Classical Hawking radiation

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Outline

1. *Classically collapsing background*: thin spherically symmetric domain wall

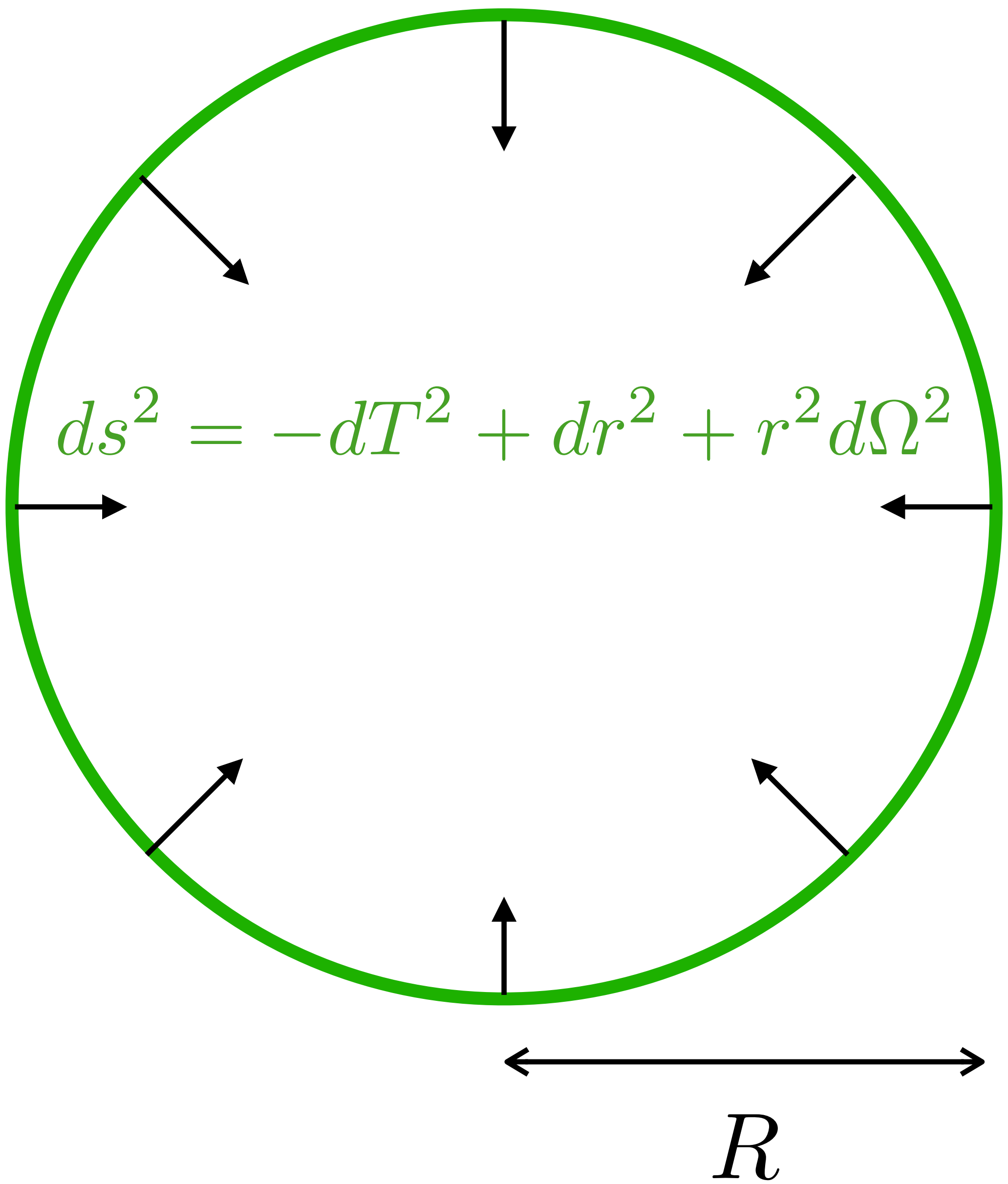
2. *Quantum radiation in this time-dependent background*: massless scalar field mode

3. *Quantum-classical correspondence*: one quantum radiative mode = two classical radiative modes

