Massive Gravity from the top down and bottom up

Claudia de Rham
Imperial College London

THE ROYAL SOCIETY
SF
CASE WESTERN RESERVE UNIVERSITY EST. 1826
think beyond the possible

PASCOS 2018
5th June 2018

Tate Deskins
(@ CWRU)

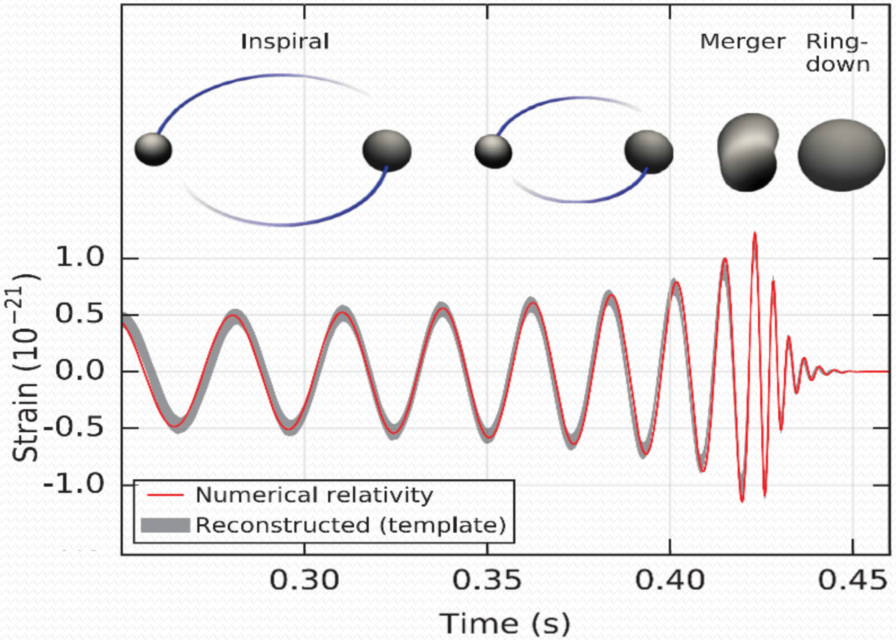
Scott Melville
(@ Imperial)

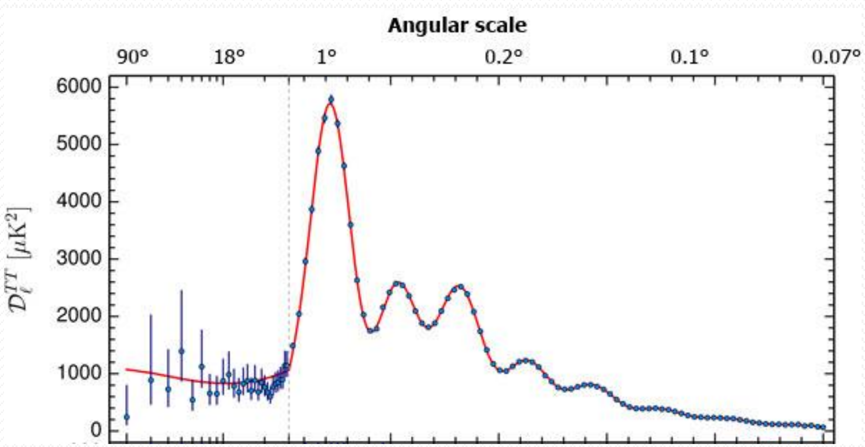
Shuang-Yong Zhou
(@ USTC)

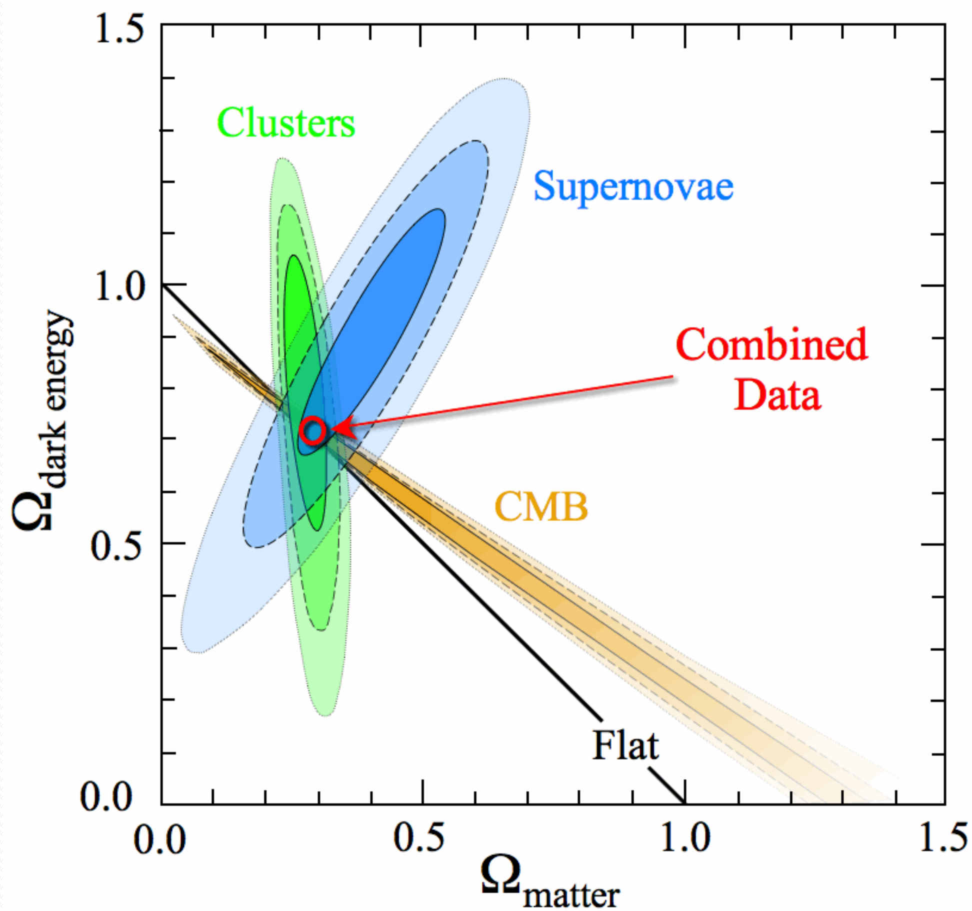
Andrew Tolley
(@ Imperial)

CdR, Deskins, Tolley & Zhou, 1606.08462, RMP
CdR, Melville, Tolley & Zhou, 1702.06134 & 1702.08577
CdR, Melville, Tolley & Zhou, 1706.02712 & 1804.10624
CdR, Melville & Tolley, 1710.09611

A century of success

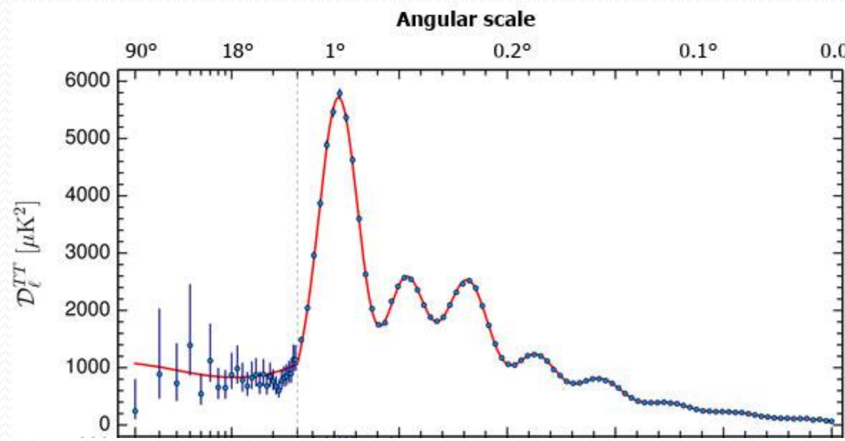






A century of success

With open questions…



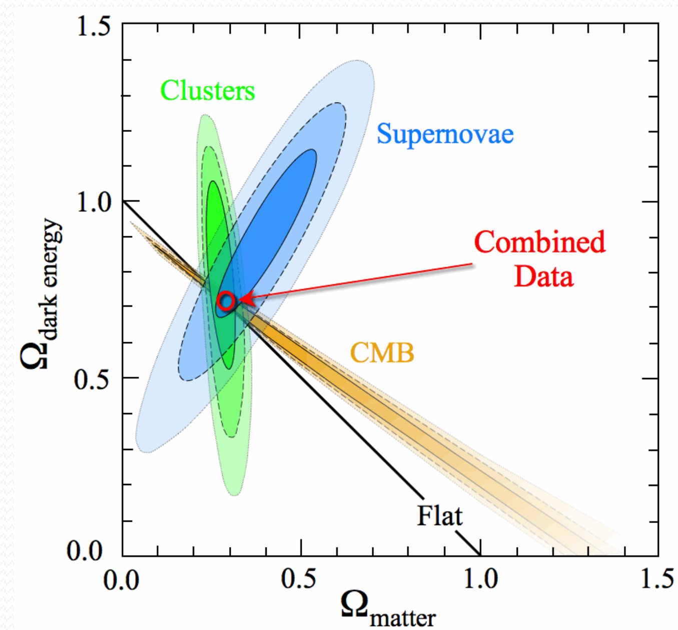
Nature of Inflaton

Pre Big Bang ? Alternatives to Inflation?

Nature of Dark Energy

Cosmological Constant Problem

Nature of Dark Matter



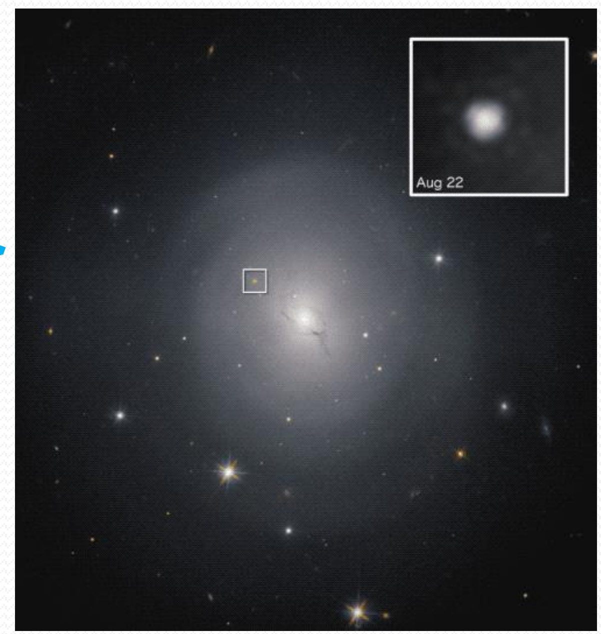
- There has recently been an explosion of models that can play important roles for cosmology
(eg. DBI, K-inflation, G-inflation, gauge inflation, ghost inflation, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, generalized galileon, Horndeski, beyond Horndeski, beyond beyond Horndeski, Fab4, beyond Fab4, EST, DHOST, K-essence, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, mimetic gravity, unimodular gravity, dipolar dark matter, …, …)
\+ Lorentz breaking models

Setting different EFTs apart

- We could simply wait for observations to tell them apart

(eg. DBI, K-inflation, G-inflation, gauge inflation, ghost inflation, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ghost condensate, Galileon, ~~generalized galileon~~, ~~Horndeski~~, beyond Horndeski, beyond beyond Horndeski, ~~Fab4~~, beyond Fab4, EST, DHOST, K-essence, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, mimetic gravity, unimodular gravity, dipolar dark matter, ..., ...)

