The Initial Correlations of the Glasma Energy-Momentum Tensor

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Rencontres QGP
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Étretat

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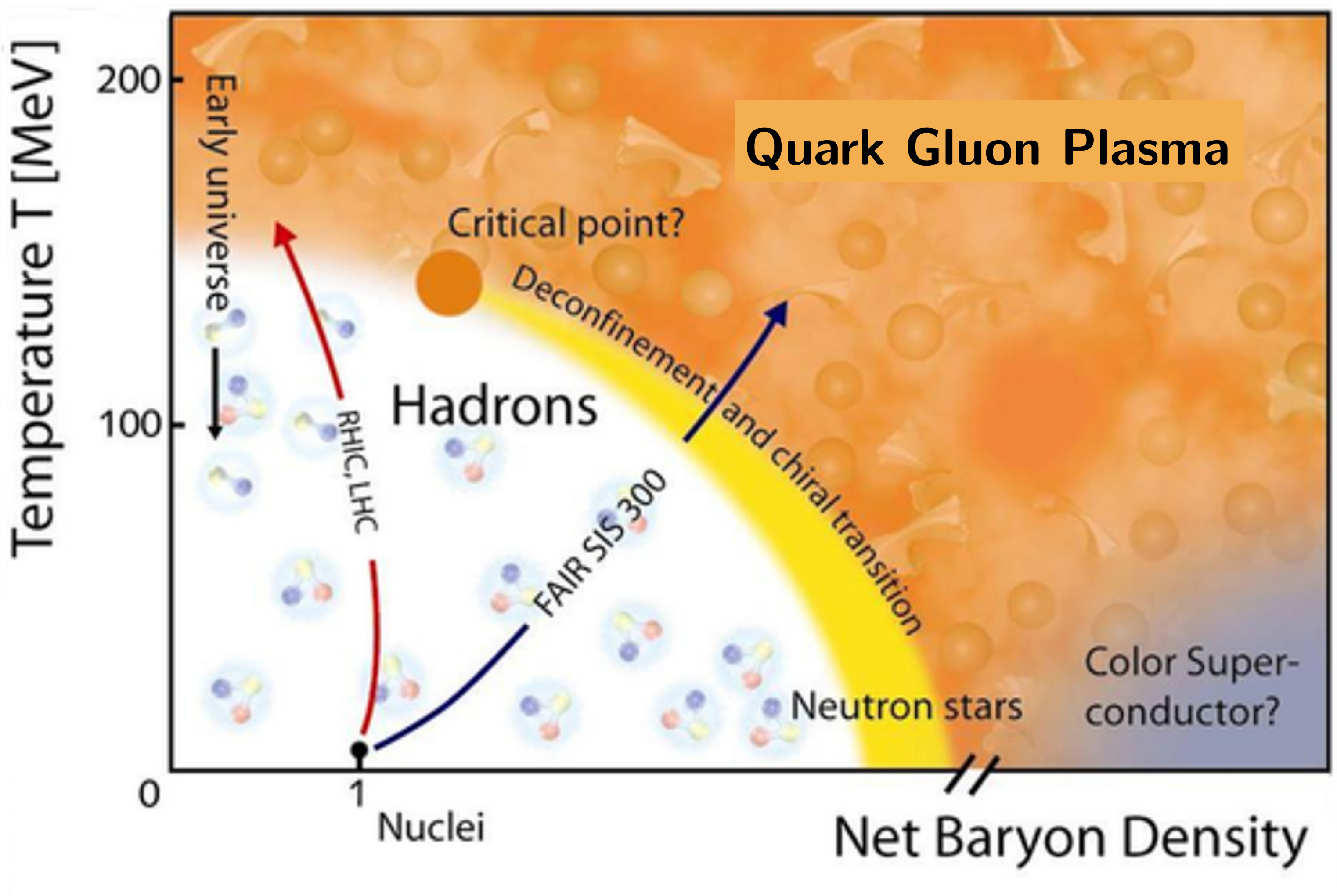
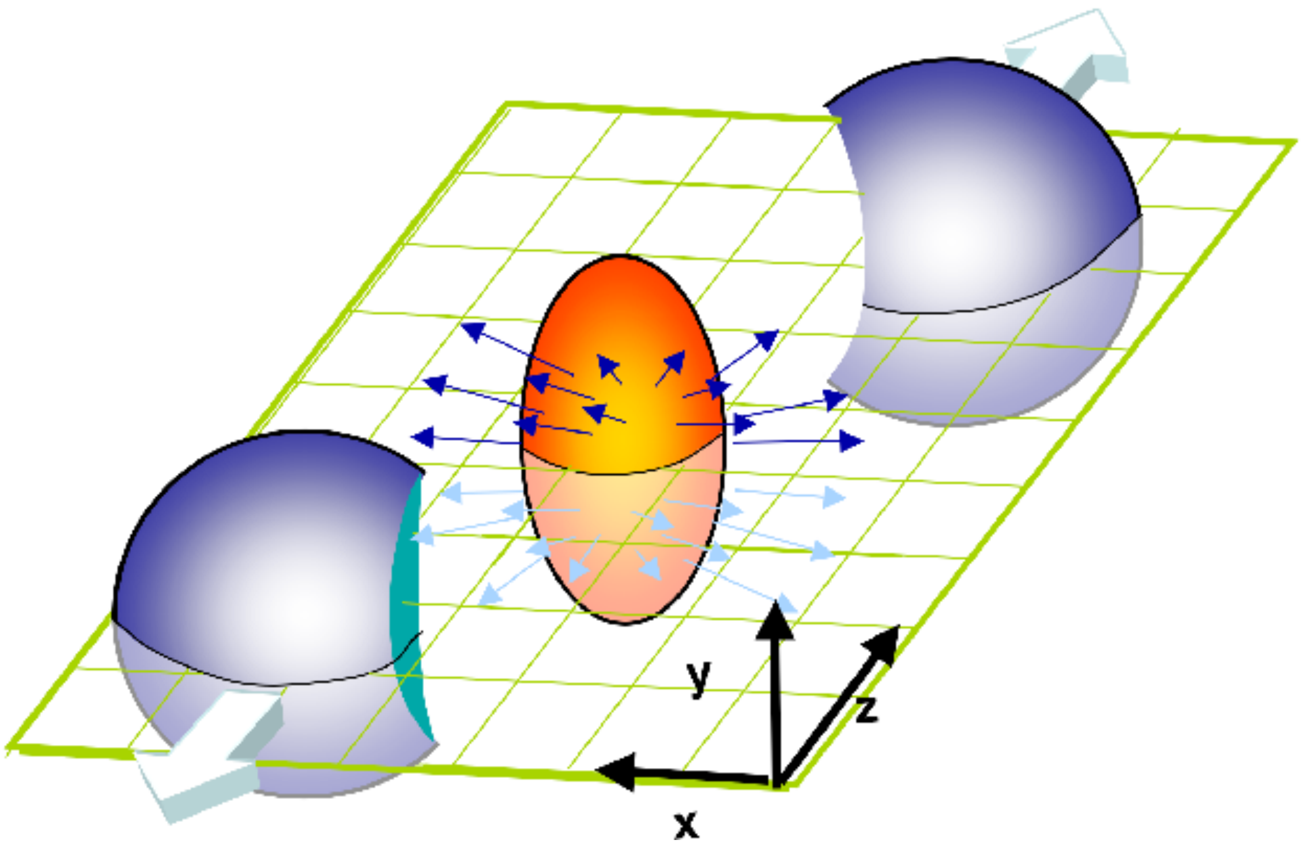
Motivation and technical framework

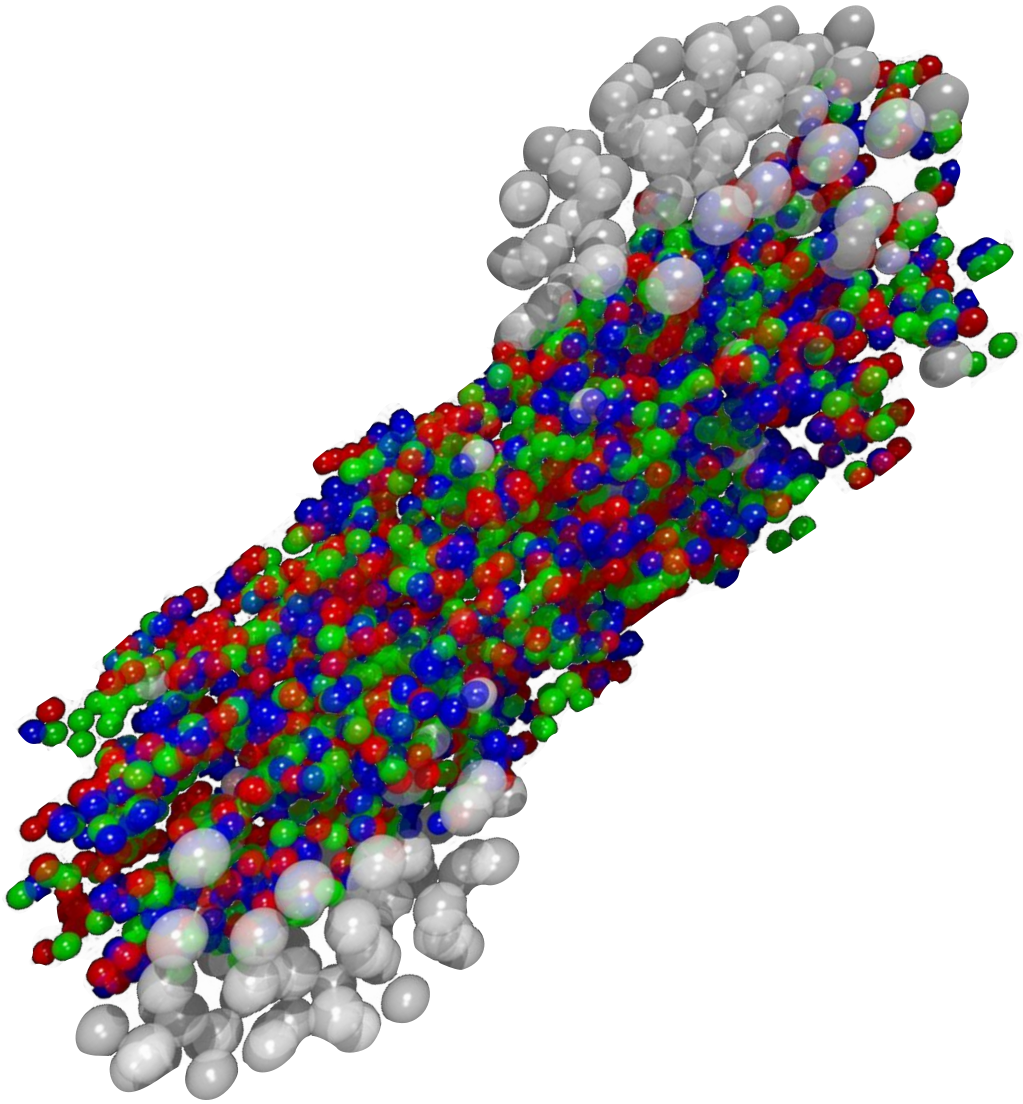


- QCD behaves differently depending on conditions of temperature and baryon density
- Low temperature and densities: **hadronic phase (confinement and spontaneously broken chiral symmetry)**
- Lattice simulations indicate a transition at high temperature to a **deconfined**, **chiral-symmetric phase:** The **QUARK-GLUON PLASMA**

