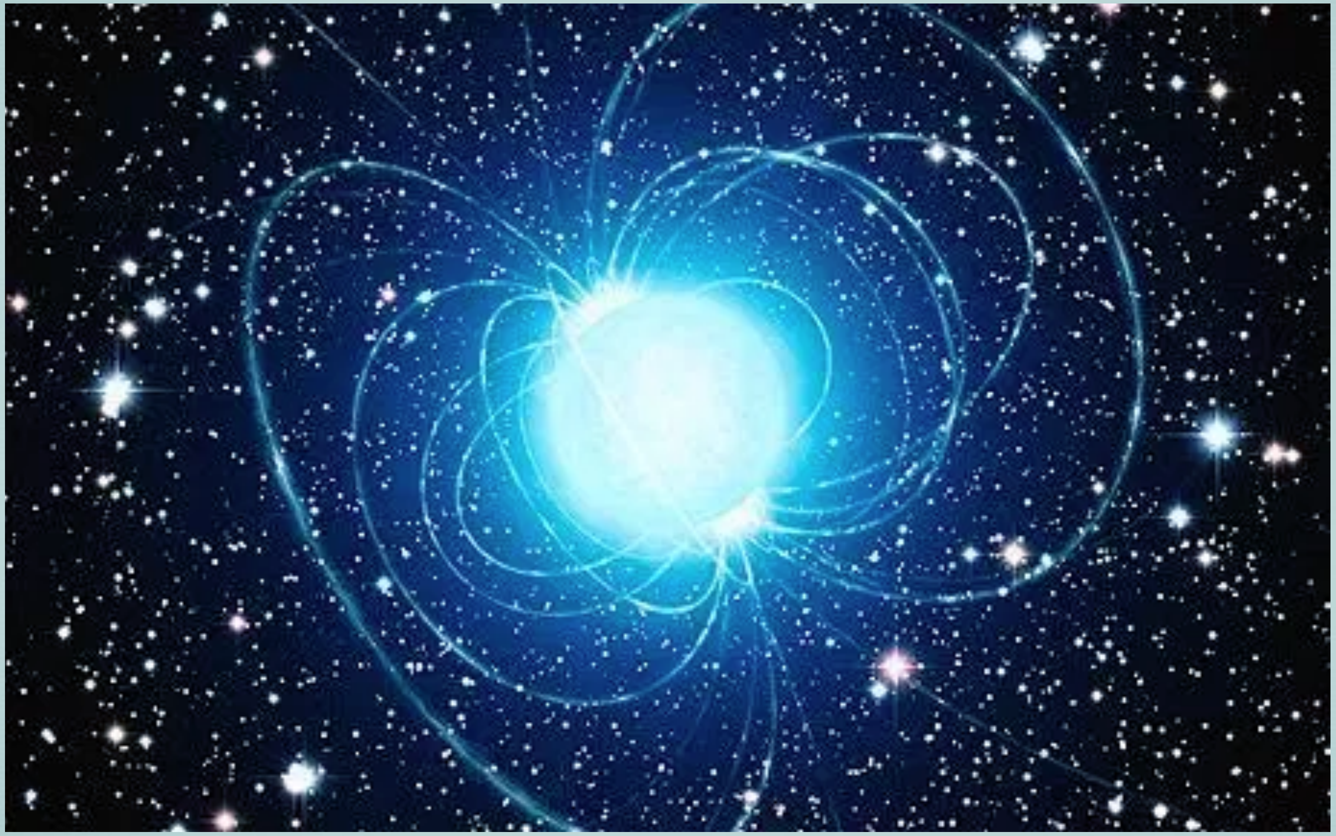
Folding-in astrophysical priors in the search for continuous gravitational waves

PASCOS, Cleveland June 7, 2018



Credit: ESO

Jing Ming (明镜)

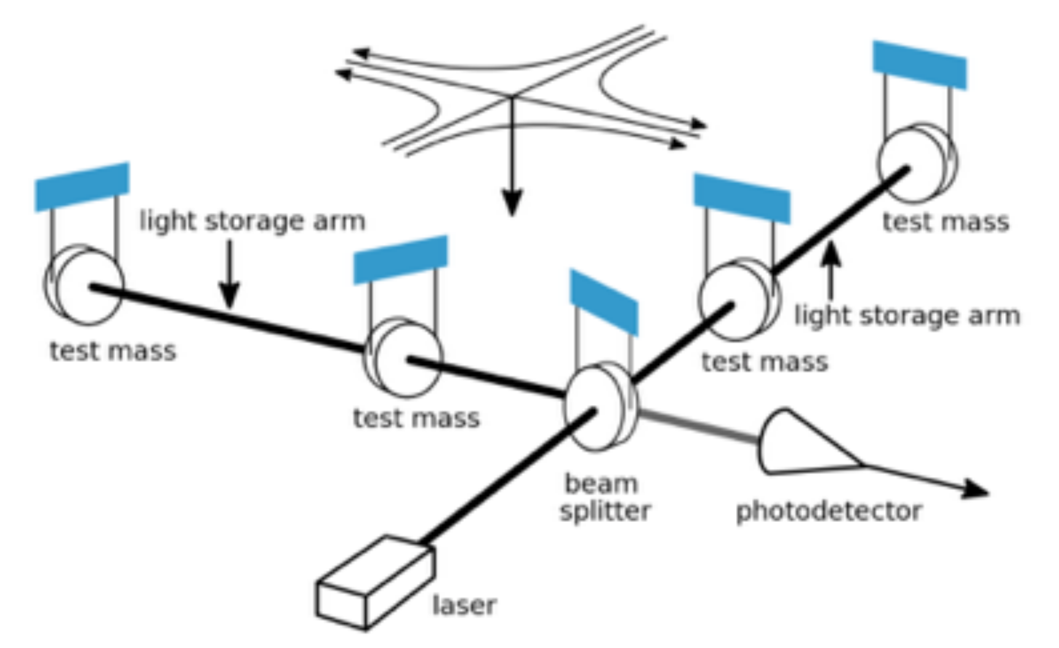
Max Planck Institute
for Gravitational Physics, Hannover

