Scaling Behavior of QCD Vertex Functions in Universal Extra Dimensions

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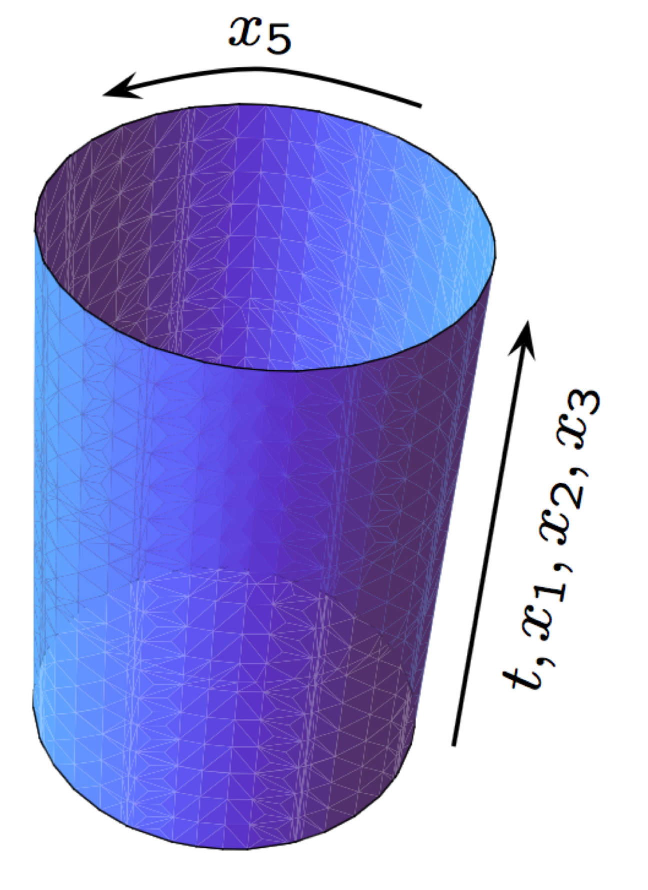
@PASCOS 2018

Fig. loop corrections governing large scale physics

Based on:

A. Freitas, DW - (arXiv: 1805.12142)

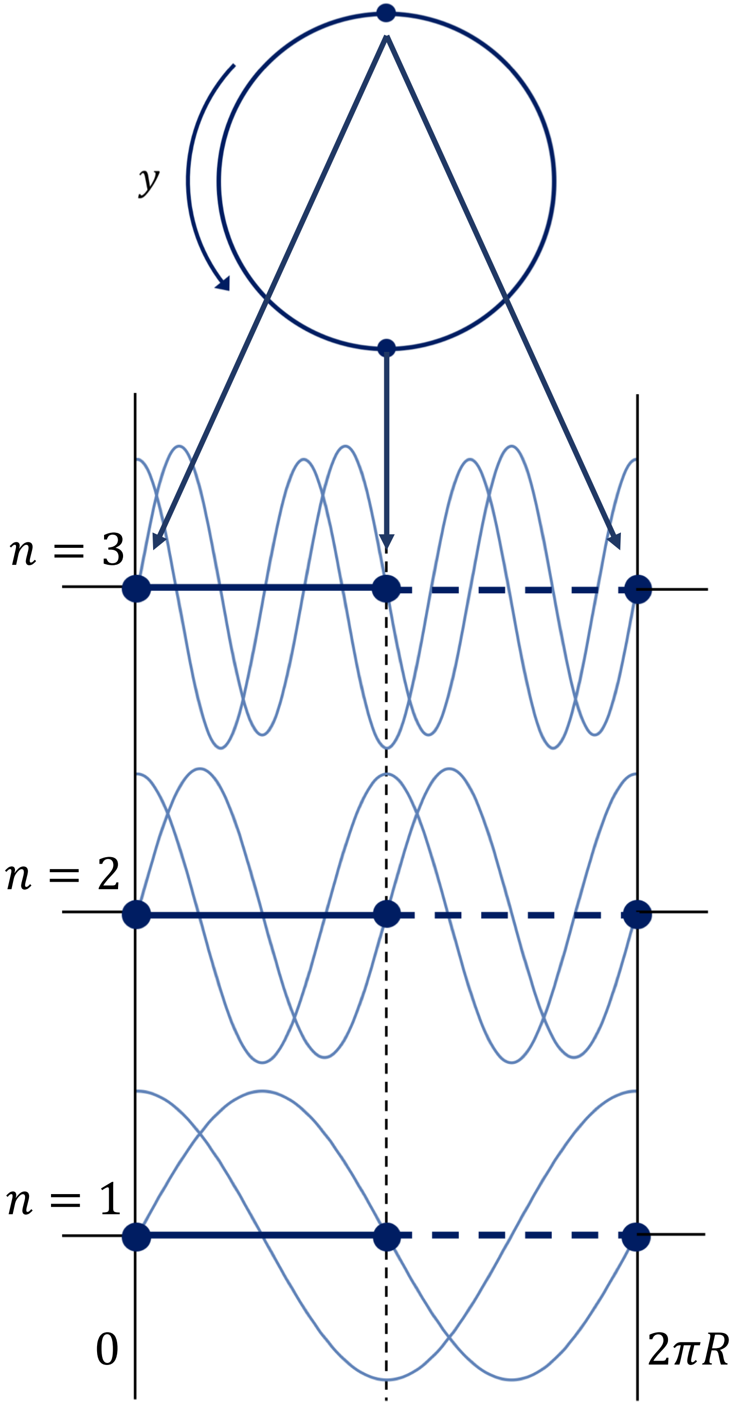
The Why, the What and the How



- **the Why**
  - Extra Dimensions are attractive **new physics** models
  - Analysis is **universally** applicable
  - **Radiative Corrections** important for new physics searches!
- **the What**
  - Infinite amount of KK-modes contributing to loop corrections
  - UED just effective theory itself → sensitivity to **UV completion**
- **the How**
  - IR behavior of vertex functions
  - Analytic Summation over tower
  - Exact renormalization group flow analysis

[1] Dienes, Dudas, Gherghetta hep-ph/9806292
J. Kubo, H. Terao, G. Zoupanos hep-ph/9910277

