

Wormholes

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Based on work in progress with:



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Fedor Popov

Related to previous work with Xiaoliang Qi



Inspired by work by Gao, Jafferis and Wall on “Traversable wormholes”

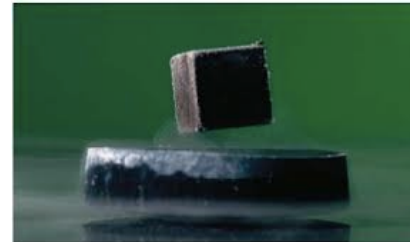
Hidden connections

Black holes

Holography
QFT = gravity

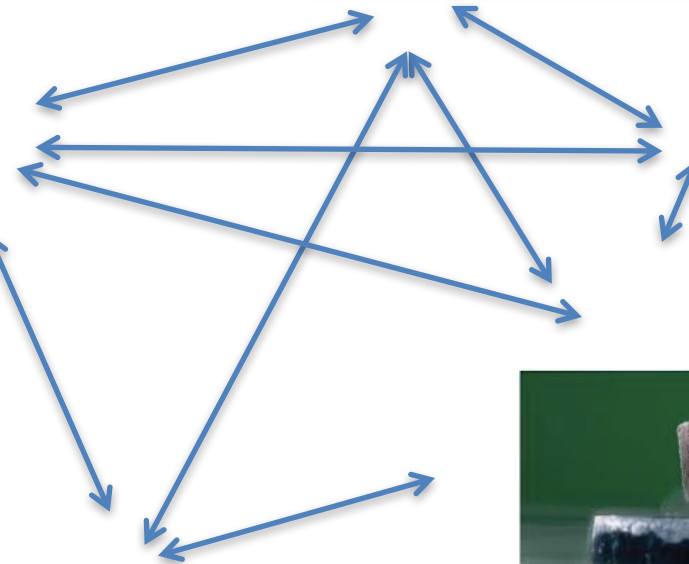
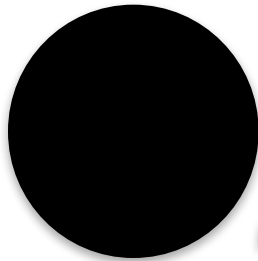


Quark gluon plasma



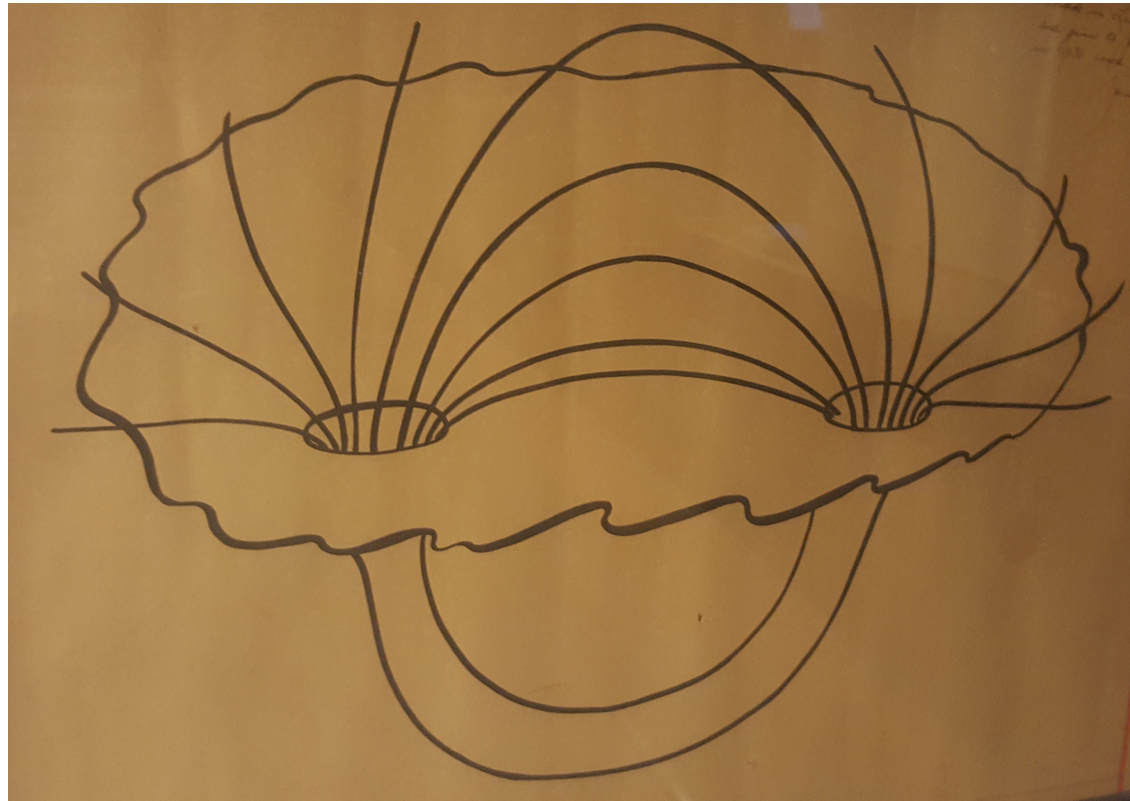
Strongly interacting
condensed matter physics

Quantum information theory



- What I will describe came about by thinking about the SYK model and its relation to near extremal black holes.
- But it can be explained without reference to that model.
- This is what I will do here.