The QCD phase space

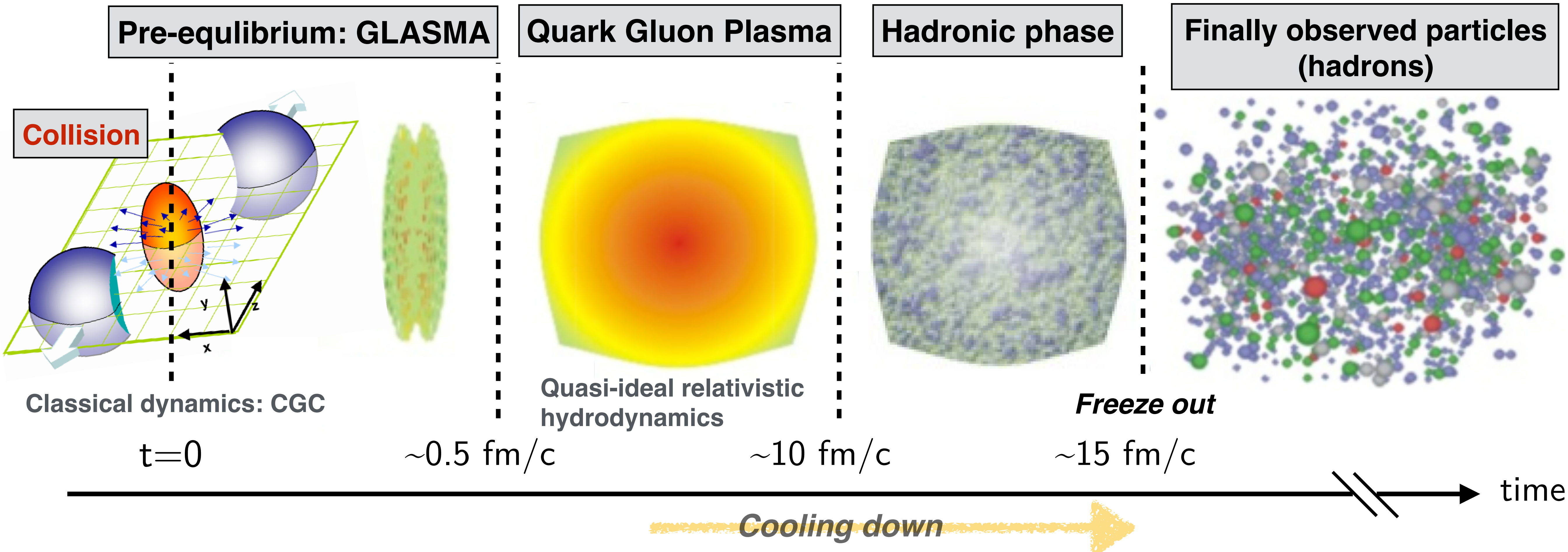
- This state of matter can be accessed in particle colliders through **Heavy Ion Collision** experiments





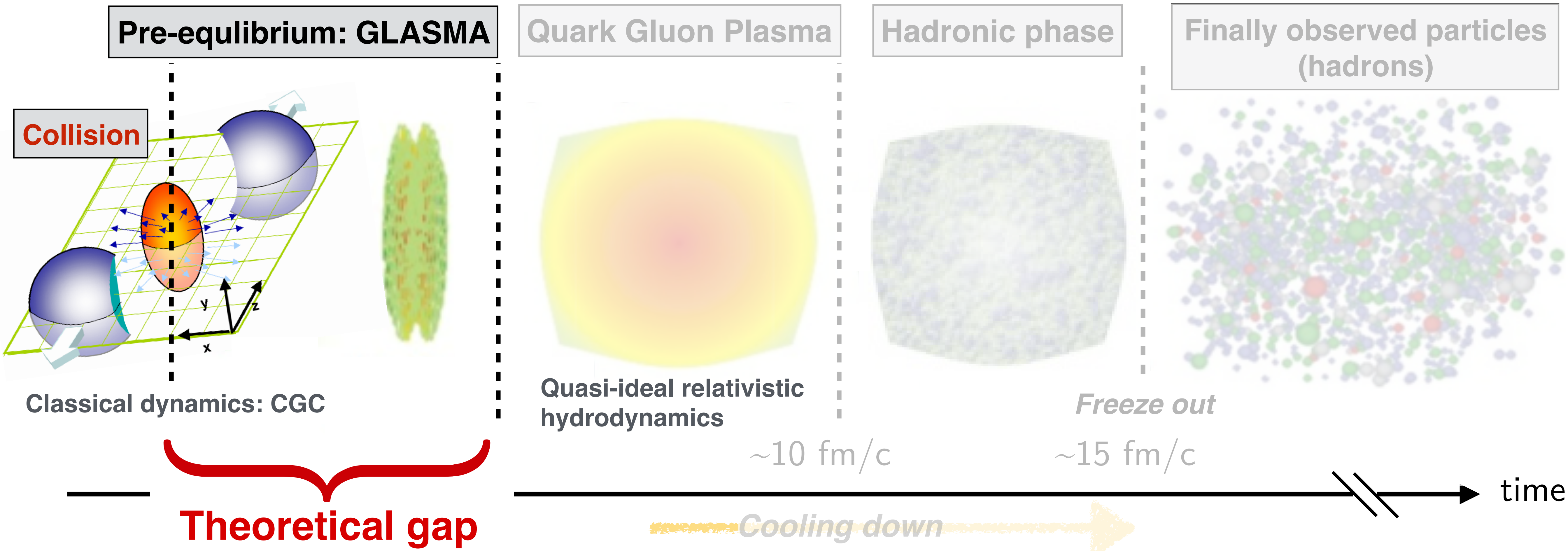
- Performed at Brookhaven National Laboratory's Relativistic Heavy Ion Collider (**RHIC**) and CERN's Large Hadron Collider (**ALICE** experiment)

Stages of a heavy ion collision



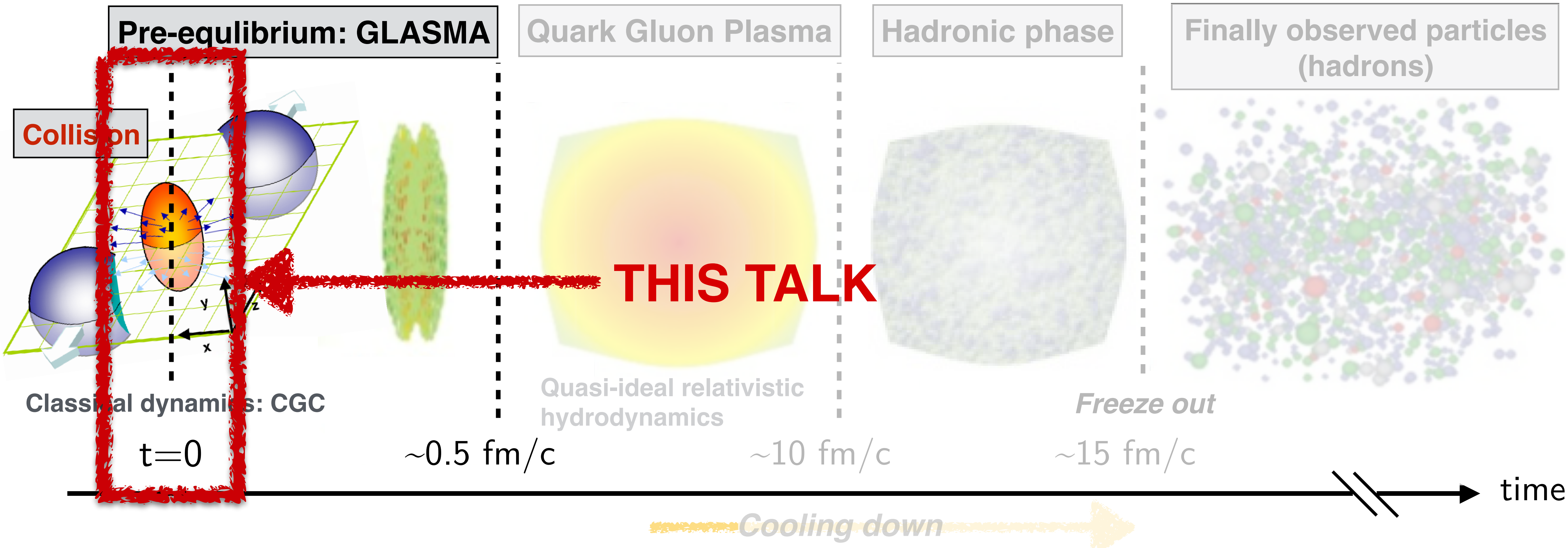
- After the collision, matter goes through different phases as it cools down
- In the last part, it reaches the hadronic phase, and this is how it appears in the detectors

Stages of a heavy ion collision



- There is a theoretical gap between the description of the early phase and the simulations of the expansion of the QGP
- Solid theoretical results are needed to mediate between both frameworks

Stages of a heavy ion collision



- There is a theoretical gap between the description of the early phase and the simulations of the expansion of the QGP
- Solid theoretical results are needed to mediate between both frameworks
- We provide a first-principles analytical calculation of:

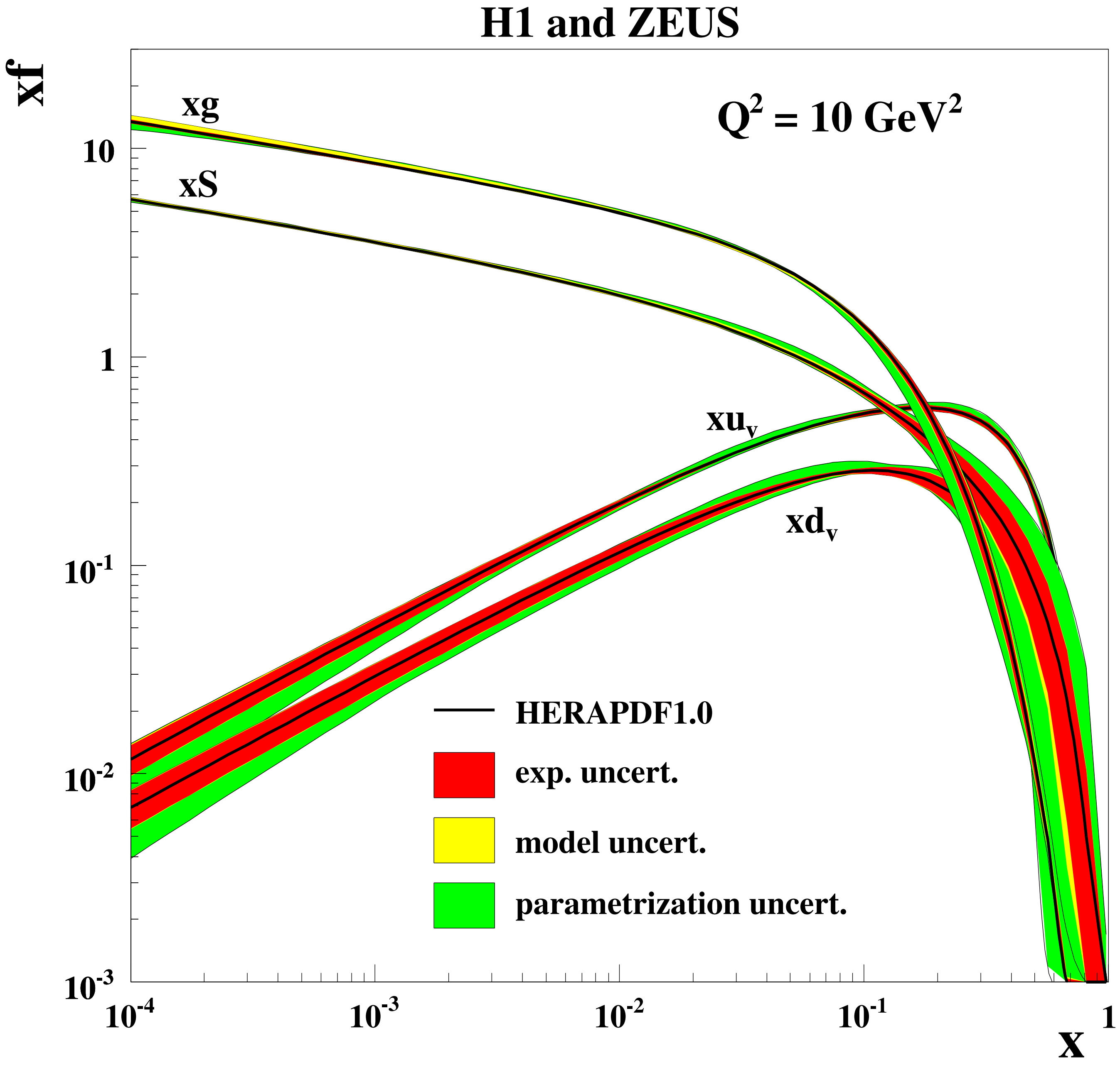
$$\langle T^{\mu\nu}(x_\perp)\rangle$$

$$\langle T^{\mu\nu}(x_\perp)T^{\mu\nu}(y_\perp)\rangle$$

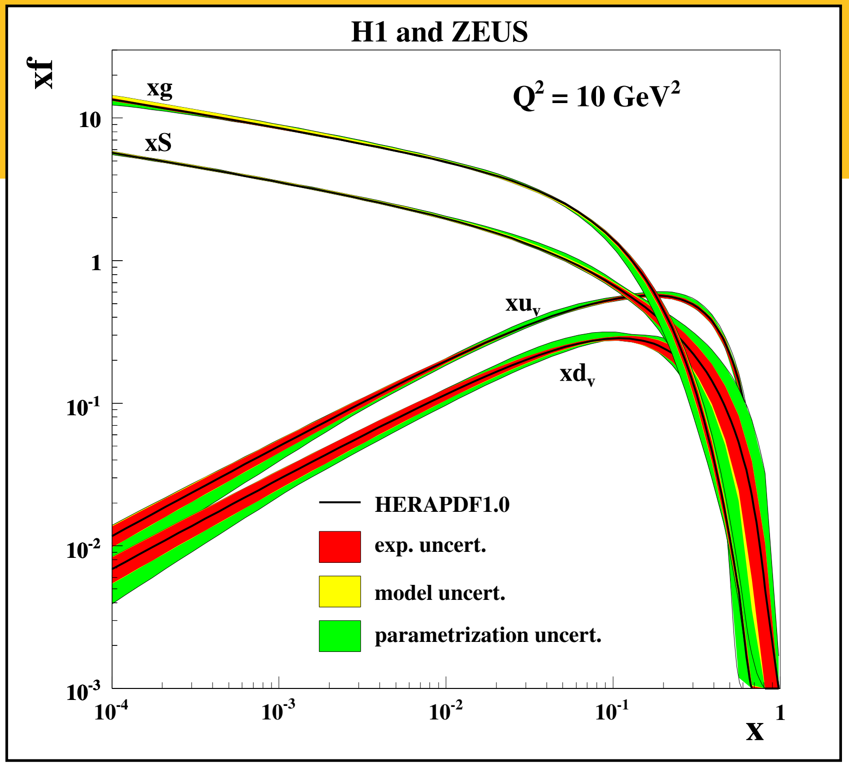
In the classical approximation (MV model)