4. **Coupling of classical radiation to collapsing background: *backreaction***

Free domain wall collapse

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$



First Israel junction condition

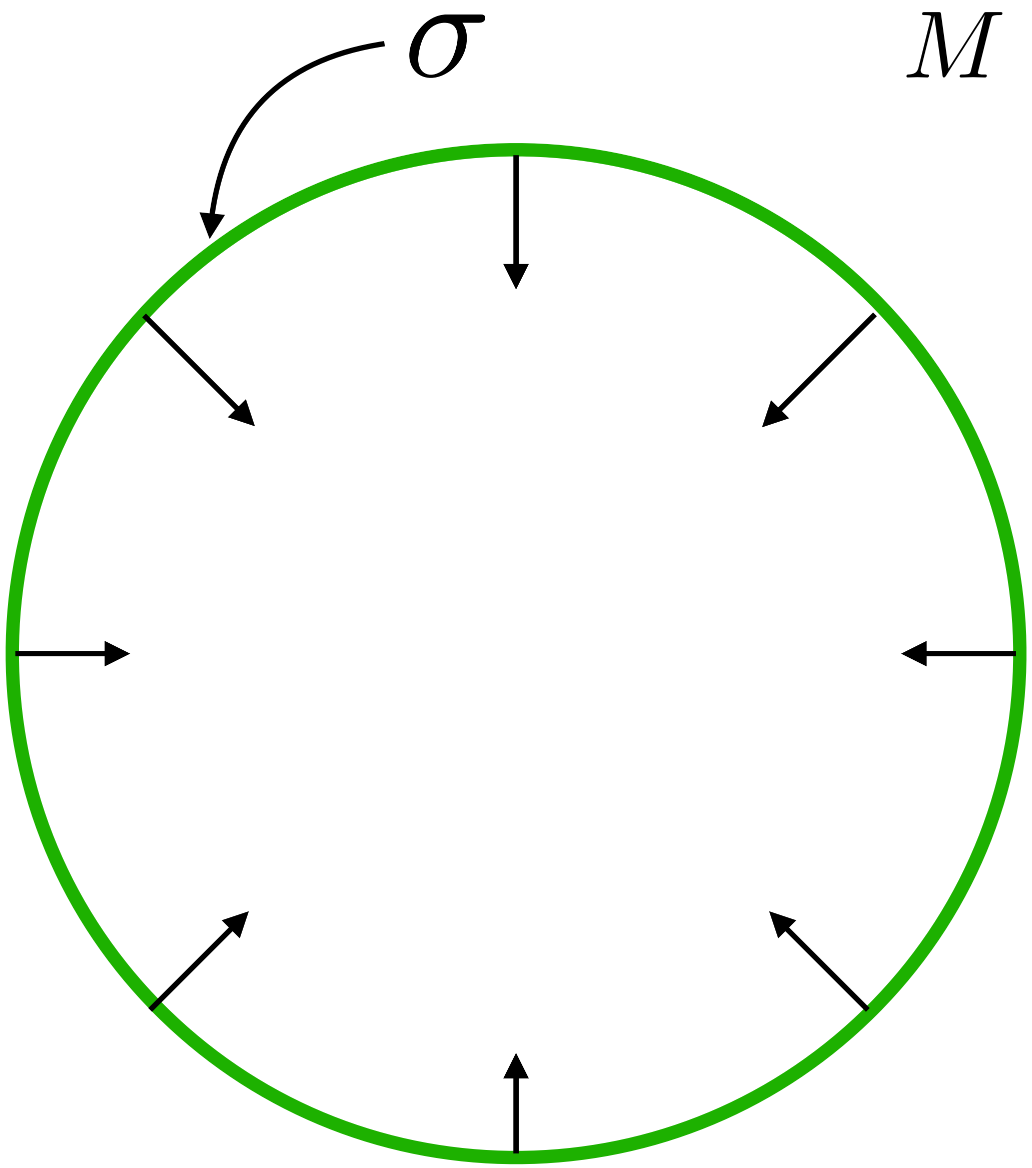
$$\dot{T} \equiv \frac{dT}{dt} = \frac{B}{\sqrt{B + (1 - B)R'^2}}$$

where

$$B \equiv 1 - 2GM/R$$

and *M* is the mass of the shell

Free domain wall collapse



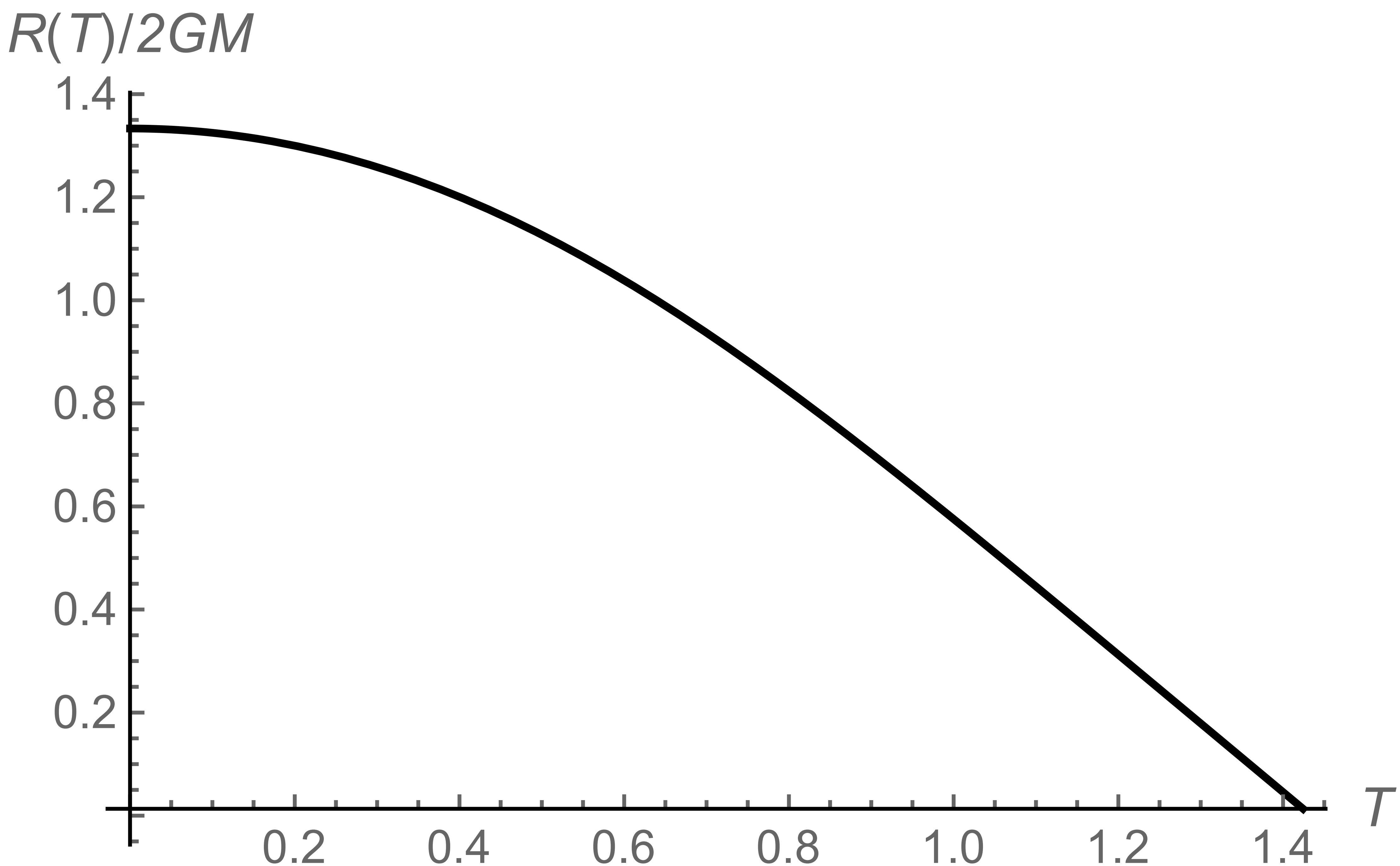
$$M = 4\pi\sigma R^2 \left[\frac{1}{\sqrt{1-R'^2}} - 2\pi G\sigma R\right]$$

