UNREALIZED REPRESENTATIONS

in nonabelian f-theory models

Nikhil Raghuram (MIT) · 5 June 2018

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BROAD MOTIVATIONS

String theory has a landscape of many vacua

- Many possible gauge groups, types of charged matter
- Would like to understand/classify full landscape

BROAD QUESTION

Which representations of light, charged matter can occur in string theory vacua?

Tough to answer:

- Many different frameworks for constructing vacua
    - Each might cover only a portion of landscape
- In all frameworks, tough to obtain reps. beyond a few simple types
    - Techniques for constructing vacua become more complex
    - Are complicated reps forbidden, or are we just not smart enough?

KEY QUESTION

Let's focus on **F-theory**

- Easy-to-use strategies for constructing vacua
- Covers relatively large portion of the string vacuum landscape
- Clear results on which (nonabelian) gauge groups can occur

LESS BROAD QUESTION

Which representations of light, charged matter can occur in F-theory vacua?

Currently, no classification of all reps. that can occur

- Certain reps. appear automatically
- Models with reps. beyond these are very complicated
  - Some reps. have no known F-theory realization

STATUS OF MATTER REPRESENTATIONS

FOR SU(N) GAUGE GROUP		
Easily obtained representations	Difficult reps. with known F-theory models	Reps. without known F-theory realization
Fundamental 2-Antisymmetric Adjoint	Symmetric 3-Anti for SU(6)-SU(8) 3-Sym for SU(2)	3-Anti of SU(9) (**84**) 4-Sym of SU(2) (**5**) 3-Sym of SU(3) (**10**)

MESSAGE TODAY **Specific obstacles to realizing last category**

- These reps. may not be realizable at all in F-theory
- These reps. appear in seemingly consistent SUGRA models

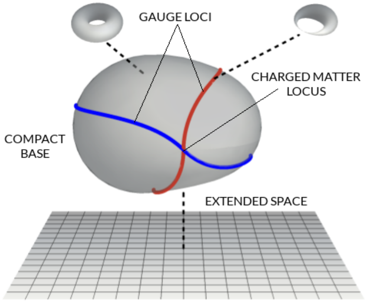
WHY IS THIS INTERESTING?

1. Important for understanding F-theory landscape, swampland
    - Could lead to new constraints, string compactification techniques, ...
    - Maybe F-theory/string theory can't give us any desired model?
2. Offers opportunities for new insights into dualities
    - e.g. duality between heterotic orbifolds and F-theory
3. Potentially interesting phenomenological implications
    - Example: Minimal Dark Matter Model **[Cirelli, Fornengo, Strumia '05]**
    - Has matter in 4-Sym (**5**) of SU(2)
    - Can such a model be realized in F-theory?

OVERVIEW OF F-THEORY

F-theory: Compactifying Type IIB with 7-branes, varying axiodilaton

1. Represent axiodilaton τ with elliptic curve at each point
   - Elliptic fibration over compactification base
2. Total space is Calabi-Yau
   - Base is not Calabi-Yau
3. Singular fibers at loci in base
   - Describe 7-brane locations
   - Give us nonabelian gauge groups
   - Codim-One $\leftrightarrow$ Gauge Bosons
   - Codim-Two $\leftrightarrow$ Charged Matter

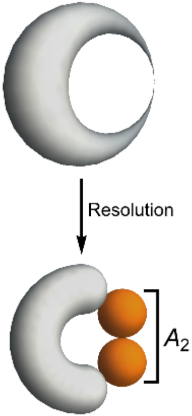


Result: Consistent compactification with minimal SUSY

SINGULARITY TYPES

Q: For codimension-one loci, how do we determine the gauge group?
A: ADE Singularity Types

- Resolve singular fiber by introducing $\mathbb{P}^1$'s
- Shrinking $\mathbb{P}^1$'s restores the singularity
  - Massless gauge bosons in shrunk limit
- $\mathbb{P}^1$'s intersect in Dynkin diagram pattern
  - Intersection matrix is negative of Dynkin matrix
  - Gives us an ADE Singularity Type
- Dynkin diagram tells you gauge algebra



EXAMPLE A_2 *singularity gives SU(3) gauge algebra*

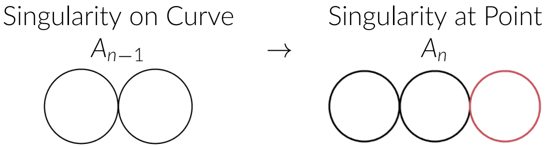
CHARGED MATTER

Typical charged matter ***Singularity type enhances at codim-two locus***

To determine the representation (after resolution)

- Have extra exceptional $\mathbb{P}^1$ at enhanced singularity
- Calculate intersection of extra $\mathbb{P}^1$ with other exceptional curves
  - Tells you Dynkin label of the representation

