Aspects of Dark Matter Axion Clumps

Mark Hertzberg

Tufts University

PASCOS Case Western, June 7 2018

QCD-Axion

$$\Delta\mathcal{L}_{qcd} \sim \theta\, G_{\mu\nu}\, \tilde{G}^{\mu\nu} \qquad\qquad |\theta| \lesssim 10^{-10}$$

QCD-Axion

$$\Delta\mathcal{L}_{qcd} \sim \theta\, G_{\mu\nu}\,\tilde{G}^{\mu\nu} \qquad\qquad |\theta| \lesssim 10^{-10}$$

$$\big\downarrow$$

(Peccei-Quinn, Weinberg, Wilczek)

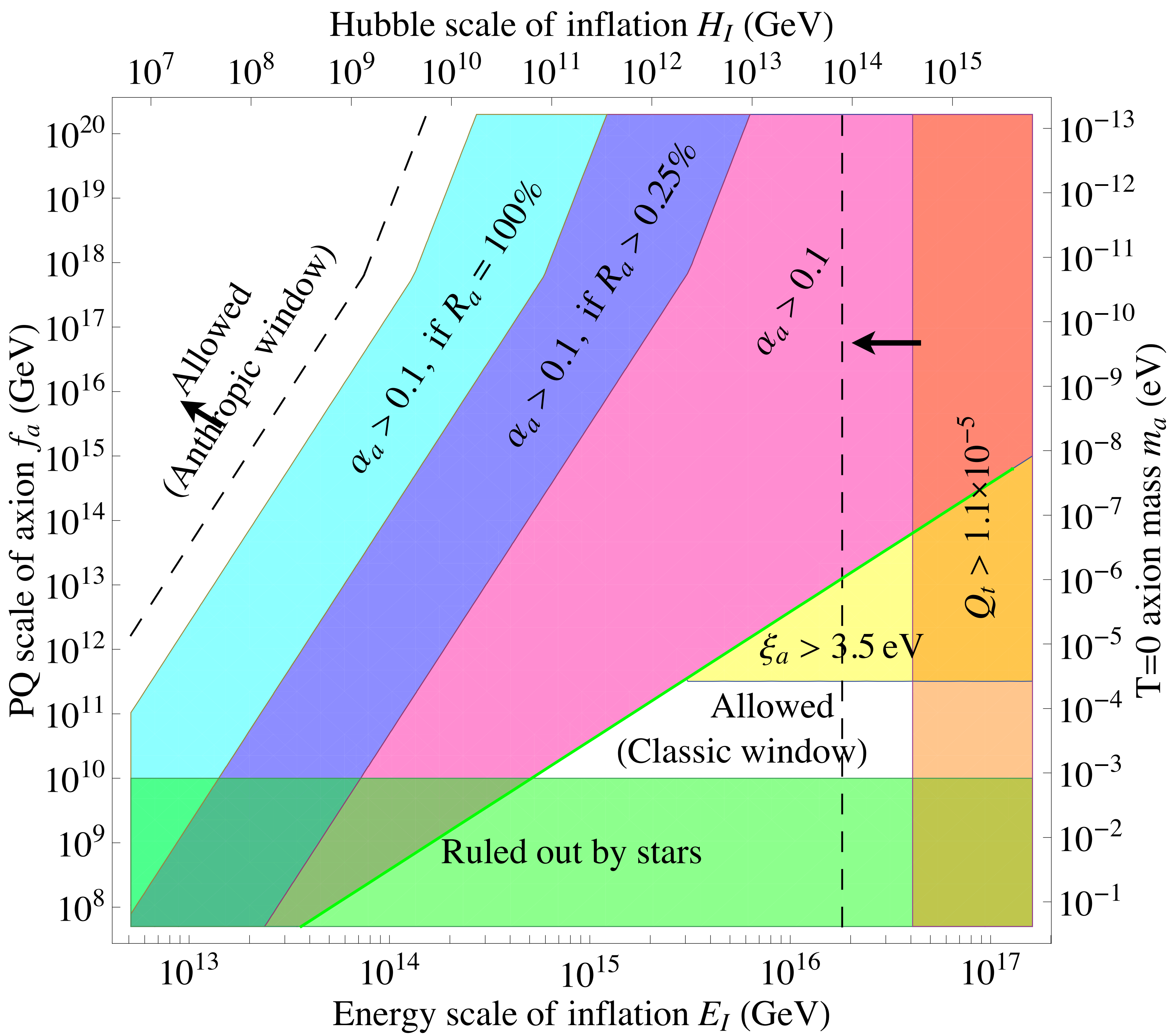
$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\, G_{\mu\nu}\,\tilde{G}^{\mu\nu} + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Periodic Potential; Expanded: $$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$

Axion mass: $$m_a \sim \frac{\Lambda_{qcd}^2}{f_a}$$

(Attractive) Self-Coupling: $$\lambda \sim -\frac{\Lambda_{qcd}^4}{f_a^4}$$

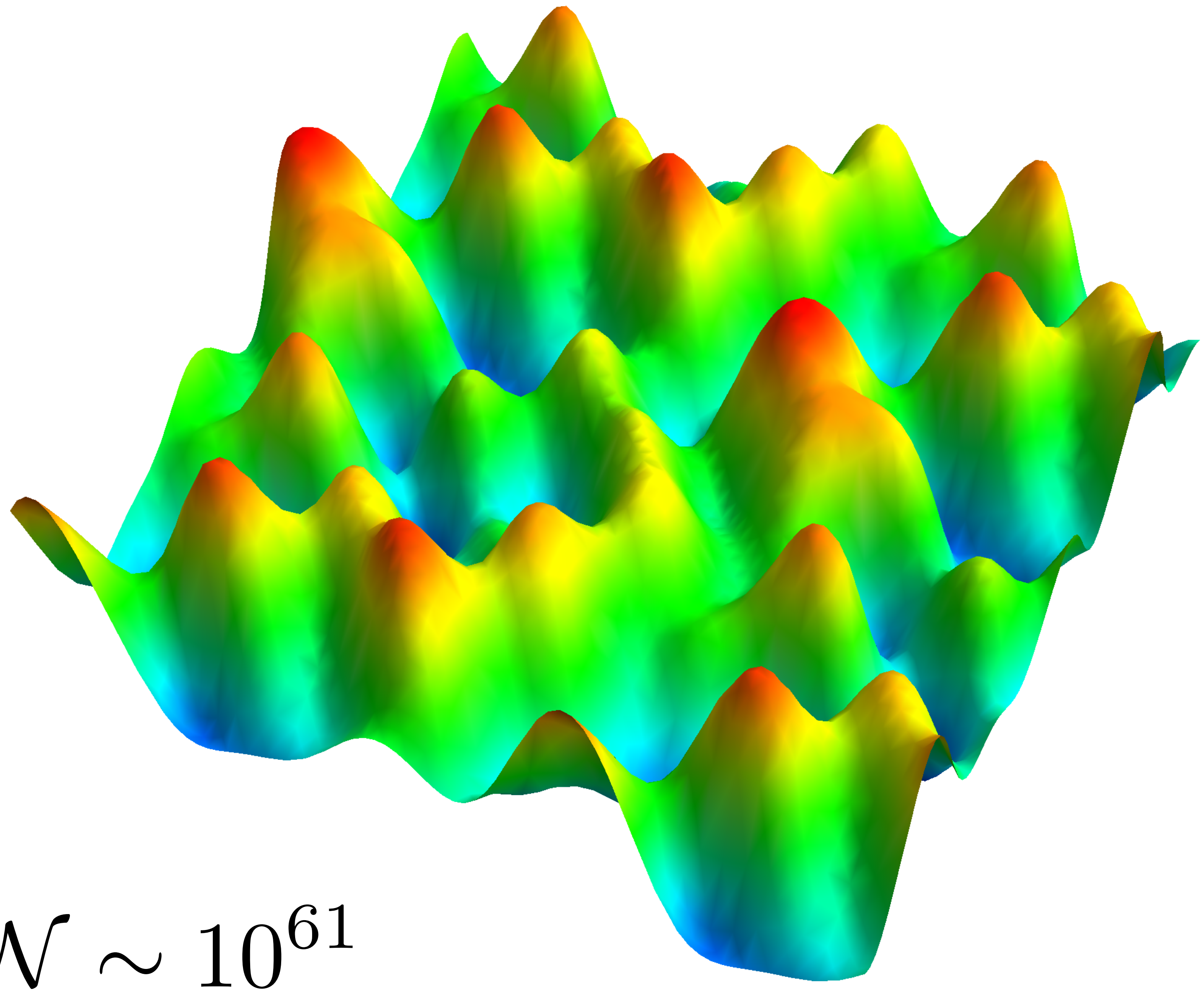
QCD-Axion Allowed Windows



Hertzberg, Tegmark, Wilczek 0807.1726

Focus on Classic Window

In Classic Window; Axion Initial Distribution



$\mathcal{N} \sim 10^{61}$

Consider Non-Relativistic Behavior $\phi \to \psi$

Hamiltonian

$$\hat{H} = \hat{H}_{\mathrm{kin}} + \hat{H}_{\mathrm{int}} + \hat{H}_{\mathrm{grav}}$$

$$\hat{H}_{\mathrm{kin}} = \int d^3x \frac{1}{2m} \nabla \hat{\psi}^\dagger \cdot \nabla \hat{\psi}$$

$$\hat{H}_{\mathrm{int}} = \int d^3x \frac{\lambda}{16m^2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi}$$

$$\hat{H}_{\mathrm{grav}} = -\frac{Gm^2}{2} \int d^3x \int d^3x' \frac{\hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}^\dagger(\mathbf{x}') \hat{\psi}(\mathbf{x}) \hat{\psi}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

Number Density

$$\hat{n}(\mathbf{x}) = \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}(\mathbf{x})$$

Gravitational Thermalization?

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

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Interaction Rate of Modes $\Gamma_k \sim \dfrac{8\pi G\, m^2\, n_{\rm ave}}{k^2}$

Collective rate from coherent Bosons

Much greater than dilute bosons $\Gamma \sim \dfrac{G^2\, m^2\, n_{ave}}{v^3}$

- Sikivie, Yang (2009), Erken, Sikivie, Tam, Yang (2011)

Gravitational Thermalization?

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Collective rate from coherent Bosons

Thermalization $\Gamma_k > H$ at late times (BEC)

- Sikivie, Yang (2009), Erken, Sikivie, Tam, Yang (2011)

Axion BEC Literature

- Sikivie, Yang (2009)
- Erken, Sikivie, Tam, Yang (2011)
- Chavanis (2012)
- Banik, Sikivie (2013)
- Davidson, Elmer (2013)
- Nouri, Saikawa, Sato, Yamaguchi (2014)
- Vega, Sanchez (2014)
- Li, Rindler-Daller, Shapiro (2014)
- Berges, Haeckel (2014)
- Banik, Christopherson, Sikivie, Todarello (2015)
- Davidson (2015)
- .....

Classical Description of BEC Phase Transition

Free Theory

$$F[\psi] = \int \frac{d^3k}{(2\pi)^3} \left[\frac{k^2}{2m} - \mu(T)\right] |\psi_k|^2$$

Number

$$\langle N \rangle = \frac{\int \mathcal{D}\psi \, N[\psi] \, \exp\left(-F[\psi]/T\right)}{\int \mathcal{D}\psi \, \exp\left(-F[\psi]/T\right)}$$

Density

$$n_{\rm th} = \int \frac{d^3k}{(2\pi)^3} \frac{T}{\frac{k^2}{2m} - \mu(T)}$$

Critical Temperature

$$T_{\rm crit} = \frac{\pi^2 \, n_{\rm tot}}{m \, k_{\rm UV}}$$

Guth, Hertzberg, Prescod-Weinstein 1412.5930

Classical Description of BEC Phase Transition

While BEC is a very quantum phenomenon
from the PARTICLE point of view

BEC is a very classical phenomenon
from the FIELD point of view

Guth, Hertzberg, Prescod-Weinstein 1412.5930

Classical vs Quantum with Interactions

What About Interactions?

Fundamental claim of Sikivie, Todarello, 1607.00949

On time scales $t > \tau = 1/\Gamma$ the classical description breaks down, requiring the full quantum theory, which is the only way to see thermalization

Toy Model

Second Quantized Language

$$\hat{H} = \sum_i \omega_i \, \hat{a}_i^\dagger \hat{a}_i + \frac{1}{4} \sum_{ijkl} \Lambda_{ij}^{kl} \, \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l \, ,$$

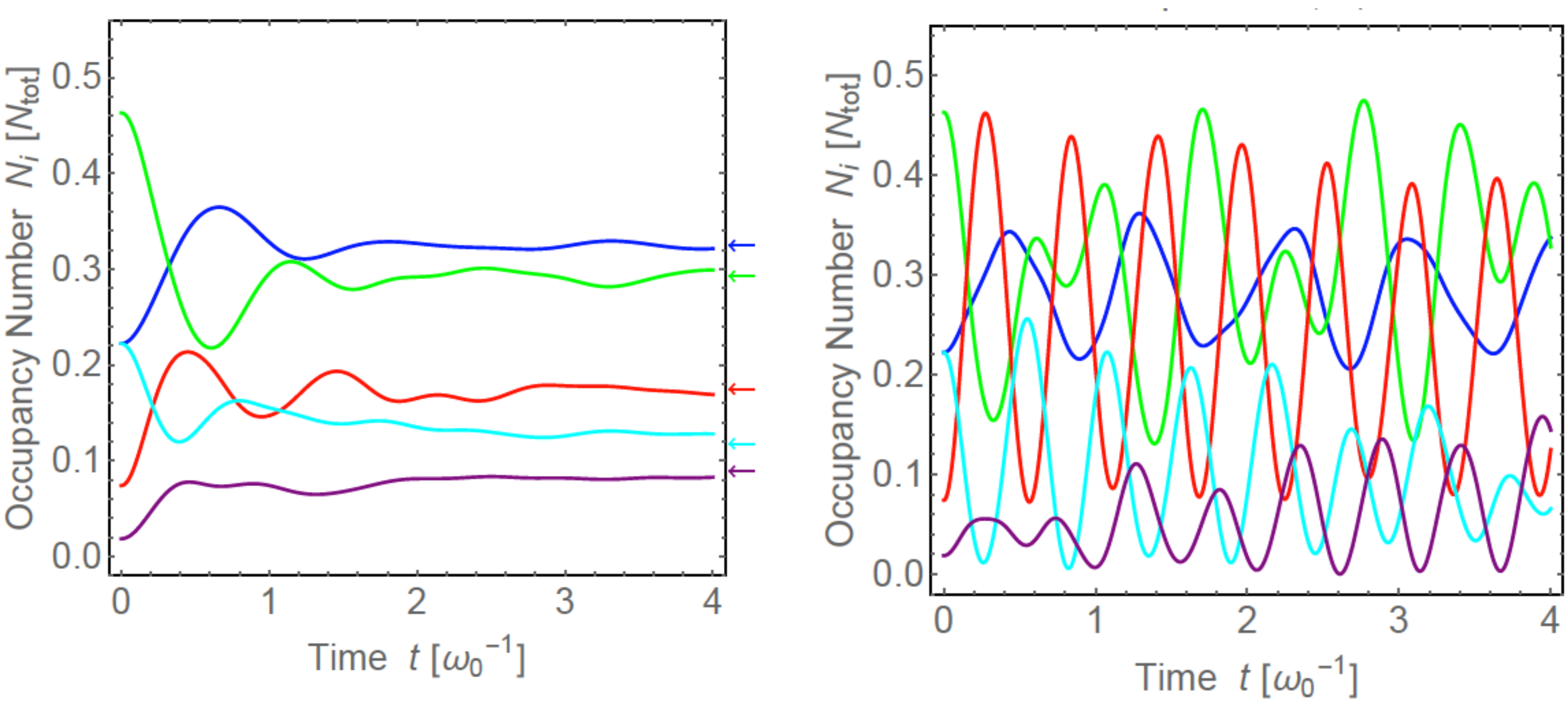
Consider just 5 oscillators for simplicity

Initial quantum state $\quad |\{N_i\}\rangle = |12, 25, 4, 12, 1\rangle$

Initial classical state $\quad a_i = \sqrt{N_i}$

Sikivie, Todarello, 1607.00949

Quantum vs Classical??



