The classical double copy for maximally symmetric spacetimes

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Based on:
1711.01296 : MCG, R.Penco, M. Trodden

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The BCJ Double Copy

Bern, Carrasco, Johansson (2008)

$$A_\mu^a : \quad \mathcal{A}_{YM} = \sum_{i\,\epsilon\,\text{trivalent}} \frac{c_i n_i}{d_i}$$

$$h_{\mu\nu},\, \phi,\, B_{\mu\nu} : \quad \mathcal{M}_G = \sum_{i\,\epsilon\,\text{trivalent}} \frac{n_i n_i}{d_i}$$

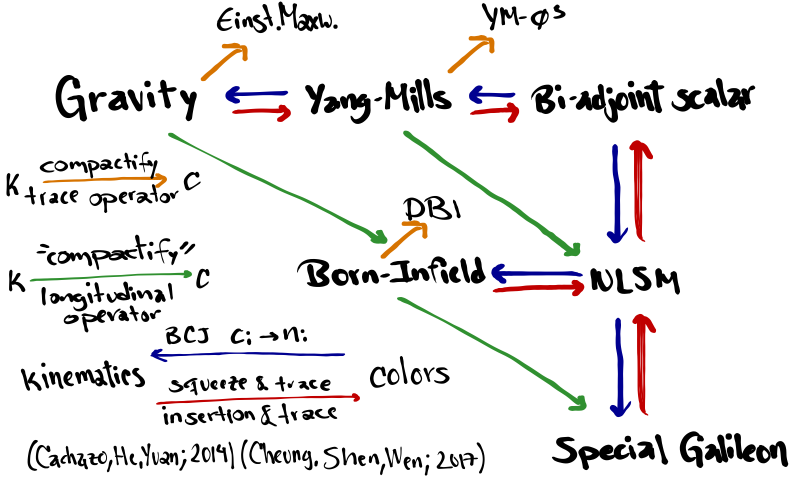
$$\phi^{a\,a'} : \quad \mathcal{A}_{\phi^3} = \sum_{i\,\epsilon\,\text{trivalent}} \frac{c_i c_i}{d_i}$$

Jacobi relations for color-kinematics duality

$$c_i + c_j + c_k = 0$$

$$n_i + n_j + n_k = 0$$

Web of Amplitudes Relations



Classical realizations of the double copy

- Perturbative case: Double copy of radiation
  - For color charges (Goldberger, Ridgway; 2016), (Goldberger, Prabhu, Thompson; 2017)
  - For bound states (Goldberger, Ridgway; 2017)
  - Including spin (Goldberger, Li, Prabhu; 2017)
  - Removing dilaton (Luna, Nicholson, O'Connell, White; 2017)
- Exact Solutions: Kerr-Schild spacetimes
  - Black holes (Monteiro, O'Connell, White; 2014)
  - Taub - NUT spacetime (Luna, Monteiro, O'Connell, White; 2015)
  - Stress tensors, energy conditions (Ridgway, Wise; 2015)
  - Accelerating black holes (Luna, Monteiro, Nicholson, O'Connell, White; 2016)
  - In curved space (Bahjat-Abbas ,Luna, White; 2017)(MCG, Penco, Trodden; 2017)
- Few special cases
  - Schockwaves (Saotome, Akhoury; 2012)
  - Linear memory effect (Chu; 2016)
  - ...

Exact solutions in the double copy (Monteiro, O'Connel, White)

Kerr-Schild metrics: $g_{\mu\nu} = \bar{g}_{\mu\nu} + k_\mu k_\nu \phi$

k_μ is a null geodetic vector

Linearizes Einstein equations

Includes black holes and waves

Color - kinematics replacements:

$$k^\mu \to c^a \quad M \to Q \quad 8\pi G \to g$$

$$h_{\mu\nu} = k_\mu k_\nu \phi \qquad A^a_\mu = c^a k_\mu \phi \qquad \phi^{a\,a'} = c^a\, c^{a'} \phi$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \nabla_\mu F^{\mu\nu} = g J^\nu \qquad \Box \phi = J$$

Invariance of Kerr-Schild metric that changes the copies

$$k_\mu \to f\, k_\mu \qquad \phi \to \phi / f^2$$

Simplest example

Schwarzschild in 4d flat space: $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$, $k_\mu = (1,1,0,0)$, $\phi = \frac{2M}{r}$

Gravity: $h_{\mu\nu} = k_\mu k_\nu \phi$ $\qquad T^\nu_\mu = \frac{M}{2}\text{diag}(0,0,1,1)\delta^3(r)$