Drawing by John Wheeler, 1966



Charge without charge.

Spatial geometry. Traversable wormhole

Recall a classic result

There are no science fiction wormholes!

- No wormhole allows you to travel faster than the speed of light in the ambient space.
- Forbidden by:
 - I) The Achronal Average Null Energy Condition

Not yet proven in a general spacetime, but believed to hold in QFT

$$\int dx^- T_{--} \geq 0$$



- II) Einstein equations.

Achronal = fastest line

Friedman Schleich, Witt, Galloway, Woolgar
Gao Wald

Longer wormholes?

- What if it takes longer to go through the wormhole ?
- Not possible in classical physics due to the Null Energy Condition.

Topological censorship: Friedman Schleich, Witt, Galloway, Woolgar

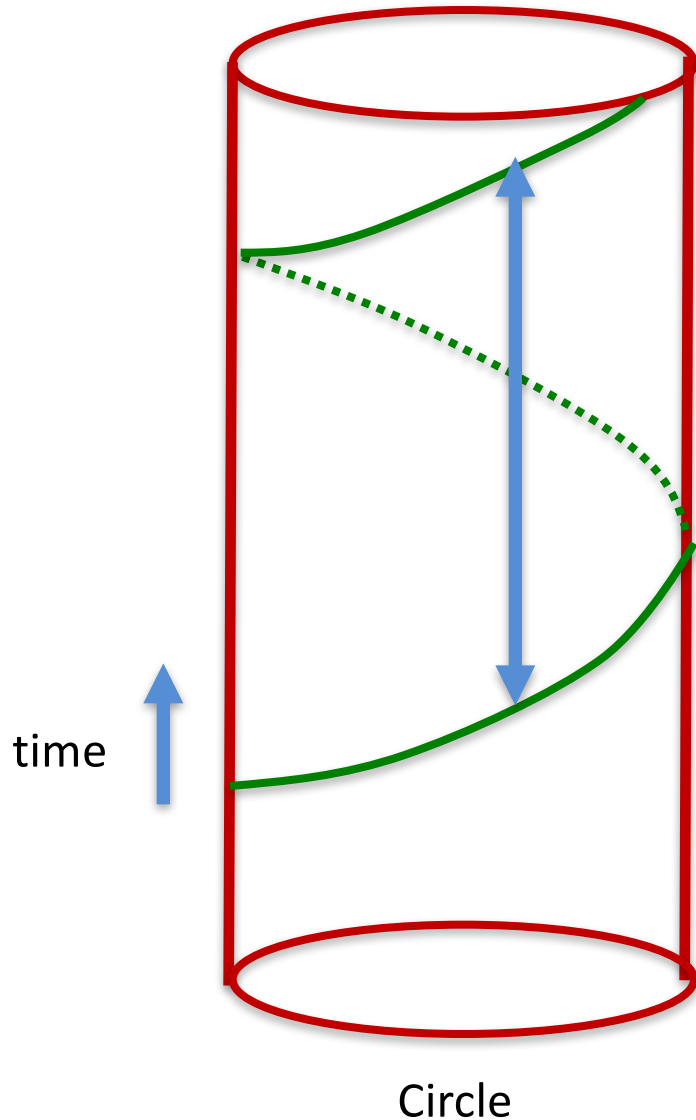
- → We need quantum effects to find a solution. Casimir-like energy.



- Can we do it in a controllable way ?

Negative energy from quantum mechanics

Eg. Two spacetime dimensions



$$T_{+++} < 0$$

$$E \propto -\frac{1}{L}$$

Negative Casimir energy

Quantum effect

The null energy condition does not hold for null lines that are not the fastest

Some necessary elements

- We need something looking like a circle to have negative Casimir energy.
- Large number of bulk fields to enhance the size of quantum effects.
- We will show how to assemble these elements in a few steps.

