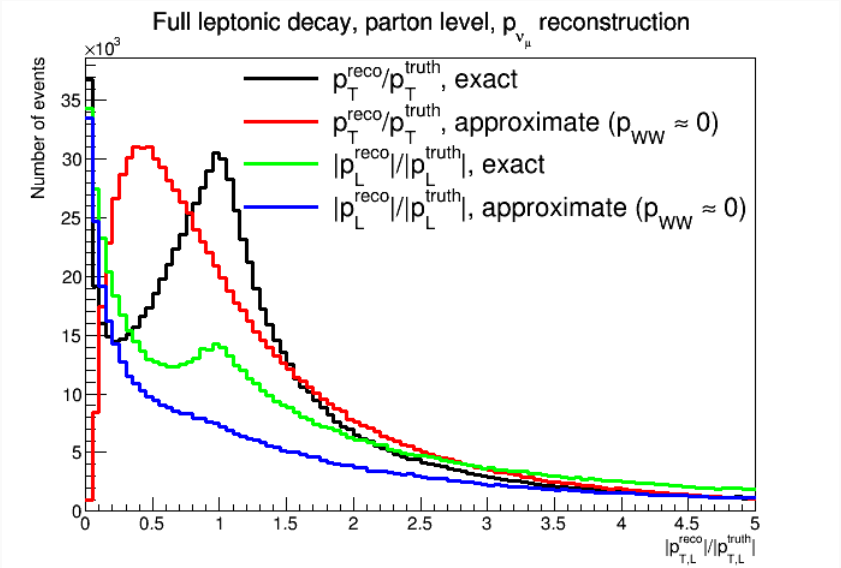


- evaluation of M_{T2} after detector smearing.

2. Reco with MAOS Truth quantities - Results

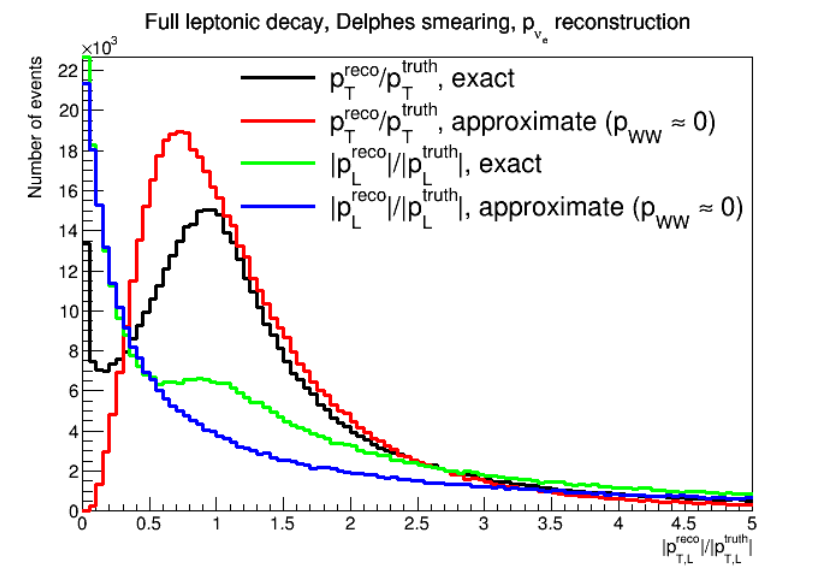


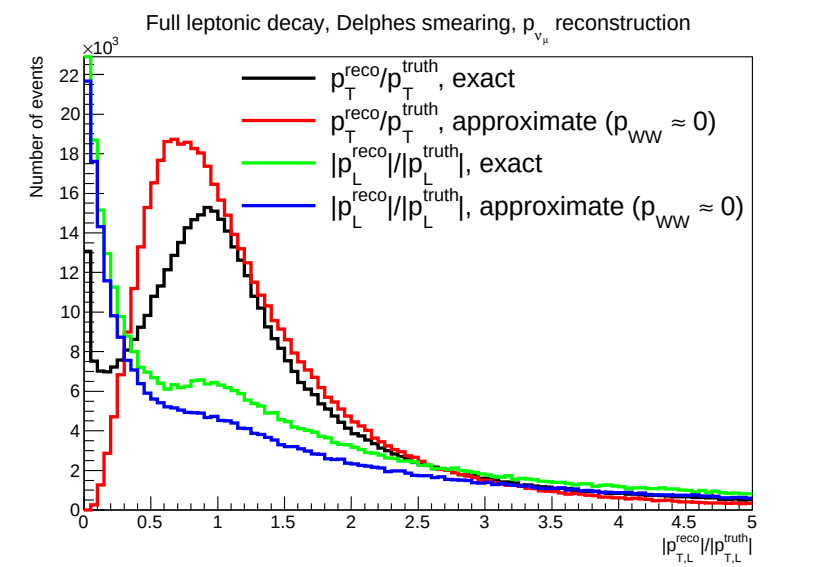
- relative longitudinal and transvers momentum reconstruction for ν_μ at parton level.



- relative longitudinal and transvers reconstruction for ν_e at parton level.