Universal Extra Dimensions



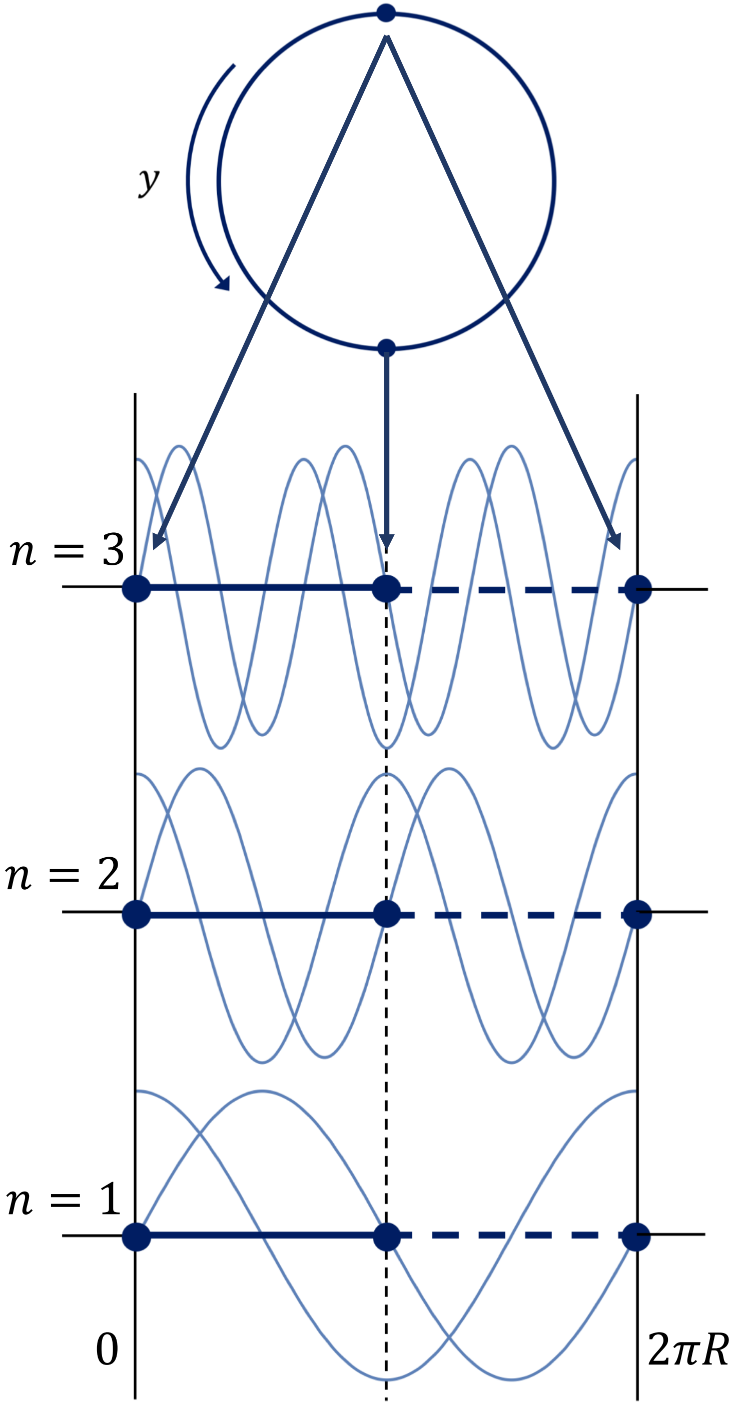
Minimal Setup: One extra dimension penetrated by all fields

$$\Phi(x,y) = \frac{1}{\sqrt{\pi R}}\left[\phi_0(x) + \sqrt{2}\sum_{n=1}^{\infty}\left(\phi_n^+(x)\cos\frac{ny}{R} + \phi_n^-(x)\sin\frac{ny}{R}\right)\right]$$

massless mode

tower of KK-excitations of mass $\frac{n}{R}$

[1] Appelquist, Cheng, Dobrescu hep-ph/0012100



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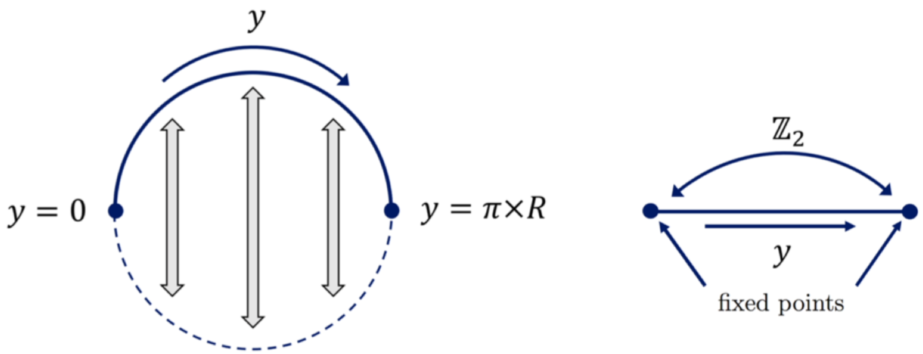
Problems: No Chiral Interactions in 5D and too many unphysical modes

Solution: Additional breaking of 5D Lorentz Invariance (**Orbifolding**)