Initial conditions: the Color-Glass Condensate

- At high energies (or equivalently, low x) the partonic content of protons and neutrons is **vastly dominated by a high density of gluons**

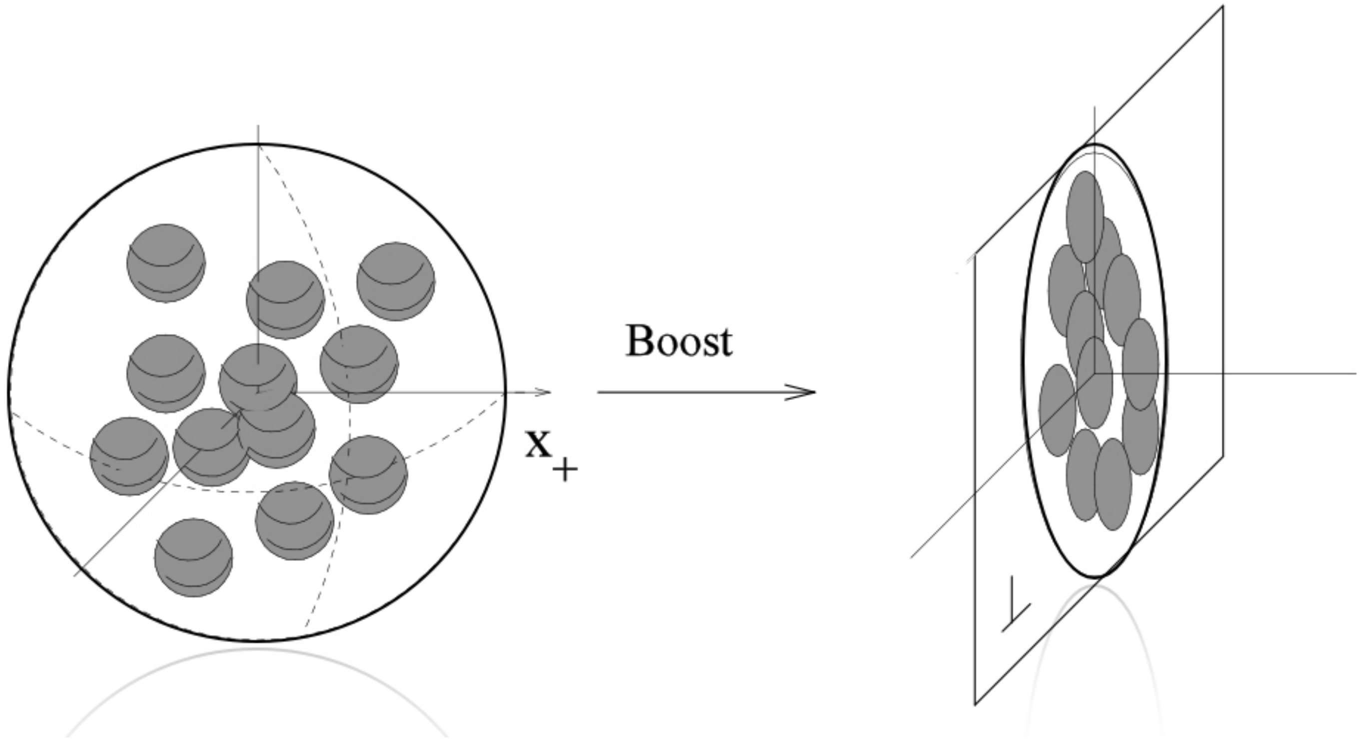


Highly Energetic Heavy Ion Collisions



- At high energies (or equivalently, low x) the partonic content of protons and neutrons is **vastly dominated by a high density of gluons**

- Relativistic kinematics: at high energies, the nuclei appear almost two-dimensional in the laboratory frame due to **Lorentz contraction**



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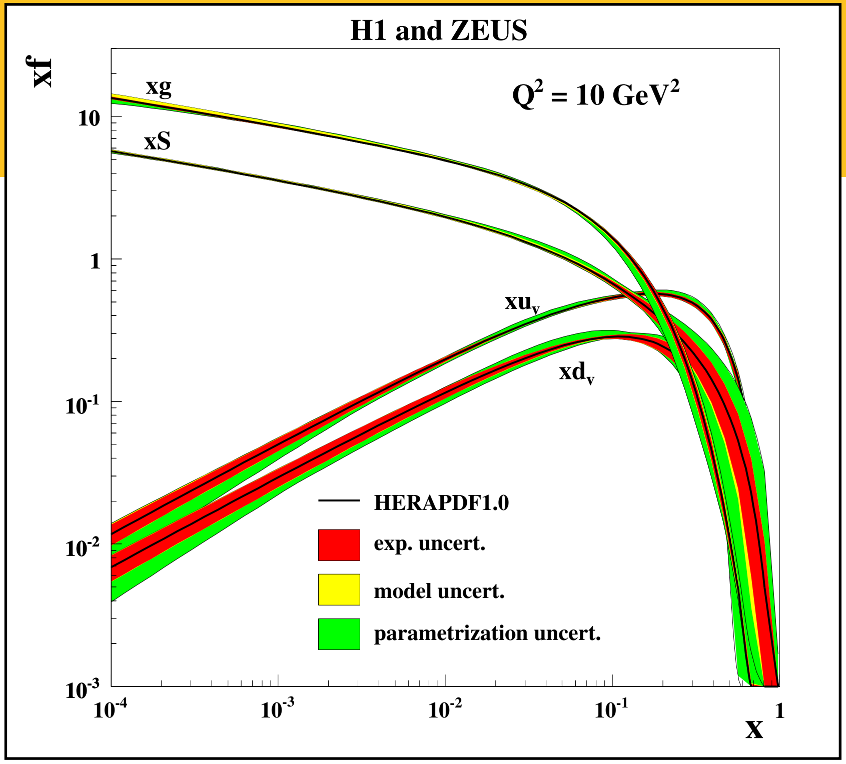

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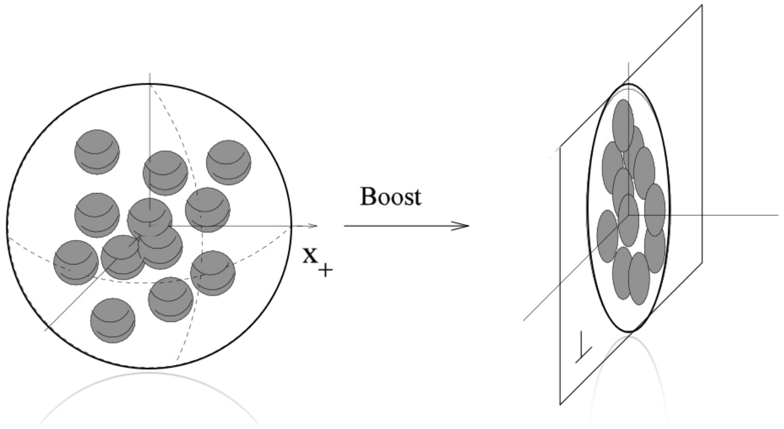
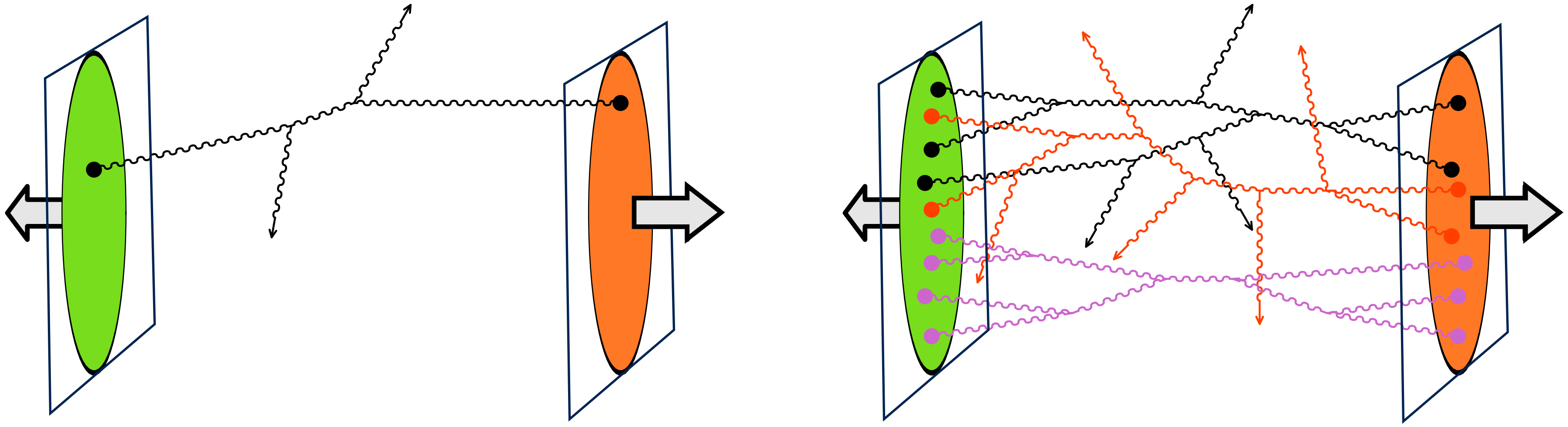
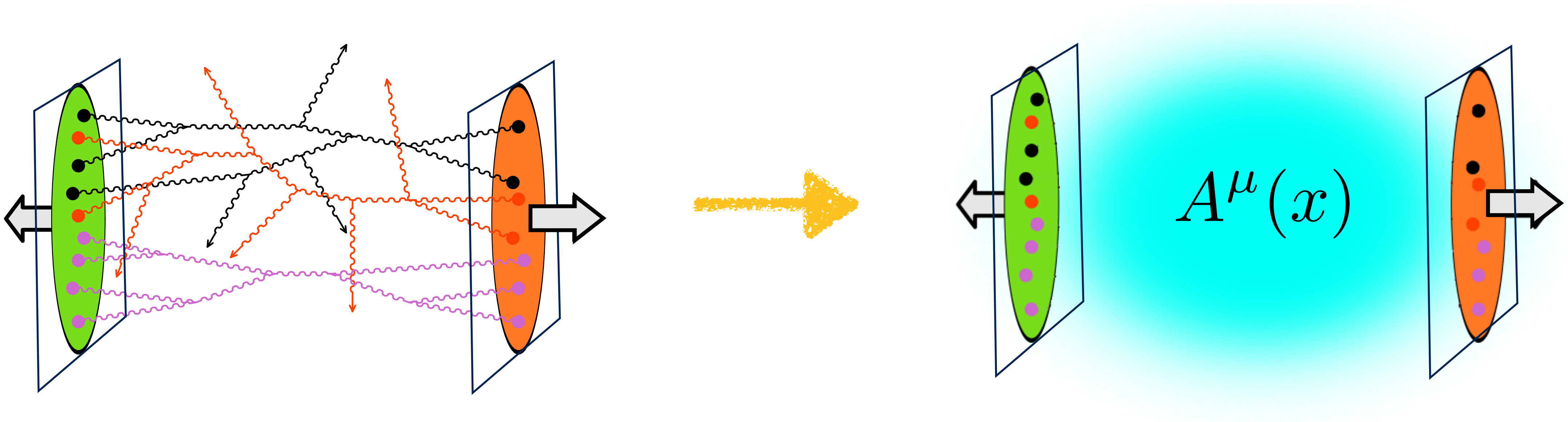


- QCD becomes **non-linear** and **non-perturbative**!