GW&GBR 170817



Setting different EFTs apart

- We could simply wait for observations to tell them apart Already doing well ! (but models evolve with observations…)

In parallel, we can question their theoretical consistency

Do these theories:

1. preserve perturbative unitarity ?
2. have any chance of ever admitting
   a standard Wilsonian UV completion ?
3. … 4. … causal, well-posedness, caustics, …

See 1703.00025
with Scott Melville

Do these theories:

1. preserve perturbative unitarity ?
2. have any chance of ever admitting
   a standard Wilsonian UV completion ?

3. … 4. … causal, well-posedness, caustics, …

work to appear with Jonathan Crabbé

“Standard” UV completion – should we care ???

- By “standard” UV completion, mean

Unitary,

Lorentz-invariant,

Local (to some extend),

Analytic

- Analyticity is implied by causality
- The absence of such a UV completion would have profound consequences for our understanding of UV physics (quantum gravity)

“Standard” UV completion ???

- The existence of a “standard” UV completion, ie

Unitary,

Lorentz-invariant,

Local (to some extend),

Analytic (implied by causality)

↓

Implies a set of constraints on any low-energy EFT that manifest as *positivity bounds*

see talks by Andrew Tolley, Austin Joyce, James Bonifacio, Scott Melville

Setting different EFTs apart

- There has recently been an explosion of models that can play important roles for cosmology

(eg. DBI, K-inflation, G-inflation, gauge inflation, ~~ghost inflation~~, Axion Monodromy, Chromo-Natural Inflation, f(R), Chameleon, Symmetron, ~~ghost condensate~~, Galileon, ~~generalized galileon~~, ~~Horndeski~~, beyond Horndeski, beyond beyond Horndeski, ~~Fab4~~, beyond Fab4, EST, DHOST, ~~K-essence~~, DGP, cascading gravity, massive gravity, minimal massive gravity, bi-gravity, multi-gravity, mass-varying massive gravity, f(R) massive gravity, mass-varying massive gravity, quasi-dilaton, extended quasi-dilaton, superfuid dark matter, Proca dark energy, generalized Proca, beyond generalized Proca, gauge field dark energy, Galileon genesis, extended Galileon genesis, SLED, ~~mimetic gravity~~, unimodular gravity, ~~dipolar dark matter~~, ..., ...)

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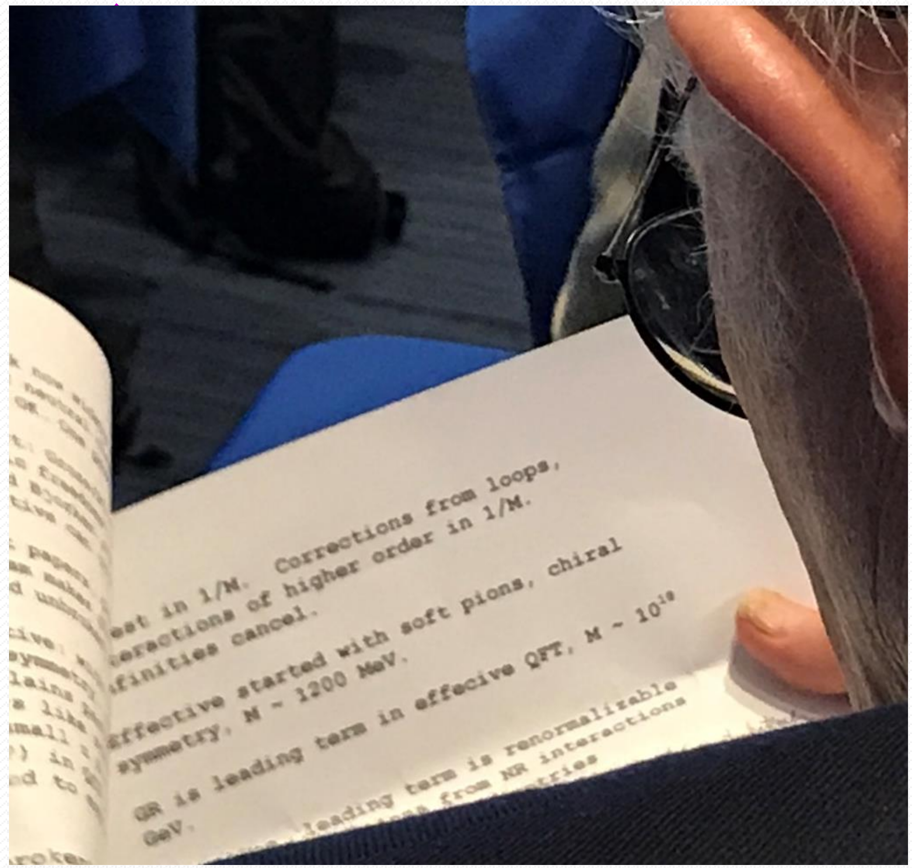
(Massive) spin-2 EFT

$$\mathcal{L} = -\tfrac{1}{2} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \tfrac{1}{2} m^2 \left(h^2 - h^2_{\mu\nu} \right) + \tfrac{1}{M_{\rm Pl}} h_{\mu\nu} T^{\mu\nu}$$

$+$ infinite set of operators

$M_{\rm Pl}$

If $m = 0$,
cutoff is M_{Pl}



Massive spin-2 EFT

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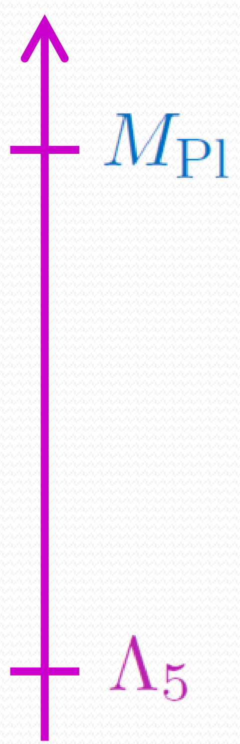
$$+ \text{ infinite set of operators}$$

If $m = 0$,
cutoff is M_{Pl}

If $m \neq 0$,
cutoff is typically

$$\Lambda_5 = (M_{\rm Pl} m^4)^{1/5} \ll M_{\rm Pl}$$



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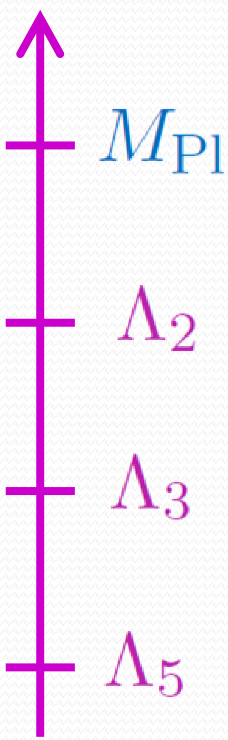


There is a very special class of non-linear theory for which the cutoff (or possibly strong coupling scale) is raised to $\Lambda_3 = (M_{\rm Pl} m^2)^{1/3} \gg \Lambda_5$

Gabadadze, Tolley & CdR, 2010

Or even $\Lambda_2 = (M_{\rm Pl} m)^{1/2}$ on AdS (see Gregory's talk) or on some backgrounds

Tolley, Zhou & CdR 2015

Massive spin-2 EFT

$$\mathcal{L}_{\text{massive spin-2 EFT}} = \mathcal{L}_{\Lambda_3} \;+\; \frac{1}{\Lambda_5^5}\,\text{other operators}$$
$$+\; \frac{1}{\Lambda_4^8}\cdots$$

Positivity bounds on Massive spin-2 EFT

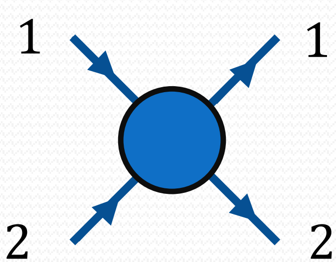
$$\mathcal{L}_{\text{massive spin-2 EFT}} = \mathcal{L}_{\Lambda_3} + \underbrace{\Delta c\,[h^3] + \Delta d\,[h^4]}_{\substack{\text{Operators that contribute} \\ \text{(as far as 2-2 tree level is concerned)}}}$$

Positivity bounds on Massive spin-2 EFT

$$\mathcal{L}_{\text{massive spin-2 EFT}} = \mathcal{L}_{\Lambda_3} + \underbrace{\cancel{\Delta c\,[h^3]} + \Delta d\,[h^4]}$$

Operators that contribute
(as far as 2-2 tree level is concerned)

Eg. of forward scattering



$$|1\rangle = |2\rangle = \alpha_{-2}|-2\rangle + \alpha_{-1}|-1\rangle + \varepsilon|0\rangle + \alpha_{+1}|+1\rangle + \alpha_{+2}|+2\rangle$$

$$\frac{\partial^2}{\partial s^2} f_{\alpha\alpha}\bigg|_{t=0} \propto \underbrace{(\alpha_1^2 - \alpha_{-1}^2)\Delta c\left(\varepsilon^2 + \mathcal{O}\varepsilon^4\right)}$$

Should be positive for any choice of $\alpha_{\pm 1}$

$\longrightarrow \Delta c = 0$

Melville, Tolley, Zhou, CdR 1804.10624

Positivity bounds on Massive spin-2 EFT

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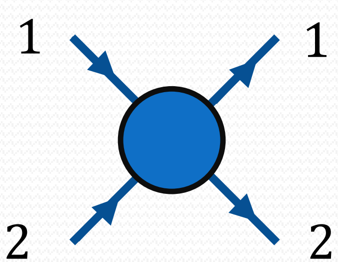
Beyond forward



$$\frac{\partial}{\partial t} f_{\tau_1 \tau_2}(s,t) \propto \frac{m^2}{\Lambda_5^{10}} \Delta d \left(s - 2m^2 + \frac{t}{2} \right) + \mathcal{O}\left(\frac{m^4}{\Lambda_5^{10}} \right) > 0$$

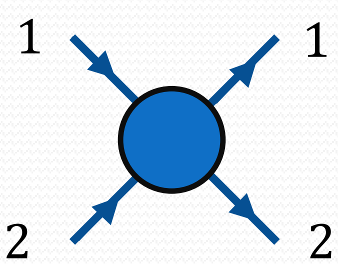
$\longrightarrow \Delta d = 0$

Melville, Tolley, Zhou, CdR 1804.10624

Positivity bounds on Massive spin-2 EFT

$$\mathcal{L}_{\text{massive spin-2 EFT}} = \mathcal{L}_{\Lambda_3} + \underbrace{\Delta c\,\cancel{[h^3]} + \Delta \cancel{d}\,[h^4]}_{\substack{\text{Operators that contribute} \\ \text{(as far as 2-2 tree level is concerned)}}}$$

Beyond forward



As far as the tree-level 2-2 scattering amplitude is concerned, any massive spin-2 EFT needs to carry the ghost-free (Λ_3) parameters to have a chance of enjoying a standard UV completion

