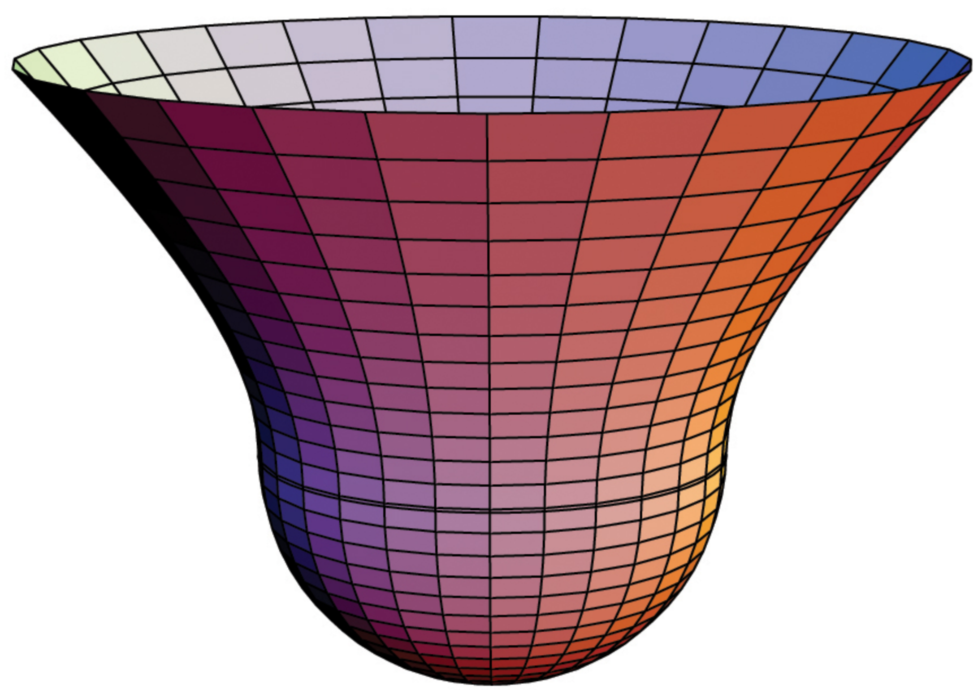
The no-boundary proposal: alive and well

Oliver Janssen (NYU,CCPP), 05/06/18 @ PASCOS2018



Based on `1705.05340`, `1804.01102` and ongoing work with Juan Diaz, Jonathan Halliwell, Jim Hartle, Thomas Hertog & Yannick Vreys

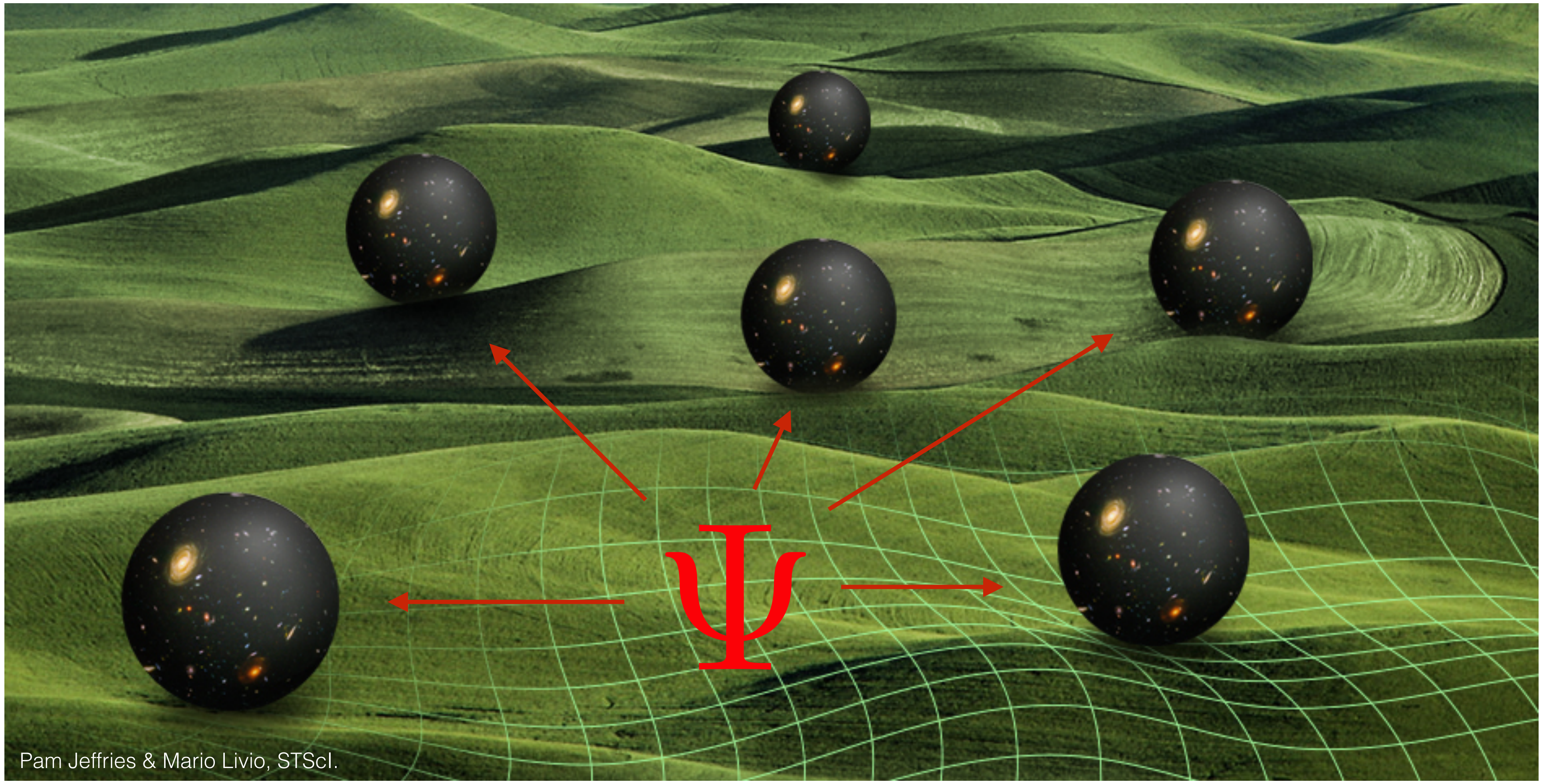

?
Pam Jeffries & Mario Livio, STScI.

