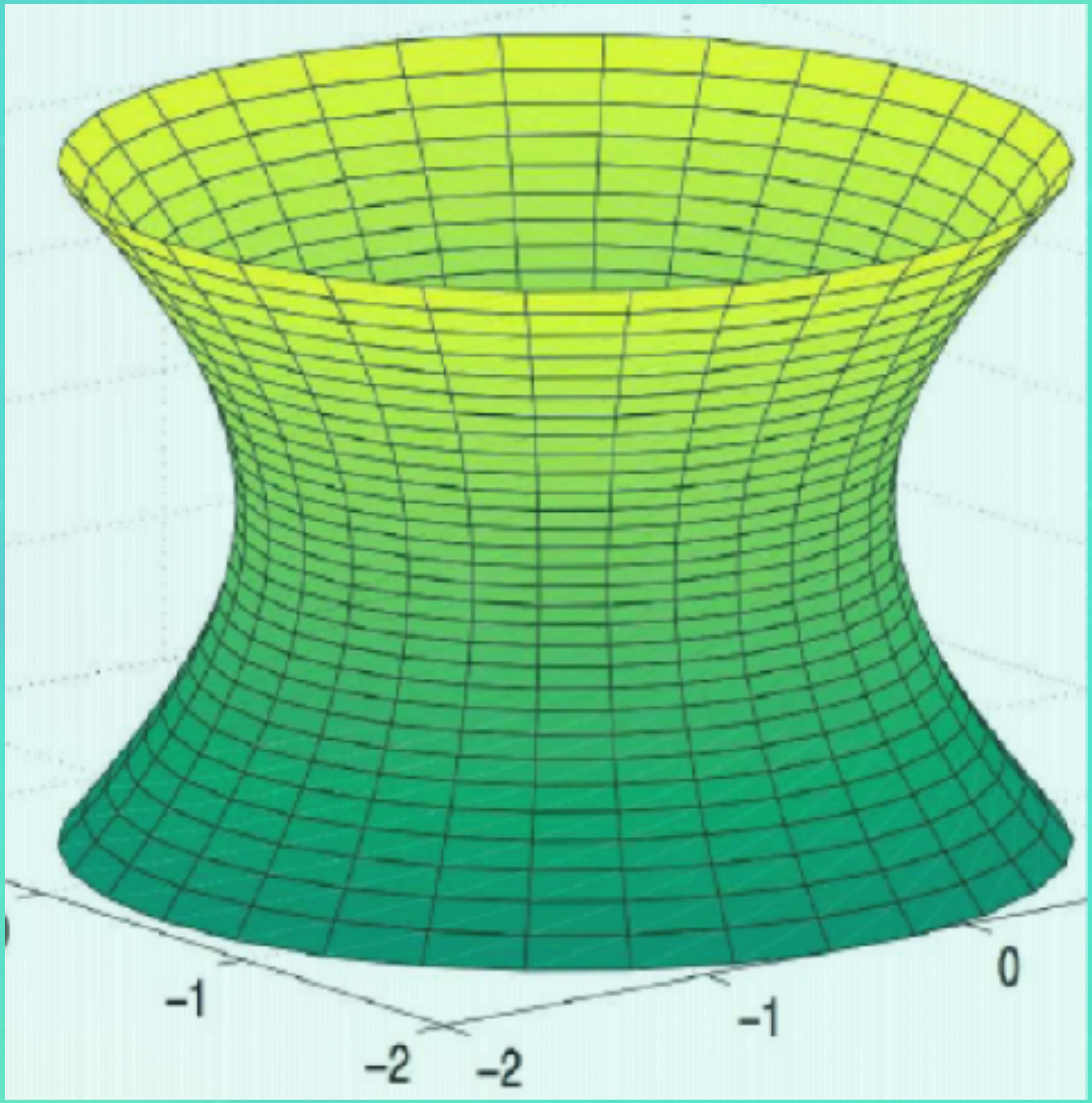
Partially Massless Gravity in de Sitter

Laura Johnson[1], Claudia deRham[2,1], Kurt Hinterbichler[1]

[1]CERCA, Department of Physics, Case Western Reserve University

[2]Theoretical Physics, Blackett Laboratory, Imperial College

Motivation



- De Sitter spacetime approximates our early universe during inflation and the phase our universe is currently entering into

- observed cosmological constant is $\dfrac{\Lambda}{M_P^2} \sim 10^{-122}$
- Partially massless symmetry ties value of cosmological constant to graviton mass

$$m^2 = 2H^2$$

Linearized Massive Gravity

$$S = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \nabla_\mu h_{\nu\lambda}\nabla^\nu h^{\mu\lambda} - \nabla_\mu h^{\mu\nu}\nabla_\nu h + \frac{1}{2}\nabla_\lambda h \nabla^\lambda h$$

