

# A UNIVERSAL BOUND ON THE STRONG COUPLING SCALE OF A GRAVITATIONALLY COUPLED MASSIVE SPIN-2 PARTICLE

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PASCOS, June 7th, 2018

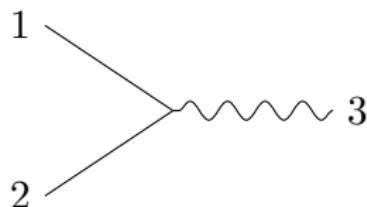
# UNIVERSAL GRAVITATIONAL INTERACTIONS

## S-MATRIX EQUIVALENCE PRINCIPLE

- ▶ Every particle that interacts with Einstein gravity must have a minimal coupling vertex with a universal coupling constant

Weinberg:

$$\mathcal{V}_{\text{cubic}} = \frac{2i}{M_p} (\epsilon_3 \cdot p_1)^2 (\epsilon_1 \cdot \epsilon_2)^s.$$

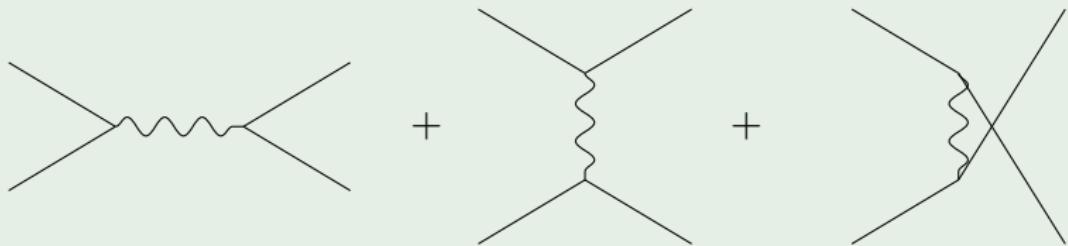


# LOW SPIN PARTICLES

## SIMPLE SCALAR EXAMPLE

- ▶ Consider a scalar minimally coupled to gravity,

$$S = \frac{1}{2} \int d^4x \sqrt{-g} (M_p^2 R - \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2).$$



- ▶ For  $E \gg m$ , the four-point amplitude is

$$\mathcal{A} = \frac{1}{4M_p^2} \frac{(s^2 + t^2 + u^2)^2}{stu}.$$

# MASSLESS HIGHER-SPIN PARTICLES

- ▶ For massless particles with  $s > 2$ , the  $S$ -matrix equivalence principle is incompatible with gauge invariance:

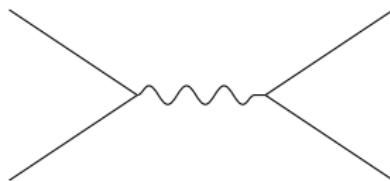
$$p_1 \cdot \frac{\partial}{\partial \epsilon_1} \left( \frac{2}{M_p} (\epsilon_3 \cdot p_1)^2 (\epsilon_1 \cdot \epsilon_2)^s + \dots \right) \neq 0.$$

## NO-GO RESULT

Massless higher-spin particles cannot couple directly or indirectly to Einstein gravity in flat spacetime.

## MASSIVE HIGHER-SPIN PARTICLES

- ▶ Massive higher-spin particles can couple to gravity, e.g. QCD and string theory.
- ▶ However, isolated massive particles with  $m \ll M_p$  and  $s > 1$  cannot remain fundamental point-like particles up to  $M_p$ .


$$\sim \frac{E^{2k}}{M_p^2 m^{2k-2}}$$

- ▶ The local effective field theory violates perturbative unitarity at some scale  $\Lambda_k \equiv (m^{k-1} M_p)^{1/k} \ll M_p$ .
- ▶ Contact terms can partially cancel the bad high-energy behaviour and raise the strong coupling scale.

# BOUNDING THE STRONG COUPLING SCALE

- ▶ How large can  $\Lambda_k$  be for a given spectrum of particles?

## STRATEGY:

1. Classify all cubic and quartic vertices
2. Construct the general four-point amplitude consistent with locality, unitarity, Lorentz invariance, gauge invariance, and crossing symmetry.
3. Find the amplitude with the highest strong coupling scale.