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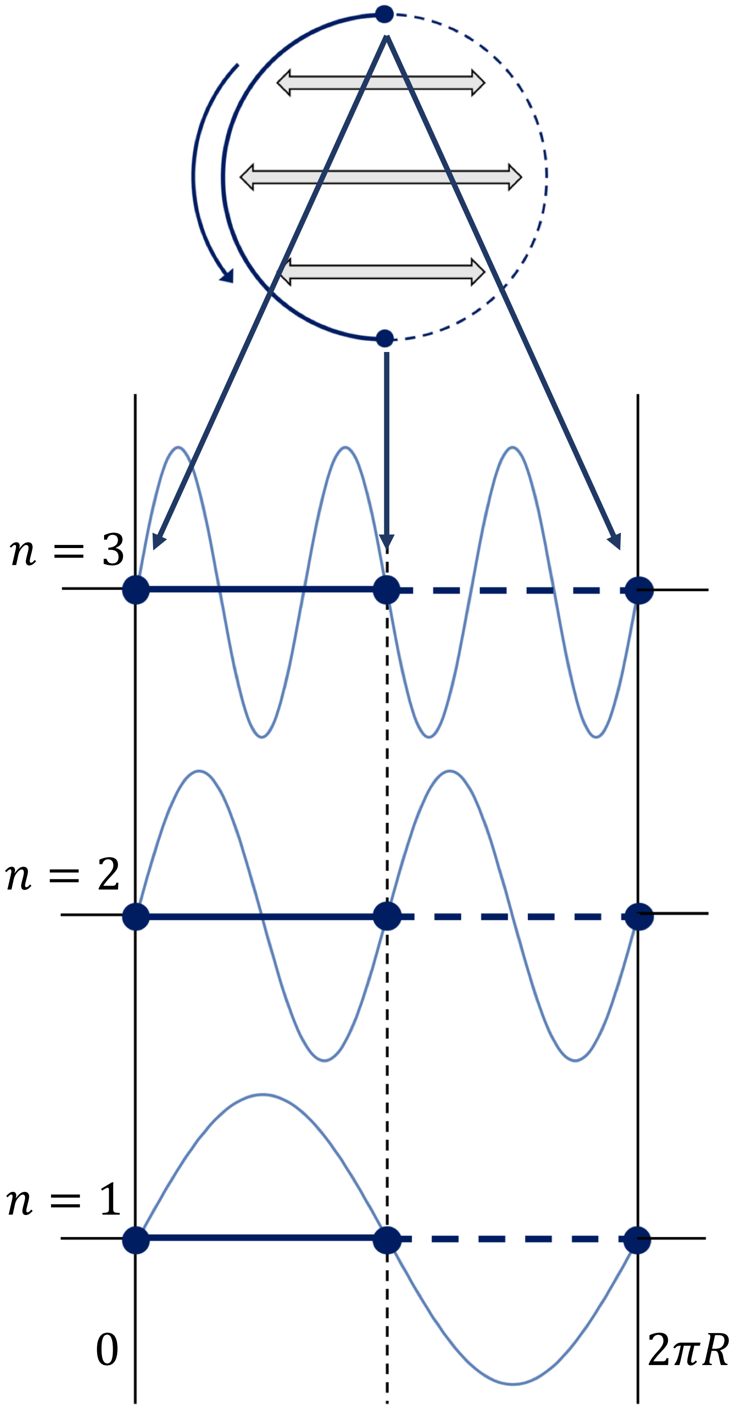

Use **boundary conditions** to project out unphysical modes (e.g. massless Goldstone mode A_5)

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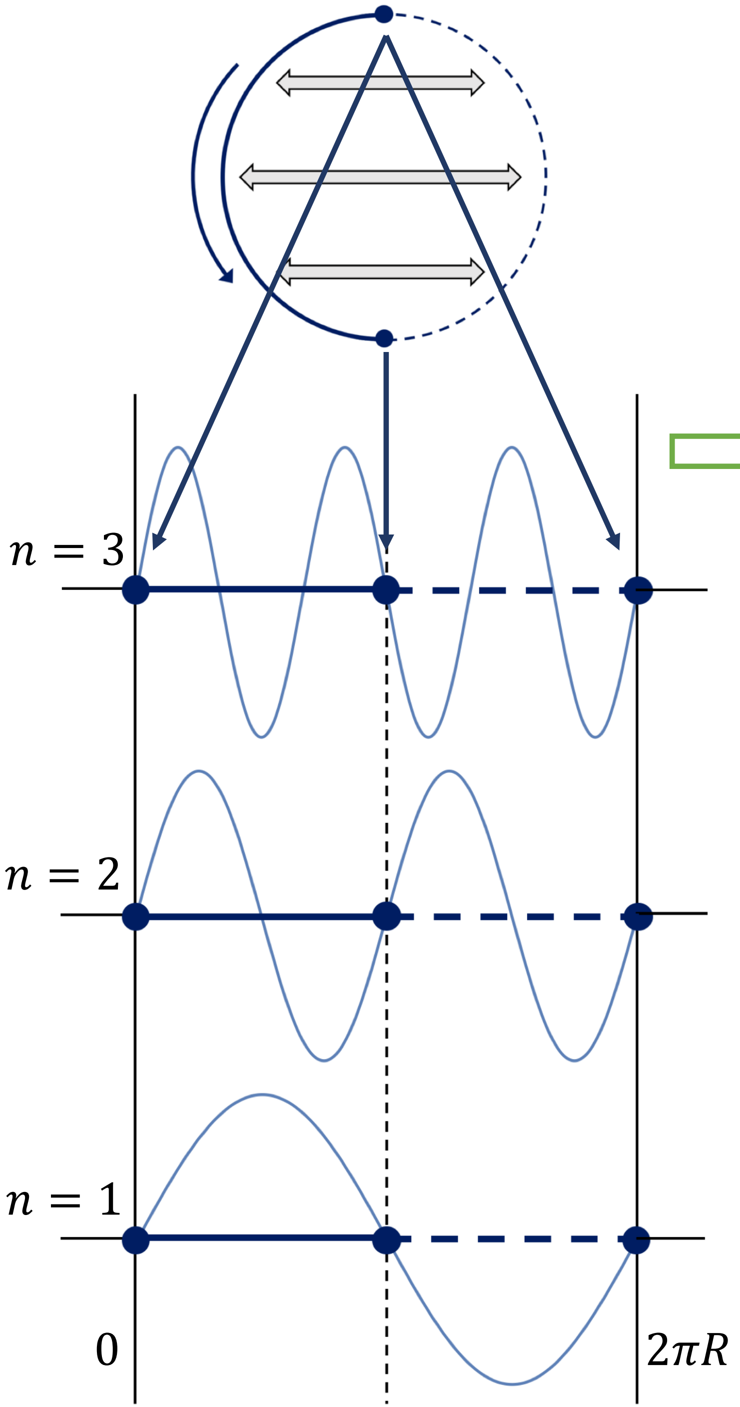

Use **boundary conditions** to project out unphysical modes (e.g. massless Goldstone mode A_5)

TWO copies for every chiral fermion (one SU(2) singlet and one doublet)