Melville, Tolley, Zhou, CdR 1804.10624

Massive Gravity

There is a special class of interactions for which the strong coupling scale is raised to $\Lambda_3=(M_{\rm Pl}m^2)^{1/3}$ (and carries no ghosts)

CdR & Gabadadze, 2010
CdR, Gabadadze & Tolley, 2011

Massive Gravity

There is a special class of interactions for which the strong coupling scale is raised to $\Lambda_3 = (M_{\rm Pl} m^2)^{1/3}$ (and carries no ghosts)

Kinetic term has to be identical as in GR

With Andrew Matas & Tolley, 2013, 2015, 2015, 2015

Massive Gravity

There is a special class of interactions for which the strong coupling scale is raised to $\Lambda_3 = (M_{\rm Pl} m^2)^{1/3}$ (and carries no ghosts)

Matter coupling has to be identical as in GR

Massive Gravity

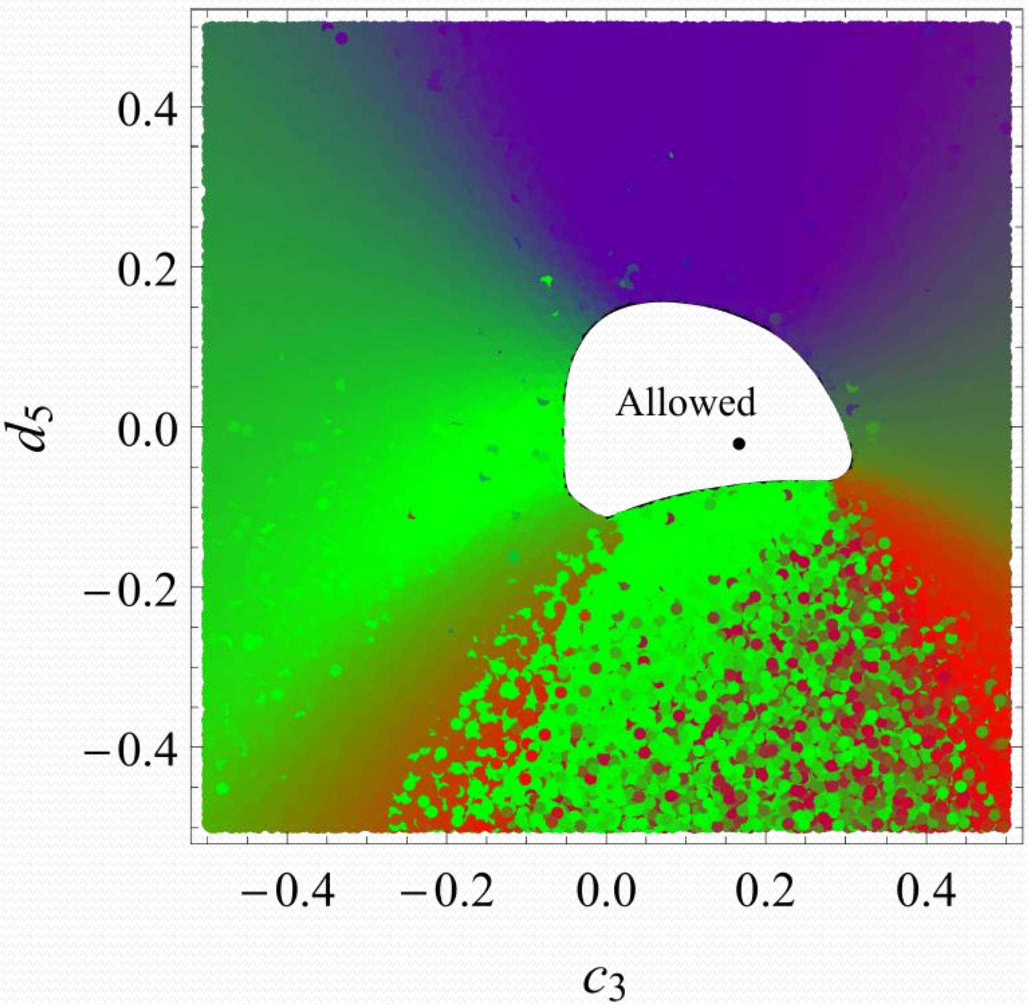
There is a special class of interactions for which the strong coupling scale is raised to $\Lambda_3 = (M_{\rm Pl} m^2)^{1/3}$ (and carries no ghosts)

Can we test such a theory ???

$+m^2 M_{\mu\nu}$

Only 2-parameters + mass scale

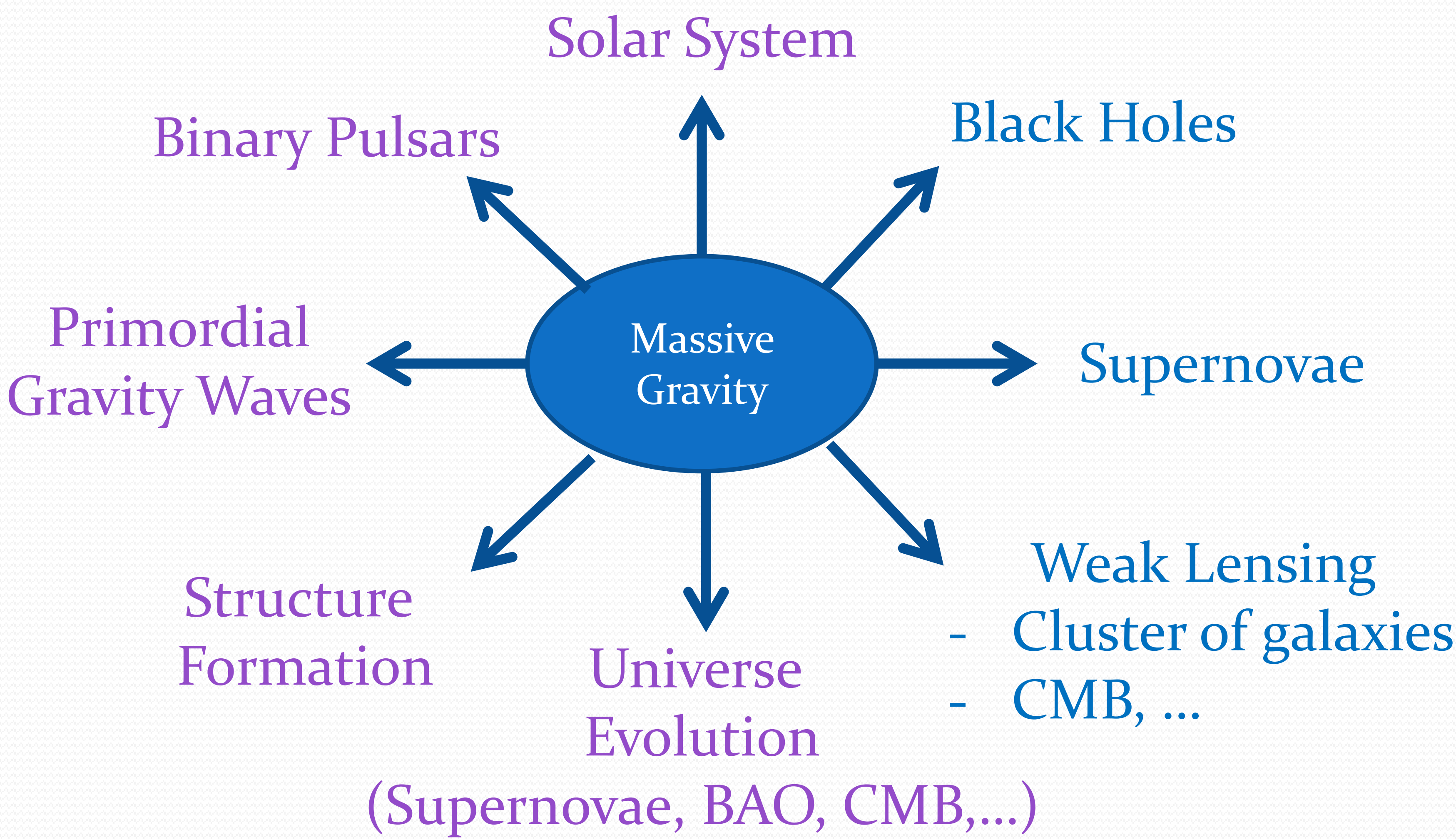
Positivity bounds on Λ_3 massive gravity



$$\mathcal{L} = \mathcal{L}_{\Lambda_3}(c_3, d_5) \cancel{+ \frac{1}{\Lambda_5^5}\text{other operators} + \frac{1}{\Lambda_4^8}\cdots}$$

Cheung & Remmen, JHEP 1604 (2016)

