# Jet fragmentation in a dense QCD medium

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# Introduction

- Jets are very important probes of the quark-gluon plasma (QGP) produced in heavy-ions collisions at LHC or RHIC.
- Understanding observables such that the jet suppression or the jet fragmentation function will help to better characterize the QGP.
- From a theoretical point of view, a complete picture of the evolution of a jet in a dense medium is still lacking.

# Motivations and goal of the talk

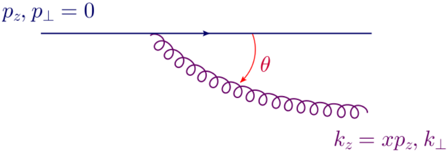
- Jet evolution in a dense medium : medium induced emissions versus vacuum-like emissions. How can we include both mechanisms ?
- Our solution is to work with the simplest possible approximation in parton shower : the **leading double-logarithm** approximation (DLA).

# Where does the double-logarithmic phase space come from ?

## Bremsstrahlung law...

Bremsstrahlung spectrum $\Longrightarrow$ energy and angle logarithms.

Formation time due to the virtuality of the parent parton :
$t_{vac} \sim \omega/k_\perp^2 \sim 1/(\omega\theta^2)$.




$$\mathrm{d}\mathcal{P} \simeq \frac{\alpha_s C_R}{\pi}\,\frac{\mathrm{d}x}{x}\,\frac{\mathrm{d}\theta^2}{\theta^2}$$



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# Where does the double-logarithmic phase space come from ?

... vs medium induced radiations

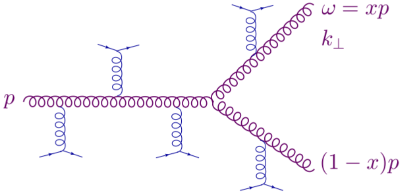




BDMPS-Z spectrum (Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov 1996–97)

NOT DOUBLE LOG !

Medium-induced formation time and broadening characteristic time scale : $t_f \sim \sqrt{\omega/\hat{q}}$ from $\langle k_\perp^2 \rangle = \hat{q}\Delta t$.




$$\mathrm{d}\mathcal{P} \simeq \bar{\alpha}\,\frac{\mathrm{d}\omega}{\omega}\,\frac{L}{t_{\mathrm{f}}(\omega)} \simeq \bar{\alpha}\,L\sqrt{\frac{\hat{q}}{\omega^3}}\,\mathrm{d}\omega$$

# Vacuum-like emission inside the medium

If $t_{vac} \ll t_f$ : emission triggered by the virtuality and not yet affected by the momentum broadening.

$\Longrightarrow$ **double-logarithmic enhancement of the probability**.

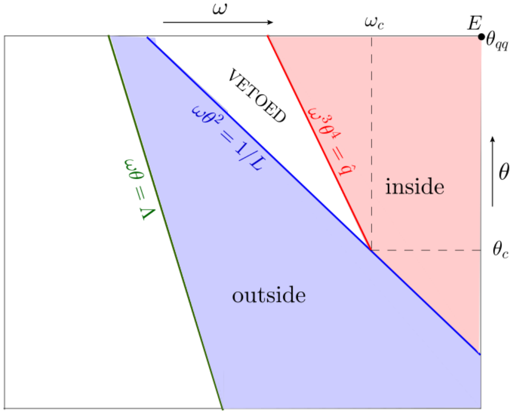
Equivalent condition

$\omega \gg (\hat{q}/\theta^4)^{1/3} \equiv \omega_0(\theta)$

# Vacuum-like emission outside the medium

- $t_{vac} \geq L \Longrightarrow$ vacuum-like emission outside the medium triggered by the virtuality of the parent parton.
- In terms of energy : $\omega \leq 1/(L\theta^2)$.

# Summary : double logarithmic phase space with a QGP




**The energy scale $\omega_c$**

The condition $t_f = L$ defines the energy scale $\omega_c = 1/2\hat{q}L^2$. Gluons with energy greater than $\omega_c$ are always vacuum like.