One chiral zero mode

Two vector KK-modes

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$\mathbb{Z}_2$
$G^M \equiv (G^\mu, G^5)$	adj.	–	–	$(+,-)$
$W^M \equiv (W^\mu, W^5)$	–	adj.	–	$(+,-)$
$B^M \equiv (B^\mu, B^5)$	–	–	adj.	$(+,-)$
(Q_L, Q_R)	**3**	**2**	$-1/6$	$(+,-)$
(u_L, u_R)	**3**	–	$+2/3$	$(-,+)$
(d_L, d_R)	**3**	–	$-1/3$	$(-,+)$
(L_L, L_R)	–	**2**	$-1/2$	$(+,-)$
(e_L, e_R)	–	–	-1	$(-,+)$
Φ	–	**2**	$+1/2$	$+$

Fig. quantum numbers of all fields in mUED

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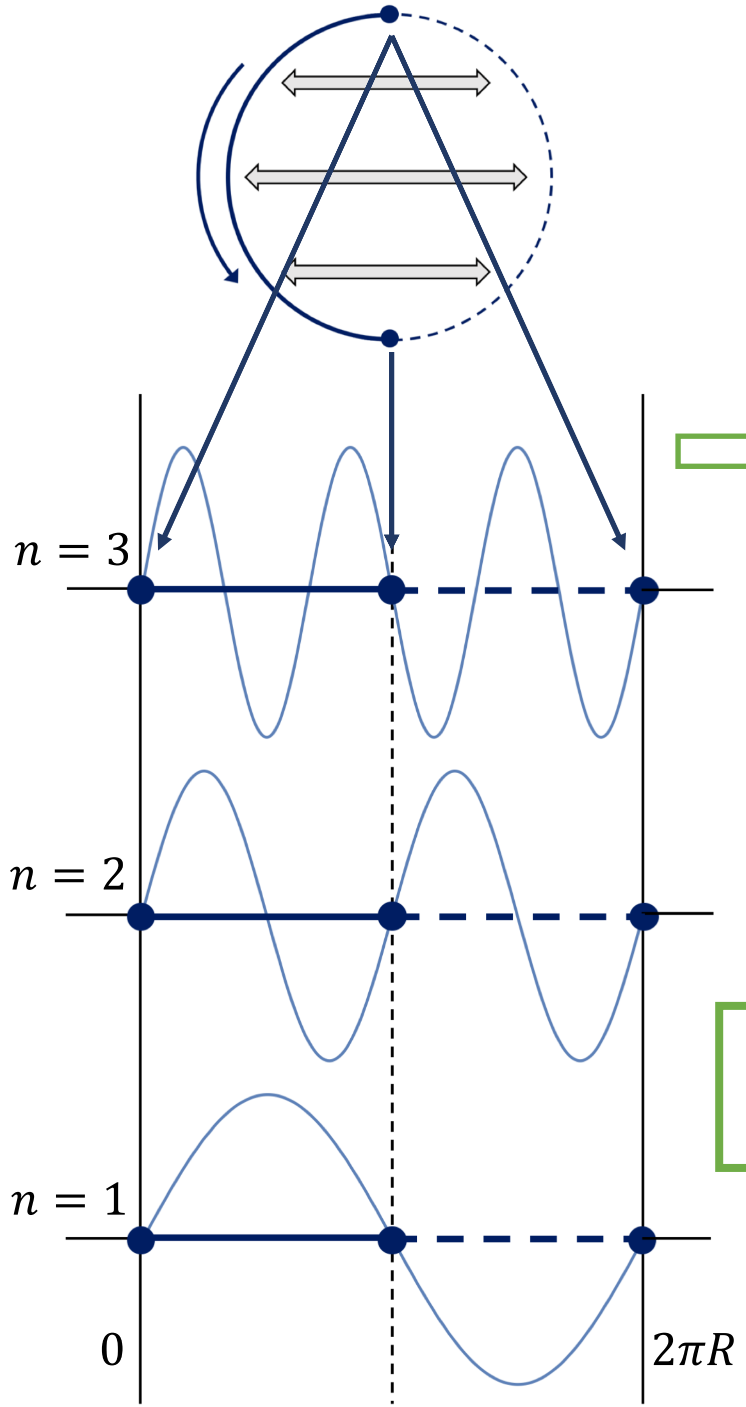

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KK-parity is good symmetry
KK-number broken at 1-loop

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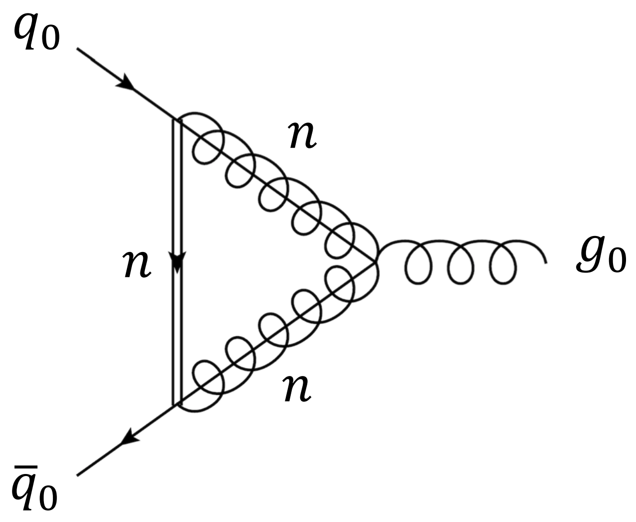
Corrections to SM Operators



<u>Renormalize Vertex for **every n**:</u>

- On-Shell for external states
- Contribution to the coupling counterterm δZ_g in $\overline{MS}$

$n = 0$

$$\beta_{\text{SM}} = \left(\frac{11}{3}C_A - \frac{4}{3}n_q T_f\right) \xrightarrow{n \geq 1} \beta_N = \left(\frac{7}{2}C_A - \frac{8}{3}n_q T_f\right)$$

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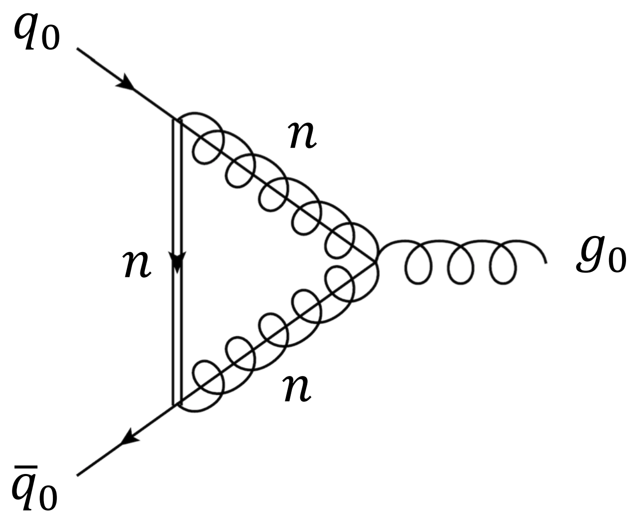