Ipser, Sikivie, 1984

Effective action

$$S_{\text{shell}} = -4\pi\sigma \int dT R^2 \left[\sqrt{1-R'^2} - 2\pi G\sigma R\right]$$

Free domain wall collapse



Massless scalar field

(in the collapsing shell background)

Model for quantum radiation in a time-dependent background

Interior

Exterior

$$S_{\rm in} = -2\pi \int dT \int_0^{R(T)} r^2 dr \left(-(\partial_T \phi)^2 + (\partial_r \phi)^2\right)$$

$$r = R(T)$$

$$S_{\rm out} = -2\pi \int dT \int_{R(T)}^{\infty} r^2 dr \left(-\frac{\dot{T}}{1-2GM/r}(\partial_T \phi)^2 + \frac{1}{\dot{T}}(\partial_r \phi)^2\right)$$

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$2GM$

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$2GM$

Near horizon limit: $R(T) \approx 2GM \qquad \dot{T} \approx \sqrt{B} \sim 0$

Quantum radiating modes

$$S_\phi \approx 2\pi \int dT \left(\int_0^{2GM} r^2 dr (\partial_T \phi)^2 - \int_{2GM}^{\infty} r^2 dr \frac{(\partial_r \phi)^2}{\dot{T}} \right)$$

Mode decomposition $\phi(T, r) = \sum_k a_k(T) f_k(r)$

$$S_\phi \approx \sum_{k,l} \int dT \left(\frac{1}{2} a'_k M_{kl} a'_l - \frac{1}{2\dot{T}} a_k N_{kl} a_l \right)$$

with *M, N* time-independent

$$S_\phi \approx \sum_k \int dT \left(\frac{1}{2} {q'_k}^2 - \frac{{\lambda_k}^2}{2\dot{T}} q_k^2 \right)$$

Quantum oscillators with time-dependent frequency

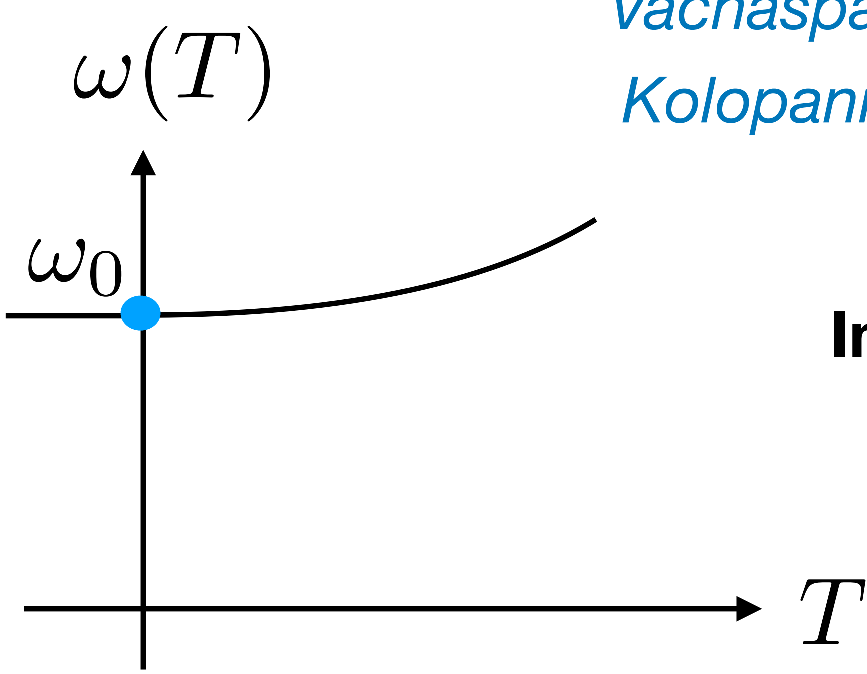
Quantum radiating mode

$$S_\phi \approx \int dT \left(\frac{1}{2} q'^2 - \frac{\omega(T)^2}{2} q^2 \right) \quad \text{with} \quad \omega(T) = \frac{\lambda}{\sqrt{\dot{T}}}$$

