A Classical-Quantum Correspondence For All

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Based on work with **George Zahariade**, arXiv:1803.08919 and ongoing.

PASCOS 2018, Case Western Reserve University

Motivation

Quantum effects in classical backgrounds —

★ Coupling of inflaton to other fields.

★ Hawking radiation during gravitational collapse.

★ Schwinger pair creation.

★ etc.

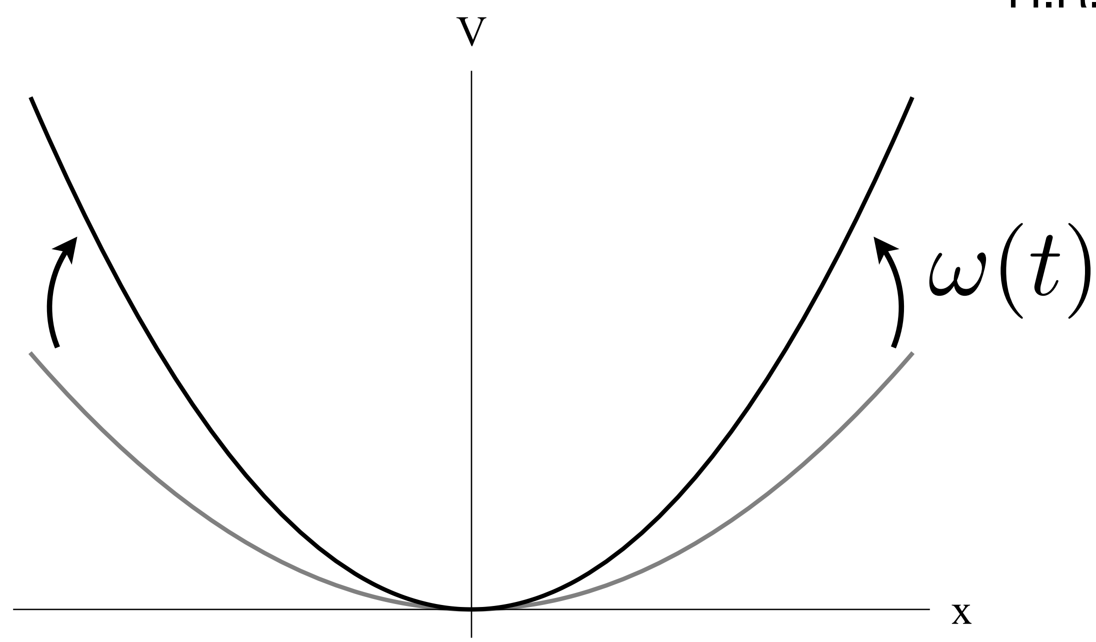
Expand field in modes: $$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \Big(c_{\mathbf{k}}(t) f_{\mathbf{k}}(\mathbf{x}) + c^{\dagger}_{\mathbf{k}}(t) f^{*}_{\mathbf{k}}(\mathbf{x}) \Big)$$

*For free fields, the mode coefficients are **simple harmonic oscillator** variables in the time-dependent classical background.*

Simple Harmonic Oscillator

(with time dependent frequency)

Early Work:
H.R. Lewis, 1968
H.R. Lewis & W.B. Riesenfeld, 1969
L. Parker, 1971



Define ladder operators:

$$\hat{a} = \frac{\hat{p} - im\omega(t)\hat{x}}{\sqrt{2m\omega(t)}}, \quad \hat{a}^\dagger = \frac{\hat{p} + im\omega(t)\hat{x}}{\sqrt{2m\omega(t)}}$$

SHO contd.

Heisenberg equations:

$$\frac{d\hat{a}}{dt} = -i[\hat{a}, H] + \frac{\partial \hat{a}}{\partial t}$$

Solution:

$$\hat{a}(t) = \frac{(p_z^* - im\omega z^*)}{\sqrt{2m\omega}}\hat{a}_0 + \frac{(p_z - im\omega z)}{\sqrt{2m\omega}}\hat{a}_0^\dagger$$