The theory

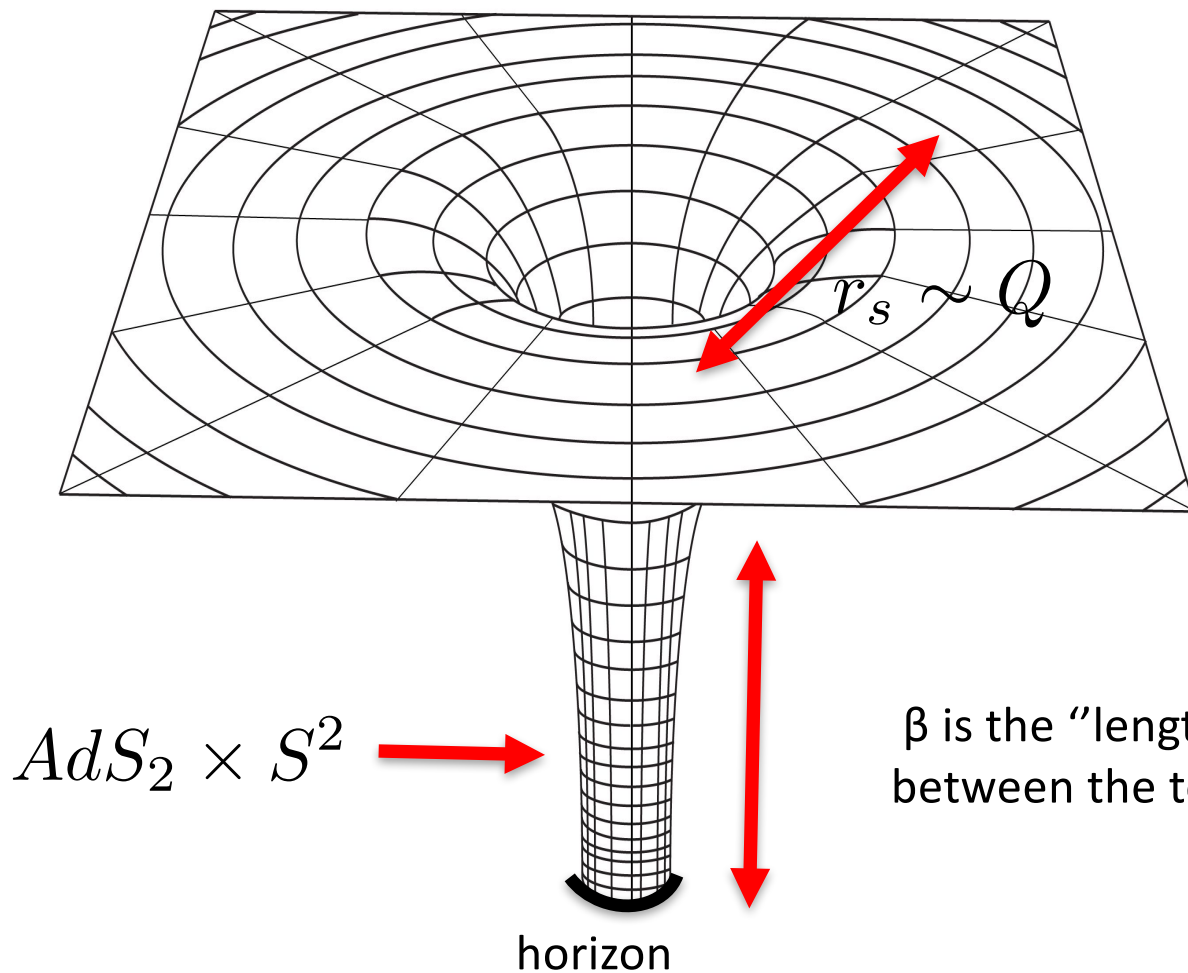
$$S = \int d^4x \sqrt{g} [R + F^2 + \bar{\psi} \not{D}\psi]$$

Einstein + U(1) gauge field + massless charged fermion

Could be the Standard Model at very small distances, smaller than the electroweak scale, where the fermions are effectively massless and the U(1) would be hypercharge. SU(3) x SU(2) x U(1).

The first solution

Extremal, or near extremal, magnetically charged black hole, magnetic charge Q .



$$M = Q + Q^3 T^2 = Q + \frac{Q^3}{\beta^2}$$

$$Q \gg 1$$



Very small

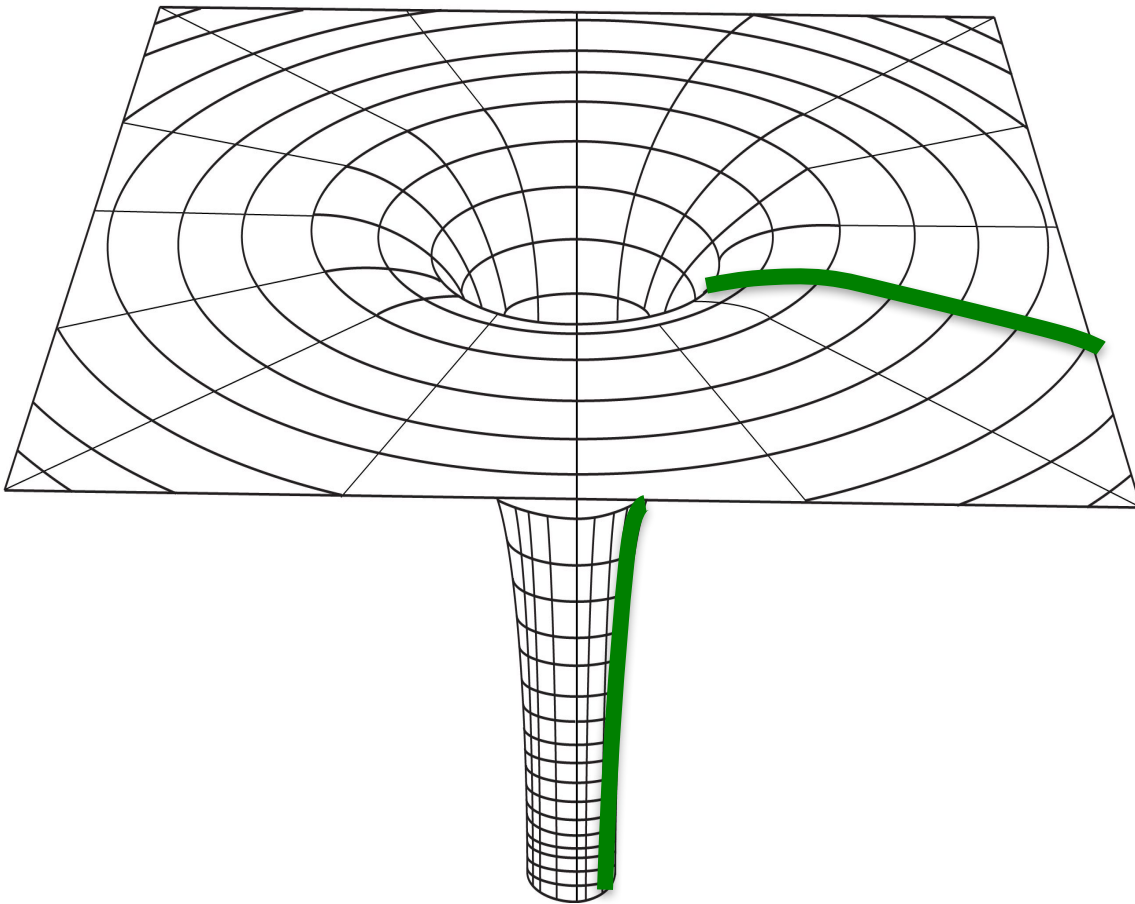
β is the "length" of the throat. Redshift factor between the top and the bottom