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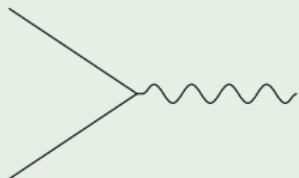
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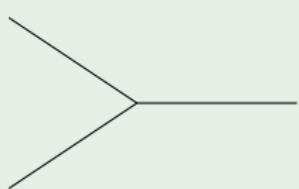
- ▶ Here we look at massive spin 2 coupled to gravity.

## EXAMPLE

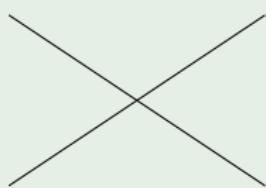
### CLASSIFYING SPIN-2 VERTICES



5 structures,



4 structures,



95 structures.

# RESULTS

- ▶ The highest strong coupling scale for a unitary gravitationally coupled massive spin-2 particle is  $\Lambda_3 = (m^2 M_p)^{1/3}$ .

## IMPLICATIONS

- ▶ Lower bound on the mass of an isolated light massive spin-2 particle: if  $\Lambda_3 \gtrsim 10^{-3}$  eV then  $m \gtrsim 10^{-18}$  eV.
- ▶ Large- $N$  QCD: if the lightest glueball has spin 2, then the next lightest state cannot be heavier than  $\Lambda_3$ .

## EXAMPLES

### $\Lambda_3$ THEORIES: SATURATING THE BOUND

- ▶ Ghost-free bigravity [Hassan & Rosen](#), a generalization of ghost-free massive gravity [de Rham, Gabadadze, & Tolley](#).
- ▶ A gravitationally coupled pseudotensor—no known theory. Uses two-derivative parity-odd vertex and exists only in  $4d$ .
- ▶ Pseudo-linear theory with massive and massless spin-2 particles (no gravity) [JB, Hinterbichler, & Johnson](#).

## EXAMPLES

### $M_p$ THEORY

- ▶ If we allow imaginary cubic couplings, i.e. ghosts, then the maximum strong coupling scale is  $M_p$ .
- ▶ The unique amplitude is generated by quadratic curvature gravity,

$$S = M_p^2 \int d^4x \sqrt{-g} \left( \frac{1}{2}R + \frac{1}{4m^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right).$$

- ▶ This amplitude is also contained in the bigravity amplitude [Paulos & Tolley](#).

## HIGHER-SPIN GENERALISATION

- ▶ **Porrati & Rahman:** for a charged massive spin- $s$  particle the strong coupling scale satisfies

$$\Lambda \leq \frac{m}{q^{1/(2s-1)}}.$$

- ▶ Assume the lightest state satisfies the weak gravity conjecture:

$$q \geq \frac{m}{M_p} \implies \Lambda \leq (m^{2s-2} M_p)^{1/(2s-1)}.$$

### CONJECTURE

The maximum strong coupling scale for a gravitationally coupled massive spin- $s$  particle is  $\Lambda_{2s-1}$  **Rahman**.

- ▶ This is true for  $s \in \{1, \frac{3}{2}, 2\}$ .

## CONCLUSIONS

- ▶ Effective field theories of gravitationally coupled massive higher-spin particles can exist, but they have low strong coupling scales.
- ▶ For spin- $s$ , the maximum strong coupling scale is conjectured to be  $\Lambda_{2s-1}$ .
- ▶ We have proven this for  $s = 2$  by studying the four-point scattering of massive particles with graviton exchange.

## FUTURE DIRECTIONS

### OPEN QUESTION

- ▶ Can we raise the cutoff to  $M_p$  by adding a finite number of additional particles?
- ▶ This works for  $s = \frac{3}{2}$ , e.g. addition of scalar and pseudoscalar gives broken  $\mathcal{N} = 1$  supergravity.
- ▶ For  $s = 2$ , we can get to  $\Lambda_{3/2} = (mM_p^2)^{1/3}$  with a Kaluza-Klein reduction of 5d gravity.