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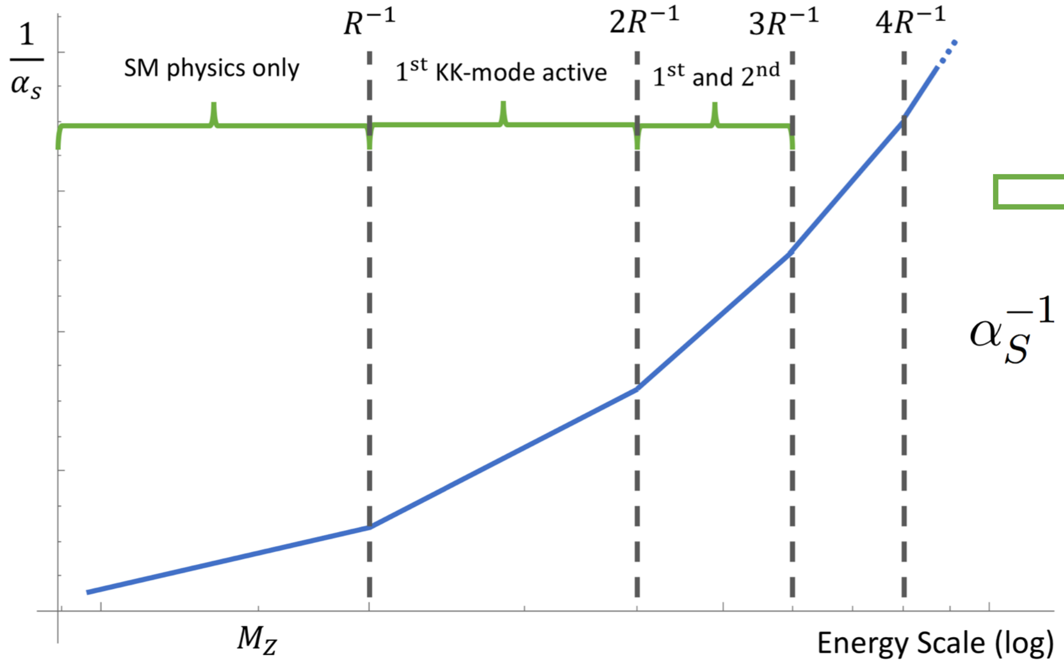
Running of coupling:

$$\alpha_S^{-1}(\mu) = \alpha_S^{-1}(M_z) + \frac{\beta_{\rm SM}}{2\pi}\log\frac{\mu}{M_Z} + \frac{\beta_N}{2\pi}\sum_{n=1}^{\Lambda R}\log\frac{\mu}{nR^{-1}}$$

SM running for $M_z \leq \mu \leq R^{-1}$

Contribution at **threshold**



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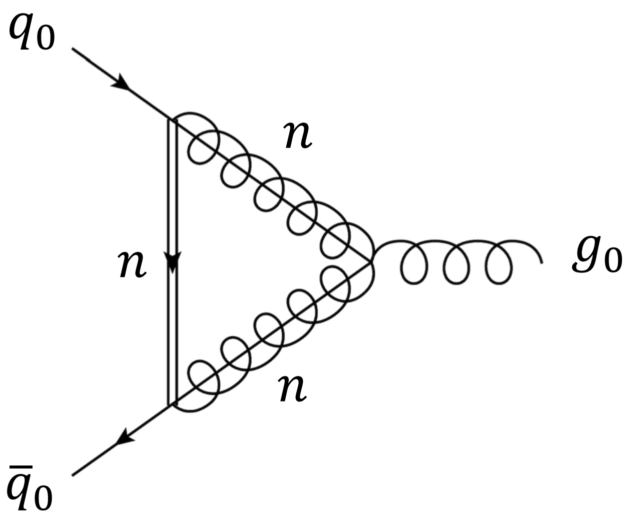

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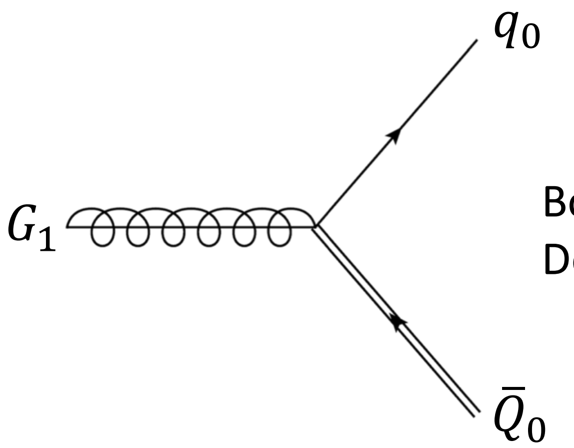
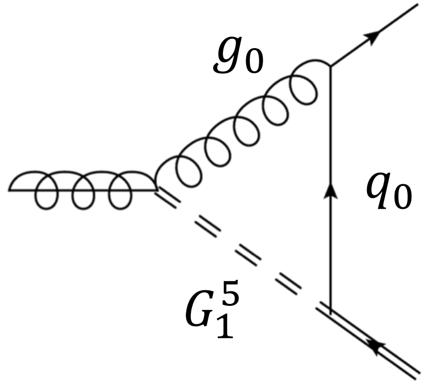
SM running for $M_z \leq \mu \leq R^{-1}$ — Contribution at **threshold** nR^{-1}

⇨ Running of Wilson Coefficient: $$C_{\text{SM}}(\mu) = \frac{g_s^3}{192\pi^2}\sum_{n=1}^{\Lambda R}\left[2C_A - (21C_A - 16n_q T_f)\log\frac{n^2}{(\mu R)^2}\right]$$