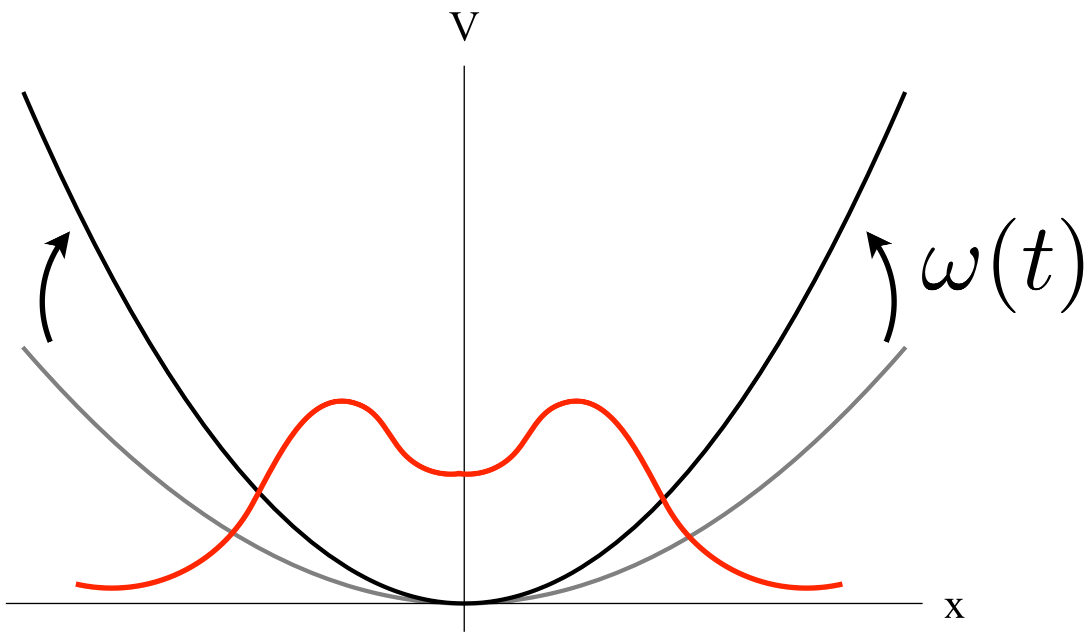
where,

$$\boxed{\ddot{z} + \omega^2(t) z = 0}$$

with initial conditions

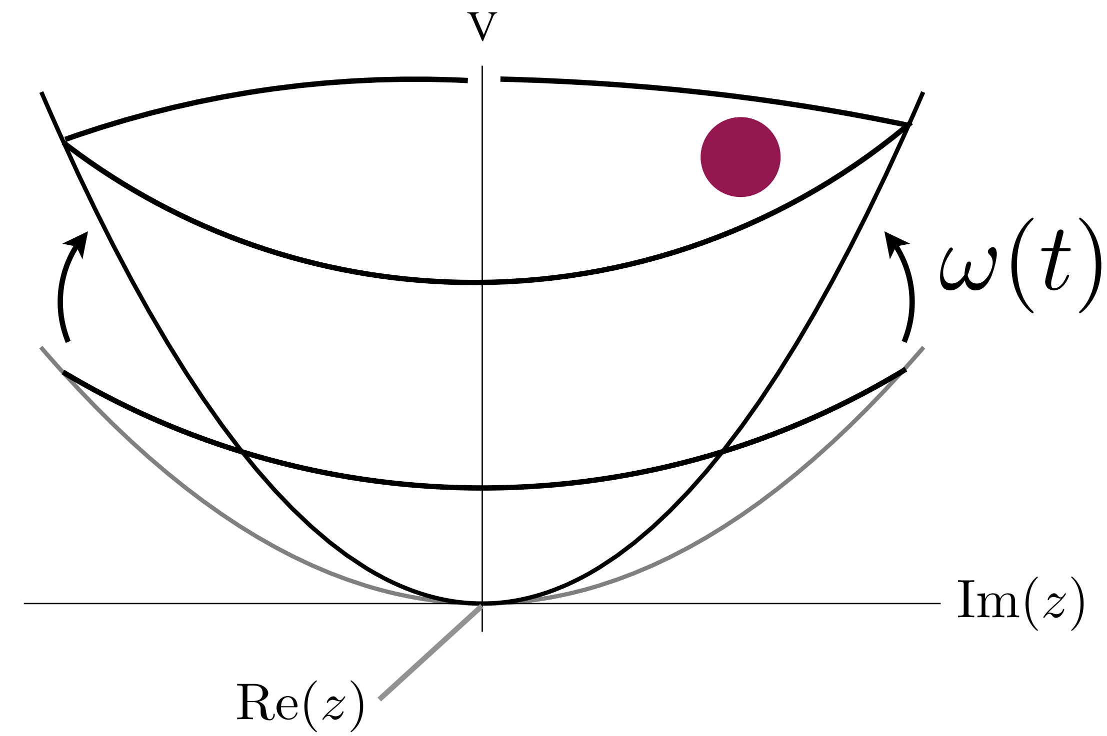
$$\boxed{z(0) = \frac{-i}{\sqrt{2m\omega_0}}, \quad \dot{z}(0) = \sqrt{\frac{\omega_0}{2m}}}$$

Classical-Quantum Correspondence (CQC)