Sikivie, Todarello, 1607.00949

Correct Classical Treatment

Initial classical state $a_i = \sqrt{N_i}\, e^{I\theta_i}, \quad \theta_i \in [0, 2\pi)$

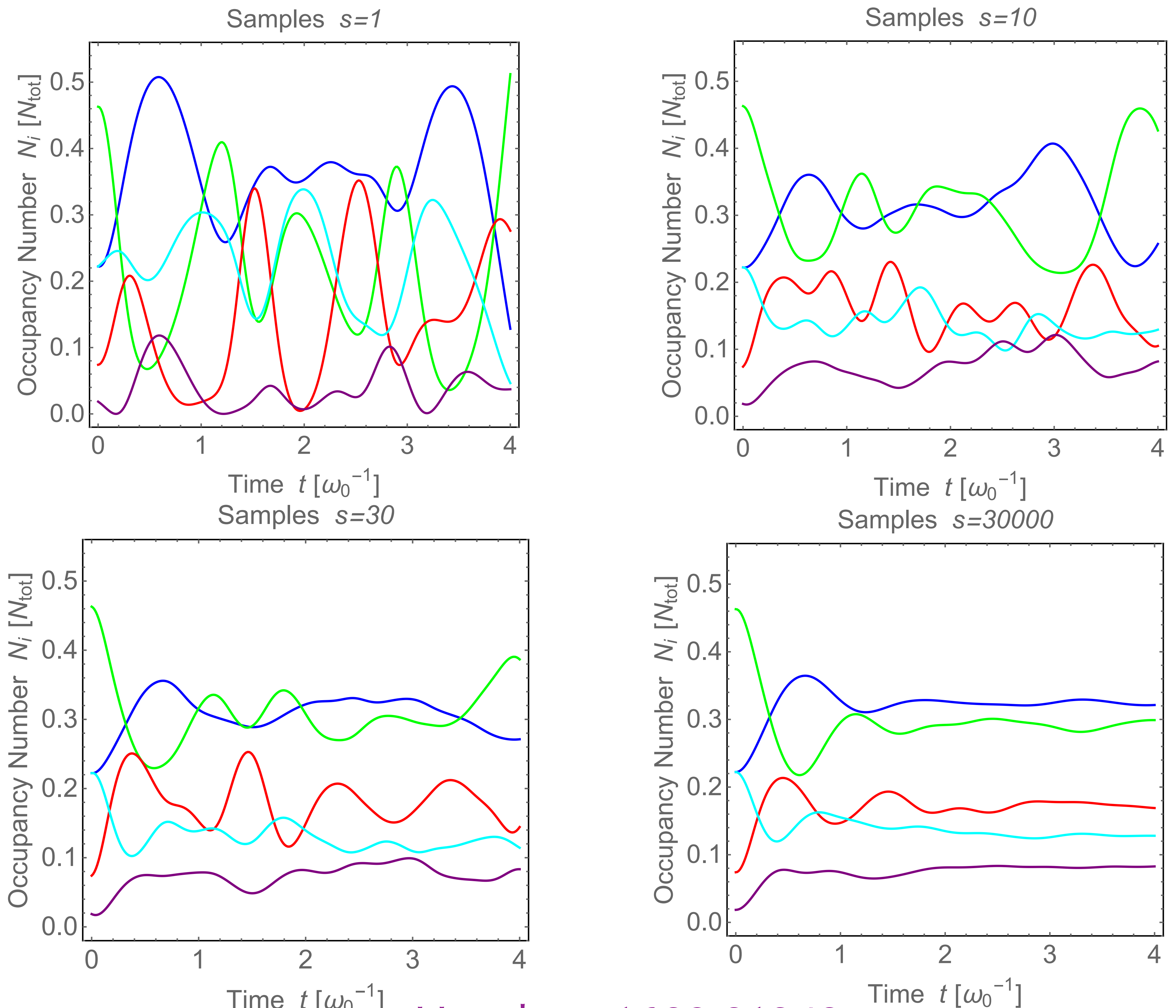
Ensemble average over random initial phases

Meaningful comparison

Connects to uncertainty in branch of wavefunction

Hertzberg 1609.01342

Correct Classical Treatment



Hertzberg 1609.01342

Implication for Correlation Functions

Implication for Correlation Functions

At high occupancy

$$\langle \{N_i\} | \hat{\psi}^\dagger(\mathbf{x}, t)\, \hat{\psi}(\mathbf{y}, t) | \{N_i\} \rangle \approx \langle \psi^*(\mathbf{x}, t)\, \psi(\mathbf{y}, t) \rangle_{ens}$$

Ergodic theorem

$$\langle \psi^*(\mathbf{x}, t)\, \psi(\mathbf{y}, t) \rangle_{ens} = \frac{1}{V} \int_V d^3z\, \psi^*_\mu(\mathbf{x} + \mathbf{z}, t)\, \psi_\mu(\mathbf{y} + \mathbf{z}, t)$$

Hertzberg 1609.01342

Implication for Correlation Functions

Correlation functions of quantum and classical micro-states agree at high occupancy, despite the spread of the quantum wave-function in these chaotic systems

(Note: this is not some trivial consequence of Ehrenfest theorem; more akin to billiard balls which exhibit chaos)

Hertzberg 1609.01342

Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

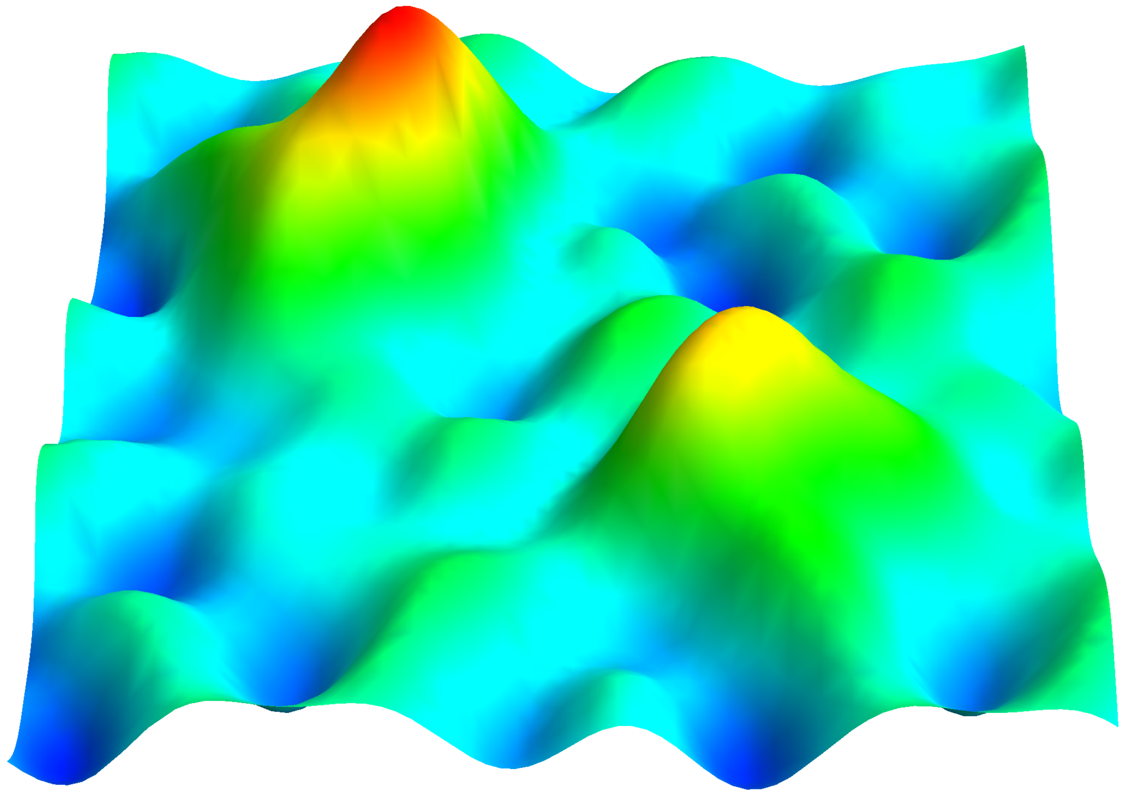
Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

Small clumps

that may populate the galaxy



Guth, Hertzberg, Prescod-Weinstein 1412.5930

Axion Clumps in Detail

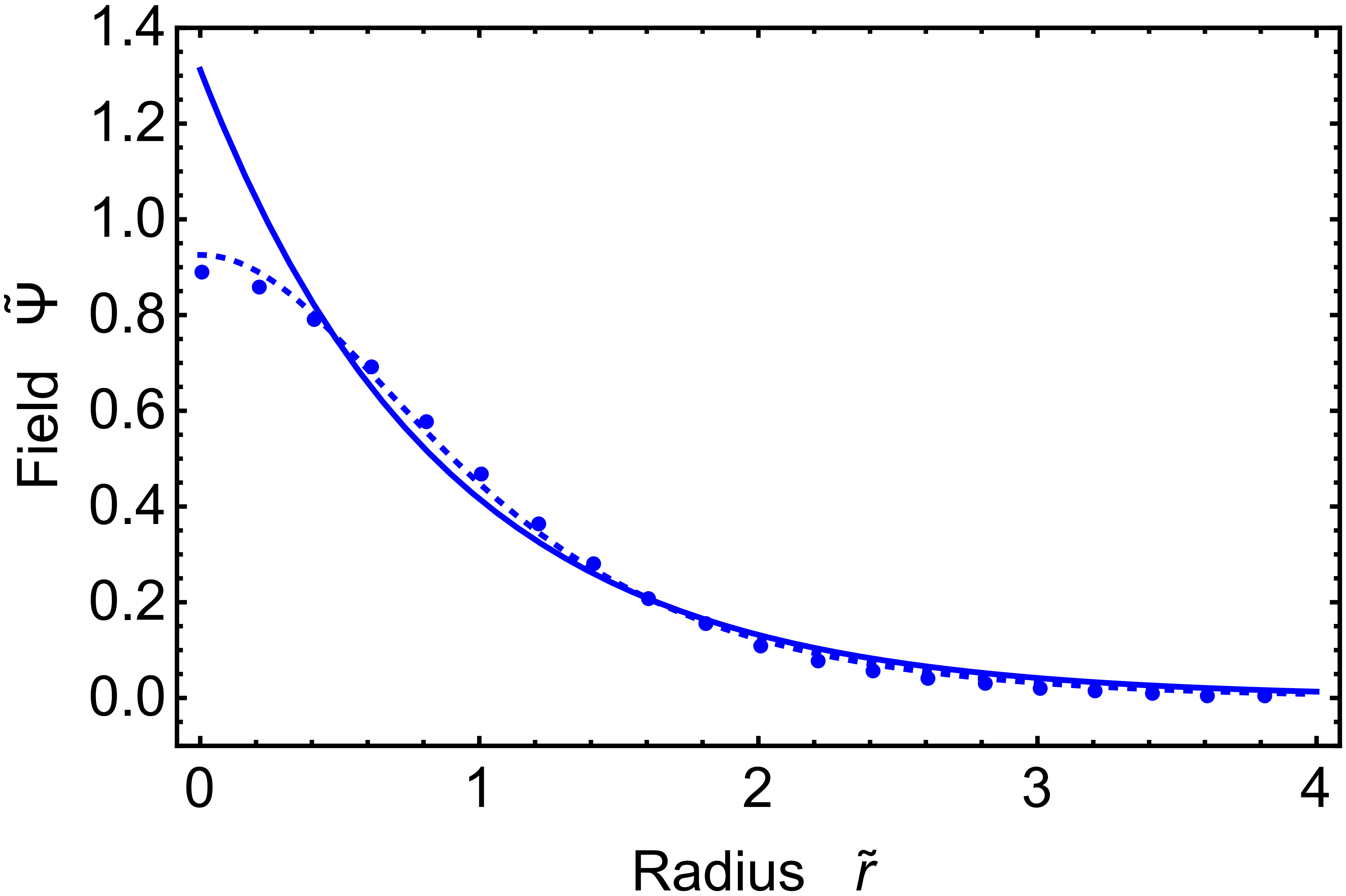
Return to Non-Relativistic Classical Field Theory

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

$$(\lambda < 0)$$

Clump Solutions (BEC) at fixed N



Schiappacasse, Hertzberg 1710.04729

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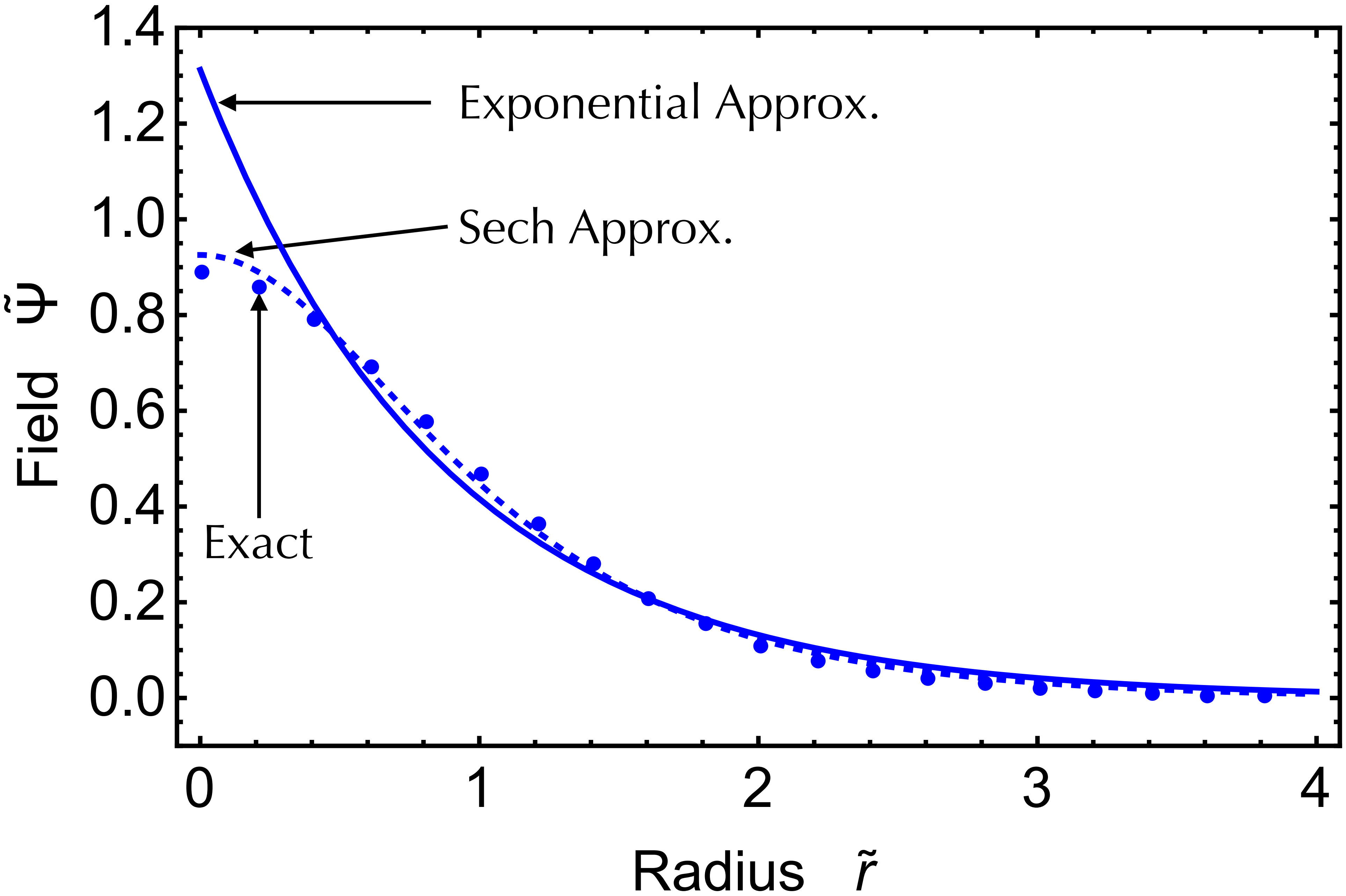

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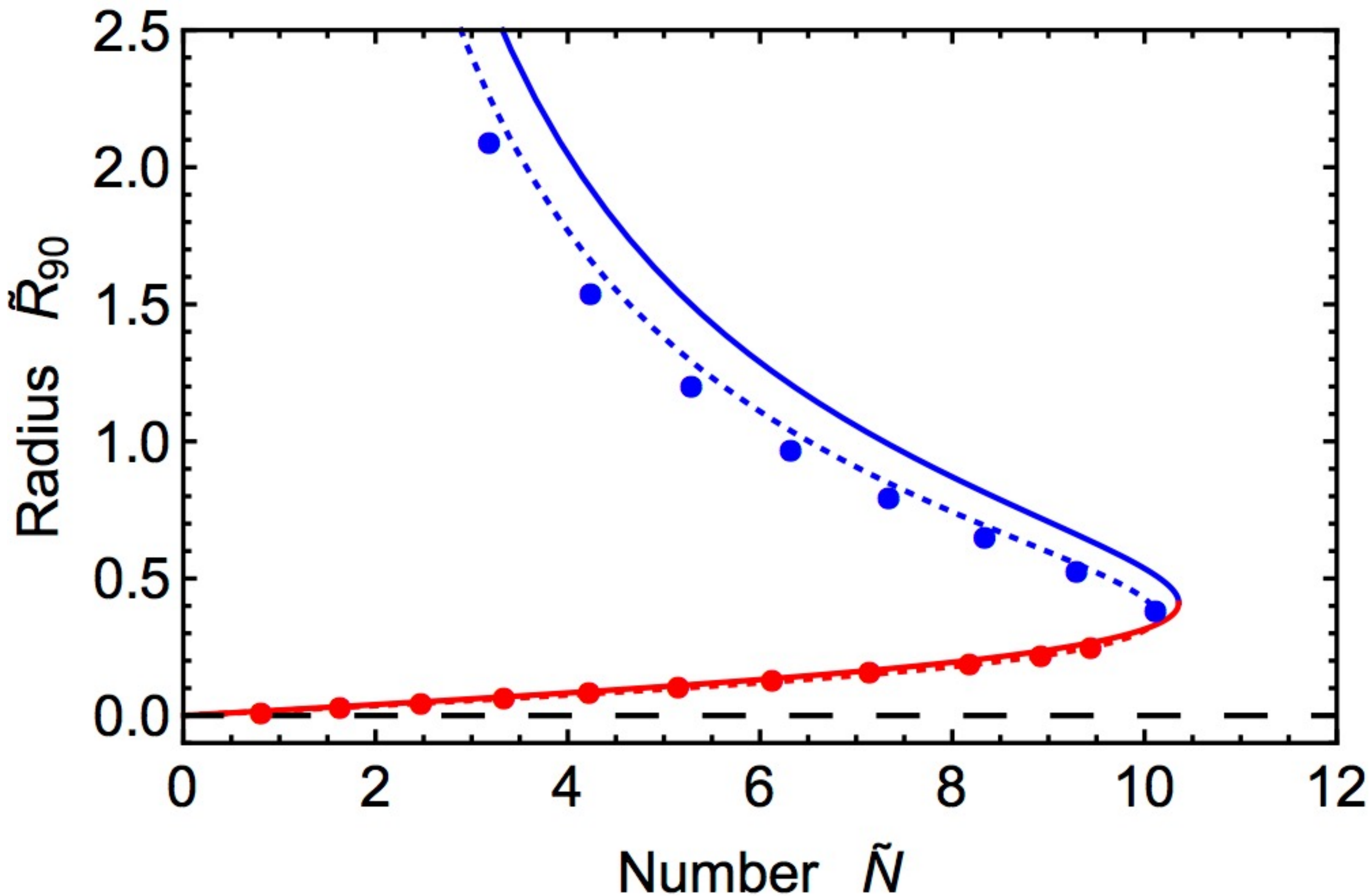
Two Branches of Solutions