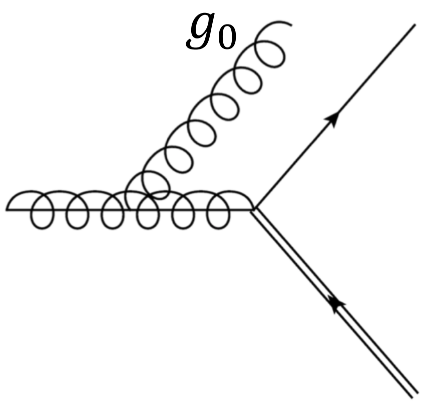
IR Behavior of KK Operators



Born
Decay



Virtual
Correction

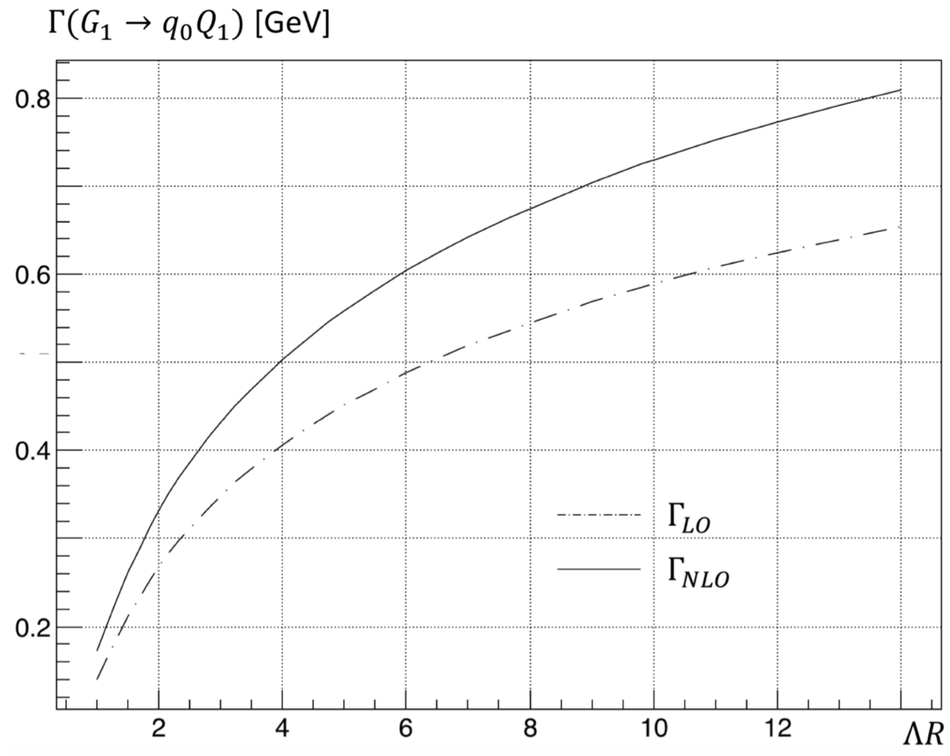


Real
Emission

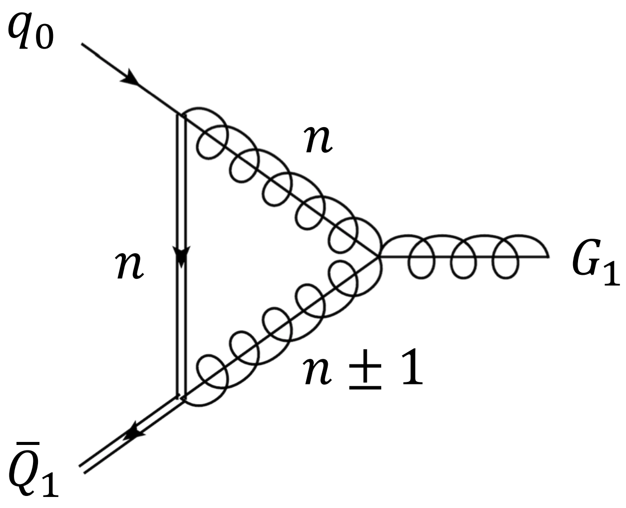
$q_0 Q_1 G_1$ Vertex has **IR divergence** at lowest order

⮡ Cancels in physical observable $\Gamma(G_1 \to q_0 Q_1)$

Technical Notes:
- **Phase Space Slicing** with two cutoffs
- Truncated mUED violates gauge invariance → **Coloron**
- Radiative **mass splitting**



Higher Corrections



Renormalization at **every level** anew – strategy as before but:
First coupling counterterm δZ_g, different!

$$\beta_{\text{Coloron}} = \left(\frac{(3 + 85C_A^2)(C_A - 2C_F)}{12} - \frac{8}{3} n_q T_f \right)$$

$n \geq 2$

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Running of coupling similar – assume α_s constant for $\mu \leq R^{-1}$

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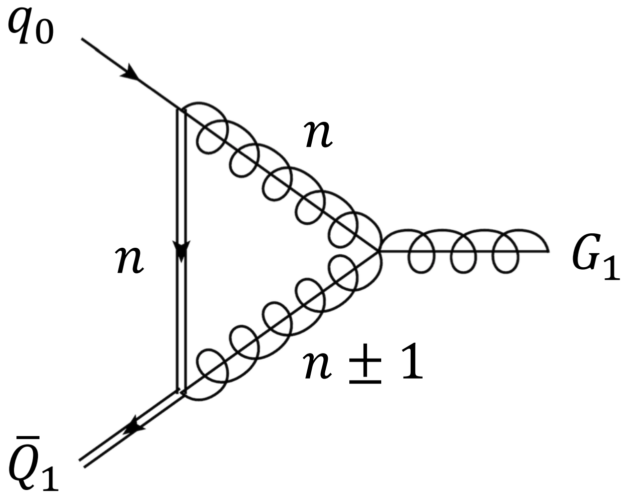

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$$C_{\text{KK}}(\Lambda) = \frac{g_s^3}{192\pi^2} \left[4\,(11C_A - 8n_q T_f)\,\Lambda R - (42C_A - 32n_q T_f + 9C_F) \log \Lambda R \right]$$

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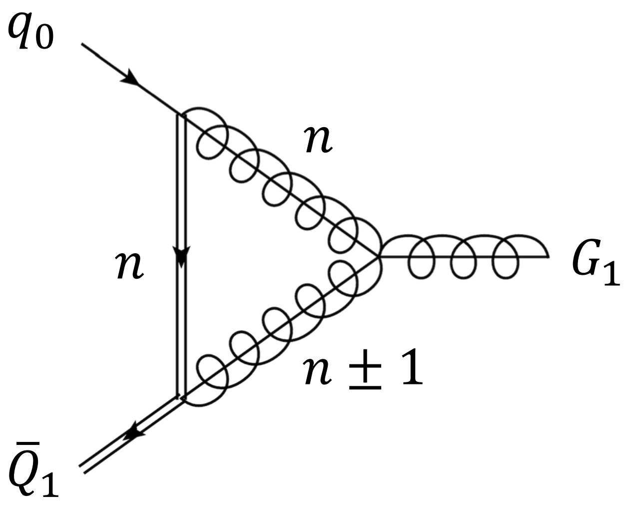

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Exact Flow Analysis

Alternative approach: Solve the 5D 1–loop ERGE

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\partial_t \mathcal{R}_k \left(\frac{\partial^2 S}{\partial\Phi\partial\Phi} + \mathcal{R}_k\right)^{-1}\right]$$

Coarse grains between IR k and UV cutoff Λ, with regulator $\mathcal{R}_k$.