Observational Tests



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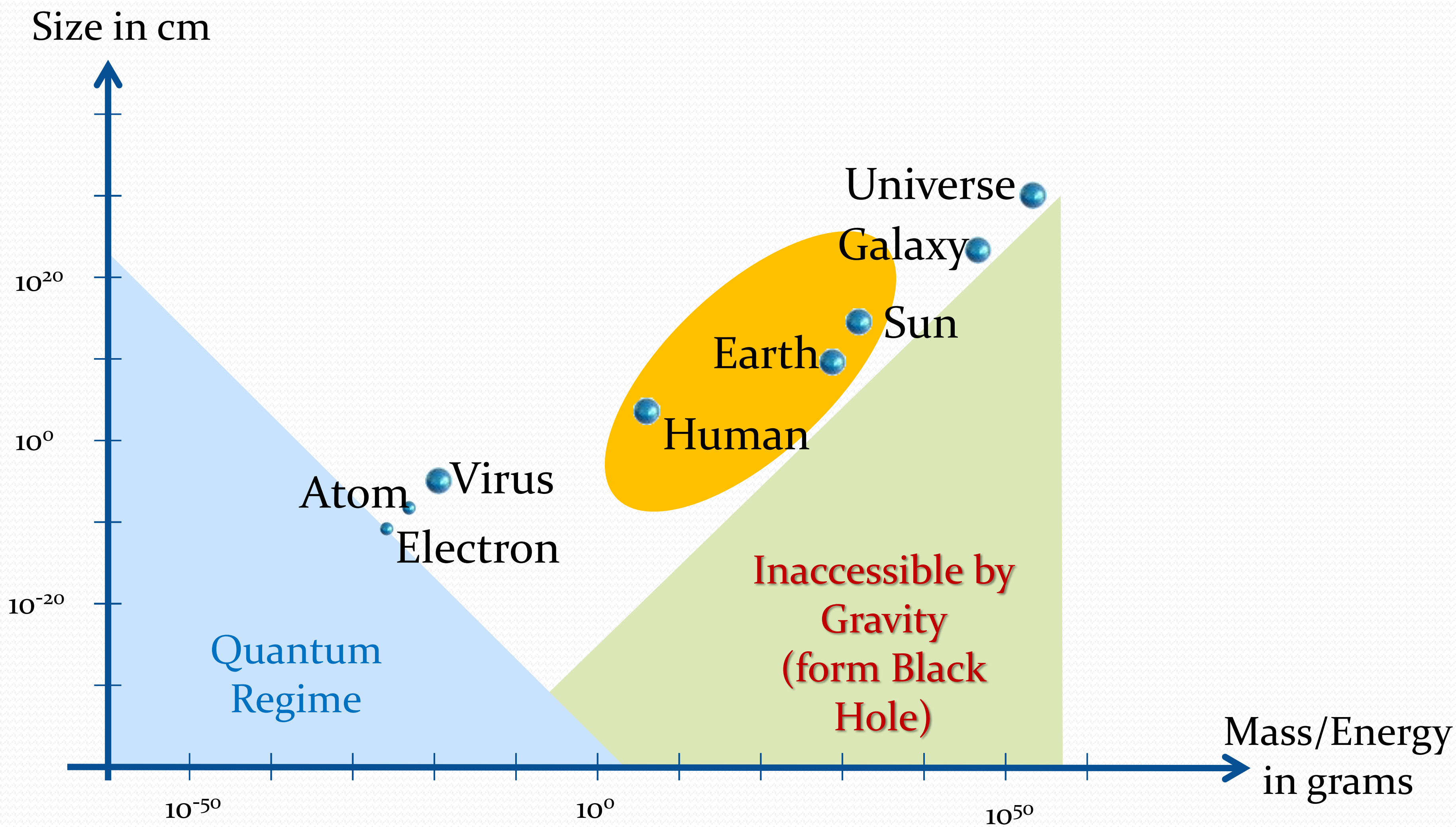
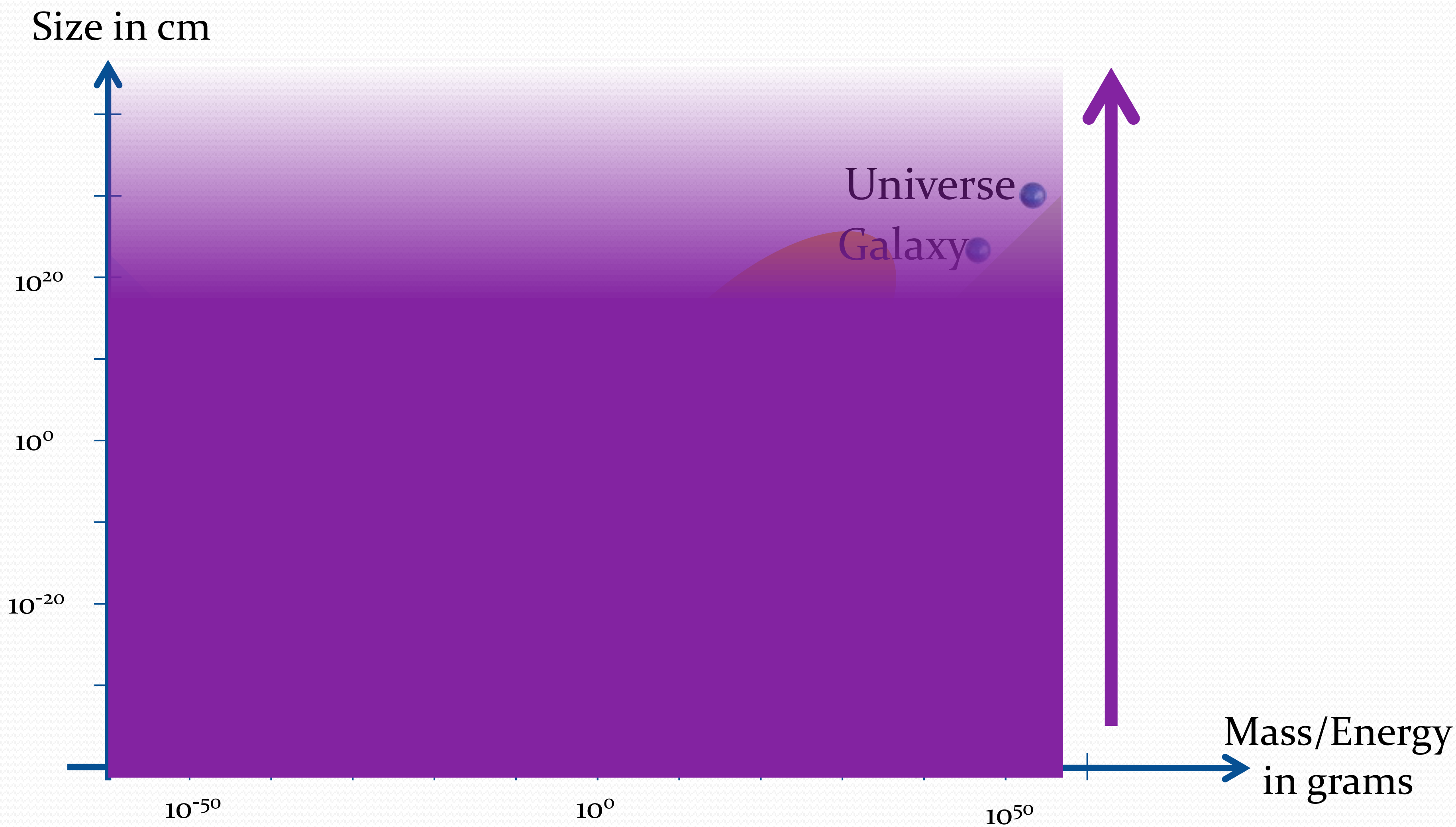


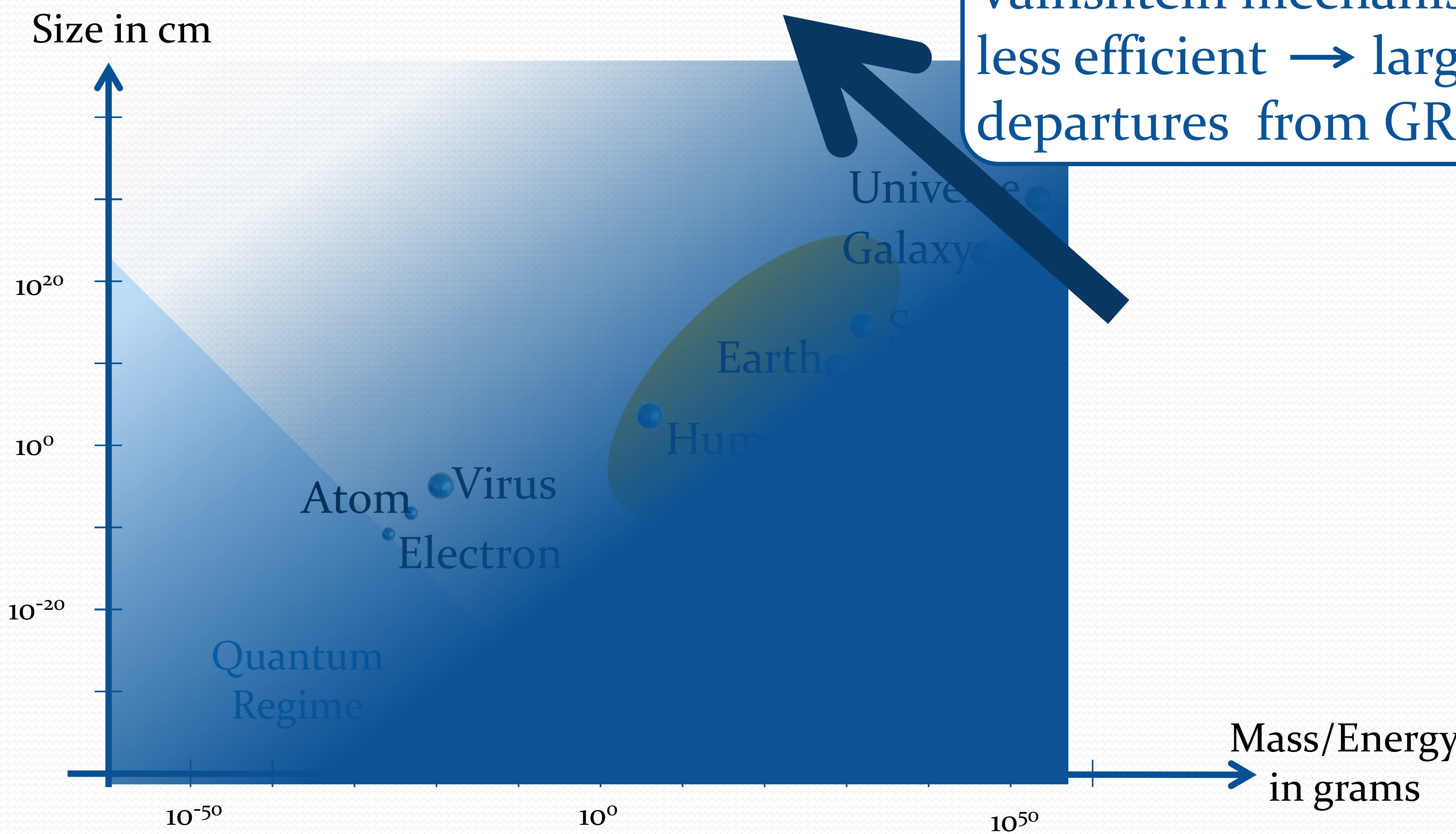
Observational Tests

Effect of mass becomes relevant



Observational Tests

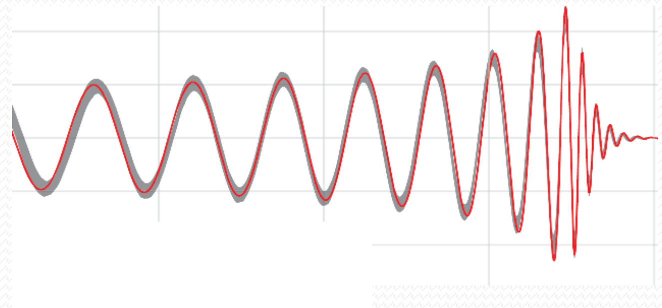
Vainshtein mechanism less efficient ⟶ larger departures from GR



How light is gravity ???

Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

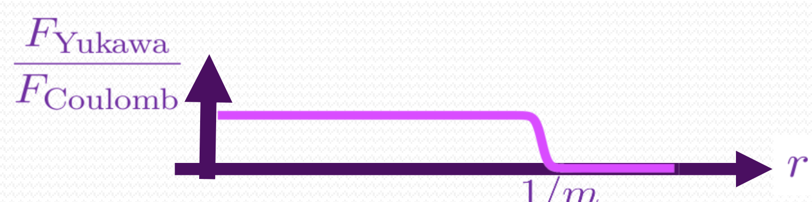


Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-29}	10^{19}	Bound clusters



Deskins, Tolley, Zhou, CdR1606.08462, RMP

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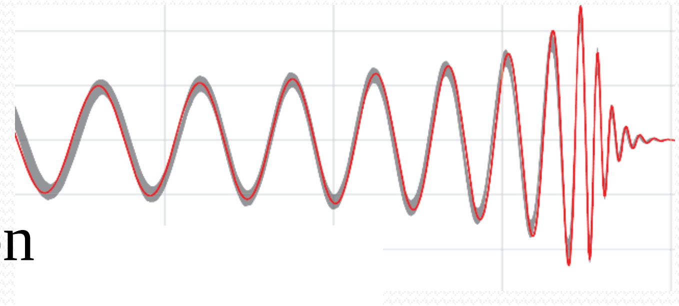
Cleanest
(least model dependent)

Only for models
that carry a helicity-0 mode
(ie. For Local and Lorentz-invariant models)

Deskins, Tolley, Zhou, CdR 1606.08462, RMP

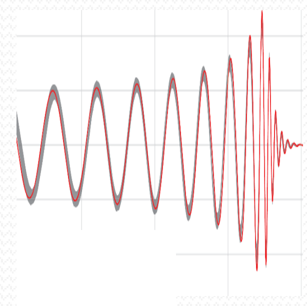
Direct detection of GWs



Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity (including DGP at the level of spectral representation)



GW signal would be more squeezed than in GR

matched filtering technique allows to determine the signal duration when emitted $\Delta\tau_e$ very accurately which can be compared with the signal duration when observed $\Delta\tau_a$.

$$\Delta t = \Delta\tau_a - \Delta\tau_e(1+z)$$

Will 1998

Direct detection of GWs

modifications of the dispersion relation put a bound on the graviton mass

$$m_g \lesssim 4 \times 10^{-22}\mathrm{eV} \left(f\Delta t \frac{f}{100\mathrm{Hz}} \frac{200\mathrm{Mpc}}{D} \right)^{1/2}$$

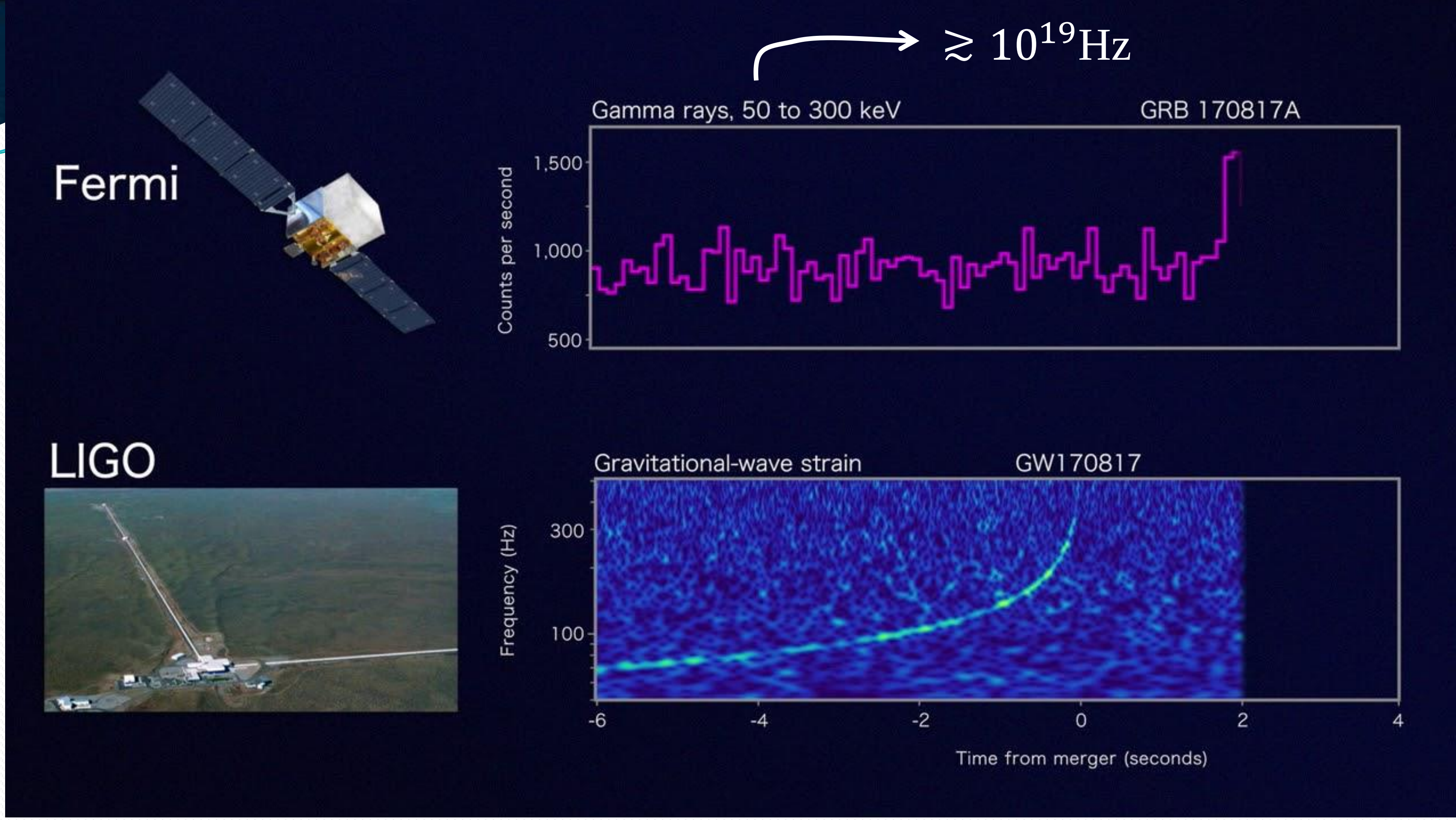
Phase distortion $f\Delta t$ can be measured up to $1/\rho$ (ρ: the signal to noise ratio)

For GW150914,

$$D \sim 400\mathrm{Mpc},\ f \sim 100\mathrm{Hz},\ \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\mathrm{eV}$$

For GW151226, ρ is smaller and the BHs are lighter so f is larger $\longrightarrow$ not as competitive

Will 1998
Abbott et al., 2016



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For GW170817 & GRB170817A

$$\Delta c = |c_\gamma - c_{\mathrm{GW}}| < 10^{-15} \quad \Rightarrow \quad m_g \lesssim 10^{-21}\mathrm{eV}$$

Direct detection of GWs

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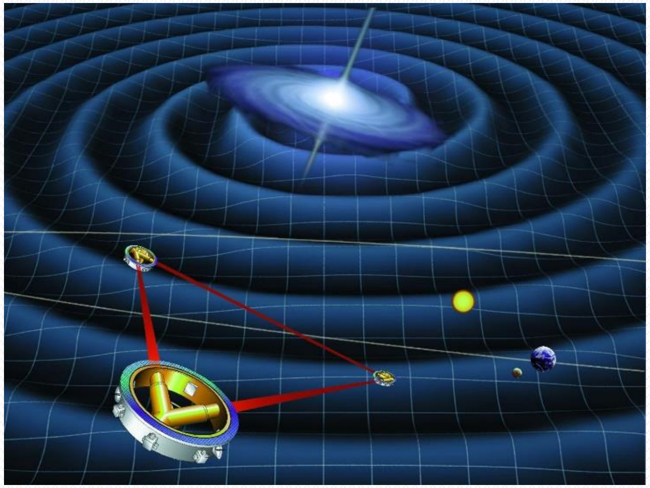


For LISA, could have

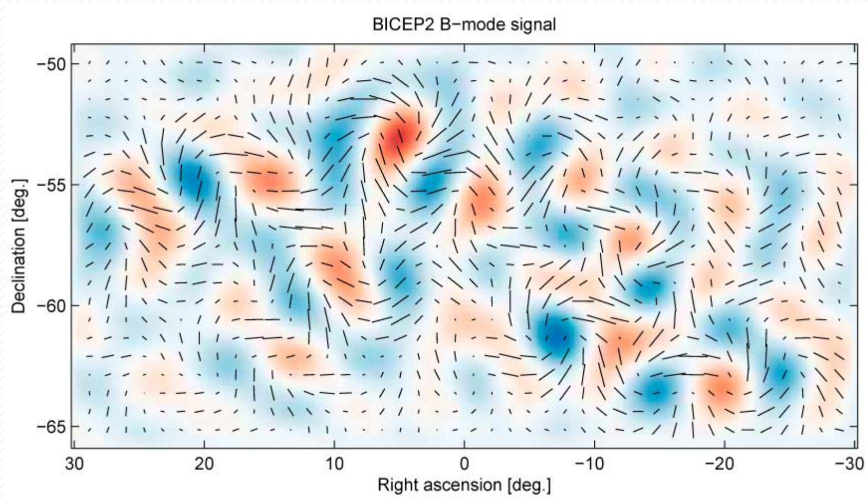
$$\rho \sim 10^3$$

$$D \sim 3\mathrm{Gpc}$$

$$f \sim 10^{-3}\mathrm{Hz}$$

$$\Longrightarrow \quad m_g \lesssim 10^{-26}\mathrm{eV}$$

Bounds from Primordial Gravitational Waves



if ever detected…

would imply the graviton is effectively massless at the time of recombination

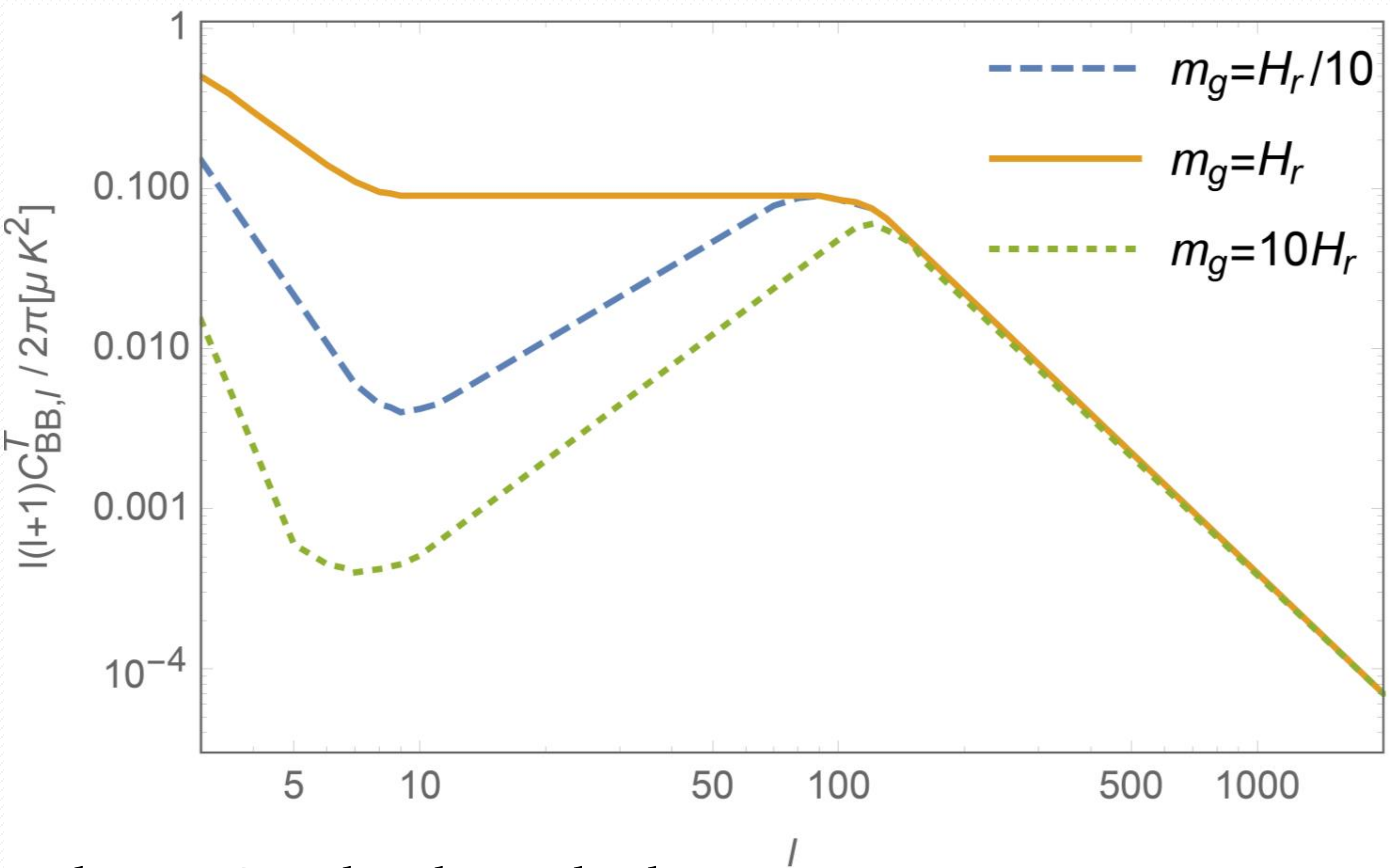
$$m_{\text{eff}} \ll 10^{-29}\text{eV}$$