LSC LIGO Scientific Collaboration

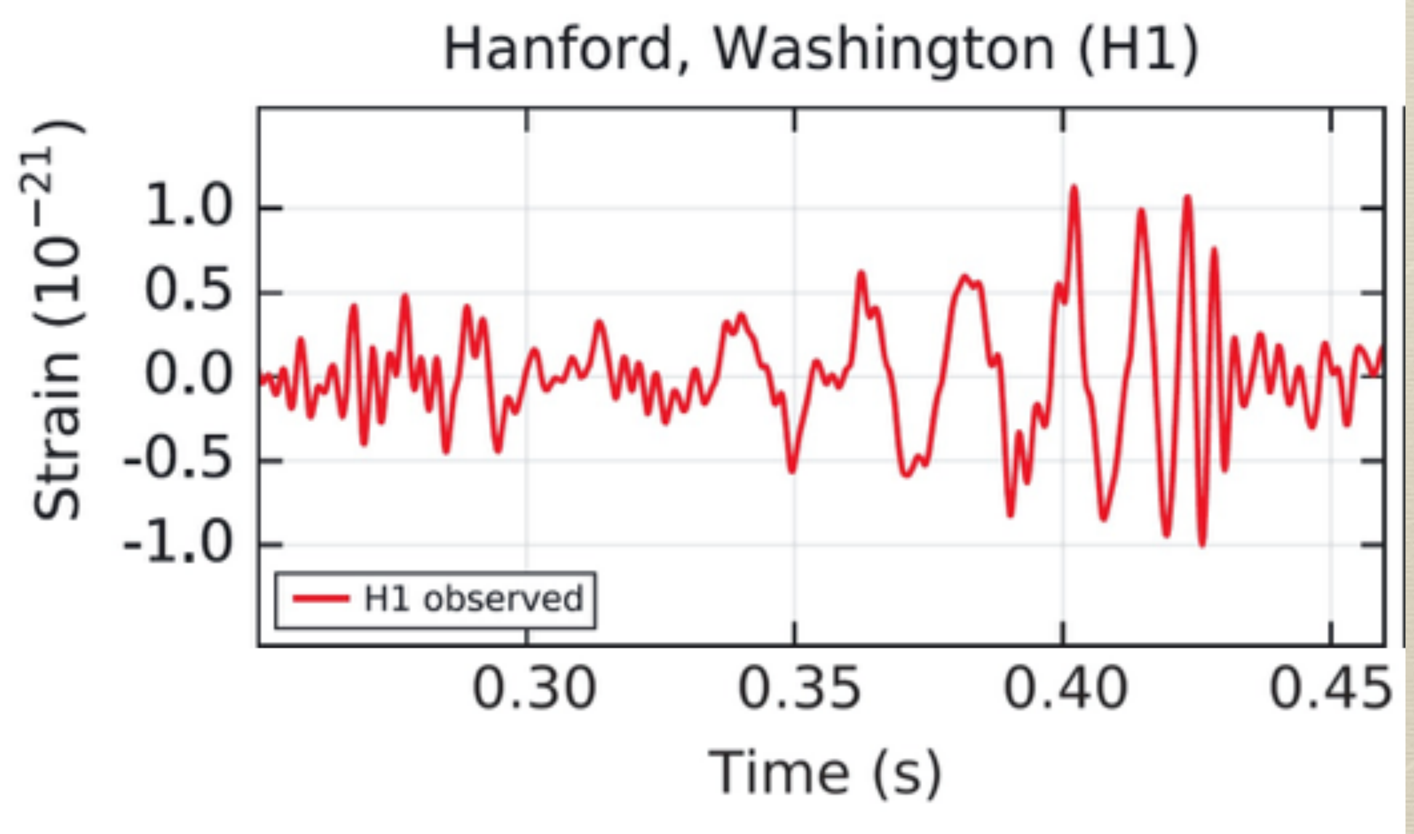
einstein@home

GW150914

Credit: LSC

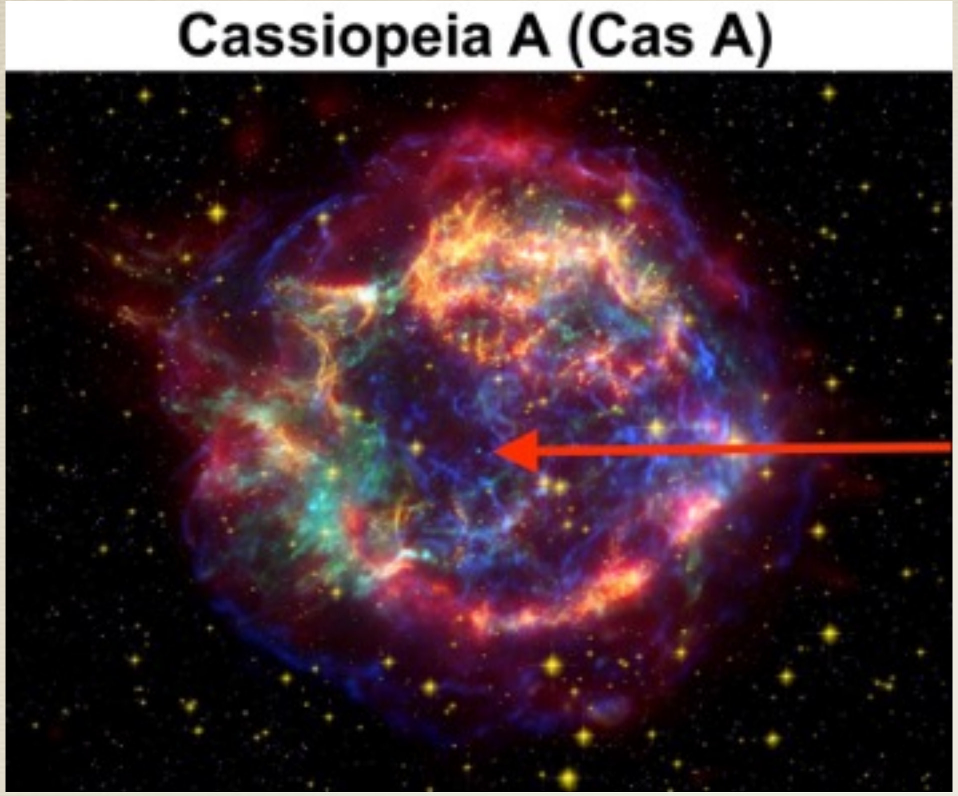


Credit: LSC



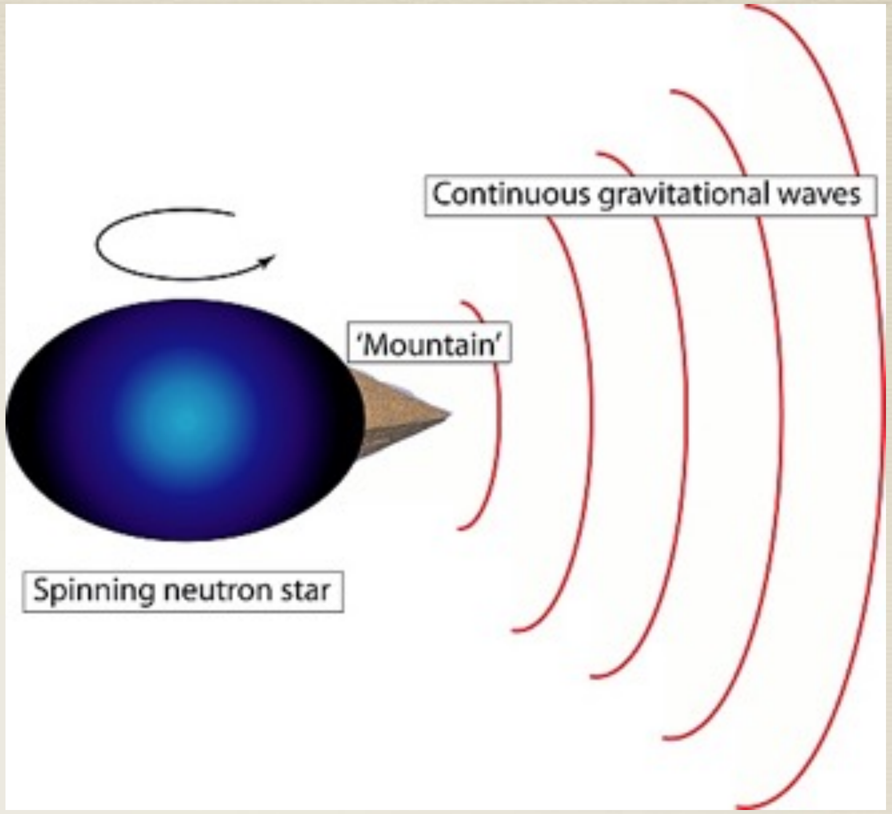
PRL 116 (6): 061102(2016)

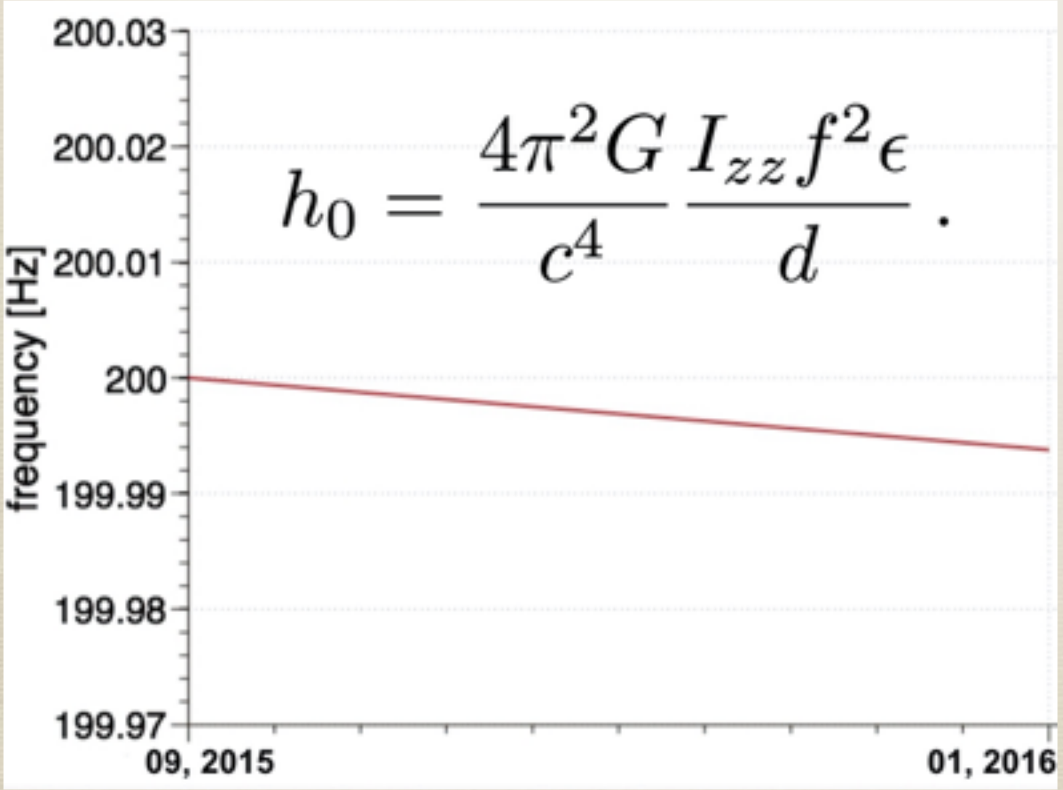
Continuous Wave (CW)



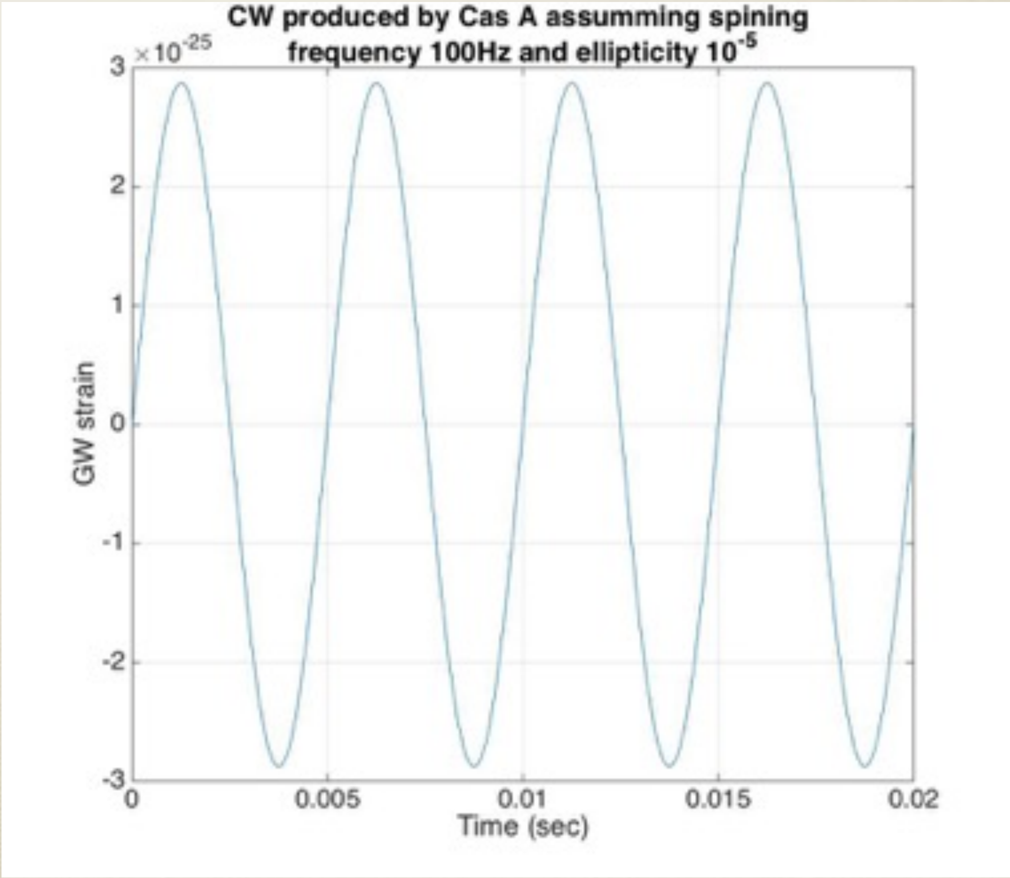
Cassiopeia A (Cas A)

Credit:NASA/JPL-





$$h_0 = \frac{4\pi^2 G}{c^4}\frac{I_{zz} f^2 \epsilon}{d}.$$



CW produced by Cas A assumming spining frequency 100Hz and ellipticity 10^{-5}

Template search

- $\mathcal{F}$-statistic: detection statistic based on matched filtering

Computing power needed in CW searches:

- 'Search template waveform' = $(\alpha, \delta, f, \dot{f}, \ddot{f}, \ldots)$
- Spacing between templates:

$$\delta f \propto 1/T_{\mathrm{obs}}$$
$$\delta \dot{f} \propto 1/T_{\mathrm{obs}}^2$$
$$\delta \ddot{f} \propto 1/T_{\mathrm{obs}}^3$$
$$\delta \alpha \propto 1/T_{\mathrm{obs}}$$
$$\delta \delta \propto 1/T_{\mathrm{obs}}$$

CW search types

$$N_{\rm temp} \propto T_{\rm obs}^6$$

Targeted search (e.g. Crab) .

copyright@stsci.edu.

Directed search (e.g. Cas A) .

copyright@NASA/JPL-Caltech

All-sky surveys $N_{\rm temp} \propto T_{\rm obs}^8$

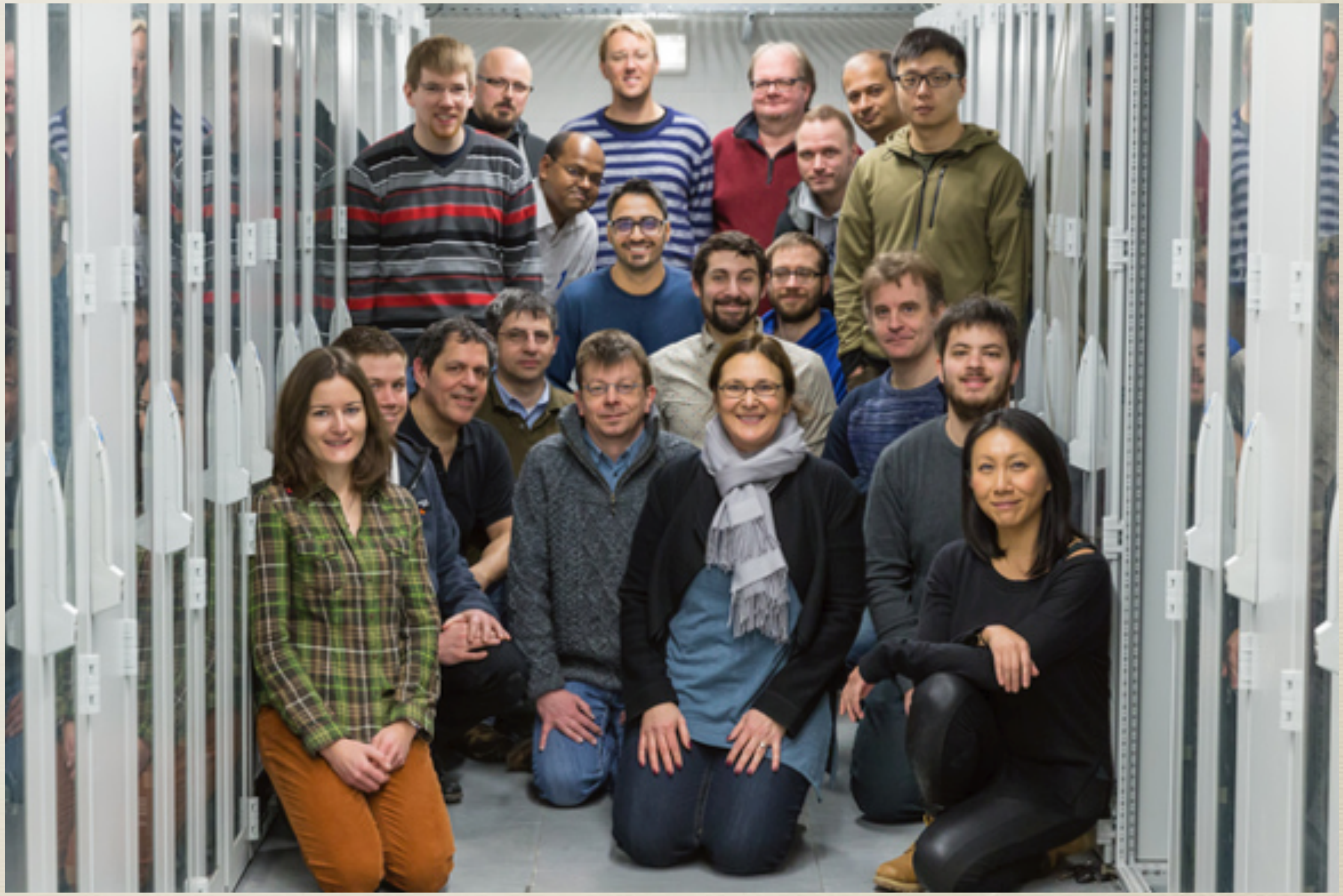
copyright@ESO

Computing power: Einstein@Home

Einstein@Home is a volunteer distributed computing project. (Like SETI@Home, but for GW data and EM data.)