[1] Wetterich arXiv:1710.05815
[2] Litim hep-th/0203006

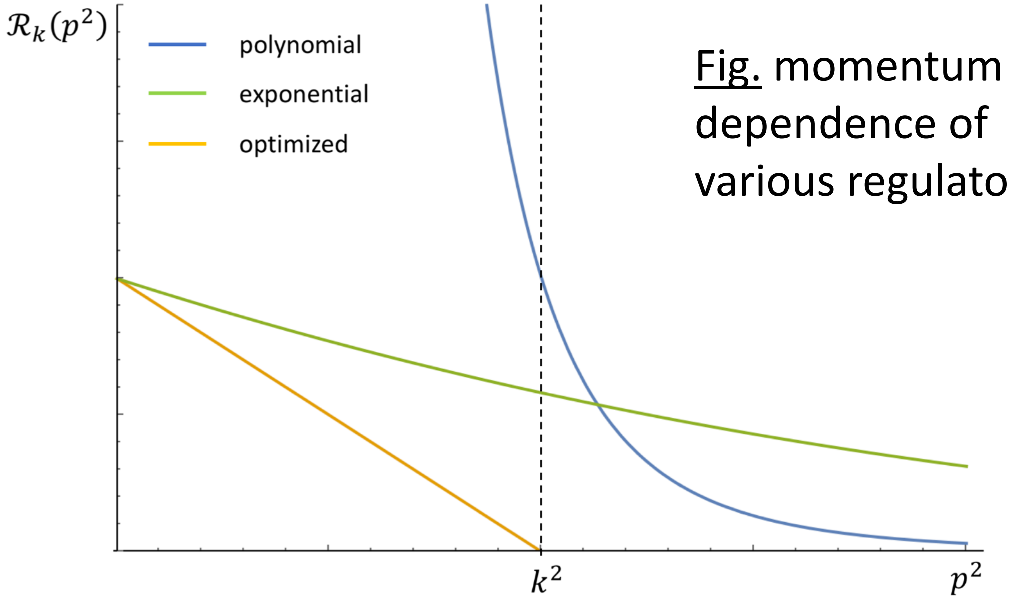

Fig. momentum dependence of various regulators

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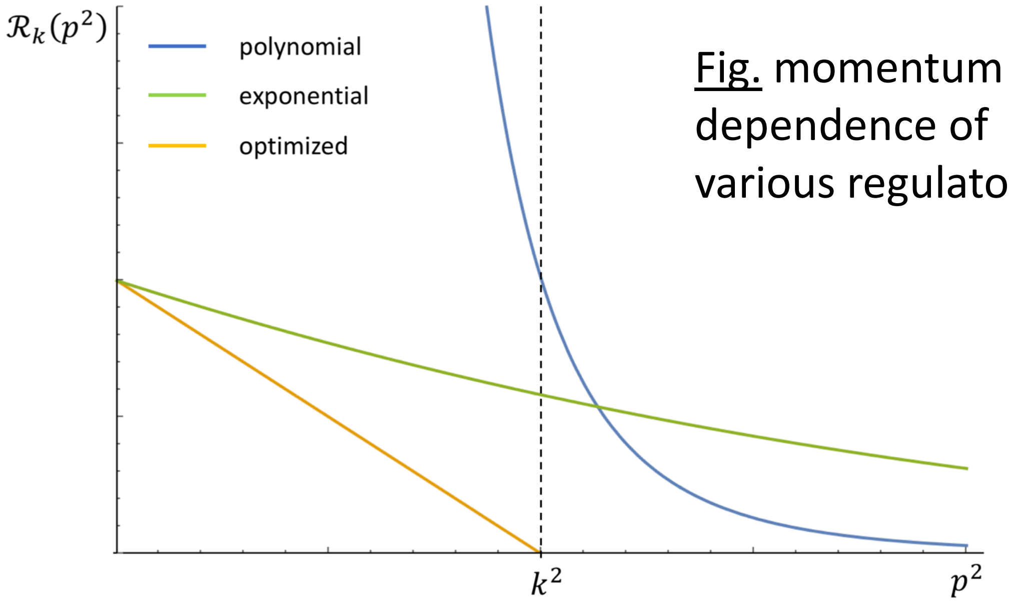
Use non-local heat kernel expansion to find effective action

$$\partial_t \Gamma_k = \frac{1}{2\,(4\pi)^{\frac{D}{2}}}\int d^D x\left[\mathrm{Tr}[\mathbb{1}]Q_{\frac{D}{2}}\left[h_k\right] + \mathrm{Tr}\left[U g_U U\right] + \mathrm{Tr}\left[\Omega_{MN} g_\Omega \Omega^{MN}\right]\right]$$