Dubovsky, Flauger, Starobinsky & Tkachev, 2010
Fasiello & Ribeiro, 2015, (for bi-gravity)
Lin&Ishak, 2016 (Testing gravity using tensor perturbations)

Bounds from Primordial Gravitational Waves

Modification to the tensor mode evolution

$$\mathcal{D}_q''(\tau) + 2\frac{a'}{a}\mathcal{D}_q'(\tau) + \left(q^2 + a^2 m_g^2\right)\mathcal{D}_q(\tau) = J_q(\tau)$$

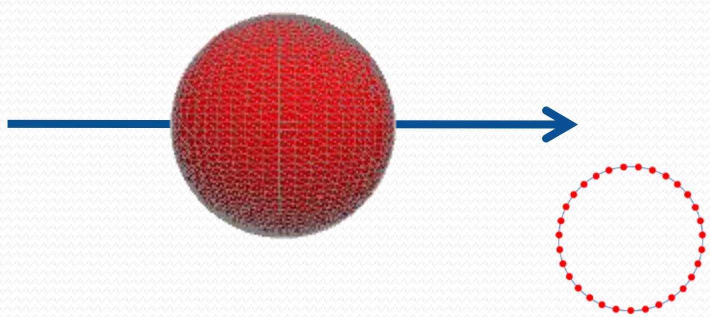


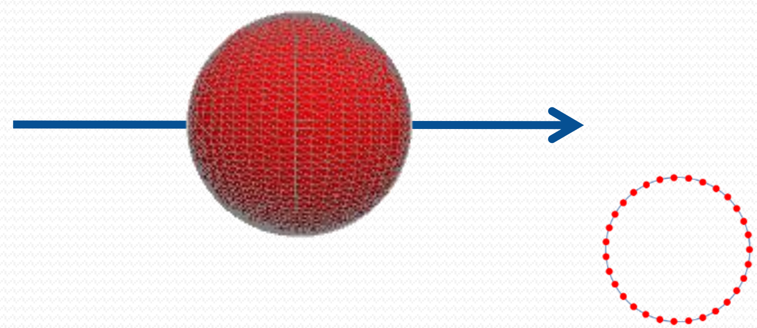
Dubovsky, Flauger, Starobinsky & Tkachev, 2010
Fasiello & Ribeiro, 2015, (for bi-gravity)

Lin&Ishak, 2016

Scalar and Vector modes of the graviton

In a Lorentz invariant theory, a massive graviton also carries:
2 helicity-1 modes and one helicity-0 mode

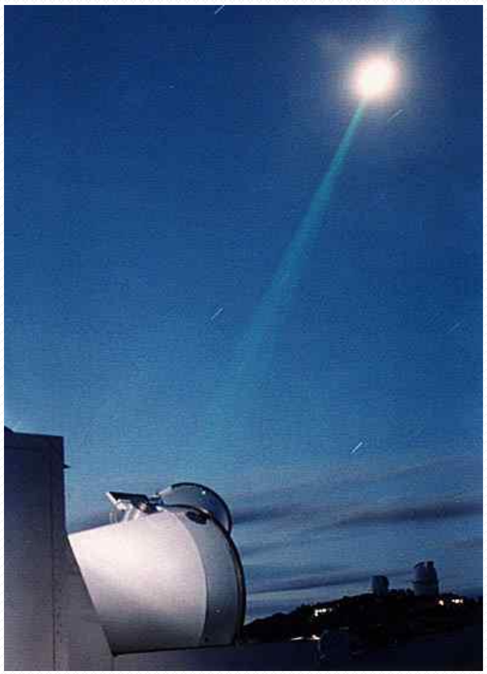




Helicity-0 mode propagates an additional gravitational force that can be very well tested (particularly in the Solar System)

Screened via a Vainshtein mechanism.
In the limit where $M_{\rm Pl} \to \infty, \quad m \to 0, \quad \Lambda_3$ fixed the helicity-0 mode behaves as a Galileon

Lunar Laser Ranging bounds



For DGP, (soft massive gravity, cubic Galileon)

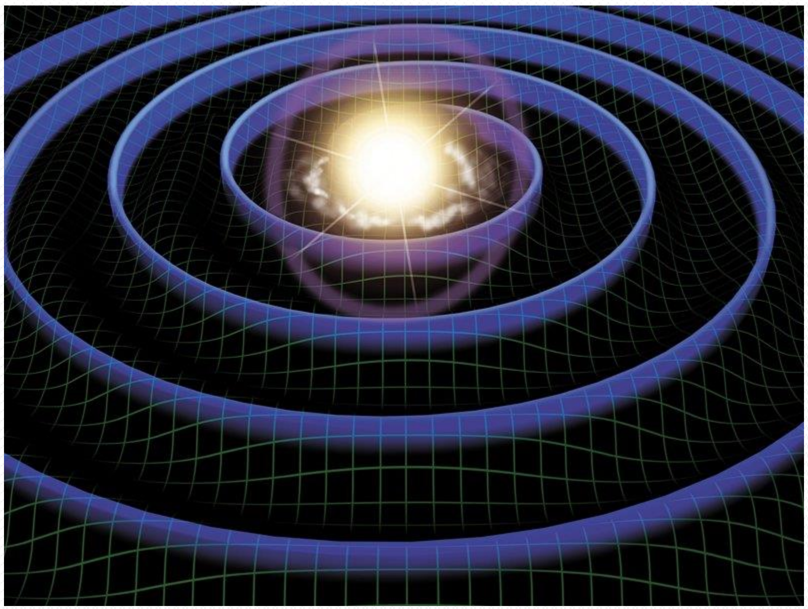
$$m_g \lesssim r^{-1}_{\mathrm{Earth-Moon}} \; \Delta\phi \; \left(\frac{r_{S,\oplus}}{r_{\mathrm{Earth-Moon}}}\right)^{1/2}$$
$$\lesssim 10^{-32}\mathrm{eV}$$

For hard mass graviton, (~ quartic Galileon)

$$m_g \lesssim r^{-1}_{\mathrm{Earth-Moon}} \; \Delta\phi^{3/4} \left(\frac{r_{S,\oplus}}{r_{\mathrm{Earth-Moon}}}\right)^{1/2}$$
$$\lesssim 10^{-30}\mathrm{eV}$$

Radiation into the scalar mode of the graviton

The existence of a scalar mode means new channels of radiation



For Binary systems:

Monopole & dipole exist but are suppressed by conservation of energy & momentum.

Quadrupole emitted by helicity-0 mode is suppressed by Vainshtein mechanism (best understood in a Galileon approximation)

Work with Furqan Dar, Tate Deskins,
John Tom Giblin & Andrew Tolley

Contours of $\dot{\phi}^2$

For the cubic Galileon:
Power still in the quadrupole as in GR
Corrections to GR are very suppressed

Galileon Quadrupole emission

$$P_{\text{Quadrupole}} \sim \frac{(\Omega_P \bar{r})^3}{(\Omega_P r_\star)^{3/2}} \frac{\mathcal{M}^2}{M_{\text{Pl}}^2} \Omega_P^2 \qquad r_*^3 = \frac{1}{M_{\text{Pl}} m^2} \frac{M_{\text{Binary}}}{M_{\text{Pl}}}$$

For the Hulse-Taylor Pulsar $\quad m_g \lesssim 10^{-27}\text{eV}$

- For the Cubic Galileon, higher multipoles are suppressed by additional powers of velocity

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- For the Cubic Galileon, higher multipoles are suppressed by additional powers of velocity
- Massive gravity and stable self-accelerating models always include *at least* a *quartic Galileon*
- In the Quartic Galileon, the angular direction is *not screened as much* as the others $\longrightarrow$ many multipoles contribute to the power with the same magnitude…

$\longrightarrow$ Multipole expansion breaks down

How light is gravity ???

Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-29}	10^{19}	Bound clusters

Cleanest
(least model dependent)

Only for models
that carry a helicity-0 mode
(ie. For Local and Lorentz-invariant models)

Deskins, Tolley, Zhou, CdR 1606.08462, RMP

Summary

- Cosmology has motivated the (re)development of entire new classes of scalar EFTs
- Observations already put strong constraints on some of these models, and particularly on the (effective) graviton mass
- (perturbative) unitarity & analyticity can allow for a better segregation
- Framework not only serves modified gravity but the whole set of EFTs used in cosmology for the description of
  - inflation, (including gauge field inflation, etc…)
  - pre big-bang/bouncing cosmology/other alternatives to inflation
  - dark energy
  - potential framework to tackle the CC problem
  - CFT's
  - …