Quantum

$$\leftrightarrow$$

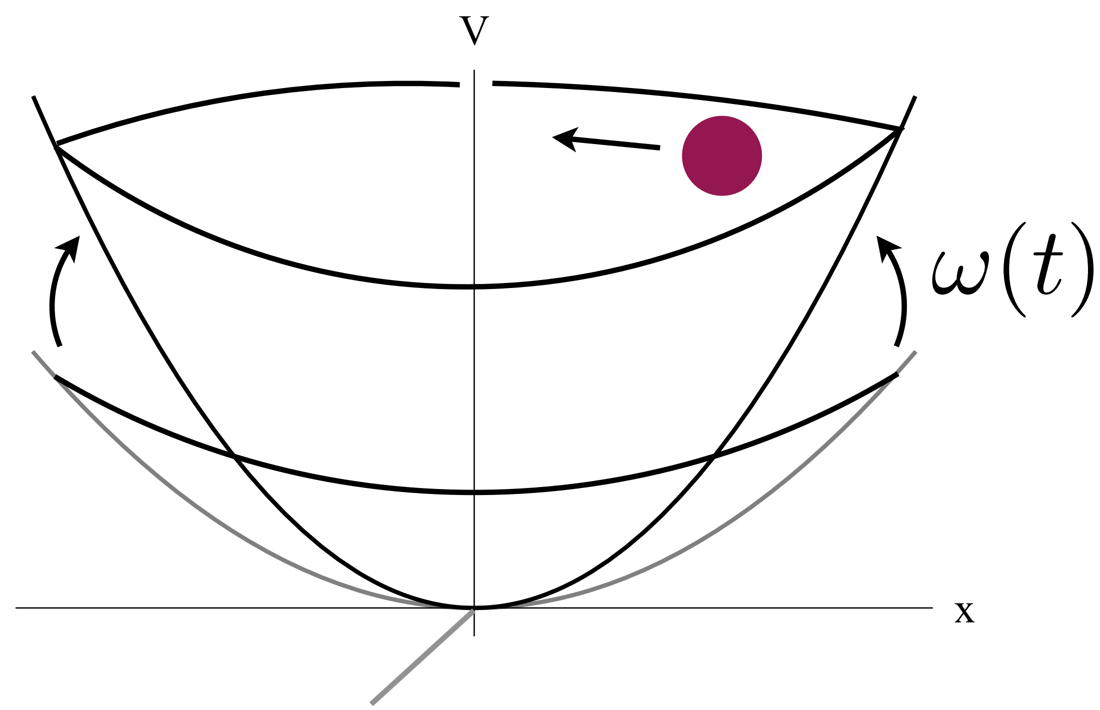


Classical

$$\hat{x} \leftrightarrow z$$

$$|0\rangle \leftrightarrow \{z_0, \dot{z}_0\}$$

Initial Conditions



Quantum ground state implies classical system must have:

- zero point energy = $\omega/2$
- angular momentum = 1/2

(In field theory, angular momentum corresponds to global charge.)

Particle Production with CQC

Particle production is usually discussed via Bogoliubov transformations.

Using CQC, we can find particle production from a classical calculation:

$$E_{\text{radiation}} = \omega \left(\underbrace{|\beta|^2}_{\text{Bogoliubov coefficient}} + \frac{1}{2} \right) = \frac{|p_z|^2}{2m} + \frac{m\omega^2}{2} |z|^2$$

Summary: Particle production in classical time-dependent backgrounds, (*e.g.* Hawking radiation, reheating during inflation,…) can *all* be calculated using a classical analysis.

Backreaction with CQC

Particle production implies backreaction on the classical background.

