Recent advances in superconformal field theories (SCFTs)

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C. Long (Northeastern), M. Martone (Austin), and others

arXiv:..., 1801.01122, 1801.06554, 1804.03152

PASCOS2018 talk

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Outline

1. Motivation
2. RG, CFT, and SCFT basics
3. 4d $\mathcal{N} = 2$ SCFTs
4. Our work
5. Lessons and problems

Motivation

Why study SCFTs?

What do we learn?

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- Pheno?

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- Pheno? Not directly ...
- Explore possible QFT dynamics/mechanisms at strong coupling.
  (NB: QFT = non-gravitational)

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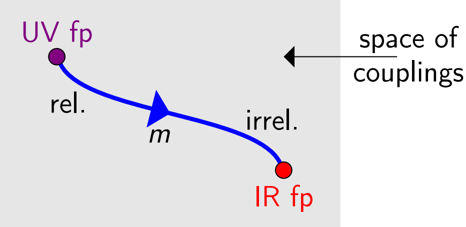
RG flow basics

("fp" = fixed point of RG flow)



Local operator flows: $S_{\text{fp}} + g \int d^d x \, \mathcal{O}(x)$

Classical fp = quadratic $\mathscr{L}(\phi, \psi_\alpha, A_\mu) + g\mathcal{O}$

$\mathcal{O}$	$3d$	$4d$	$5d, 6d$
$\phi^4, \psi^2\phi$	rel.	marg.-irrel.	irrel.
$\phi^2 A^2, \psi^2 A, A^4$	rel.	marginal	irrel.
$\phi^6, \psi^2\phi^2$	marg.-rel.	irrel.	irrel.

Q: Which fp's can come from/flow to classical (lagrangian) fp's?

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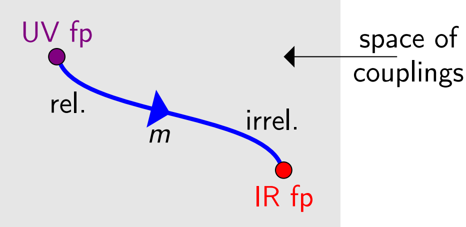

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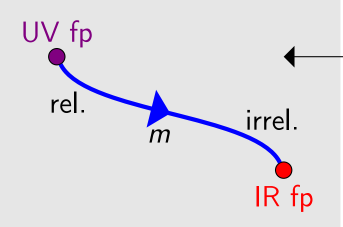

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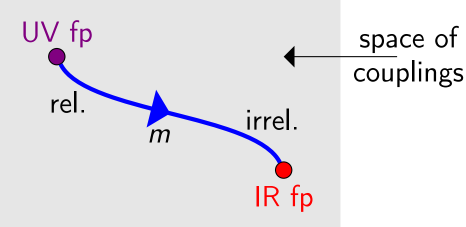

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(1) Between dimensions: $\mathrm{d}_{\mathrm{UV}} > \mathrm{d}_{\mathrm{IR}}$ (e.g., compactify on circles)

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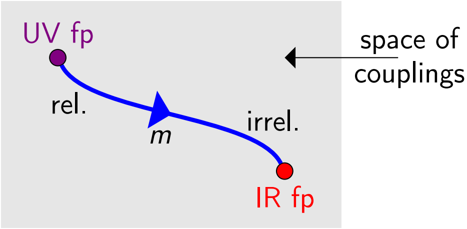

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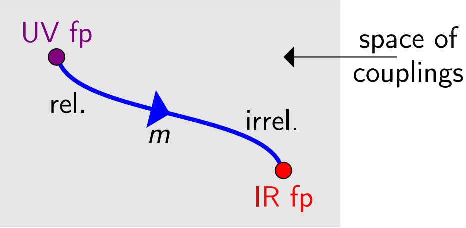

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(2) Spontaneous breaking of scale invariance: (e.g., by bc's at ∞)

$\Rightarrow$ Nambu-Goldstone boson = dilaton

$\Rightarrow$ *manifold* of vacua = "moduli space"

Q: When does a CFT have a moduli space/dilaton?