Quantum cosmology

Wave function of the universe Ψ

Hartle & Hawking's proposal for Ψ : "ground state of quantum gravity"

$\longrightarrow$ theory of initial conditions for cosmology



Recently in the news

- The mathematical and physical foundations underlying the no-boundary proposal have been claimed ill-defined, in a series of four papers (Feldbrugge, Lehners & Turok, `1703.02076`, `1705.00192`, `1708.05104` and `1805.01609`)

- One claims to have introduced a "new element of rigor", Picard-Lefschetz theory, that would put the theory on firmer mathematical footing. One also insists on a purely Lorentzian formulation of the theory

- Using this, one claims that *any* implementation of the no-boundary idea via a semiclassical path integral leads to *unsuppressed* fluctuations around isotropic backgrounds, concluding that the idea in this form should be discarded

Proposal for Ψ (Hartle & Hawking '83)

$$\Psi_0[\text{three-geometry}] = \sum_{\text{four-geometries}} \exp(-I[g])$$

Sum over (all Euclidean) four-geometries g which have a boundary on which the induced three-geometry is the argument of the wave function, with I the Euclidean gravitational action

If the three-geometry is closed (this talk), the four-geometries are compact and have as *only boundary* the three-manifold on which the argument of the wave function lives

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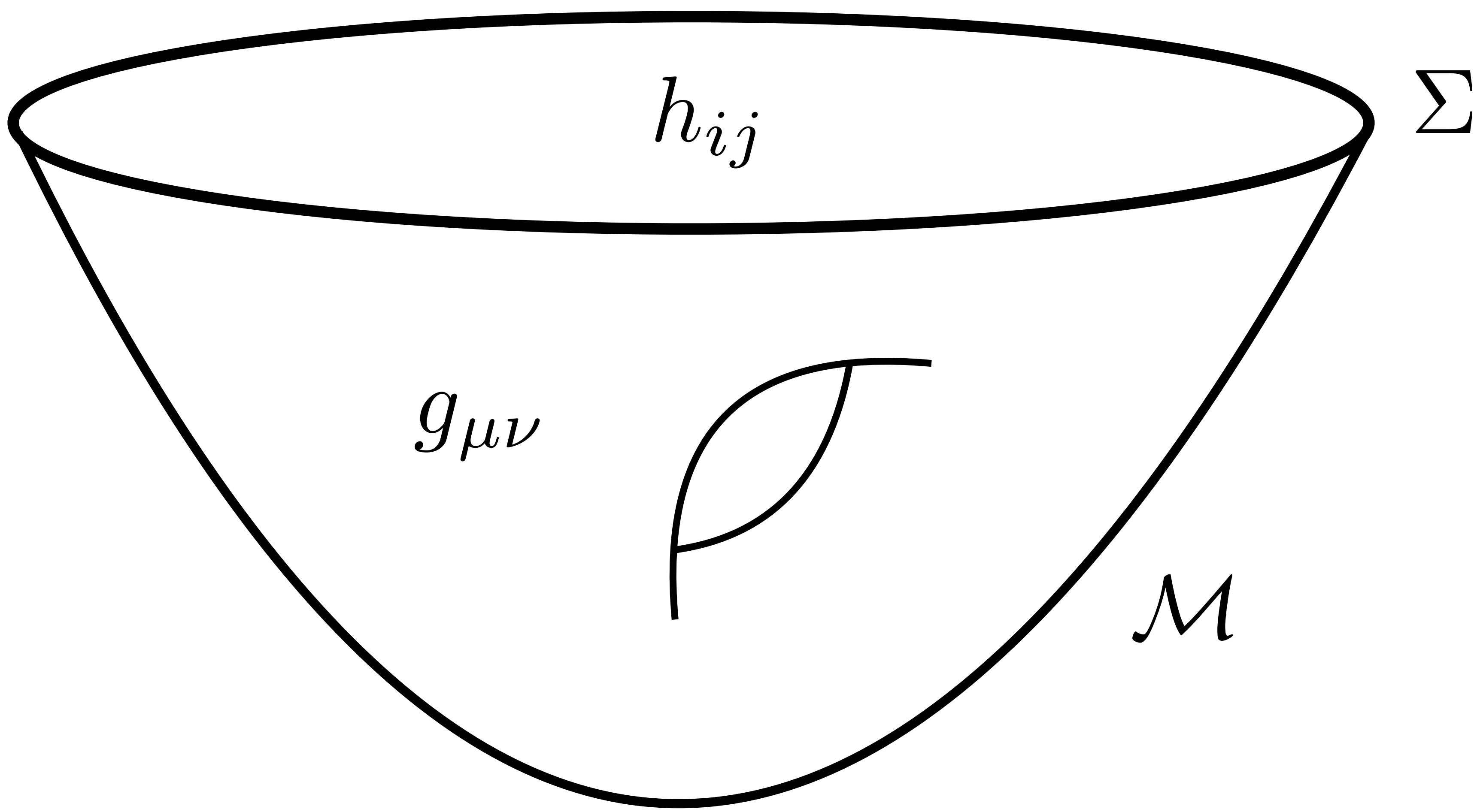
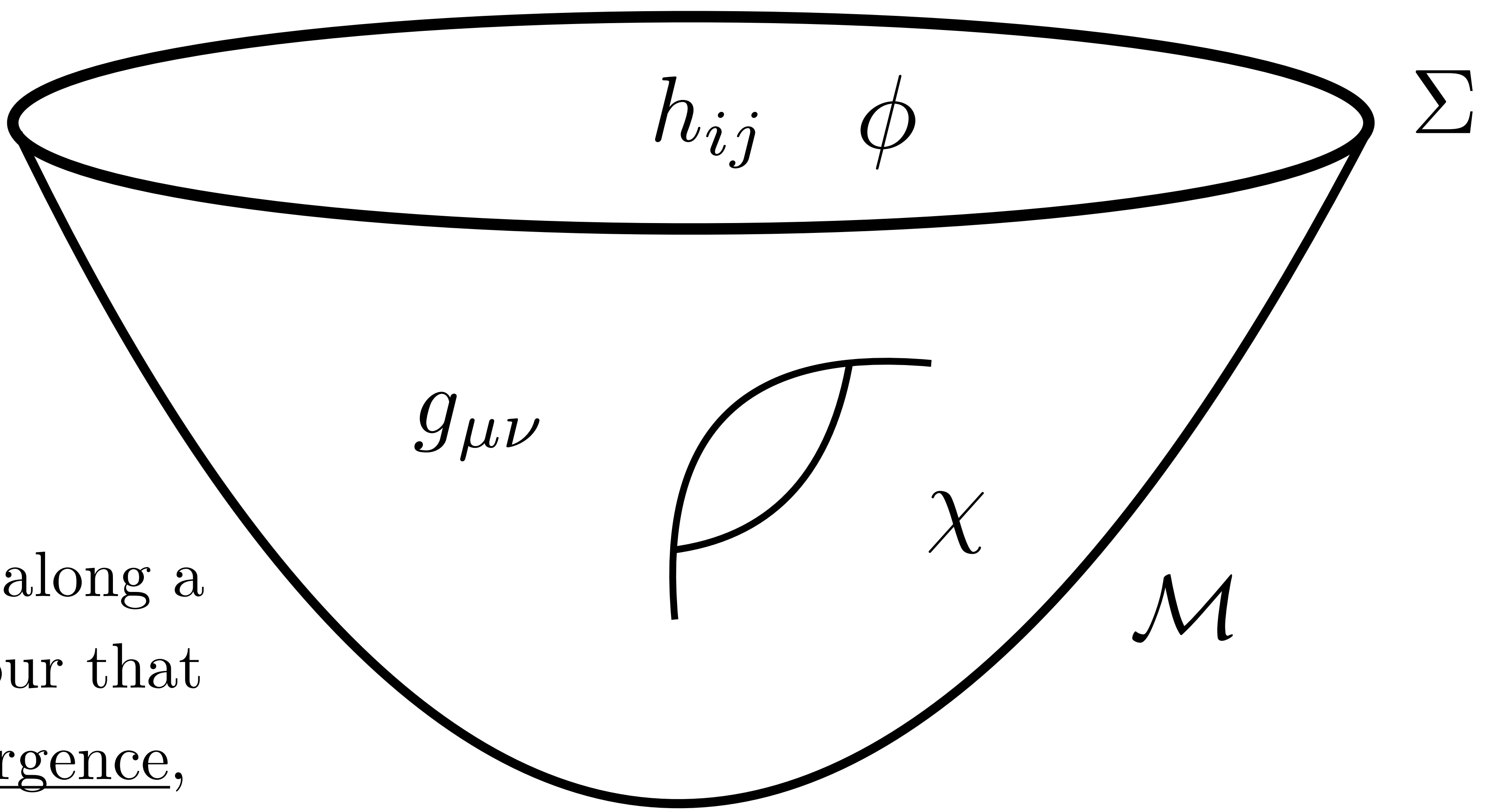