Color Glass Condensate: McLerran-Venugopalan model

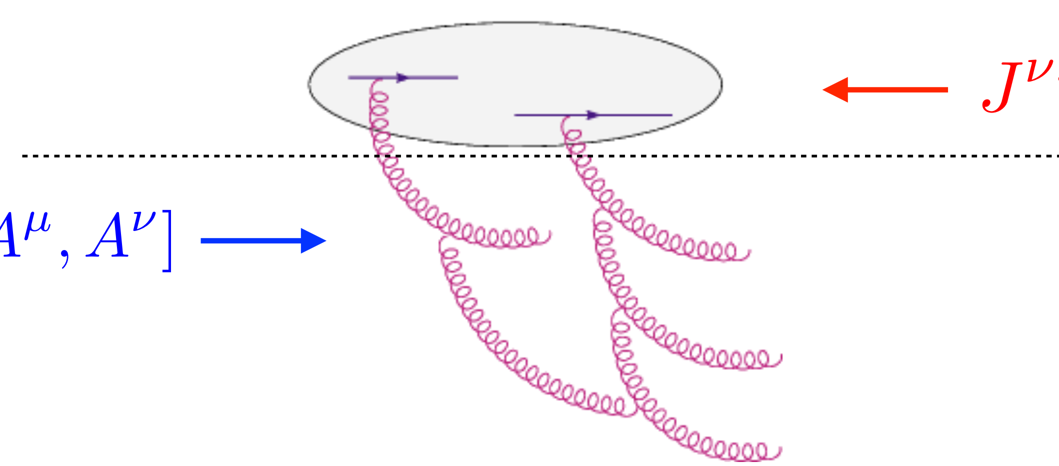
- We use an approximation of QCD for high gluon densities where we replace the gluons with a **classical field** generated by the valence quarks



- Dynamics of the field described by **Yang-Mills** classical equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho(x)$$

$J^{\nu,a} = \delta^{\nu+}\rho^a(x^-, x_\perp)$



$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\,[A^\mu, A^\nu]$

Separation of d.o.f.

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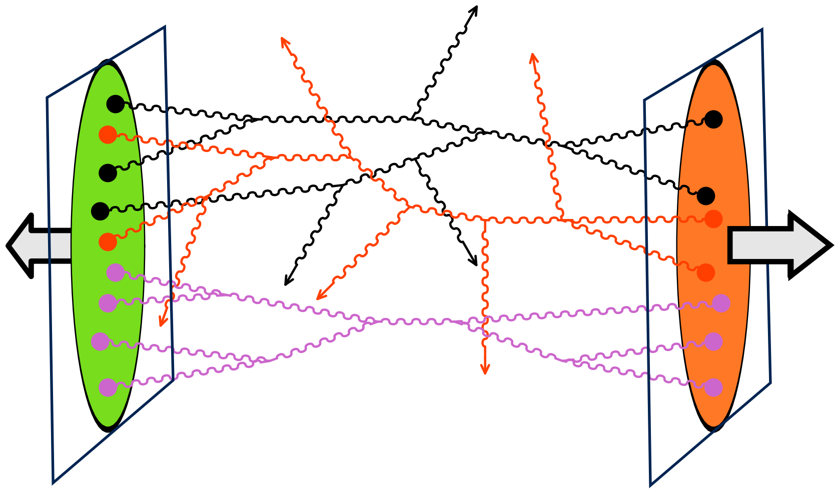

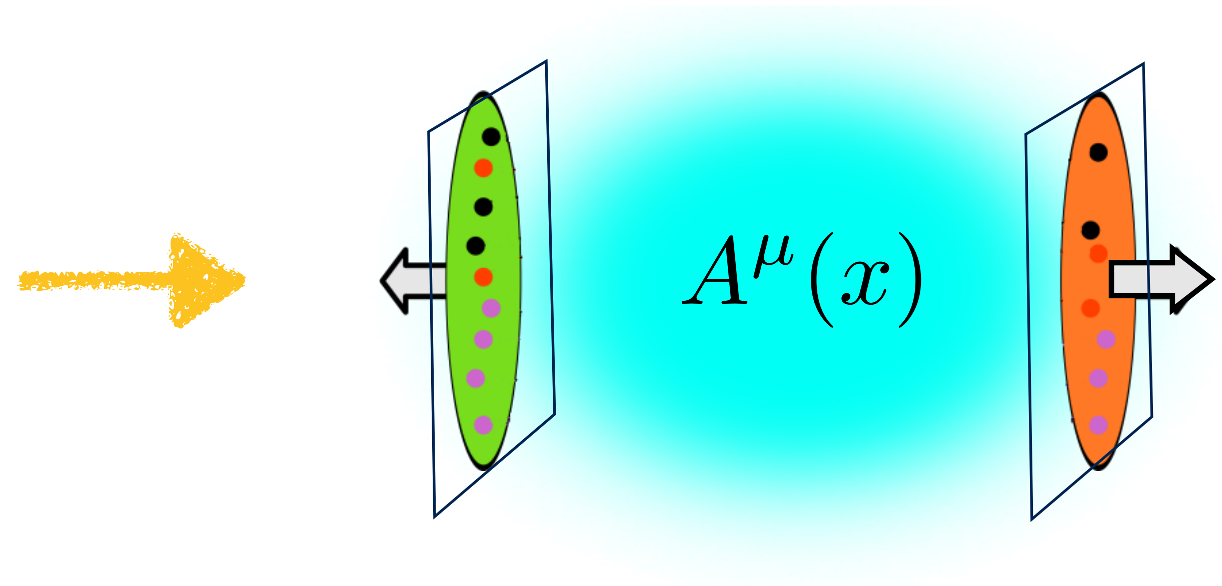

- Dynamics of the field described by **Yang-Mills** classical equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho(x)$$

- Calculation of observables: **average** over background classical fields

$$\langle \mathcal{O}[\rho] \rangle = \int [d\rho] \exp\left\{ -\int dx \, \mathrm{Tr}\left[\rho^2\right] \right\} \mathcal{O}[\rho]$$

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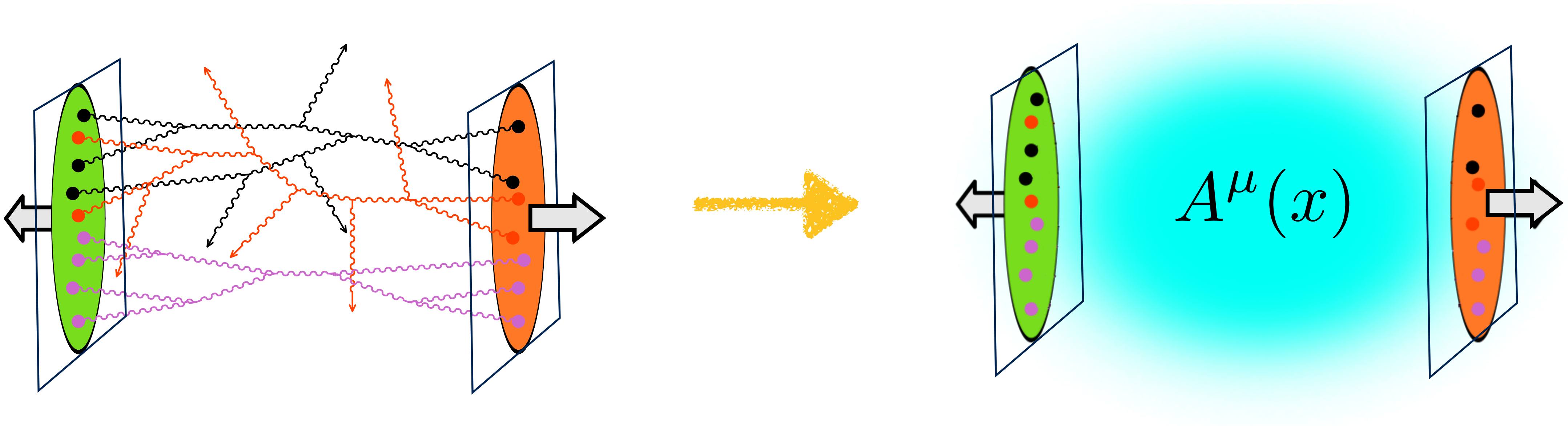

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$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho(x)$$

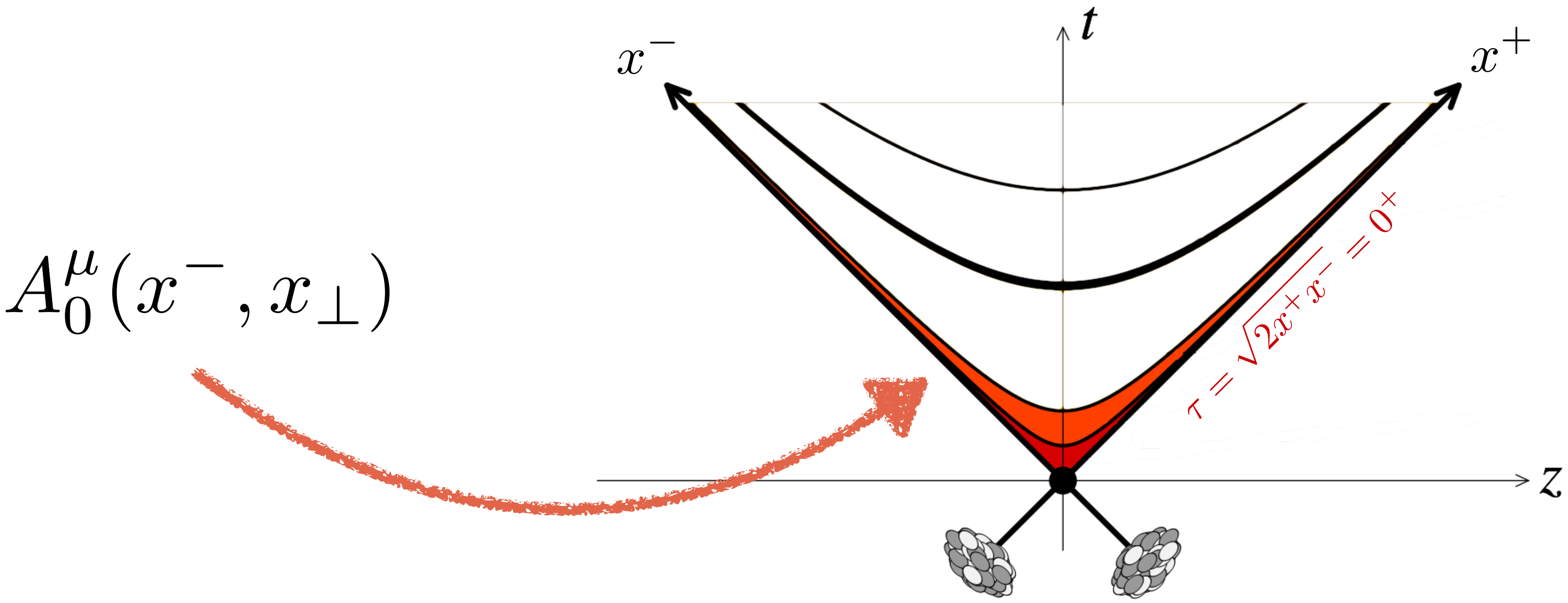
- Calculation of observables: **average** over background classical fields

- Basic building block: **2-point correlator (McLerran-Venugopalan)**

$$\langle \rho^a(x^-, x_\perp) \rho^b(y^-, y_\perp) \rangle = \mu^2(x^-) \delta^{ab} \delta(x^- - y^-) \delta^{(2)}(x_\perp - y_\perp)$$

Steps for the calculation

1) Calculate the gluon fields at early times in a HIC



2) Build the energy-momentum tensor

$$T_0^{\mu\nu}(x_\perp) = 2\,\mathrm{Tr}\left\{\frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} - F^{\mu\alpha}F^{\nu}_{\ \alpha}\right\}_0$$

3) Average over the color source distributions

$$\langle T_0^{\mu\nu}(x_\perp)\rangle = \int [d\rho_1]W_1[\rho_1][d\rho_2]W_2[\rho_2]T_0^{\mu\nu}(x_\perp)[\rho_1, \rho_2]$$

$$\langle T_0^{\mu\nu}(x_\perp)T_0^{\sigma\gamma}(y_\perp)\rangle = \int [d\rho_1]W_1[\rho_1][d\rho_2]W_2[\rho_2]\, T_0^{\mu\nu}(x_\perp)T_0^{\sigma\gamma}(y_\perp)[\rho_1, \rho_2]$$

