Semiclassical Eternal Inflation

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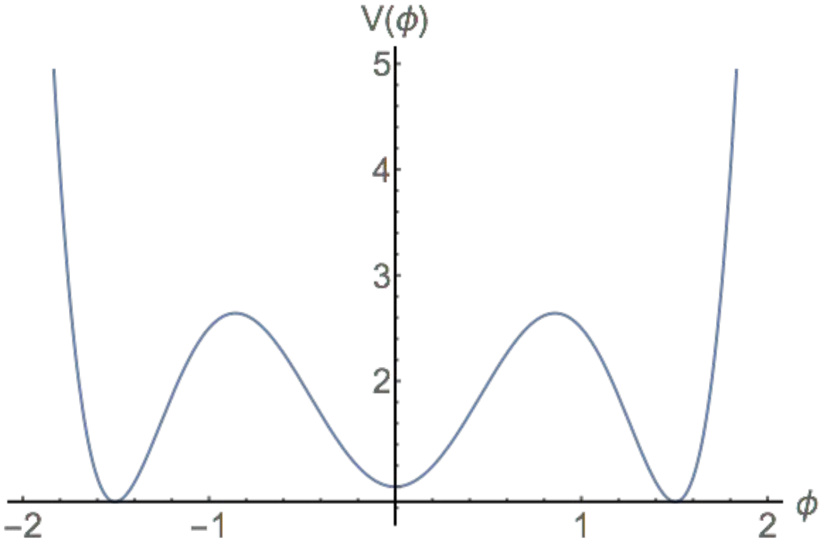
MAX-PLANCK-GESELLSCHAFT

Studienstiftung des deutschen Volkes

Eternal inflation - the usual picture

False vacuum eternal inflation



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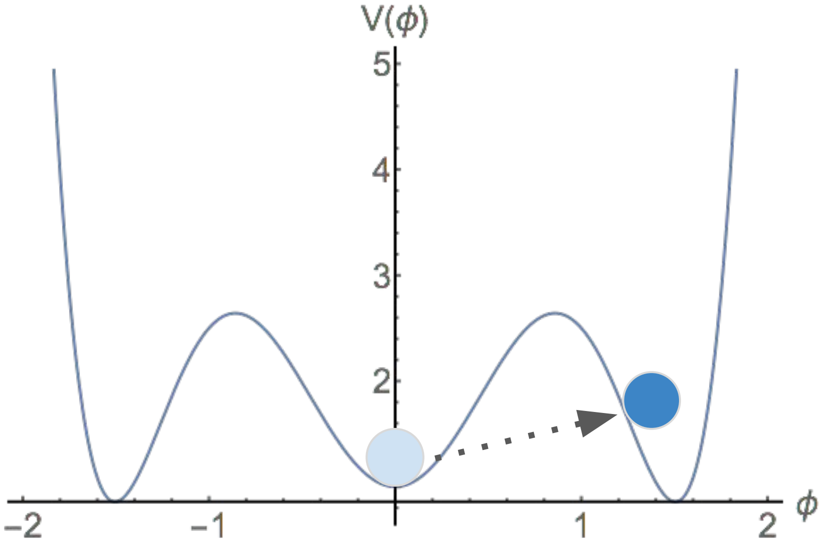

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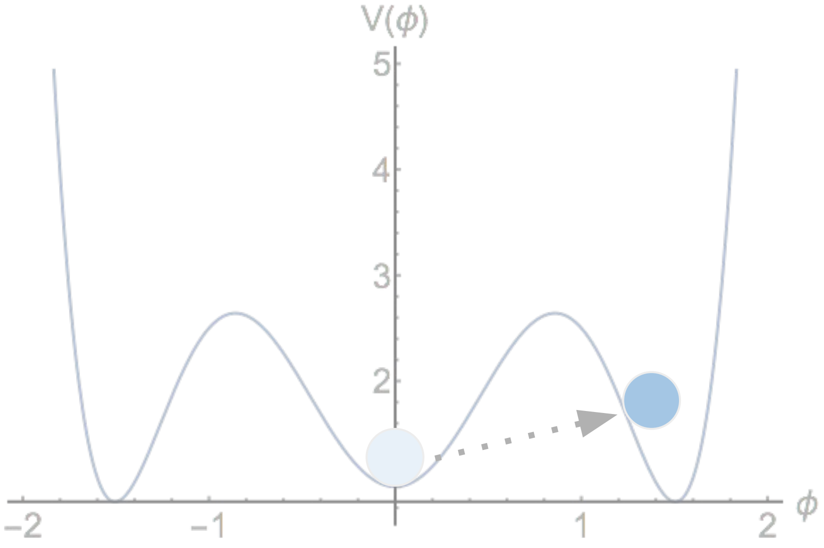
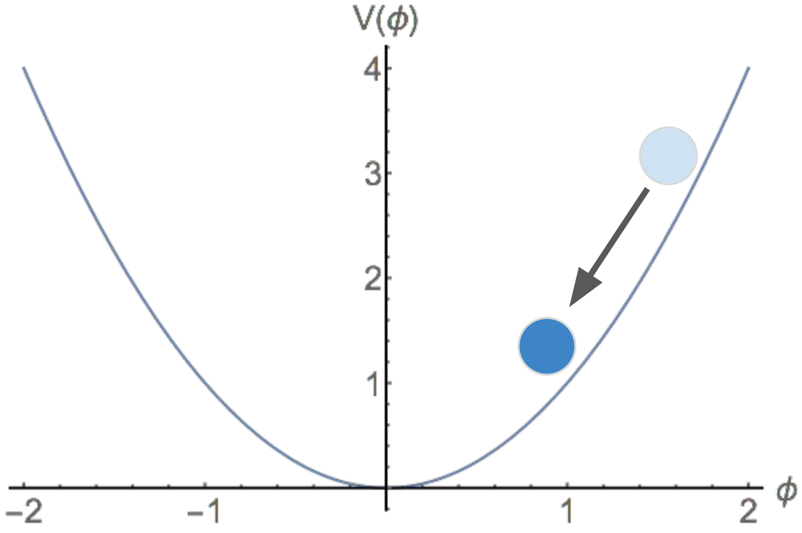


Slow roll eternal inflation

Classical evolution:



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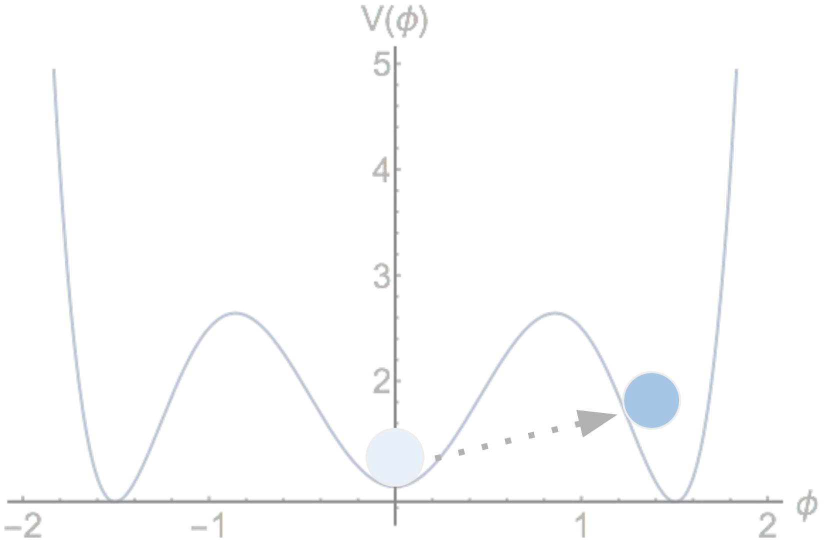
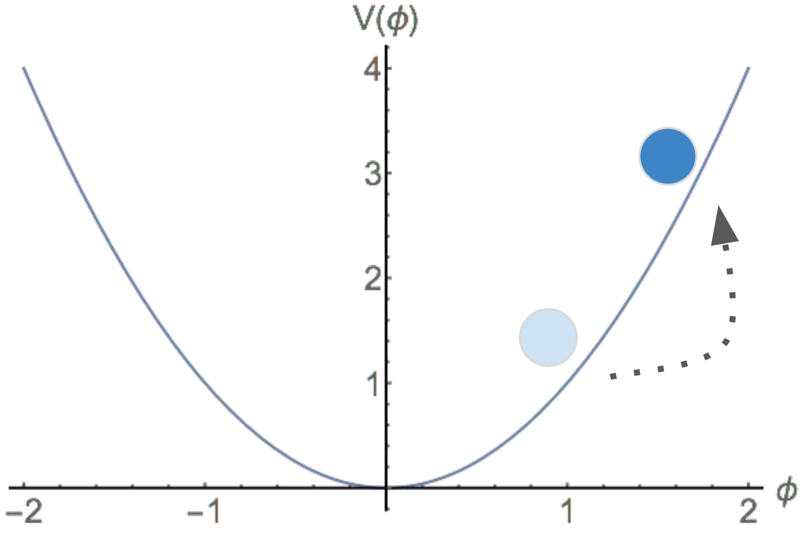


Slow roll eternal inflation

Quantum evolution:



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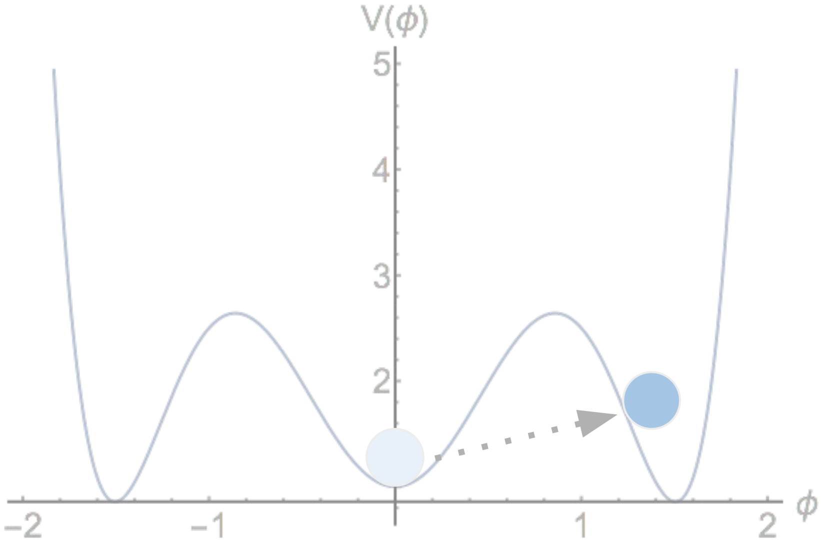
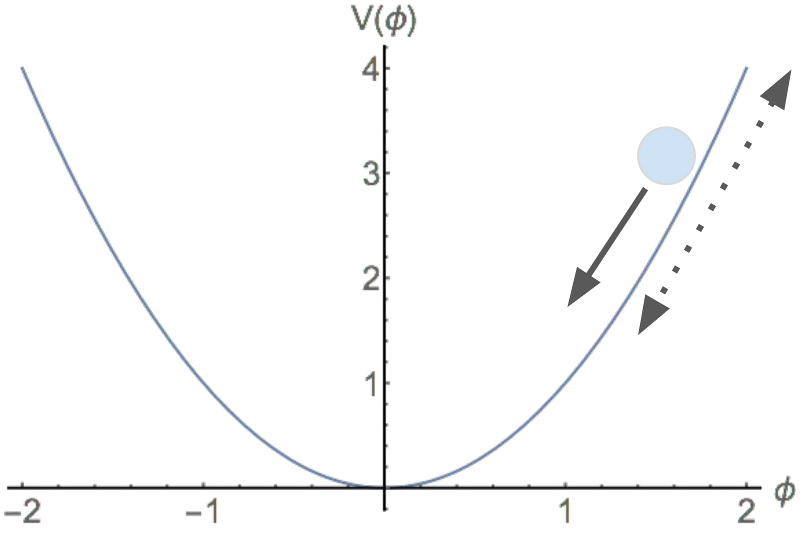
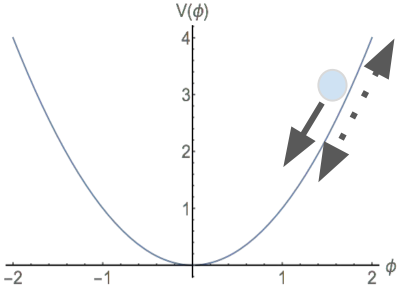


Slow roll eternal inflation

Combined:



Eternal inflation - the usual calculation

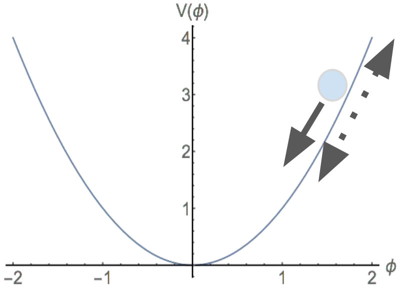


Evolution of the scalar field:

$$\Delta\phi = \Delta\phi_{cl} + \Delta\phi_{qu}$$

$$\Delta\phi_{cl} = \frac{\dot{\phi}}{H}$$

Eternal inflation - the usual calculation



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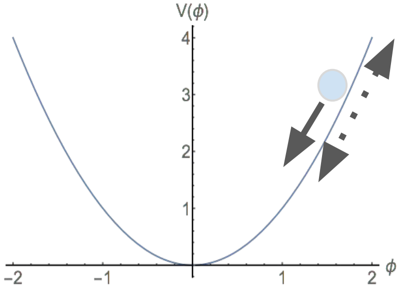

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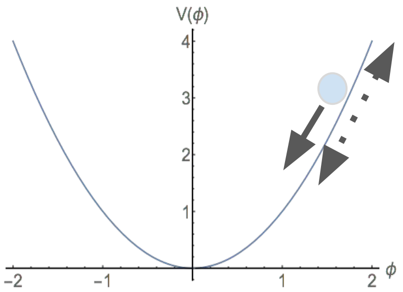

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Need $\Delta\phi_{qu} > \Delta\phi_{cl}$

$\frac{H}{\sqrt{8\pi\epsilon}} > 1$ <-> $\Delta^2_{\mathcal{R}} > 1$

Breakdown of linear pert. theory!

The path integral framework

Feynman propagator for minisuperspace of gravity and a scalar field:

$$G[a_1, \phi_1; a_0, \phi_0] = \int_{0^+}^{\infty} dN \int_{a(0)=a_0}^{a(1)=a_1} \int_{\phi(0)=\phi_0}^{\phi(1)=\phi_1} \mathcal{D}a\mathcal{D}\phi e^{\frac{i}{\hbar} S(N,a,\phi)}$$

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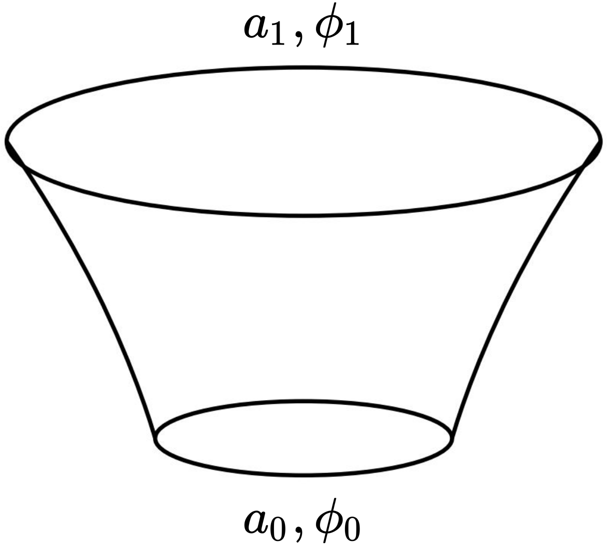

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Hard to solve in general

Solving the path integral

$$S = \frac{6\pi^2}{\kappa^2} \int dt_p N \left(-\frac{a\dot{a}^2}{N^2} + ka + \frac{\kappa^2 a^3}{3} \left(\frac{1}{2} \frac{\dot{\phi}^2}{N^2} - V \right) \right)$$

Matter content ϕ, choose potential

$$V(\phi) = \alpha \cosh \sqrt{\frac{2}{3}} \kappa\phi + \beta \sinh \sqrt{\frac{2}{3}} \kappa\phi$$

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$$V(\phi) = \alpha \cosh \sqrt{\frac{2}{3}} \kappa\phi + \beta \sinh \sqrt{\frac{2}{3}} \kappa\phi$$

Define coordinates

$$x(t) = a^2 \cosh \sqrt{\frac{2}{3}} \kappa\phi$$

$$y(t) = a^2 \sinh \sqrt{\frac{2}{3}} \kappa\phi$$

Solving the path integral

$$S = \int_0^1 dt \frac{N}{\kappa^2}\left[3k + \frac{3}{4N^2}(\dot{y}^2 - \dot{x}^2) - \kappa^2(\alpha x + \beta y)\right]$$

Decoupled EoM -> Solveable

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Derivatives are quadratic

Can now solve the path integrals over x and y exactly to be left with a single integral over the lapse (here I ignore prefactors for simplicity)

$$G[x_1, y_1; x_0, y_0] = \int_{0^+}^{\infty} dN e^{iS(x_0, x_1, y_0, y_1, N)}$$

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$$S = \frac{1}{36} N^3 \kappa^2 \left(\alpha^2 - \beta^2\right) + N \left(\frac{3k}{\kappa^2} - \frac{1}{2}\alpha(x_0 + x_1) - \frac{1}{2}\beta(y_0 + y_1) \right) + \frac{3}{4\kappa^2} \frac{1}{N} \left((y_1 - y_0)^2 - (x_1 - x_0)^2 \right)$$

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four saddle points to the N integral

An example

Cosh potential

Boundary conditions: $a(0) = 11$, $\phi(0) = 1/10$, $a(1) = 33$, $\phi(1) = \frac{1}{2}$

Four real saddle points with two contributing to the integral

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Boundary conditions: a(0) = 11, ϕ(0) =1/10, a(1) = 33, ϕ(1) = ½

Four real saddle points with two contributing to the integral

But recall the action:

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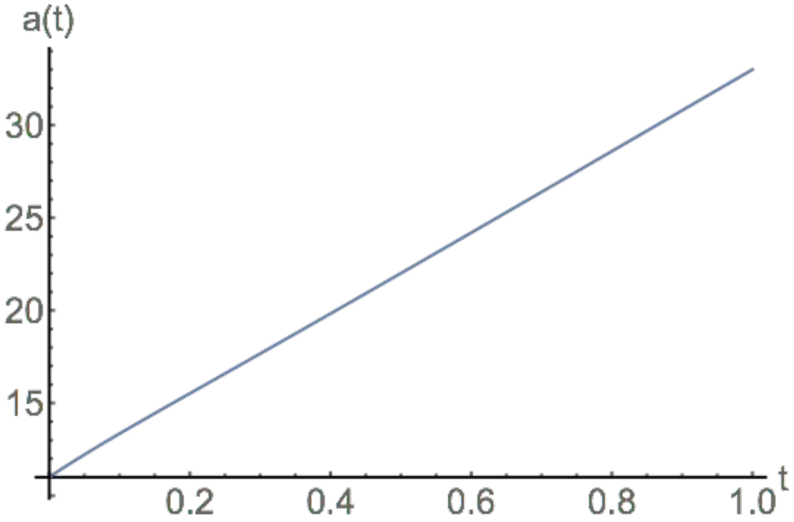
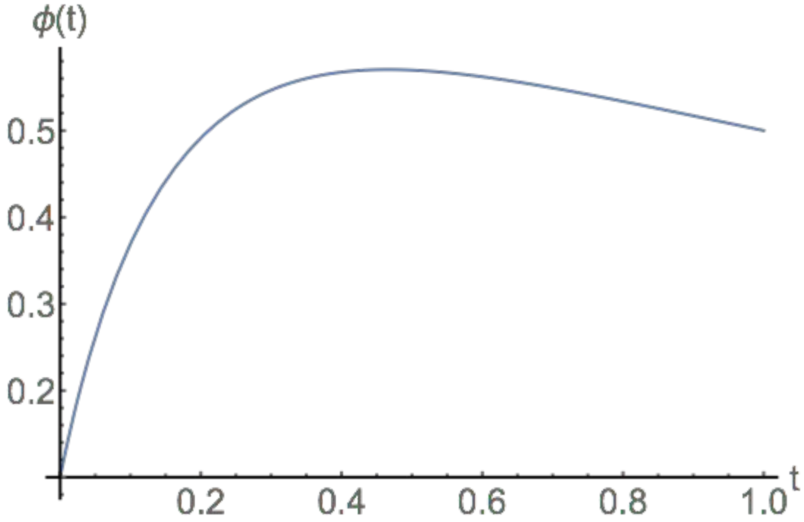
This is also real -> classical evolution

How can this be?

An example

Look at saddle point geometry:





Scalar field already has large kinetic energy in the beginning

How to ask the right question?

The path integral picks out solutions with the wrong momenta!

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Switch from position-position representation to a hybrid by convolving with an initial state

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$$\Downarrow$$

$$G[x_1, y_1; \psi_0] = \int \int G[x_1, y_1; x_0, y_0] \psi_0(x_0, y_0) dx_0 dy_0$$

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$$\psi_0 = e^{\frac{i}{\hbar}(p_x x_0 + p_y y_0) - \frac{(x_0 - x_i)^2}{4\sigma_x^2} - \frac{(y_0 - y_i)^2}{4\sigma_y^2}}$$

$$G[x_1, y_1; \psi_0] = \int\int G[x_1, y_1; x_0, y_0]\psi_0(x_0, y_0) dx_0 dy_0$$

Does this formulation make sense?

Test case: classical evolution during inflation

$$\phi_1 < \phi_0$$

$$a_1 > a_0$$

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Region IV -> two relevant real saddle points

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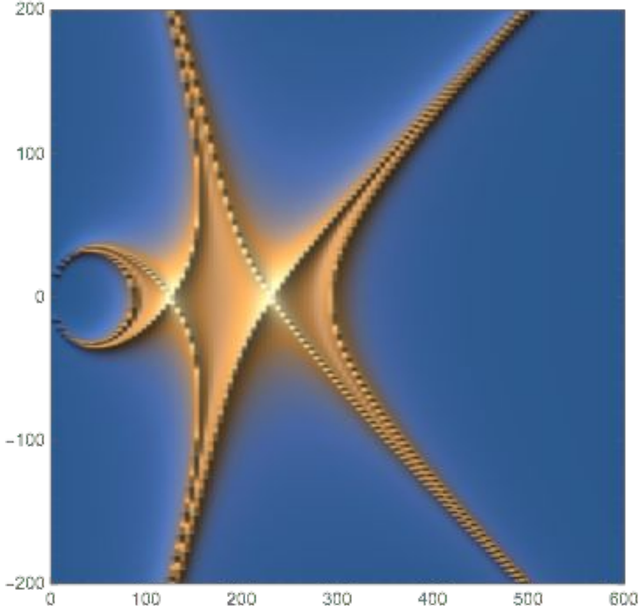
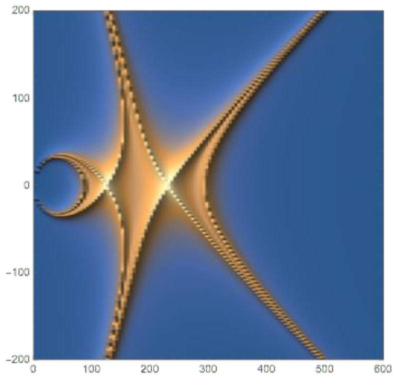
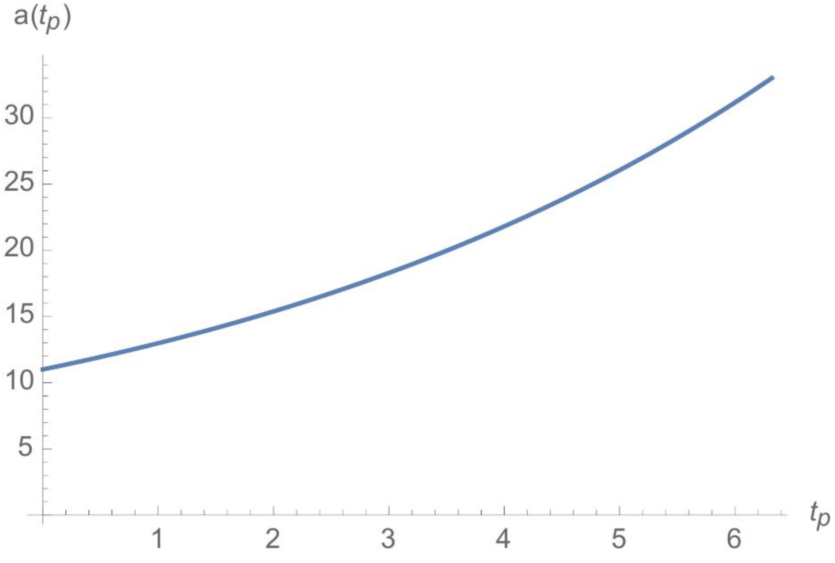