Active users: 40,000
Computing power: 1.7 PFLOPS

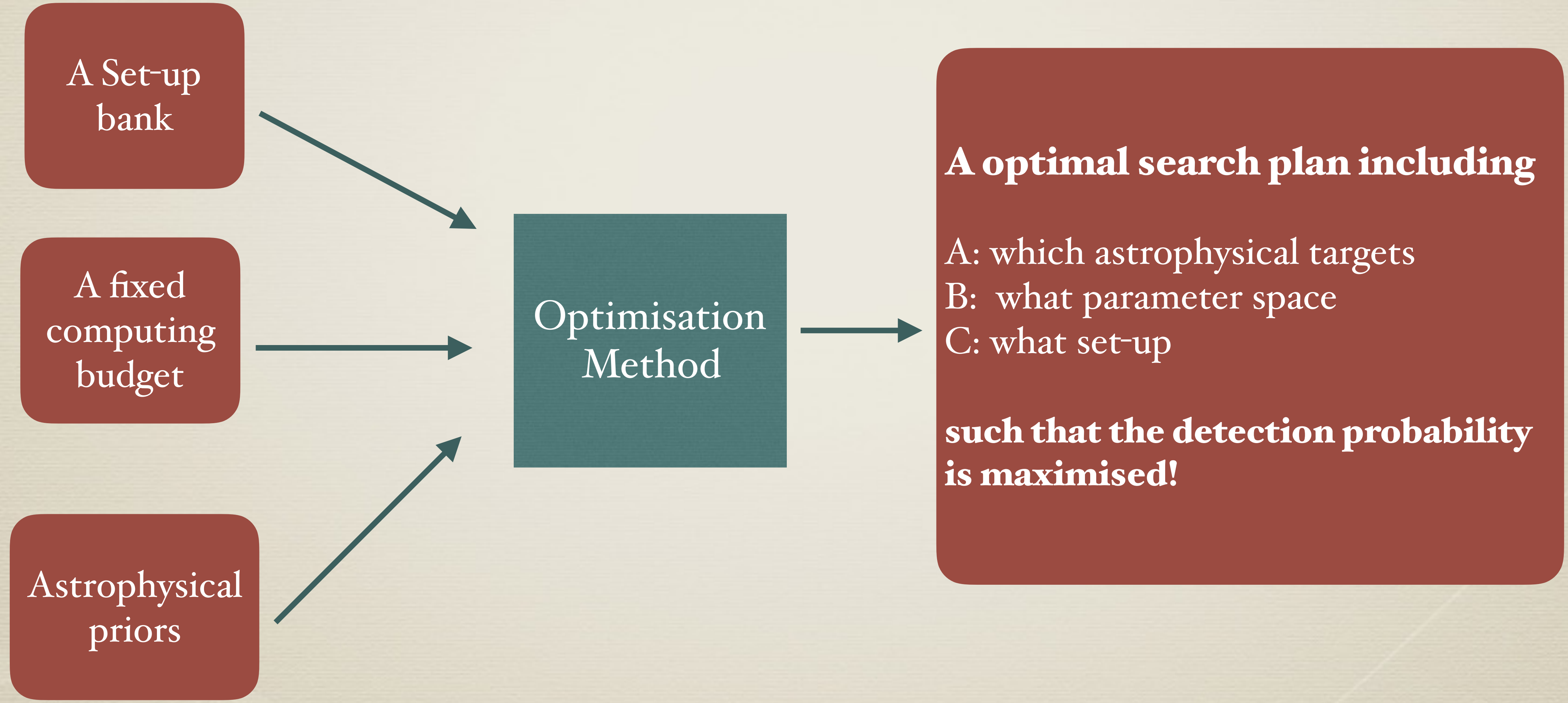
EM means Einstein@Home-month.
Budget: a few EM



Support for Windows, Mac OSX, Linux clients.
Please sign up your computers to Einstein@Home
https://einsteinathome.org/

The optimisation method

J. Ming, B. Krishnan, M. A. Papa, C. Aulbert, and H. Fehrmann.
'Optimal directed searches for continuous gravitational waves'. Physical Review D, 93(6):064011, Mar. 2016.



A Set-up bank

A fixed computing budget

Astrophysical priors

Optimisation Method

A optimal search plan including

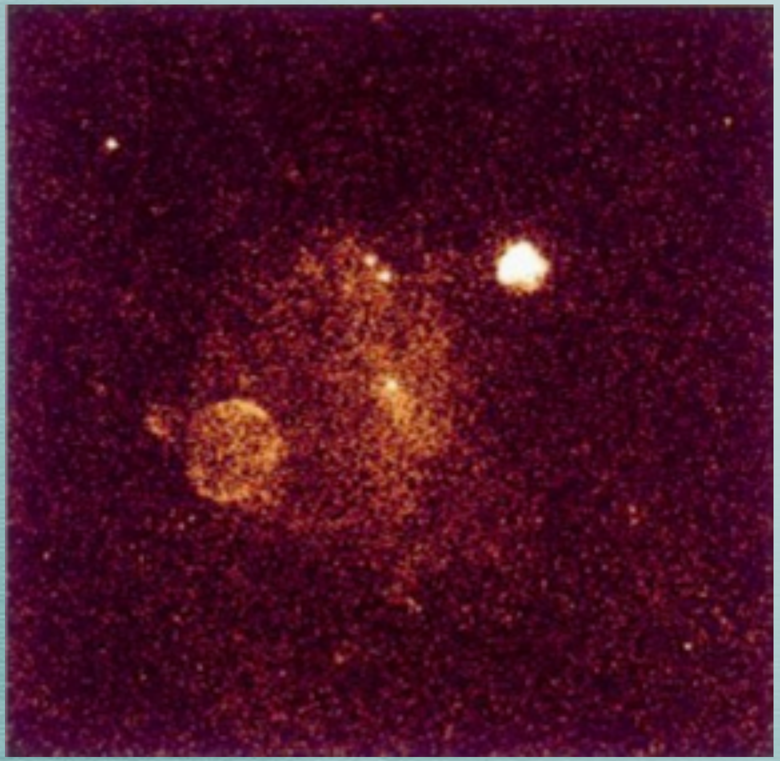
A: which astrophysical targets
B: what parameter space
C: what set-up

such that the detection probability is maximised!

An Example:

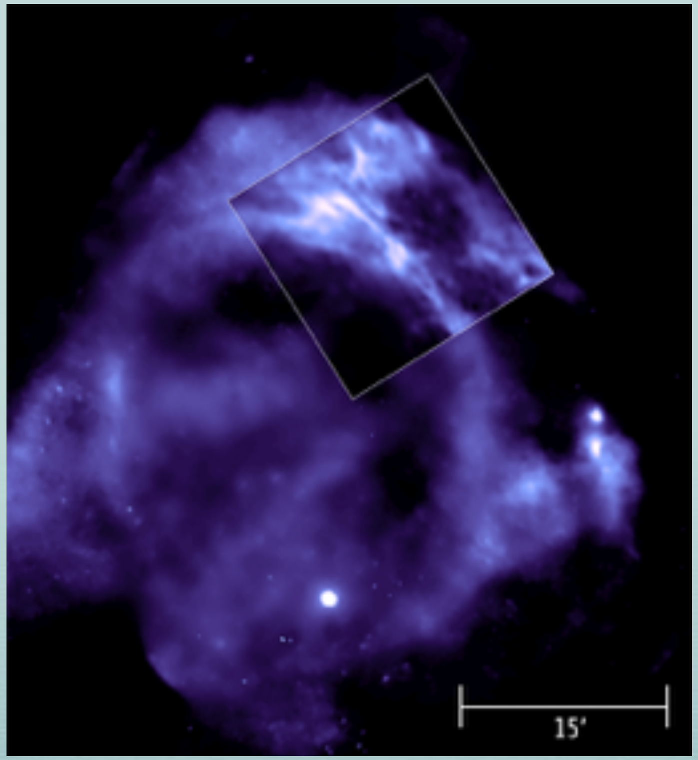
The directed search for 3 targets in LIGO O1 data

J. Ming, M. A. Papa, B. Krishnan, R. Prix, C. Beer, S. J. Zhu, H.-B.Eggenstein, O. Bock, and B. Machenschalk. 'Optimally setting up directed searches for continuous gravitational waves in advanced LIGO O1 data'. Phys. Rev. D 97, 024051 (2018)



Vela Jr

Nature 396, 141-142(1998)