GR terms $\longrightarrow$ $$+\frac{1}{4}R(h_{\mu\nu}h^{\mu\nu} - h^2)$$

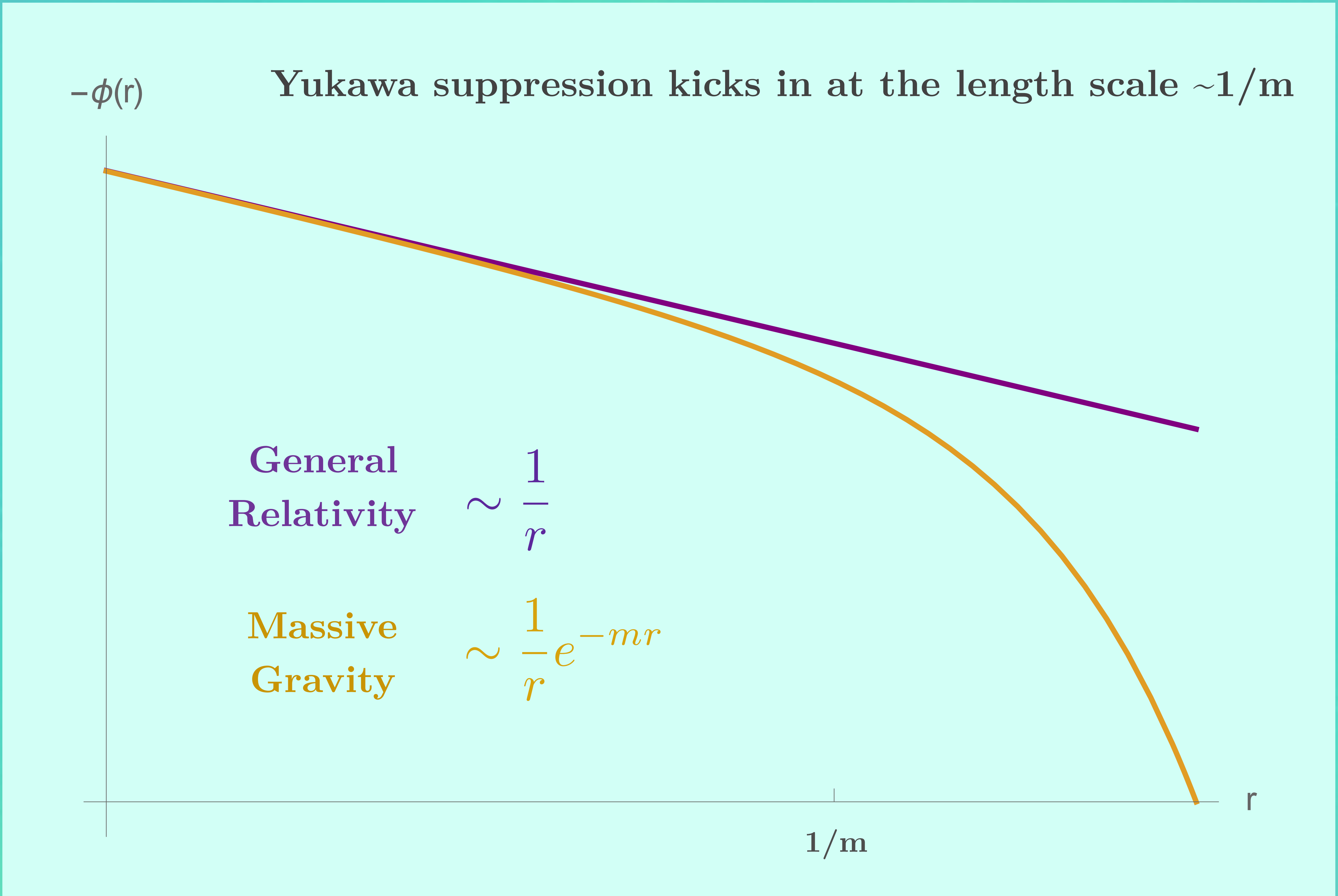
$$-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\Big]$$ $\longleftarrow$ **mass terms**

- The linearized GR piece is invariant under

$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- This is broken by the mass term