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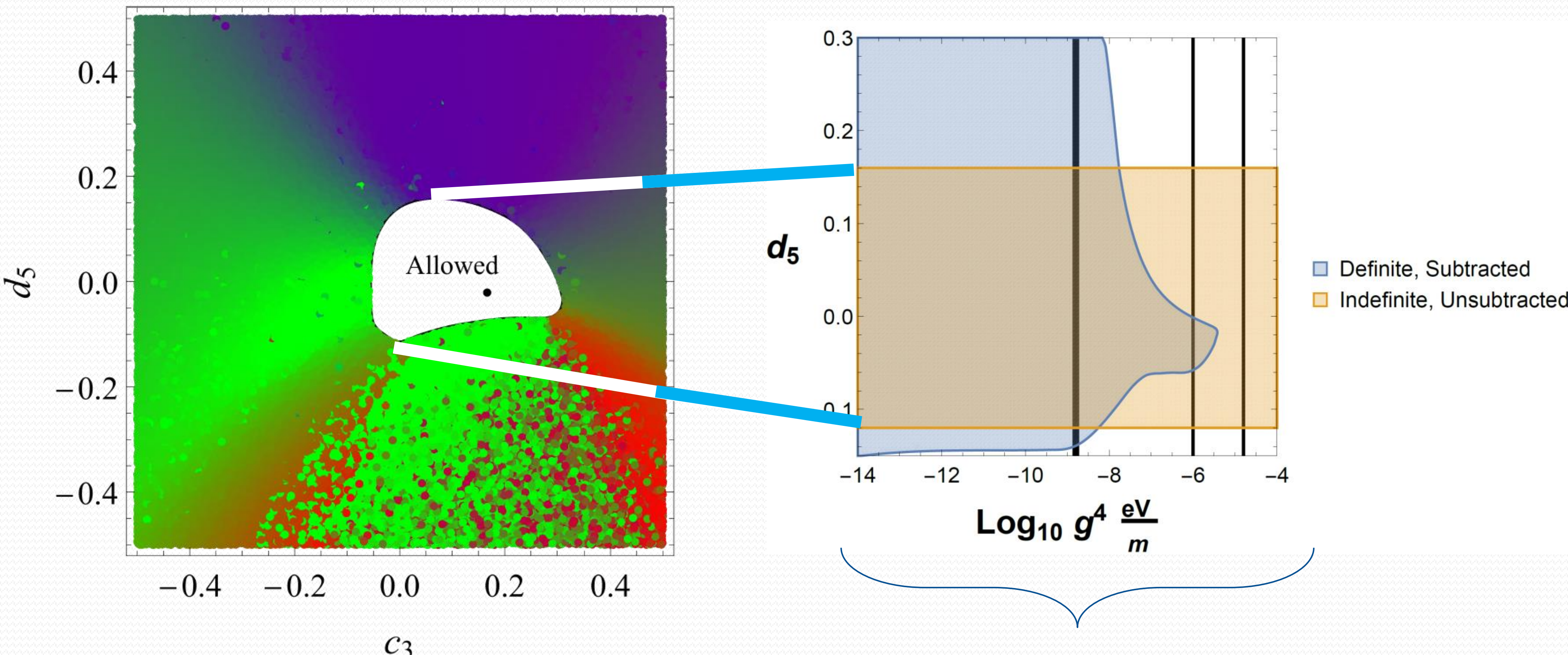
Improved positivity bounds

$$B^{(2,0)}(0) > \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu-2m^2)^3} = \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \sqrt{1-\frac{4m^2}{\mu^2}}\,\frac{\mu\sigma_{\mathrm{total}}(\mu)}{(\mu-2m^2)^3}\,.$$

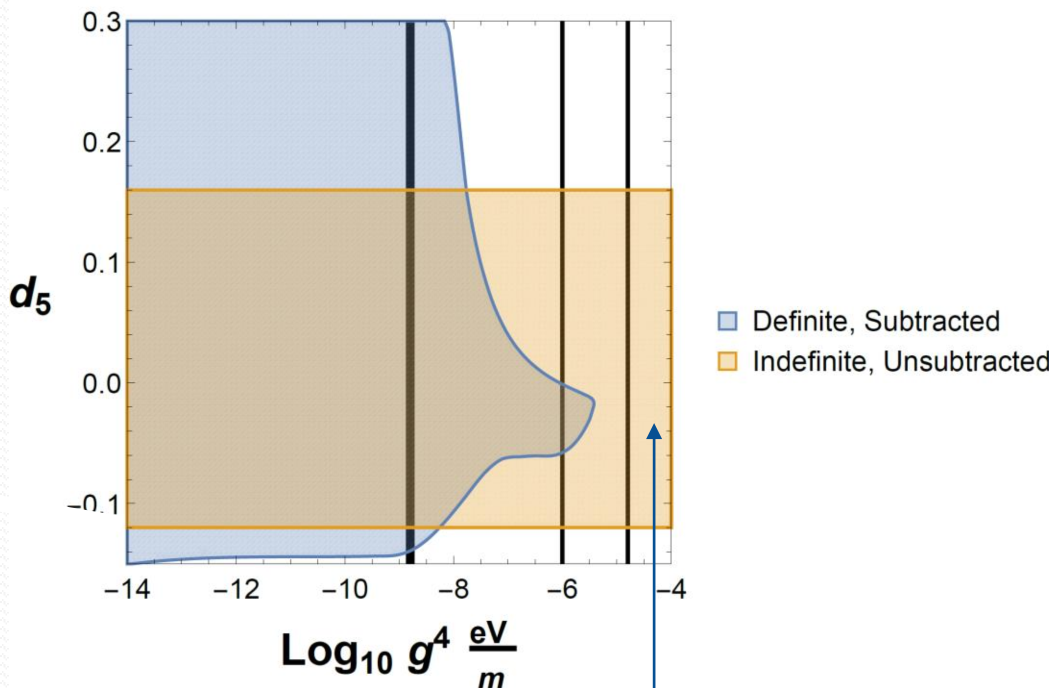


Effectively measures the scale of the cutoff

CdR, Melville, Tolley, 1710.09611

Improved positivity bounds

$$B^{(2,0)}(0) > \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\,A(\mu,t)}{(\mu-2m^2)^3} = \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu\,\sqrt{1-\frac{4m^2}{\mu^2}}\,\frac{\mu\sigma_{\mathrm{total}}(\mu)}{(\mu-2m^2)^3}\,.$$

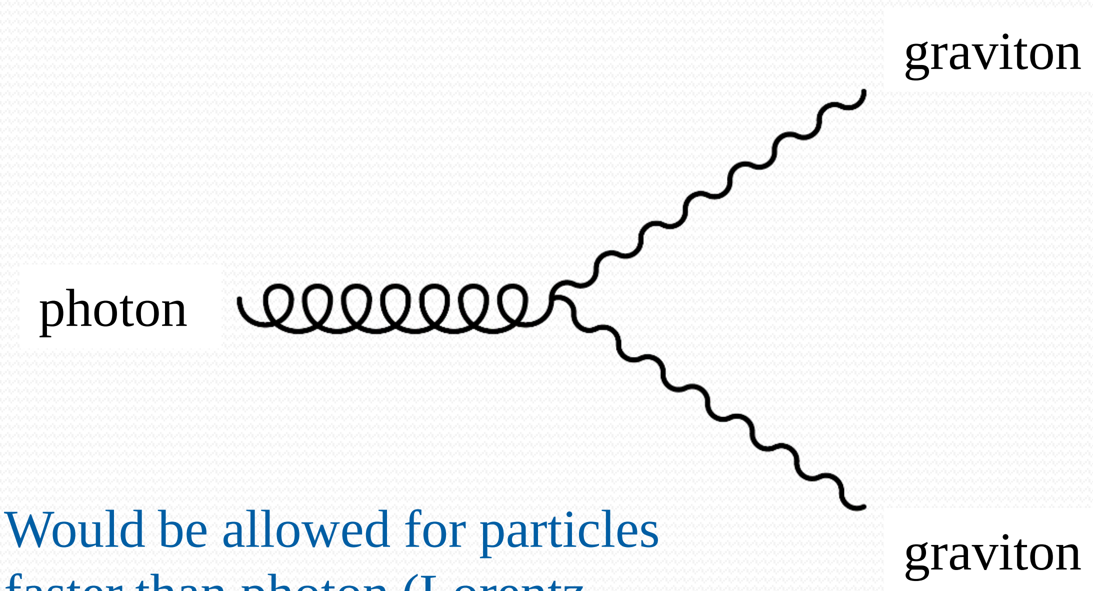


Bellazzini, Riva, Serra, Sgarlata 1710.0253
Assuming a large enough g, ***the improved positivity*** bounds can rule out the allowed parameter space

CdR, Melville, Tolley, 1710.09611: the **improved positivity bounds** should be seen as a **constrain on the value of the cutoff !**

Cherenkov Radiation

Particles traveling faster than GWs could decay into GWs



Forbidden process in
Lorentz invariant models
(if the photon is massless)

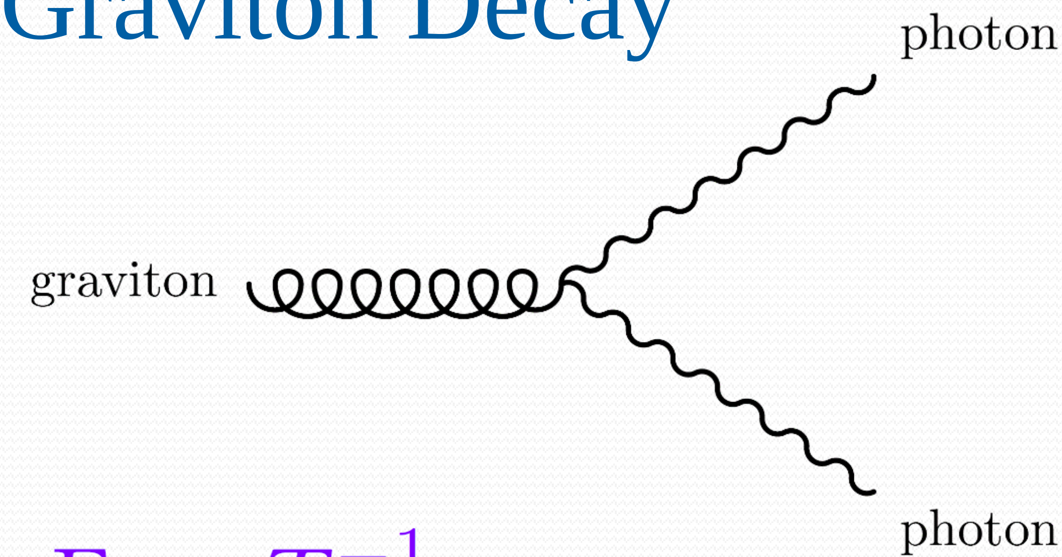
Would be allowed for particles
faster than photon (Lorentz
violating models)

eg. Blas, Ivanov, Sawicki, Sibiryakov1602.04188

Can be used to put bounds on the difference of speeds
but those translate into very weak bounds on the graviton mass

Graviton Decay

If the graviton has a mass:



aLIGO direct detection: $\Gamma \ll T^{-1}_{\mathrm{GW\ travel\ time}}$

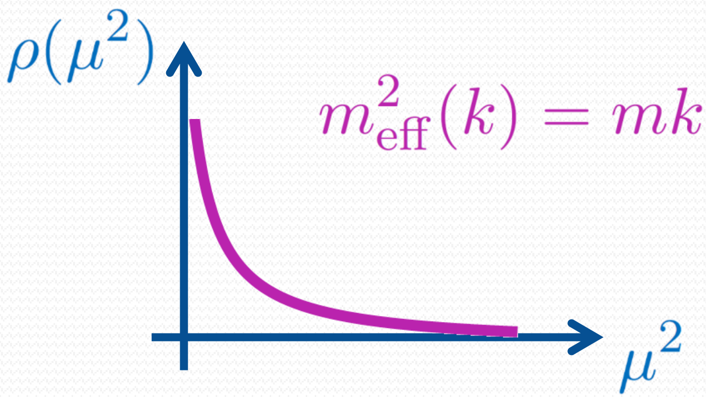
Very weak bound…

Constraints from cosmology: $\Gamma \ll H_{\mathrm{today}}$

$$\mathrm{Im}[m_g^2] \ll H_{\mathrm{today}} \sqrt{\mathrm{Re}[m_g^2]}$$

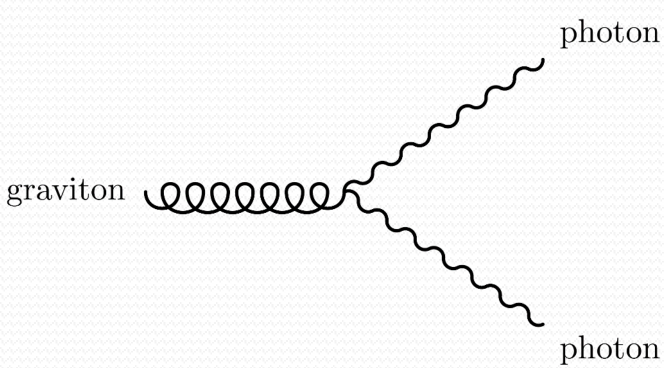
Graviton Decay

If the graviton is a resonance (eg. in DGP, Cascading Gravity,…)