Only known examples are SCFTs. See also: Hellerman Maeda & al (1706.05743,1710.07336,1804.01535), Beem & Rastelli (1707.07679), Karananas & Shaposhnikov (1708.02220)

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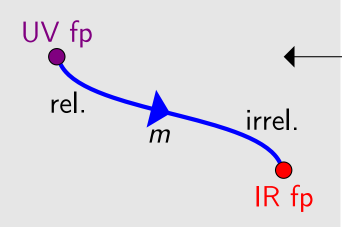

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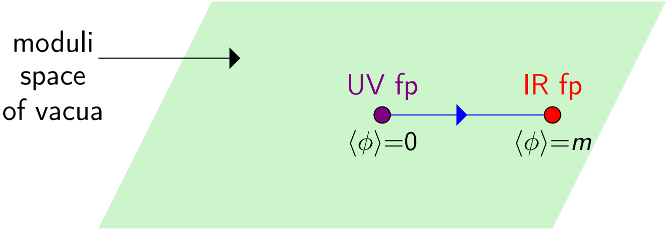

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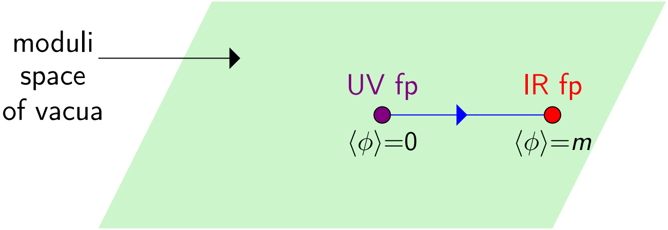

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CFT basics

massive QFT	CFT
Poincaré $\simeq SO(d-1,1) \ltimes \mathbb{R}^d$	Conformal $\simeq SO(d,2)$
(quasi-)particles and S-matrix in IR	no particles (unless nearly free)
RG flow of couplings with scale	no scale, no flow

CFT tools:

- Correlators $\langle \phi_1(x_1) \cdots \phi_n(x_n) \rangle$ of local, line, etc operators.
- Local operators: primaries $\phi_i \leftrightarrow \{\Delta_i, R_i\} = \{\text{dimension, spin}\}$
  & descendants $\propto \partial_\mu^n \phi_i$.
- Local op. algebra (OPE): $\phi_i(x)\phi_j(y) = \sum_k c_{ij}^k \mathscr{D}_{ij}^k(x-y, \partial_y)\phi_k(y)$.
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SCFT basics

Combining supersymmetry and conformal symmetry is restrictive:
no SCFTs in $d > 6$ (Nahm 1978).

# susys	3d	4d	5d	6d
4	$\mathcal{N}=2$	$\mathcal{N}=1$		
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SCFT tools:

- SC algebra representation theory
- Bootstrap
- Chiral operators
- Protected local op algebras
- a-theorem, a-maximization

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SCFT tools:

- SC algebra representation theory: classifies local ops in unitary SCFTs $\Rightarrow$ classifies possible local deformations of SCFTs even if no lagrangian [Cordova et al 1602.01217, 1612.00809]
  Q: similar classification for non-local (line, surface, etc) operators?
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SCFT tools:

- SC algebra representation theory
- Bootstrap: in progress... [eg Beem, Lemos, Liendo, Peelaers, Rastelli, ... last 4 years]
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SCFT tools:

- SC algebra representation theory
- Bootstrap
- Chiral operators: chiral rings, SC index $\Leftrightarrow$ moduli space complex structure, RG flow invariants [... going back to the late 80's]
- Protected local op algebras
- a-theorem, a-maximization

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- SC algebra representation theory
- Bootstrap
- Chiral operators
- Protected local op algebras: central charges, partition function, ∞ OPE subalgebras $\Leftrightarrow$ fp invariants [eg Beem et al 1312.5344, ...]
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- SC algebra representation theory
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- a-theorem, a-maximization: (4d) $\Leftrightarrow$ RG flow ordering [eg Intriligator & Wecht 0304128, ...]

4d $\mathcal{N} = 2$ SCFTs

$\mathcal{N} = 2$ in 4d is "sweet spot" for SCFTs:

4d $\Rightarrow \exists$ UV & IR $\mathscr{L}$ fp's, w pt, line, surface ops.

$\mathcal{N}=2 \Rightarrow$ moduli spaces, w BPS particles, & 2d χ algebra $\subset$ OPE.

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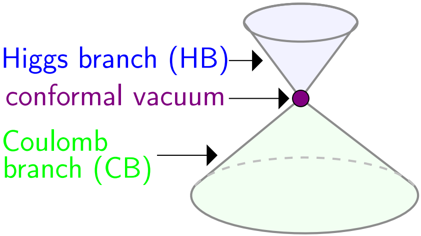
Moduli spaces:



HB: • $\mathcal{N}$=2 σ-model in IR.
• hyperkähler $\Leftarrow$ 2dχalg.
• lifted by most rel. defos.

CB: • $U(1)^r$ $\mathcal{N}$=2 QED in IR.
• special Kähler $\Leftarrow$ SW theory.
• persists under defos.