Potential in the static, weak-field limit



Helicity Analysis

Introduce helicity fields

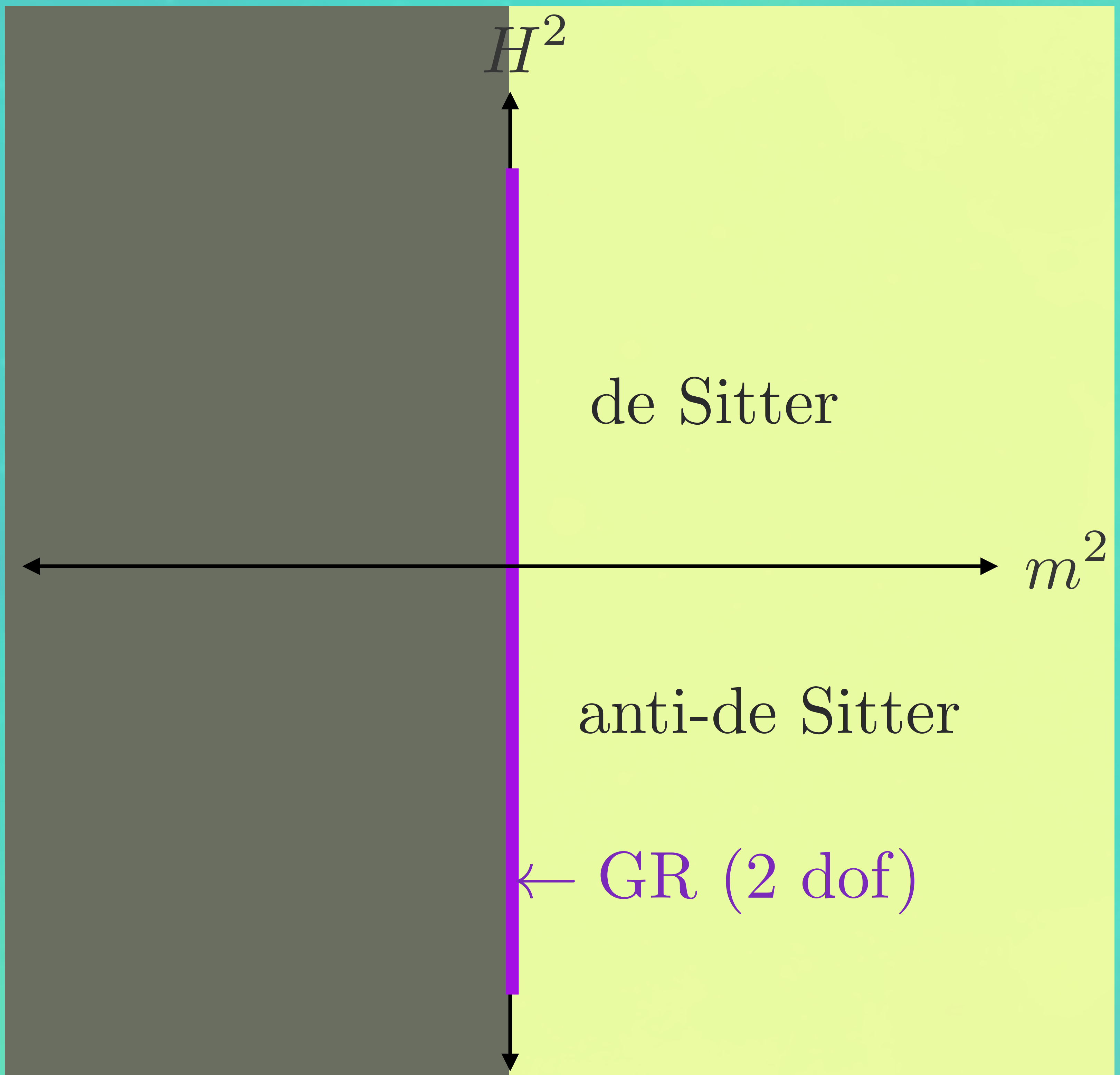
$$h_{\mu\nu} \to h_{\mu\nu} + \nabla_\mu A_\nu + \nabla_\nu A_\mu + 2\nabla_\mu \nabla_\nu \phi$$

and take the relativistic limit

$$\left\{ \begin{array}{ccc} h_{\mu\nu} & \text{helicity } \pm 2 & 2 \text{ dof} \\ A_\mu & \text{helicity } \pm 1 & 2 \text{ dof} \\ \phi & \text{helicity } \quad 0 & 1 \text{ dof} \end{array} \right\}$$

giving a total of 5 degrees of freedom.