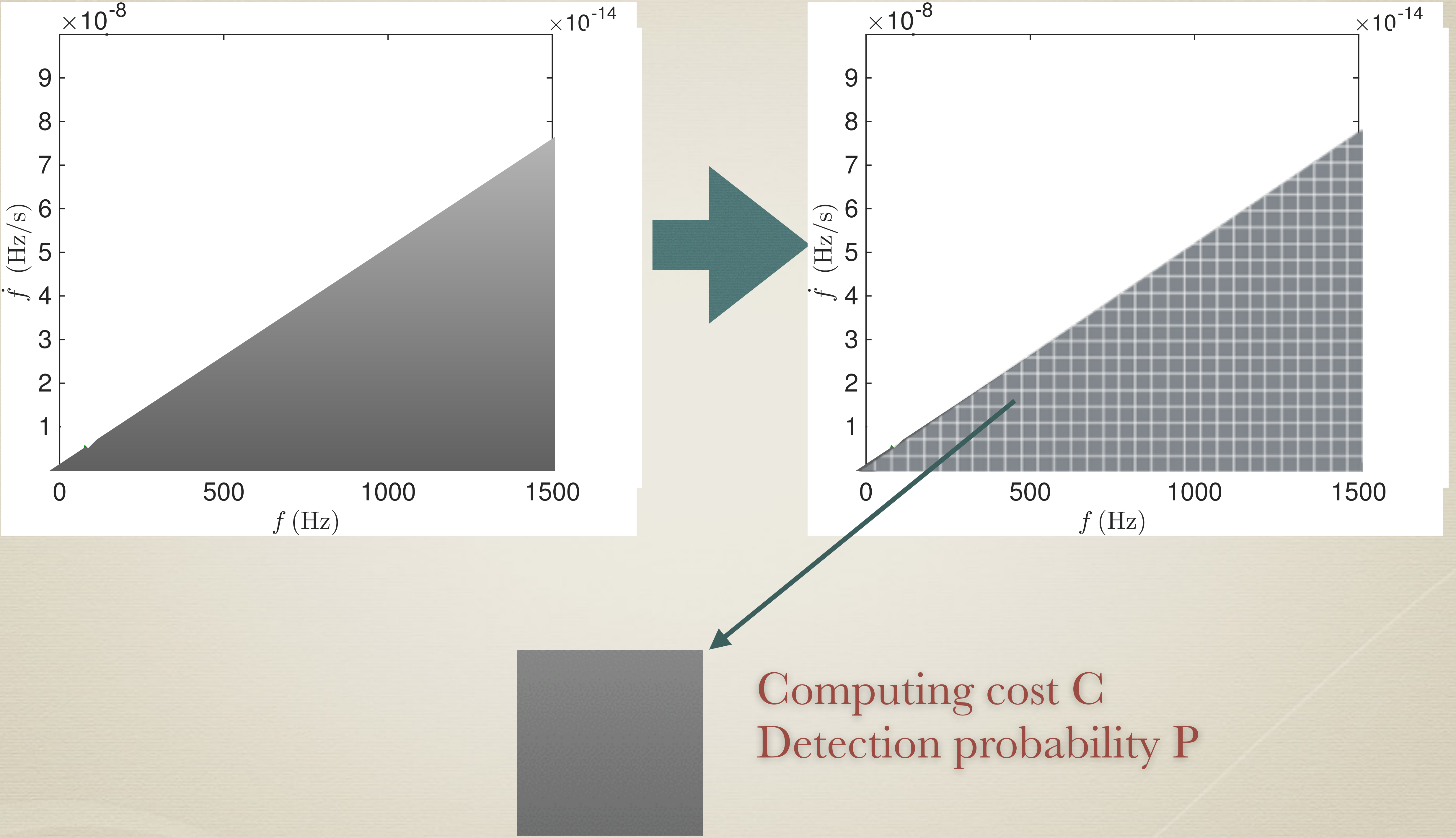
G347.3

Credit: Chandra&XMM-Newton

Cas A

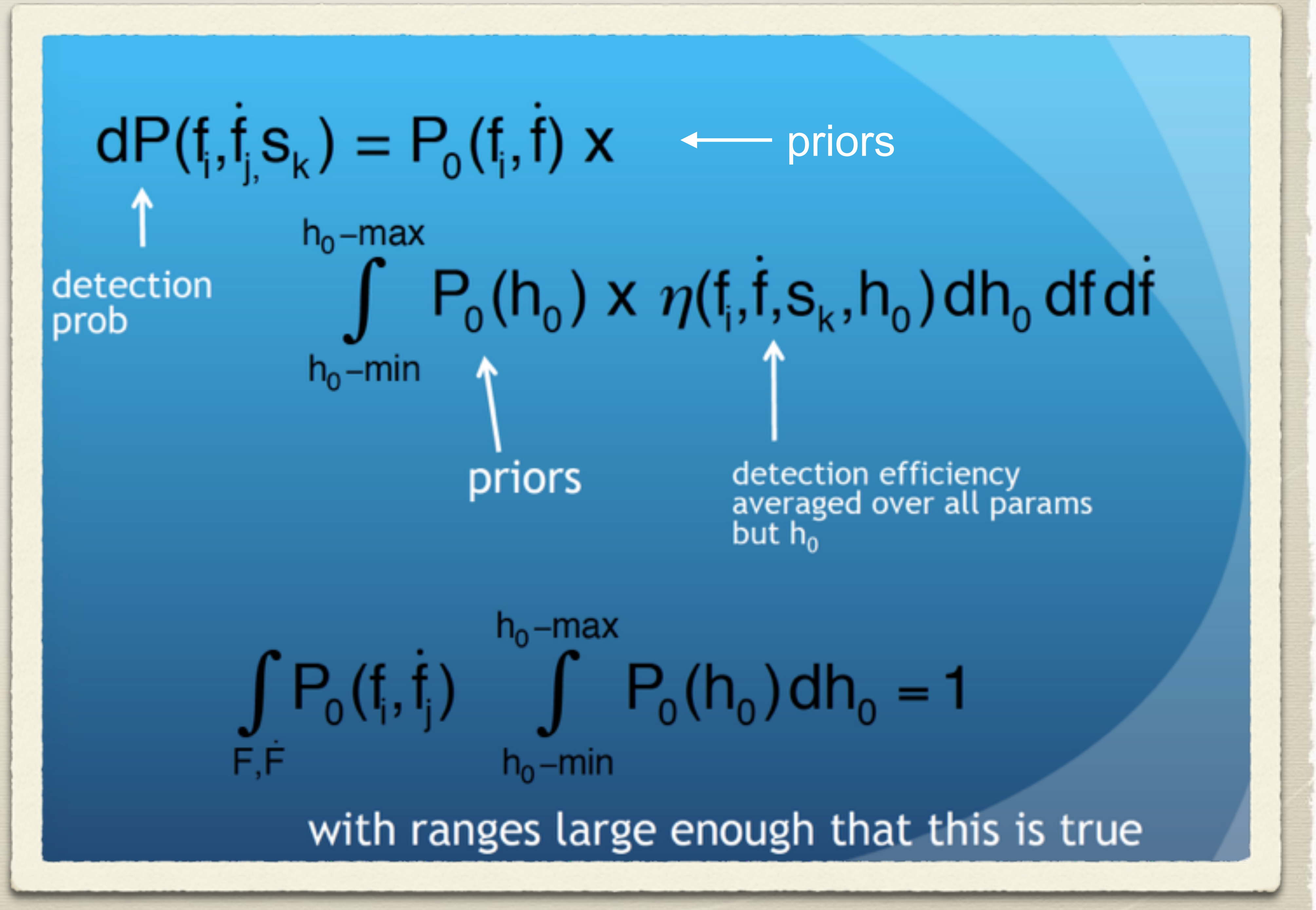
copyright@NASA/JPL-Caltec

Optimisation scheme



Computing cost C
Detection probability P

The detection probability



$$dP(f_i,\dot{f}_{j,}s_k) = P_0(f_i,\dot{f}) \text{ x}$$

priors

detection prob

$$\int_{h_0-min}^{h_0-max} P_0(h_0) \text{ x } \eta(f_i,\dot{f},s_k,h_0)\,dh_0\,df\,d\dot{f}$$

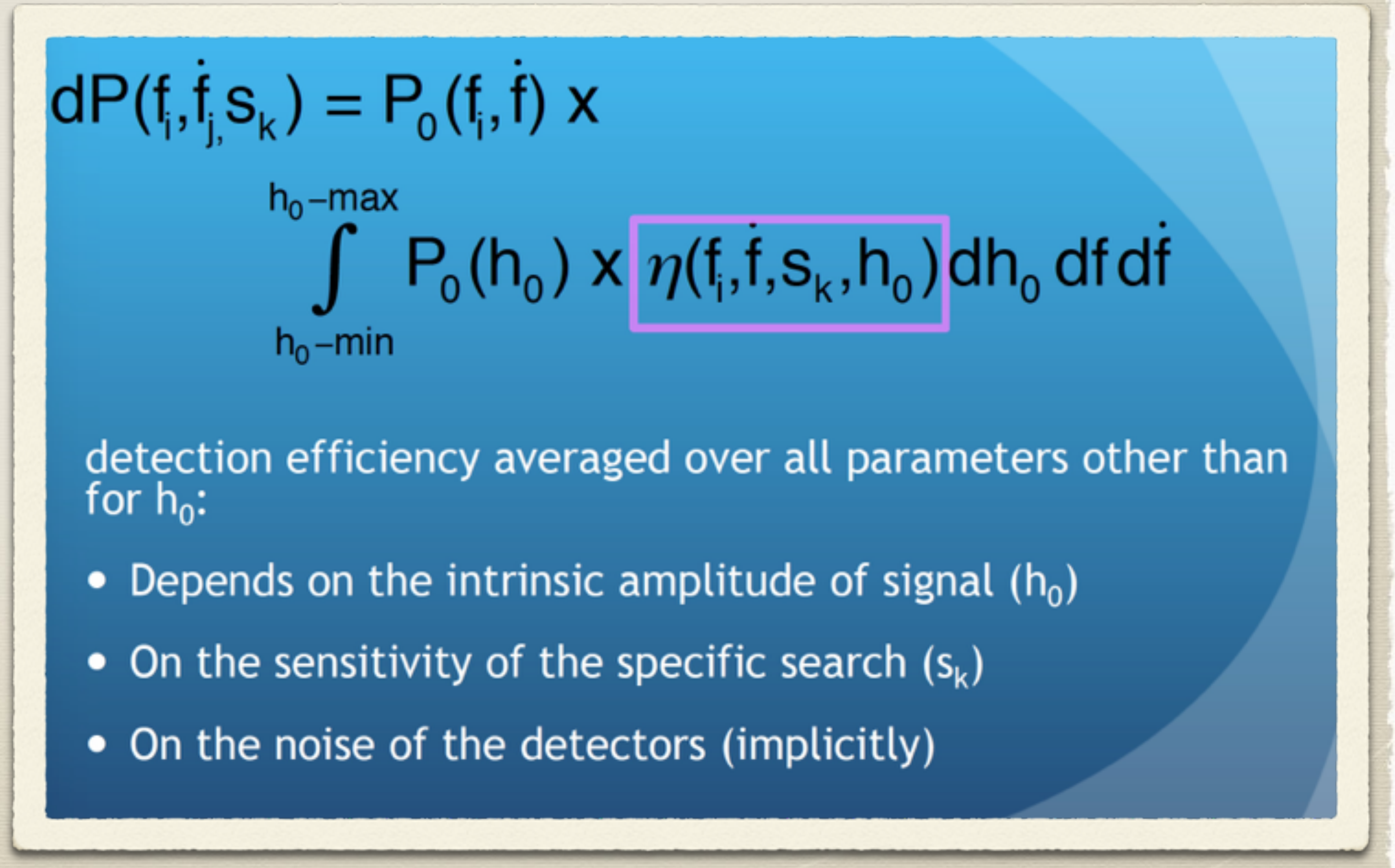
priors

detection efficiency averaged over all params but h_0

$$\int_{F,\dot{F}} P_0(f_i,\dot{f}_j) \int_{h_0-min}^{h_0-max} P_0(h_0)\,dh_0 = 1$$

with ranges large enough that this is true

The detection probability

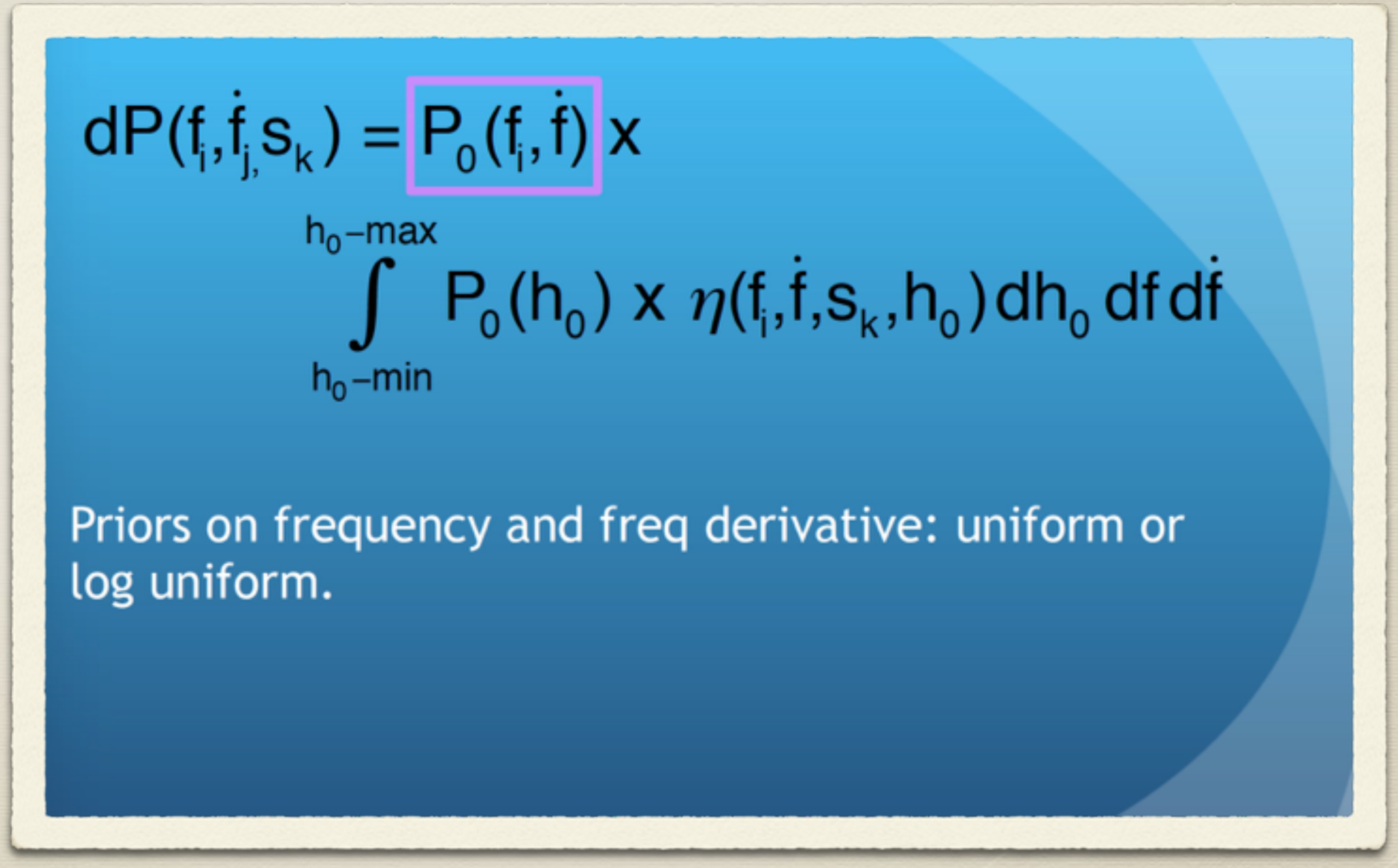


$$dP(f_i,\dot{f}_j,s_k) = P_0(f_i,\dot{f}) \times \int_{h_0-min}^{h_0-max} P_0(h_0) \times \boxed{\eta(f_i,\dot{f},s_k,h_0)}\, dh_0\, df\, d\dot{f}$$

detection efficiency averaged over all parameters other than for h_0:

- Depends on the intrinsic amplitude of signal (h_0)
- On the sensitivity of the specific search (s_k)
- On the noise of the detectors (implicitly)