# How to resum these double logarithms in the medium ?

Iteration of vacuum-like emissions

### Large $N_c$ limit

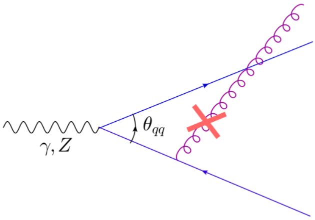
Emission of a soft gluon by an antenna $\Leftrightarrow$ splitting of the parent antenna into two daughter antennae.

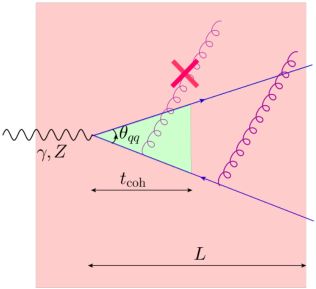
### Decoherence time

- Reminder : color coherence is responsible for **angular ordering** in vacuum cascades
- In the medium, an antenna loses its color coherence after a time $t_{coh} = 1/(\hat{q}\theta^2_{q\bar{q}})^{1/3}$.

(Mahtar-Tani, Salgado, Tywoniuk, 2010-11 ; Casalderrey-Solana, Iancu, 2011)

# Coherence in vacuum vs (de)coherence in the medium




## The angular scale $\theta_c$

The condition $t_{coh} = L$ gives the definition of the critical angle $\theta_c = 2/\sqrt{\hat{q}L^3}$. Antennae with angles greater than $\theta_c$ always lose their coherence propagating over a distance $L$.

# How to resum these double logarithms in the medium ?

In the leading double-logarithmic approximation, **successive in-medium vacuum-like emissions form angular-ordered cascades**.

## Proof

- First case : $t_{vac}(\omega_i, \theta_i^2) \leq t_{coh}(\omega_{i-1}, \theta_{i-1}^2)$, the parent antenna did not lose its coherence during the time required by the next antenna to be formed $\Rightarrow$ $\theta_i^2 \ll \theta_{i-1}^2$.
- Second case : $t_{vac}(\omega_i, \theta_i^2) \geq t_{coh}(\omega_{i-1}, \theta_{i-1}^2) \Rightarrow$ $t_{vac}(\omega_i, \theta_i^2) \geq t_f(\omega_i, \theta_i^2)$ or $\theta_i^2 \leq \theta_{i-1}^2 \Rightarrow \theta_i^2 \leq \theta_{i-1}^2$

# Consequences on the emissions outside the medium

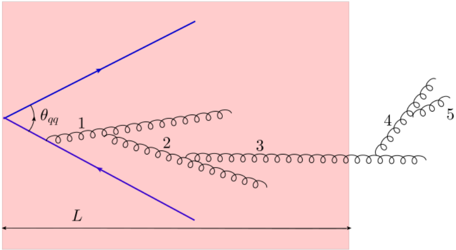
- The precedent proof does **not** apply if the antenna $i-1$ is the **last inside the medium**.
- In that case, the formation time of the next antenna is larger than $L$.

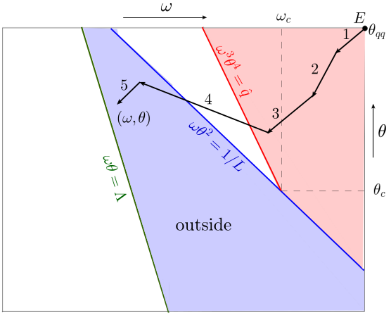
## Last emission inside the medium

- If $\theta_{i-1}^2 \leq \theta_c^2$ : the decoherence time is also larger than $L$ $\Rightarrow$ angular ordering is preserved.
- If $\theta_{i-1}^2 \geq \theta_c^2$ : the antenna has lost its coherence during the formation time of the next antenna $\Rightarrow$ **no constraint** on the angle of the next antenna.