2. Reco with MAOS quantities from Delphes - Results



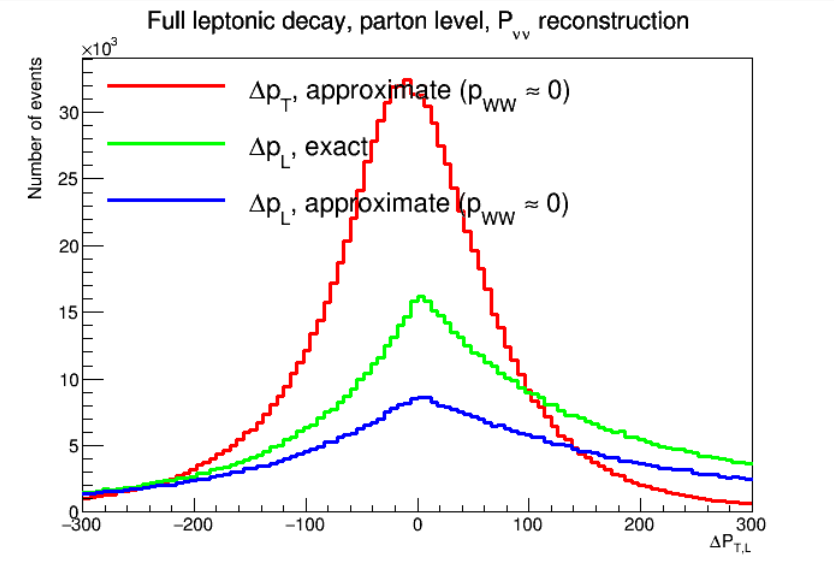


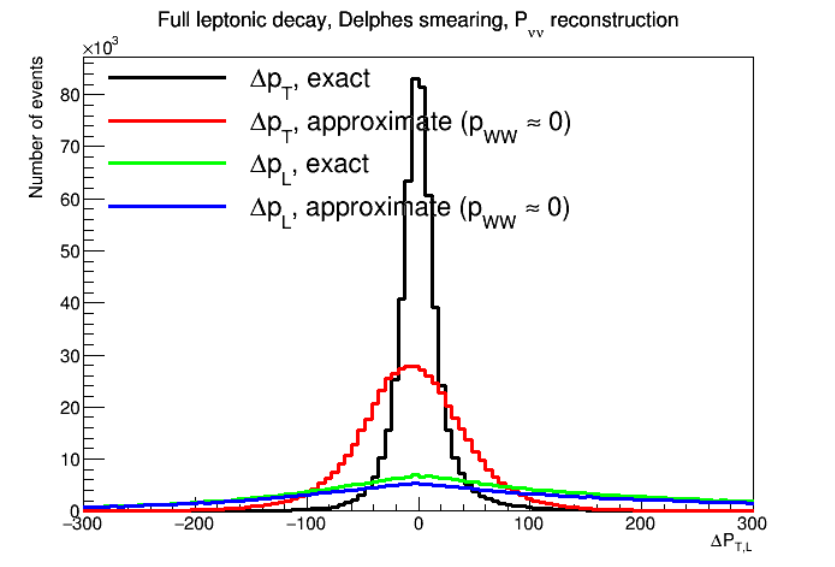
- relative longitudinal and transvers momentum reconstruction for ν_μ after detector interaction. [5]

- relative longitudinal and transvers reconstruction for ν_e after detector interaction.

[5] We used standard `Delphes` ATLAS card

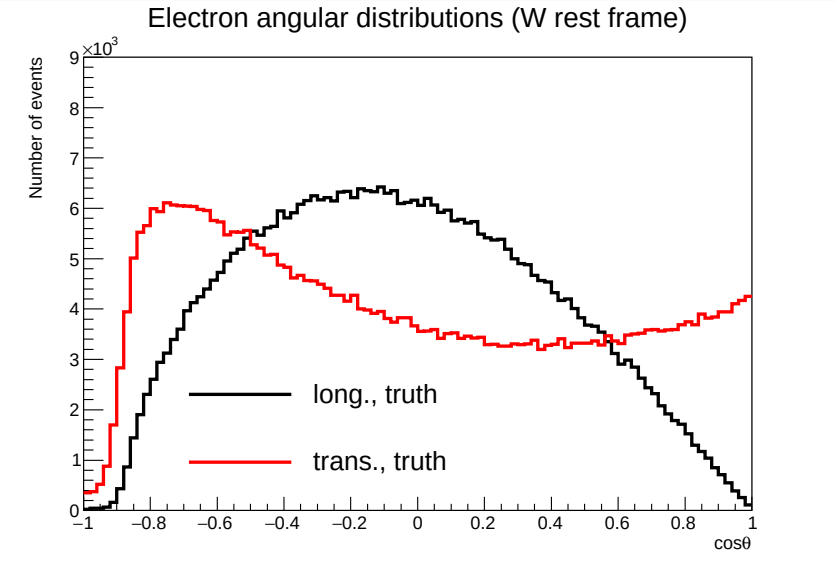
2. Reco with MAOS quantities Truth Vs Delphes - Results



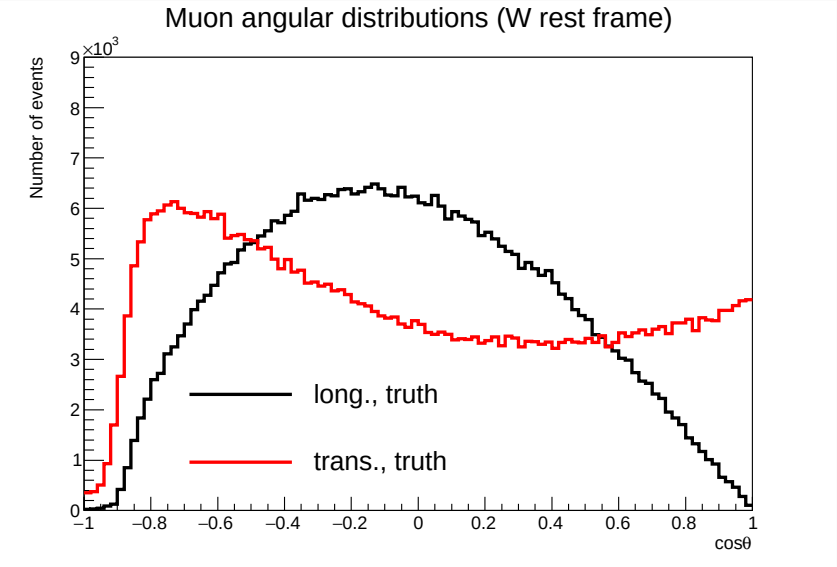


- reconstruction of longitudinal and transvers momentum at parton level of $P_{\nu\nu}$.
- reconstruction of longitudinal and transvers momentum after detector smearing of $P_{\nu\nu}$.

2. Reco evolution of MAOS quantities - Truth Level

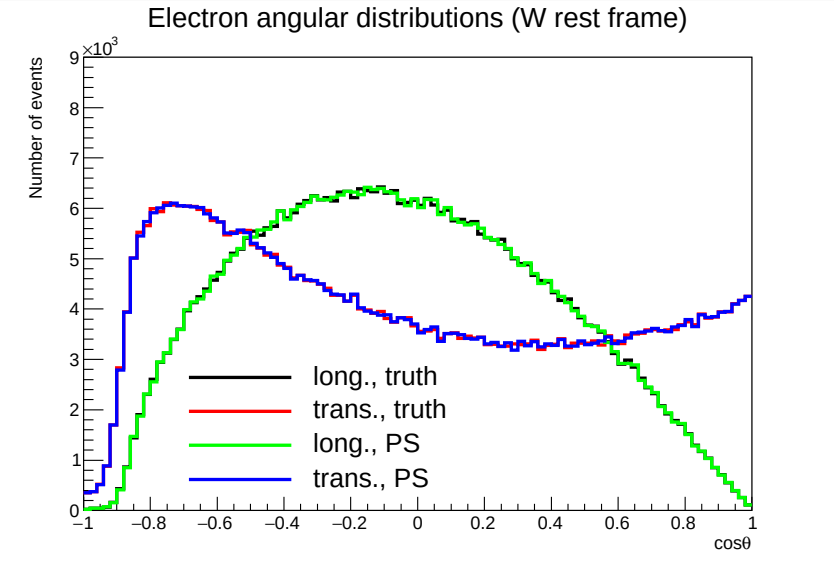


- $cos\theta$ distribution of longitudinal and transverse component of the $electron$ at parton level.