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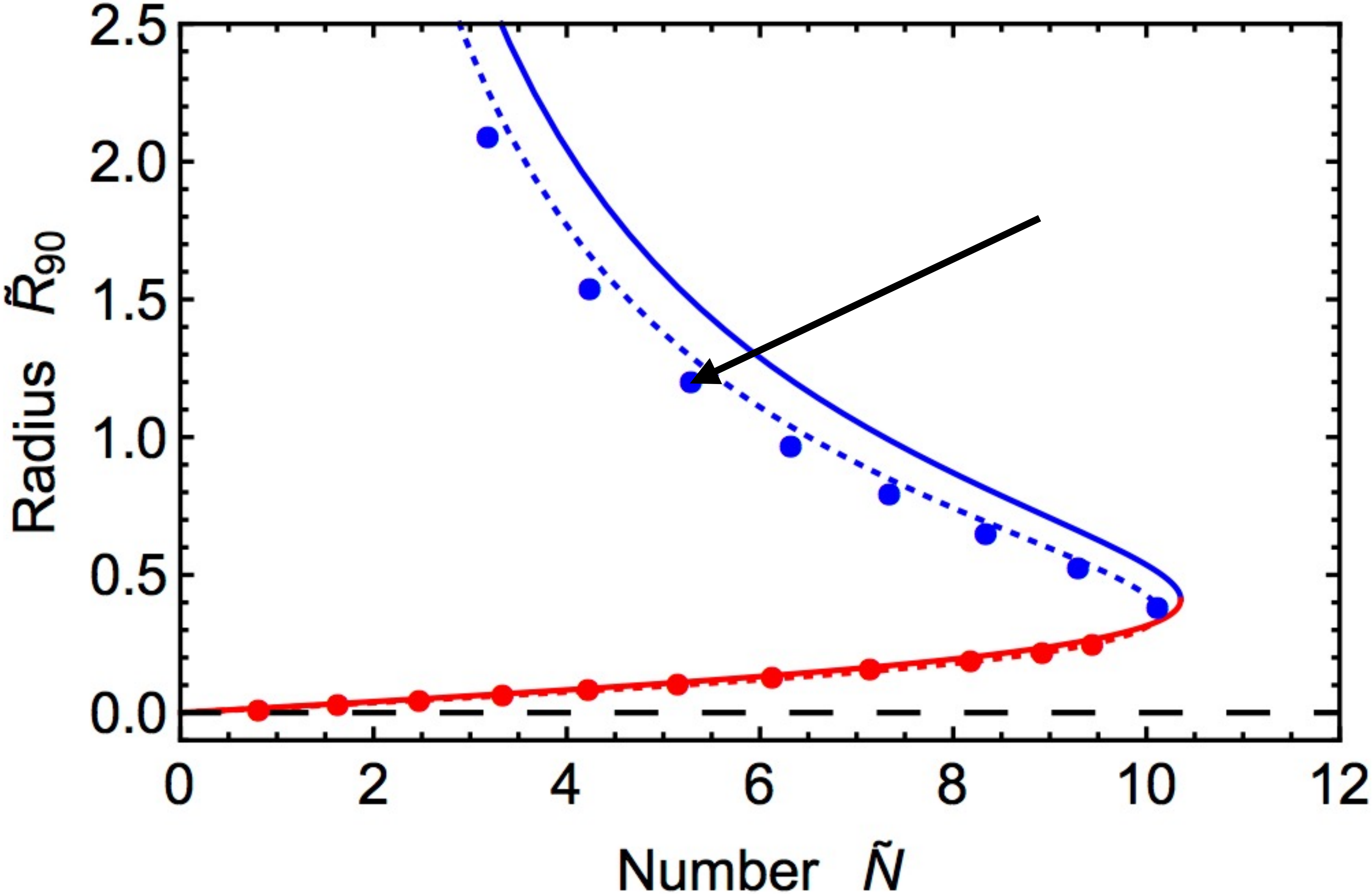

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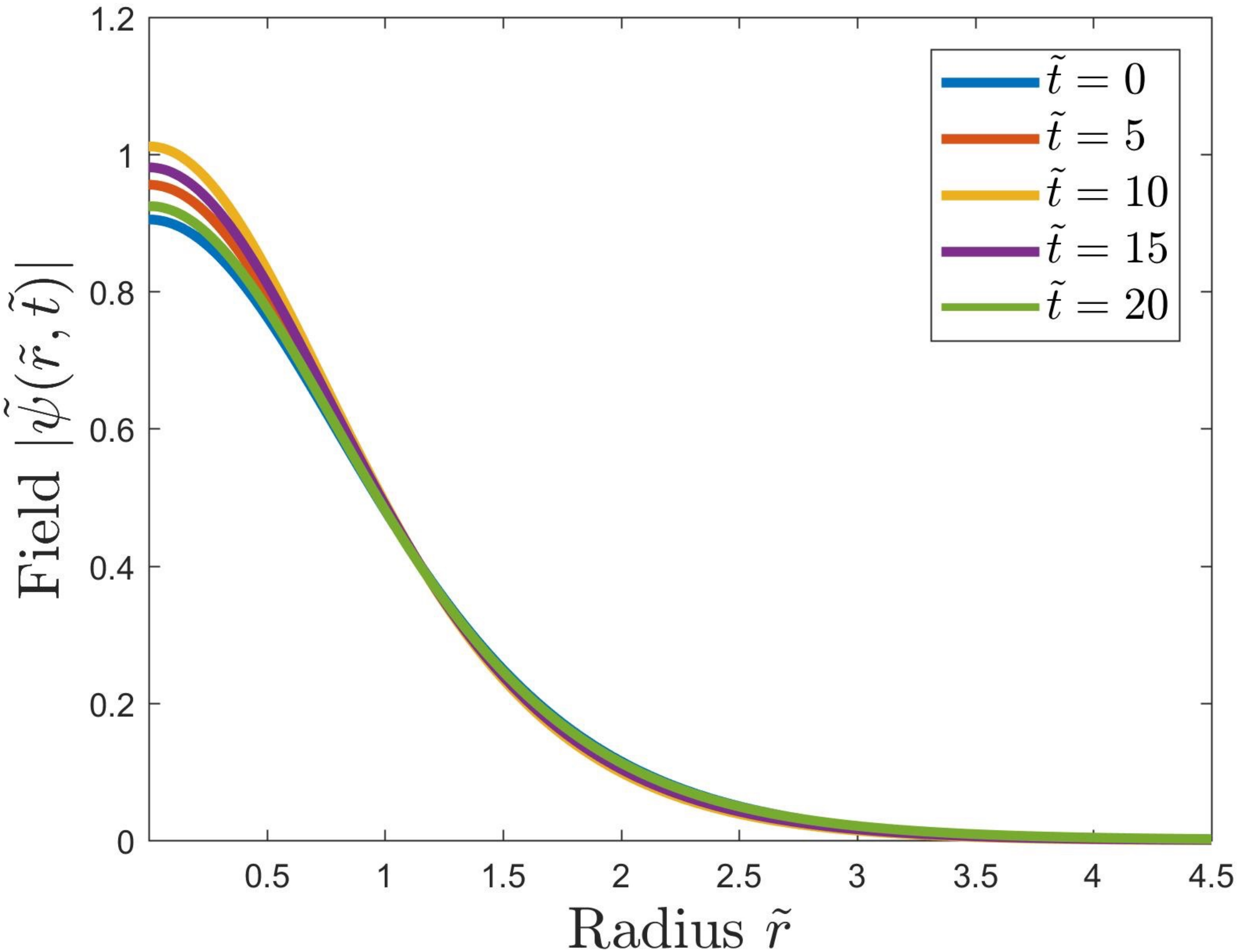
Perturbing Upper Branch



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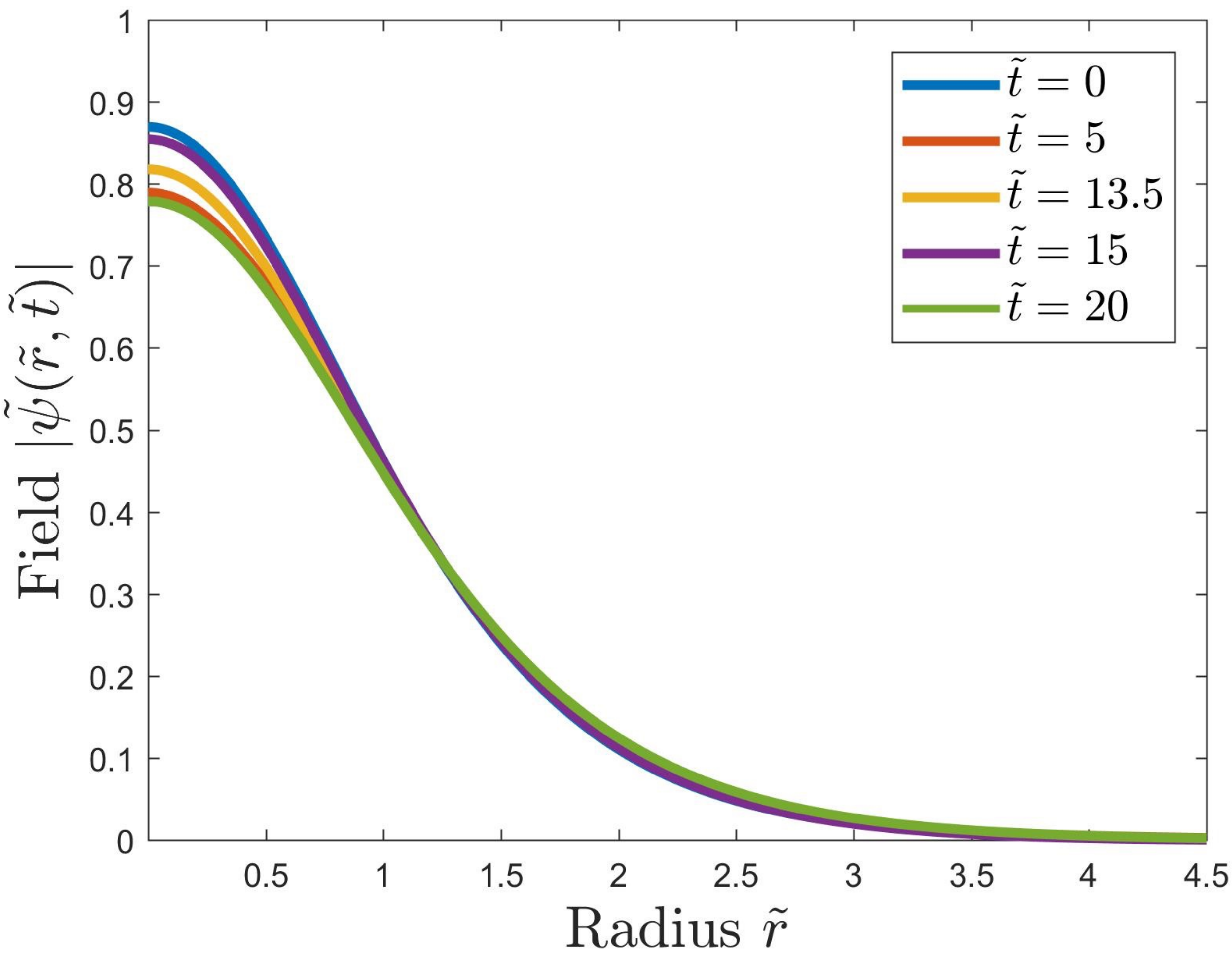

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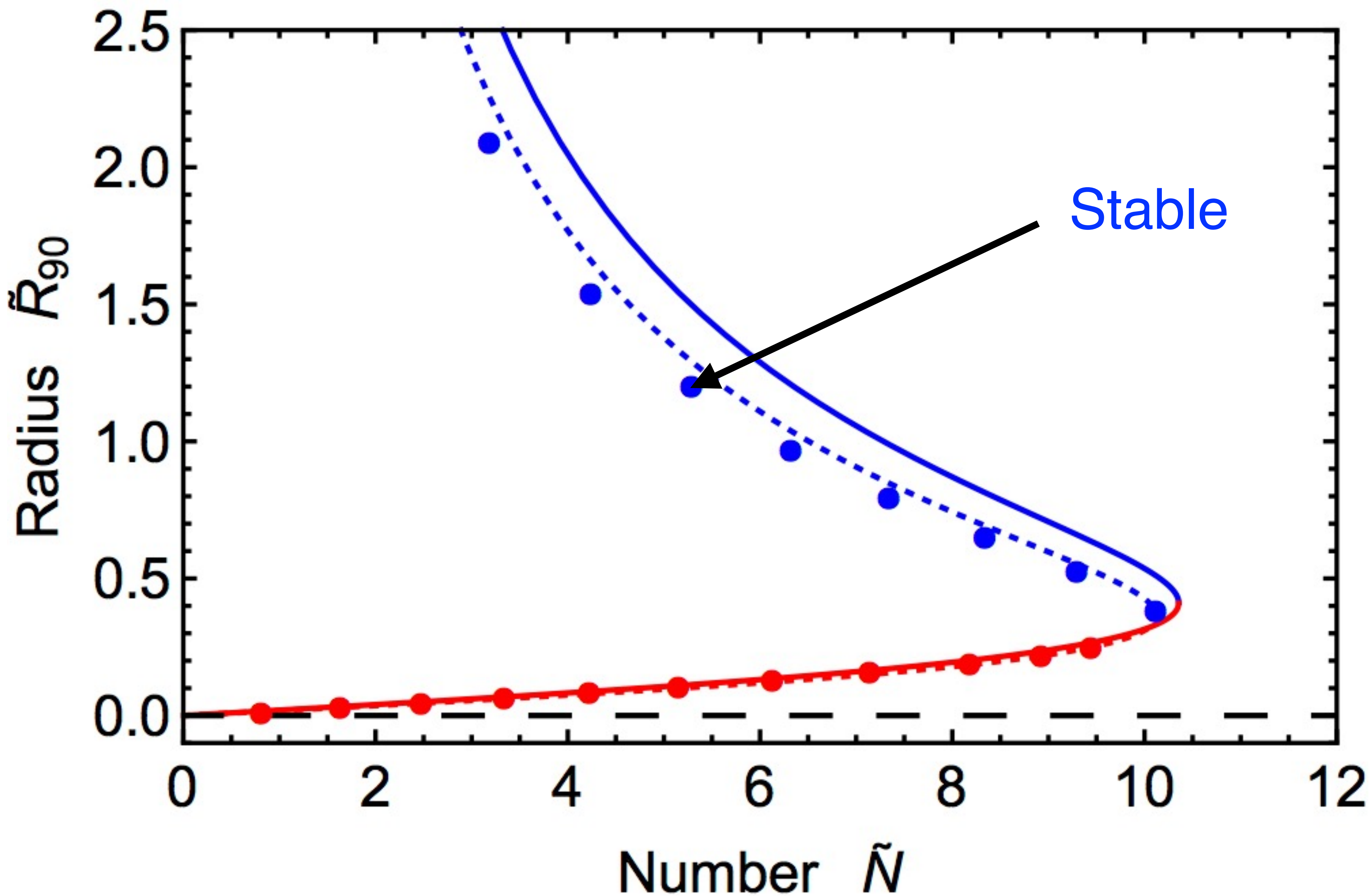

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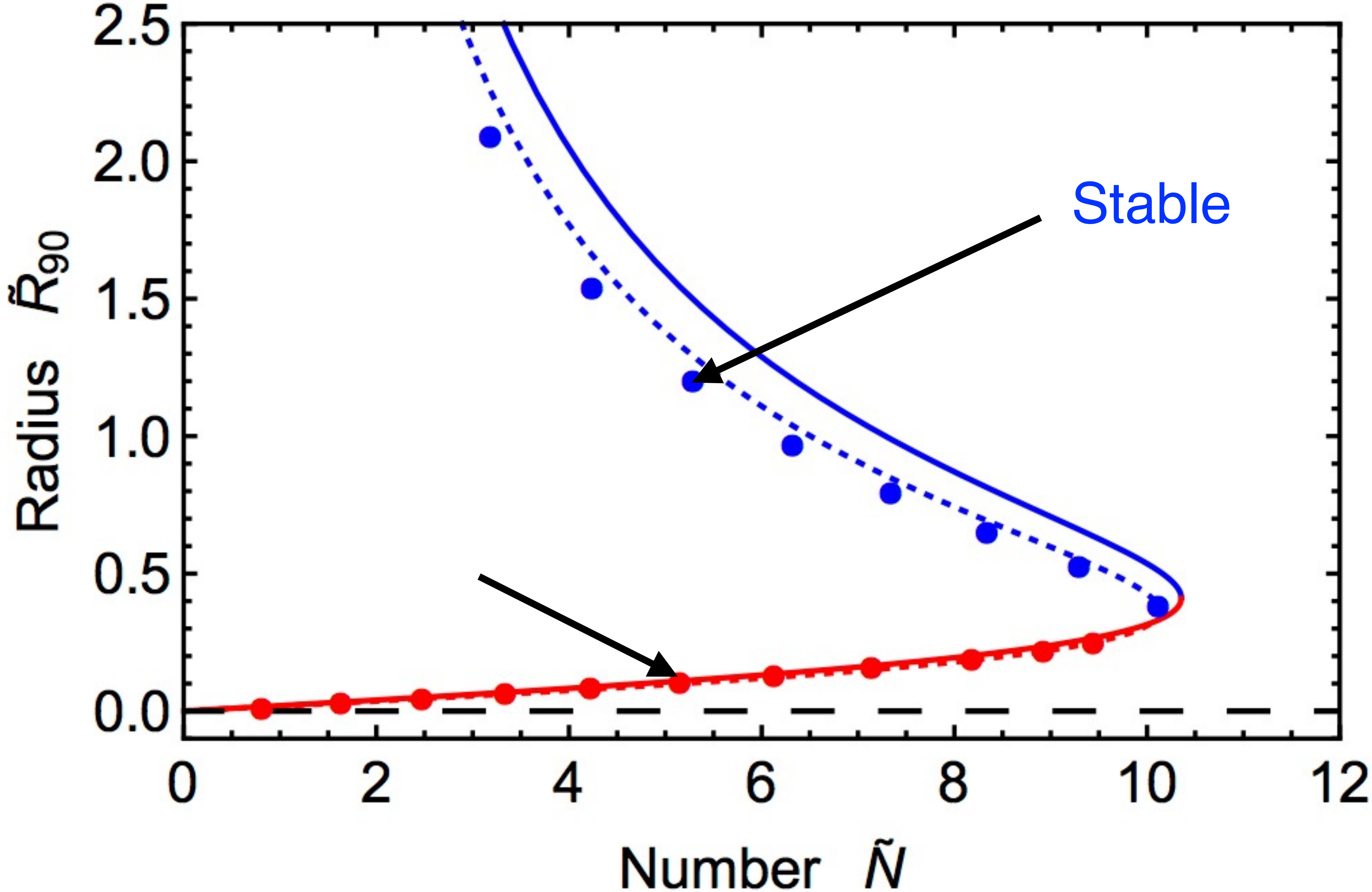