Curvature vs. Mass



General Relativity 2 dof

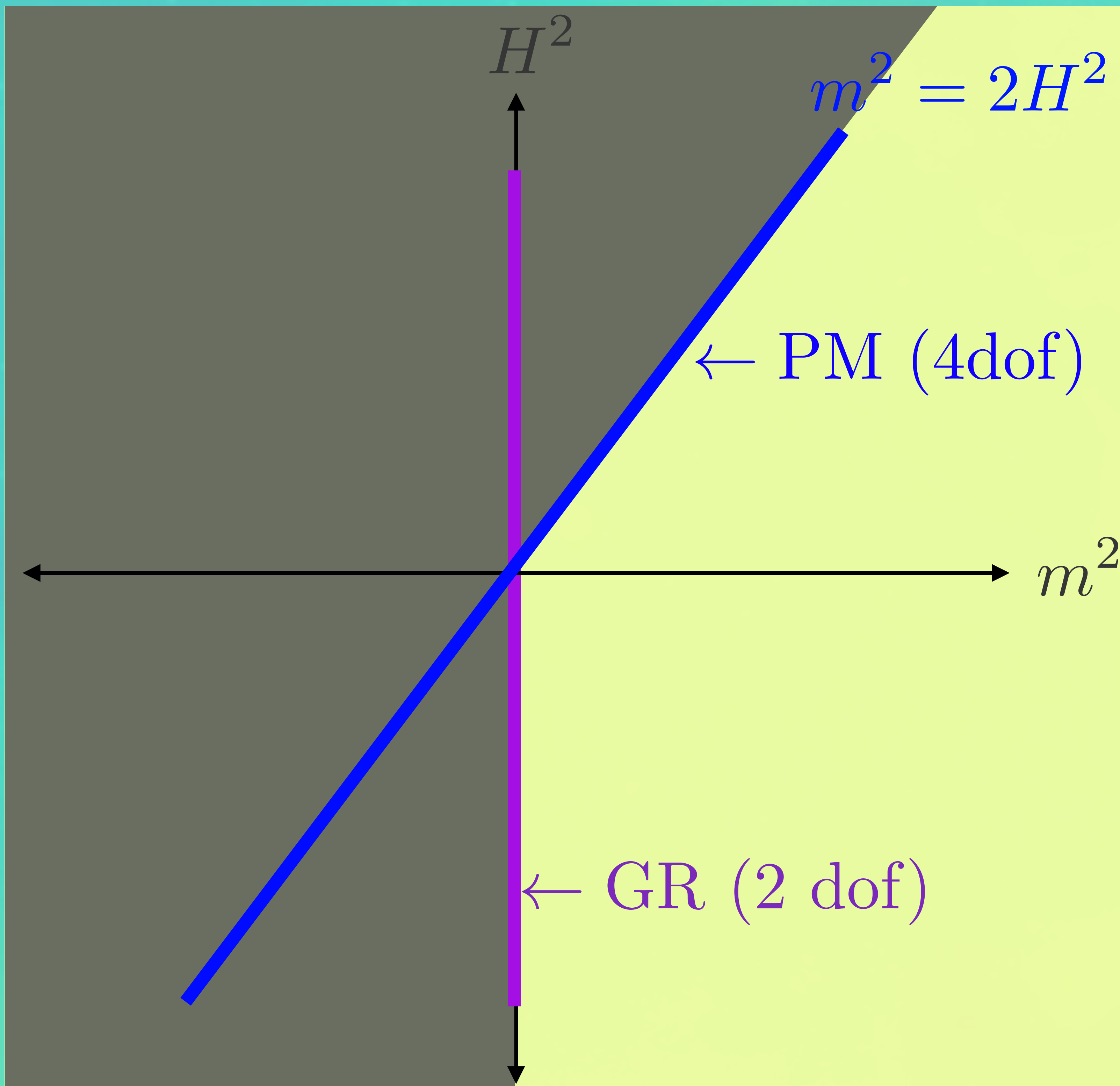
Massive Gravity 5 dof

Linearized Massive Gravity in de Sitter

$$S = \int d^4x \mathcal{L}_{m=0} + \sqrt{-g}\Big[-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$
$$-\frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} + 6m^2H^2A^\mu A_\mu - 2m^2(h_{\mu\nu}\nabla^\mu A^\nu - h\nabla_\mu A^\mu)$$
$$+6m^2(m^2 - 2H^2)(\phi\nabla_\mu A^\mu + \frac{1}{2}h\phi - \frac{1}{2}(\partial\phi)^2 + m^2\phi^2)\Big]$$