$$l_{\text{Planck}} = 1$$

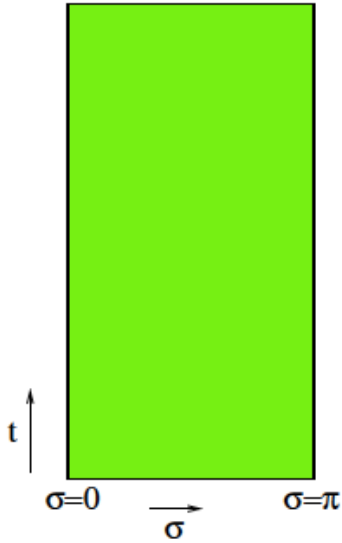
Motion of charged fermions

- Magnetic field on the sphere.
- There is a Landau level with precisely zero energy.
- Orbital and magnetic dipole energies precisely cancel.
- Degeneracy Q = flux of the magnetic field on the sphere
- We effectively get Q massless two dimensional fermions along the time and radial direction.
- We can think of each of them as following a magnetic field line.

Q massless two dimensional fields, along field lines.

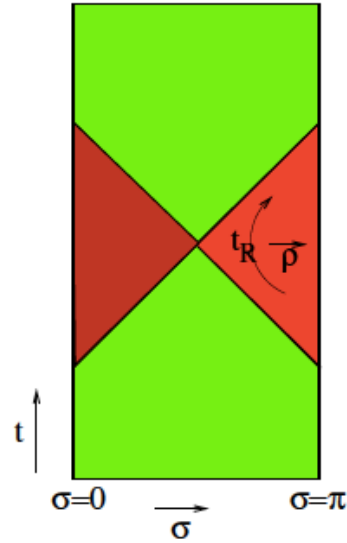


AdS₂



$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$$

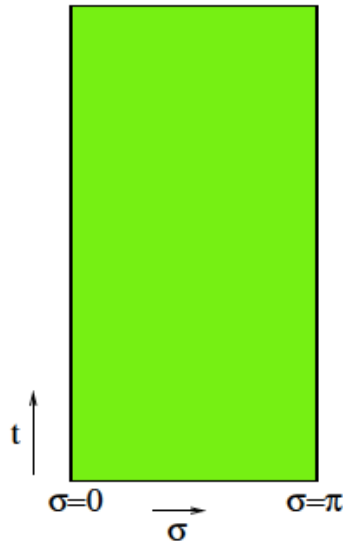
Global



$$ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{(r^2 - 1)}$$

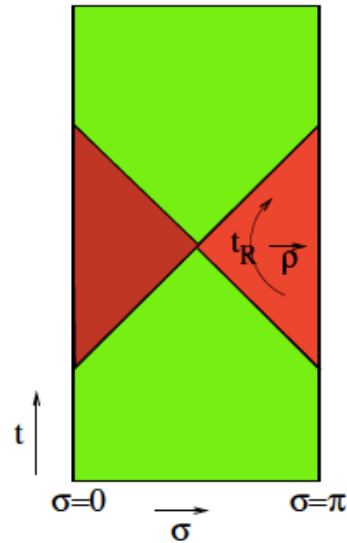
Thermal/Rindler

AdS₂



$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$$

Global



$$ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{(r^2 - 1)}$$

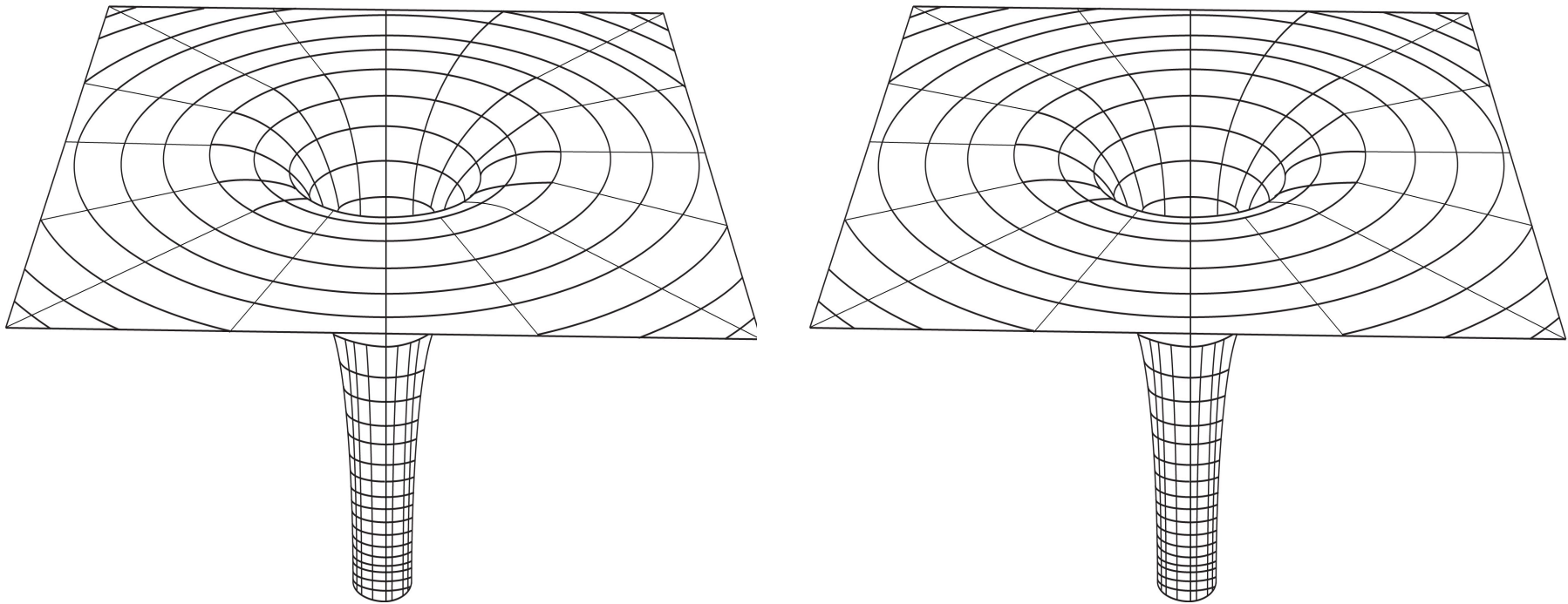
Thermal/Rindler

Two black holes connected in various ways. All equally valid solutions in the exact extremal limit (infinite length throat).