EXAMPLE ***Fundamental of SU(n)***



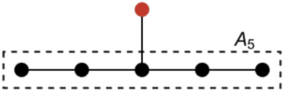
Dynkin Label: $[0, \ldots, 0, 1]$

3-ANTISYMMETRICS OF SU(N)

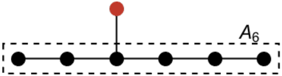
We can realize 3-Anti.'s of SU(6)-SU(8)

- [Morrison, Taylor '11], [Anderson, Gray, NR, Taylor '15]

20 of SU(6) $A_5 \to E_6$



35 of SU(7) $A_6 \to E_7$



56 of SU(8) $A_7 \to E_8$

What might 3-Anti. of SU(9) look like?

84 of SU(9) $A_8 \to \hat{E}_8$

3-ANTI OF SU(9)

84 of SU(9) $A_8 \to \hat{E}_8$

PROBLEM **Attempts to obtain 3-Anti of SU(9) lead to codim-2 singularities that are too severe**

- Cannot be resolved through fiber resolution procedure
  - Requires blow-up on the base
- Different interpretation than typical codimension-2 singularities
  - Associated with tensionless strings, SCFTs
  - Not like typical matter in F-theory
  - Also see **talk by Oehlmann**
- I'll refer to them as ***superconformal loci***
  - Name comes from behavior of algebraic expression for elliptic fibration

OTHER REPRESENTATIONS

Issue with superconformal loci occurs for other representations as well

REPRESENTATION	HYPOTHETICAL ENHANCEMENT
4-Anti. of SU(8) (**70**)	$A_7 \to \hat{E}_7$
4-Sym. of SU(2) (**5**)	$A_1^4 \to \hat{D}_4$
3-Sym. of SU(3) (**10**)	$A_2^3 \to \hat{E}_6$

COMMON FEATURE Enhancements to *extended* Dynkin diagrams

RULES FOR REALIZABLE REPS

RULE **Reps must involve embedding in standard Dynkin diagram**

- Embedding in extended Dynkin diagram not allowed

ALLOWED

3-Anti of SU(8) (56)

$A_7 \to E_8$

DISALLOWED

3-Anti of SU(9) (84)

$A_8 \to \hat{E}_8$

RULE FOR REALIZABLE REPS II

RULE **Reps must involve embedding in standard Dynkin diagram**

- Embedding in extended Dynkin diagram not allowed

REASON

- Resolution introduces exceptional $\mathbb{P}^1$'s
  - Intersection matrix is negative of Dynkin matrix for Dynkin diagram
- Light matter in limit where $\mathbb{P}^1$'s shrink
- **To contract set of $\mathbb{P}^1$'s, intersection matrix must be negative definite**
- Dynkin matrix for extended Dynkin diagrams not positive definite
  - Cannot shrink these $\mathbb{P}^1$'s

Hypothetical 3-Anti. of SU(9)

$A_8 \rightarrow \hat{E}_8$

$$\begin{pmatrix} -2 & 1 & 0 & 0 & 0 & \\ 1 & -2 & 1 & 0 & 0 & \\ 0 & 1 & -2 & 1 & 1 & \cdots \\ 0 & 0 & 1 & -2 & 0 & \\ 0 & 0 & 1 & 0 & -2 & \\ & & \vdots & & & \ddots \end{pmatrix}$$

Intersection Matrix for $\hat{E}_8$

NON-REALIZABLE REPS

Examples of non-realizable reps include

- 3-antisym. of SU(9) (**84**)
- 4-sym. of SU(2) (**5**)
- 3-sym. of SU(3) (**10**)
- 4-antisym. of SU(8) (**70**)

even though **they seem to be allowed in consistent SUGRA models**

OTHER GROUPS

Matter spectra for Sp, SO, exceptional groups even more limited

- Suggests F-theory can only realize “standard” nonabelian reps plus a few exotics

HETEROTIC ORBIFOLDS

3-Antisymmetric of SU(9) (**84**) violates our rule

- Requires $A_8 \to \hat{E}_8$ enhancement
- All attempts in F-theory lead to superconformal loci

But it does occur in abelian heterotic orbifolds

EXAMPLE **HETEROTIC $E_8 \times E_8$ ON $T^4/\mathbb{Z}_3$**
[Honecker, Trapletti '06]

- 6D SUGRA model with SU(9)$\times E_8$ gauge group
- **84** matter occurs in the untwisted sector
- If you smooth orbifold to K3, SU(9) Higgsed to SU(8) w/ **56** matter
  - SU(8) model can be realized in F-theory
  - Can explicitly understand Het/F-theory duality for SU(8) model

Something interesting happening in orbifold limit.