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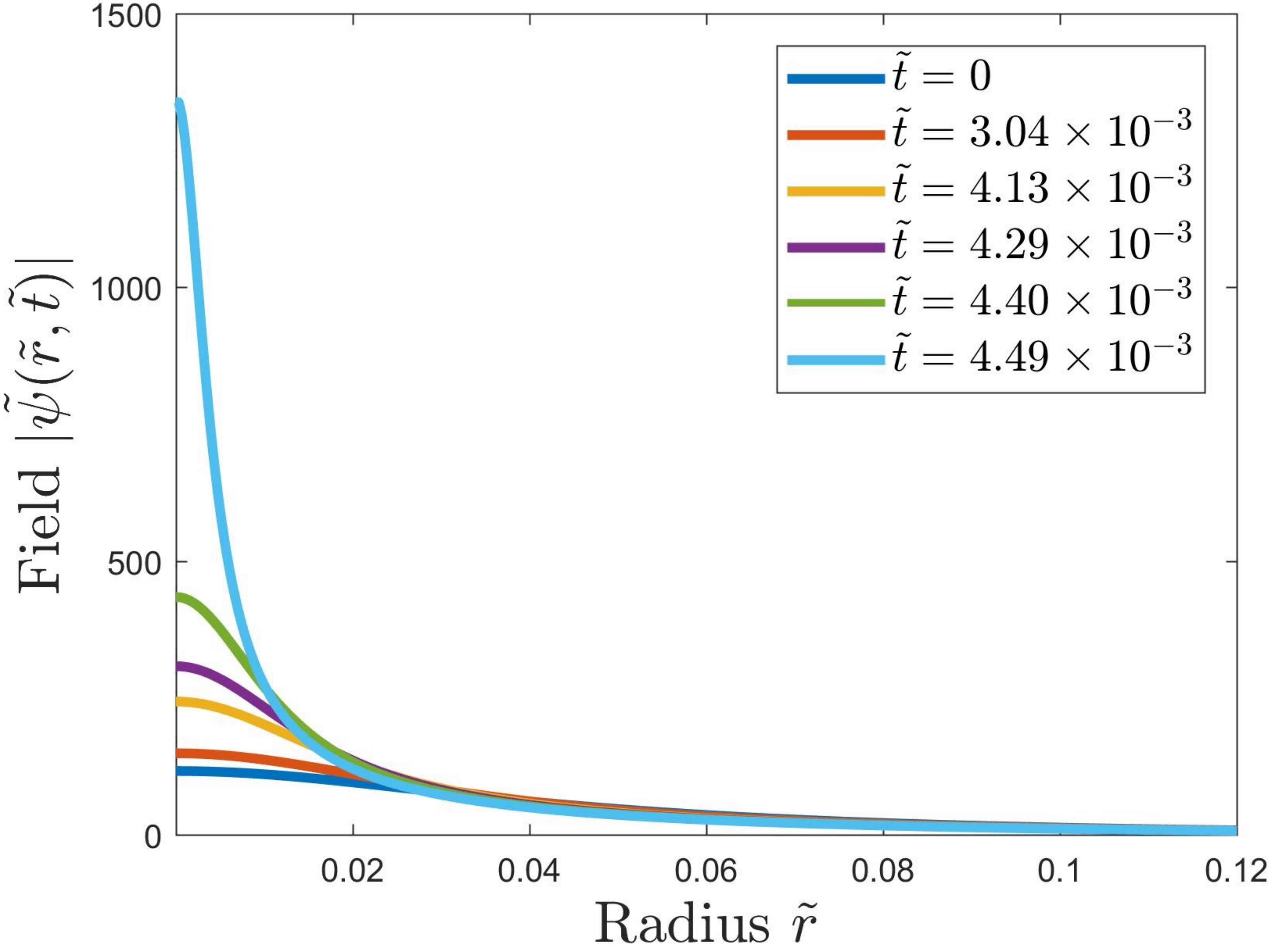
Perturbing Lower Branch



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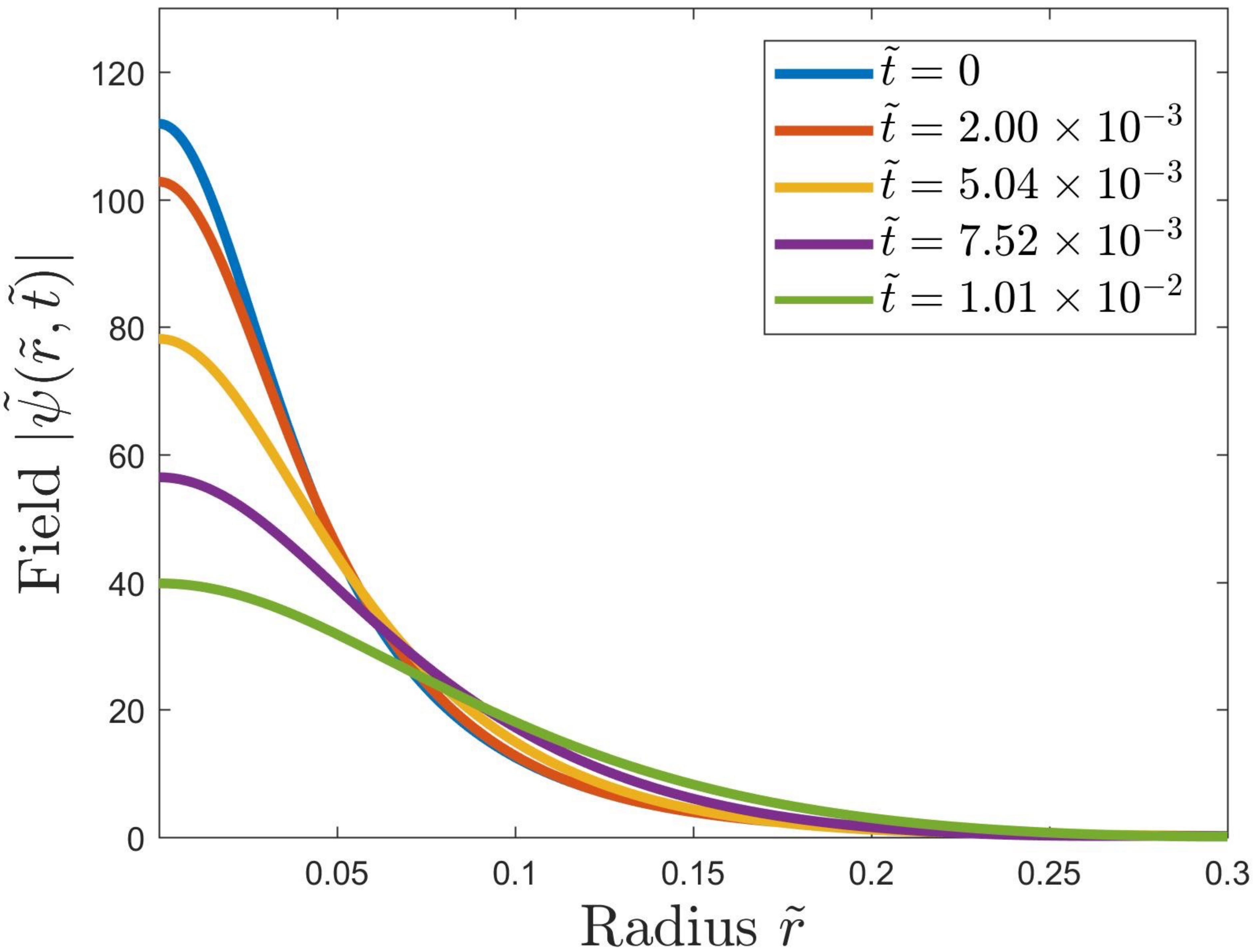

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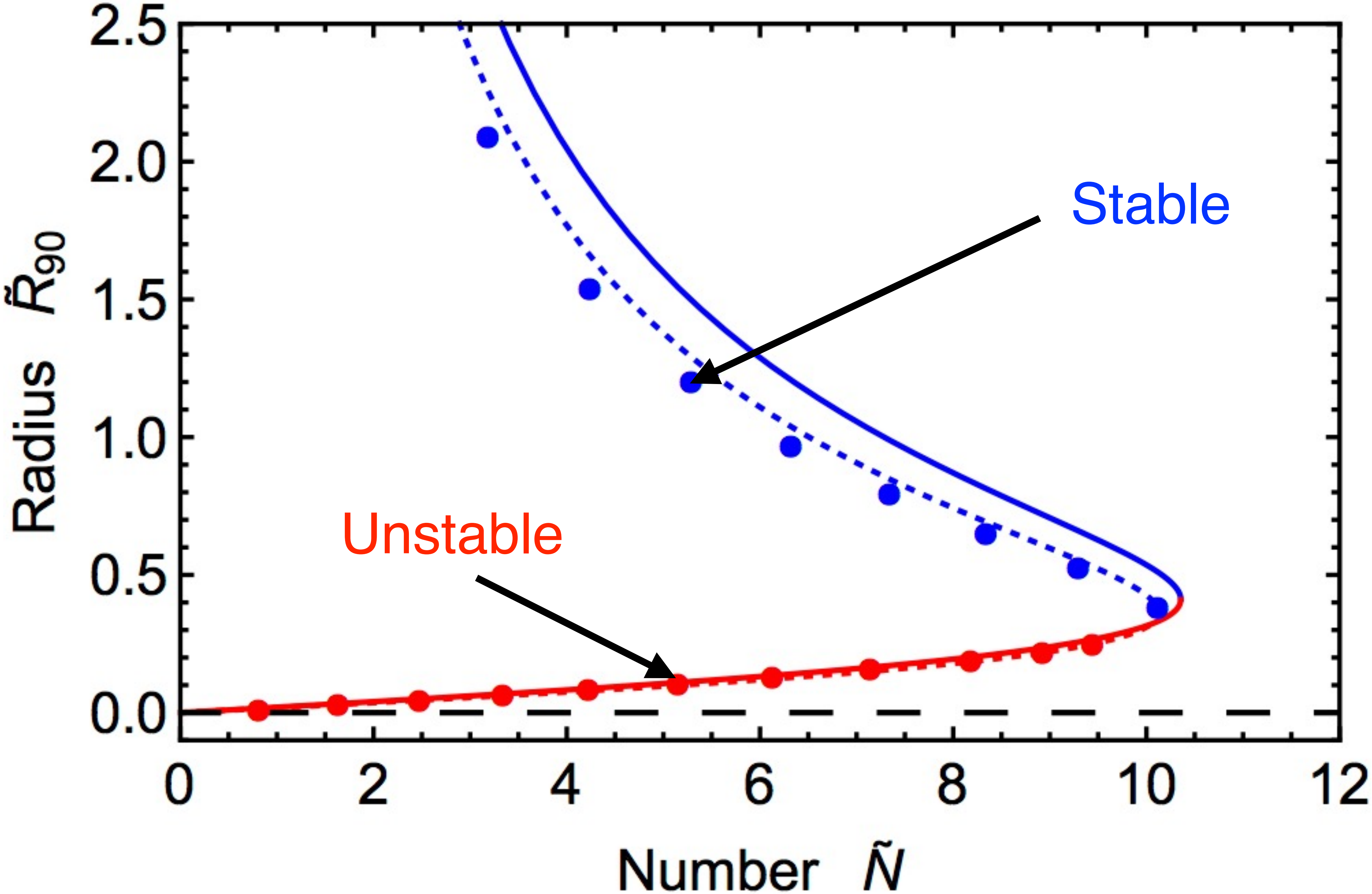

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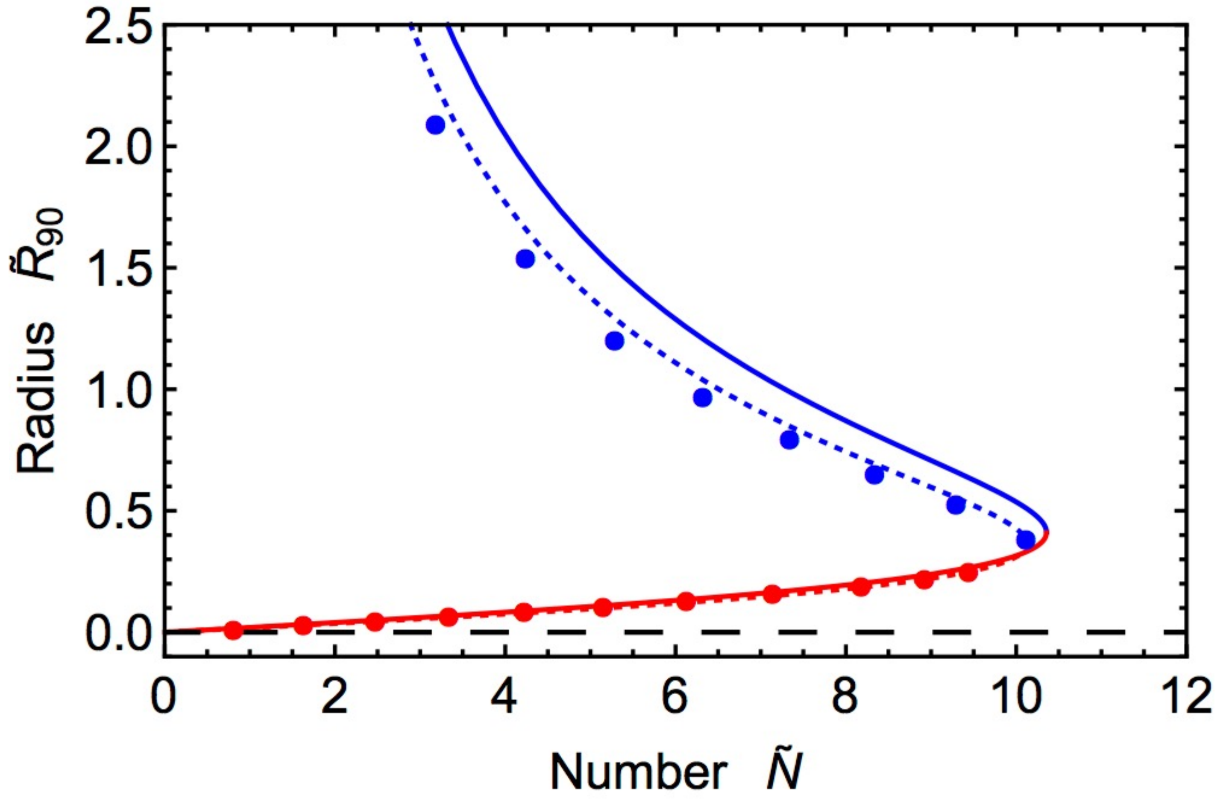

$$N_{max} = \frac{f_a}{m^2\sqrt{G}}\tilde{N}_{max} \sim 8\times 10^{59}\,(\tilde{m}^{-2}\tilde{f}_a)\,,$$

$$M_{max} = N_{max}\,m \qquad \sim 1.4\times 10^{19}\,\mathrm{kg}\,(\tilde{m}^{-1}\tilde{f}_a)\,,$$

$$R_{90,min} = \frac{a\,(\tilde{R}_{90}/\tilde{R})}{bN_{max}Gm^3} \quad \sim 130\,\mathrm{km}\,(\tilde{m}^{-1}\tilde{f}_a^{-1})\,,$$

where $\tilde{f}_a \equiv f_a/(6\times 10^{11}\,\mathrm{GeV})$ and $\tilde{m} \equiv m/(10^{-5}\,\mathrm{eV})$.