(Y. Mehtar-Tani, K. Tywoniuk, Physics Letters B 744, 2015)

# Parton shower in a QGP



# Analytical study of jets at DLA

## Double differential gluon distribution

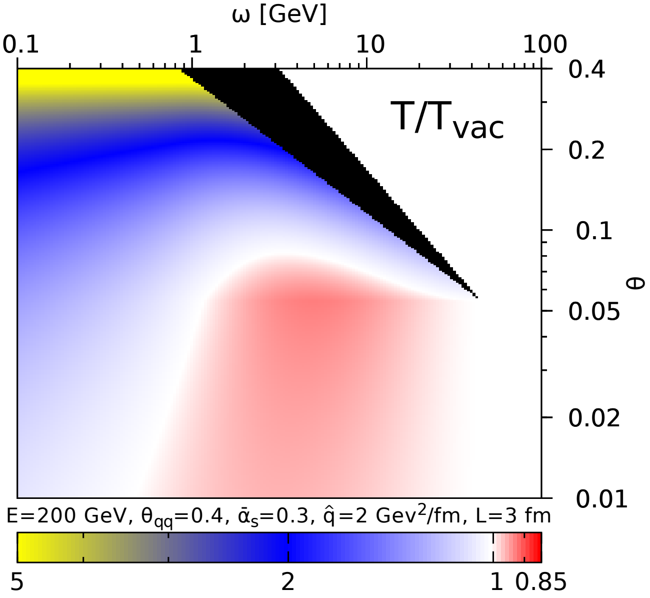
$T(\omega, \theta^2 \mid E, \theta^2_{q\bar{q}}) \equiv \omega\theta^2 \frac{d^2N}{d\omega d\theta^2}$
$\Rightarrow$ probability of emission of a gluon with energy $\omega$ and angle $\theta^2$ from an antenna with energy $E$ and opening angle $\theta^2_{q\bar{q}}$.

In the vacuum at DLA, this quantity satisfies the simple master equation

$$T_{vac}(\omega, \theta^2 \mid E, \theta^2_{q\bar{q}}) = \bar{\alpha}_s + \int_{\theta^2}^{\theta^2_{q\bar{q}}} \frac{d\theta_1^2}{\theta_1^2} \int_{\omega/E}^{1} \frac{dz_1}{z_1} \bar{\alpha}_s T_{vac}(\omega, \theta^2 \mid z_1 E, \theta_1^2)$$

With a medium, this equation holds only inside the medium $\Rightarrow$ mathematically, one must take into account "jumps" over the vetoed region.

# Numerical results : ratio $T(\omega, \theta^2)/T_{vac}(\omega, \theta^2)$



# Fragmentation function with fixed-coupling

## Definition

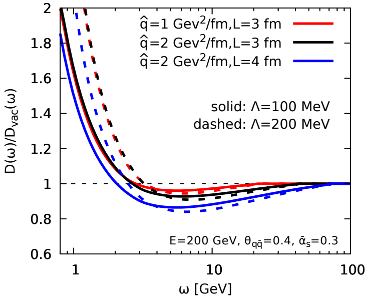
Integral over angle between the $k_\perp$ cut-off and $\theta_{q\bar{q}}$

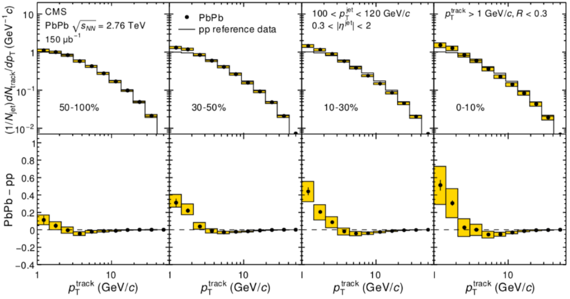
$\Rightarrow D(\omega) \equiv \omega \frac{dN}{d\omega} = \int_{\Lambda^2/\omega^2}^{\theta^2_{q\bar{q}}} \frac{d\theta^2}{\theta^2} T(\omega, \theta^2)$

## Remarks

- Formula reliable only for $\omega \ll E$ at DLA.
- Different from the fragmentation function given by experimentalists represented as a function of the ratio $\omega/E$ where $E$ is the total energy of the jet. Here, "our" $E$ is an unobservable parameter since in practice, the jet loses energy via medium-induced radiations.

