Warped Massive Gravity

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PASCOS, CWRU, June 5, 2018

Massive gauge and gravitational fields:

- Mass due to "matter" in a lab

  Plasma, superconductors, charged condensates,..

- Mass due to a relativistic Higgs vacuum

  Our universe permiatted by the Higgs VEV

  Degrees of freedom of the massive gauge boson/graviton: often there are more degrees of freedom, beyond the longitudinal modes of massive gauge bosons, e.g., the ion acoustic wave in plasma, Higgs boson in a relativistic systems

  For a relativistic theory the Higgs boson needed for a weakly coupled UV completion

Minimal number of degrees of freedom:

- $SU(2)$ massive gauge fields (Vainshtein, Khriplovich '71):

$$\frac{\partial\pi^a\partial\pi^a}{(1+\pi^a\pi^a/v^2)^2}$$

  the strong interaction scale is $v = m/g$

- In pure massive gravity (de Rham, GG, Tolley)

$$\Lambda_3 = (M_{\rm P}\, m^2)^{1/3}$$

- Massive graviton plus new degrees of freedom (this talk)

$$\Lambda_* \sim (M_{\rm P}\, m\bar{H})^{1/3} >> \Lambda_3$$

General considerations (Arkani-Hamed, Georgi, Schwartz)

The longitudinal mode gets kinetic term via mixing with a tensor:

$$\mathcal{L}_2 = (\partial h)^2 + m^2 h\partial\partial\pi + hT$$

Mass scale is irrelevant in the leading approxomation:

$$\mathcal{L}_2 = (\partial \hat{h})^2 - m^4(\partial\pi)^2 + \hat{h}T + m^2\pi T$$

and rescale, $\pi \to \pi/m^2$.

$$m^4\pi(\partial\partial\pi)^2 \to \frac{\pi(\partial\partial\pi)^2}{M_{\mathrm{P}} m^2}\,.$$

On curved backgrounds, e.g., AdS:

(Porrati; Kogan, Mouslopoulos, Papazoglou)

On curved backgrounds, e.g., on AdS_4, with $-\Lambda < 0$, one obtains, $-\Lambda m^2 (\partial \pi)^2$. This would raise the strong scale as long as the magnitude of the cosmological constant is large, $\Lambda >> m^2$.

A way to use the above while being in Minkowski, GG

4D massive gravity is embedded into a D-dimensional ($D = 4 + n > 4$) massive gravity with a large curvature scale, $\bar{\Lambda}$; D-dimensional kinetic term for the D-dimensional longitudinal mode, $\Pi(x^\mu, z^1, z^2, ..., z^n)$,

$$-M_D^{2+n} \bar{m}^2 \bar{\Lambda} \left(\partial_D \Pi(x^\mu, z^1, z^2, ..., z^n) \right)^2$$

where M_D and $\bar{m}$ are the higher-dimensional Planck and graviton mass respectively.

The action and coupling to the matter:

$$\tilde{\mathcal{L}}_2 = (\partial\hat{h})^2 - m^4(\partial\pi)^2 - M_D^{2+n}L^n\bar{m}^2\bar{\Lambda}(\partial\pi)^2 + \hat{h}T + m^2\pi T$$

As long as, $M_D^{2+n}L^n\bar{m}^2\bar{\Lambda} >> m^4$, (in $M_{\mathrm{P}} = 1$ units), all OK