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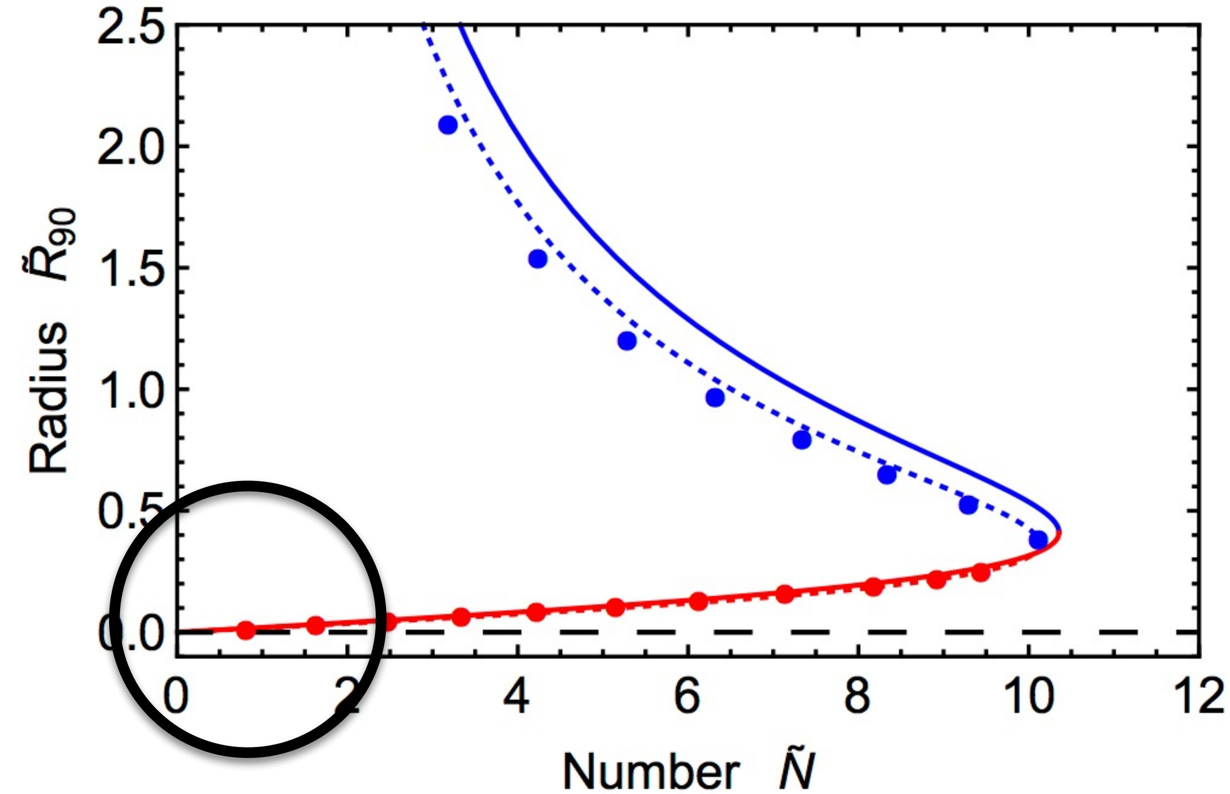
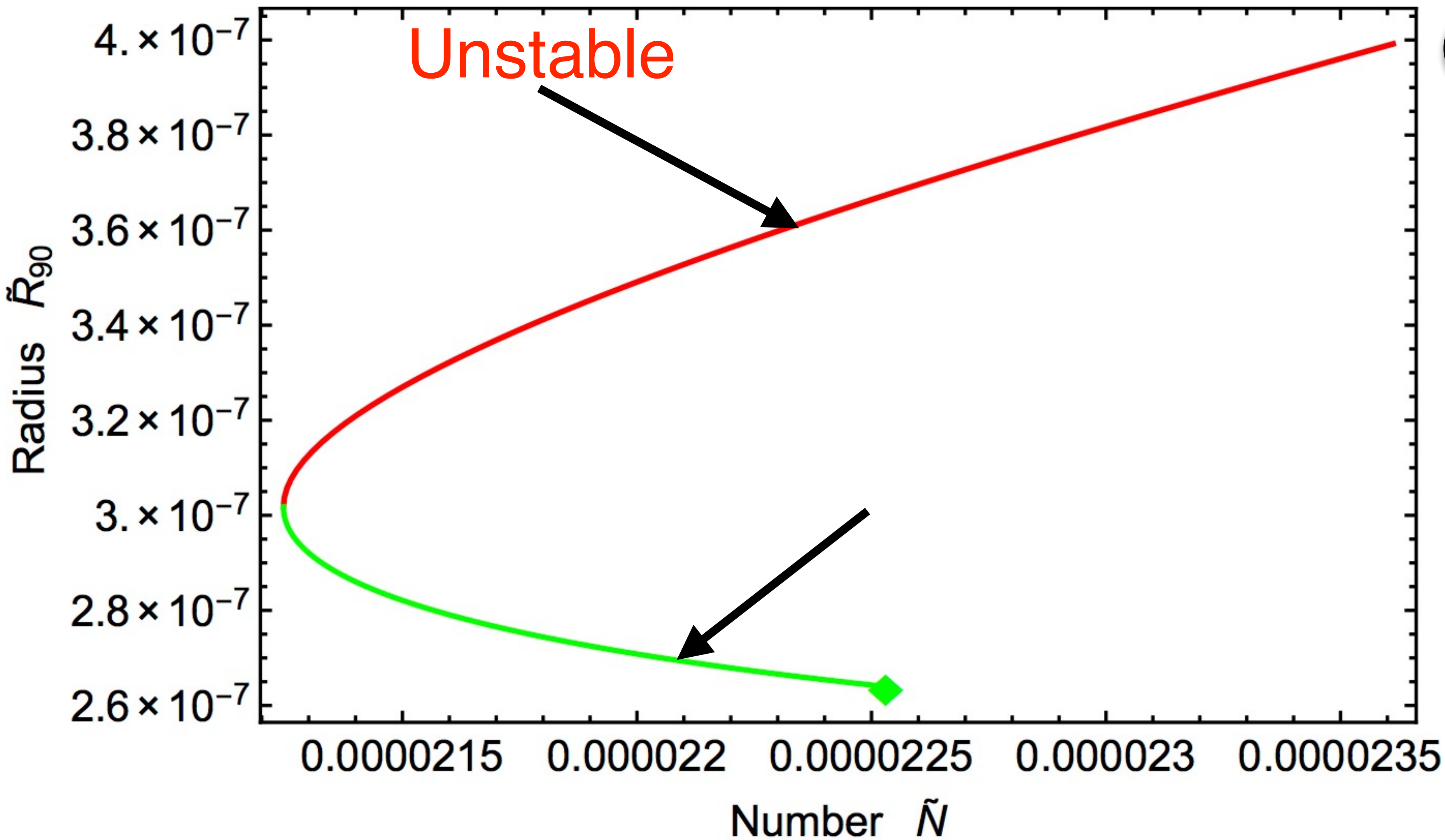
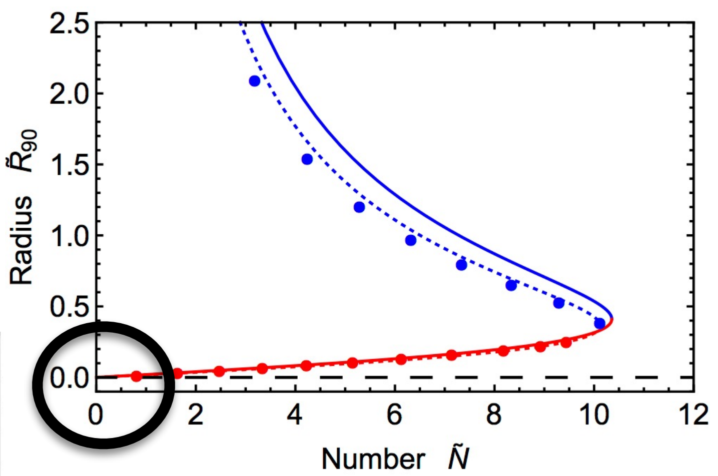
Relativistic Branch (Axiton)



Relativistic Branch (Axiton)





Kolb, Tkachev astro-ph/9311037, Schiappacasse, Hertzberg 1710.04729, Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910

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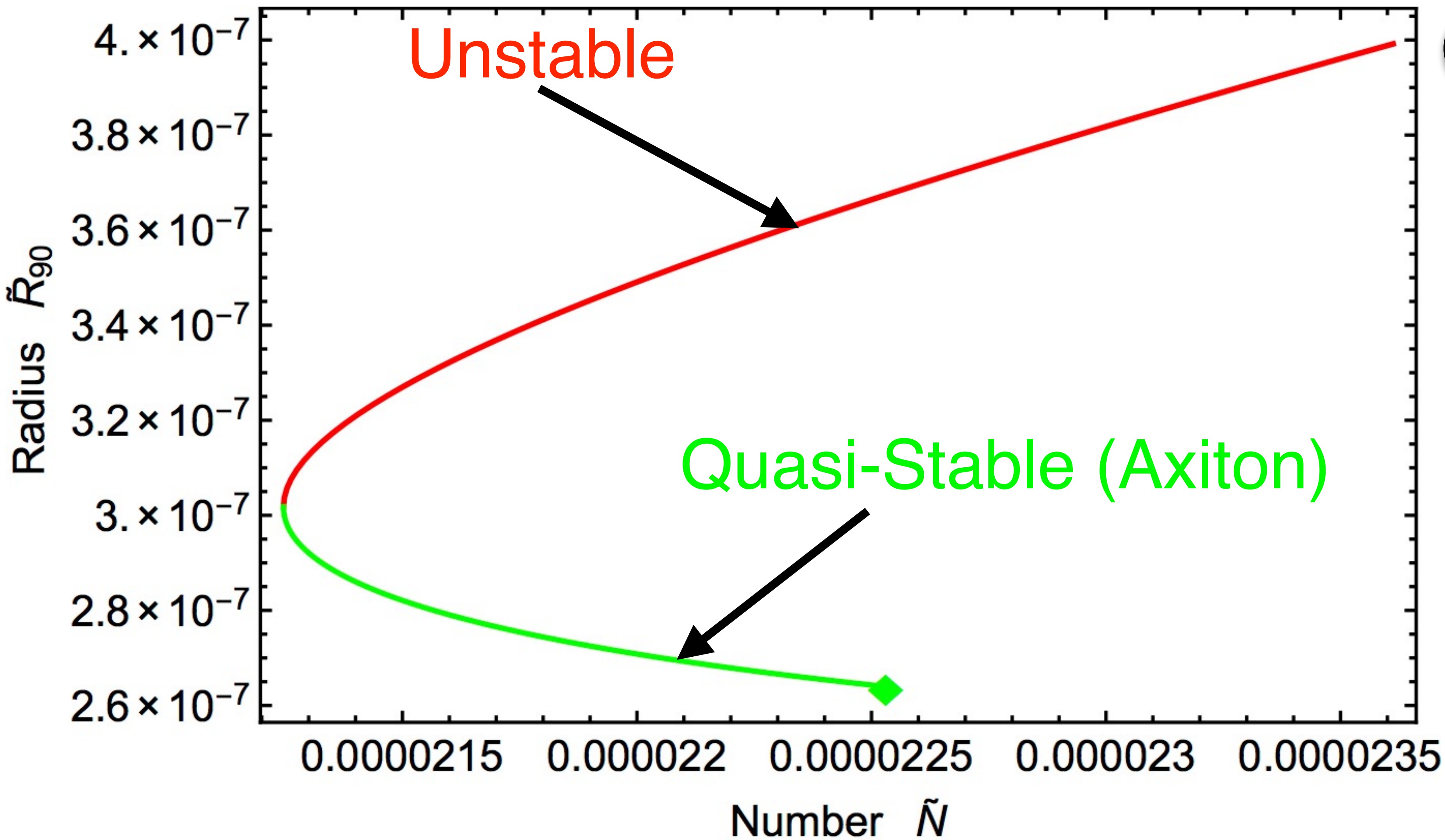

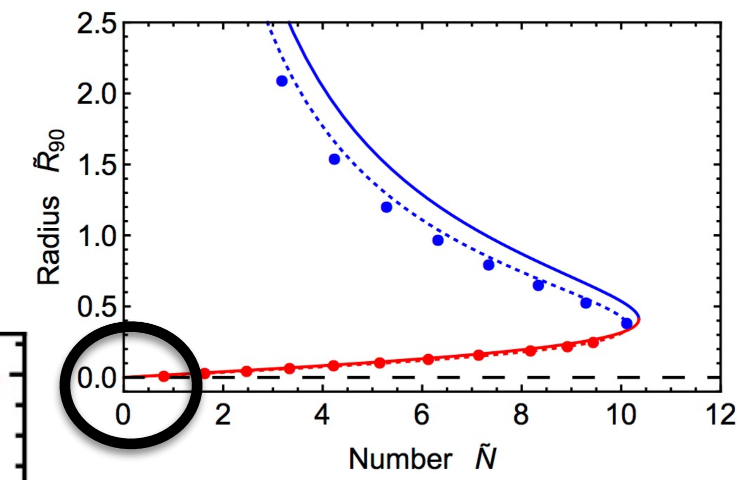

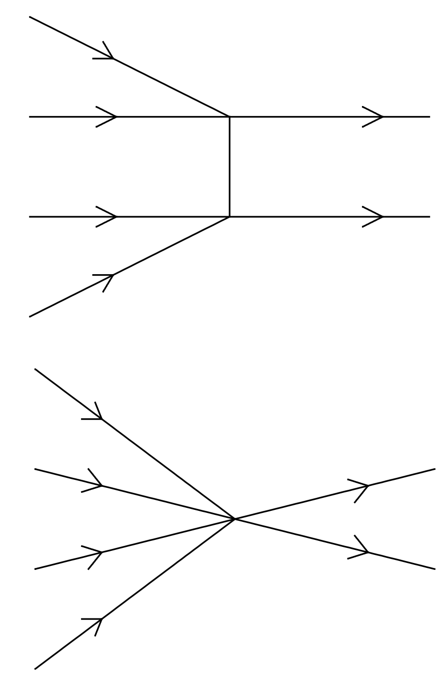

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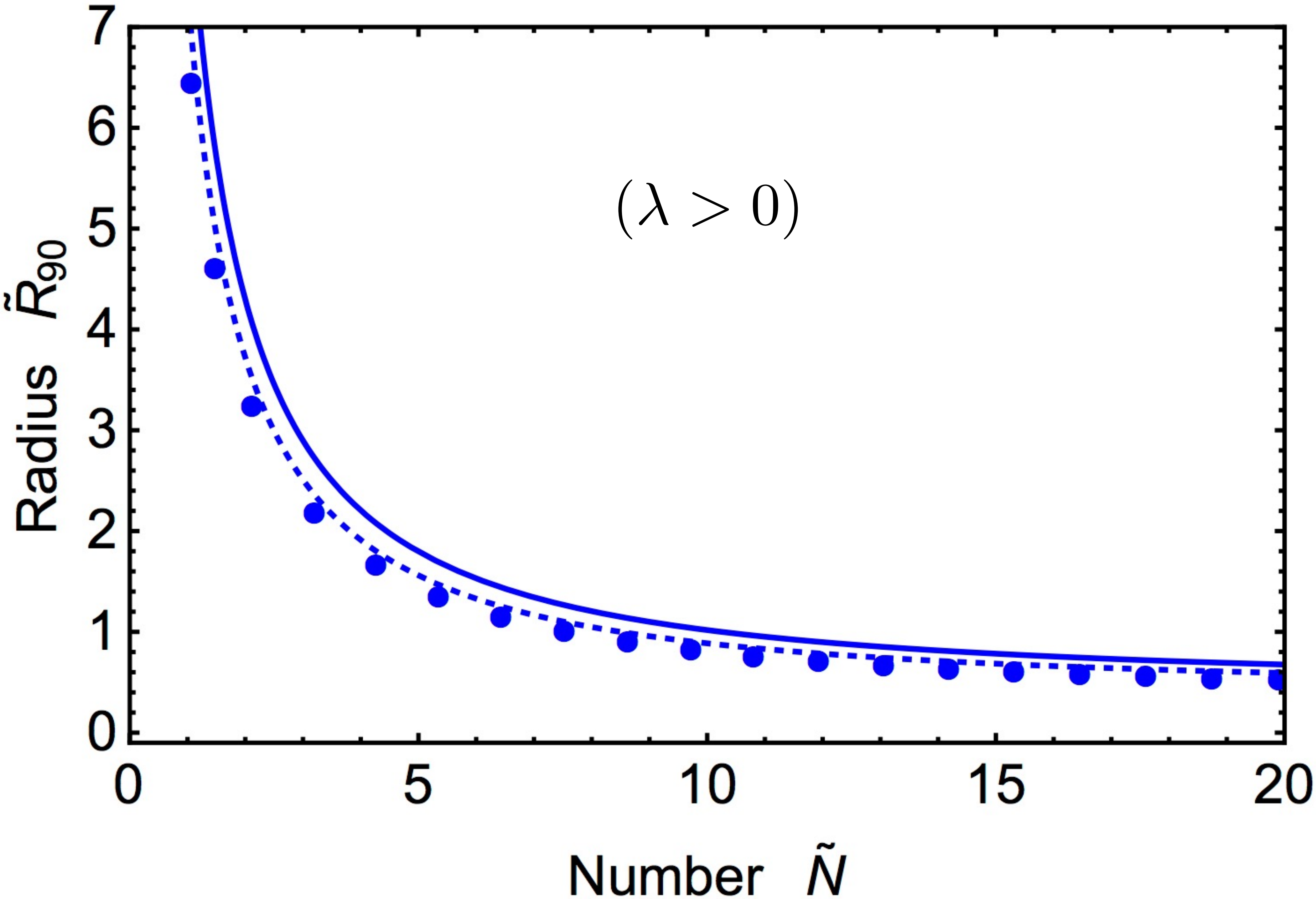
Repulsive Self Interactions