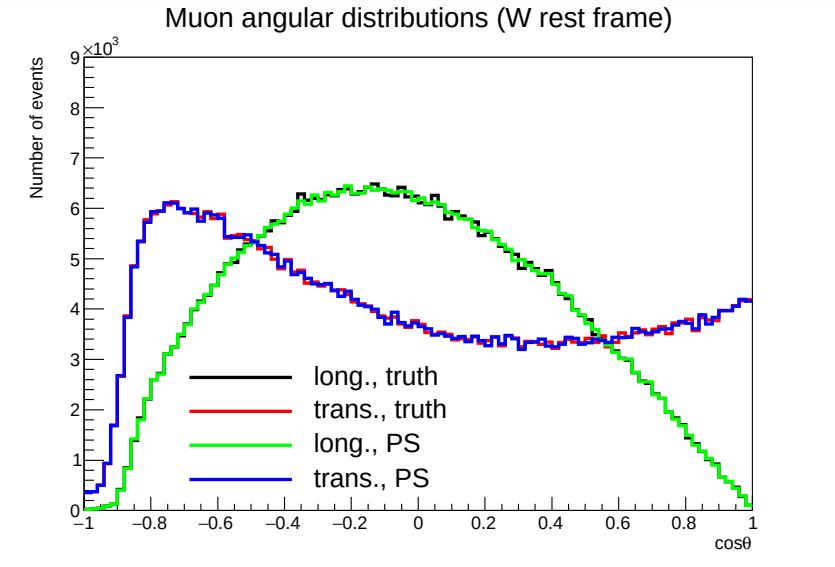


- $cos\theta$ distribution of longitudinal and transverse component of the μ at parton level.

2. Reco evolution of MAOS quantities - After Parton Shower

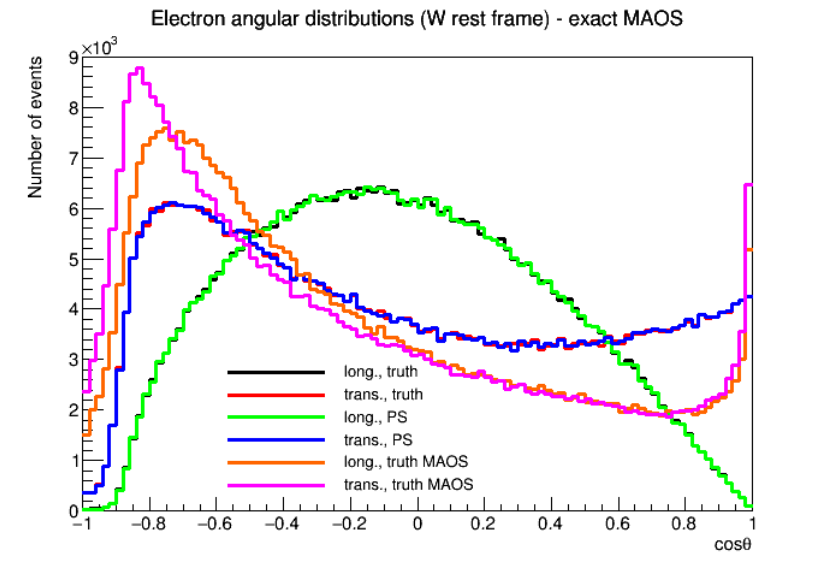


- $cos\theta$ distribution of longitudinal and transverse component of the $electron$ after parton shower.

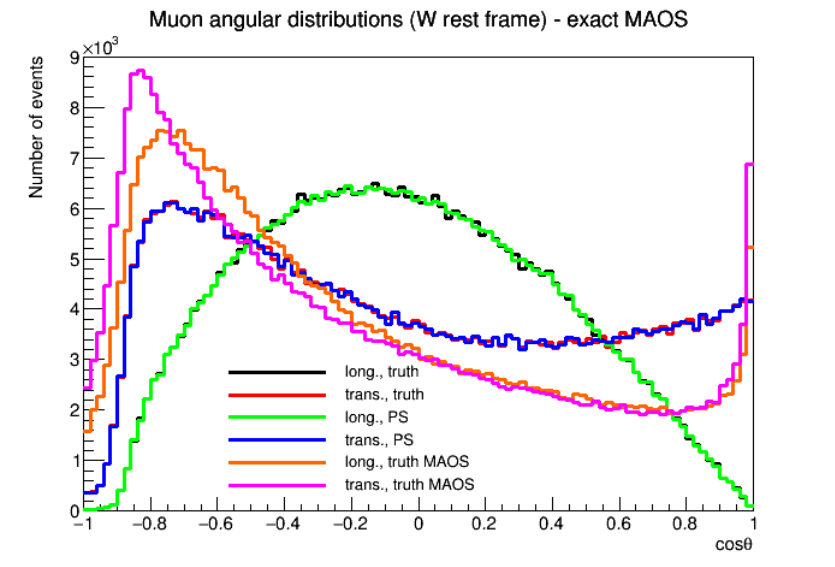


- $cos\theta$ distribution of longitudinal and transverse component of the μ after parton shower.