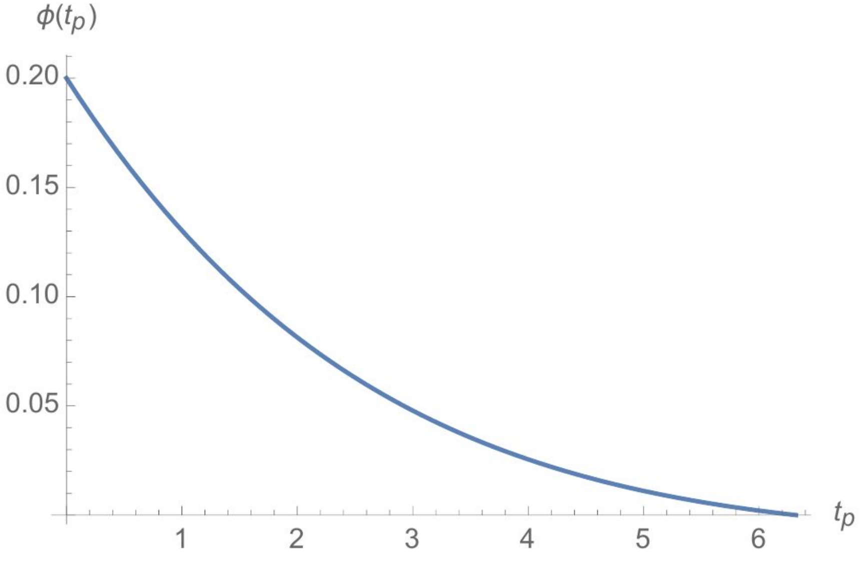
Two saddle points contribute?



Test case: classical evolution during inflation

Saddle point 1:





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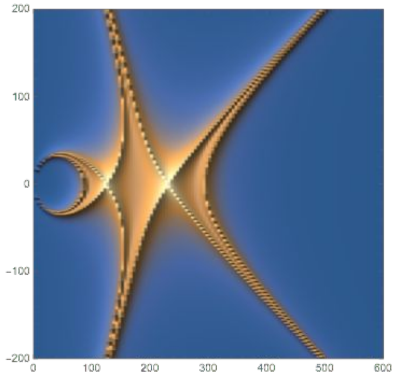

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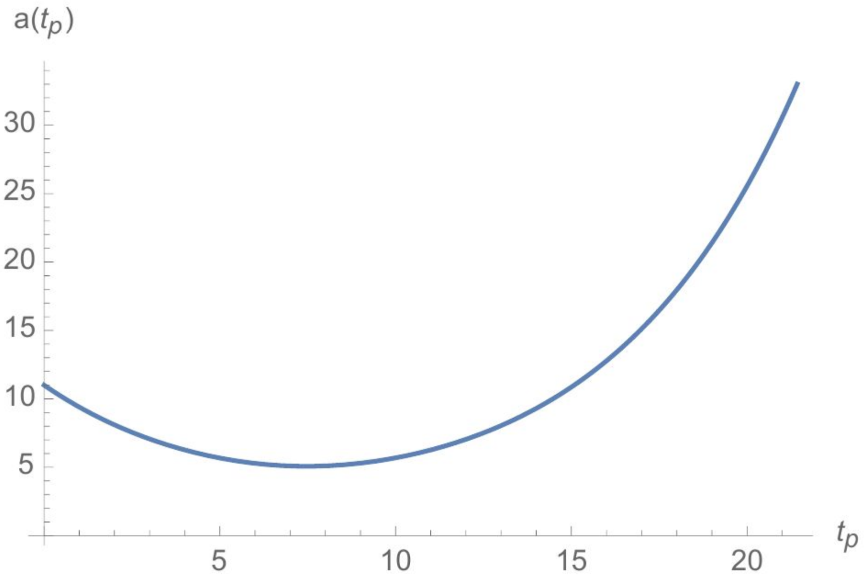
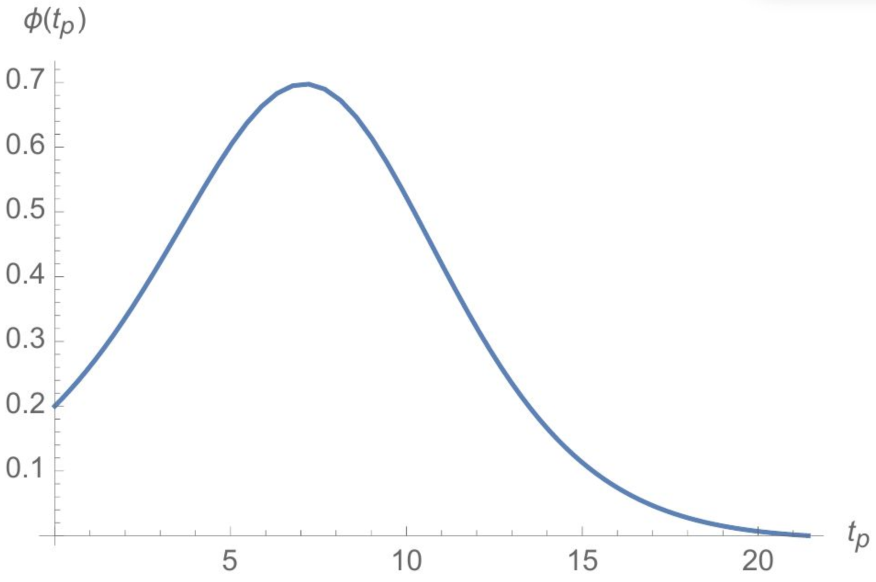
Saddle point 2:





Two saddle points contribute?

Test case: classical evolution during inflation

Now increase σ and choose inflationary momenta

$$G[x_1, y_1; \psi_0] = \int \int G[x_1, y_1; x_0, y_0] \psi_0(x_0, y_0) dx_0 dy_0$$

Two saddle points contribute?