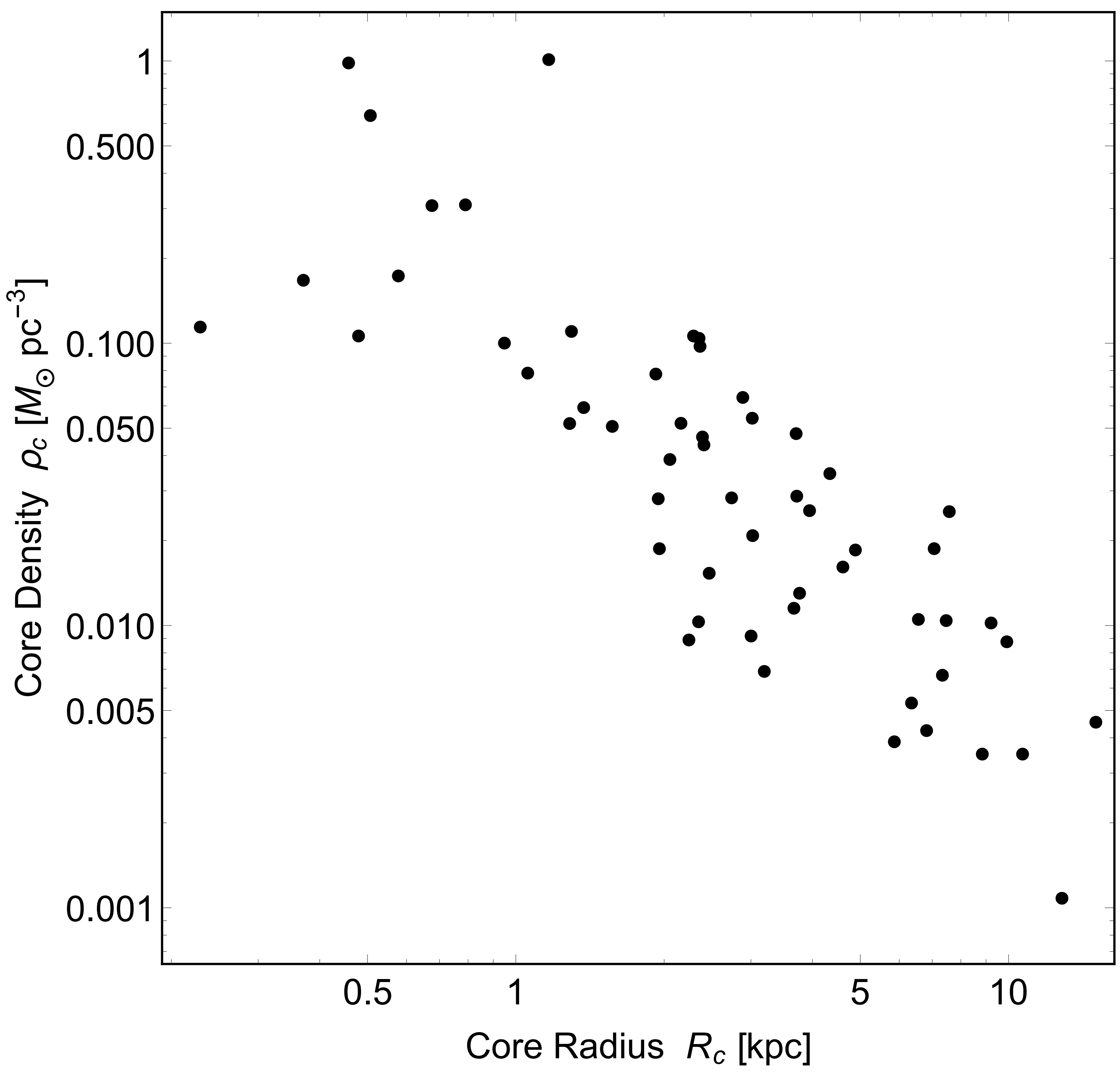
Repulsive Self Interaction (Axion-Like Particle)



Schiappacasse, Hertzberg 1710.04729

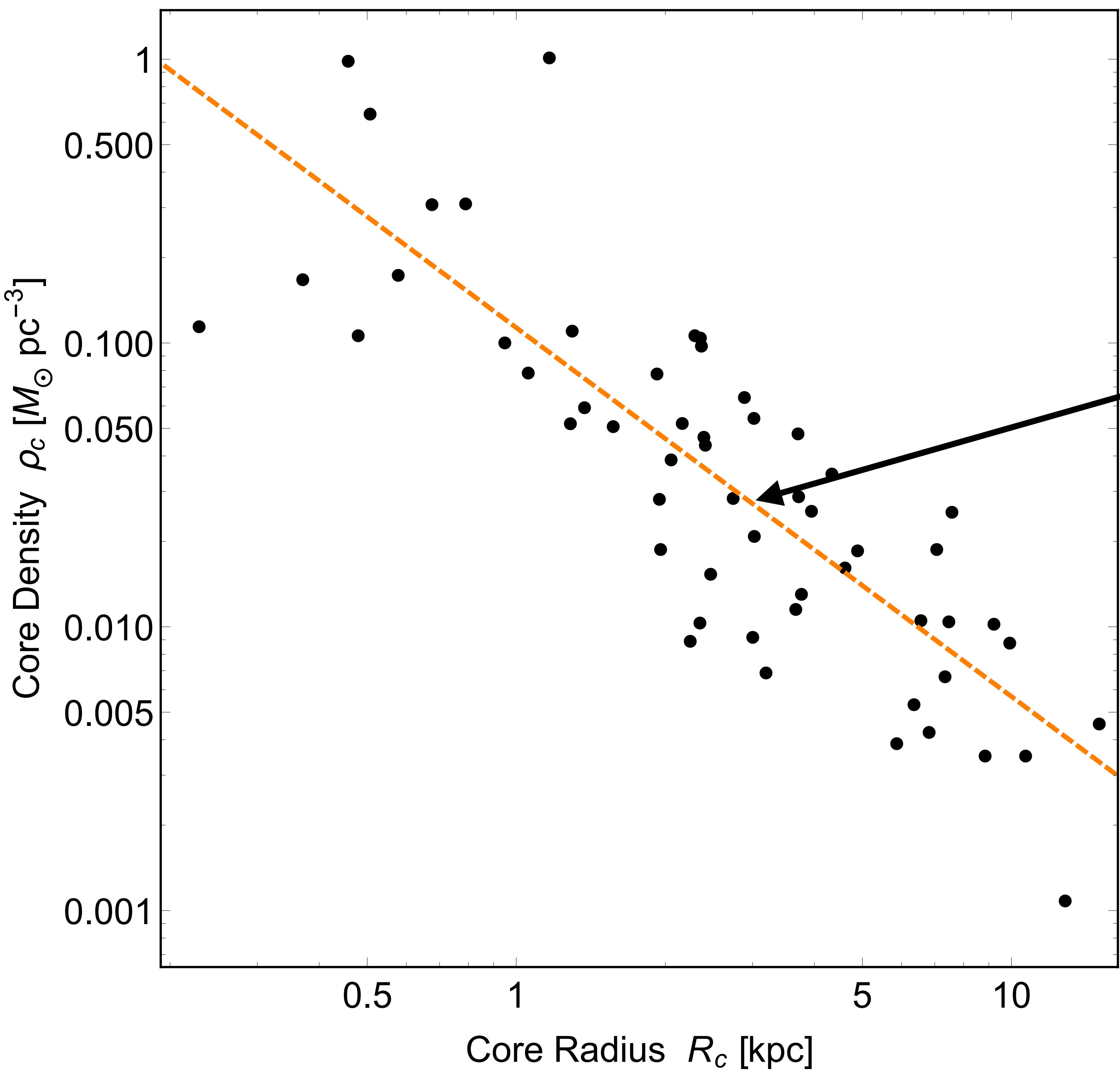
Implications for Fuzzy Dark Matter

Core Density Vs Core Radius (Data)



e.g., see Rodriguez et al 1701.02698

Core Density Vs Core Radius (Data)

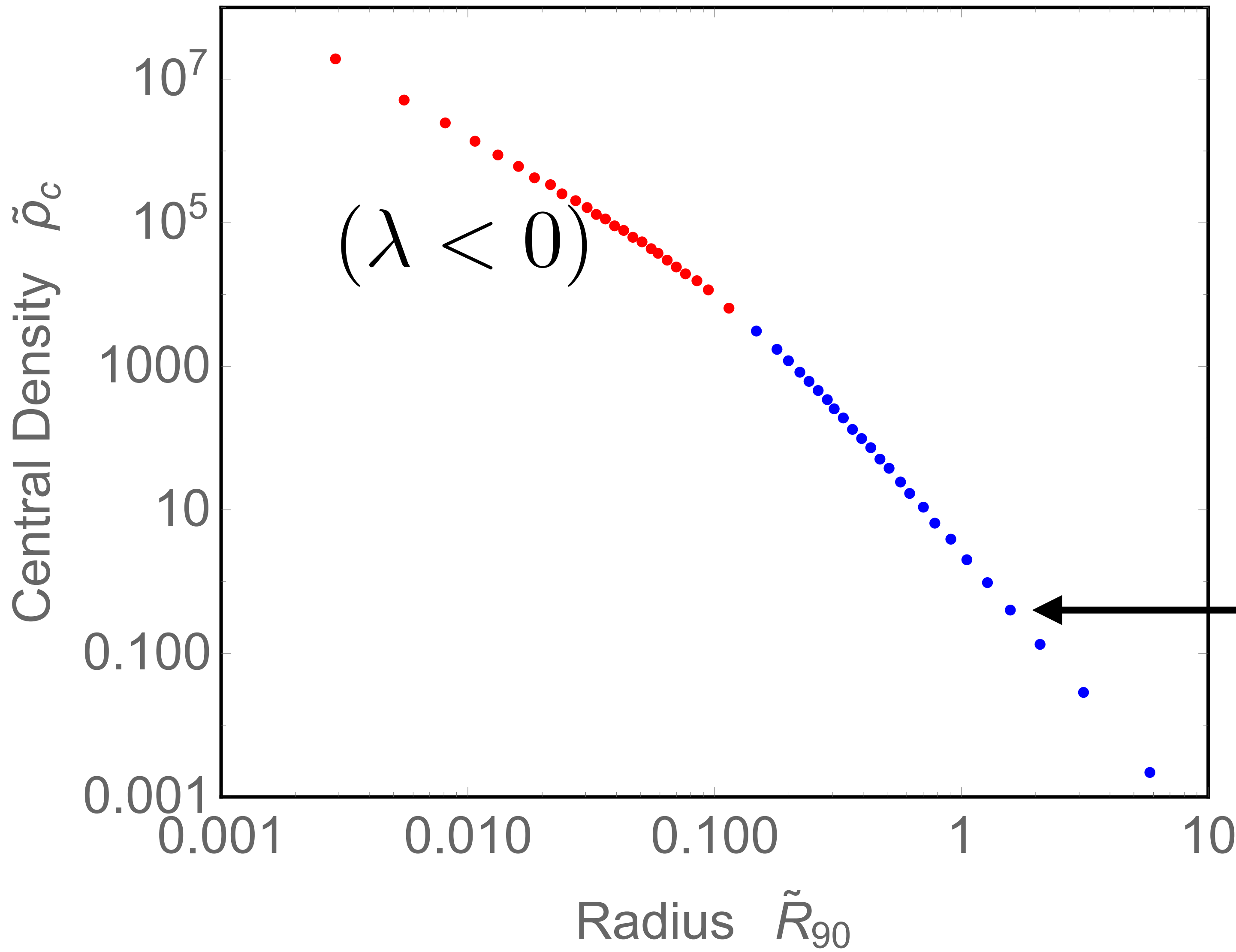


$$\rho_c \sim \frac{1}{R_c^\beta}$$

$$\beta \approx 1$$

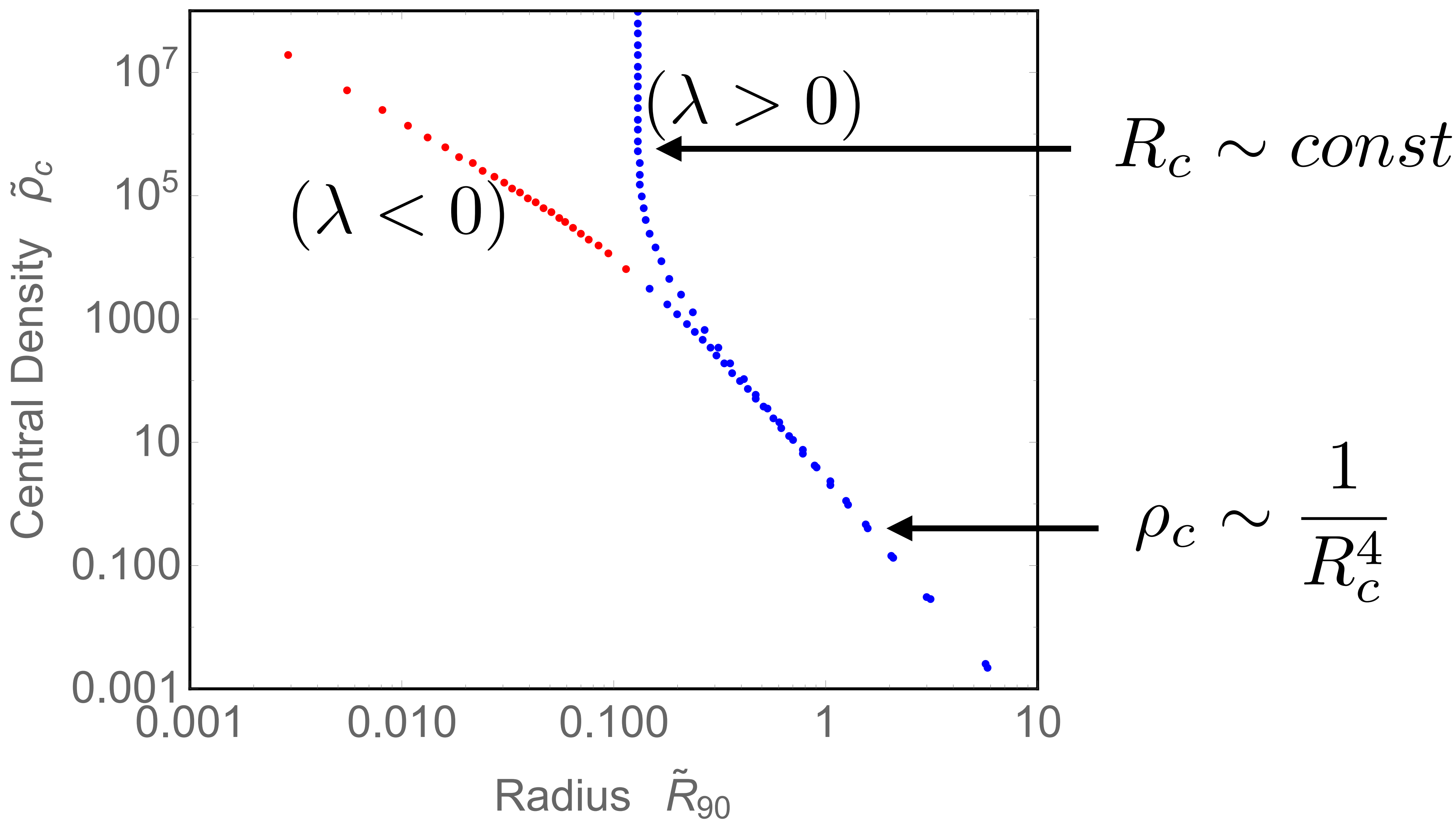
e.g., see Rodriguez et al 1701.02698

Core Density Vs Core Radius (Light Scalar in BEC)



$$\rho_c \sim \frac{1}{R_c^4}$$

Core Density Vs Core Radius (Light Scalar in BEC)

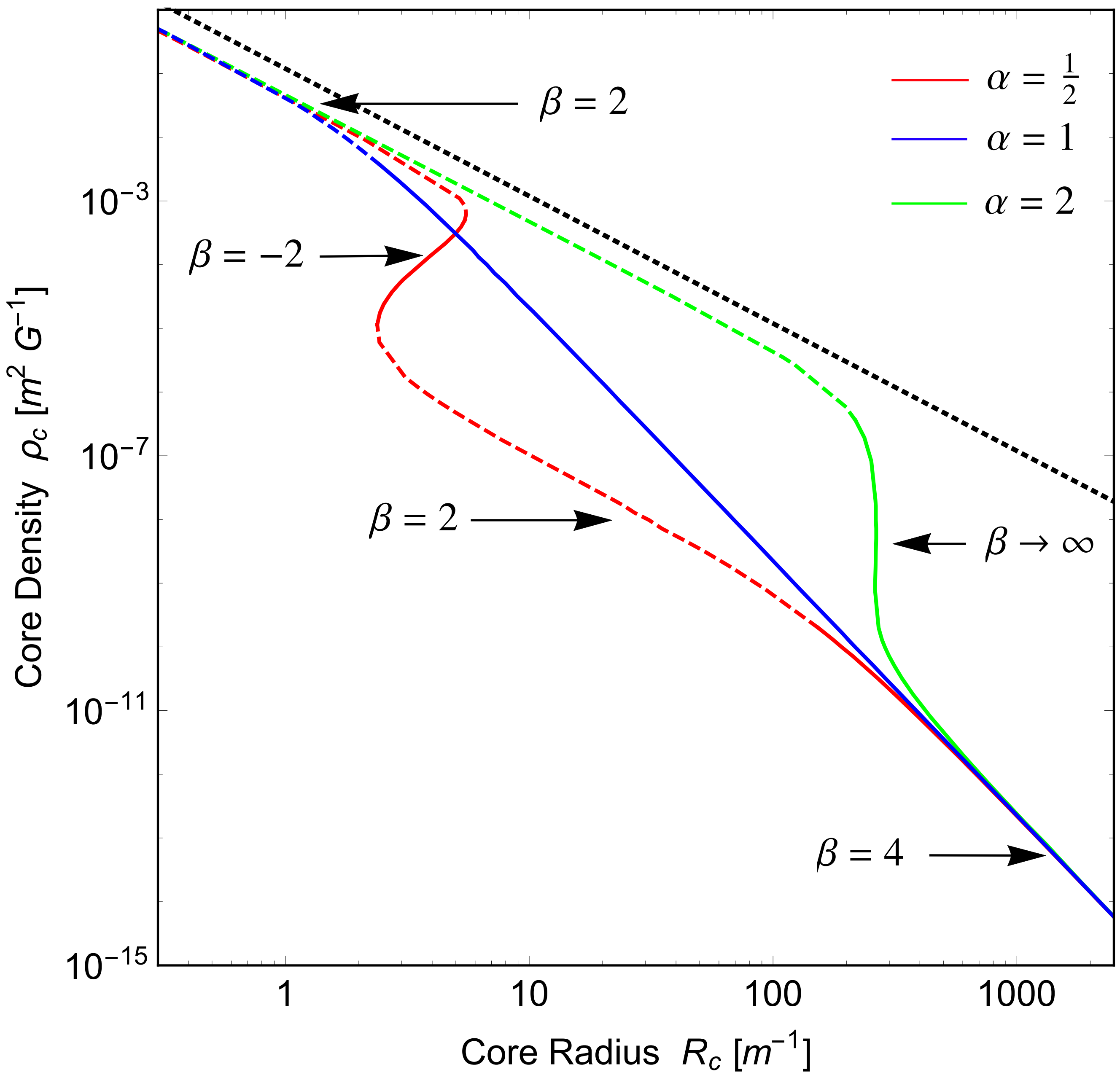


Core Density Vs Core Radius (Light Scalar in BEC)