2. Reco evolution of MAOS quantities - Truth Level

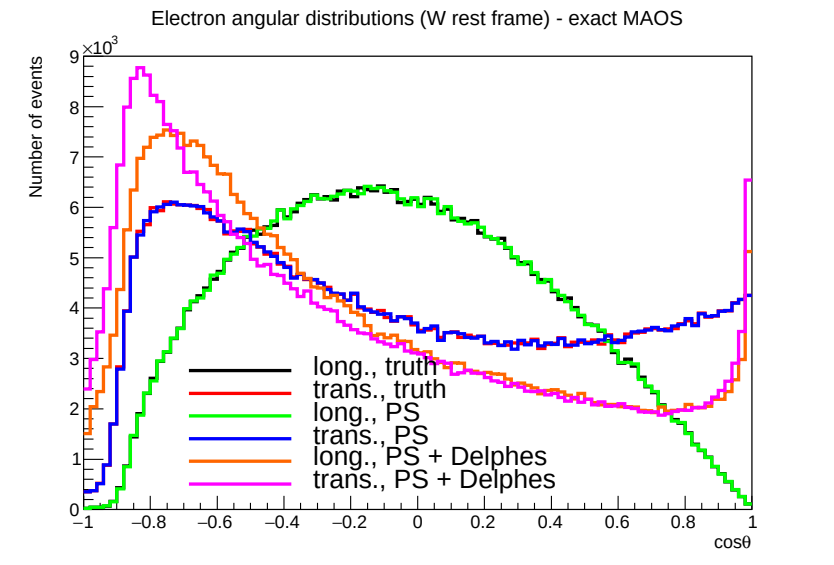


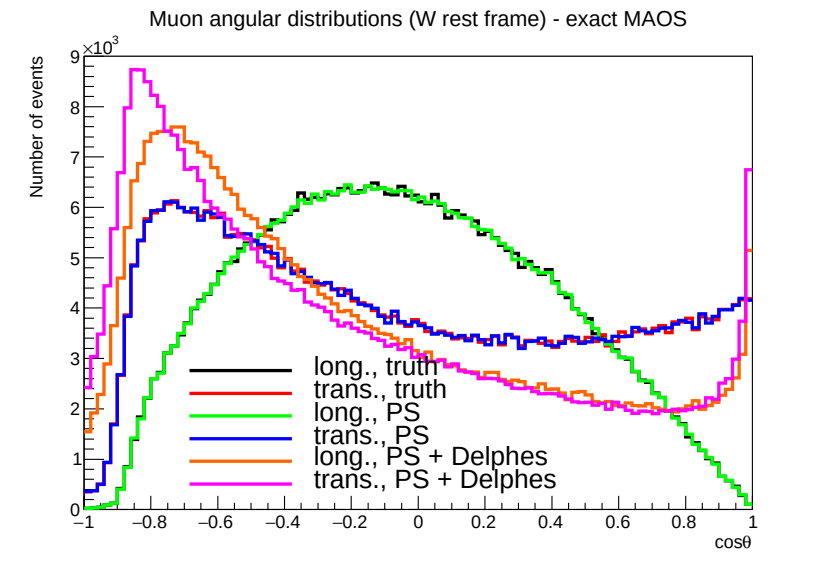
- $cos\theta$ distribution of longitudinal and transverse component of the $electron$ using $MAOS$ algorithm.



- $cos\theta$ distribution of longitudinal and transverse component of the μ using $MAOS$ algorithm.

2. Reco evolution of MAOS quantities - Detector Smearing

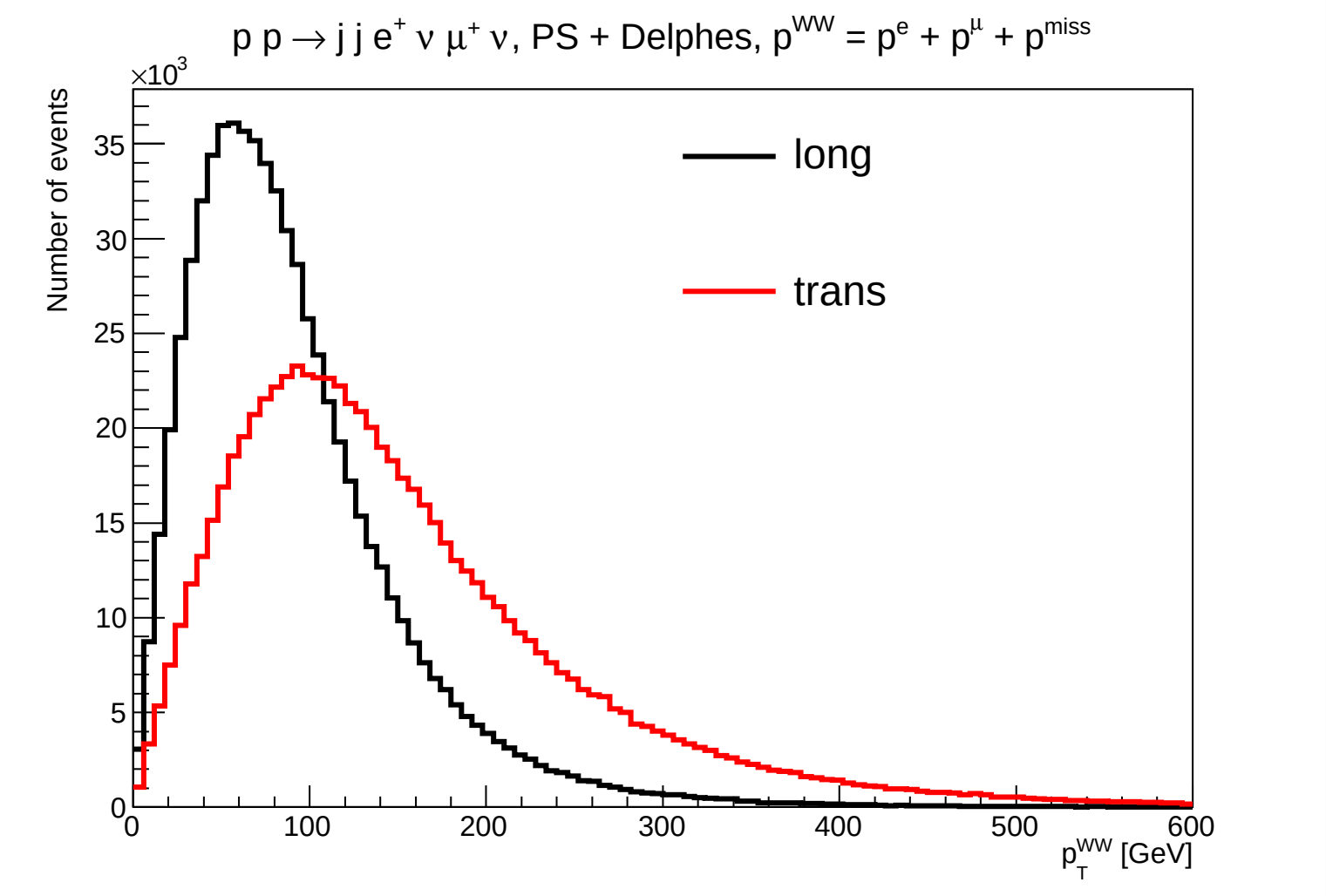




- $cos\theta$ distribution of longitudinal and transverse component of the $electron$ after detector simulation.

- $cos\theta$ distribution of longitudinal and transverse component of the μ after detector simulation.

VBS Fully Leptonic W boson reconstruction



Conclusions

CONCLUSIONS: WW reconstruction

- In semi-leptonic channel the reconstruction of the W reference frame can be obtained up to a sign ambiguity
- In semi-leptonic channel the use of selection criteria represents the most promise way to select correct solution
- In full-leptonic the reconstruction is worse wrt semi-leptonic case
- Examination of leptons angular distribution gives no differences in polarization (transverse and longitudinal)
- This method could be improved studying different selection criteria for longitudinal solution
- According to current scenario W polarization separation is not feasible in W reference frame and we should find a variable (combination) in lab system, i.e. p_T t of WW
- Next: Improve study on `Delphes` smearing effect
- Next: Trying different algorithm in addition to MAOS

References

[1] arXiv:hep-ph/0603011

[2] arXiv:0801.3359 [hep-ph]