- Similar phenomena seem to occur for other orbifolds
- Example: $T^4/\mathbb{Z}_2$ with $E_7 \times \mathrm{SU}(2) \times E_8$ group, $(\mathbf{56}, \mathbf{2}, \mathbf{1})$ matter

CONCLUSIONS

- Obstructions to realizing many representations in F-theory
  - Reps. must involve enhancement to standard Dynkin diagram, not enhanced Dynkin diagram
- Attempts to obtain unrealizable reps. lead to superconformal loci
  - Associated with tensionless strings, SCFTs
- "Problematic" reps. seem to occur naturally in heterotic orbifolds

FUTURE DIRECTIONS

- Further tests of Dynkin diagram rule
- Better understand Het/F-theory for these orbifolds

Thank you!

BACKUP SLIDES

A MORE WELL STUDIED EXAMPLE

Consider Heterotic on $T^4/\mathbb{Z}_2$ with

- $E_7 \times SU(2) \times E_8$ gauge group
- $(\mathbf{56}, \mathbf{2})$ matter in untwisted sector charged under $E_7 \times SU(2)$

In F-theory, intersections between E_7, SU(n) give superconformal loci

- Known example of "conformal matter"
  - These loci give SCFTs, not typical matter
  - [Del Zotto, Heckman, Tomasiello, Vafa '14]
- Also violates Dynkin diagram rule

$$E_7 \times A_1 \to \hat{E}_8$$

- Suggests you cannot get $E_7 \times SU(2)$ $(\mathbf{56}, \mathbf{2})$ matter in F-theory

A MORE WELL STUDIED EXAMPLE II

F-theory dual of $T^4/\mathbb{Z}_2$ orbifold constructed in [Ludeling, Ruehle '14]

- Resulting F-theory model has (4,6) singularities
- SU(2) Higgsed when orbifold smoothed to K3
  - Can construct F-theory model without (4,6) loci for Higgsed situation

Situation is remarkably similar to the 3-Anti. of SU(9)

- In both cases, "problematic" matter occurs in untwisted sector
- After smoothing orbifold to K3, can construct well-behaved F-theory duals

Hints at some general aspect of heterotic orbifolds and F-theory

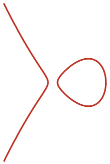
- Should be understood better

HIGHER GENUS REPRESENTATIONS

- Certain reps. involve 7-branes wrapped on higher genus divisors
- Exotic reps. can be localized at singular loci

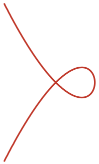
FOR SU(N)

Smooth Curve of Genus g



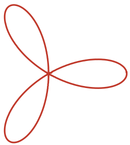
g Adjoints

Double Point Singularity



One Adjoint
or
Sym $+$ ***2-Anti***

Triple Point Singularity



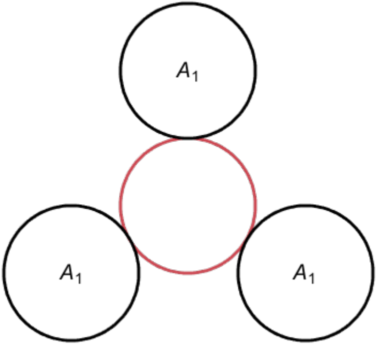
3 Adjoints
or
$\frac{1}{2}$ ***3-Sym*** $+ 2 \times$ ***Fund***

FIBERS FOR HIGHER GENUS REPS

In enhanced fiber, see multiple copies of gauge Dynkin diagram

- Close to singular points, gauge divisor looks like multiple intersecting copies

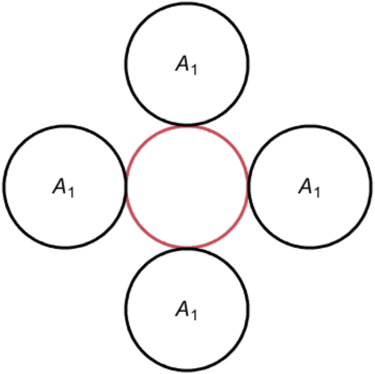
EXAMPLE 3-SYMMETRIC (4) OF SU(2)



Enhancement: $A_1^3 \to D_4$

4-SYMMETRIC OF SU(2)

The 4-Symmetric (5) of SU(2) involves enhancement $A_1^4 \to \hat{D}_4$



This violates Dynkin diagram rule.

HIGHER LEVEL REPRESENTATIONS

Reps like the 4-Sym (5) of SU(2) are higher level representations

- Sum of Dynkin labels is greater than 1

Generally tough to get in heterotic

- Might be possible in free fermion heterotic constructions
- See **[Lewellen '89]**, **[Dienes, March-Russell '96]**