Scalar field vanishes when $m^2 = 2H^2$

Curvature vs. Mass in Linear Massive Gravity



General Relativity 2 dof

Massive Gravity 5 dof

Partially Massless Gravity 4 dof

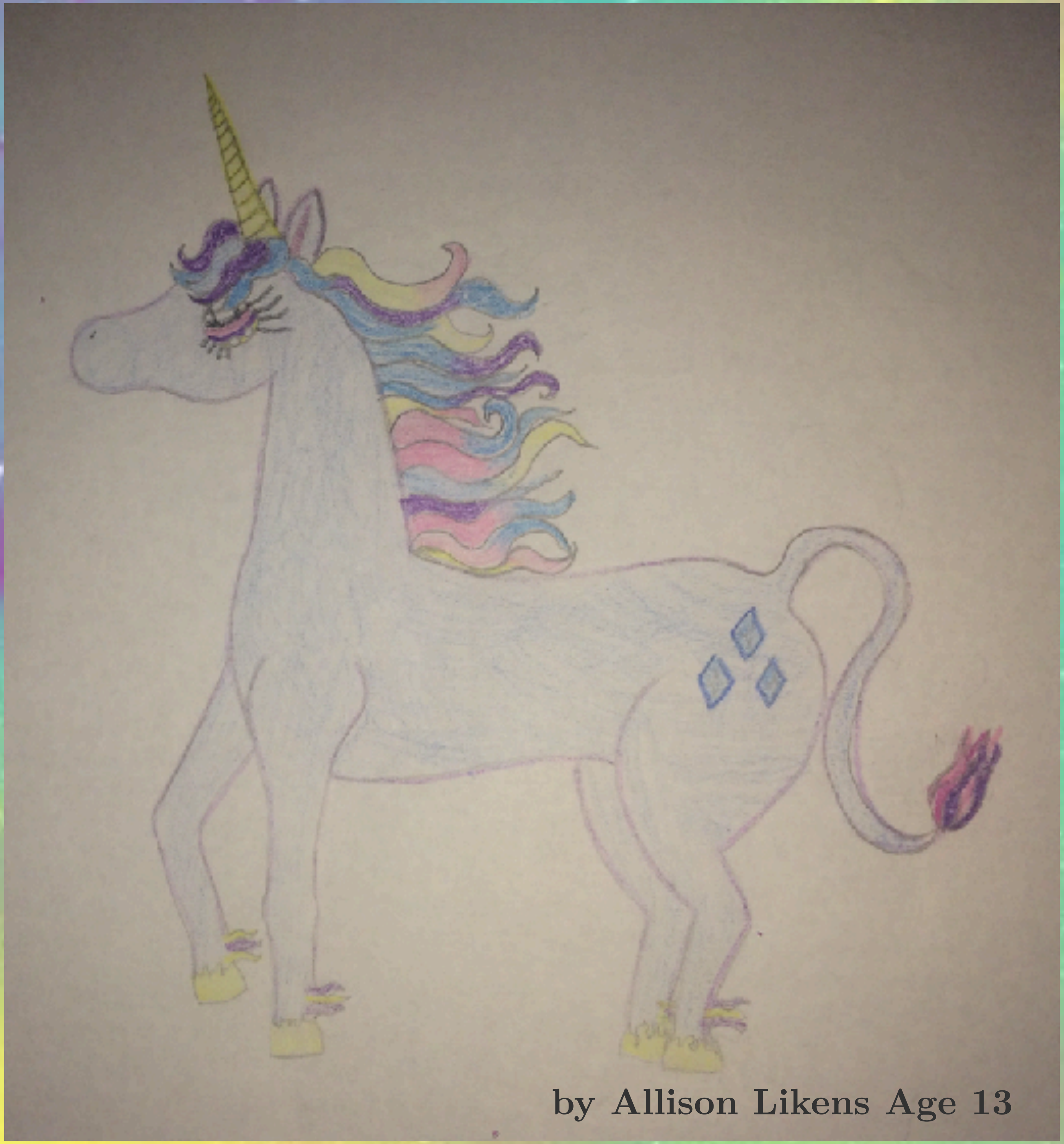
Gauge Symmetry

- When $m^2 = 2H^2$ the scalar field vanishes leaving 4 degrees of freedom.

- There is an additional gauge symmetry.

$$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \lambda + \frac{1}{2} m^2 \lambda g_{\mu\nu}$$

Non-linear Partially Massless Gravity (4 Degrees of Freedom)



- helicity-0 mode is absent giving only 4 degrees of freedom
- removes issues related to superluminalities associated with Galileon-like interactions
- no vDVZ discontinuity and no need for a Vainstein mechanism.

Massive Gravity Non-Linear Interactions

$$S = \frac{1}{2} M_P^2 \int d^4x \sqrt{-g} \Big[R - \frac{1}{4} m^2 V(g,h) \Big]$$