The acquire non-zero energy when the throat has finite length

$$M - Q = Q^3 T^2 = \frac{Q^3}{\beta^2}$$

Connect a pair black holes

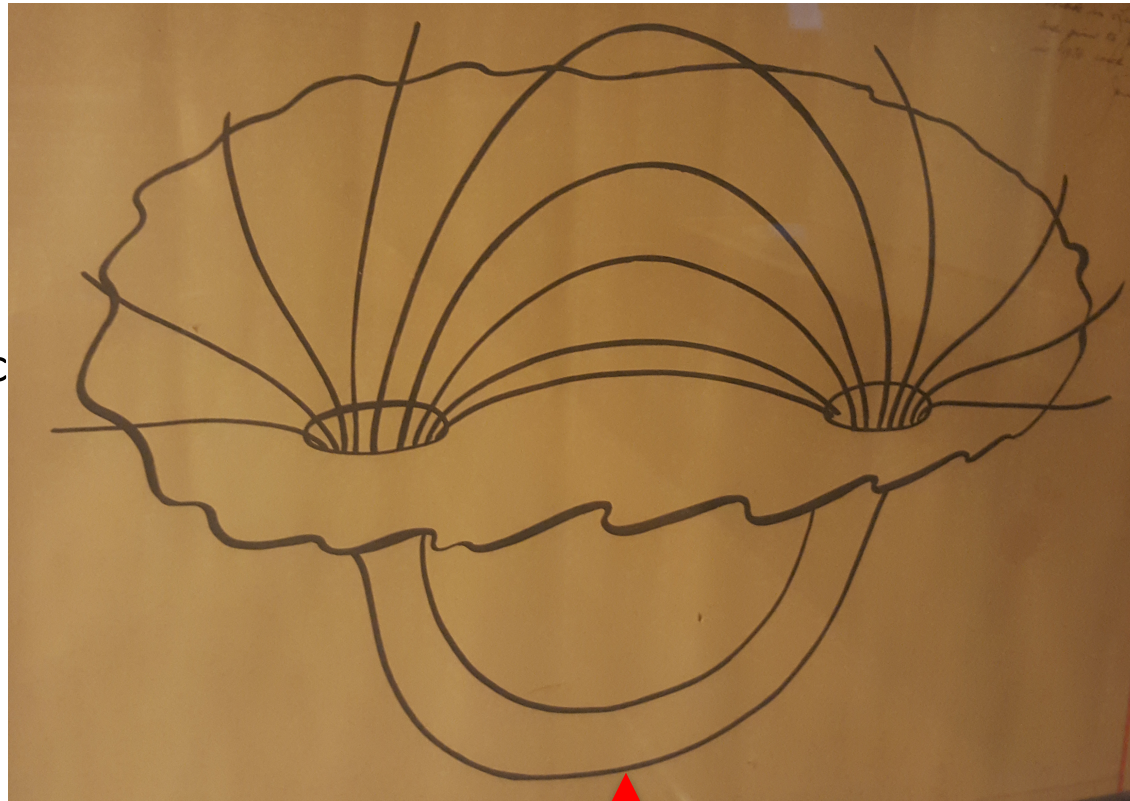


connect

Connect a pair black holes

Positive magnetic charge