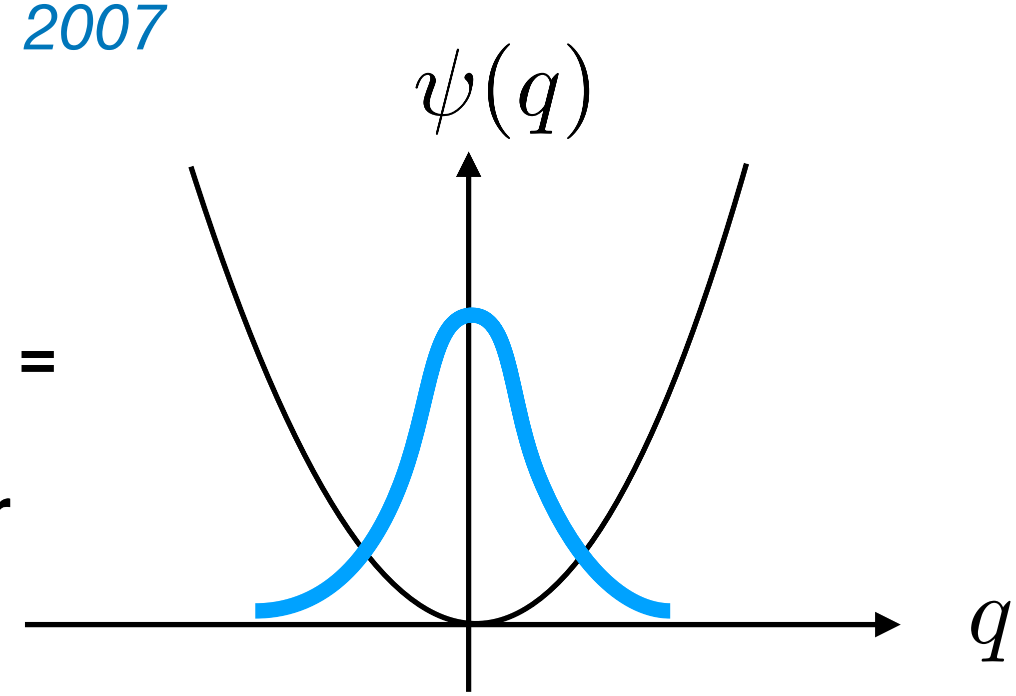
Explicit time-dependence $\Rightarrow$ Excited states get populated

Vachaspati, Stojkovic, Krauss, 2007

Kolopanis, Vachaspati, 2013



Initial quantum state = ground state of harmonic oscillator



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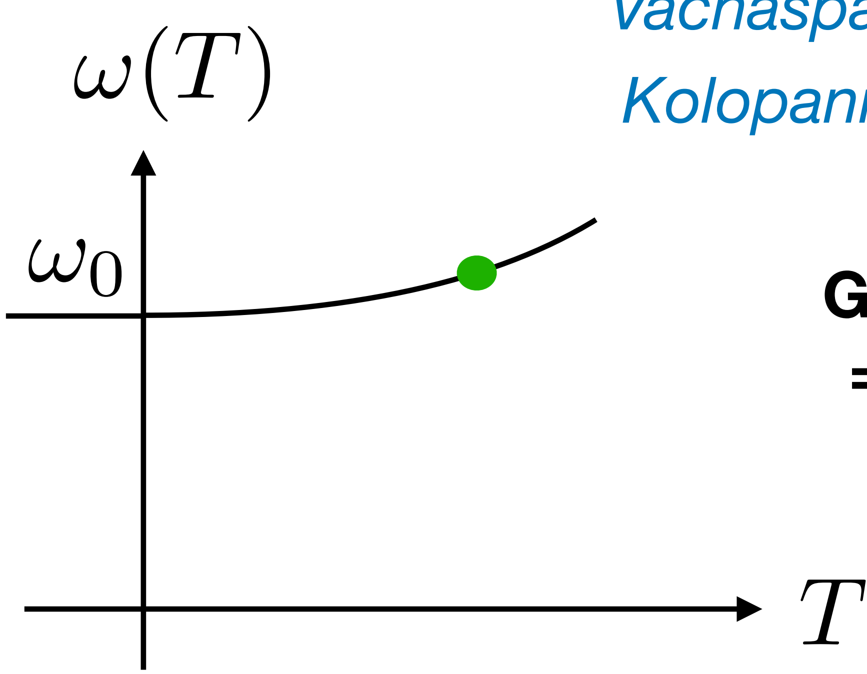
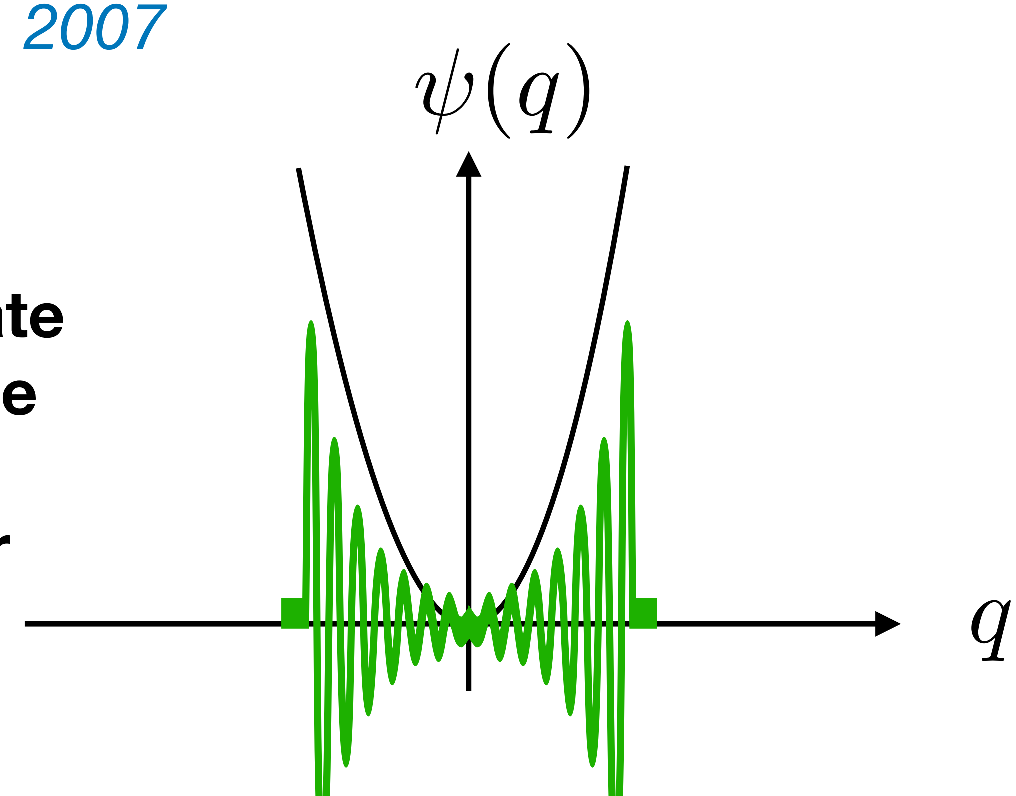