Proposal for Ψ (Hartle & Hawking '83)

$$\Psi_{\mathrm{HH}}\left[h_{ij},\phi\right]_{\Sigma} \sim \sum_{\mathcal{M}} \int_{\mathcal{C}} \mathcal{D}g_{\mu\nu}\mathcal{D}\chi \; e^{-S_E[g_{\mu\nu},\chi;\mathcal{M}]/\hbar}$$



integration is along a *complex* contour that ensures convergence, amongst other requirements

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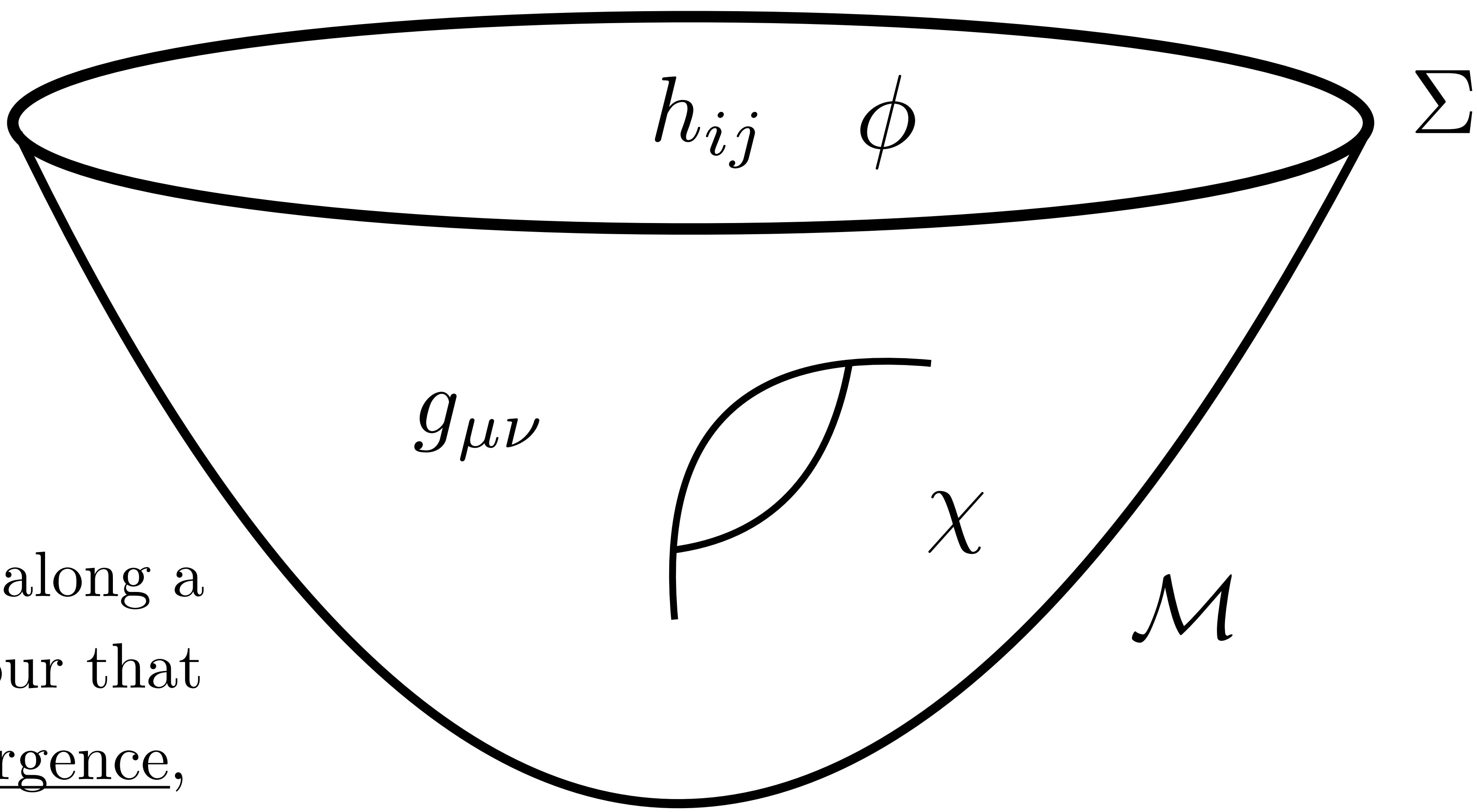

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Quantum gravity?