[3] The DELPHES 3 collaboration -JHEP

[4] arXiv:1710.09339

[5] arXiv:1205.2484

[6] arXiv:hep-ph/9406381

[7] arXiv:hep-ph/9406381

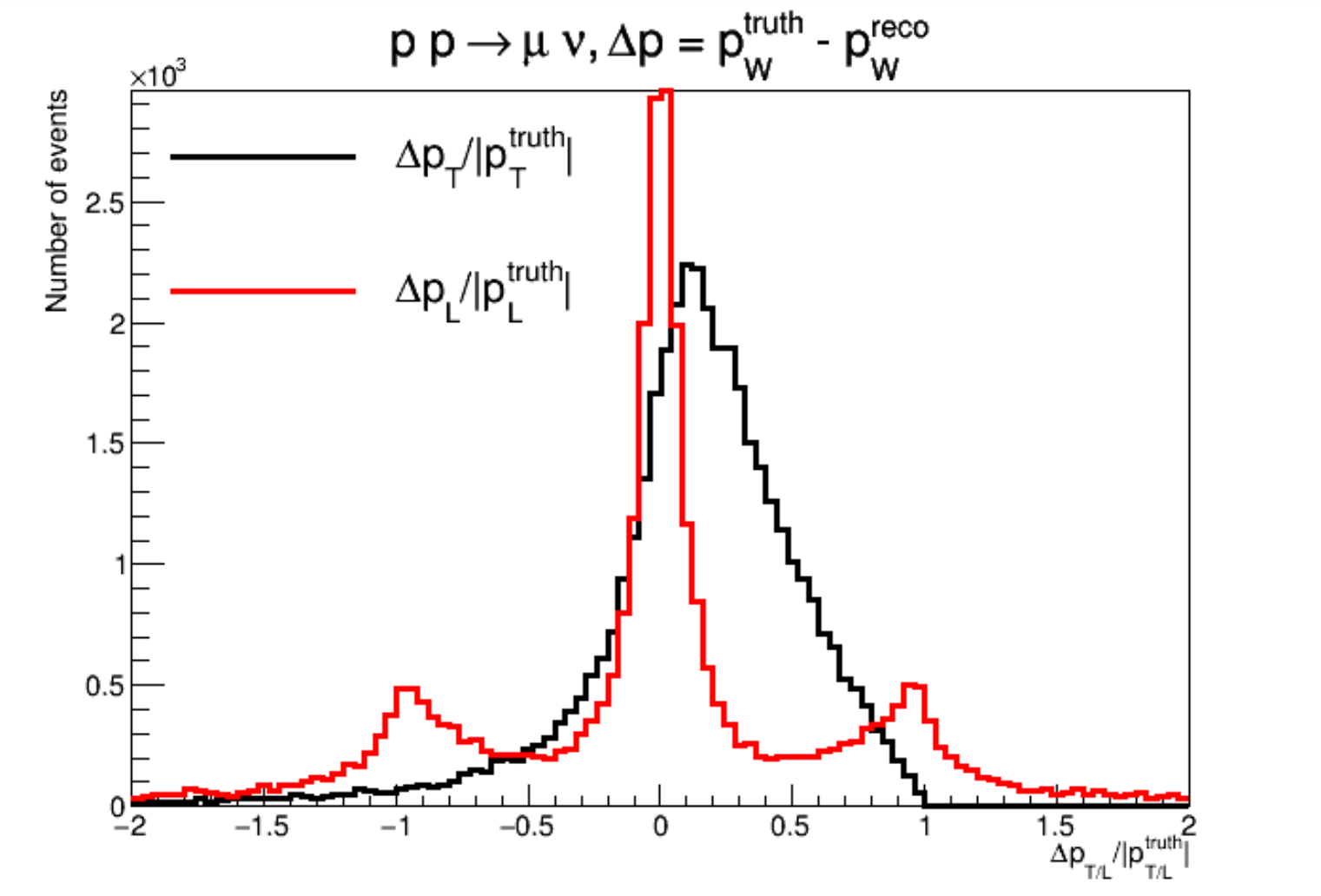
[8] arXiv:0908.0079

[9] B. Hoonhout, K. Oussoren, S. Bentvelse: *Higgs spin analysis in Collins-Soper frame using opening angles of different-flavour final state leptons* (link)

Backup

W distribution in $pp \to \mu^+ \nu_\mu$

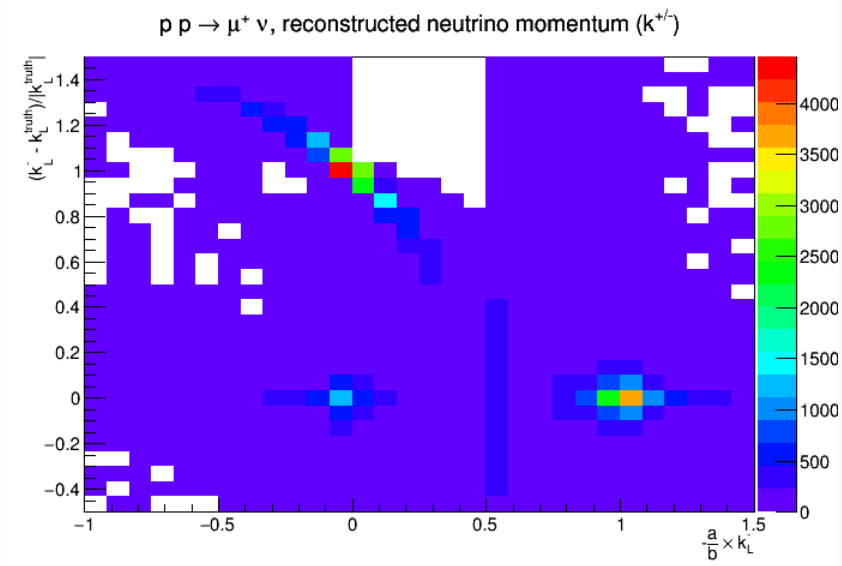
Reconstruction of W according to different criteria at parton level, chosing different $p\nu_L$ solution

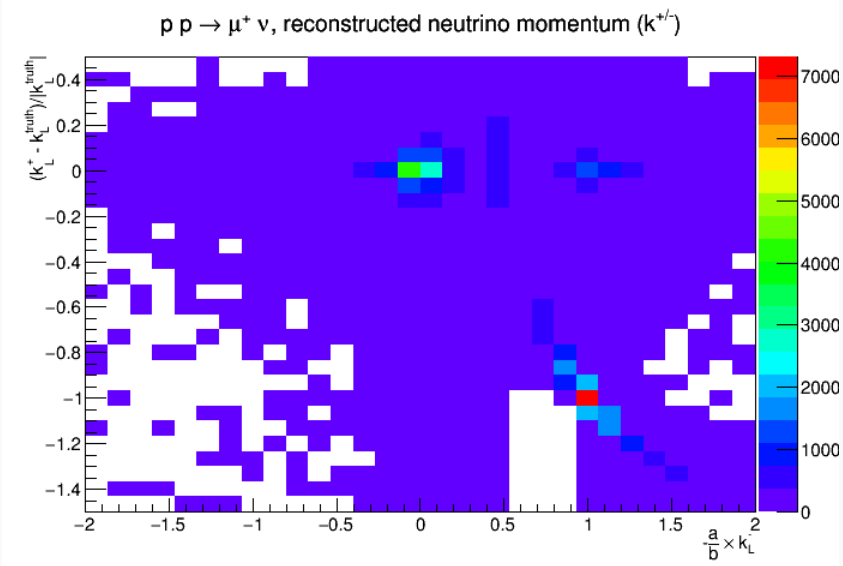


Criteria Selection in $pp \to \mu^+\nu_\mu$

Different criteria to choose correct sign of the neutrino longitudinal component:

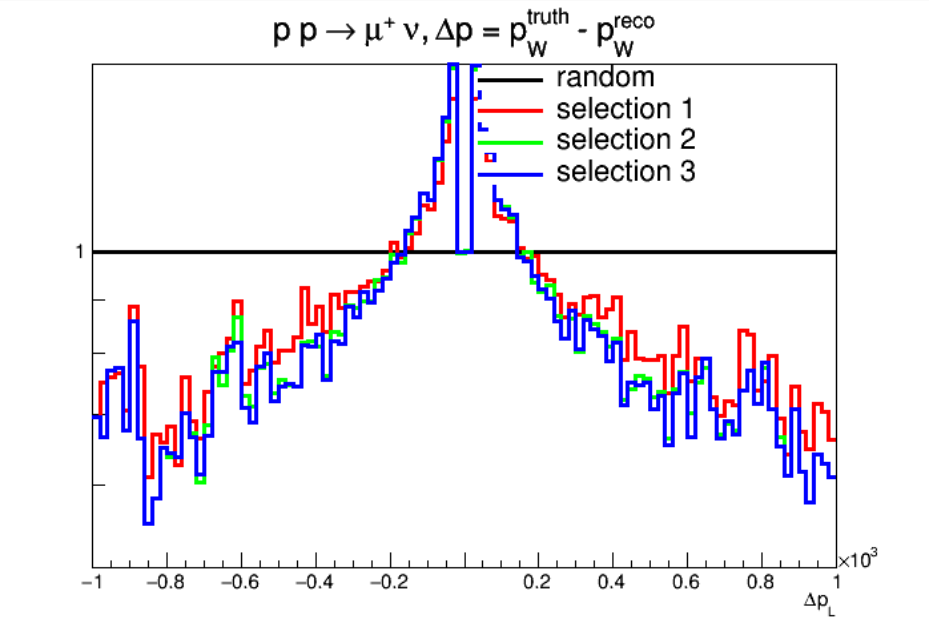
- **selection 1** discards solutions which have absolute value smaller than 50 GeV. If both solutions lie under or above 50 GeV, random solution is chosen.
- **selection 2** algorithm discards all the solutions K_L for which $K_L * a/b > 0.5$ or $K_L * a/b < 1.5$ (a, b, parameters of quadratic equation). If both solutions pass or don't pass this criterion, random solution is chosen.

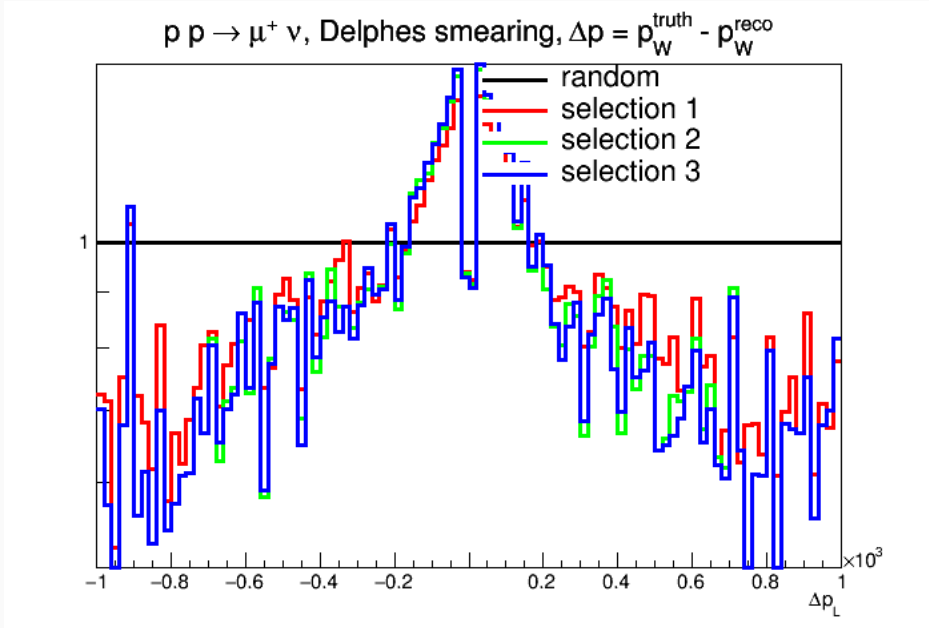




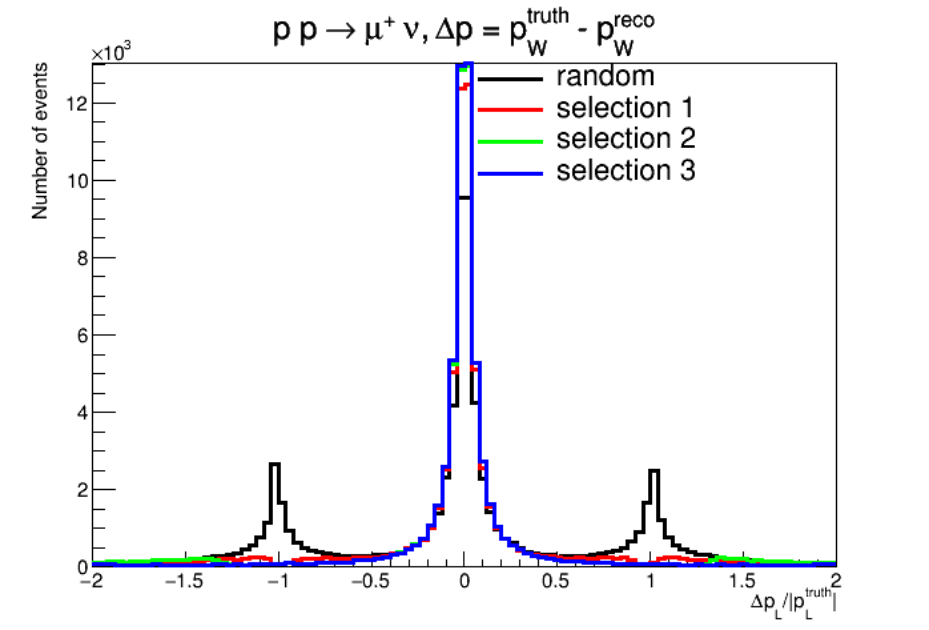
Criteria Selection in $pp \to \mu^+\nu_\mu$