Calculation of the gluon fields

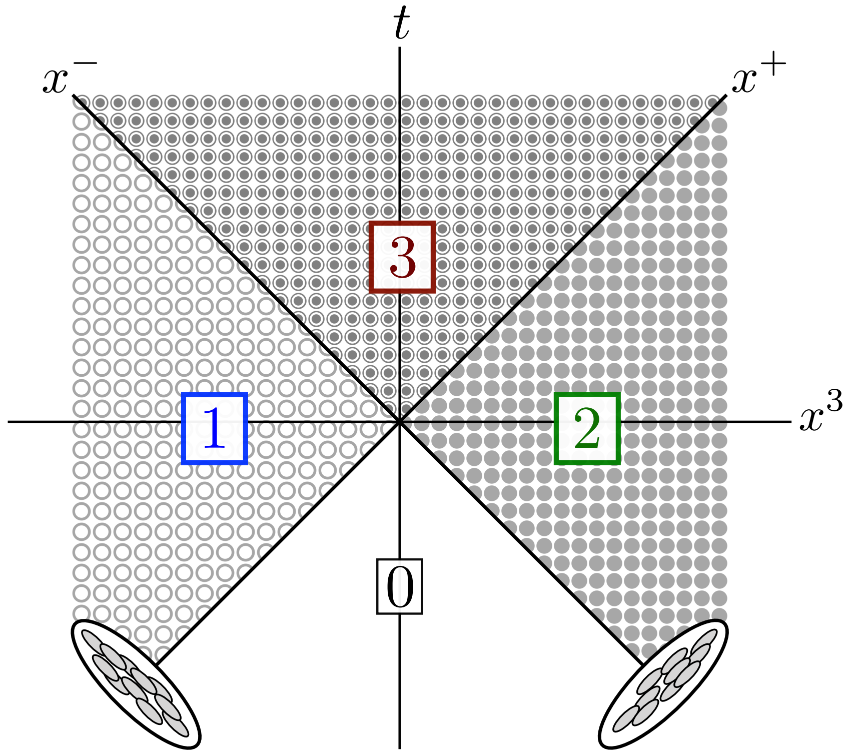
The gluon fields at τ=0⁺ in HICs

[Kovner, McLerran, Weigert 1998]

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu$$

$$\mathbf{J_1^\nu} = \rho_{\mathbf{1}}(\mathbf{x}_\perp)\delta(\mathbf{x}^-)\delta^{\nu+}$$

$$\mathbf{J_2^\nu} = \rho_{\mathbf{2}}(\mathbf{x}_\perp)\delta(\mathbf{x}^+)\delta^{\nu-}$$



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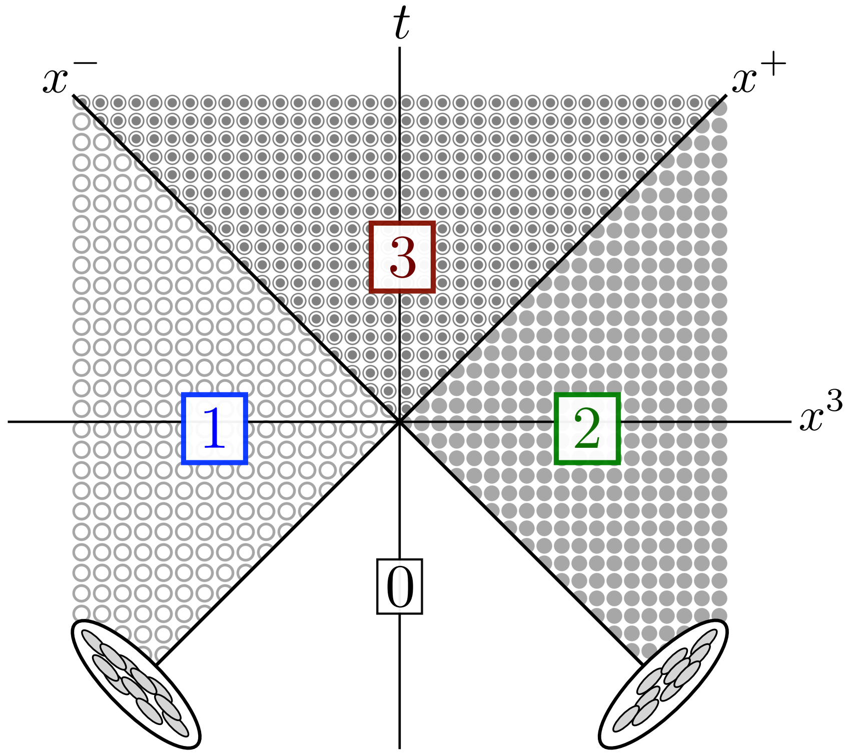

[1, 2] Single nucleus solution

$$A_1^\pm = 0$$

$$A_1^i = \theta(x^-)\int_{-\infty}^{\infty} dz^- U_1^\dagger(z^-, x_\perp)\frac{\partial^i \tilde{\rho}_1(z^-, x_\perp)}{\nabla^2} U_1(z^-, x_\perp) \equiv \theta(x^-)\alpha_1^i(x_\perp)$$

$$U_1(x^-, x_\perp) = \mathrm{P}^- \exp\left\{-ig\int_{x_0^-}^{x^-} dz^- \frac{1}{\nabla^2}\tilde{\rho}_1(z^-, x_\perp)\right\}$$

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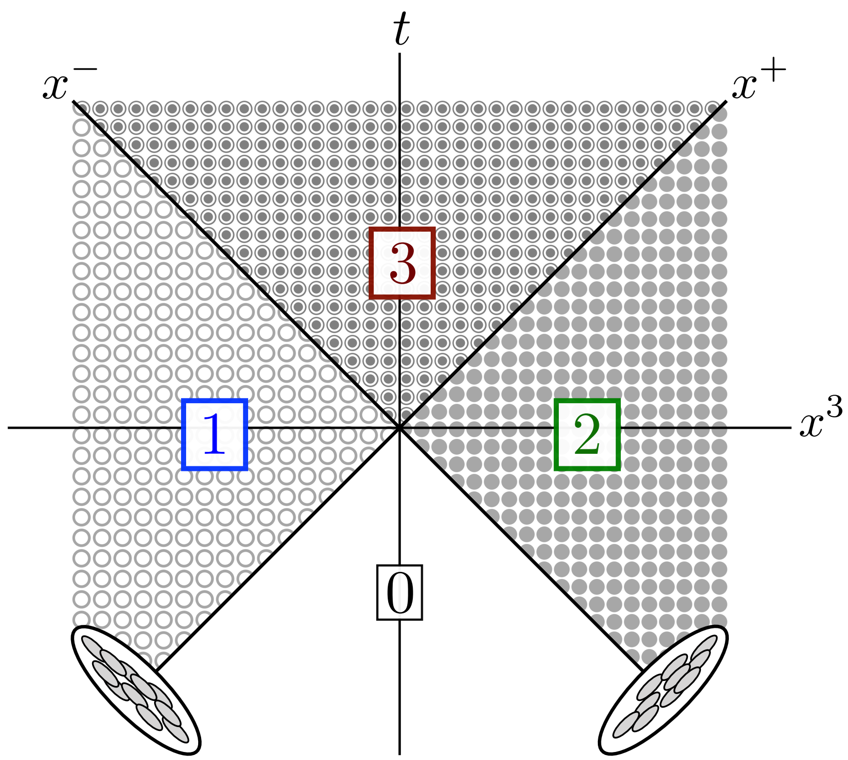

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[3] Forward light cone $\tau = 0^+$

$$A^\pm = \pm x^\pm \alpha(\tau = 0^+, x_\perp)$$

$$A^i = \alpha^i(\tau = 0^+, x_\perp)$$

$$\alpha^i(\tau = 0^+, x_\perp) = \alpha_1^i(x_\perp) + \alpha_2^i(x_\perp)$$

$$\alpha(\tau = 0^+, x_\perp) = \frac{ig}{2}\left[\alpha_1^i(x_\perp), \alpha_2^i(x_\perp)\right]$$

Calculation of the energy-momentum tensor $T^{\mu\nu}(\tau = 0^+)$

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu$$

$$\mathbf{J_1^\nu} = \rho_1(\mathbf{x}_\perp)\delta(\mathbf{x}^-)\delta^{\nu+}$$

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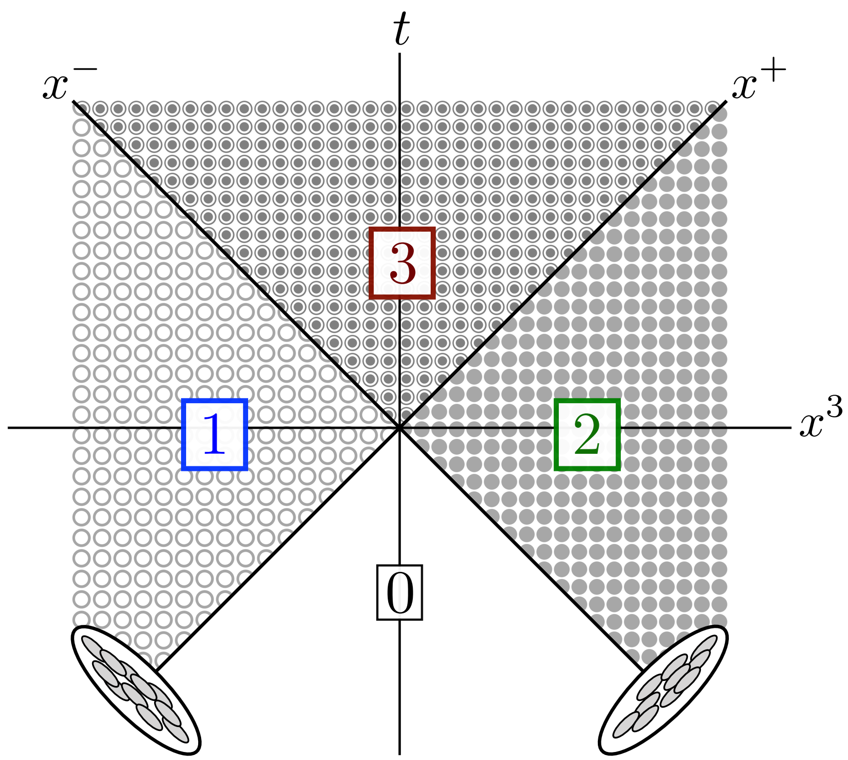

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$$A^\pm = \pm x^\pm \alpha(\tau = 0^+, x_\perp) \qquad \alpha^i(\tau = 0^+, x_\perp) = \alpha_1^i(x_\perp) + \alpha_2^i(x_\perp)$$

$$A^i = \alpha^i(\tau = 0^+, x_\perp) \qquad \alpha(\tau = 0^+, x_\perp) = \frac{ig}{2}\left[\alpha_1^i(x_\perp), \alpha_2^i(x_\perp)\right]$$

- We can obtain the early-time energy-momentum tensor as:

$$T_0^{\mu\nu} = \frac{1}{4}g^{\mu\nu}F^{\alpha\beta,a}F^a_{\alpha\beta} - F^{\mu\alpha,a}F^{\nu,a}_{\alpha}$$

$$= -\frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left([\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\right) \times \mathrm{diag}(1, 1, 1, -1)$$

$$\equiv \epsilon_0 \times \mathrm{diag}(1, 1, 1, -1) \equiv \epsilon_0 \times t^{\mu\nu}$$

Correlators of the energy-momentum tensor at $\tau = 0^+$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0 \rangle\, t^{\mu\nu}$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\begin{aligned}
\langle \epsilon_0 \rangle &= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\right\}\right\rangle \\
&= -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a, t^b\right]\left[t^c, t^d\right]\right\} \\
&= \frac{g^2}{2}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl}) f^{abm} f^{cdm} \left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1 \left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2
\end{aligned}$$

- For the 1-point correlator of $T^{\mu\nu}$:

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- We momentarily take two different transverse coordinates:

$$\langle \alpha^{i,a}(x_\perp)\alpha^{j,b}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dz^{-\prime} \left\langle \frac{\partial^i \tilde{\rho}^{a'}(z^-, x_\perp)}{\nabla^2} \underbrace{U^{a'a}(z^-, x_\perp)}_{\sim e^{i\rho}} \frac{\partial^j \tilde{\rho}^{b'}(z^{-\prime}, y_\perp)}{\nabla^2} \underbrace{U^{b'b}(z^{-\prime}, y_\perp)}_{\sim e^{i\rho}} \right\rangle$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle\epsilon_0\rangle\, t^{\mu\nu}$

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$$= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\}$$
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Luckily, Wilson lines and (external) color source densities factorize