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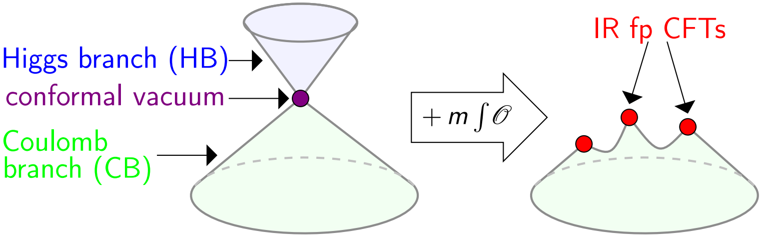
$\mathcal{N} = 2$ in 4d is "sweet spot" for SCFTs:

4d $\Rightarrow \exists$ UV & IR $\mathscr{L}$ fp's, w pt, line, surface ops.

$\mathcal{N}$=2 $\Rightarrow$ moduli spaces, w BPS particles, & 2d χ algebra $\subset$ OPE.

Moduli spaces:



HB:
- $\mathcal{N}$=2 σ-model in IR.
- hyperkähler $\Leftarrow$ 2dχalg.
- lifted by most rel. defos.

CB:
- $U(1)^r$ $\mathcal{N}$=2 QED in IR.
- special Kähler $\Leftarrow$ SW theory.
- persists under defos.

deformed HB:
"map" of RG flow.

(NB: "rank" r=CB cplx dim)

4d $\mathcal{N} = 2$ SCFTs: some questions

- Are they all connected to 4d $\mathscr{L}$ fp's by RG flows?
- Are they all connected to higher-d SCFTs by RG flows?
- Are all their marginal couplings gauge couplings?
- Are all 4d $\mathcal{N} = 4$ SCFTs YM theories?

- Do they all have a CB?
- Do CB + HB geometries uniquely characterize SCFT?
- Are all CB $\simeq \mathbb{C}^r$ as complex spaces?
- Are there a finite number at each rank?
- Are all special Kähler geometries CBs of SCFTs?

4d $\mathcal{N} = 2$ SCFTs: some questions

- Are they all connected to 4d $\mathscr{L}$ fp's by RG flows? probably not
- Are they all connected to higher-d SCFTs by RG flows? probably not
- Are all their marginal couplings gauge couplings? ??
- Are all 4d $\mathcal{N} = 4$ SCFTs YM theories? ??

- Do they all have a CB? probably
- Do CB + HB geometries uniquely characterize SCFT? no
- Are all CB $\simeq \mathbb{C}^r$ as complex spaces? no
- Are there a finite number at each rank? probably
- Are all special Kähler geometries CBs of SCFTs? no, but...

Our work

PCA Long Lotito Lü Martone: 1505.04814, 1601.00011, 1602.02764, 1609.04404, 1611.08602, 1704.05110, 1801.01122, 1801.06554, 1804.03152
Caorsi Cecotti: 1801.04542, 1803.00531
Bourget Pini Rodriguez-Gómez: 1804.01108

Idea: For fixed CB rank, classify all possible special Kähler geometries.
But: CB geometries are singular, and rules at singularities need to be discovered.
Find: Possible kinds and numbers of singularities are limited by:

- consistency with EM duality of low energy $U(1)^r$ gauge theory (Seiberg Witten '94)
- unitarity of SCFT
- underlying complex structure of the CB
- consistency with Dirac quantization condition in low energy $U(1)^r$ gauge theory
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Results:

1. For rank $= 1$ and CB $\simeq_{\mathbb{C}} \mathbb{C} \Rightarrow$ 28 possible geometries
   - 2 are known gauge theories
   - 1 is a gauge theory with non-maximal spectrum of line operators
   - 23 are "known" isolated ($=$ strongly-coupled, non-$\mathscr{L}$) SCFTs
   - 2 have no known SCFT
2. For rank $= 1$ but CB $\not\simeq_{\mathbb{C}} \mathbb{C}$
   - number of geometries is seemingly (denumerably) infinite
   - if exist, form a separate set under RG flows
3. For rank ≥ 2
   - if CB $\simeq_{\mathbb{C}} \mathbb{C}^r$, number of geometries or whether it is finite is unknown
   - but, if CB $\simeq_{\mathbb{C}} \mathbb{C}^r$, we compute a finite, rational, spectrum of CB scaling dimensions
   - exist $\mathscr{L}$ theories with disconnected gauge groups with CB $\not\simeq_{\mathbb{C}} \mathbb{C}^r$

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Conclusion

Basic lessons:

- "most" QFTs are non-lagrangian, ie, have no semi-classical limit
- likely denumerable number of SCFTs with 8 supersymmetries for $d \geq 3$

Basic difficulties:

- lack first principles connection between CFT data and moduli space geometries
  - OPE algebra $\Rightarrow$ Higgs branch geometries (Beem Rastelli 1707.07679)
  - but unlikely to be true for CB, since need to include line operators (eg, worldlines of charged BPS states)
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End

Thanks!