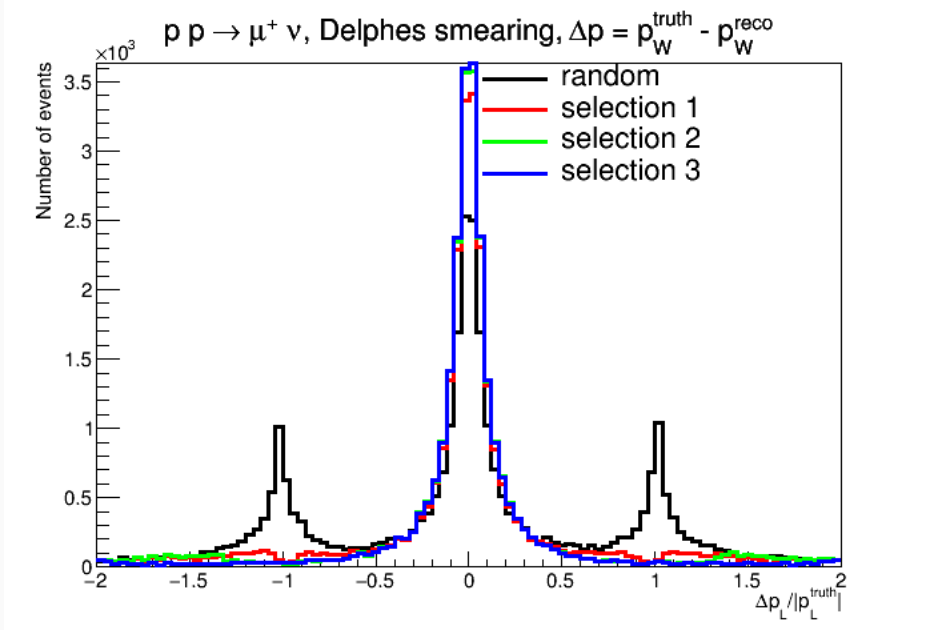
- **selection 3** algorithm discards all the solutions for which $k_L * a/b > 0.5$ or $K_L a/b < 1.5$. If this is the case for both solutions, the solution which $K_L * a/b$ is closer to one is chosen. If no solution violates described criterium random solution is chosen.





Criteria Selection in $pp \to \mu^+\nu_\mu$

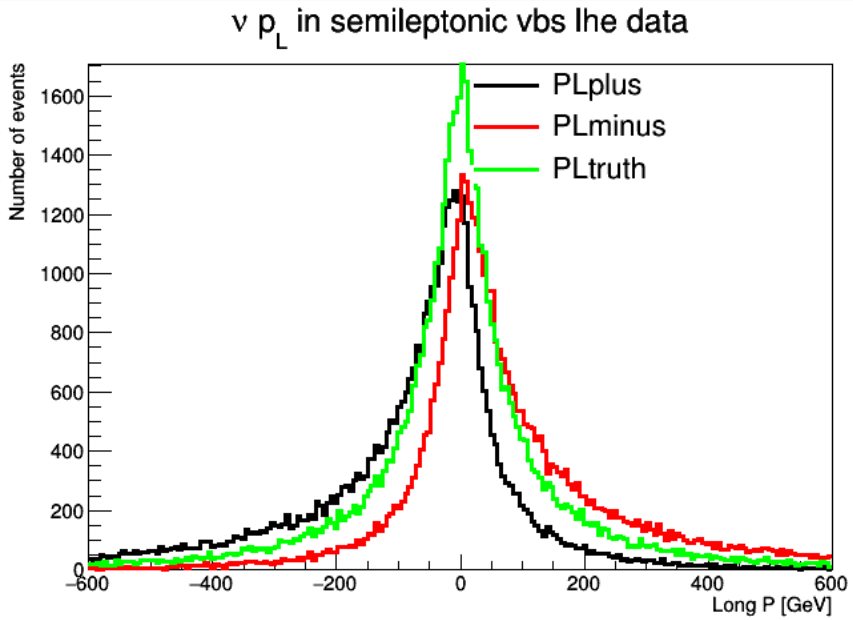


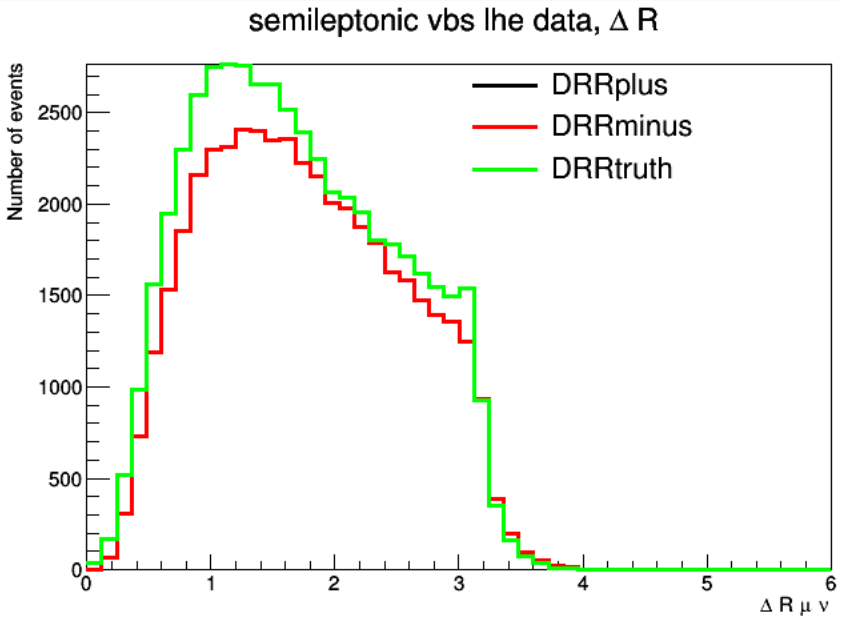


Collection of selection criteria as relative to the truth longitudinal component.
On the left at parton level, on the right after detector simulation (`Delphes`)
$\to$ **Selection 3 seems the best choice**

VBS Semileptonic - Parton Level

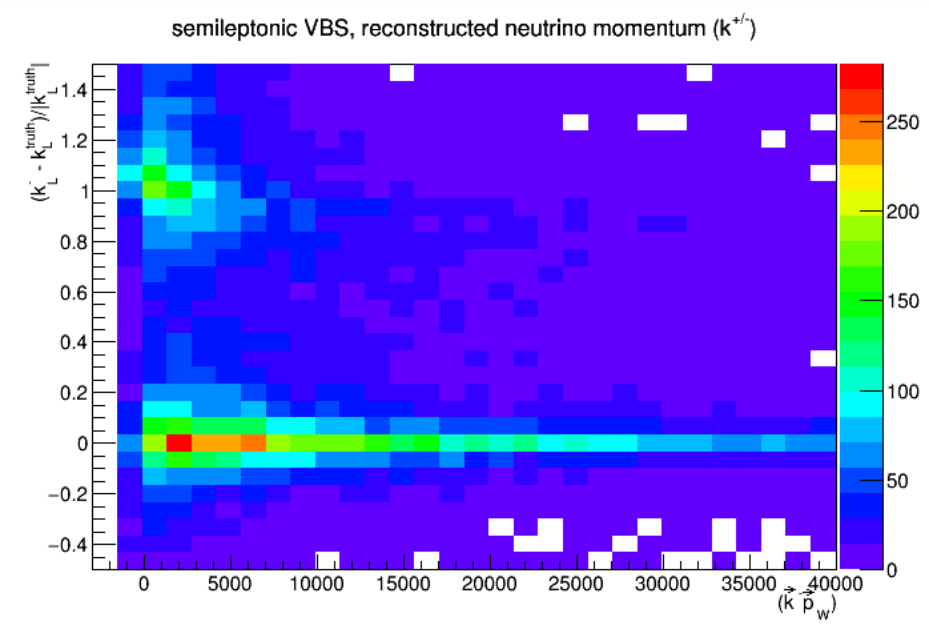
Plot of $p_{\nu L}$ and of ΔR from truth data according to different solution