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-> Stokes phenomenon

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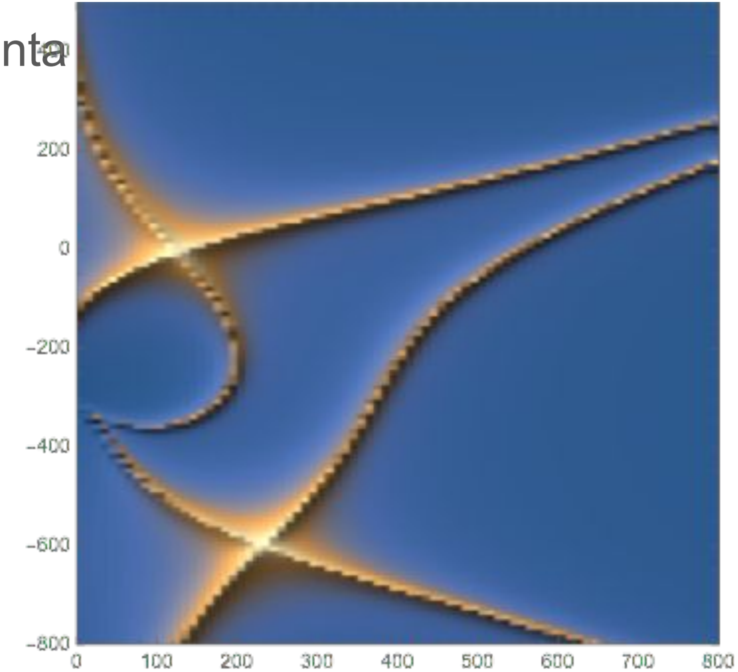

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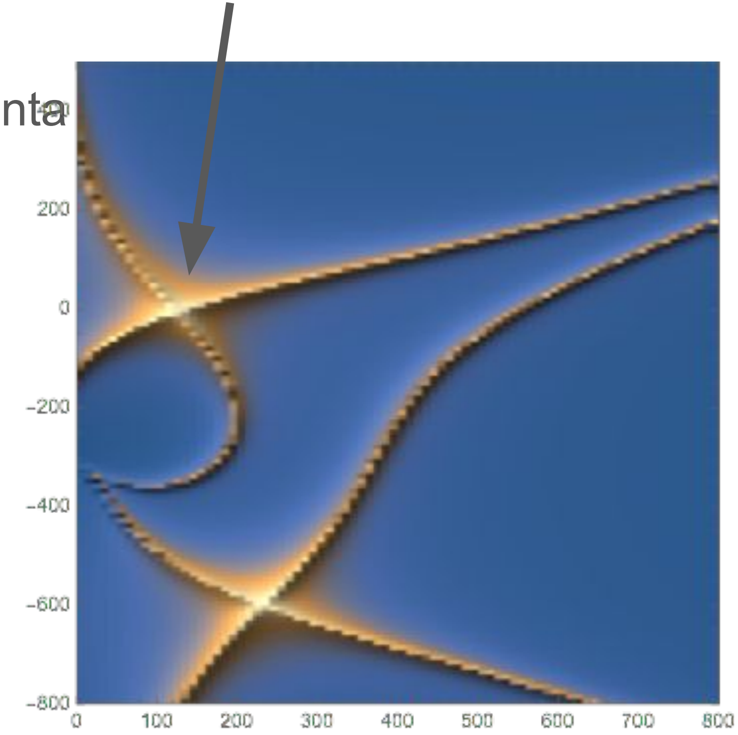
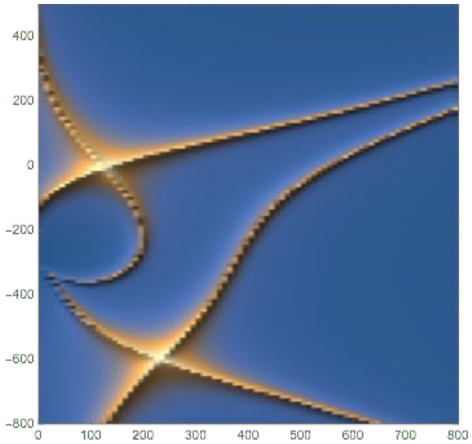
only one saddle contributes

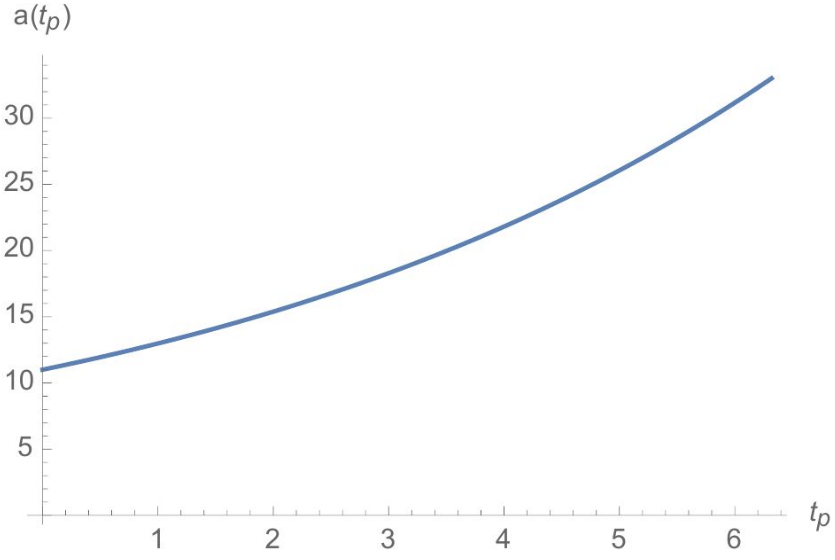


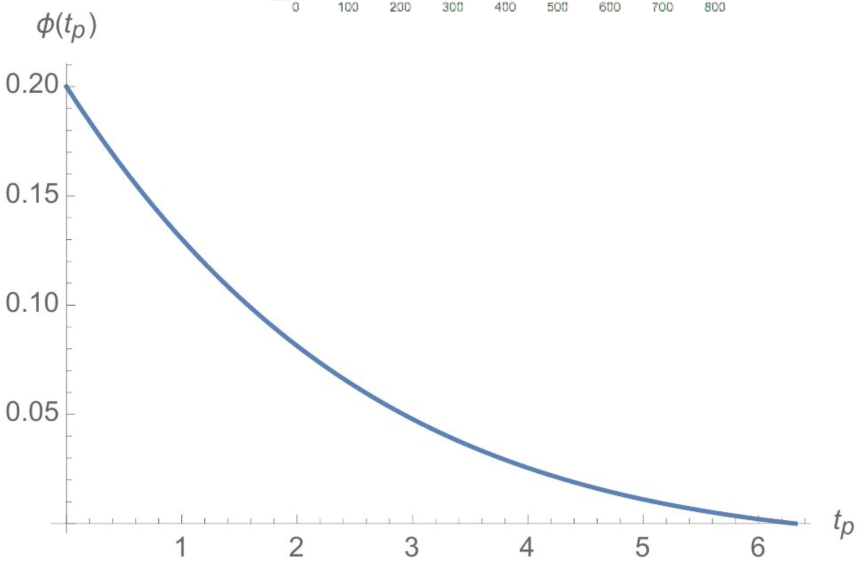
Geometry of remaining saddle?