# Numerical results for the fragmentation function



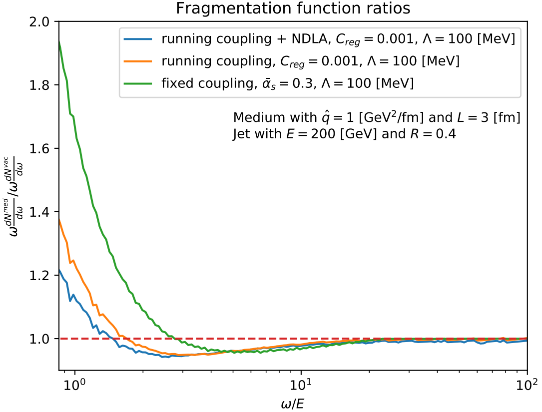




(CMS collaboration, Phys. Rev. C 90, 2014)

# Results beyond DLA

## Preliminary results

- Running coupling + DLA : $\bar{\alpha}_s P_{gg}(z) \rightarrow \bar{\alpha}_s(k_\perp^2)\frac{1}{z}$.
- Running coupling + NDLA : $\bar{\alpha}_s P_{gg}(z) \rightarrow \bar{\alpha}_s(k_\perp^2)\frac{1}{z}\left(1 - \frac{11}{12}z\right)$.



# Conclusion

## In perspective

- Estimate the energy loss by a jet at next-to-double-log accuracy.
- Monte-Carlo simulation : build an event generator which will include the full splitting functions (hence, energy conservation) for the vacuum-like cascades and the medium-induced cascades.

Thank you for listening !

# What about the energy loss ?

Energy loss is **negligible** for any parton of the cascade inside the medium (except for the last one)

- $\omega_{loss} \sim \hat{q}t^2$ energy of the hardest medium induced emission that can develop during $t$.
- By the inequality $t_{vac}(\omega_i, \theta_i^2) \ll t_f(\omega_i, \theta_i^2)$, one finds that $\omega_{loss} \ll \omega_i$.

However...

- Energy loss is not negligible for the last antenna inside the medium since it will cross the medium along a distance of order $L$.
- Medium induced gluon cascades are important for large angle radiations.

# Mathematical interlude : calculation of $\omega\theta^2 \frac{d^2N}{d\omega d\theta^2}$

The starting point is the basic formula for the multiplicity in the vacuum

$$\omega\theta^2 \frac{d^2N}{d\omega d\theta^2} \equiv T_{vac}(\omega, \theta^2 \mid E, \theta^2_{q\bar{q}}) = \bar{\alpha}_s I_0\left(2\sqrt{\bar{\alpha}_s \log(E/\omega) \log(\theta^2_{q\bar{q}}/\theta^2)}\right)$$

Then, crossing the vetoed region and violating the angular ordering is implemented by a convolution in both energy/angle of the last gluon inside the medium and the first gluon outside the medium.

**Cascade inside the medium + cascade outside**

$$T(\omega, \theta^2) = \bar{\alpha}_s \int_{\theta_c^2}^{\theta^2_{q\bar{q}}} \frac{d\theta_1^2}{\theta_1^2} \int_{\omega_0(\theta_1^2)}^{E} \frac{d\omega_1}{\omega_1} T_{vac}(\omega_1, \theta_1^2 \mid E, \theta^2_{q\bar{q}})$$
$$\int_{\theta^2}^{\min(\theta^2_{q\bar{q}}, \theta_L^2(\omega))} \frac{d\theta_2^2}{\theta_2^2} \int_{\omega}^{\min(\omega_1, \omega_L(\theta_2^2))} \frac{d\omega_2}{\omega_2} T_{vac}(\omega_2, \theta_2^2 \mid \omega, \theta^2)$$

# Sketch of the mathematical formalism of QCD with medium

- The quark-gluon plasma is described in its rest frame by a static density of color charges, following a gaussian distribution. The resolution of the Yang-Mills equation in light-cone coordinates $x^{\pm} = (x^0 \pm x^3)/\sqrt{2}$ and light-cone gauge gives the statistical distribution of the gauge field associated $\mathcal{A}_a^-$.