In the absence of a complete theory of quantum gravity it is reasonable to try and identify principles which specify a wave function of the universe which is adequate for predictions on scales larger than the Planck scale using a low-energy effective gravitational theory. The hope would be that such principles could be generalized to a complete theory.

(Halliwell & Hartle '91)

but cf. `1111.6090`, "Holographic no-boundary measure"
`1711.10037`, "Higher Spin de Sitter Hilbert Space"

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Some principles:

1. Integral should converge
2. Ψ should satisfy constraints implementing diff. invariance (Wheeler-DeWitt eq.)
3. Classical spacetime on familiar scales should be implied when the universe is large
4. Reproduction of QFT for matter when spacetime is approx. classical, i.e. matter fluctuations on a fixed (curved) background should be stable

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Feldbrugge et al. don't satisfy these points because of insistence on Lorentzian contour

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Minisuperspace quantum cosmology: the general programme

- Path integral over all physical dof. in GR too difficult technically. The best we have been able to do is "truncate" the theory down to a small number of fields of a single variable: *minisuperspaces*

- Typically only time reparametrization invariance and isotropic fields are kept. Manifold decomposed into 3+1, and we consider metric Anzätze / action

$$\mathrm{d}s^2 = -N(\tau)^2\mathrm{d}\tau^2 + h_{ij}[q^\alpha(\tau)]\mathrm{d}x^i\mathrm{d}x^j$$

$$S[q,\Pi;N] = \int_{\mathcal{M}} \mathrm{d}^4x \left(\frac{R}{2} - \Lambda\right) = \int_0^1 \mathrm{d}\tau \, (\Pi_\alpha \dot{q}^\alpha - NH)$$

$$H(q,\Pi) \equiv \frac{1}{2} f^{\alpha\beta}(q)\Pi_\alpha\Pi_\beta + U(q)$$

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- Classical reparametrization invariance leads to constraint $H = 0$

$$\delta N = \dot{f}$$

$$\delta q^\alpha = f\{q^\alpha, H\} \qquad f(0) = 0 = f(1)$$

$$\delta p_\beta = f\{p_\beta, H\}$$

Minisuperspace quantum cosmology: the general programme

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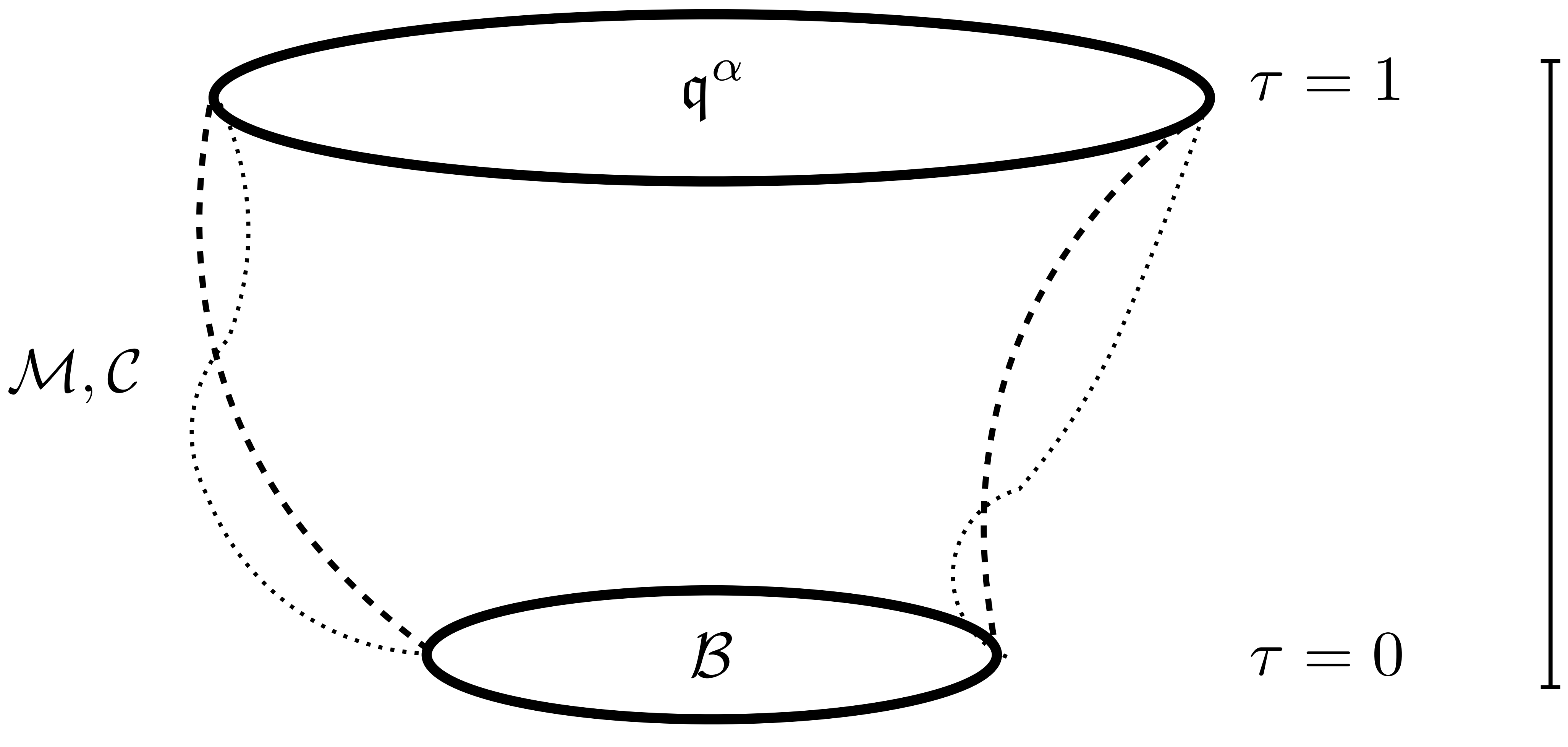
- Classical reparametrization invariance leads to constraint $H = 0$
- In quantum theory: physical state $\Psi(\mathfrak{q}^\alpha)$ annihilated by the operator version of the constraint

$$\hat{H}\Psi = 0 \qquad \text{(Wheeler-DeWitt eq.)}$$

In position space: $$\hat{H} = -\frac{\hbar^2}{2}\nabla^2 + \hbar^2 \xi R + U \, , \quad \xi = \frac{2 - D}{8(D-1)}$$