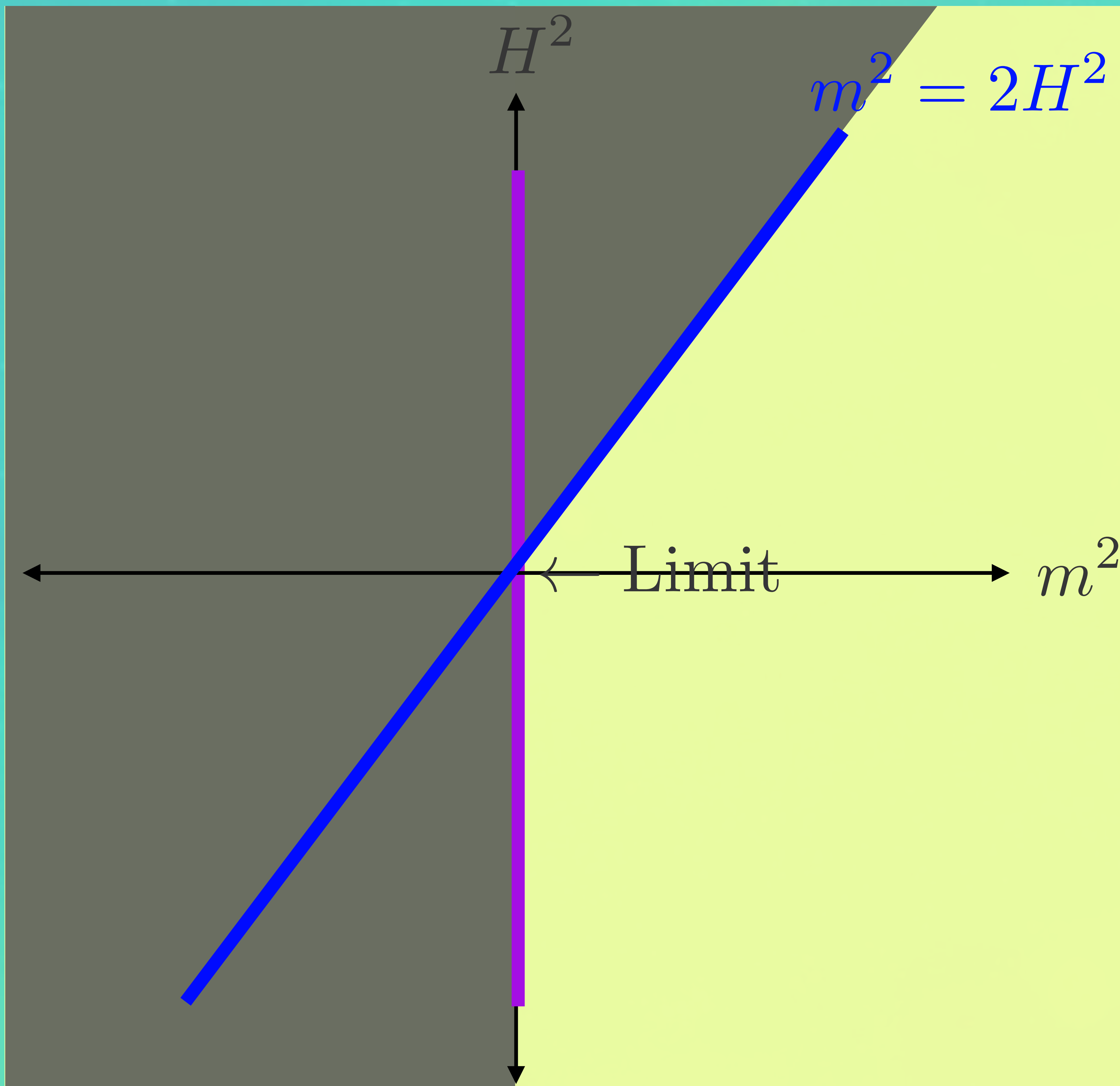
$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots ,$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2 \rangle - \langle h \rangle^2, \\
V_3(g,h) &= +c_1 \langle h^3 \rangle + c_2 \langle h^2 \rangle \langle h \rangle + c_3 \langle h \rangle^3, \\
V_4(g,h) &= +d_1 \langle h^4 \rangle + d_2 \langle h^3 \rangle \langle h \rangle + d_3 \langle h^2 \rangle^2 + d_4 \langle h^2 \rangle \langle h \rangle^2 + d_5 \langle h \rangle^4, \\
V_5(g,h) &= +f_1 \langle h^5 \rangle + f_2 \langle h^4 \rangle \langle h \rangle + f_3 \langle h^3 \rangle \langle h \rangle^2 + f_4 \langle h^3 \rangle \langle h^2 \rangle + f_5 \langle h^2 \rangle^2 \langle h \rangle \\
&\quad + f_6 \langle h^2 \rangle \langle h \rangle^3 + f_7 \langle h \rangle^5, \\
&\ \vdots
\end{aligned}$$

Generic massive gravity has a cutoff scale of Λ_5

$$\sim \frac{(\partial^2 \phi)^3}{\Lambda_5^5}, \quad \Lambda_5 = (M_p m^4)^{1/5}$$

Massless Limit in Non-Linear Theory



- Take limit as $m \to 0$

Tuning Coefficients to Raise Cutoff (dRGT)

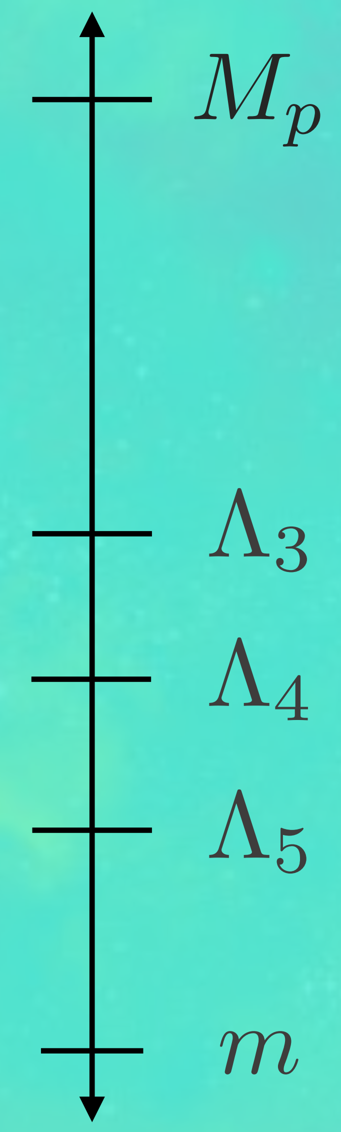
- Can tune the parameters to remove interactions coming in at:

$$\sim \frac{(\partial^2\phi)^3}{\Lambda_5^5}, \quad \Lambda_5 = (M_p m^4)^{1/5}$$