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0 \rangle \, t^{\mu\nu}$

- For the 1-point correlator of $T^{\mu\nu}$:

$$\langle \epsilon_0 \rangle = -g^2(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i, \alpha_2^j][\alpha_1^k, \alpha_2^l]\right\}\right\rangle$$

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$$\delta^{a'b'}\mu^2(x^-)\delta(x^- - y^-)\partial_x^i \partial_y^j L(x_\perp - y_\perp)$$

Where:

$$L(x_\perp - y_\perp) = \int d^2 z_\perp G(x_\perp - z_\perp) G(y_\perp - z_\perp).$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle \epsilon_0 \rangle\, t^{\mu\nu}$

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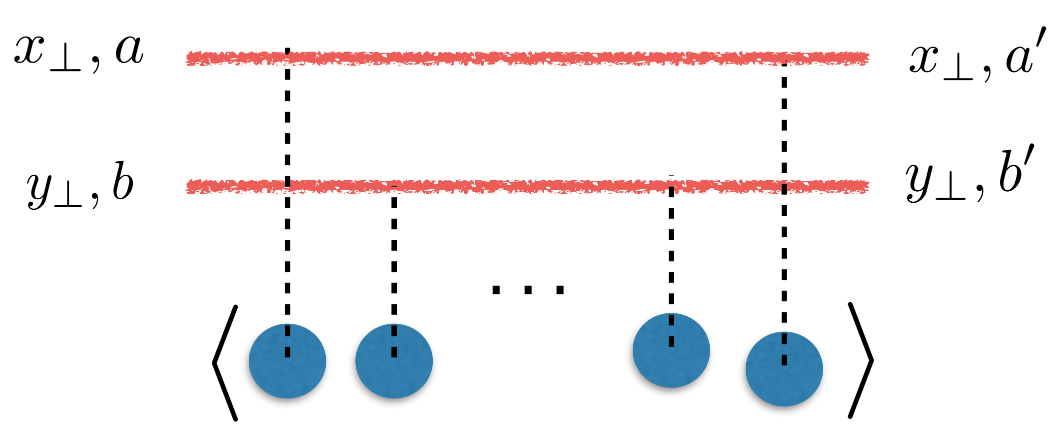

$$\frac{\delta^{ab}\delta^{a'b'}}{N} \exp\left[-g^2 \frac{N}{2}\Gamma(x_\perp, y_\perp)\bar{\mu}^2(x^-)\right]$$

$\langle T^{\mu\nu}(x_\perp)\rangle = \langle\epsilon_0\rangle\, t^{\mu\nu}$

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$$
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\langle\epsilon_0\rangle &= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \mathrm{Tr}\left\{[\alpha_1^i,\alpha_2^j][\alpha_1^k,\alpha_2^l]\right\}\right\rangle \\
&= -g^2(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\left\langle \alpha_1^{i,a}\alpha_2^{j,b}\alpha_1^{k,c}\alpha_2^{l,d}\right\rangle \mathrm{Tr}\left\{\left[t^a,t^b\right]\left[t^c,t^d\right]\right\} \\
&= \frac{g^2}{2}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})f^{abm}f^{cdm}\left\langle \alpha_1^{i,a}(x_\perp)\alpha_1^{k,c}(x_\perp)\right\rangle_1\left\langle \alpha_2^{j,b}(x_\perp)\alpha_2^{l,d}(x_\perp)\right\rangle_2 \\
&= \frac{g^2}{8}f^{abm}f^{cdm}(\delta^{ij}\delta^{kl}+\epsilon^{ij}\epsilon^{kl})\delta^{ac}\delta^{ik}\delta^{bd}\delta^{jl}\bar{\mu}_1^2\bar{\mu}_2^2(\partial^2L(0_\perp))^2 \\
&= g^2C_A^2C_F\bar{\mu}_1^2\bar{\mu}_2^2(\partial^2L(0_\perp))^2 \\
&= \frac{C_F}{g^2}\bar{Q}_{s1}^2(x_\perp)\bar{Q}_{s2}^2(x_\perp)\left(4\pi\,\partial^2L(0_\perp)\right)^2
\end{aligned}
$$

Notation:

$$\bar{\mu}^2 = \int_{-\infty}^{\infty} dz^-\mu^2(z^-)$$

- Here we have introduced a **momentum scale** characterizing each nucleus:

$$\bar{Q}_s^2 = \alpha_s N_c\,\bar{\mu}^2(x_\perp)$$

- In the MV model the factor $\partial^2L(0_\perp)$ yields a **logarithmic IR divergence**.

$\langle T^{\mu\nu}(x_\perp) T^{\sigma\rho}(y_\perp)\rangle = \langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle\, t^{\mu\nu}\, t^{\sigma\rho}$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'} + \epsilon^{i'j'}\epsilon^{k'l'}) f^{abn} f^{cdn} f^{a'b'm} f^{c'd'm}$$
$$\times \langle \alpha_{1\,x}^{i\,a}\alpha_{1\,x}^{k\,c}\alpha_{1\,y}^{i'a'}\alpha_{1\,y}^{k'c'}\rangle\langle \alpha_{2\,x}^{j\,b}\alpha_{2\,x}^{l\,d}\alpha_{2\,y}^{j'b'}\alpha_{2\,y}^{l'd'}\rangle$$

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- The building block:

$$\langle\alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle = \int_{-\infty}^{\infty} dz^- dw^- dz^{-\prime} dw^{-\prime} \Big\langle \frac{\partial^i\tilde{\rho}^e(z^-,x_\perp)}{\nabla^2}U^{ea}(z^-,x_\perp) \frac{\partial^k\tilde{\rho}^f(w^-,x_\perp)}{\nabla^2}U^{fc}(w^-,x_\perp)\frac{\partial^{i'}\tilde{\rho}^{e'}(z^{-\prime},y_\perp)}{\nabla^2}U^{e'a'}(z^{-\prime},y_\perp)\frac{\partial^{k'}\tilde{\rho}^{f'}(w^{-\prime},y_\perp)}{\nabla^2}U^{f'c'}(w^{-\prime},y_\perp)\Big\rangle.$$

$\langle T^{\mu\nu}(x_\perp)T^{\sigma\rho}(y_\perp)\rangle = \langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle\, t^{\mu\nu}\, t^{\sigma\rho}$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

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$$\times \langle \alpha^{i\,a}_{1\,x}\alpha^{k\,c}_{1\,x}\alpha^{i'a'}_{1\,y}\alpha^{k'c'}_{1\,y}\rangle\langle \alpha^{j\,b}_{2\,x}\alpha^{l\,d}_{2\,x}\alpha^{j'b'}_{2\,y}\alpha^{l'd'}_{2\,y}\rangle$$

- Technical difficulties:
  - The **expansion of the correlator** $\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle$ is far more difficult than that of $\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(y_\perp)\rangle$. Schematically: *[Fillion-Gourdeau & Jeon '09]*

$$\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle = \underbrace{\langle \rho^4\rangle\langle U^4\rangle}_{\text{3 terms (Wick's theorem)}} + \langle \rho^2\rangle\underbrace{\langle \rho^2 U^4\rangle_c}_{\text{4 terms}}$$

$\langle T^{\mu\nu}(x_\perp) T^{\sigma\rho}(y_\perp)\rangle = \langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle \, t^{\mu\nu}\, t^{\sigma\rho}$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

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$$\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle = \langle \rho^4\rangle\langle U^4\rangle + \langle \rho^2\rangle\langle \rho^2 U^4\rangle_c$$

  - Instead of having to calculate the adjoint Wilson line dipole, we need the much more complex adjoint **Wilson line quadrupole** *[Kovner & Wiedemann '01]*

$$\left\langle U^{ab}(z^-, x_\perp) U^{cd}(z^-, y_\perp) U^{ef}(z^-, x'_\perp) U^{gh}(z^-, y'_\perp)\right\rangle$$

$\langle T^{\mu\nu}(x_\perp)T^{\sigma\rho}(y_\perp)\rangle = \langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle\, t^{\mu\nu}\, t^{\sigma\rho}$

- For the 2-point correlator of $T^{\mu\nu}$: prepare for trouble and make it double

$$\langle \epsilon(x_\perp)\epsilon(y_\perp)\rangle = \frac{g^4}{4}(\delta^{ij}\delta^{kl} + \epsilon^{ij}\epsilon^{kl})(\delta^{i'j'}\delta^{k'l'} + \epsilon^{i'j'}\epsilon^{k'l'})f^{abn}f^{cdn}f^{a'b'm}f^{c'd'm}$$
$$\times \langle \alpha_{1\,x}^{i\,a}\alpha_{1\,x}^{k\,c}\alpha_{1\,y}^{i'a'}\alpha_{1\,y}^{k'c'}\rangle\langle \alpha_{2\,x}^{j\,b}\alpha_{2\,x}^{l\,d}\alpha_{2\,y}^{j'b'}\alpha_{2\,y}^{l'd'}\rangle$$

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  - The **color structure** of this object is frustratingly complex. Even with all parts analytically calculated, the contraction of the color indices demands a computational treatment (via FeynCalc)

$\mathrm{Cov}[\epsilon_0](x_\perp, y_\perp) = \langle \epsilon_0(x_\perp)\epsilon_0(y_\perp)\rangle - \langle \epsilon_0(x_\perp)\rangle\langle \epsilon_0(y_\perp)\rangle$

$$
\begin{aligned}
\mathrm{Cov}[\epsilon](\tau=0^+;x_\perp,y_\perp) = &-\frac{\partial_x^i\Gamma\partial_y^i\Gamma(N_c^2-1)A(4A^2-B^2)}{16N_c^2\Gamma^5 g^4}(f_1g_2+f_2g_1) \\
&+\frac{(N_c^2-1)(16A^4+B^4)}{2N_c^2\Gamma^4 g^4}f_1f_2+\frac{(\partial_x^i\Gamma\partial_y^i\Gamma)^2(N_c^2-1)A^2}{64N_c^2\Gamma^6 g^4}g_1g_2 \\
&+\frac{(4A^2+B^2)^2r^2}{N_c^2\Gamma^4g^4}\left(\left[\frac{1}{2}Q_{s1}^2Q_{s2}^2r^2+4Q_{s2}^2e^{-\frac{Q_{s1}^2r^2}{4}}-4Q_{s1}^2\right]+[1\leftrightarrow 2]\right) \\
&+\frac{(N_c^2-1)(4A^2+B^2)}{2N_c^2\Gamma^2g^4}\left(4\pi\,\partial^2L(0_\perp)\right)^2\left(\left[\bar{Q}_{s1}^4(Q_{s2}^2r^2-4+4e^{-\frac{Q_{s2}^2r^2}{4}})\right]+[1\leftrightarrow 2]\right) \\
&+\frac{(4A^2+B^2)^2}{\Gamma^4g^4N_c^2(N_c^2-1)^2(N_c^2-4)^2}\Bigg(\Bigg[-4(N_c^2-1)(N_c^2-4)(N_c^6-3N_c^4-26N_c^2+16)e^{-\frac{Q_{s1}^2r^2}{4}} \\
&+(N_c-3)(N_c+1)^3(N_c+2)^2N_c^3\left((N-2)e^{\frac{Q_{s1}^2r^2}{4}}-2(N-1)\right)e^{-\frac{r^2\left(N_cQ_{s1}^2+2(N_c-1)Q_{s2}^2\right)}{4N_c}} \\
&+(N_c+3)(N_c-1)^3(N_c-2)^2N_c^3\left((N+2)e^{\frac{Q_{s1}^2r^2}{4}}-2(N+1)\right)e^{-\frac{r^2\left(N_cQ_{s1}^2+2(N_c+1)Q_{s2}^2\right)}{4N_c}} \\
&+4(N_c^2-8)(N_c^2-1)^3(N_c^2+4)e^{-\frac{1}{4}r^2\left(Q_{s1}^2+Q_{s2}^2\right)} \\
&+\frac{1}{2}(N_c-2)^2(N_c-1)^3(N_c+3)N_c^4e^{-\frac{(N_c+1)r^2\left(Q_{s1}^2+Q_{s2}^2\right)}{2N_c}} \\
&+\frac{1}{2}(N_c-3)(N_c+1)^3(N_c+2)^2N_c^4e^{-\frac{(N_c-1)r^2\left(Q_{s1}^2+Q_{s2}^2\right)}{2N_c}}\Bigg]+[1\leftrightarrow 2] \\
&+2\left(N_c^2-4\right)^2\left(N_c^6+2N_c^4-19N_c^2+8\right)\Bigg)
\end{aligned}
$$

with:

$$f_{1,2} = e^{-\frac{Q_{s1,2}^2r^2}{4}}(Q_{s1,2}^2r^2+4)-4$$

$$g_{1,2} = e^{-\frac{Q_{s1,2}^2r^2}{4}}\left(Q_{s1,2}^4r^4+8Q_{s1,2}^2r^2+32\right)-32.$$