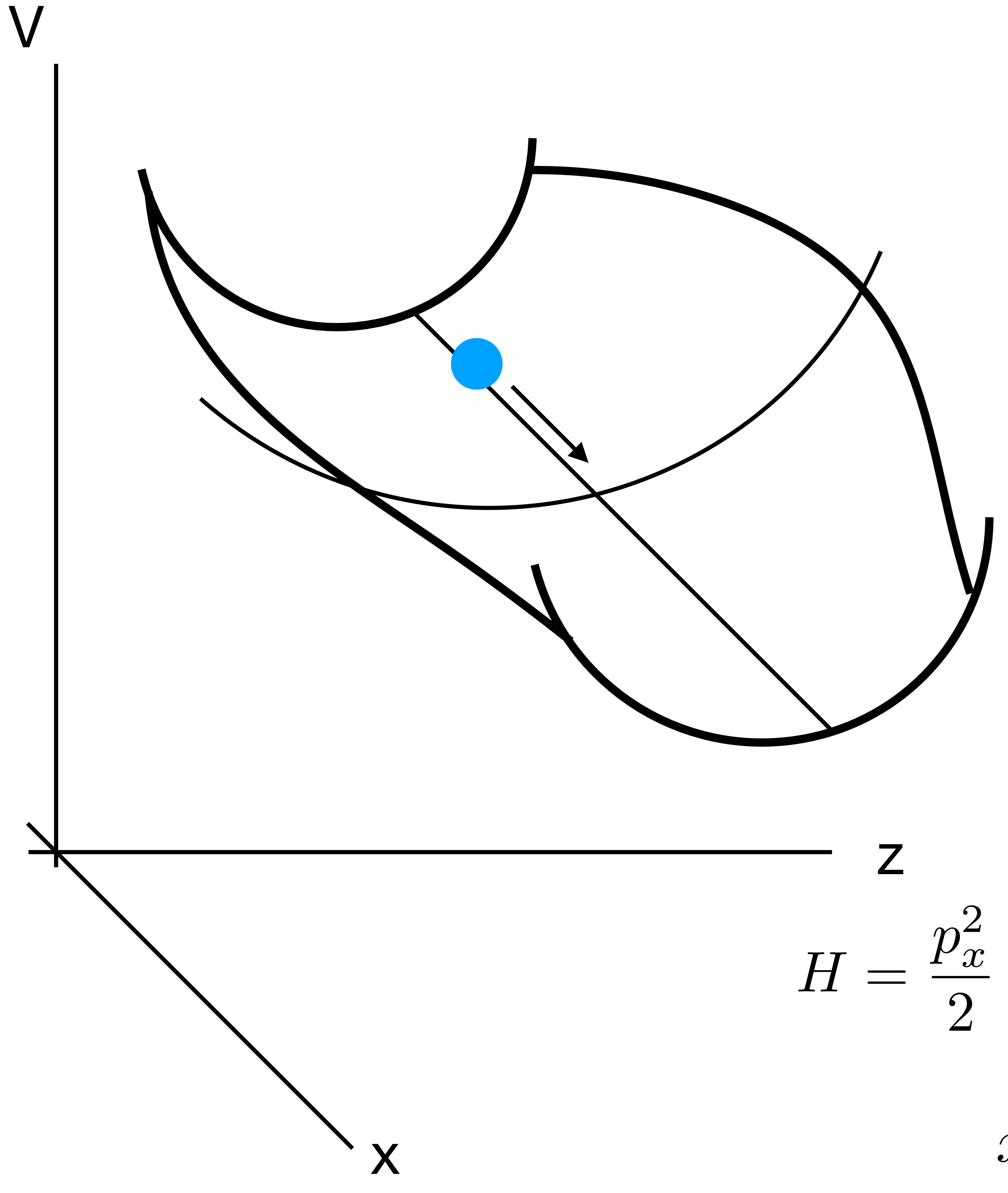
- evaporation of black holes,
- slow-down of inflaton dynamics,
- screening of electric fields,
- etc..

CQC provides a classical framework to address backreaction.

Two Examples:

- Rolling background.
- Collapsing shell.

Example 1: Rolling Background



$$H = \frac{p_x^2}{2} - ax + \frac{p_z^2}{2} + \frac{1}{2}\omega_0^2 z^2 + \frac{\lambda}{2} x^2 z^2$$

$$x_{cl} = \frac{1}{2} a t^2, \quad z_{cl} = 0$$

Analyses

CQC Analysis: $\ddot{x} = a - \lambda x|z|^2, \qquad \ddot{z} = -(\omega_0^2 + \lambda x^2)z$ (z=complex)

$$x(0) = 0, \quad \dot{x}(0) = 0, \quad z_R(0) = 0, \quad \dot{z}_R(0) = \sqrt{\frac{\omega_0}{2}}, \qquad z_I(0) = \frac{-1}{\sqrt{2\omega_0}}, \quad \dot{z}_I(0) = 0$$

Simple numerical problem — takes few seconds with Mathematica.