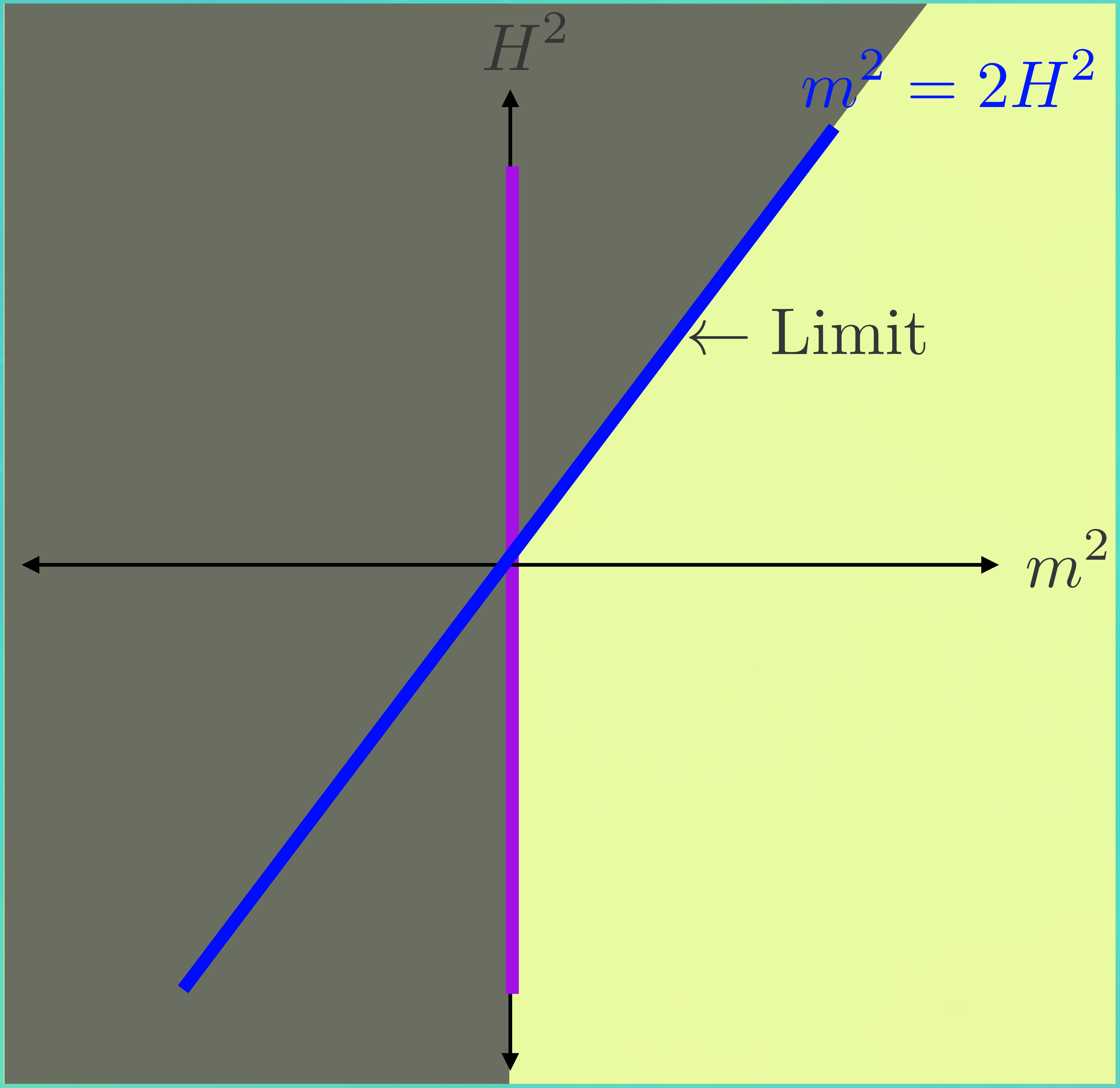
$$\sim \frac{h(\partial^2\phi)^4}{\Lambda_4^8}, \frac{\partial A(\partial^2\phi)^2}{\Lambda_4^4}, \quad \Lambda_4 = (M_p m^3)^{1/4}$$

- This raises the cutoff scale to Λ_3

$$\sim \frac{h(\partial^2\phi)^n}{\Lambda_3^{3(n-1)}}, \frac{\partial A(\partial^2\phi)^n}{\Lambda_3^n}, \quad \Lambda_3 = (M_p m^2)^{1/3}$$



Partially Massless Limit in Non-Linear Theory



- Set $m^2 = 2H^2 + \Delta^2$
- Take limit as $\Delta \to 0$

Non-linear Interactions

- Ghost free massive gravity (dRGT) has two free parameters, α_3, α_4

- and a cutoff of $\boldsymbol{\Lambda_4}$

$$\sim \frac{\partial^5 \hat{\phi}^3}{\Lambda_4^4}, \quad \Lambda_4 = (M_P \Delta^3)^{1/4}$$