**Correlation functions**

$\langle \mathcal{A}_a^-(x^+, x_\perp)\mathcal{A}_b^-(y^+, y_\perp)\rangle_m = g^2 n_0 \delta_{ab}\delta(x^+ - y^+)\gamma(x_\perp - y_\perp)$
with $\gamma(x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{\exp(ik_\perp x_\perp)}{(k_\perp^2 + m_D^2)^2}$

- The medium is assumed to be very dense, with **density** $n_0 \gg 1$. Every observable calculated from the generating functional with the external field $\mathcal{A}_a$ has to be calculated resuming every order of $g^2 n_0$.

# Momentum broadening 1/3

- Neglecting for now coherence effects between the two legs of the antenna, we want to know how a highly energetic particle propagates through a dense medium.
- Within the eikonal approximation, the resummation of Feynman diagrams is given by a Fourier transform of a Wilson line through the medium field $\mathcal{A}$

$$\mathcal{M}_{\beta\alpha}(k,p) = 4\pi\delta(k^+ - p^+)p^+ \int dx_\perp e^{ix_\perp(p_\perp - k_\perp)} W_{\beta\alpha}(x_\perp)$$

with

$$W_{\beta\alpha}(x_\perp) = \mathcal{P}\left[e^{ig\int_{-\infty}^{\infty} \mathcal{A}_a^-(x^+,x_\perp)t^a dx^+}\right]_{\beta\alpha}$$

$\Rightarrow$ **color rotation**

# Momentum broadening 2/3



The probability $\frac{d\mathcal{P}_{broad}(k_\perp|p_\perp)}{dk_\perp}$ of ending up with a quark with momentum $k_\perp$ due to momentum broadening knowing that its initial transverse momentum was $p_\perp$ is given by the modulus square of the matrix element $\mathcal{M}(k,p)$.

$$\frac{d\mathcal{P}_{broad}(k_\perp \mid p_\perp)}{dk_\perp} \propto \frac{1}{N_c} \int dk^+ \mathrm{Tr}\Big\langle \mid \mathcal{M}(k,p) \mid^2 \Big\rangle_m$$

One sees that this calculation involves the medium average

$$\mathrm{Tr}\Big\langle W(x_\perp) W^\dagger(y_\perp) \Big\rangle_m$$

# The dipole S-matrix $\mathrm{Tr}\langle W(x_\perp)W^\dagger(y_\perp)\rangle_m$

The external field $\mathcal{A}$ has an given extent $L$ in the $x^+$ direction, the "length" of the medium.
A first order calculation in $g^2 n_0$ gives

$$\mathrm{Tr}\Big\langle W(x_\perp)W^\dagger(y_\perp)\Big\rangle_m \simeq 1 - g^2 n_0 C_R L[\gamma(0)-\gamma(x_\perp - y_\perp)]$$

Resumming to all orders, the dipole total cross sections is

$$\mathrm{Tr}\Big\langle W(x_\perp)W^\dagger(y_\perp)\Big\rangle_m = e^{-g^2 n_0 C_R L[\gamma(0)-\gamma(x_\perp - y_\perp)]}$$

# Momentum broadening 3/3

- The parameter $\hat{q}$ : under the harmonic approximation $g^2 C_R(\gamma(0) - \gamma(r_\perp)) \simeq \frac{1}{2}\hat{q}r_\perp^2$

- Then $\frac{d\mathcal{P}_{broad}(k_\perp|p_\perp)}{dk_\perp} = \frac{1}{\pi\hat{q}L}\exp\left(-\frac{(k_\perp - p_\perp)^2}{\hat{q}L}\right)$

- **Physical interpretation** given by the average transverse momentum squared acquired by collisions with the medium during a time $\Delta t$.

$$\langle k_\perp^2 \rangle = \hat{q}\Delta t$$