Generic quantum state = excited state of the instantaneous harmonic oscillator



A quantum-classical correspondence

Coupling of quantum mode to classical background?

Backreaction?

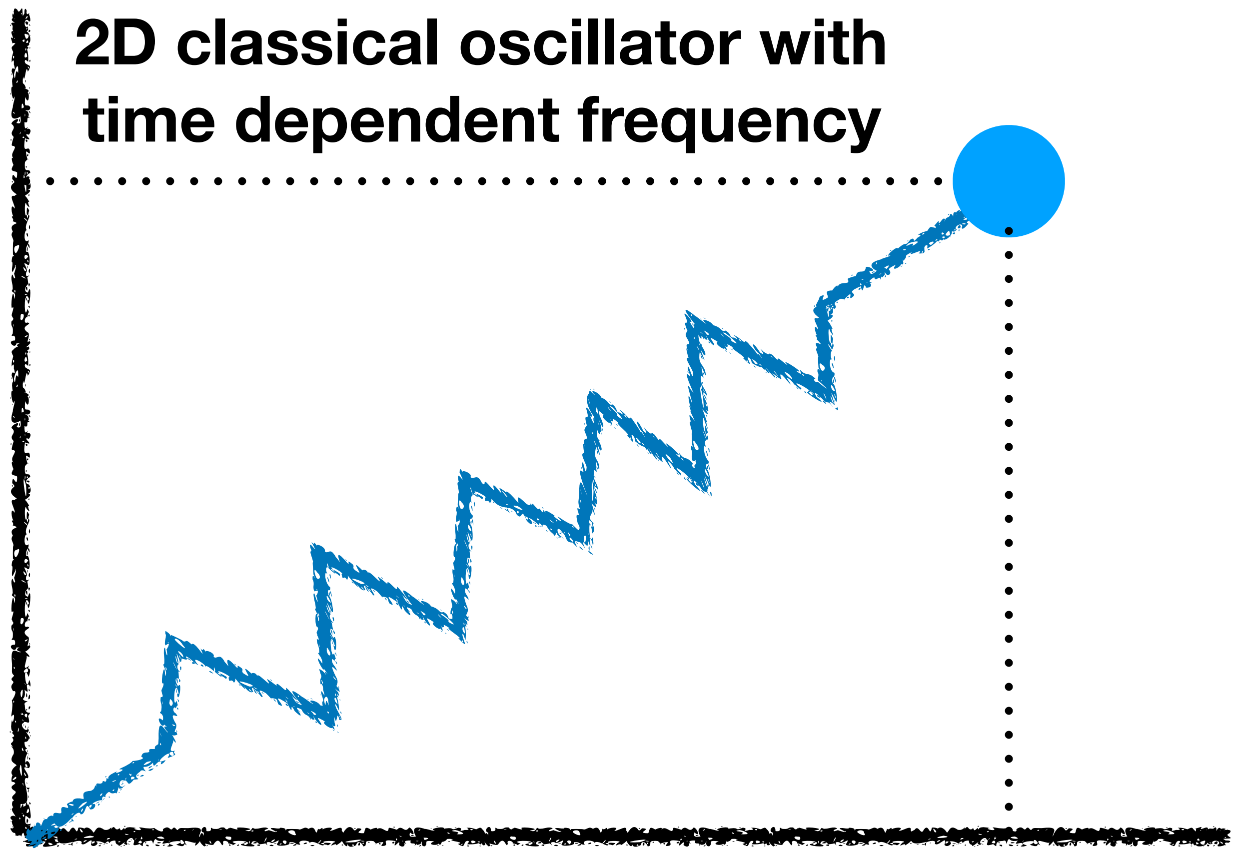
Average energy of the quantum system (starting in its ground state)

$$\Updownarrow$$

Energy of a corresponding classical system (with properly chosen initial conditions)