Minisuperspace quantum cosmology: the general programme

$$\Psi(\mathfrak{q}^\alpha) = \sum_{\mathcal{M}} \int_{\mathcal{C}} \mathrm{d}N \int_{\mathcal{B}}^{q^\alpha(1)=\mathfrak{q}^\alpha} \mathcal{D}q^\alpha\, \mathcal{D}\Pi_\alpha\; e^{iS[q,\Pi;N]/\hbar}$$



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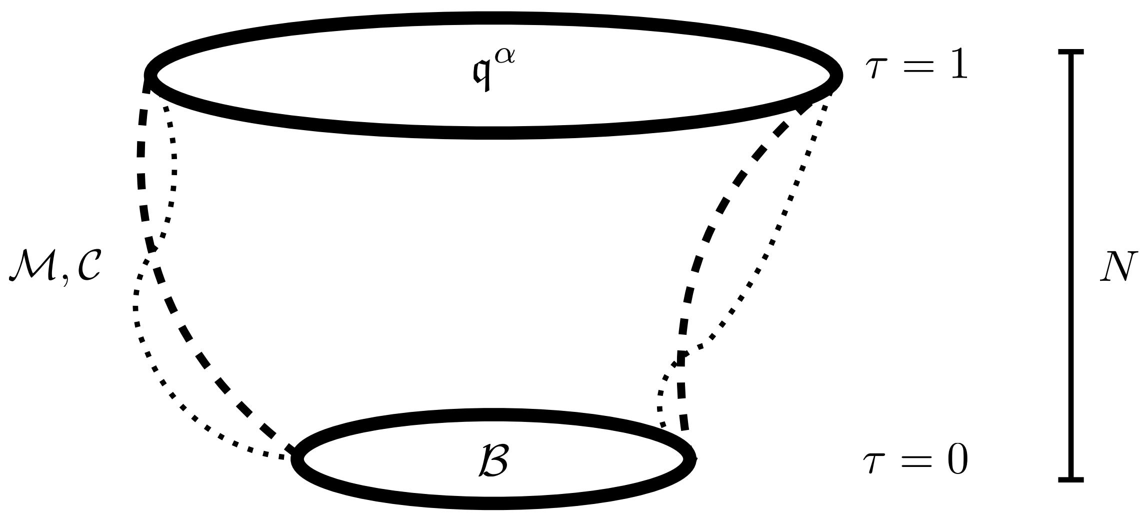
↓

propagator $K(\mathfrak{q}^\alpha, N; \mathcal{B}, 0)$



Schrödinger equation:

$$HK = i\hbar\,\partial_N K$$

With suitable contour $\mathcal{C}$,
Ψ solves the WDW equation

$$H\Psi = 0$$

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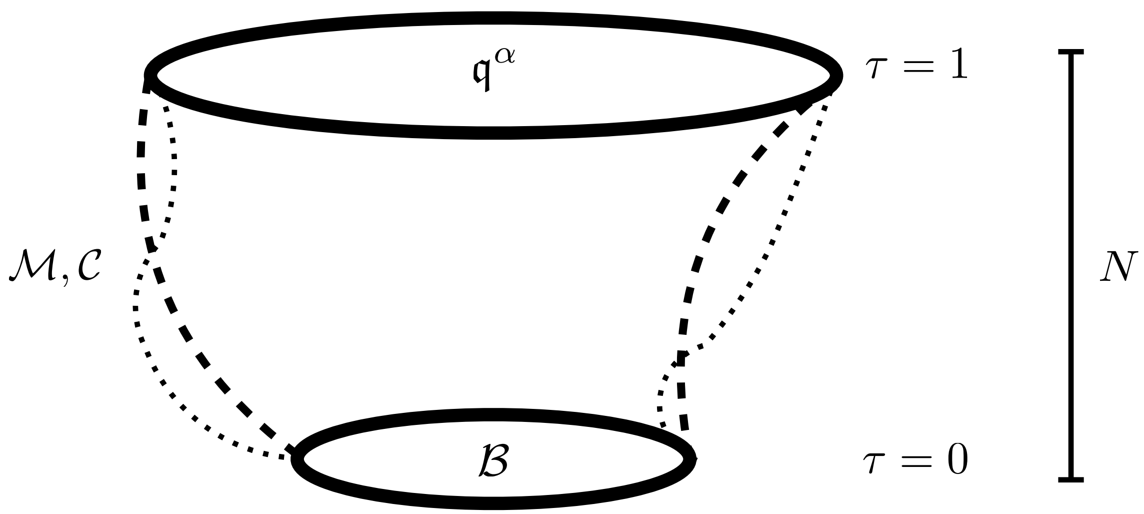

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Feldbrugge et al.: $H\Psi \propto \delta$

The no-boundary proposal in minisuperspace models

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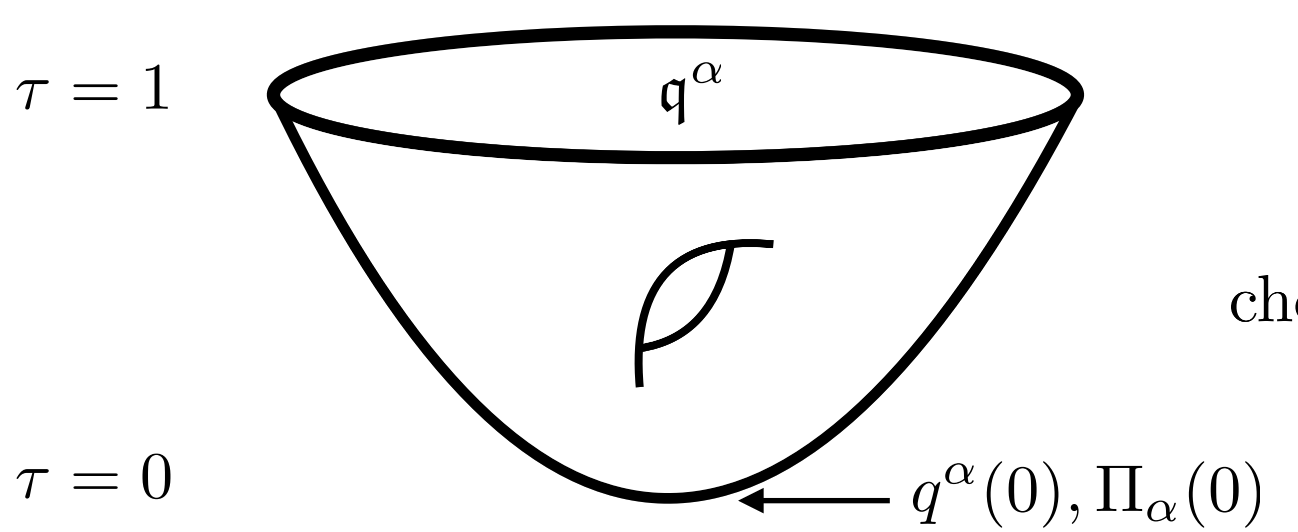
Three-volume should go to zero, in a regular way (at least, classically)