Pocket formulae

- Omitting for the moment the issues with the r->0 divergencies (GBW-model)

r-> 0

$$\lim_{r\to 0} \mathrm{Cov}[\epsilon](0^+; x_\perp, y_\perp) = \frac{3C_F}{g^4 2N_c} Q_{s1}^4 Q_{s2}^4$$

$$\lim_{r\to 0} \frac{\mathrm{Cov}[\epsilon](0^+; x_\perp), y_\perp)}{\langle \epsilon_0(x_\perp)\rangle \langle \epsilon_0(y_\perp)\rangle} = \frac{3}{(N_c^2 - 1)}$$

r-> ∞

$$\lim_{r\to \infty} \mathrm{Cov}[\epsilon](0^+; x_\perp, y_\perp) = \frac{2(N_c^2-1)\left(Q_{s1}^4 Q_{s2}^2 + Q_{s1}^2 Q_{s2}^4\right)}{g^4 N_c^2 r^2}$$

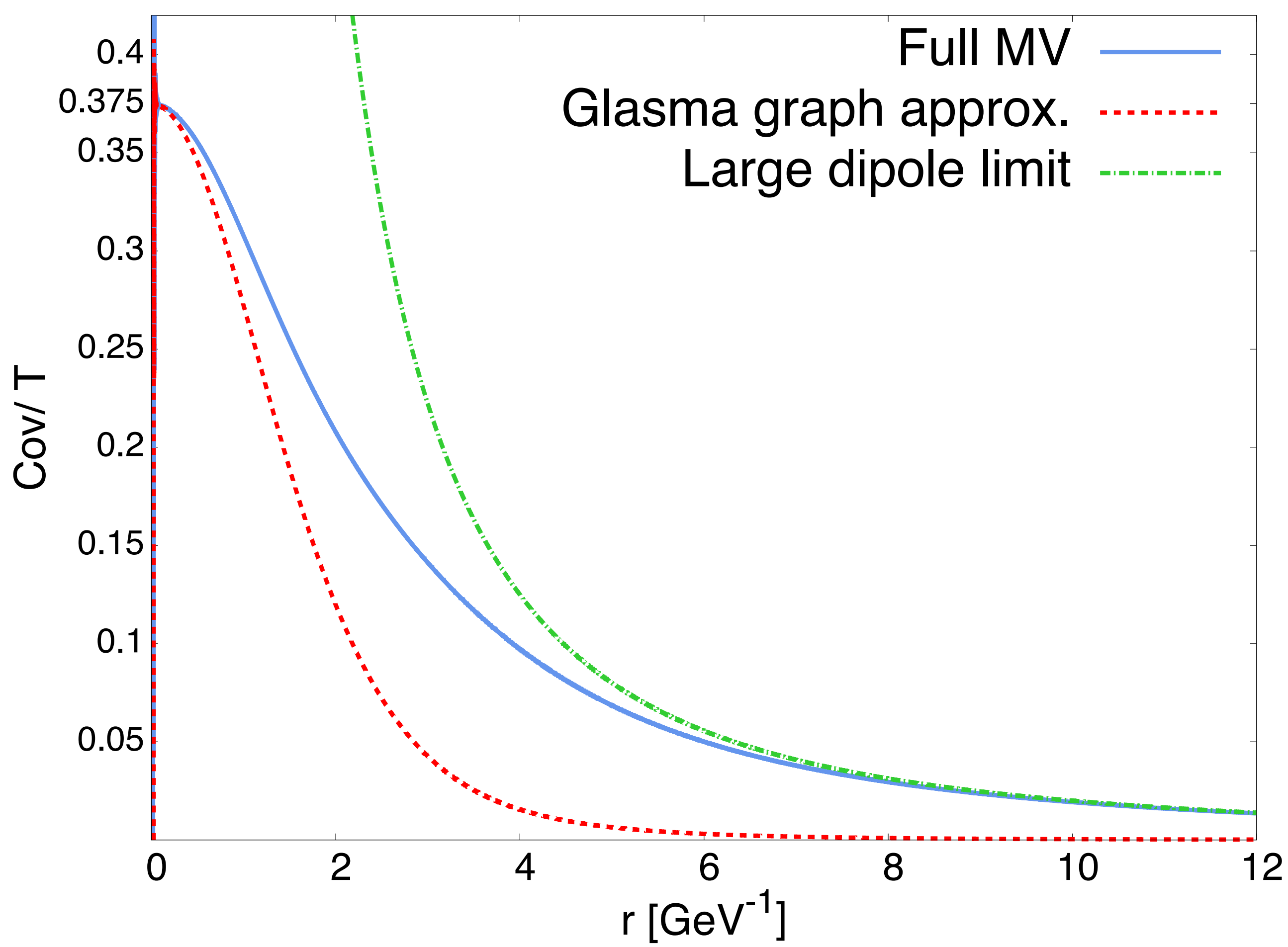
$$\lim_{r\to \infty} \frac{\mathrm{Cov}[\epsilon](0^+; x_\perp, y_\perp)}{\langle \epsilon_0(x_\perp)\rangle \langle \epsilon_0(y_\perp)\rangle} = \frac{1}{2(N_c^2-1)r^2}\left(\frac{1}{Q_{s1}^2} + \frac{1}{Q_{s2}^2}\right)$$

Comparison with the 'Glasma Graph' approximation

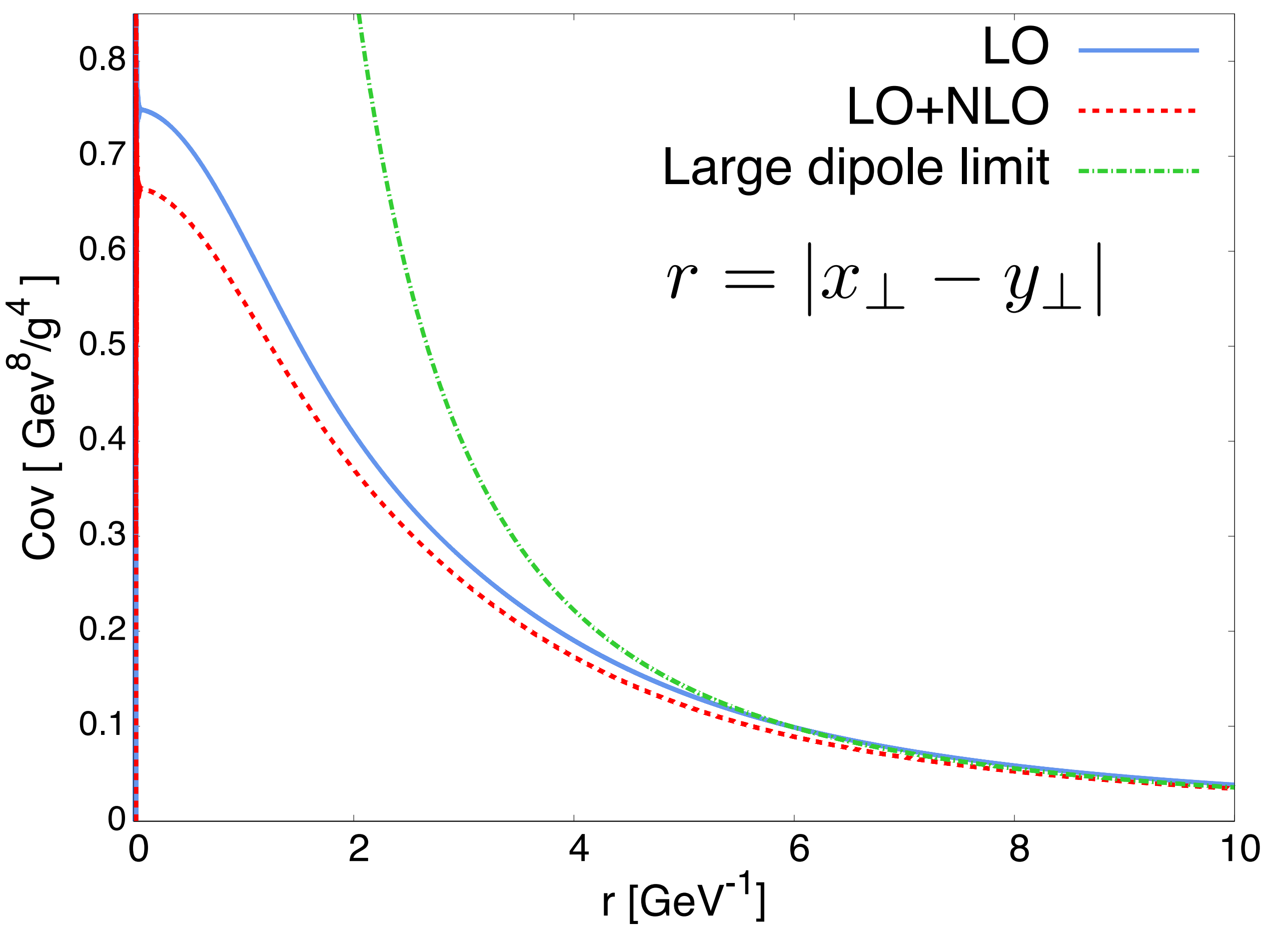
- Glasma Graph approximation *[Lappi & Schlichting 2018, Muller & Schaefer 2012].* Assume Gaussian distribution of the produced gluon fields:

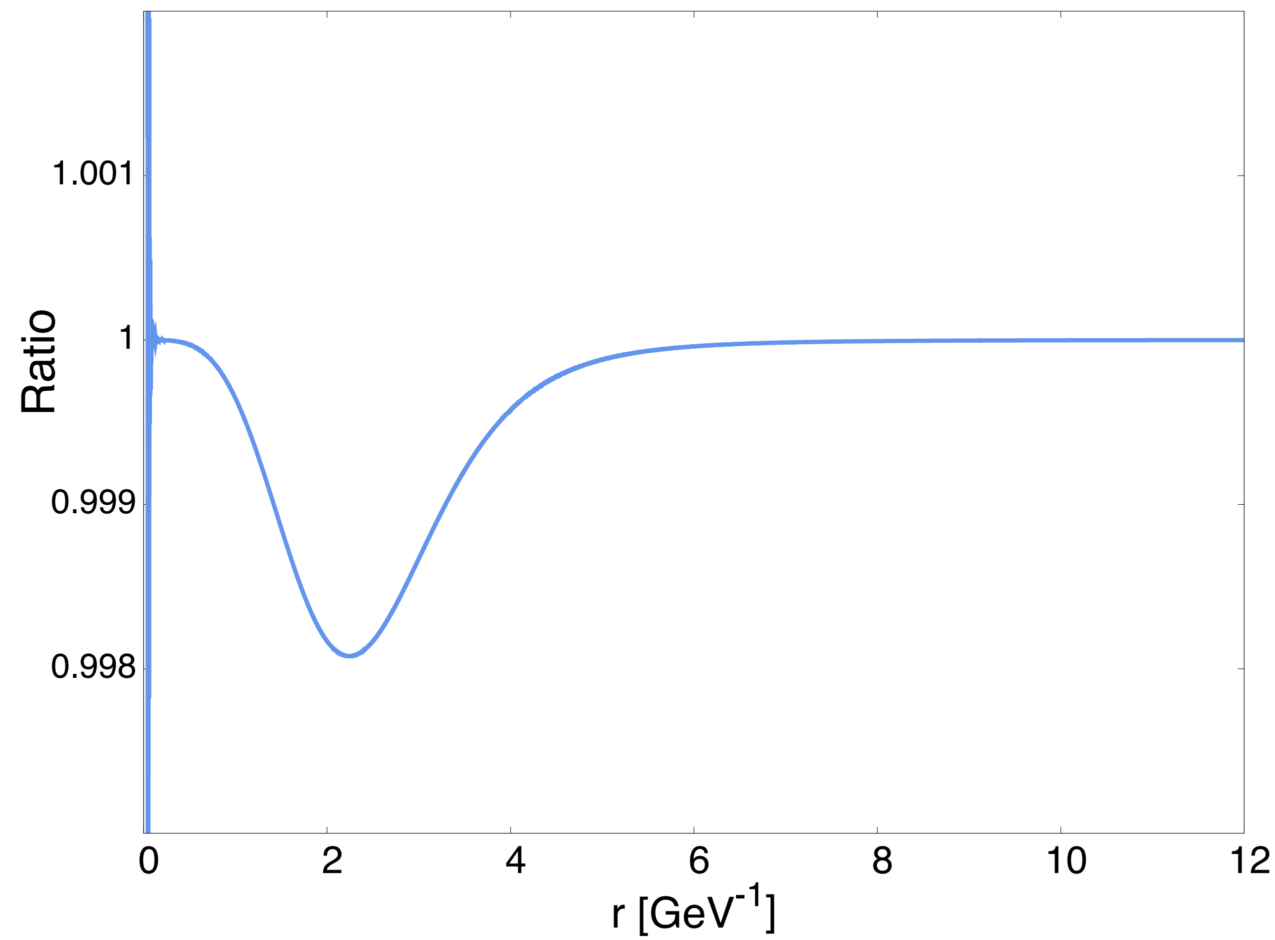
$$\begin{aligned}\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle_{\rm GG} = &\langle \alpha^{i\,a}(x_\perp)\alpha^{k\,c}(x_\perp)\rangle\langle \alpha^{i'a'}(y_\perp)\alpha^{k'c'}(y_\perp)\rangle \\ &+\langle \alpha^{i\,a}(x_\perp)\alpha^{i'a'}(y_\perp)\rangle\langle \alpha^{k\,c}(x_\perp)\alpha^{k'c'}(y_\perp)\rangle \\ &+\langle \alpha^{i\,a}(x_\perp)\alpha^{k'c'}(y_\perp)\rangle\langle \alpha^{k\,c}(x_\perp)\alpha^{i'a'}(y_\perp)\rangle.\end{aligned}$$

- Agreement with full result in the r->0 limit. Strong discrepancies in the r-> ∞ limit



Nc expansion





Sum of the first two orders of the N-expansion of the energy density covariance for N=3 in the classical MV model.