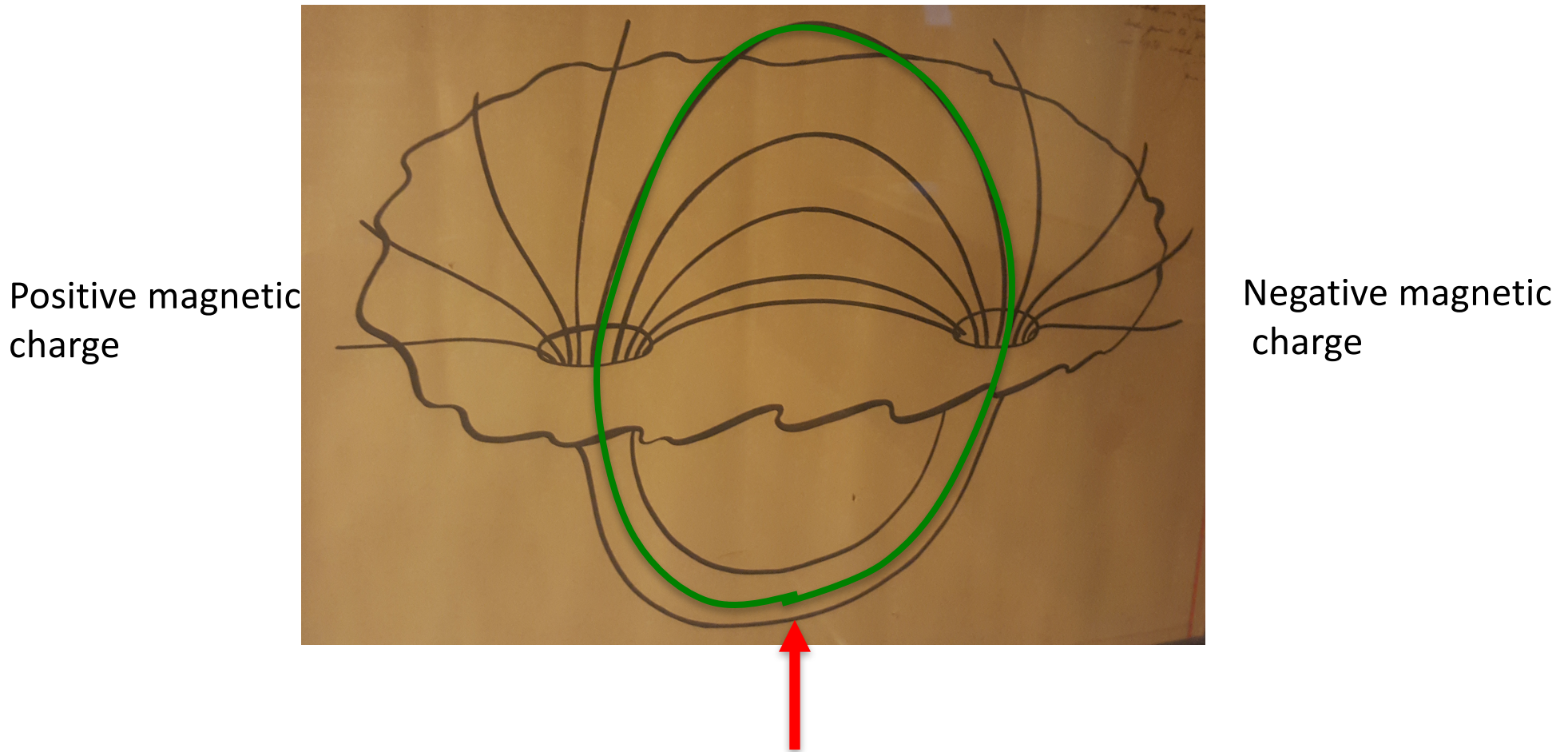
Negative magnetic charge



Not a solution yet.
Not a black hole.

Nearly $AdS_2 \times S^2$ wormhole of finite length

Fermion trajectories

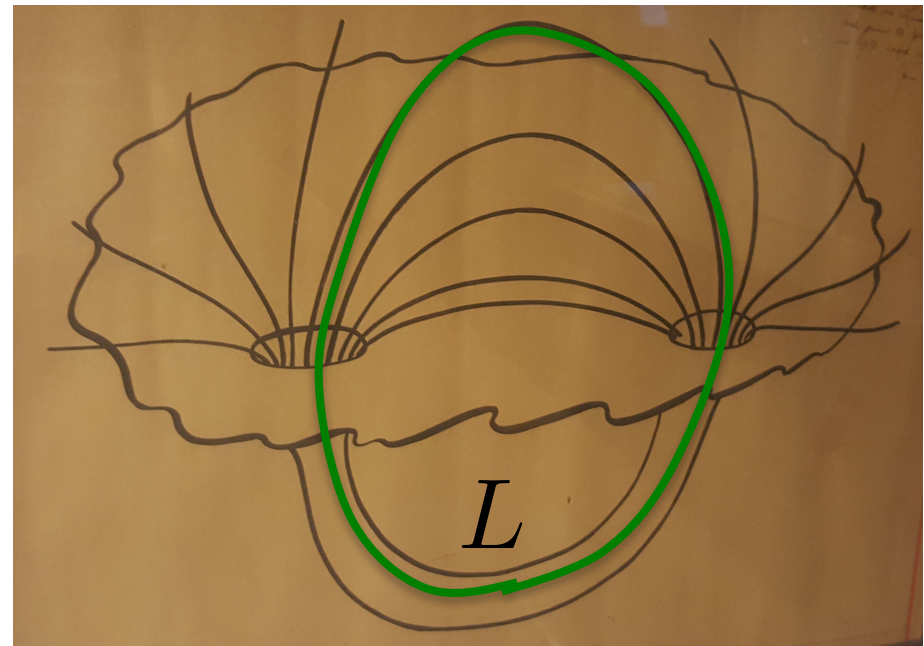


Charged fermion moves along this closed circle.