choose this consistent with QM

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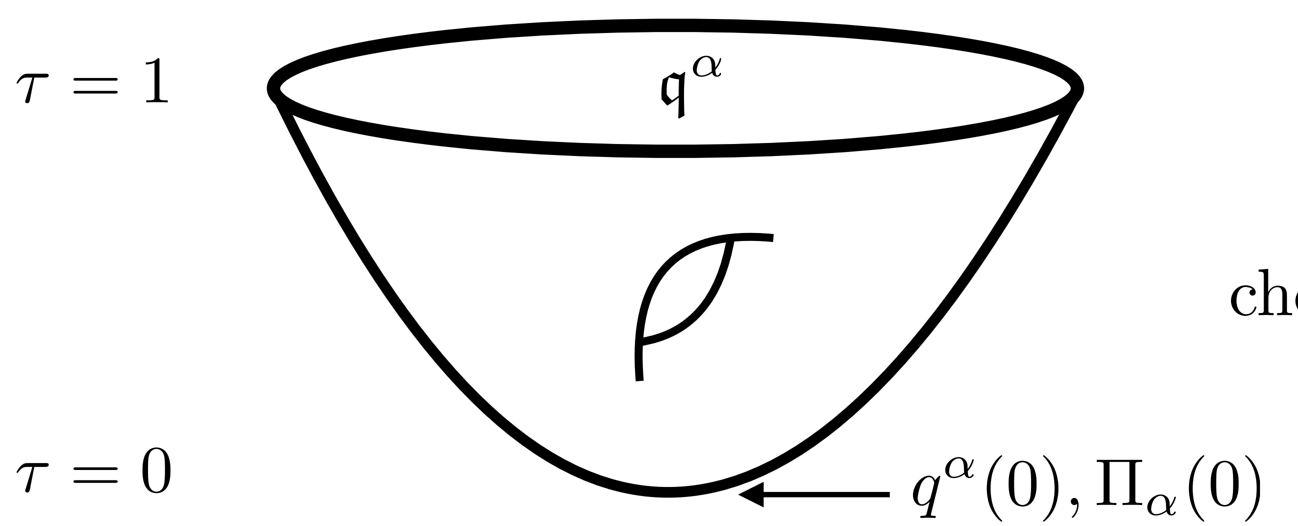

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Ψ_{HH} in an anisotropic minisuperspace

We studied a two-dimensional, anisotropic minisuperspace model (Bianchi type IX cosmology)

$$\mathrm{d}s^2 = -\frac{N(\tau)^2}{q(\tau)}\mathrm{d}\tau^2 + \frac{p(\tau)}{4}\left(\sigma_1^2 + \sigma_2^2\right) + \frac{q(\tau)}{4}\sigma_3^2$$

This is a non-linear completion of the model studied by Feldbrugge et al.

$$\Psi_{\mathrm{HH}}(\mathfrak{p}, \mathfrak{q}) = \sum_{\mathcal{M}} \int_{\mathcal{C}} \mathrm{d}N \int_{\mathcal{B}}^{(p,q)(1)=(\mathfrak{p},\mathfrak{q})} \mathcal{D}p\, \mathcal{D}q\, \mathcal{D}\Pi_p \mathcal{D}\Pi_q \; e^{iS[p,q,\Pi_p,\Pi_q;N]/\hbar}$$

four-ball ($\mathcal{M}$)

closed contour around origin ($\mathcal{C}$)

$p(0) = 0,\ \Pi_q(0) = -i$ ($\mathcal{B}$)

Ψ_{HH} in an anisotropic minisuperspace

$$\Psi_{\mathrm{HH}}(\mathfrak{p},\mathfrak{q}) = \oint \mathrm{d}N \int_{(p,\Pi_q)(0)=(0,-i)}^{(p,q)(1)=(\mathfrak{p},\mathfrak{q})} \mathcal{D}p\,\mathcal{D}q\,\mathcal{D}\Pi_p\mathcal{D}\Pi_q\; e^{iS[p,q,\Pi_p,\Pi_q;N]/\hbar}$$

Instantons: (part of) Taub-NUT-de Sitter with complex NUT parameter

$$p = 4\left(\tau^2 - L^2\right) \qquad \Delta = (\tau - L)^2 - \frac{\Lambda}{3}(\tau + 3L)(\tau - L)^3$$