Ratio between the full result and the sum of the first two orders of the N-expansion, which turns out to be a very good approximation.

Conclusions

- We have performed an exact analytical calculation of the covariance of the energy momentum tensor of the **Glasma** at $\tau = 0^+$, in the framework of the **Color Glass Condensate**.

- We expect to be able to **generalize this framework** by introducing an impact parameter dependence and relaxing some of the original assumptions, which could potentially open the door to **phenomenological applications**.

- The following steps are computing the **time evolution** of our result towards thermalization time $\tau \sim 1/Q_s$, where it can serve as input for **hydro QGP simulations**.

$$T^{\mu\nu} = T_0^{\mu\nu} + T_1^{\mu\nu}\tau + T_2^{\mu\nu}\tau^2 + \dots$$

Conclusions

- We have performed an exact analytical calculation of the covariance of the energy momentum tensor of the **Glasma** at $\tau = 0^+$, in the framework of the **Color Glass Condensate**.
- We expect to be able to **generalize this framework** by introducing an impact parameter dependence and relaxing some of the original assumptions, which could potentially open the door to **phenomenological applications**.
- The following steps are computing the **time evolution** of our result towards thermalization time $\tau \sim 1/Q_s$, where it can serve as input for **hydro QGP simulations**.

Thanks for your attention

Back-up: Expressions of two first orders of Nc expansion

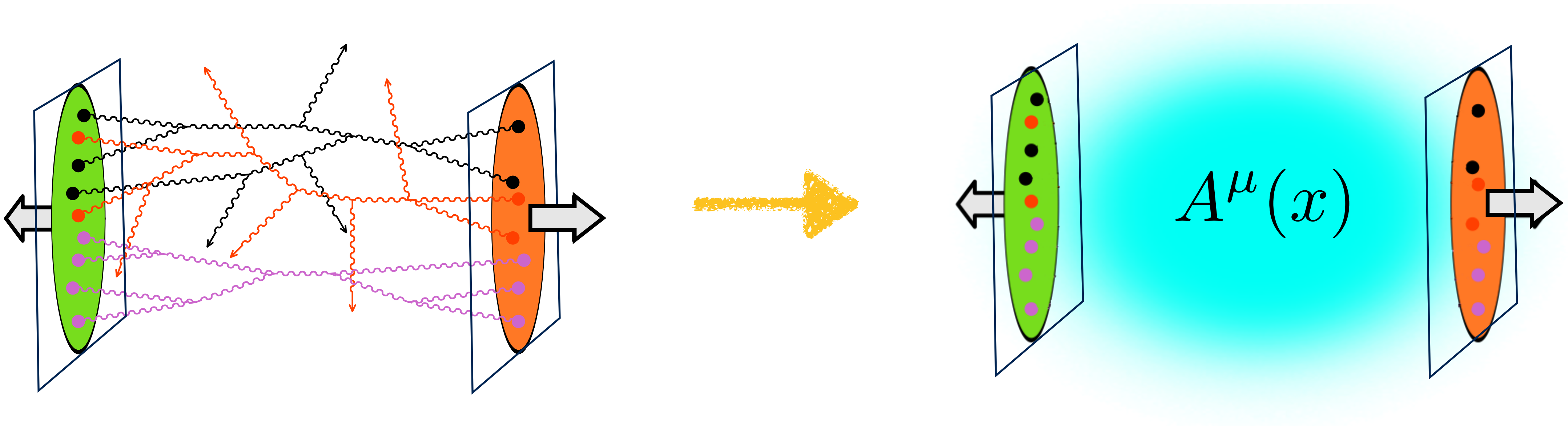
- Leading order:

$$\left[\mathrm{Cov}[\epsilon_{\mathrm{MV}}](0^+;x_\perp,y_\perp)\right]_{N_c^0} = \frac{1}{4g^4\,r^8} e^{-\frac{r^2}{2}\left(Q_{s1}^2+Q_{s2}^2\right)} \Bigg(128 + 128\left(e^{\frac{Q_{s1}^2 r^2}{2}} + e^{\frac{Q_{s2}^2 r^2}{2}}\right)$$
$$- \left[256 e^{\frac{Q_{s1}^2 r^2}{4}} + 16 e^{\frac{r^2}{4}\left(2Q_{s1}^2+Q_{s2}^2\right)}\left(Q_{s2}^4 r^4 + 8Q_{s2}^2 r^2 - 2Q_{s1}^4 r^4 + 48\right)\right] - [1 \leftrightarrow 2]$$
$$-e^{\frac{r^2}{4}\left(Q_{s1}^2+Q_{s2}^2\right)}\Big[Q_{s1}^4 Q_{s2}^4 r^8 + 4Q_{s1}^2 Q_{s2}^2 r^6\left(Q_{s1}^2+Q_{s2}^2\right)$$
$$+ 128 r^2\left(Q_{s1}^2+Q_{s2}^2\right) + 16 r^4\left(Q_{s1}^2+Q_{s2}^2\right)^2 + 1024\Big]$$
$$+8 e^{\frac{r^2}{2}\left(Q_{s1}^2+Q_{s2}^2\right)}\left[-4r^4\left(Q_{s1}^4+Q_{s2}^4\right) + Q_{s1}^2 Q_{s2}^2 r^6\left(Q_{s1}^2+Q_{s2}^2\right) + 80\right]\Bigg)$$

- First correction:

$$\left[\mathrm{Cov}[\epsilon_{\mathrm{MV}}](0^+;x_\perp,y_\perp)\right]_{N_c^{-2}} = \frac{1}{4N_c^2 g^4\,r^8} e^{-\frac{r^2}{2}\left(Q_{s1}^2+Q_{s2}^2\right)} \Bigg(16\left(Q_{s1}^2 r^2 + Q_{s2}^2 r^2 + 8\right)^2$$
$$+ \left[16 Q_{s1}^2 r^2 (8 + Q_{s1}^2 r^2) e^{\frac{Q_{s2}^2 r^2}{2}} - 32(8 + Q_{s1}^2 r^2)(4 + Q_{s1}^2 r^2) e^{\frac{Q_{s2}^2 r^2}{4}}\right] + [1 \leftrightarrow 2]$$
$$+ \left[16\, r^2 e^{\frac{1}{4} r^2\left(Q_{s1}^2+2Q_{s2}^2\right)}\left(r^2\left(Q_{s1}^4 - 2Q_{s2}^4\right) + 8\left(Q_{s1}^2 + 2Q_{s2}^2\right)\right)\right] + [1 \leftrightarrow 2]$$
$$-8 e^{\frac{1}{2} r^2\left(Q_{s1}^2+Q_{s2}^2\right)} r^2\left(Q_{s1}^2+Q_{s2}^2\right)\left(Q_{s1}^2 Q_{s2}^2 r^4 - 4r^2\left(Q_{s1}^2+Q_{s2}^2\right) + 32\right)$$
$$+ e^{\frac{1}{4} r^2\left(Q_{s1}^2+Q_{s2}^2\right)}\Big[Q_{s1}^4 Q_{s2}^4 r^8 + 4Q_{s1}^2 Q_{s2}^2 r^6\left(Q_{s1}^2+Q_{s2}^2\right) + 128 r^2\left(Q_{s1}^2+Q_{s2}^2\right)$$
$$+16 r^4\left(Q_{s1}^2+Q_{s2}^2\right)^2 - 1024\Big]\Bigg)$$

- We use an approximation of QCD for high gluon densities where we replace the gluons with a **classical field** generated by the valence quarks



- Dynamics of the field described by **Yang-Mills** classical equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho(x)$$

- Calculation of observables: **average** over background classical fields

- Basic building block: (**generalized**) **2-point correlator**

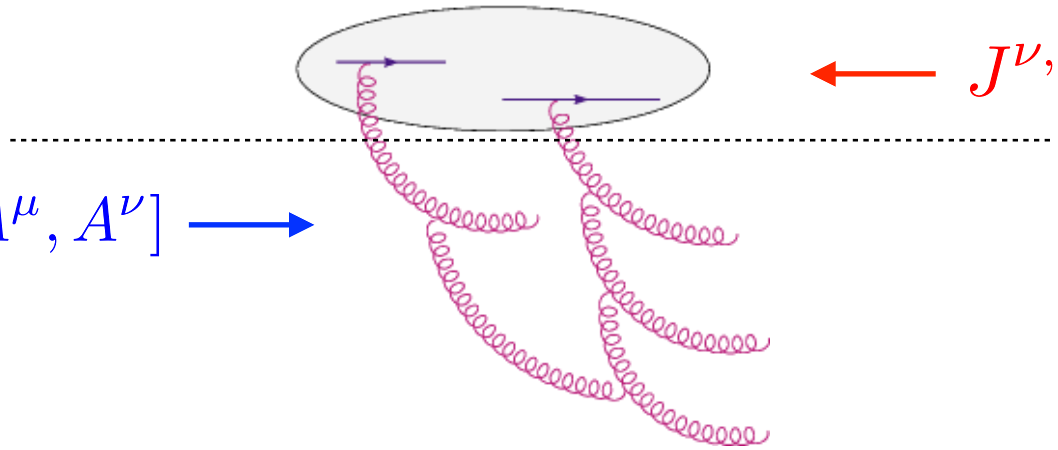
Non-Gaussianities

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)h(b_\perp)\delta^{ab}\delta(x^- - y^-)f(x_\perp - y_\perp)$$

Back-up: More about the Color Glass Condensate

- Separation of 'slow' and 'fast' degrees of freedom

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig\,[A^\mu, A^\nu]$ ⟶



⟵ $J^{\nu,a} = \delta^{\nu +}\rho^a(x^-, x_\perp)$

- Dynamic relation given by solution to classical Yang-Mills equations:

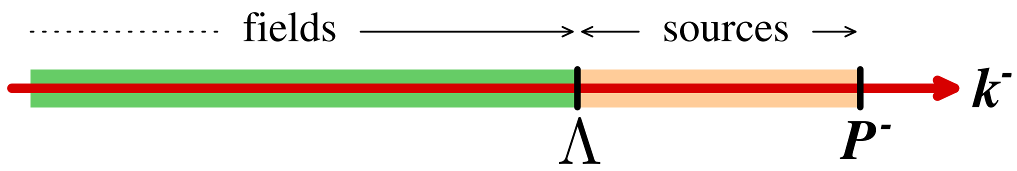
$$[D_\mu,\, F^{\mu\nu,a}] = J^{\nu,a}$$

- Calculation of observable quantities: average over color sources

$$\langle \mathcal{O} \rangle = \int [D\rho]\, W_\Lambda[\rho]\mathcal{O}[\rho]$$

- Scale (in)dependence: JIMWLK equations

$$\frac{\partial W_\Lambda}{\partial \log \Lambda} = \mathcal{H} W_\Lambda$$



- McLerran-Venugopalan model: W_Λ is a Gaussian distribution

$$\langle \rho^a(x^-, x_\perp)\rho^b(y^-, y_\perp)\rangle = \mu^2(x^-)\delta^{ab}\delta^2(x_\perp - y_\perp)\delta(x^- - y^-)$$