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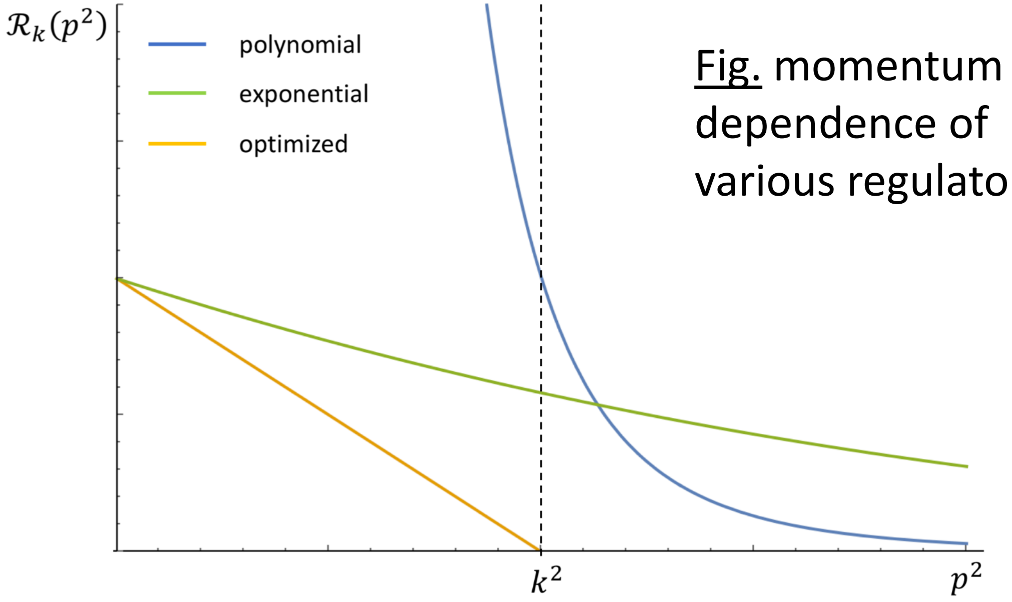
QED

$$\partial_k \alpha_s^{-1} = -\frac{2T_f}{3\pi} R$$

QCD

$$\partial_k \alpha_s^{-1} = \frac{7C_A}{4\pi} R$$



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Comparison of Schemes

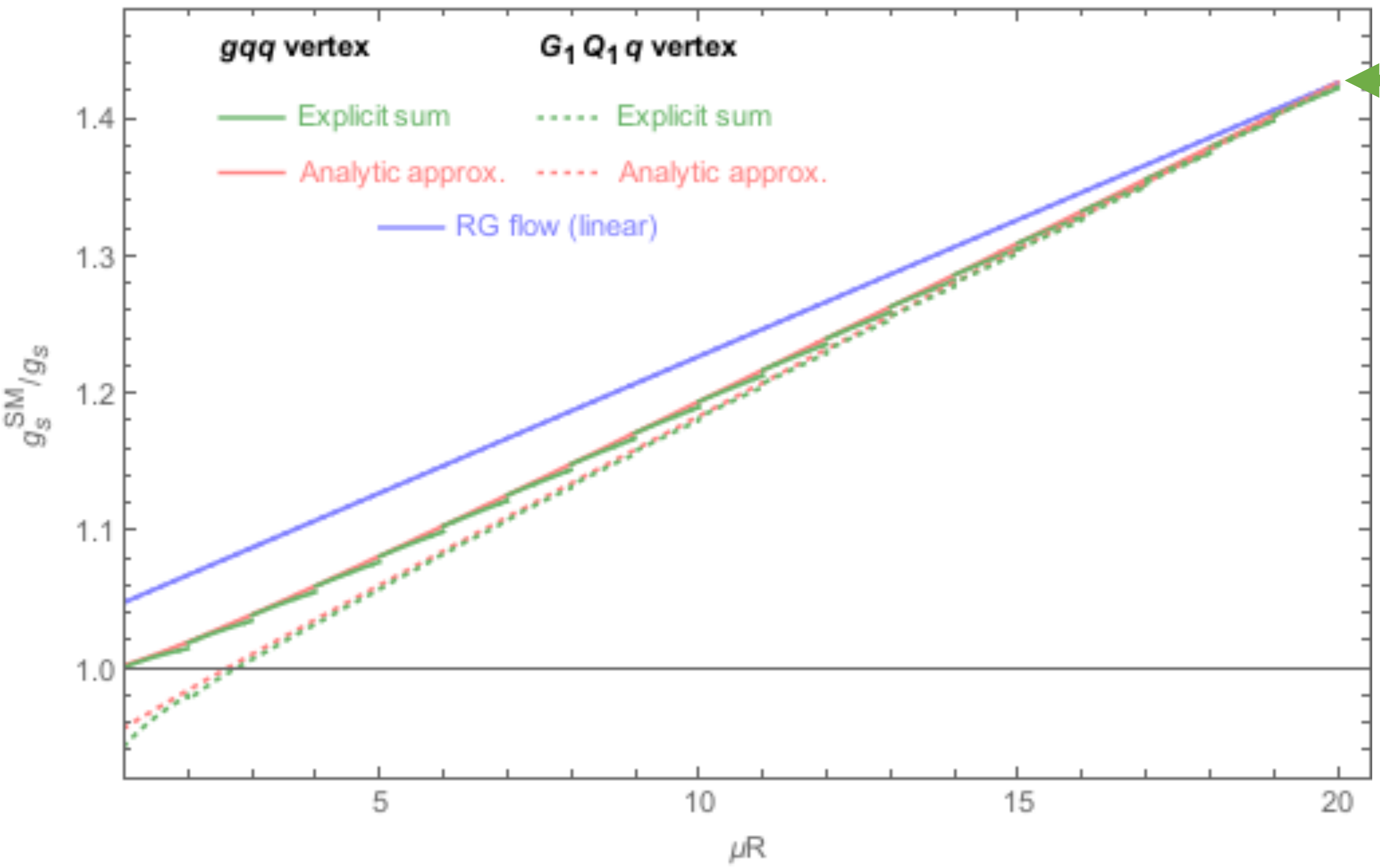

Fig. Cutoff scale-dependence of the vertices for different schemes

Couplings fixed at $\Lambda R = 20$

Lessons to learn:

- Large Cutoff – no distinction between vertices (% level)

$$\frac{g_{g_0 Q_1 G_1}(R^{-1})}{g_{q_0 q_0 g_0}(R^{-1})} \approx \frac{23 g_s^2}{192\pi^2} \log \Lambda R$$

- Leading (Linear) behavior independent of compactification (**Threshold corrections**!)

… and now what?

UED leads to interesting and rich Pheno!

- UED signatures can be probed at LHC and beyond
- Cutoff dependence is **universal**
- Sensitivity to **UV completion** (numerically modest)

⇒ Measure (future) decay width(s) for further information!

- We showed:

> - Large scale behavior for all vertices similar
> - FRGE describes linear running **independent** of compactification
> - Universal behavior for large class of models

Thanks!