$$q = \frac{16L^2\Delta}{\tau^2 - L^2} \qquad L = \frac{iN_s}{4}$$

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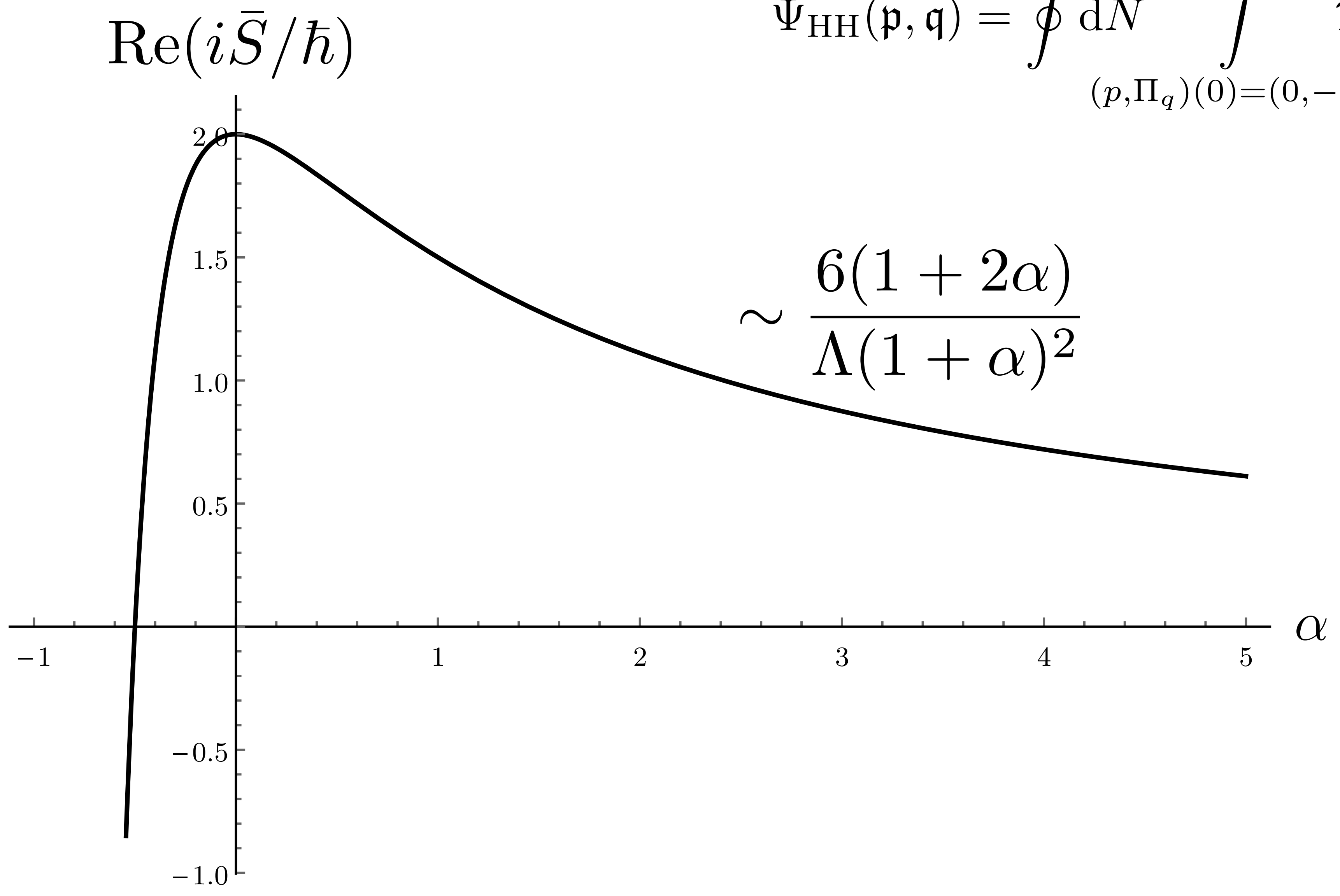

$$\alpha = \mathfrak{p}/\mathfrak{q} - 1$$

squashing parameter

Fluctuations around isotropy are suppressed in the no-boundary state

Comments and conclusions

- When implementing the NBP in detail in toy models of quantum gravity we are faced with ambiguities, such as the choice of contour for PI and QM boundary conditions. These are constrained by mathematical and physical requirements. In simple models there is a unique semiclassical no-boundary wave function

- We did not mention Picard-Lefschetz theory

- We calculated the/a no-boundary wave function in an anisotropic minisuperspace and saw that fluctuations are suppressed. The calculation of Feldbrugge et al. is plagued by the breakdown of perturbation theory

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Thank you!

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Feldbrugge et al.: what went wrong?

Minisuperspace: tensor perturbations of FLRW metric for a homogeneous and isotropic closed spacetime

$$\mathrm{d}s^2 = -\frac{N(\tau)^2}{q(\tau)}\mathrm{d}\tau^2 + q(\tau)\left(\Omega_{ij} + \varepsilon_{ij}\right)\mathrm{d}\Omega^i\mathrm{d}\Omega^j$$

$$\varepsilon_{ij} = 2\sum_{n,l,m} \varphi_{nlm}(\tau)\ \left(G_{ij}\right)^n_{lm}(\mathbf{\Omega})$$

$n = 2, 3, \ldots$

$l \in \{2, 3, \ldots, n\}$

$m \in \{-l, \ldots, l\}$

$$\nabla^2 \left(G_{ij}\right)^n_{lm} = -(n^2 + 2n - 2)\left(G_{ij}\right)^n_{lm}$$

transverse traceless tensor harmonics on S^3

So in principle $q^\alpha = \{q, \varphi_{nlm}\}\,, \quad S = S_{\mathrm{EH}} + \Lambda\,, \quad \ldots$

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- Expand S to quadratic order in φ_{nlm} and attempt to evaluate PI semicl.

$$S \to S_{\mathrm{background}} + \int_0^1 \mathrm{d}\tau\, N\left(\frac{1}{2N^2}q^2\dot{\varphi}^2_{nlm} - \frac{n(n+2)}{2}\varphi^2_{nlm}\right)$$

(idem massless scalar)

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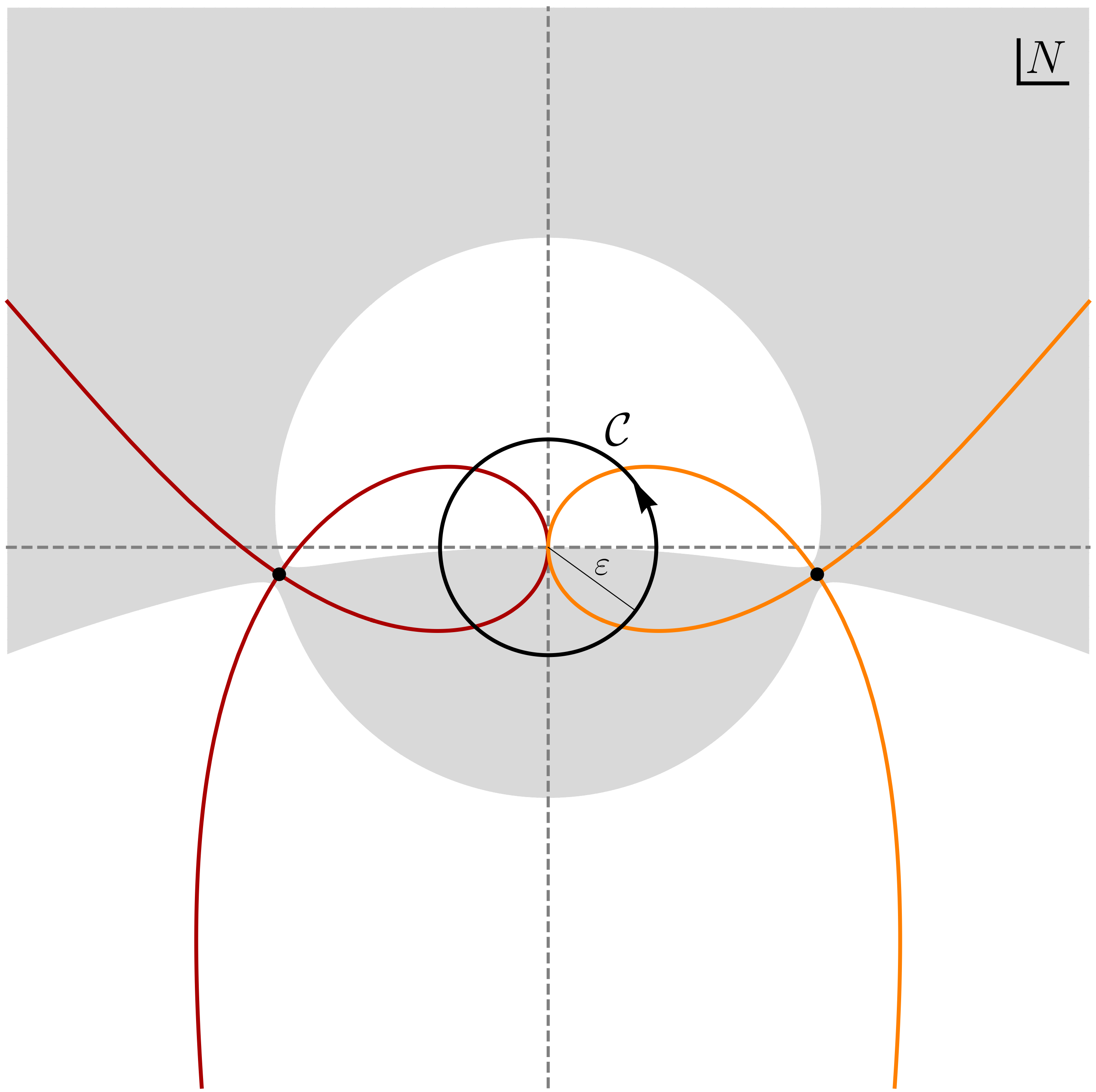