Yang-Mills: $A_\mu = k_\mu \phi$ $\qquad J^\mu = Q\delta^\mu_0 \delta^3(r)$

Bi-adjoint scalar: ϕ $\qquad J = M\delta^3(r)$

Double Copy in (A)dS

4d Schwarzschild in (A)dS: $\bar{g}_{\mu\nu} = (A)dS, \;\; k_\mu = \left(1, \frac{1}{1-\frac{\Lambda}{3}r^2}, 0, 0\right), \;\; \phi = \frac{2M}{r}$

Gravity: $G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$ $\qquad T^{\nu}_{\mu} = \dfrac{M}{2}\mathrm{diag}(0,0,1,1)\delta^3(r)$

Yang-Mills: $\nabla_\mu F^{\mu\nu} = gJ^\nu$ $\qquad J^\mu = Q\delta^\mu_0\delta^3(r)$

Bi-adjoint scalar: $\left(\Box - \dfrac{R}{6}\right)\phi = J$ $\qquad J = M\delta^3(r)$

New eom for scalar, inclusion of mass term

In 4d it is conformally invariant, but it is NOT in other dimensions.

YM and ϕ^3 eom from Einstein equations

Einstein equations

$$-16\pi G\left(T^\mu_\nu - \delta^\mu_\nu \frac{T}{d-2}\right) = 2(\bar{R}^\mu_\nu - R^\mu_\nu) = \left[\bar{\nabla}_\lambda F^{\lambda\mu} + \frac{(d-2)}{d(d-1)}\bar{R}A^\mu\right] k_\nu + X^\mu{}_\nu + Y^\mu{}_\nu$$

$$X \propto \bar{\nabla}_\lambda k^\lambda, k^\lambda \bar{\nabla}_\lambda \phi$$

YM eom: Apply Killing vector V^ν

$$\bar{\nabla}_\lambda F^{\lambda\mu} + \frac{(d-2)}{d(d-1)}\bar{R}A^\mu + \frac{V^\nu}{V^\lambda k_\lambda}(X^\mu{}_\nu + Y^\mu{}_\nu) = 8\pi G\, J^\mu$$

$$J^\mu \equiv -\frac{2V^\nu}{V^\rho k_\rho}\left(T^\mu{}_\nu - \delta^\mu_\nu \frac{T}{d-2}\right)$$

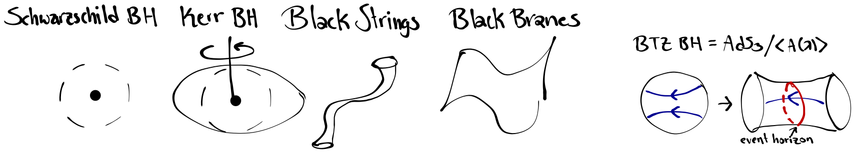
ϕ^3 eom: Apply Killing vector again $V^\nu V^\mu$

$$\bar{\nabla}^2\phi = j - \frac{(d-2)}{d(d-1)}\bar{R}\phi - \frac{V_\nu}{(V^\mu k_\mu)^2}\left(V^\mu X^\nu{}_\mu + V^\mu Y^\nu{}_\mu + Z^\nu\right)$$

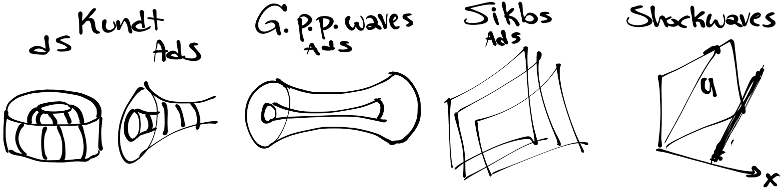
$$j = (V_\nu J^\nu)/(V^\rho k_\rho)$$

Examples in (A)dS

Stationary: $\nabla_\mu F^{\mu\nu} = gJ^\nu$ $\qquad \Box\phi - \frac{2(d-3)}{d(d-1)}\bar{R}\phi = J$



Waves: $\nabla_\mu F^{\mu\nu} + \frac{(d-2)}{d(d-1)}\bar{R}A^\mu = gJ^\nu$ $\qquad \Box\phi - \frac{(d-4)}{d(d-1)}\bar{R}\phi = J$



Open questions:

- Relation to amplitudes double copy
- Choice of k_μ and ϕ
- Dependence with dimensionality
- Broken gauge symmetry for waves
- Missing dilaton and 2-form

Future directions

- Other classical double copy examples:
  - NLSM^2=Special Galileon ? (MCG, Penco, Trodden; in progress)