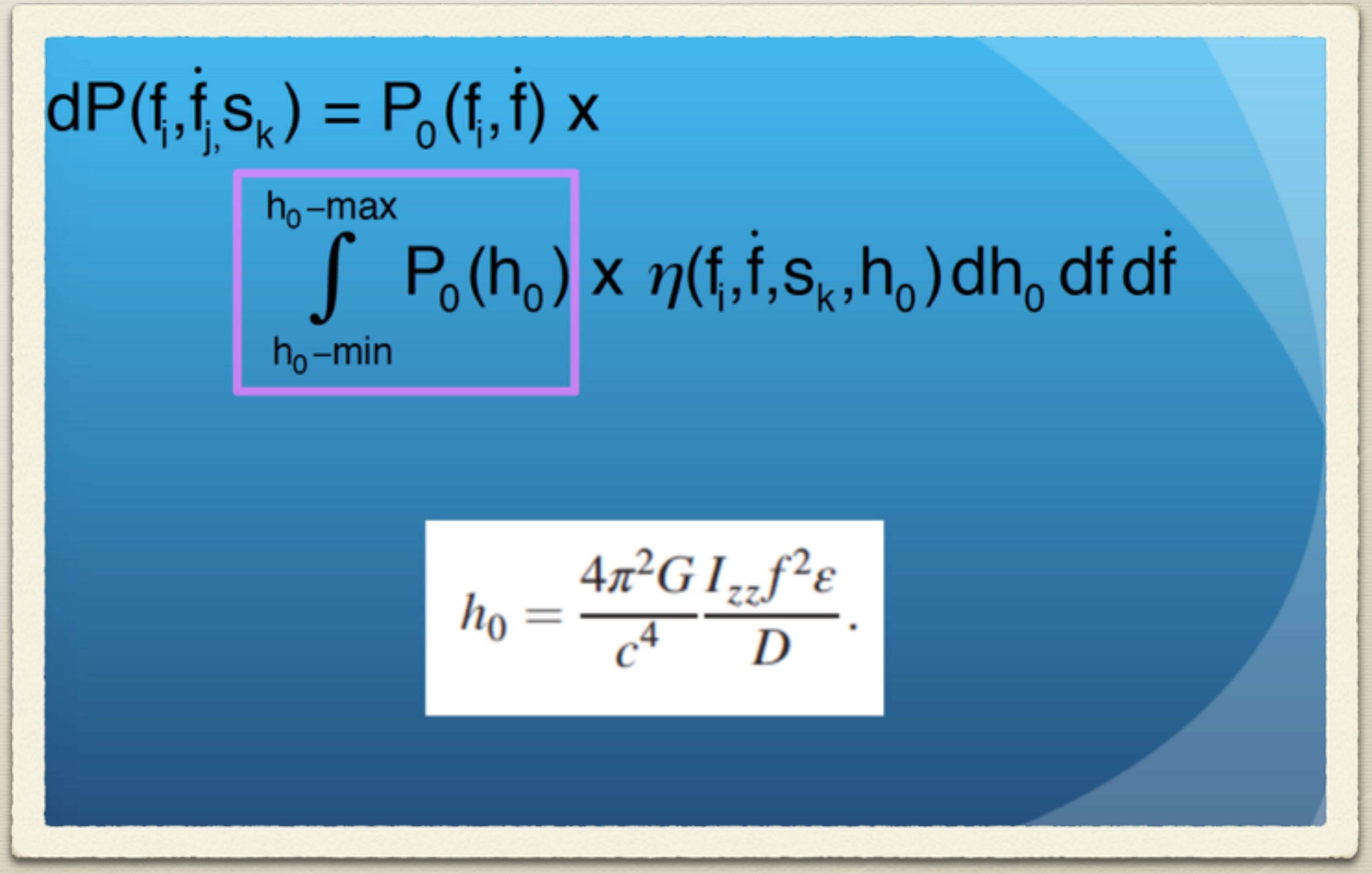
The detection probability



$$dP(f_i,\dot{f}_{j,}s_k) = \boxed{P_0(f_i,\dot{f})}\ \text{x} \int_{h_0-\text{min}}^{h_0-\text{max}} P_0(h_0)\ \text{x}\ \eta(f_i,\dot{f},s_k,h_0)\,dh_0\,df\,d\dot{f}$$

Priors on frequency and freq derivative: uniform or log uniform.

The detection probability

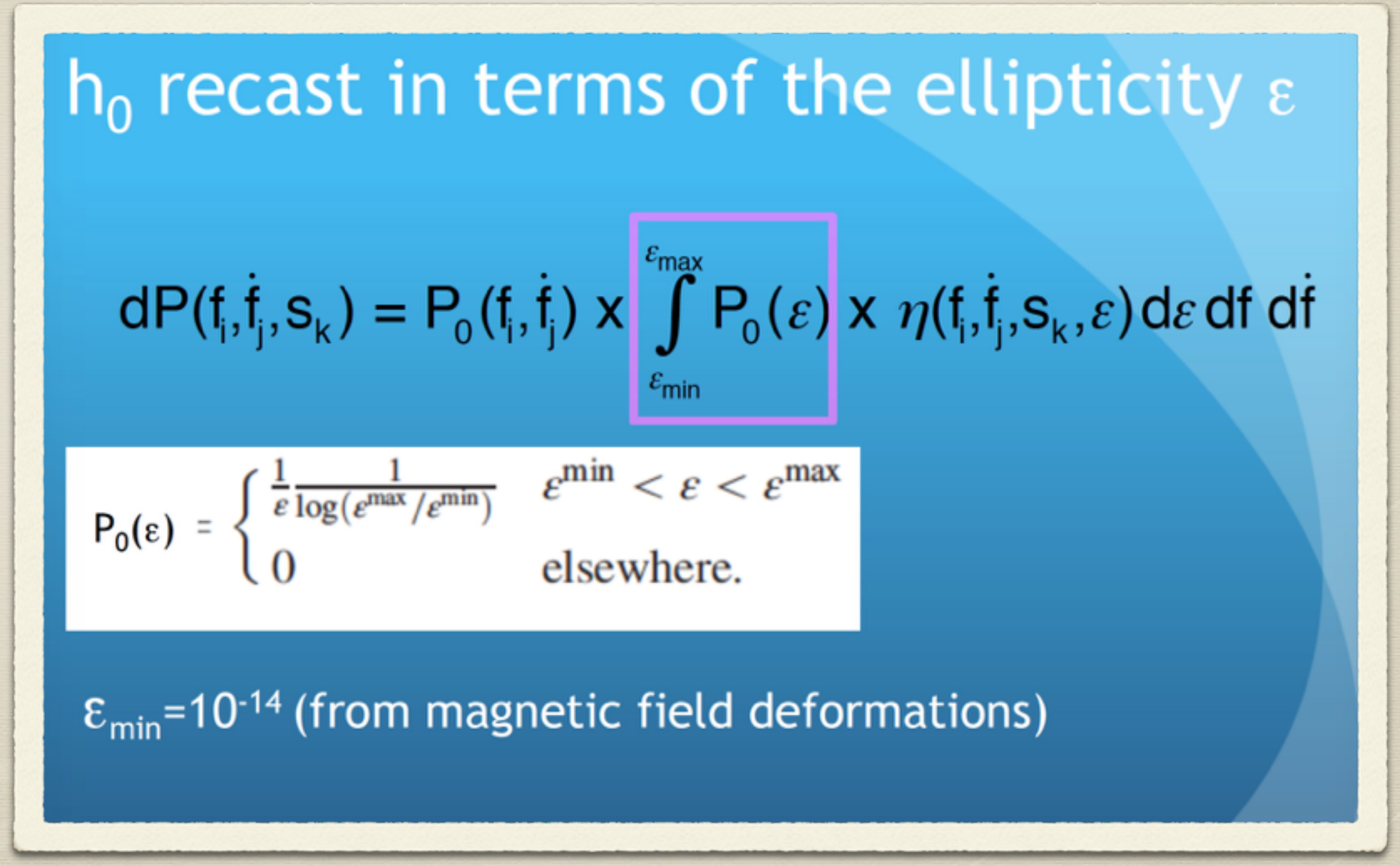


$$dP(f_i,\dot{f}_j,s_k) = P_0(f_i,\dot{f}) \times$$

$$\boxed{\int_{h_0-\mathrm{min}}^{h_0-\mathrm{max}} P_0(h_0)} \times \eta(f_i,\dot{f},s_k,h_0)\,dh_0\,df\,d\dot{f}$$

$$h_0 = \frac{4\pi^2 G}{c^4}\frac{I_{zz} f^2 \varepsilon}{D}.$$

The detection probability



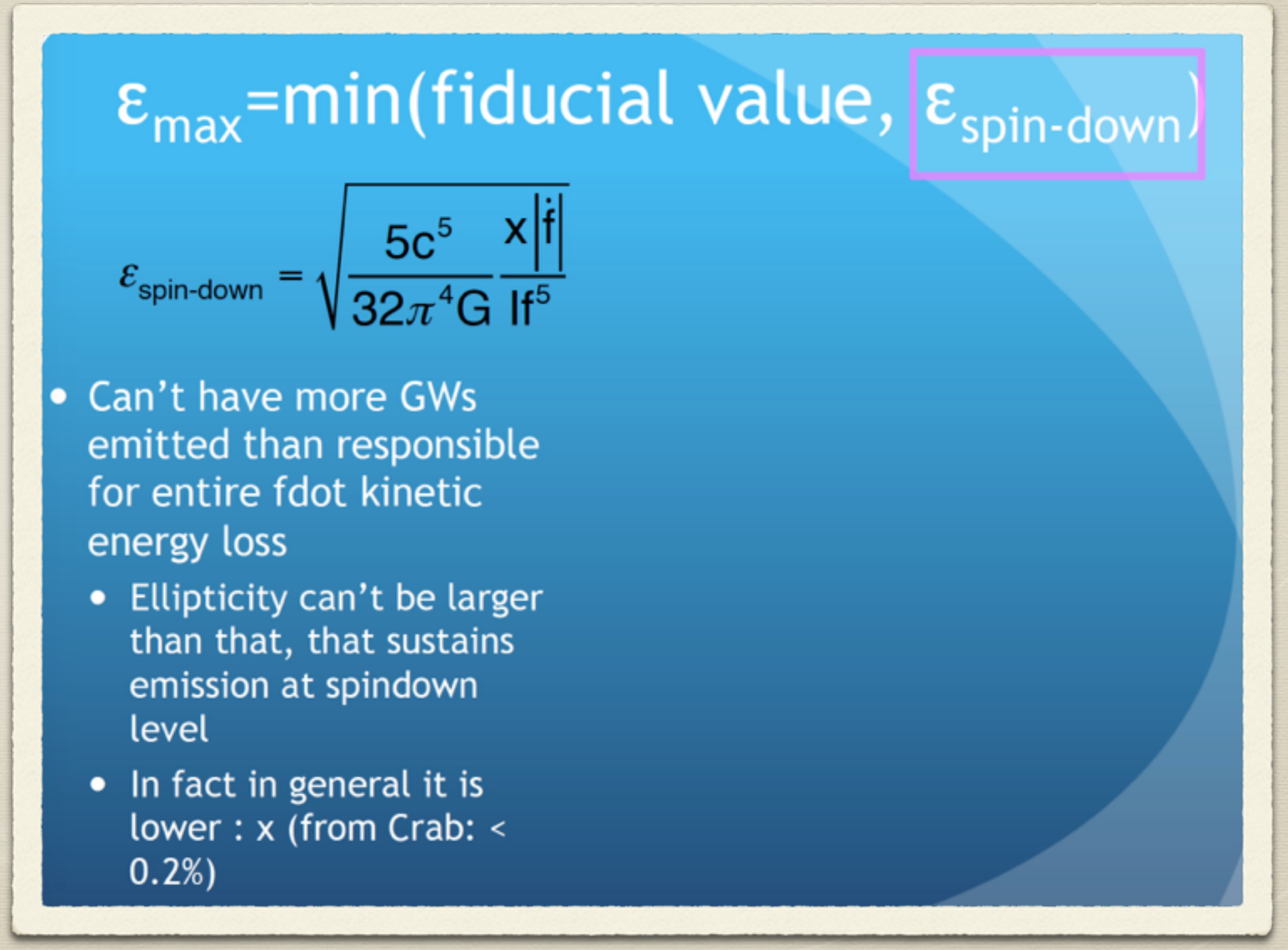
h_0 recast in terms of the ellipticity ε

$$dP(f_i,\dot{f}_j,s_k) = P_0(f_i,\dot{f}_j) \times \boxed{\int_{\varepsilon_{max}}^{\varepsilon_{min}} P_0(\varepsilon)} \times \eta(f_i,\dot{f}_j,s_k,\varepsilon)\, d\varepsilon\, df\, d\dot{f}$$

$$P_0(\varepsilon) = \begin{cases} \frac{1}{\varepsilon}\frac{1}{\log(\varepsilon^{max}/\varepsilon^{min})} & \varepsilon^{min} < \varepsilon < \varepsilon^{max} \\ 0 & \text{elsewhere.} \end{cases}$$

$\varepsilon_{min} = 10^{-14}$ (from magnetic field deformations)