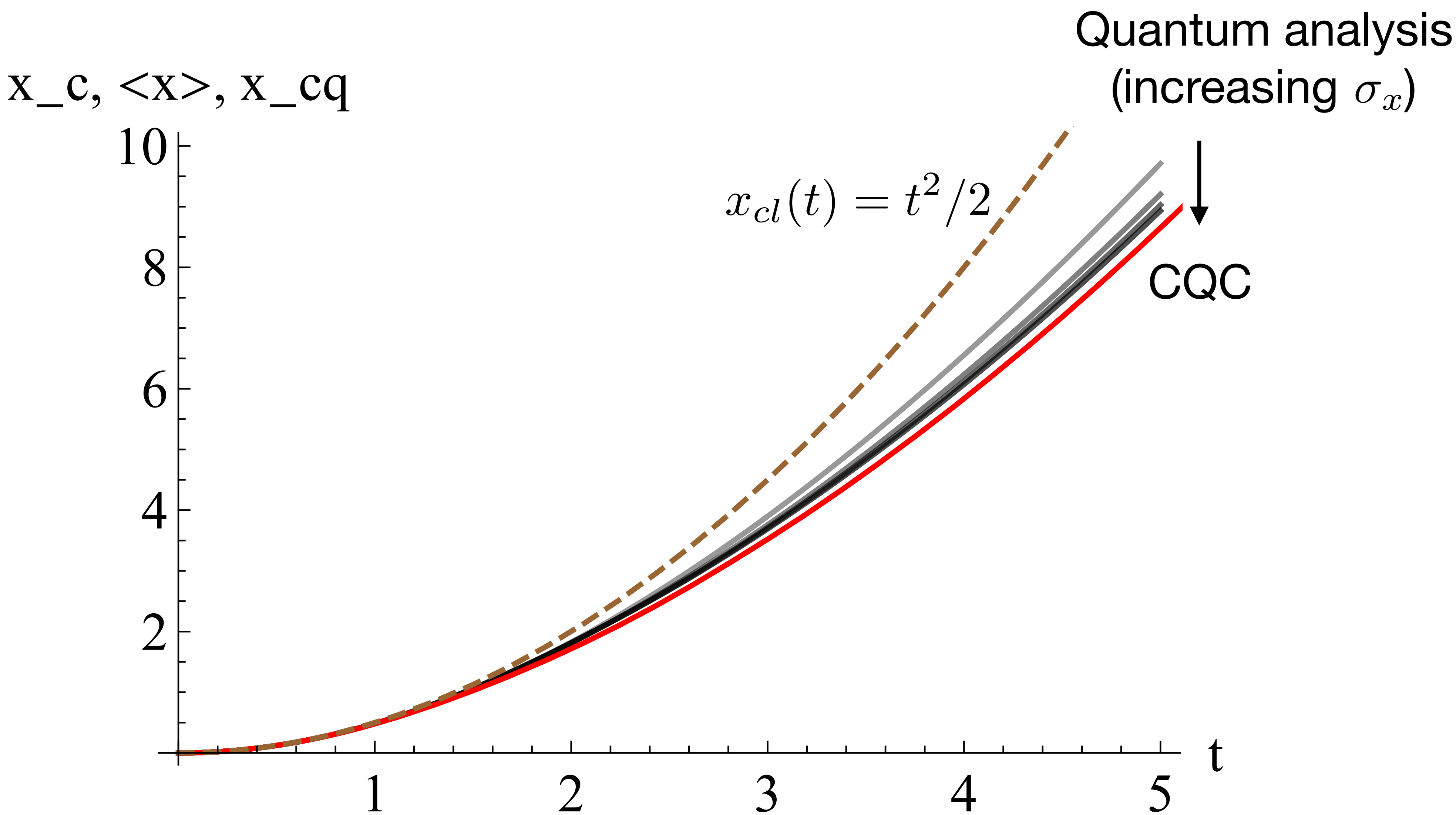
Full Quantum Analysis: $H\psi(x,z,t) = i\frac{\partial}{\partial t}\psi(x,z,t)$ (z=real)

$$\psi(t=0,x,z) = \left(\frac{1}{\pi\sigma_x^2}\right)^{1/4} e^{-x^2/(2\sigma_x^2)} \times \left(\frac{\omega_0}{\pi}\right)^{1/4} e^{-\omega_0 z^2/2}$$