However, the general argument must be incomplete

below the scale of new physics still the old theory; it has to be that

$$m_{KK} < \Lambda_3$$

Example: warped massive gravity, GG

4D massive gravity embedded in 5D AdS massive gravity:
The 5D massive action *(C. de Rham, GG, Tolley)*

$$S_5 = M_5^3 \int d^4x\, dz\, \sqrt{-\bar{g}}\, \left(\bar{R}(\bar{g}) + 2\bar{\Lambda} + 2\bar{m}^2 \mathcal{V}(\bar{\mathcal{K}}^M_N) \right)$$

where

$$\mathcal{V}(\bar{\mathcal{K}}) = \det_2(\bar{\mathcal{K}}) + \beta_3 \det_3(\bar{\mathcal{K}}) + \beta_4 \det_4(\bar{\mathcal{K}}) + \beta_5 \det_5(\bar{\mathcal{K}})$$

with the definition

$$\bar{\mathcal{K}}^A{}_B = \delta^A_B - \sqrt{\bar{g}^{AM} \bar{f}_{MB}}, \qquad \bar{f}_{MN} = \partial_M \Phi^I \partial_N \Phi^J \tilde{f}_{IJ}(\Phi)$$

and $\Phi^J(x^\mu, z)$, $(I, J = 0, 1, 2, 3, 5)$, five scalar Stückelberg fields.
(F. Hassan, R. A. Rosen, arbitrary fiducial metric, bigravity)

The total action:

$$S_{total} = S_5 + S_4 + S_{GH} \,,$$

with S_5 and S_4 defined similarly in 5D and 4D respectively; S_{GH} is the Gibbons-Hawking boundary term.

Bulk boundary connection

$$\bar{g}_{\mu\nu}(x,z)|_{z=0} = g_{\mu\nu}(x)$$

$$\delta^a_J \Phi^J(x,z)|_{z=0} = \varphi^a(x)$$

$$\delta^I_a \delta^J_b \tilde{f}_{IJ}(\Phi)|_{\Phi^z=0} = \eta_{ab}$$

Classical solutions:

The fiducial metric is assumed to be AdS (justified a posteriori)

$$ds^2_{Fid} = \tilde{f}_{IJ} d\Phi^I d\Phi^J = \frac{L^2}{(\Phi^z + L)^2}\left[\eta_{ab} d\Phi^a d\Phi^b + (d\Phi^z)^2\right]$$

Then, the physical metric has a solution ($z > 0$, $\Phi^z > 0$)

$$ds^2 = \bar{g}_{AB} dx^A dx^B = A^2(z)\left[\eta_{\mu\nu} dx^\mu dx^\nu + dz^2\right], \quad A(z) \equiv \frac{L}{z+L}$$

This could be obtained in bigravity, with a weak coupling of the massless graviton

$$\tilde{M}_5^3 \int d^5\Phi \sqrt{\tilde{f}(\Phi)}\left(R(\tilde{f}(\Phi)) + 2\bar{\Lambda}\right)$$

Linearized theory – the action:

$$\mathcal{L}_{5D} = M_5^3 \sqrt{\bar{g}^{AdS}} \left(-\bar{h}_{AB} \bar{\mathcal{E}}^{ACBD} \bar{h}_{CD} - \frac{\bar{m}^2}{2} (\tilde{h}_{AB}^2 - \tilde{h}^2) \right)$$

$\bar{\mathcal{E}}^{ACBD}$ is the Einstein operator on AdS_5; the Stückelberg fields, $\Phi^J \delta_J^A = x^A + \frac{1}{\bar{m}} V^A$, enter the Lagrangian via

$$\tilde{h}_{AB} \equiv \bar{h}_{AB} - \frac{1}{\bar{m}} (\nabla_A V_B + \nabla_B V_A) \,,$$

The quadratic part of the 4D Lagrangian reads as follows:

$$\mathcal{L}_{4D} = M_4^2 \left(-h_{\mu\nu} \mathcal{E}^{\mu\alpha\nu\beta} h_{\alpha\beta} - \frac{m^2}{2} ({h'}_{\mu\nu}^2 - {h'}^2) \right) + h_{\mu\nu} T^{\mu\nu}$$

$\mathcal{E}^{\mu\alpha\nu\beta}$ is the Einstein operator for 4D Minkowski space, h' is

$${h'}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{\bar{m}} (\partial_\mu v_\nu + \partial_\nu v_\mu)$$

all contractions by $\eta^{\mu\nu}$. The boundary terms (GG, Pirtskhalava)