The detection probability

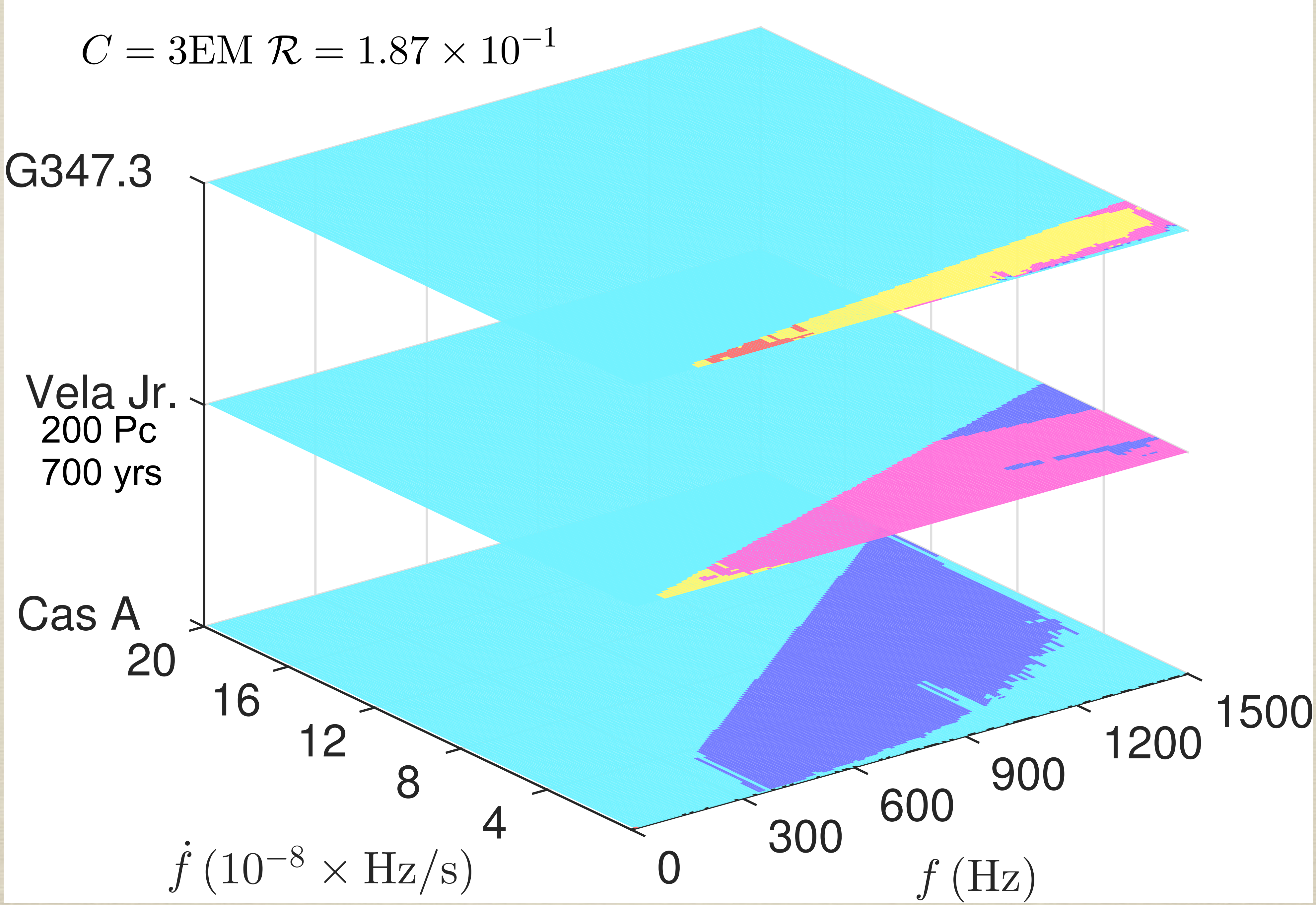


ε_{max}=min(fiducial value, $\varepsilon_{spin\text{-}down}$)

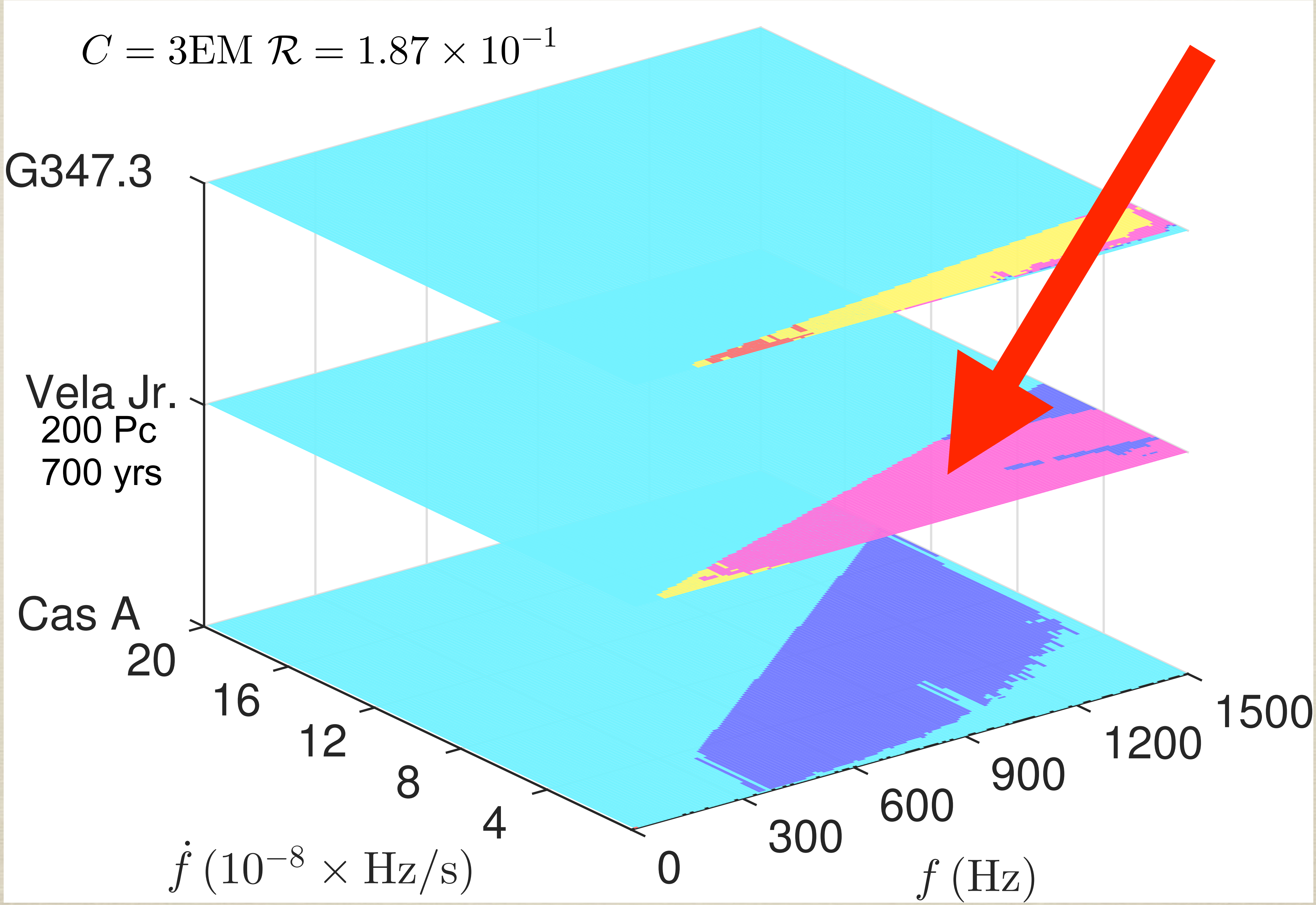
$$\varepsilon_{spin\text{-}down} = \sqrt{\frac{5c^5}{32\pi^4 G}\frac{x\left|\dot{f}\right|}{I f^5}}$$

- Can't have more GWs emitted than responsible for entire fdot kinetic energy loss
  - Ellipticity can't be larger than that, that sustains emission at spindown level
  - In fact in general it is lower : x (from Crab: < 0.2%)