Test case: classical evolution during inflation

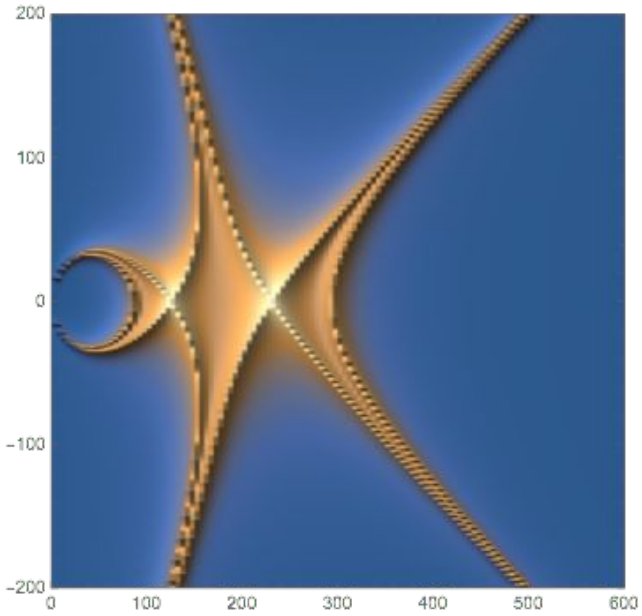
Saddle point geometry







Return to the SREI case



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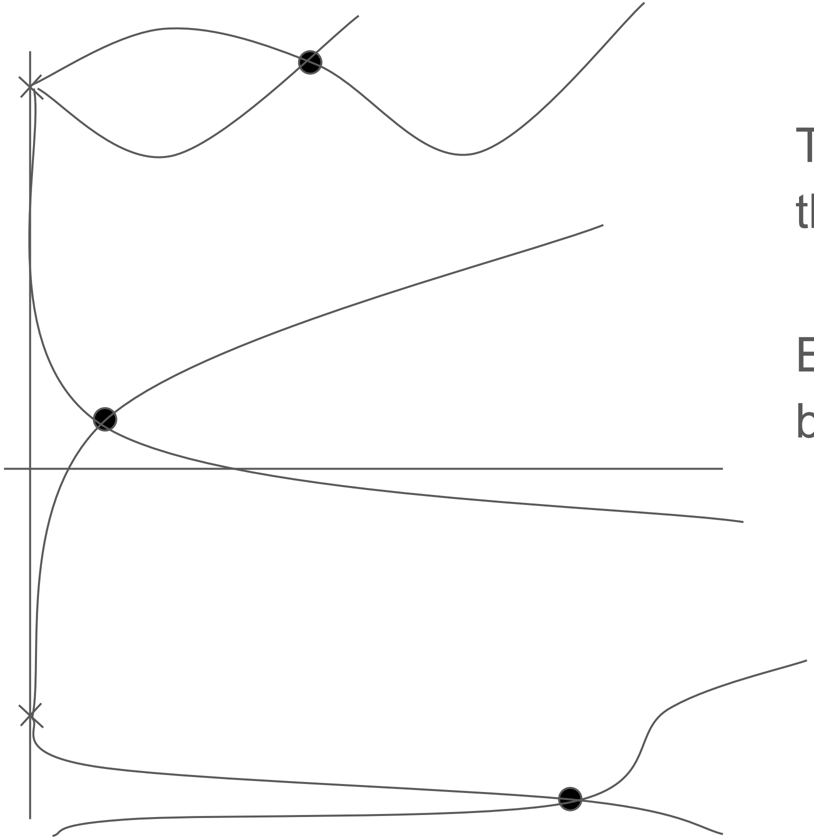

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Emergence of a new saddle point that becomes relevant

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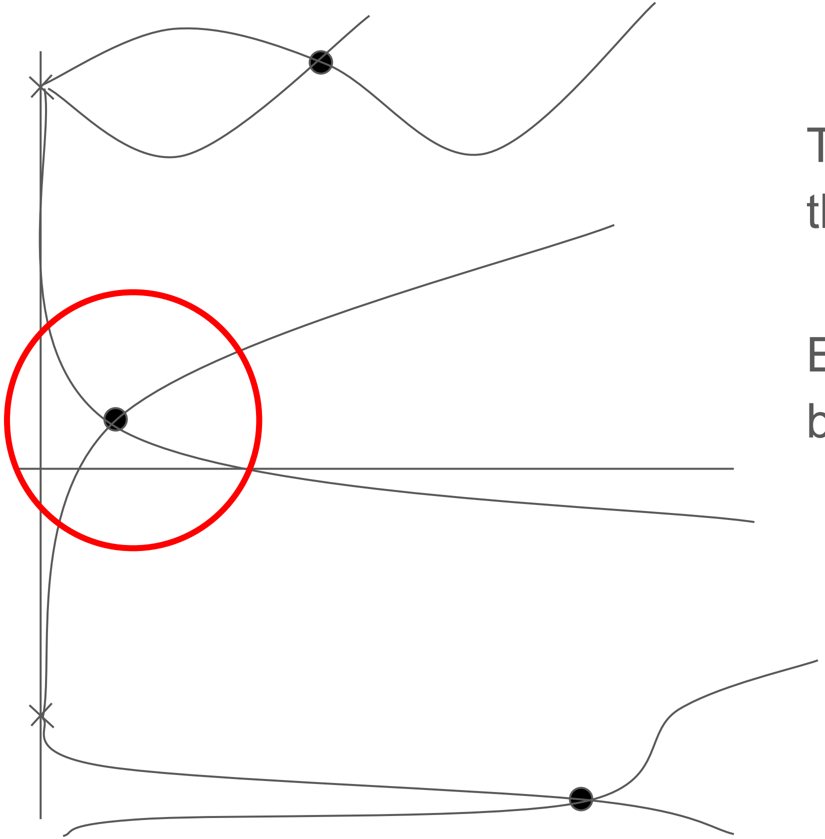