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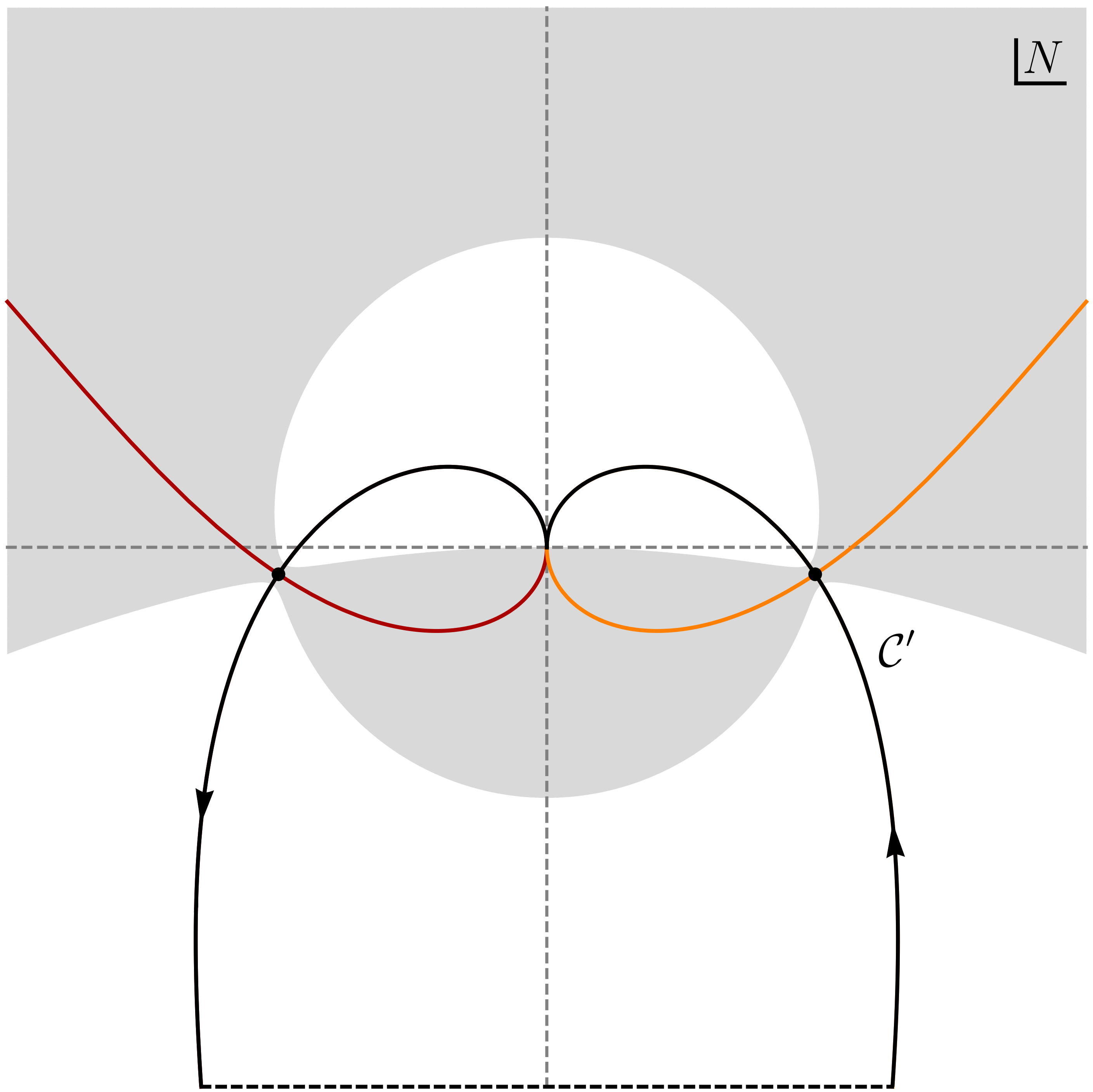

Interpretation of Ψ

3. Classical spacetime on familiar scales should be implied when the universe is large

$$\Psi[h_{ij},\phi] \approx \mathcal{A}\exp\left(\frac{i}{\hbar}S[h_{ij},\phi]\right)$$
$$= \mathcal{A}\exp\left(-\frac{S_{\rm I}}{\hbar}\right)\exp\left(\frac{iS_{\rm R}}{\hbar}\right)$$

S satisfies the Lorentzian HJ eq., and if $|\nabla S_{\rm I}| \ll |\nabla S_{\rm R}|$ so does $S_{\rm R}$

LHJ: $\nabla S_{\rm I} \perp \nabla S_{\rm R}$

To leading order in $\hbar$, we can assign a "probability" $\exp(-2S_{\rm I}/\hbar)$ to the integral curves of $S_{\rm R}$

→ classical *histories*