Extension to general potentials, polytropes, full relativistic

$$V(\phi) \propto ((1 + \phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable



$$\rho_c \sim \frac{1}{R_c^\beta}$$

Deng, Hertzberg, Namjoo, Masoumi 1804.05921

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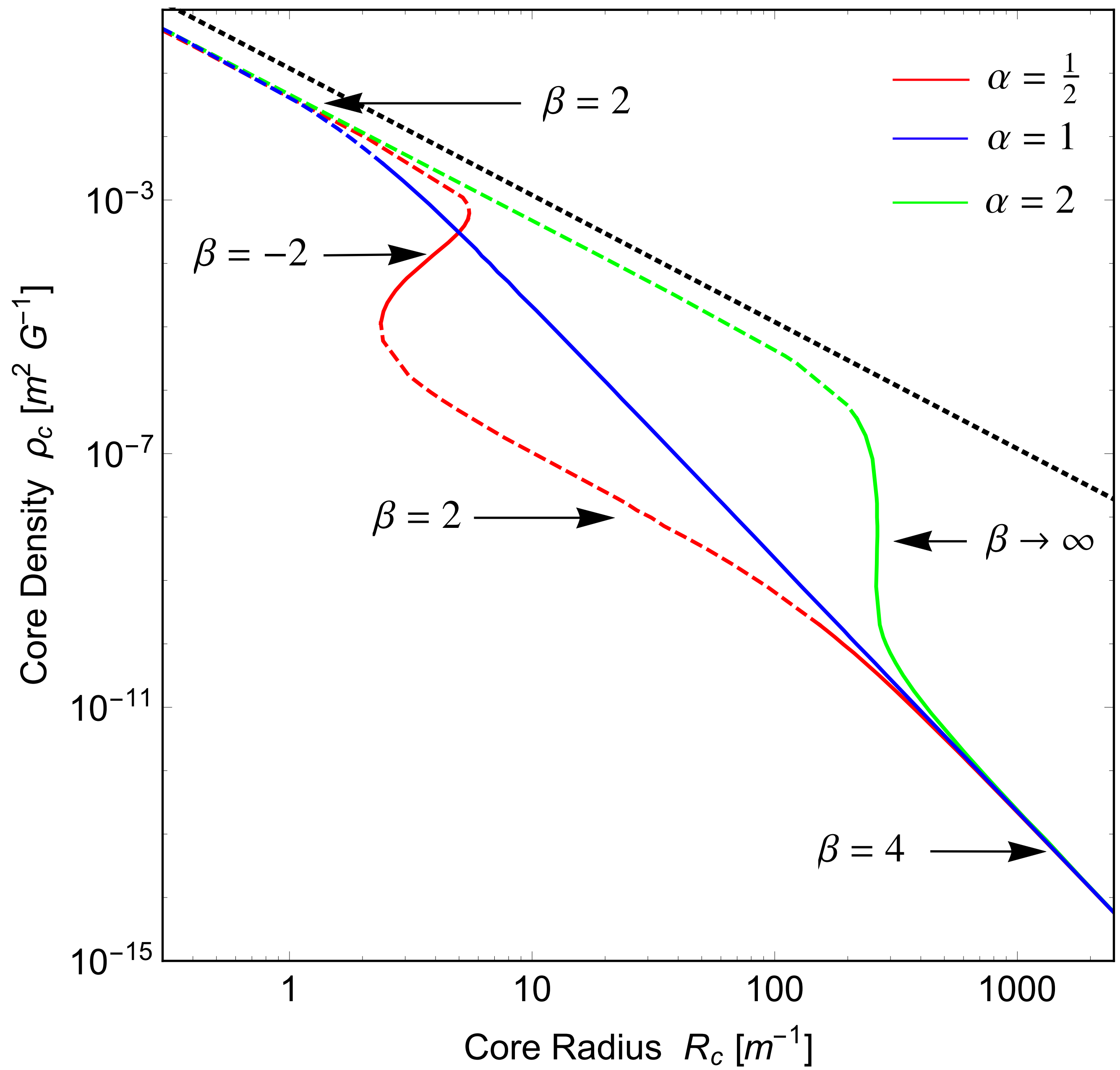

$$\rho_c \sim \frac{1}{R_c^\beta}$$

Never obtain $\beta \sim 1$ and stable

Deng, Hertzberg, Namjoo, Masoumi 1804.05921

Axion Clump Resonance into Photons

Consider Axion to Photon Coupling

Photon Lagrangian $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{g_{a\gamma}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Equation of motion $$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + g_{a\gamma}\, \partial_t \phi\, \nabla \times \mathbf{A} = 0$$

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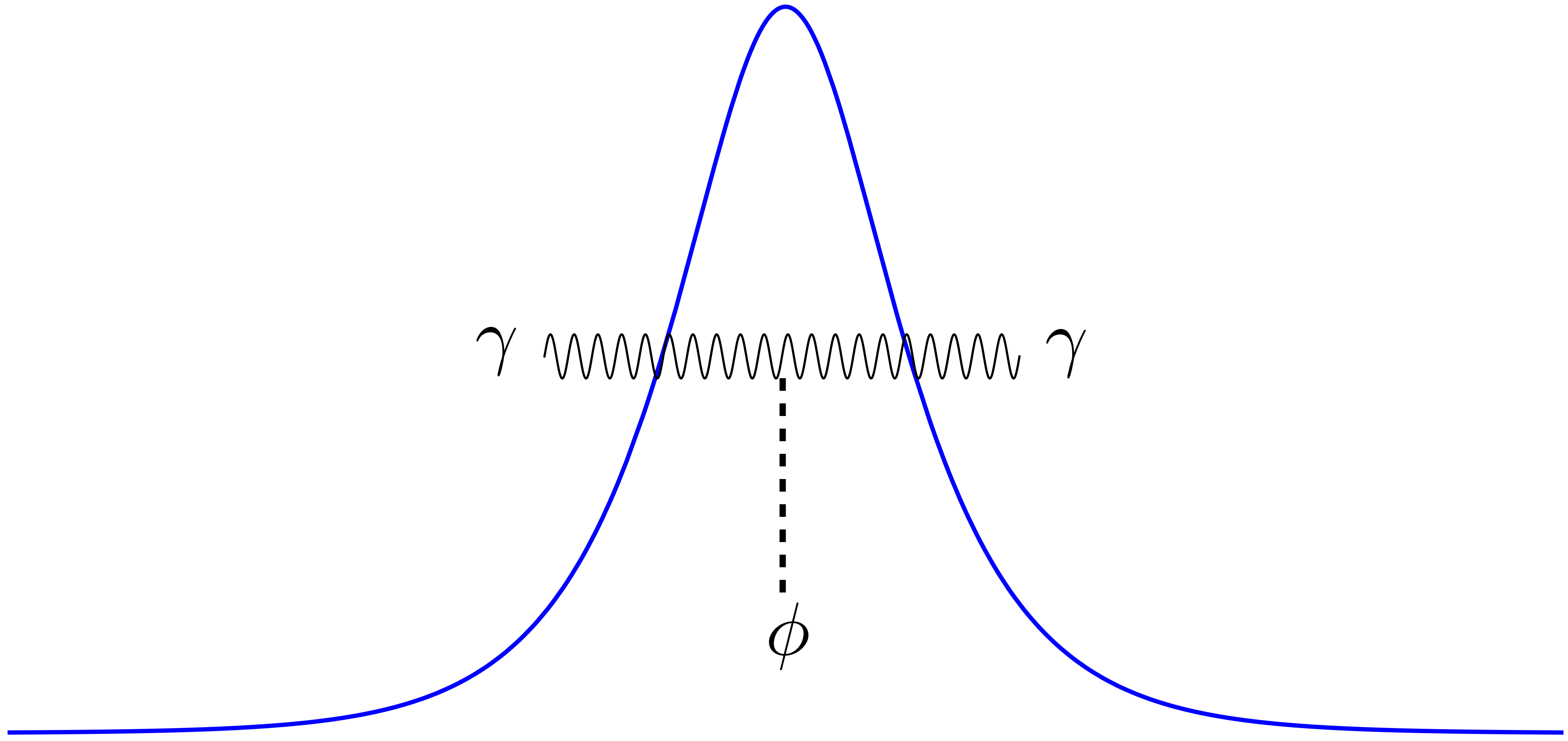


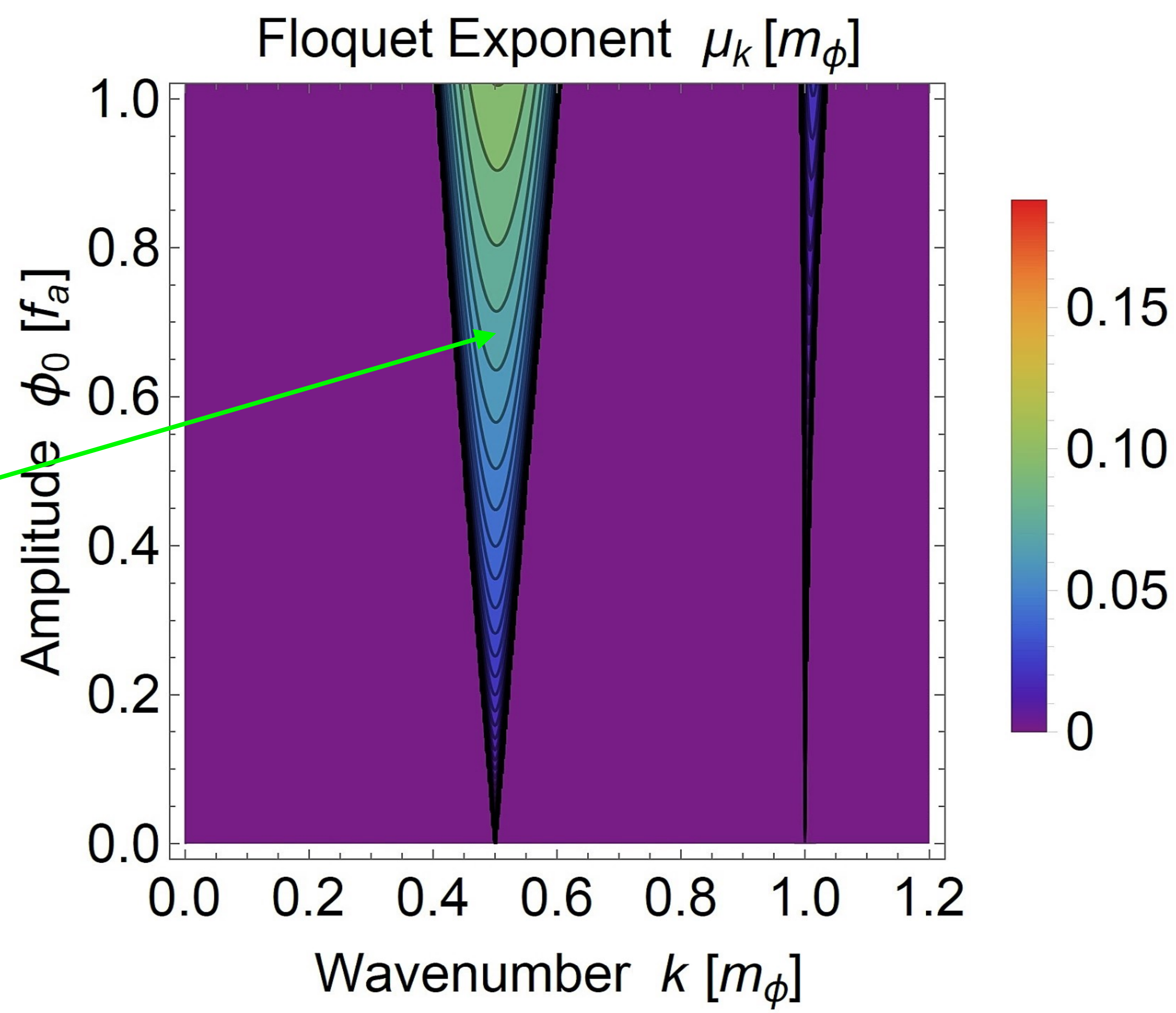
Homogeneous Axion Field

Mathieu Equation

$$\ddot{\mathbf{A}}^T_{\mathbf{k}} + k^2 \mathbf{A}^T_{\mathbf{k}} + g_{a\gamma}\, k\, \partial_t \phi(t) \mathbf{A}^T_{\mathbf{k}} = 0$$

Parametric resonance always present

$$k \approx \frac{m_a}{2}$$



Inhomogeneous (Spherical) Axion Clump

Decomposition into vector spherical harmonics

$$\mathbf{A}(\mathbf{r},t)=\sum_{lm}\int\frac{d^3k}{(2\pi)^3}\left[a_{lm}(k,t)\mathbf{N}_{lm}(k,\mathbf{r})+b_{lm}(k,t)\mathbf{M}_{lm}(k,\mathbf{r})\right]$$

where

$$\mathbf{M}_{lm}(k,\mathbf{r})=i\frac{j_l(kr)}{\sqrt{l(l+1)}}\nabla\times\left[Y_{lm}(\theta,\varphi)\mathbf{r}\right]$$

$$\mathbf{N}_{lm}(k,\mathbf{r})=\frac{i}{k}\nabla\times\mathbf{M}_{lm}$$

Hertzberg, Schiappacasse 1805.00430

Inhomogeneous (Spherical) Axion Clump

Dominant instability $l=1, m=0, \quad b_{10}=-i\, a_{10}$

$$\ddot{a}_{10}(k,t)+k^2 a_{10}(k,t)+g_{a\gamma}k\int\frac{dk'}{(2\pi)}\partial_t\tilde{\phi}(k-k')a_{10}(k',t)\,=0$$

Hertzberg, Schiappacasse 1805.00430

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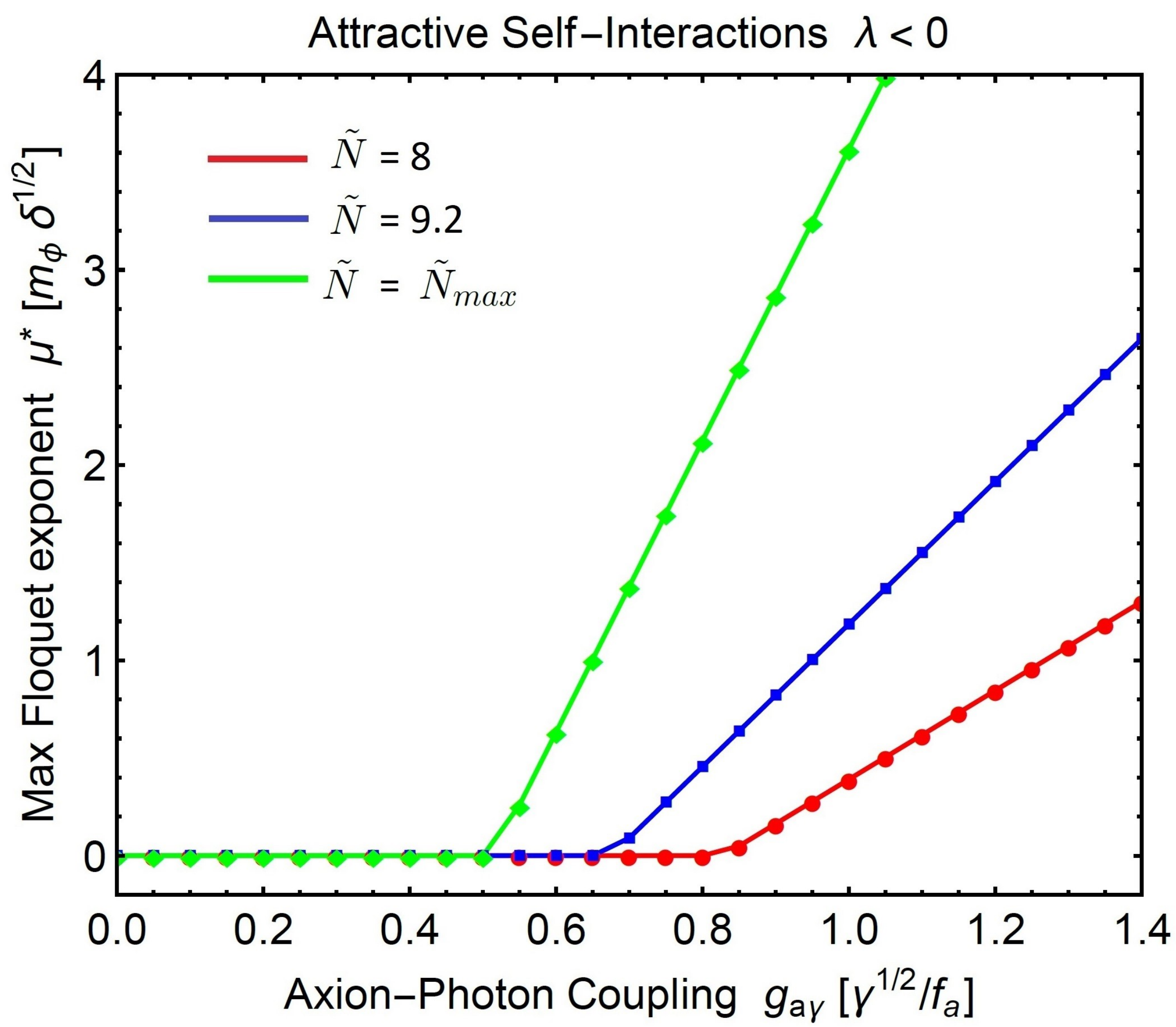