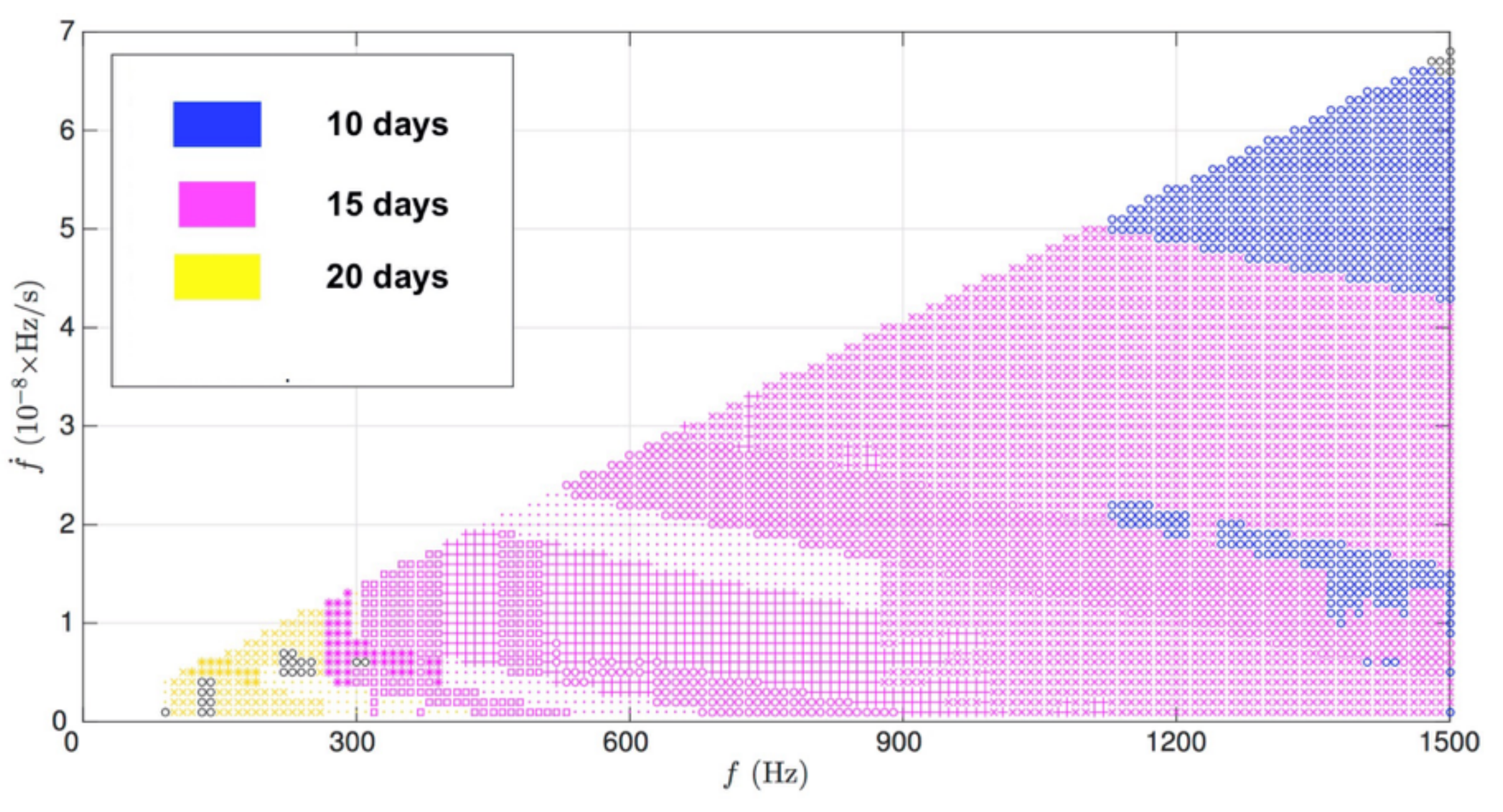
Optimisation results



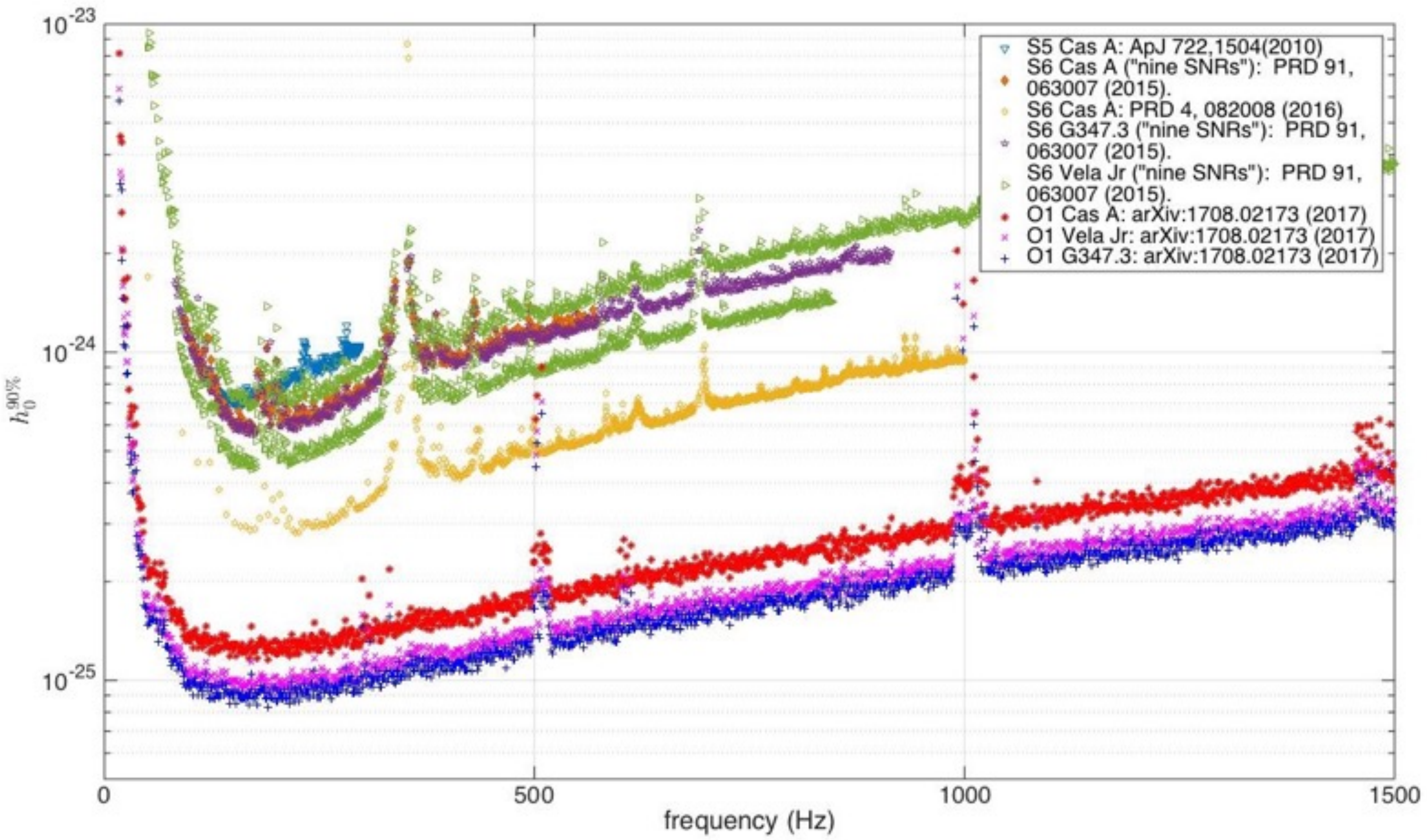
Optimisation results



Optimisation results



Final chosen set-ups



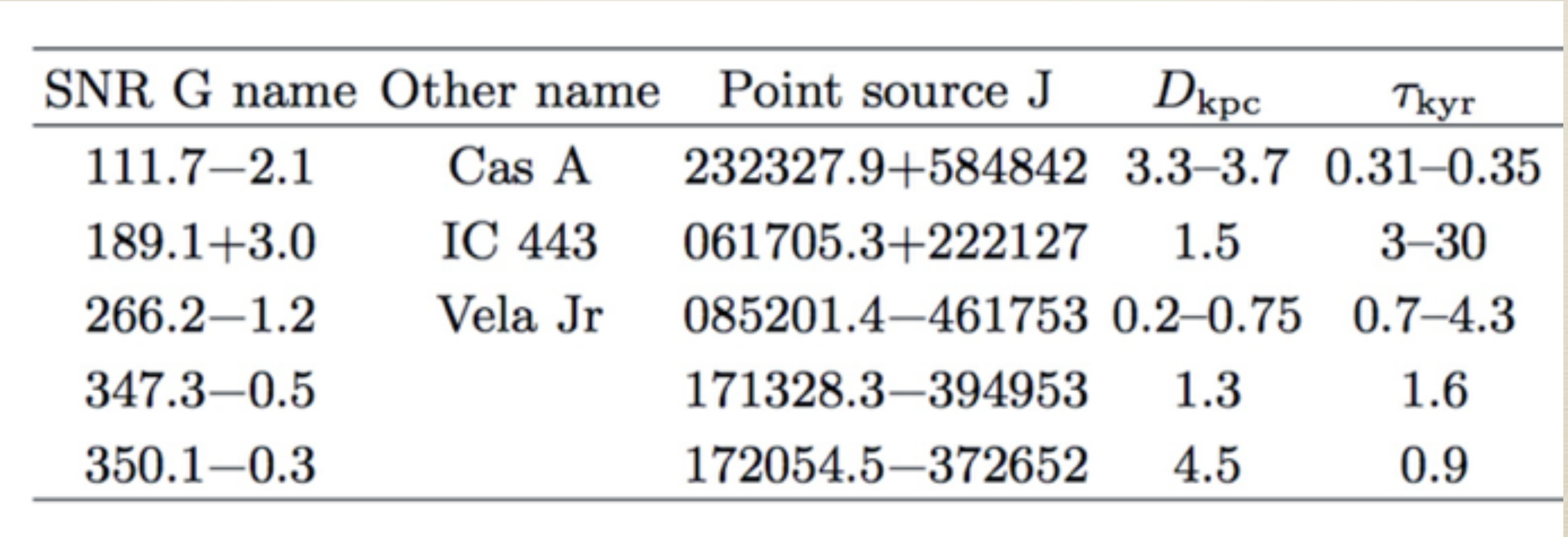
Bring-home message:

- We have a scheme to fold-in priors on the source parameters and ensure an optimal search set-up.
- Any prior on the maximum ellipticity or ellipticity distribution of a specific object, or a population of objects, is most welcomed.
- Priors on frequency, frequency derivatives and the ranges of the latter in relation to frequency and age.
- Would like to use this sort of approach to determine parameter space and search set-up for blind surveys: additional search parameter, sky position

Thank you

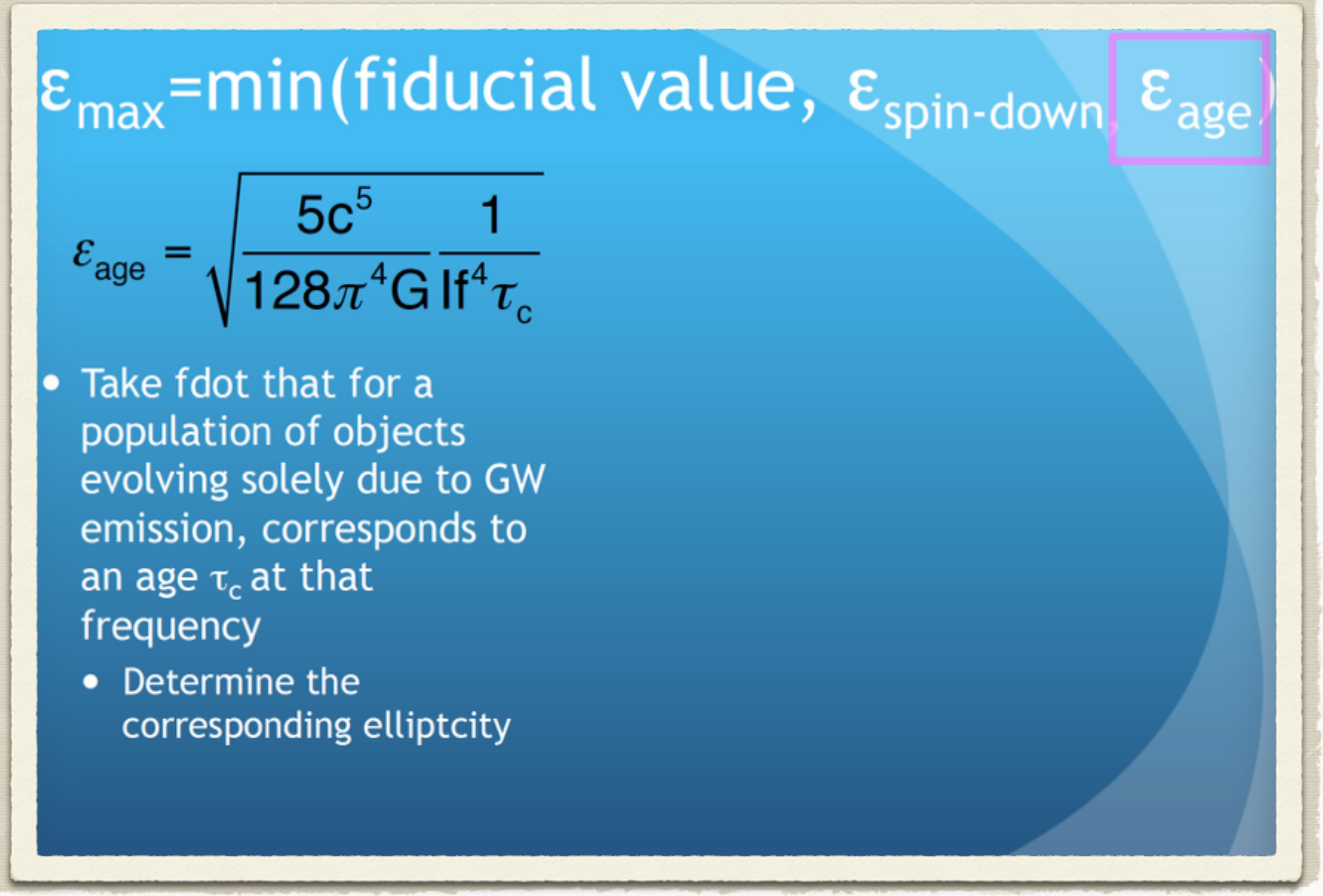
Astrophysical targets

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} f^2 \epsilon}{d} \, .$$



SNR G name	Other name	Point source J	D_{kpc}	τ_{kyr}
111.7−2.1	Cas A	232327.9+584842	3.3–3.7	0.31–0.35
189.1+3.0	IC 443	061705.3+222127	1.5	3–30
266.2−1.2	Vela Jr	085201.4−461753	0.2–0.75	0.7–4.3
347.3−0.5		171328.3−394953	1.3	1.6
350.1−0.3		172054.5−372652	4.5	0.9

The detection probability



$$\varepsilon_{max}=\min(\text{fiducial value}, \varepsilon_{\text{spin-down}}, \boxed{\varepsilon_{age}})$$

$$\varepsilon_{age}=\sqrt{\frac{5c^5}{128\pi^4 G}\frac{1}{If^4\tau_c}}$$

- Take fdot that for a population of objects evolving solely due to GW emission, corresponds to an age τ_c at that frequency
  - Determine the corresponding elliptcity

Ellipticity and GW amplitude

Distribution of ellipticity ϵ :

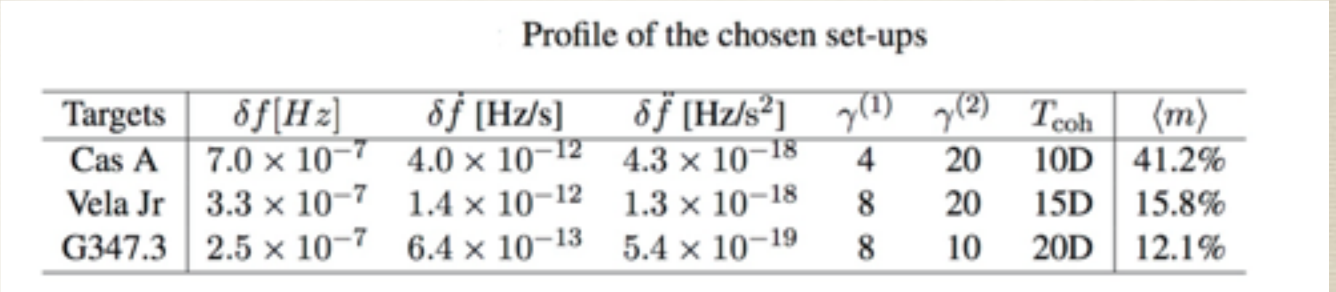
$$p(\epsilon) = \begin{cases} \frac{1}{\epsilon} \frac{1}{\log(\epsilon^{max}/\epsilon^{min})} & \epsilon^{min} < \epsilon < \epsilon^{max} \\ 0 & \text{elsewhere} \, . \end{cases}$$

$$\epsilon^{max} = \min(10^{-4}, \ \epsilon^{sd}, \ \epsilon^{age}), \quad \epsilon^{min} = 10^{-14}.$$