Casimir energy

Assume: “Length of the throat” is larger than the distance.

L_{out}



Casimir energy is of the order of

$$E \propto -\frac{Q}{L + L_{\text{out}}} \sim -\frac{Q}{L}$$

Finding the solution

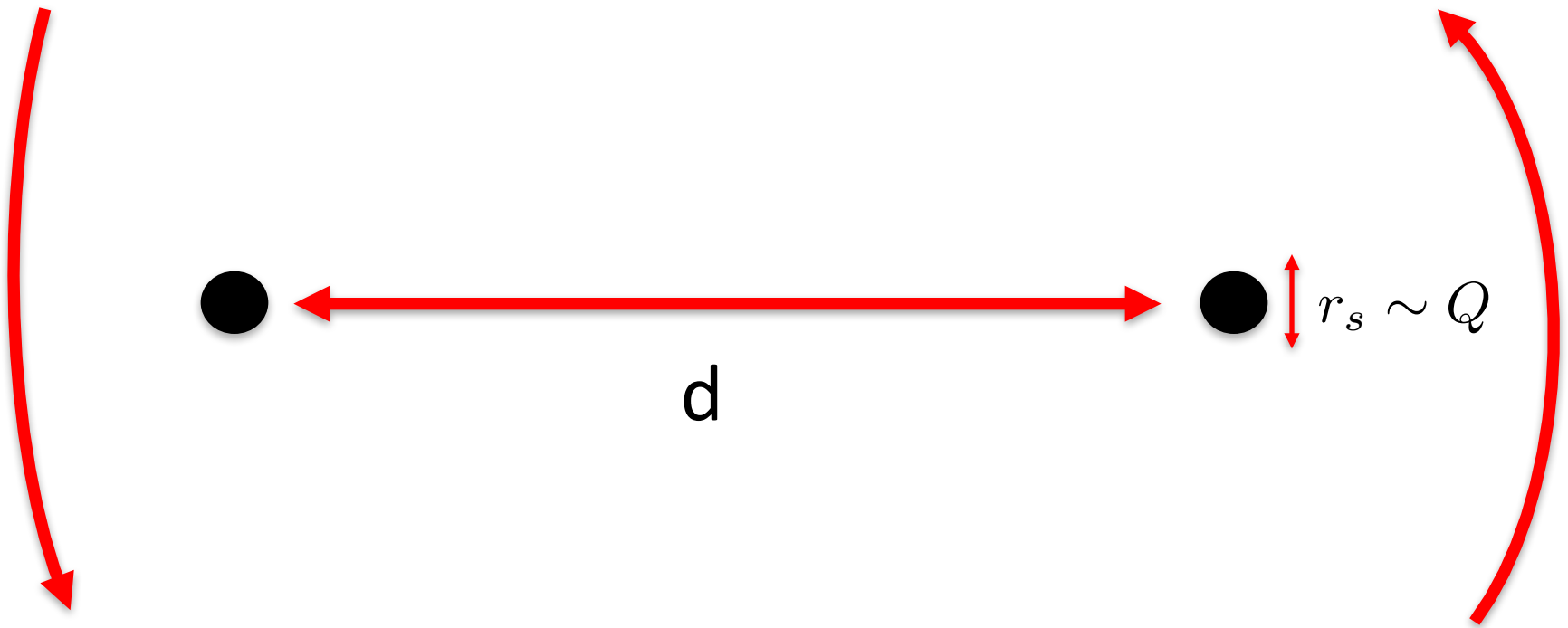
Balance the classical curvature + gauge field energy vs the Casimir energy.

$$E = Q + \frac{Q^3}{L^2} - \frac{Q}{L}, \quad \frac{\partial E}{\partial L} = 0 \longrightarrow L \sim Q^2, \quad E_{\min} - Q \sim -\frac{1}{Q} \sim -\frac{1}{r_s}$$

Now the throat is stabilized. Negative binding energy.