Linearized theory – the spectrum:

The linearized theory is continuous in the following massless limit:

$$m \to 0, \quad \bar{m} \to 0, \quad m/\bar{m} \to 0, \quad \bar{\Lambda} = \mathit{fixed}$$

In the above limit the spectrum consists of:
RS zero mode, KK gravitons, KK vectors and scalars

Away from the limit: the RS zero mode disappears, a resonance in the KK tower (similar to a scalar, Dubovsky, Rubakov, Tinyakov)

The strong coupling is due to the longitudinal mode of the resonance graviton; the latter gets a large kinetic term due to the background

$$-M_5^3 \bar{\Lambda} (\partial \Pi)^2, \quad \Pi = \frac{\Pi^c}{\sqrt{M_5^3 \bar{\Lambda}}} = \frac{\Pi^c}{M_5^{3/2} \bar{H}}$$

Nonlinear interactions:

Bulk generic (C. de Rham, GG, 10)

$$M_5^3 \bar{m}^2 \bar{h} \left(\left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^2 + \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^3 + \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^4 \right)$$

Bulk special (related to total derivatives)

$$M_5^3 \bar{m}^2 \bar{\Lambda} \left(\left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right) \ldots + \left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\Pi}{\bar{m}} \right) \left(\frac{\nabla\nabla\Pi}{\bar{m}} \right)^3 \right)$$

The Π field needs to be canonically normalized

As a result, the 5D strong scale is

$$\Lambda_{5D} \simeq (M_5^{3/2}\bar{m}\bar{H})^{2/7} = \Lambda_{7/2}\left(\frac{\bar{H}}{\bar{m}}\right)^{2/7} >> \Lambda_{7/2}$$

How about the 4D strong scale?
Effective kinetic term of the longitudinal mode, captures all KK's

$$-\frac{M_5^3\bar{\Lambda}}{2}\,\pi(x)\,\sqrt{-\Box_4}\,\frac{K_1(L\sqrt{-\Box_4})}{K_2(L\sqrt{-\Box_4})}\,\pi(x) \tag{1}$$

In the low energy approximation $L\sqrt{-\Box_4} << 1$

$$L\,\frac{M_5^3\bar{\Lambda}}{2}\,\pi(x)\,\Box_4\,\pi(x) \tag{2}$$

The 4D strong scale:

$$-LM_5^3\bar{\Lambda}(\partial\pi)^2 + LM_5^3\bar{m}^2 h\left(\Sigma_{n=1}^3\left(\frac{\partial\partial\pi}{\bar{m}}\right)^n\right) + \frac{LM_5^3\bar{\Lambda}}{\bar{m}}(\partial\pi)^2(\partial\partial\pi)^2\cdots$$

$$\Lambda_* \simeq (M_5^{3/2}\bar{m}\bar{H}^{1/2})^{1/3}$$

$$\Lambda_* \sim (M_{\mathrm{P}}\bar{m}\bar{H})^{1/3} = (\Lambda_2^2\bar{H})^{1/3}$$

For: $\bar{H} \sim 10^{16}\ GeV$, $M_5 \sim 10^{18}\ GeV$, and $\bar{m} \sim m \sim 10^{-42}\ GeV$

$$\Lambda_{5D} \sim GeV, \quad \Lambda_* \sim MeV$$

is some 19 orders of magnitude greater than $\Lambda_3 \sim 10^{-19}\ MeV$.

Outlook

- Can the strong scale be raised even further?

  Tuning between free parameters of the theory in the bulk and on the brane might lead to cancellation of some of the leading nonlinearities and give a higher scale (GG, Pirtskhalava, in progress).

  Inducing the bulk graviton mass by quantum corrections in the 5D AdS bulk (Porrati; Duff, Liu); additional states in the 5D bulk with special boundary conditions. This could lead to a theory with the strong scale at M_5, which would be an ultimate goal (GG, Older, Pirtskhalava, in progress).