Stay tuned for Tanmay Vachaspati's talk for more details…

A quantum-classical correspondence



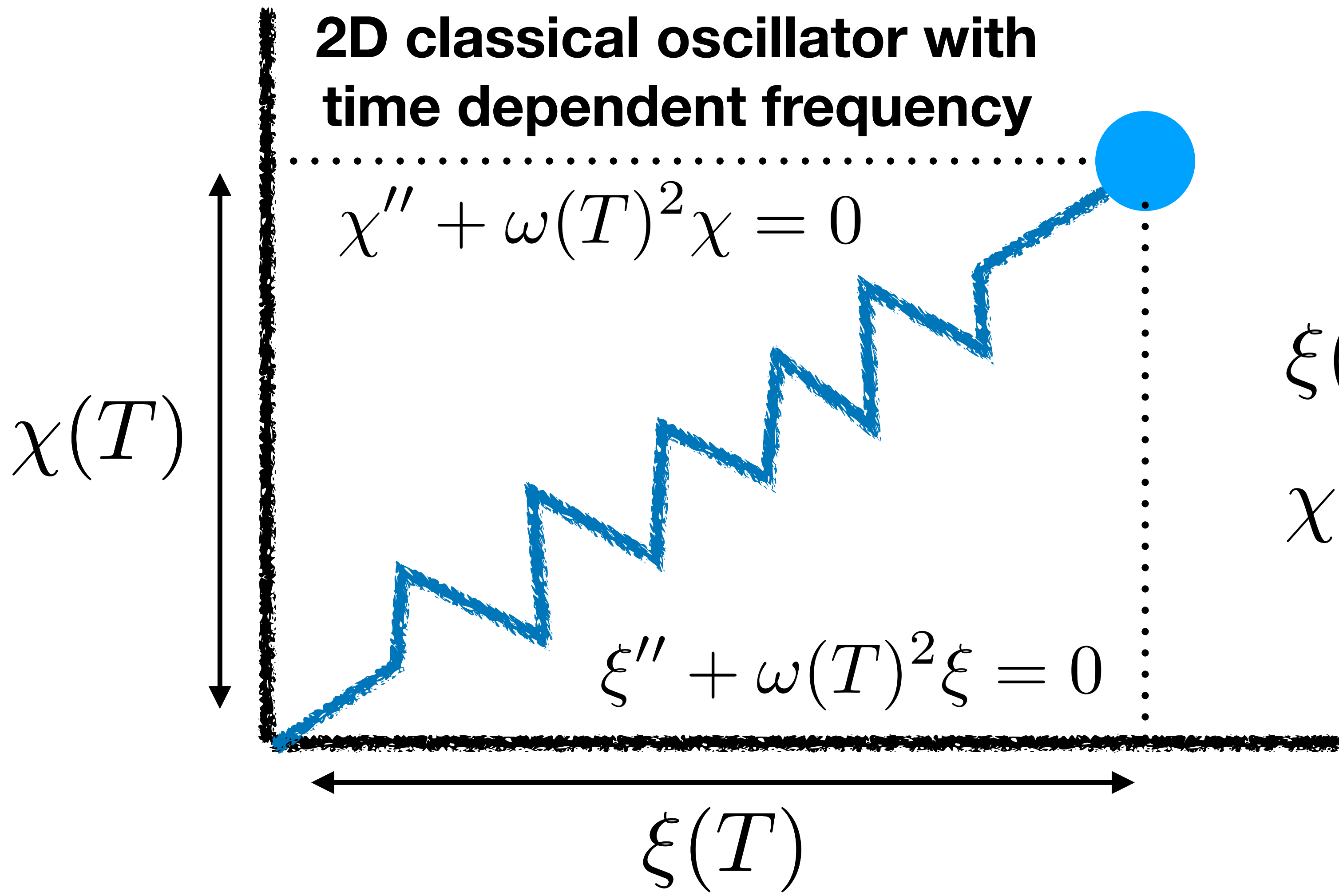
Initial conditions

$$E_{\text{spring}}(T=0) = \omega_0/2$$

$$L_{\text{spring}}(T) = 1/2$$

$$\langle\psi|H(T)|\psi\rangle = \text{ energy of 2D classical spring}$$

A quantum-classical correspondence



Initial conditions

$$\xi(0) = 1/\sqrt{2\omega_0}\ , \quad \xi'(0) = 0$$

$$\chi(0) = 0\ , \quad \chi'(0) = \sqrt{\omega_0/2}$$

$$\langle\psi|H(T)|\psi\rangle = \frac{1}{2}\xi'^2 + \frac{1}{2}\omega(T)^2\xi^2 + \frac{1}{2}\chi'^2 + \frac{1}{2}\omega(T)^2\chi^2$$

Toy model for backreaction

Coupling of corresponding classical system to classical background

$$S_{\rm toy} = S_{\rm shell} + \int dT \left(\frac{1}{2}\xi'^2 - \frac{1}{2}\omega(T)^2\xi^2 + \frac{1}{2}\chi'^2 - \frac{1}{2}\omega(T)^2\chi^2 \right)$$