This is not yet a solution: The two objects attract and would fall on to each other

Adding rotation



Some necessary inequalities

$$L \sim Q^2 \quad \text{From stabilized throat solution}$$

$$d \ll L \longrightarrow d \ll Q^2 \quad \text{Black holes close enough to that Casimir energy computation was correct.}$$

$$\sqrt{\frac{Q}{d^3}} = \Omega \ll \frac{1}{L} \sim \frac{1}{Q^2} \longrightarrow Q^{\frac{5}{3}} \ll d \quad \text{Black holes far enough so that they rotate slowly compared to the energy gap.}$$

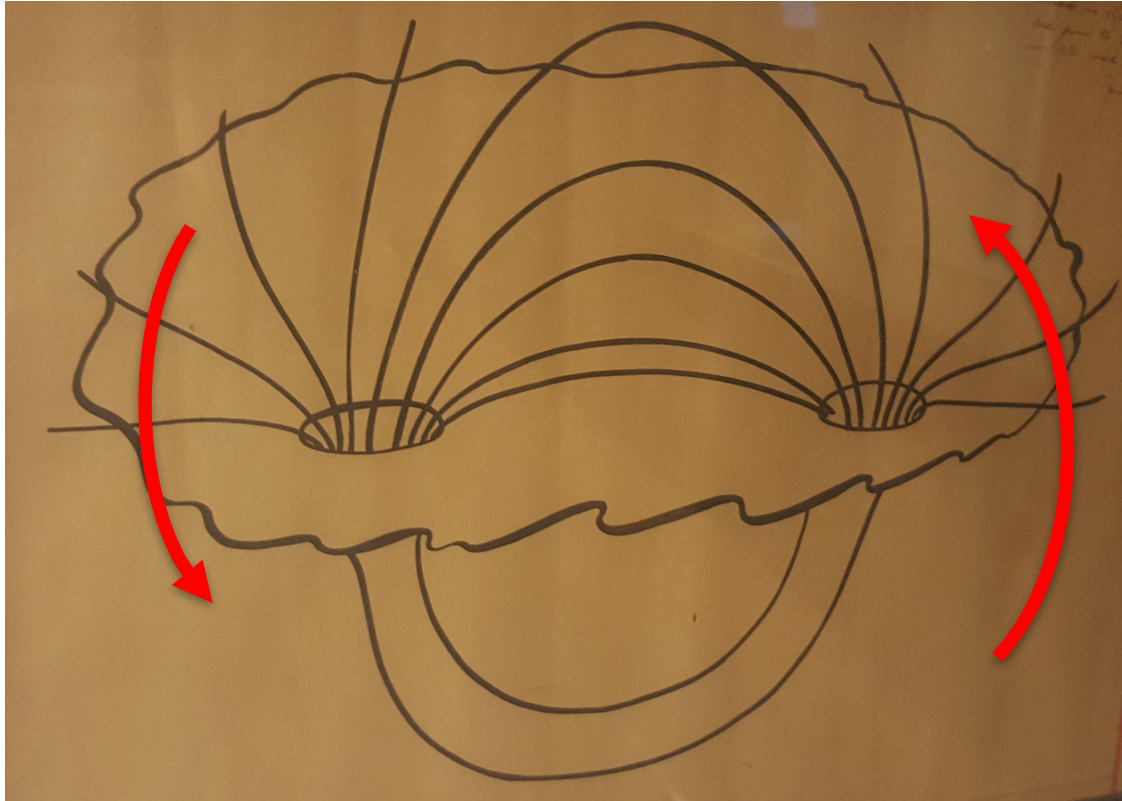
Kepler
rotation frequency

Unruh-like temperature less than energy gap

They are compatible $Q^{\frac{5}{3}} \ll d \ll Q^2$

Other effects we could think off are also small :
can allow small eccentricity, add electromagnetic and gravitational radiation, etc.

Final solution



Looks like two near extremal black holes if you do not get to the middle of wormhole
But there is no horizon!. Zero entropy solution.
It has a small binding energy.

Rotation \rightarrow temperature

$$T \sim \Omega \ll E_{\text{gap}} \sim \frac{1}{L}$$



Temperature does not create particles
in the throat

$$\text{Two Black Holes : } F = -TQ^2$$

$$\text{Wormhole : } F = -E_{\text{binding}} = -\frac{1}{Q}$$

Wormhole is the stable thermodynamic phase for $T < 1/Q^3$

For the solution we described so far: Wormhole is metastable.

Thermodynamically stable wormholes

$$d > Q^2 \longrightarrow L \sim d$$

distance inside is comparable to
distance outside

$$E_{\text{binding}} \sim -Q/d, \quad E_{\text{gap}} = \frac{1}{d}$$

$$\text{Stability} \rightarrow \Omega \sim T < (-E_{\text{Binding}})/S_0 \sim \frac{1}{dQ} \rightarrow d > Q^3$$

For such large wormhole lengths the quantum gravity effects are large in the throat.
Maybe they exist!.

Wormholes in the Standard Model

It could exist if nature is described by the Standard Model at short distances and d is smaller than the electroweak scale,

$$1 \ll Q \ll 10^8$$

If the standard model is not valid \rightarrow it is possible that similar ingredients are present in the true theory.

That it can exist, does not mean that it is easily produced by some natural or artificial process.



They are connected through a wormhole!

Much smaller than the ones LIGO or the LHC can detect!

Pair of entangled black holes.

Question

- Suppose we start from two near extremal black holes. They are not connected.
- How fast do they reconnect ?
- Can't be too fast because of the 2nd law and the large initial entropy.
- It is a process that would involve topology change!

Conclusions

- We displayed a solution of an Einstein Maxwell theory with charged fermions.
- It is a traversable wormhole in four dimensions and with no exotic matter.
- It balances classical and quantum effects.
- It has a non-trivial spacetime topology, which is forbidden in the classical theory.
- It does not violate causality.
- It has no horizon and no entropy.
- Can be viewed as a pair of entangled black holes.