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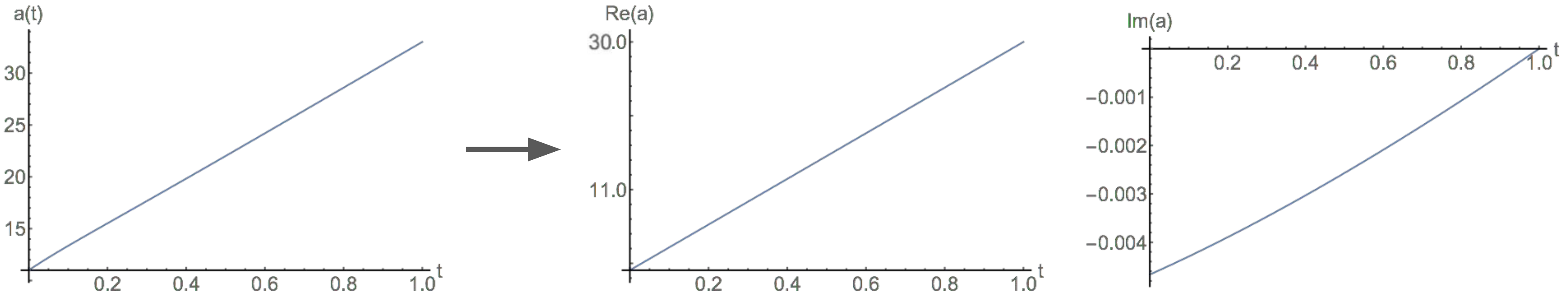

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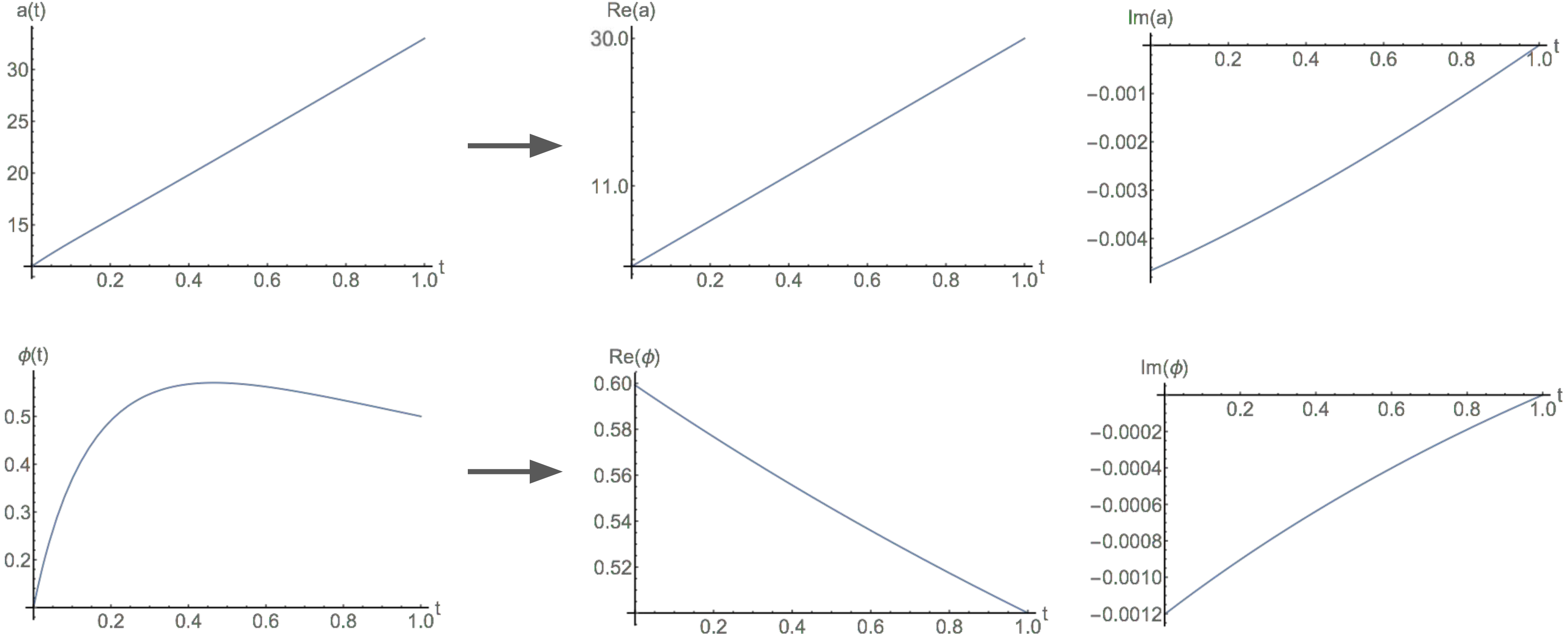

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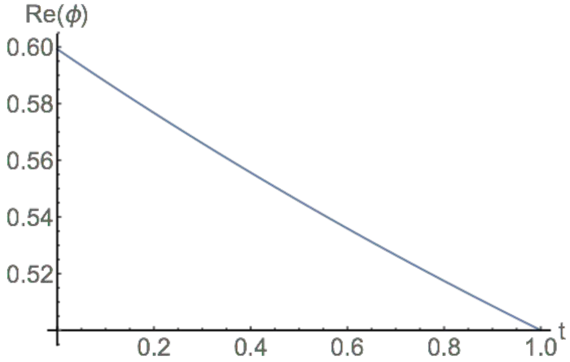

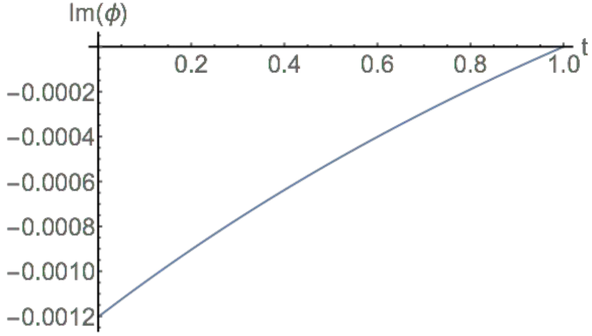

Saddle fulfills criteria for momenta but shifts position to simply roll classically!

Implications

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