Tuning Coefficients to Raise Cutoff

- Tuning the parameters removes interactions coming in at:

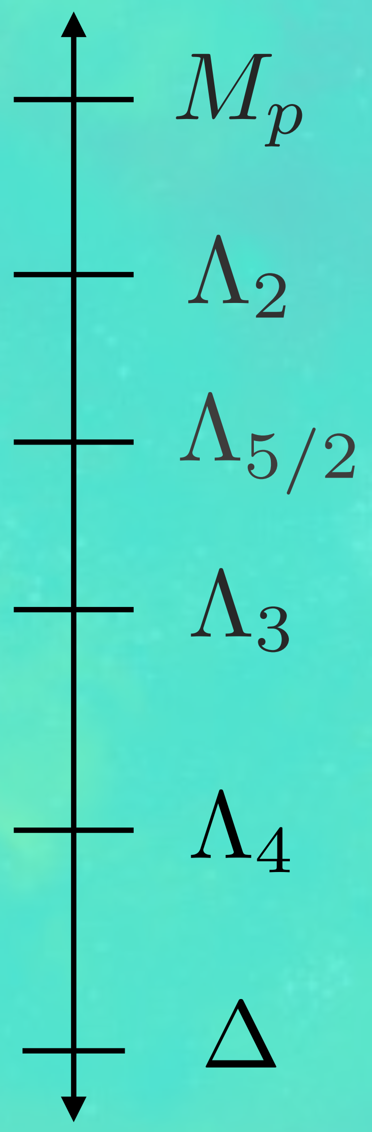
$$\sim \frac{\partial^5 \hat{\phi}^3}{\Lambda_4^4}, \quad \Lambda_4 = (M_P \Delta^3)^{1/4}$$

$$\sim \frac{\partial^4 \hat{h} \hat{\phi}^2}{\Lambda_3^3}, \frac{\partial^6 \hat{\phi}^4}{\Lambda_3^6}, \quad \Lambda_3 = (M_P \Delta^2)^{1/3}$$

$$\sim \frac{\partial^5 \hat{h} \hat{\phi}^3}{\Lambda_{5/2}^5}, \quad \Lambda_{5/2} = (M_P^2 \Delta^3)^{1/5}$$

- This raises the cutoff scale to Λ_2

$$\sim \frac{(\nabla^2 \phi)^n}{\Lambda_2^{n-2}}, \frac{h^2 (\nabla^2 \phi)^{n-2}}{\Lambda_2^{n-2}}, \quad \Lambda_2 = (M_p \Delta)^{1/2}$$



Partially Massless Limit of Massive Gravity

$$\mathcal{L}_{dSGal} = -\frac{3}{16}\Big((\partial\phi)^2 - 4H^2\phi^2)\Big) - \frac{3}{64}\frac{1}{\Lambda_2}\Big((\partial\phi)^2\Box\phi + 6H^2\phi(\partial\phi)^2 - 8H^4\phi^3\Big)$$
$$+ \frac{1}{256}\frac{1}{\Lambda_2^2}\Big[(\partial\phi)^2\Big([\Pi^2] - [\Pi]^2\Big) - 6H^2\phi(\partial\phi)^2\Box\phi - \frac{1}{2}H^2(\partial\phi)^4$$
$$- 18H^4\phi^2(\partial\phi)^2 + 12H^6\phi^4\Big]$$

$$\mathcal{L}_{h^2} = -\frac{1}{4}|detV|\Big[\frac{1}{2}F_{\mu\alpha a}F_{\nu\beta b}(V^{-2})^{\mu\nu}(V^{-2})^{\alpha\beta}\gamma^{\alpha\beta}$$
$$- (2F_{\mu ab}F_{\nu\alpha\beta} - F_{\mu\alpha a}F_{\nu b\beta})(V^{-2})^{\mu\nu}(V^{-1})^{\alpha\beta}(V^{-1})^{ab}\Big]$$

where $$F_{abc} = \nabla_a h_{bc} - \nabla_b h_{ac}$$
$$V_{\mu\nu} = \gamma_{\mu\nu} + \frac{1}{\Lambda_2}\Big(\nabla_\mu\nabla_\nu\phi + H^2\phi\gamma_{\mu\nu}\Big)$$

Summary

- scalar mode in the full non-linear theory does not completely decouple (still have 5 dof)
- strong coupling scale is raised, raising the range of applicability of the theory
- remaining Lagrangian has a cutoff of Λ_2 and enjoys the partially massless symmetry

$$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \lambda + \frac{1}{2} m^2 \lambda g_{\mu\nu}$$

Future Work

- Look for spherical solutions (black holes)
- See how Vainshtein radius is affected (radius inside which general relativity is restored)
- See what sort of implications this theory would have for cosmology