with $\quad \omega(T) = \dfrac{\lambda}{\sqrt{\dot{T}}}$

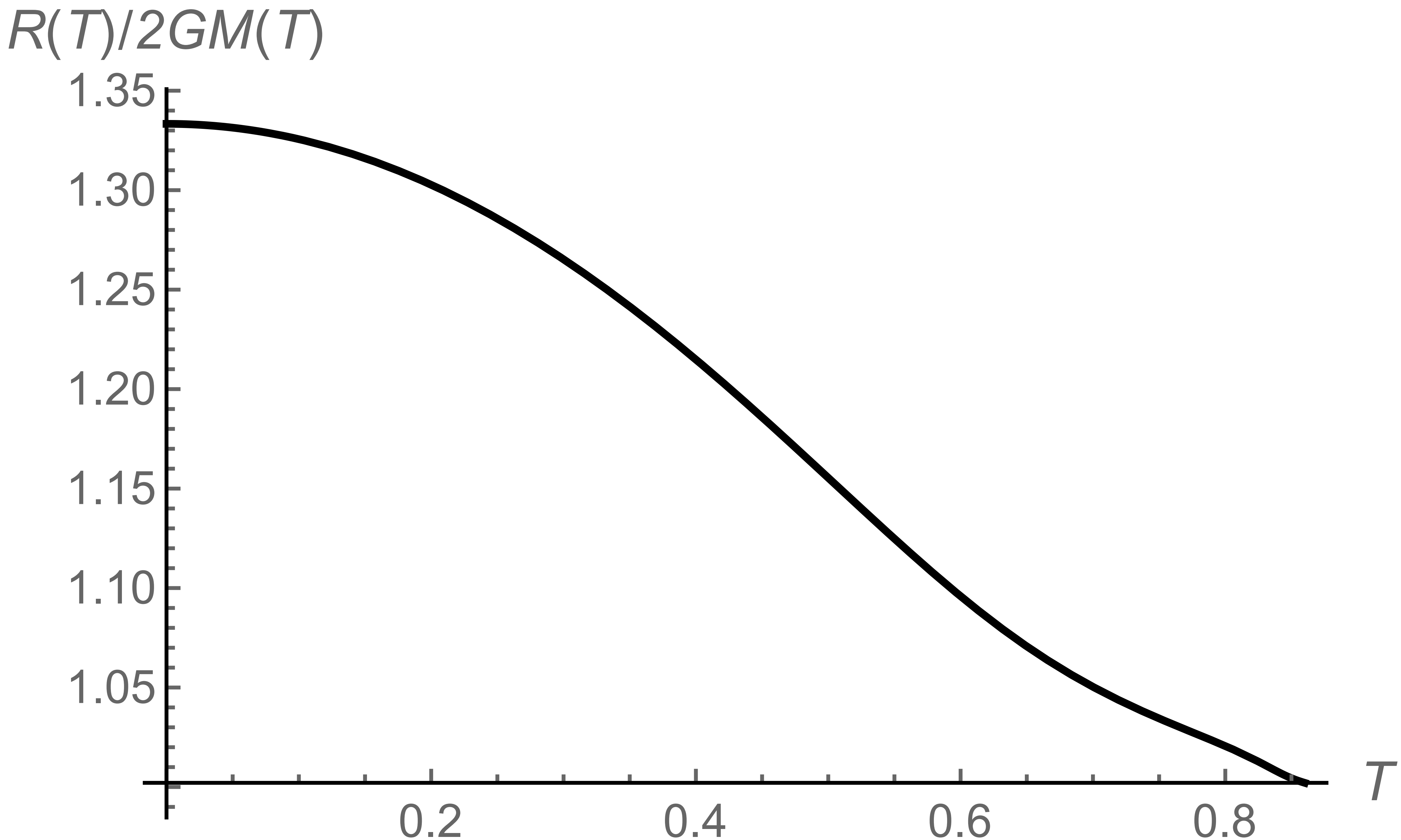
⇨ Equations of motion for $R(T)$, $\xi(T)$ and $\chi(T)$

BUT evaporation leads to time-dependent shell mass…

⇨ Additional equation

$$M'(T) = -\frac{d}{dT}\left[\frac{1}{2}\xi'^2 + \frac{1}{2}\omega(T)^2\xi^2 + \frac{1}{2}\chi'^2 + \frac{1}{2}\omega(T)^2\chi^2 \right]$$

Numerical results



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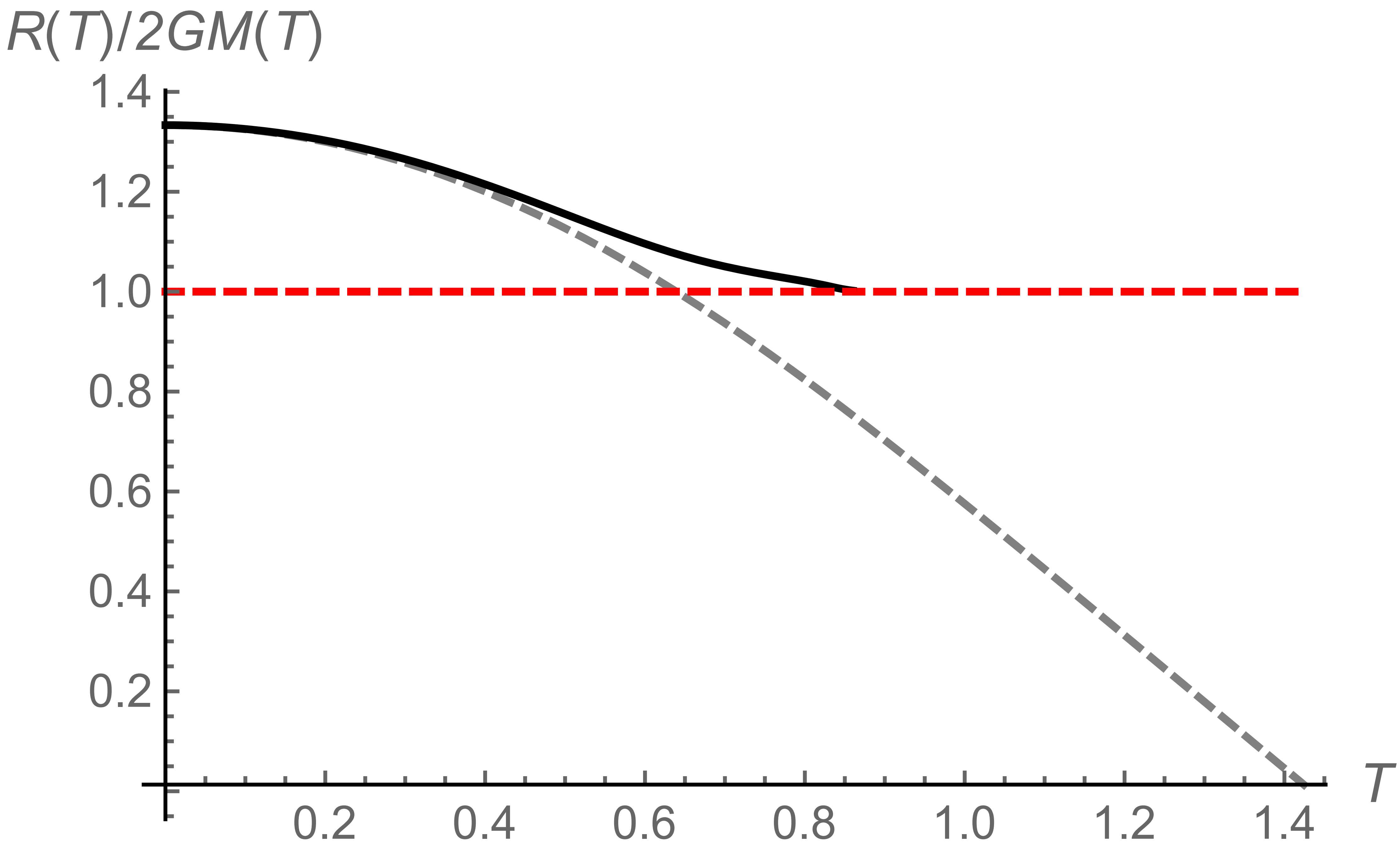