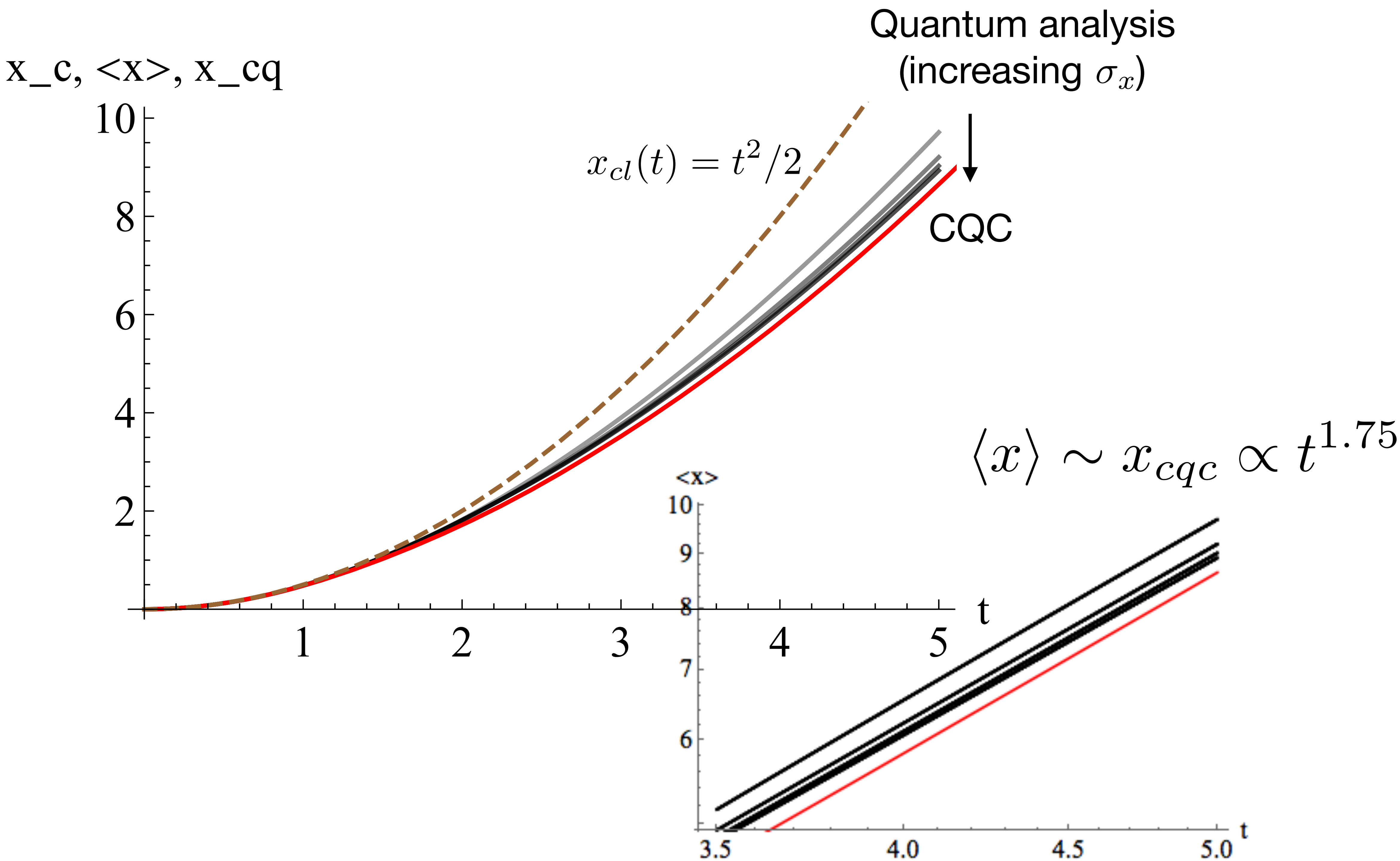
free parameter

Challenging and laborious numerical problem — takes several days on cluster.

Rolling with Backreaction



Rolling with Backreaction



Quantum vs. CQC

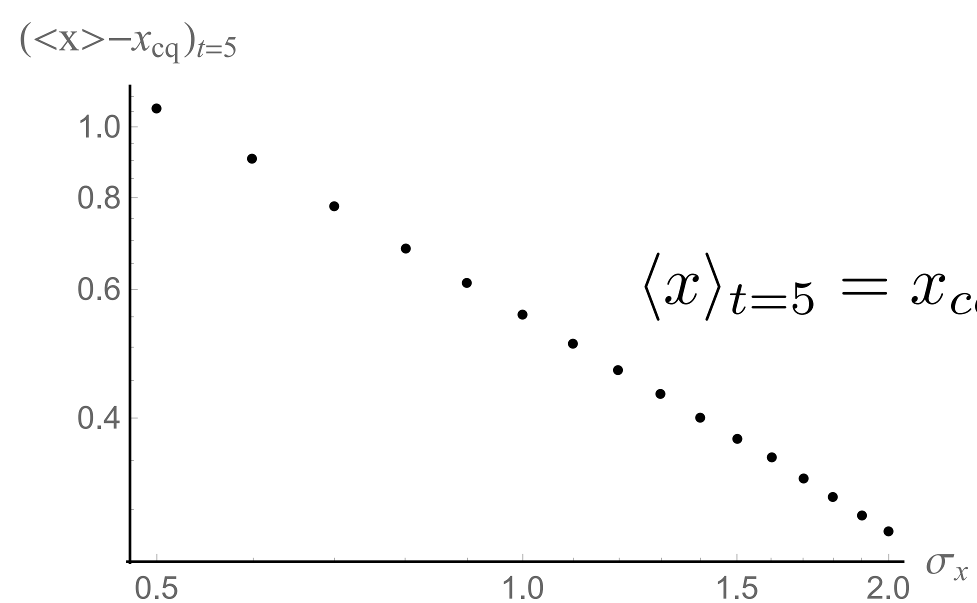
CQC is *exact* if the background is classical.

The wavepacket of the rolling particle moves down the potential and spreads.

The dynamics is classical if the rolling is faster than the spreading:

$$at \gg \frac{1}{2m\sigma_x}$$

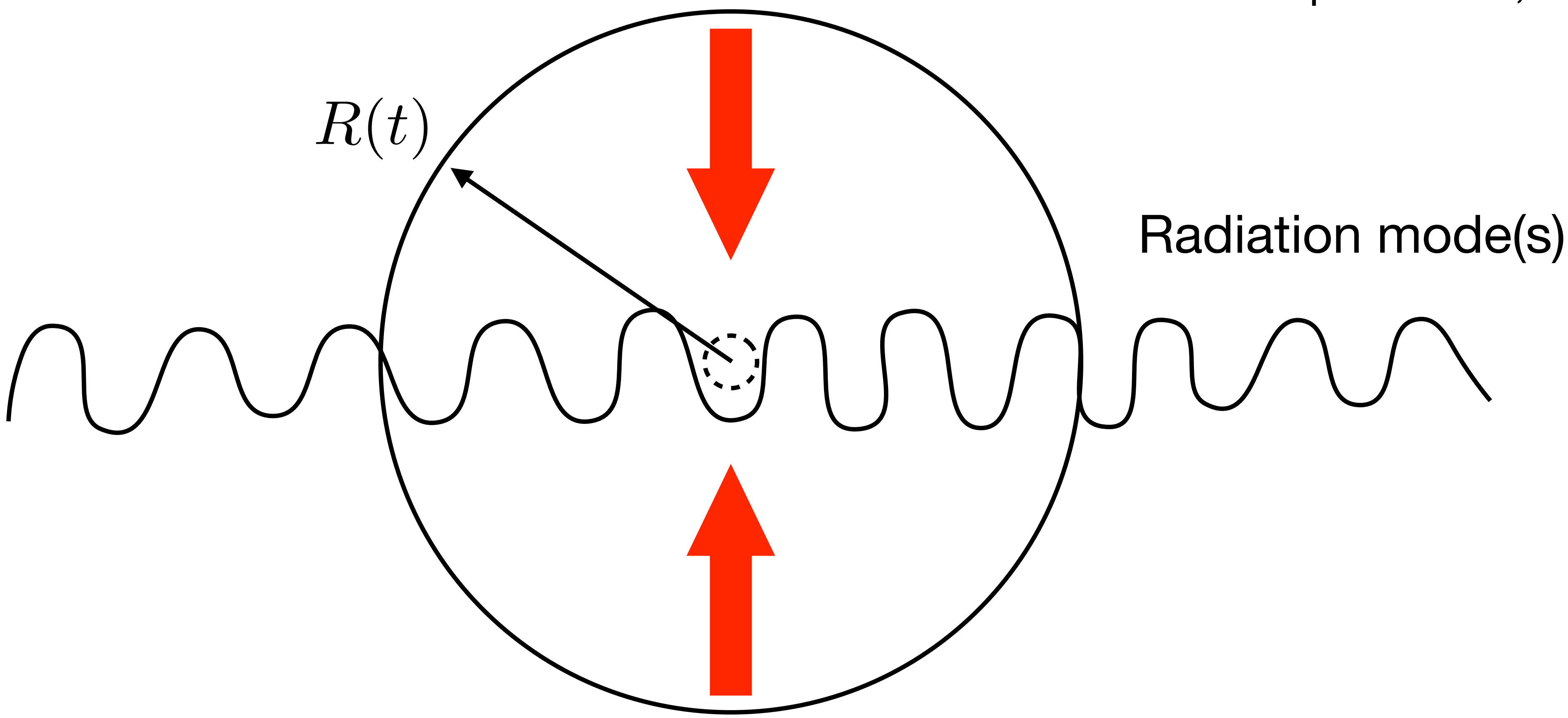
Therefore quantum goes to CQC for wide wavepackets at late times.



$$\langle x\rangle_{t=5} = x_{cq}(t=5) + \frac{2}{\sigma_x}$$

Example 2: Collapsing Shell

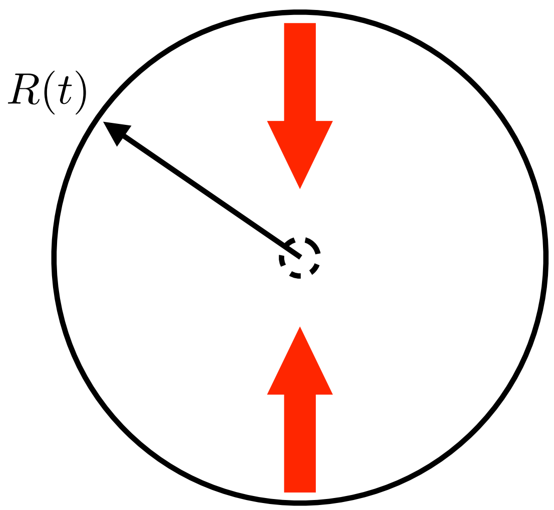
TV, Stojkovic & Krauss, 2007
Kolopanis & TV, 2013



Changing shell metric leads to quantum radiation and shell evaporation.

Recast as simple harmonic oscillators with time-dependent frequencies.

Shell Dynamics



Ipser & Sikivie, 1984

$$g_{\mu\nu} = \begin{cases} \text{Minkowski } (T, r, \theta, \phi), & r \leq R(T) \\ \text{Schwarzschild } (t, r, \theta, \phi), & r > R(T) \end{cases}$$

Metric is time-dependent because the shell collapses.

Scalar field modes get excited due to changing metric.

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} \Big(c_{\mathbf{k}}(t) f_{\mathbf{k}}(\mathbf{x}) + c^\dagger_{\mathbf{k}}(t) f^*_{\mathbf{k}}(\mathbf{x}) \Big)$$

SHO with time-varying frequency.

*Scalar field changes the form of the metric but for now we restrict attention to just the shell radius and the mode excitation.