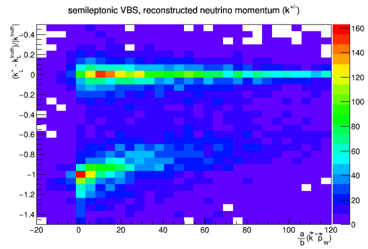
Criteria Selection at Parton Level

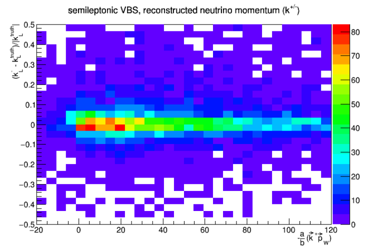
Beyond *plus* or *minus* solution we can find different criteria for selecting the correct sign in calculation of the ν_L in this channel.

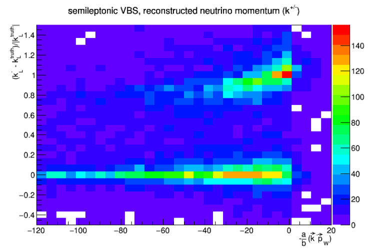


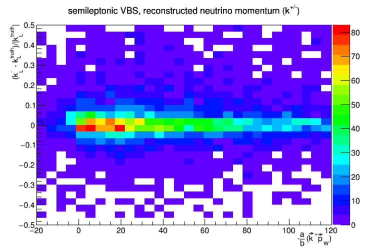
- **selection 1** if the scalar product of the reconstructed neutrino three-momentum (solution is taken for the longitudinal component) with the reconstructed W three-momentum is smaller than 5000 GeV^2, the solution is discarded. If both solutions lie under or above 5000 GeV^2, random solution is chosen.

Criteria Selection at Parton Level







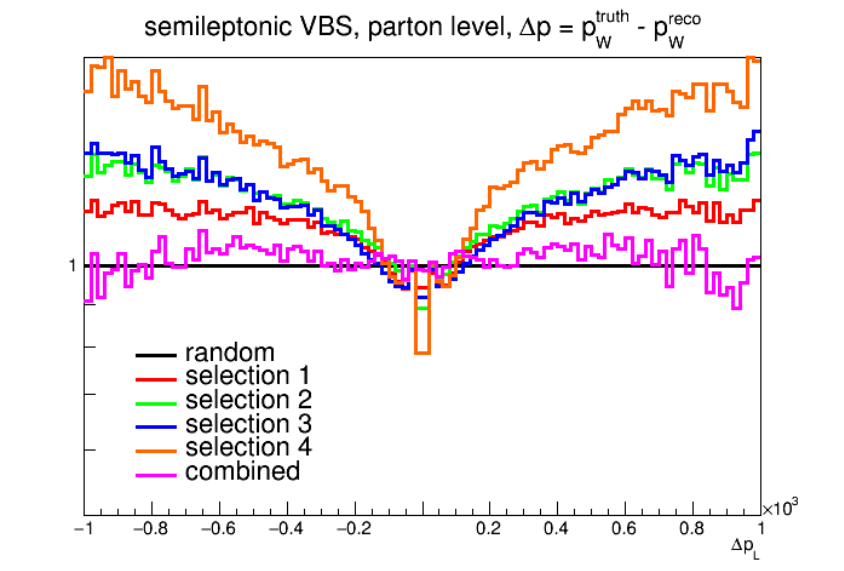


Criteria Selection at Parton Level

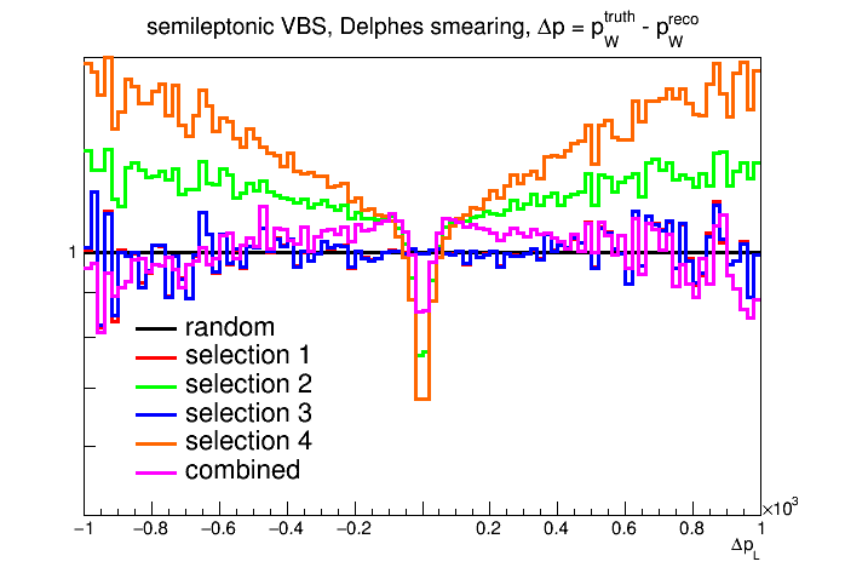
- **selection 3** absolute value of the scalar product of the reconstructed neutrino three-momentum with the reconstructed W three-momentum, multiplied by a/b, is smaller than 25 GeV, the solution is discarded.
- **selection 4** algorithm discards all the solutions for which $-sol * a/b < 0.5$. If both solutions pass or don't pass this criterium, random solution is chosen
- **combined** algorithm discards the solutions which have absolute value smaller than 50 GeV. If both solutions lie under 50 GeV, selction 3 is applied. If both solutions lie above 50 GeV, solution with lower absolute value of the scalar product of the reconstructed neutrino three-momentum with the reconstructed W three-momentum, multiblied by a/b, is taken. In this plot the events which have both solution above 50 GeV are shown.

Criteria Application Outcome

Reconstruction of W momentum at parton level according to different criteria (log scale).



Reconstruction of W momenta after detector interaction according to different criteria (log scale).



Best criteria: Combined

2. $cos\theta$ distribution for approximative and exact solution