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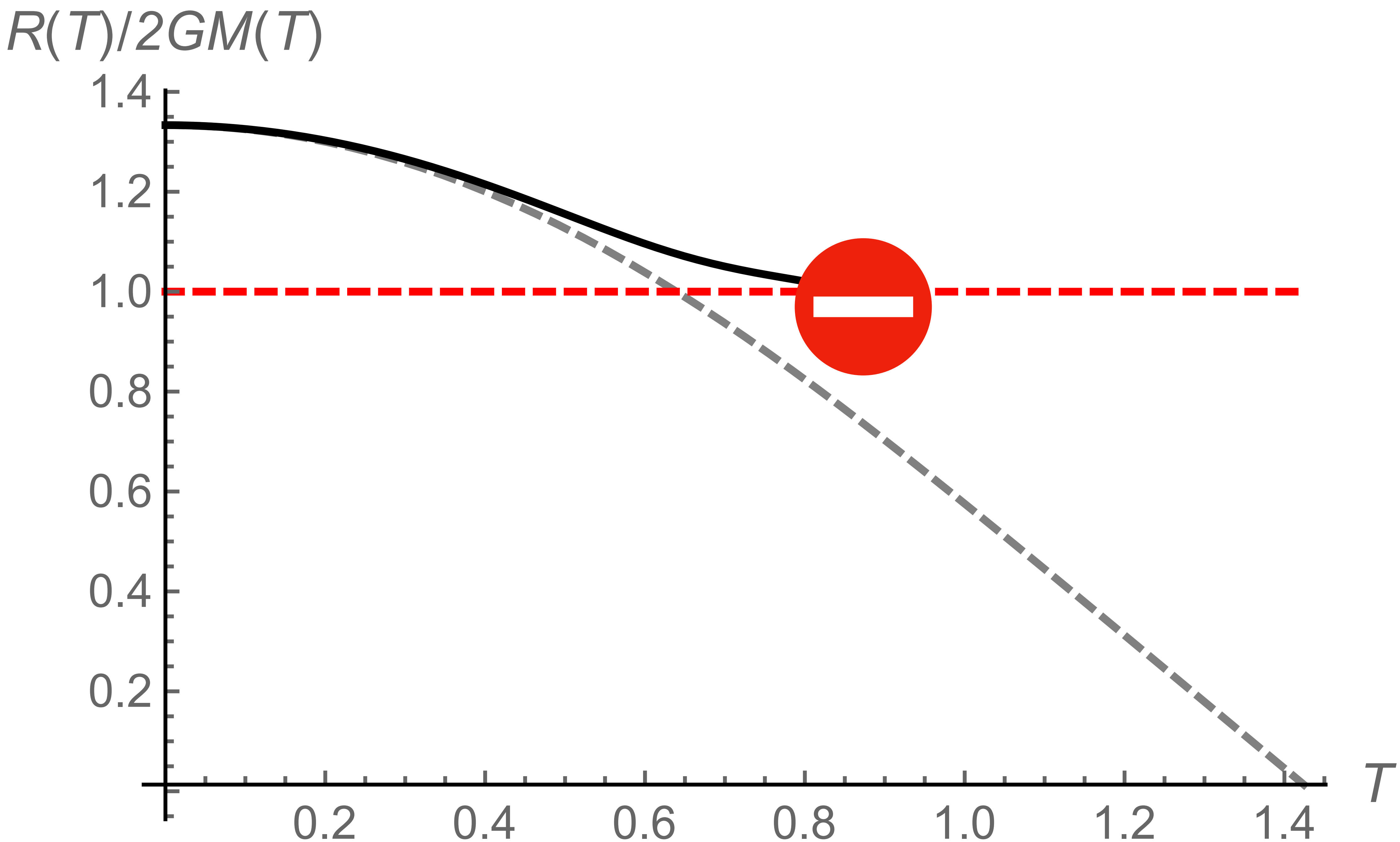

Outlook

- Toy model for backreaction of quantum radiation in collapsing background
- Potential applicability to other areas of physics: *see Tanmay Vachaspati's talk!!!*
- Limitations: near horizon limit approximations, only one mode, interactions…
- Full classical field theory treatment: Einstein-Hilbert action + thick domain wall + 1 complex scalar field (with global charge)