Hertzberg, Schiappacasse 1805.00430

Inhomogeneous (Spherical) Axion Clump

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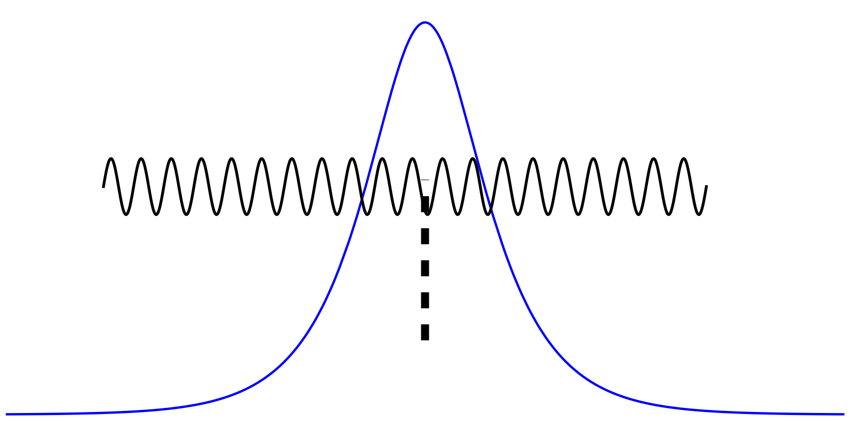
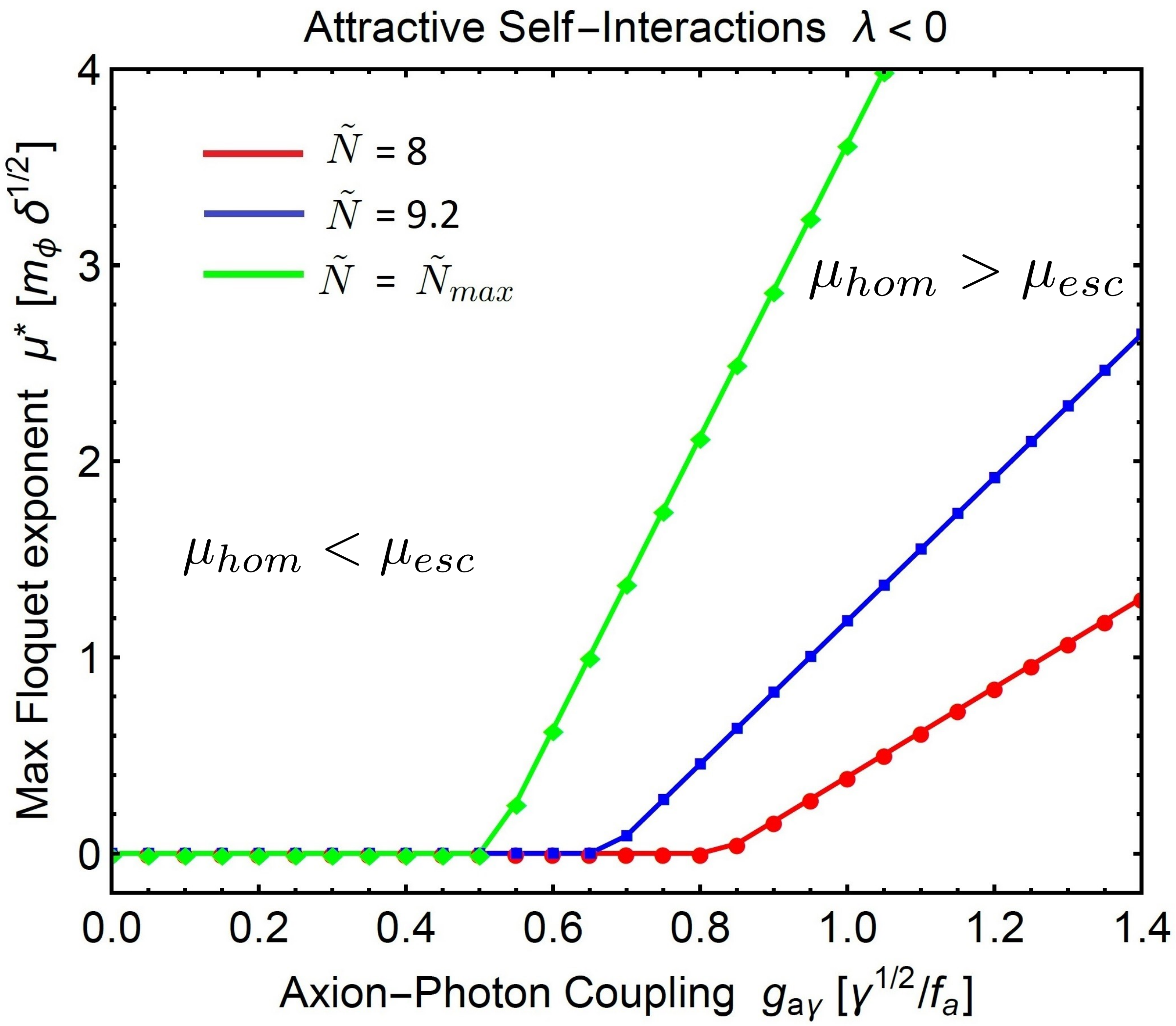
$$\ddot{a}_{10}(k,t) + k^2 a_{10}(k,t) + g_{a\gamma} k \int \frac{dk'}{(2\pi)} \partial_t \tilde{\phi}(k-k') a_{10}(k',t) = 0$$

$\mu_{hom} > \mu_{esc}$





Hertzberg, Schiappacasse 1805.00430

Resonance Condition (Spherical) Axion Clump

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a}} \qquad (\lambda < 0)$$

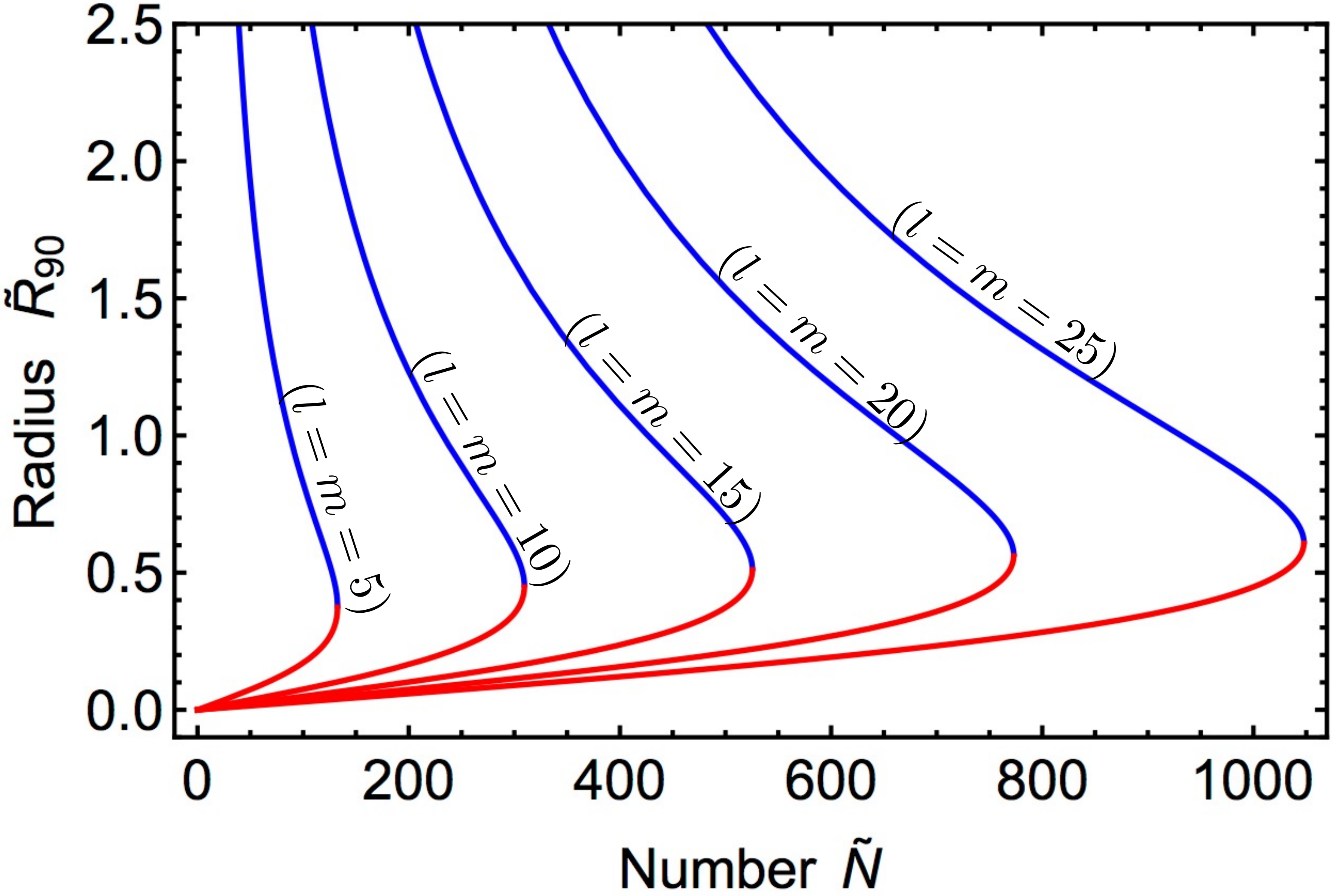
No resonance for standard QCD axion-photon coupling $\qquad g_{a\gamma} \sim \dfrac{\alpha}{f_a}$

Allowed for models with enhanced couplings, decay to hidden sectors, or repulsive interactions

Hertzberg, Schiappacasse 1805.00430

Including Angular Momentum

Two Branches of Solutions (with Angular Momentum)



High angular momentum allows higher amplitude at core, which helps for resonance into photons

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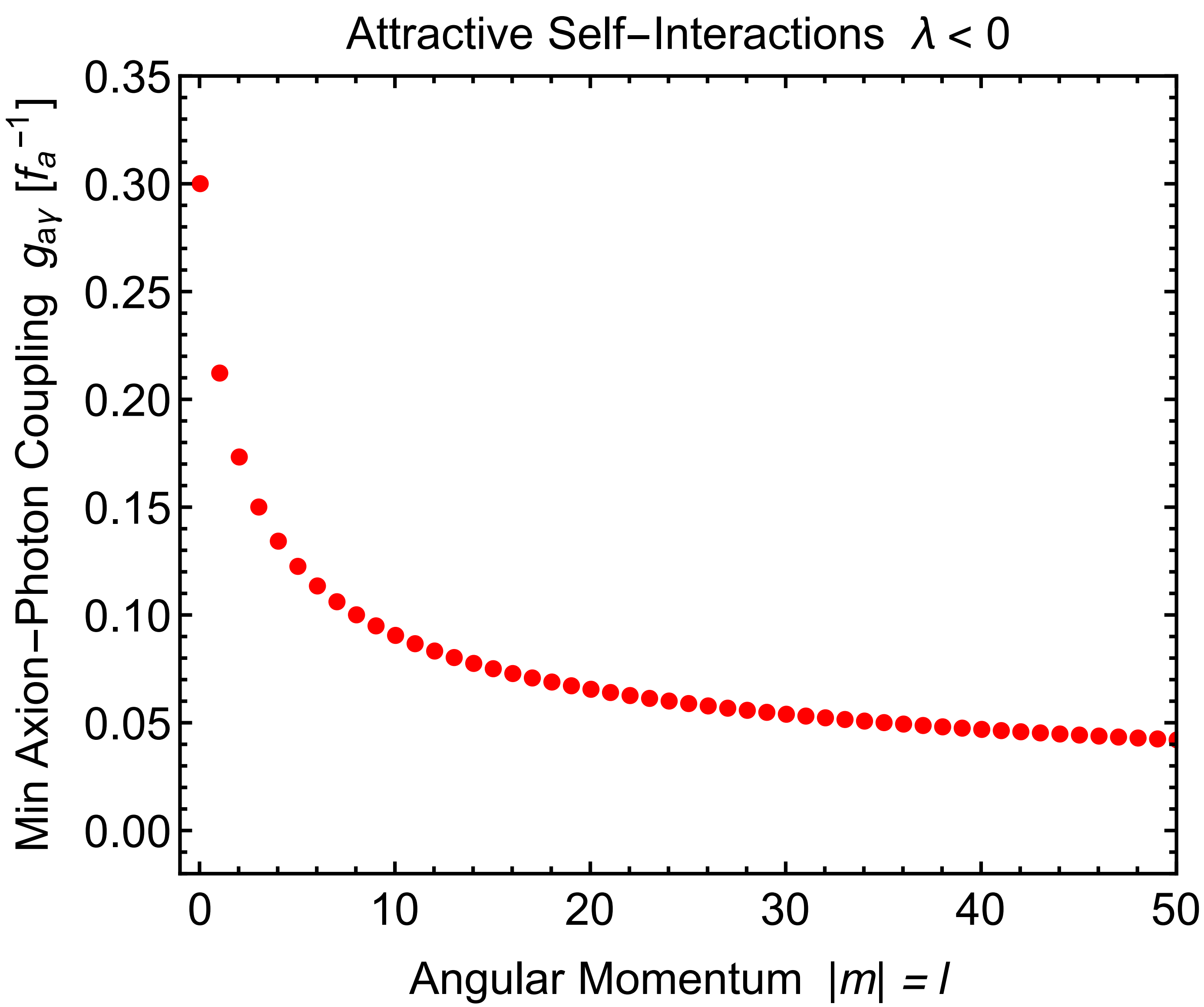
Resonance Condition (Non-Spherical) Axion Clump

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a\sqrt{l+1}}} \qquad (\lambda < 0)$$



Resonance allowed for standard QCD axion-photon couplings, with high angular momentum

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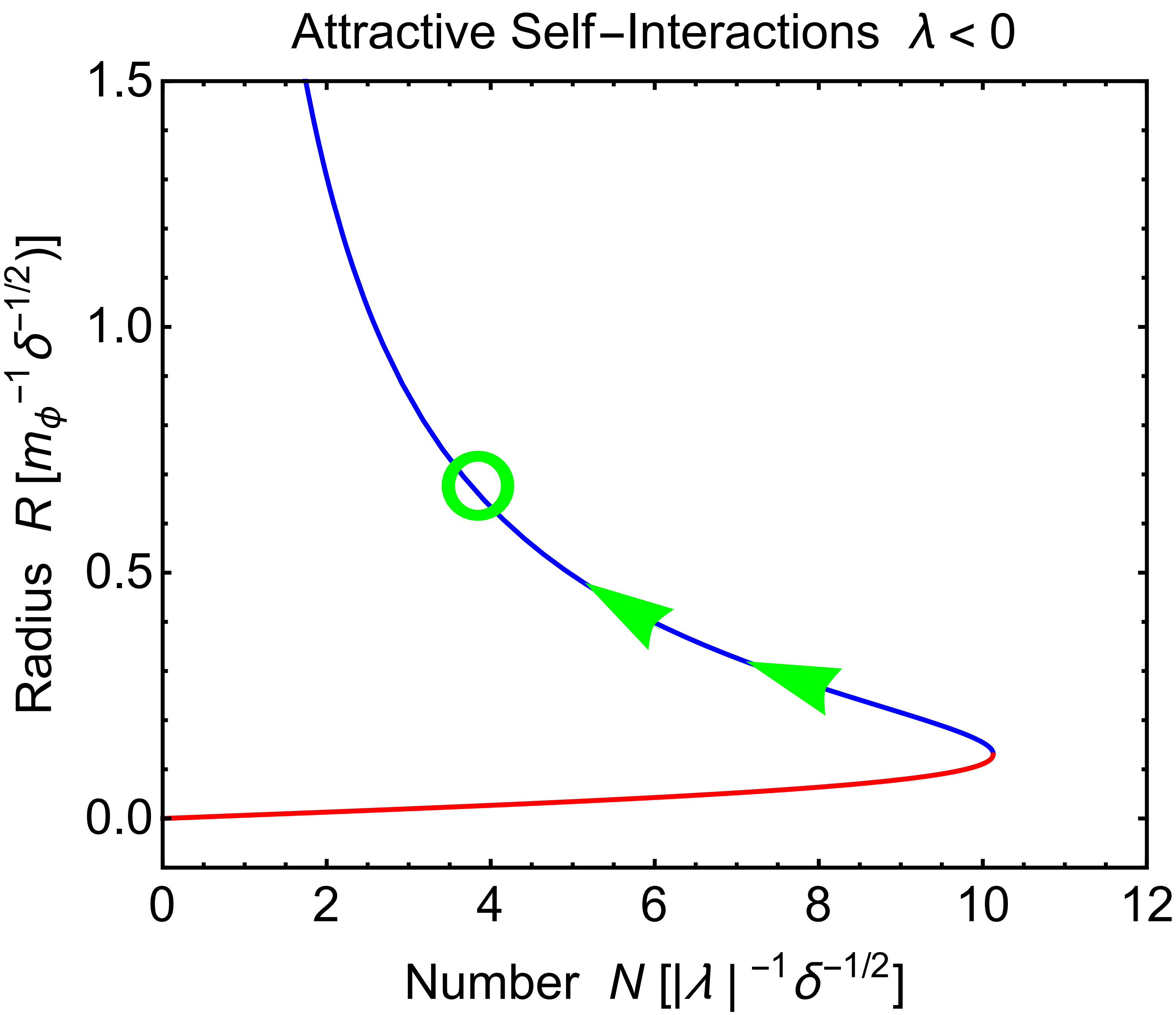
Astrophysical Consequences



(i) Mass Pile Up

(ii) Late Time Mergers; Radiowave Bursts

$$\lambda_{EM} = \frac{2\pi}{k} \approx \frac{4\pi}{m_a}$$

$$= \mathcal{O}(1)\, meters$$

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Thank you