The graviton already has a finite lifetime even without taking into account its possible decay into photons

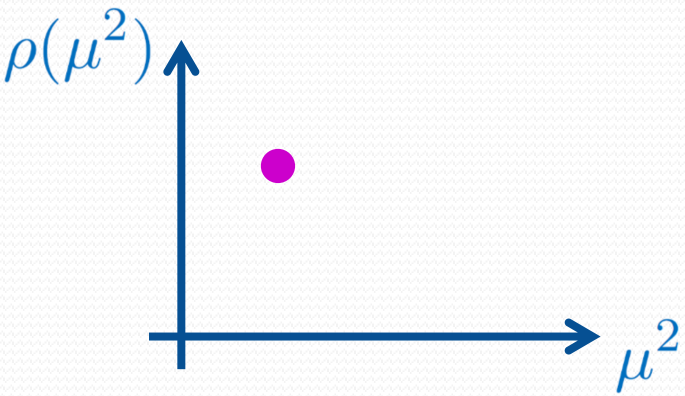
$$m \lesssim H_{\rm today}$$



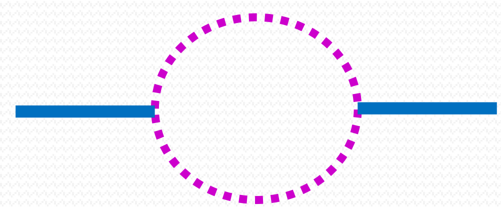
Graviton Decay

For a hard mass graviton

At tree-level, $\mathrm{Im}[m_g^2] = \Gamma = 0$



loop-effect on graviton self-energy

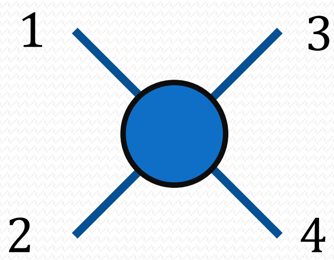


$$\Gamma \sim N \frac{m_g^3}{M_{\mathrm{Pl}}^2}$$

N: total number of light particles that may exist
(photon + axion, hidden sector not subject to SM constraints,…)

$$m_g \lesssim 10^7 \mathrm{eV} \times N^{-1/3}$$

Causality vs Analyticity



$$\mathcal{A} = \int \mathrm{d}^4 z \; \underbrace{e^{ik^0(z^0 - \vec{e}.\vec{z})}}_{\geq 0} \; \langle k_3 \left| G_{\mathrm{ret}}(z) \right| k_1 \rangle$$

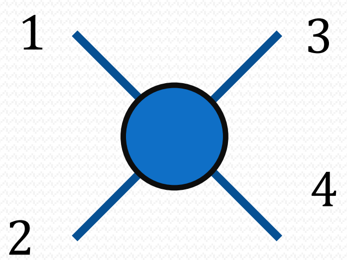
when $G_{\mathrm{ret}}(z) \neq 0$

$\mathcal{A}$ is analytic in k^0 in upper ½ of plane k^0

$$k^0 = \frac{1}{\sqrt{4m^2 - t}} \left(s + t/2 - 2m^2 \right)$$

For fixed $t < 4m^2$
$\mathcal{A}$ is analytic in s in upper ½ of $\mathbb{C}$ plane

Analyticity and Causality



$$\langle k_3 k_4 | \hat{S} | k_1 k_2 \rangle = \langle k_3 | \hat{a}_{k_4} \hat{S} \hat{a}^\dagger_{k_2} | k_1 \rangle$$

Consider the lightest massive particle,
a single such particle is stable against decay $\hat{S}|k\rangle = |k\rangle$

$$\hat{S} = \hat{\mathbb{I}} + i\hat{T}$$

$$i\langle k_3 | \hat{a}_{k_4} \hat{T} \hat{a}^\dagger_{k_2} | k_1 \rangle = \langle k_3 | \left[\hat{a}_{k_4}, \hat{S} \right] \hat{a}^\dagger_{k_2} | k_1 \rangle$$

$$\left[\hat{a}, \hat{S} \right] \sim \frac{\partial \hat{S}}{\partial a^\dagger} \sim -\epsilon(k^0) \int \mathrm{d}^4 x e^{-ikx} \frac{\delta \hat{S}}{\delta \phi(x)}$$

Response to a source (current) $\hat{J}$: $$\frac{\delta \hat{S}}{\delta \hat{\phi}(x)} = -i \hat{J}(x) \hat{S}$$

$$\langle k_3 k_4 | \hat{T} | k_1 k_2 \rangle = (2\pi)^4 \delta^{(4)}(k) \int \mathrm{d}^4 z \, e^{i(k_2 + k_4)z/2} \left\langle k_3 \left| \frac{\delta \hat{J}(-z/2)}{\delta \hat{\phi}(z/2)} \right| k_1 \right\rangle$$

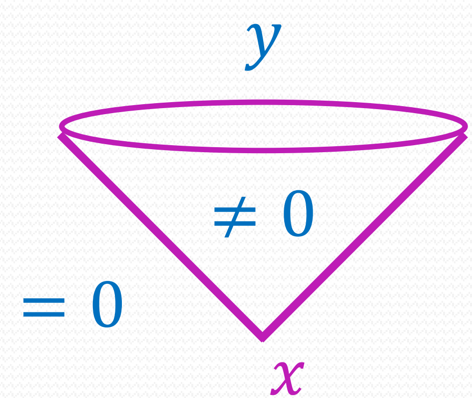
Bogoliubov-Shirkov Causality

$$\langle k_3 k_4 | \hat{T} | k_1 k_2 \rangle = (2\pi)^4 \delta^{(4)}(k) \int \mathrm{d}^4 z \, e^{i(k_2+k_4)z/2} \left\langle k_3 \left| \frac{\delta \hat{J}(-z/2)}{\delta \hat{\phi}(z/2)} \right| k_1 \right\rangle$$

$$\frac{\delta \hat{J}(y)}{\delta \phi(x)} = 0$$

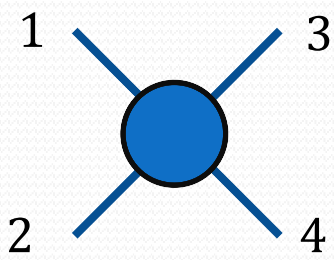
if $\quad x^0 < y^0$

or $\quad (x-y)^2 > 0$



$$\langle k_3 k_4 | \hat{T} | k_1 k_2 \rangle = (2\pi)^4 \delta^{(4)}(k) \overbrace{\int \mathrm{d}^4 z \, e^{i(k_2+k_4)z/2} \langle k_3 \left| G_{\mathrm{ret}}(z) \right| k_1 \rangle}^{\text{Amplitude } A}$$

Breit coordinate system



$$\mathcal{A} = \int \mathrm{d}^4 z \, e^{i(k_2+k_4)z/2} \, \langle k_3 \,| G_{\mathrm{ret}}(z) |\, k_1 \rangle$$

$$k_2 = \left(k^0, -\vec{p} + \lambda \vec{e}\right) \qquad |\vec{e}| = 1, \ \vec{e}.\vec{p} = 0$$

$$k_4 = \left(k^0, \ \ \vec{p} + \lambda \vec{e}\right) \qquad \lambda = \sqrt{k_0^2 - \vec{p}^{\,2} - m^2} \longrightarrow k^0 \text{ at high energy}$$

$$k_1 = \left(\sqrt{\vec{p}^{\,2} + m^2}, \ \ \vec{p}\right)$$

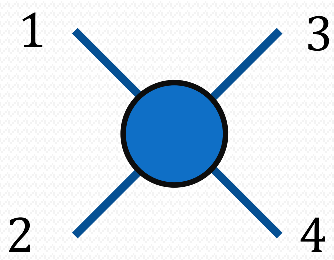
$$k_3 = \left(\sqrt{\vec{p}^{\,2} + m^2}, -\vec{p}\right)$$

$$e^{i(k_2+k_4)z/2} = e^{ik^0 \underbrace{(z^0 - \vec{e}.\vec{z})}_{\geq 0}}$$

is analytic in k^0 in upper ½ of plane k^0

when $G_{\mathrm{ret}}(z) \neq 0$

Breit coordinate system



$$\mathcal{A} = \int \mathrm{d}^4 z\, e^{i(k_2+k_4)z/2} \left\langle k_3 \left| G_{\mathrm{ret}}(z) \right| k_1 \right\rangle$$

$$k_2 = \left(k^0, -\vec{p} + \lambda\vec{e}\right)$$

$$k_4 = \left(k^0, \quad \vec{p} + \lambda\vec{e}\right)$$

$$k_1 = \left(\sqrt{\vec{p}^2 + m^2}, \quad \vec{p}\right)$$

$$k_3 = \left(\sqrt{\vec{p}^2 + m^2}, -\vec{p}\right)$$

$$k^0 = \frac{1}{\sqrt{4m^2 - t}}\left(s + t/2 - 2m^2\right)$$

For fixed $t < 4m^2$

$\mathcal{A}$ is analytic in s in upper ½ of ℂ plane

$\mathcal{A}$ is analytic in k^0 in upper ½ of plane k^0

Weak Coupling

$$A_{\text{tree}}(s,0) \sim c_1 \frac{s^2}{\Lambda^4} + c_2 \frac{s^4}{\Lambda^8} + \dots$$

- Positivity bound imposes $c_1 > 0$
- At one loop, renormalization contributes at order Λ^{-8}
  → would need to specify how to separate tree and loops before we can impose $c_2 > 0$
- Issue if there is only one scale in the problem (theory strongly coupled at Λ)

1. Weak Coupling (UV theory is weakly coupled)

$$\mathcal{A}_{\text{tree}}(s,0) \sim g\left(\tilde{c}_1 \frac{s^2}{\Lambda^4} + \tilde{c}_2 \frac{s^4}{\Lambda^8} + \cdots\right) \qquad \mathcal{A}_{1-\text{loop}} \sim g^2$$

Alternatives to Weak Coupling

2. Massive Galileons (or $P(X)$)

$$A_{\text{tree}}(s,\theta) \sim \left(d_1(\theta)\frac{m^2 s^2}{\Lambda^6} + d_2(\theta)\frac{s^3}{\Lambda^6} + d_3(\theta)\frac{s^4}{\Lambda^8} \dots \right)$$

while $\displaystyle A_{1-\text{loop}}(s,\theta) \sim \sum_{n=0}^{3} \frac{\tilde{d}_n(\theta) m^{2n} s^{6-2n}}{\Lambda^{12}} + \dots$

Tree-level bounds apply at least up to order Λ^{-10} (without invoking weak coupling)

3. Including the Loops to a given order