Shell Radiation with CQC

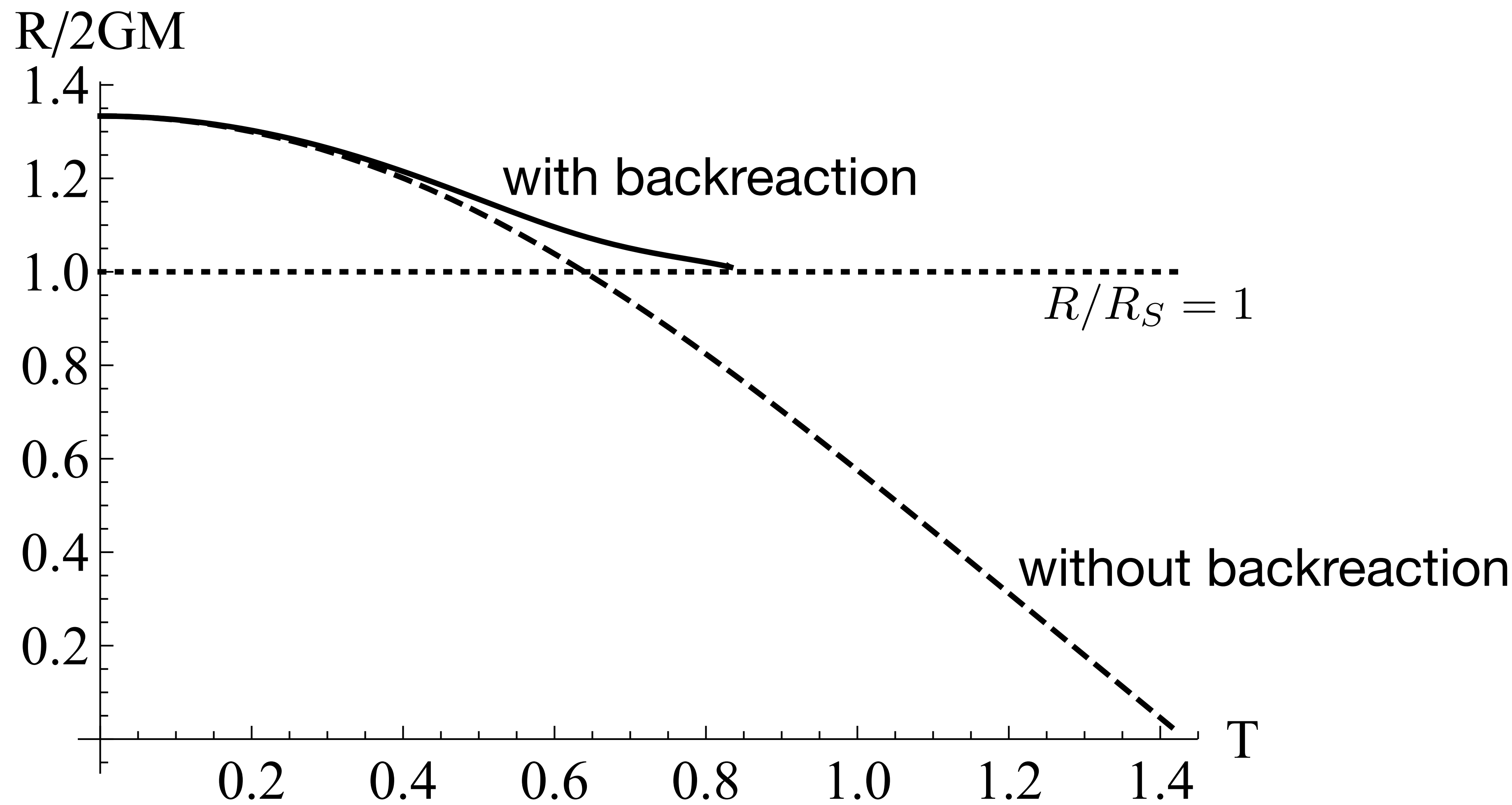
$$S = -4\pi\sigma \int dT R^2 \left[\underbrace{\sqrt{1 - R_T^2} - 2\pi G\sigma R}_{\text{Shell dynamics.}} \right] + \frac{1}{2\omega_0} \int dT \left[\underbrace{\frac{|\dot{z}|^2}{2} - \frac{\kappa^2 |z|^2}{2\dot{T}}}_{\text{One radiation mode.}} \right]$$

$$\dot{T} \equiv \frac{dT}{dt} = \frac{B}{\sqrt{B + (1-B)R_T^2}}$$

$$B \equiv 1 - \frac{2GM}{R} = 1 - \frac{R_S}{R}$$

$$\boxed{\omega^2(T) = \frac{\kappa^2}{\dot{T}} \to \infty, \quad \text{as } B \to 0.}$$

Shell Evaporation with CQC



Conclusions

- ★ **Classical Quantum Correspondence (CQC):** Quantum excitations of a real variable in a time-varying background can be calculated by solving classical equations for a complex variable with specified initial conditions.

- ★ **CQC defines a backreaction problem:** Comparison with full quantum backreaction in the rolling problem shows excellent agreement.

- ★ **CQC can be applied to a wide set of systems ("for all").**

- ★ **Evaporation during gravitational collapse slows down the collapse:** A full implementation requires numerical relativity+complex scalar field (ongoing).