Thus we have the distribution of GW amplitude h0:

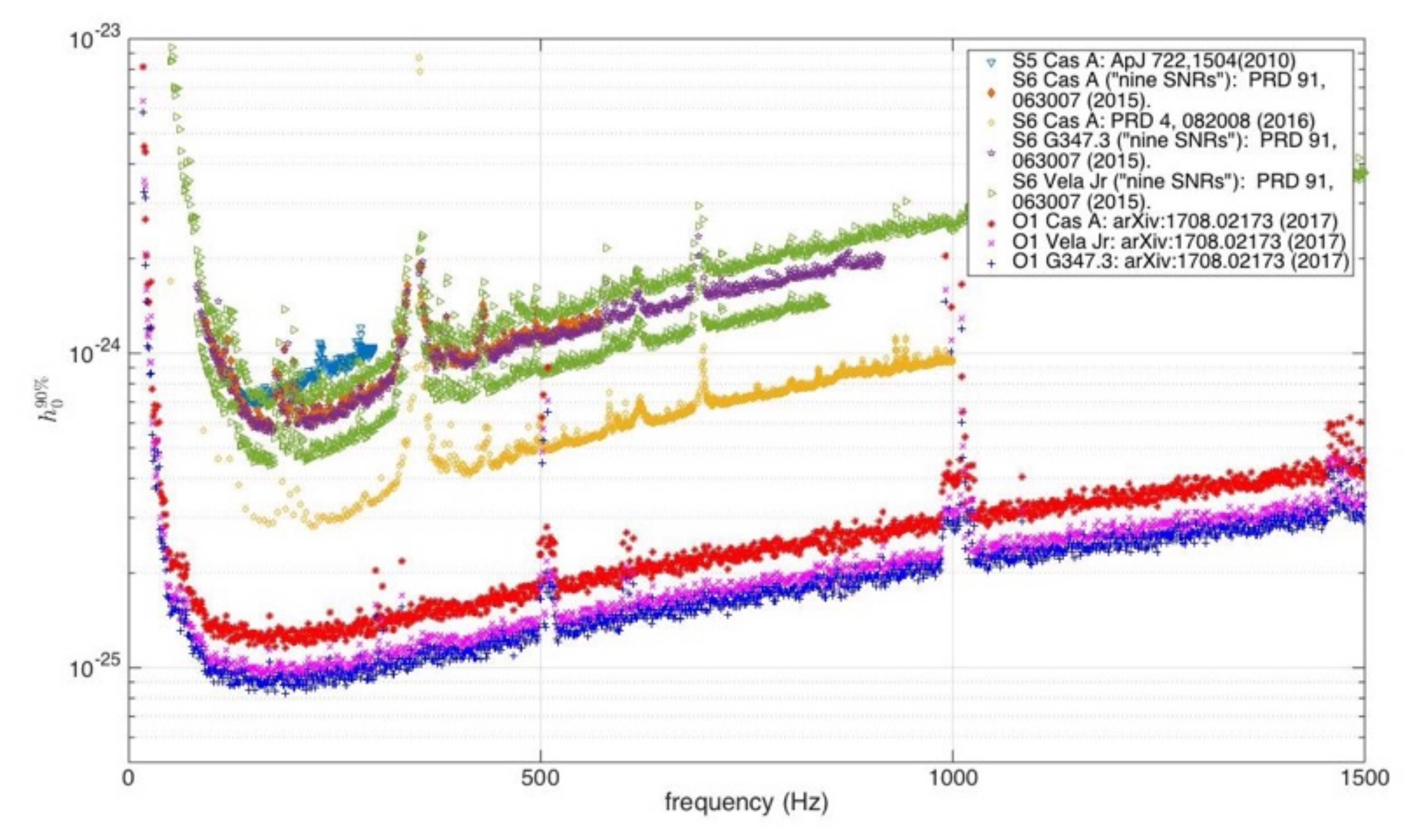
$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} f^2 \epsilon}{d} \, .$$

Final chosen set-ups

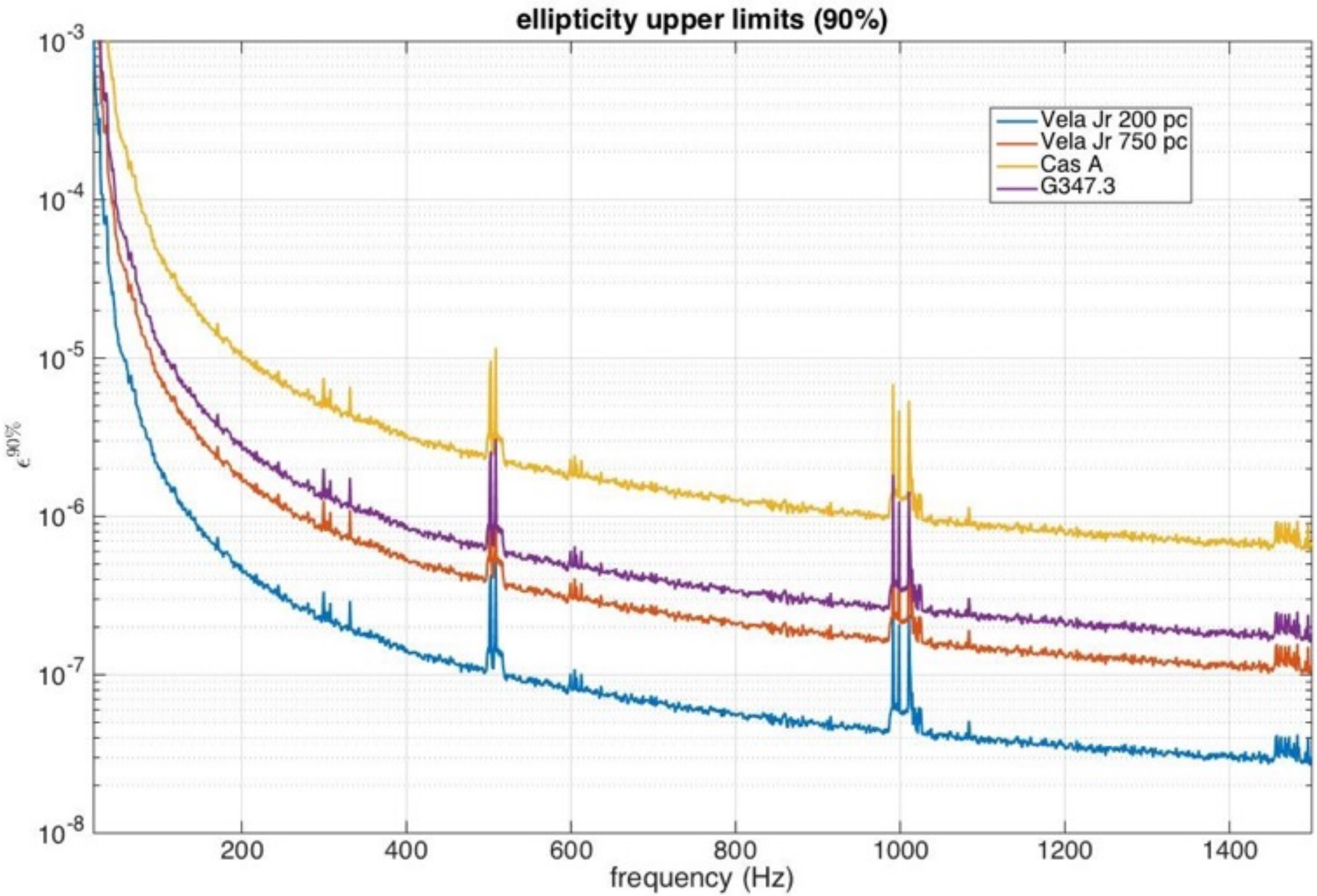


Profile of the chosen set-ups

Targets	$\delta f [Hz]$	$\delta \dot{f}$ [Hz/s]	$\delta \ddot{f}$ [Hz/s^2]	$\gamma^{(1)}$	$\gamma^{(2)}$	$T_{\rm coh}$	$\langle m \rangle$
Cas A	7.0×10^{-7}	4.0×10^{-12}	4.3×10^{-18}	4	20	10D	41.2%
Vela Jr	3.3×10^{-7}	1.4×10^{-12}	1.3×10^{-18}	8	20	15D	15.8%
G347.3	2.5×10^{-7}	6.4×10^{-13}	5.4×10^{-19}	8	10	20D	12.1%

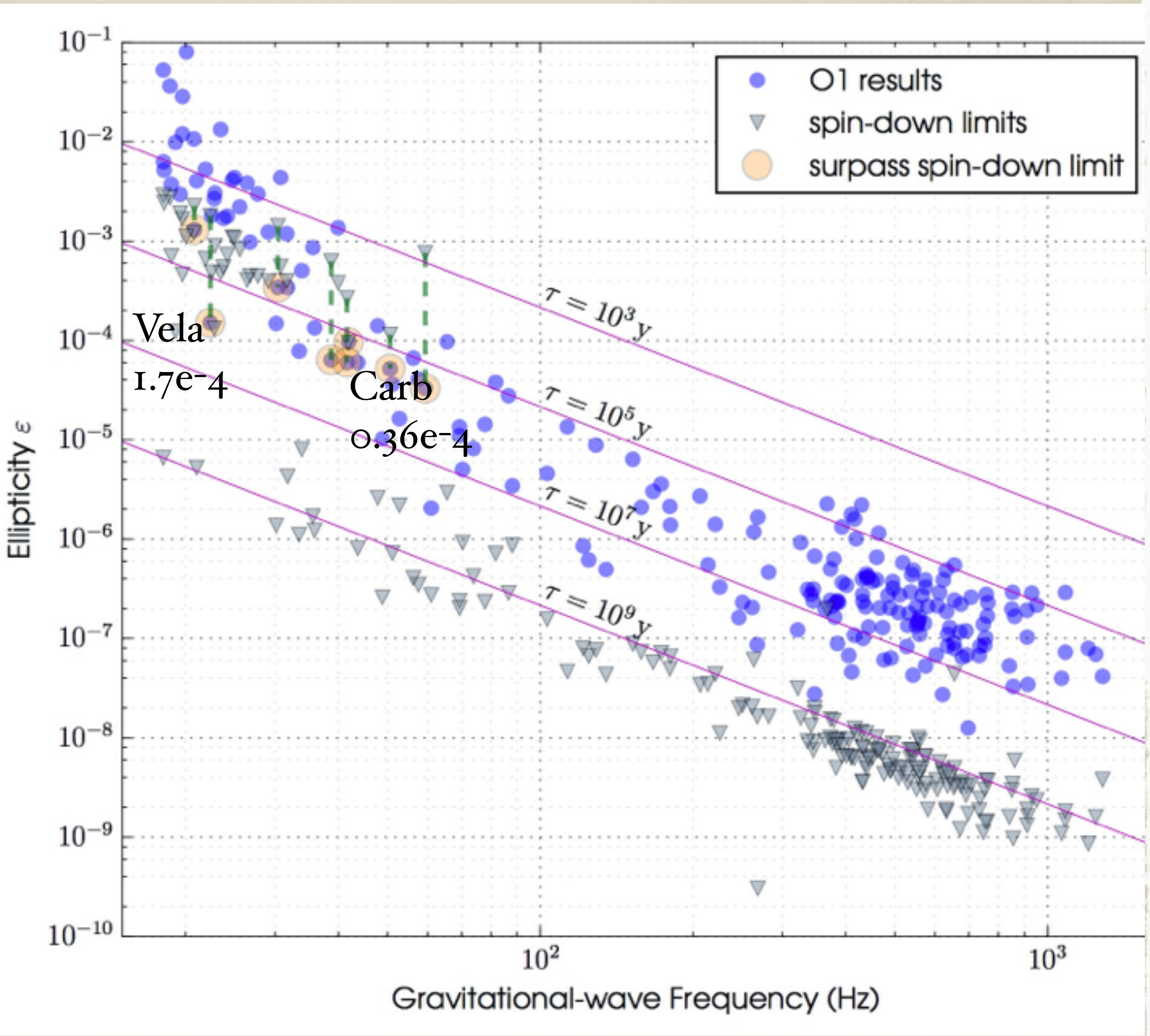


Ellipticity upper limits(90%)



Ellipticity upper limits from known pulsars

The Astrophysical Journal, 839:12, Apr. 2017.



Vela
1.7e-4

Carb
0.36e-4

Spin-down upper limit of ellipticity

Emitting CWs at a fraction x of the spindown energy:

$$h_0^{\mathrm{sd}} = \frac{1}{D}\sqrt{\frac{5GI}{2c^3}\frac{|\dot{f}|}{f}} \qquad \text{and} \qquad h_0 \leq h_0^{\mathrm{sd}}\,.$$

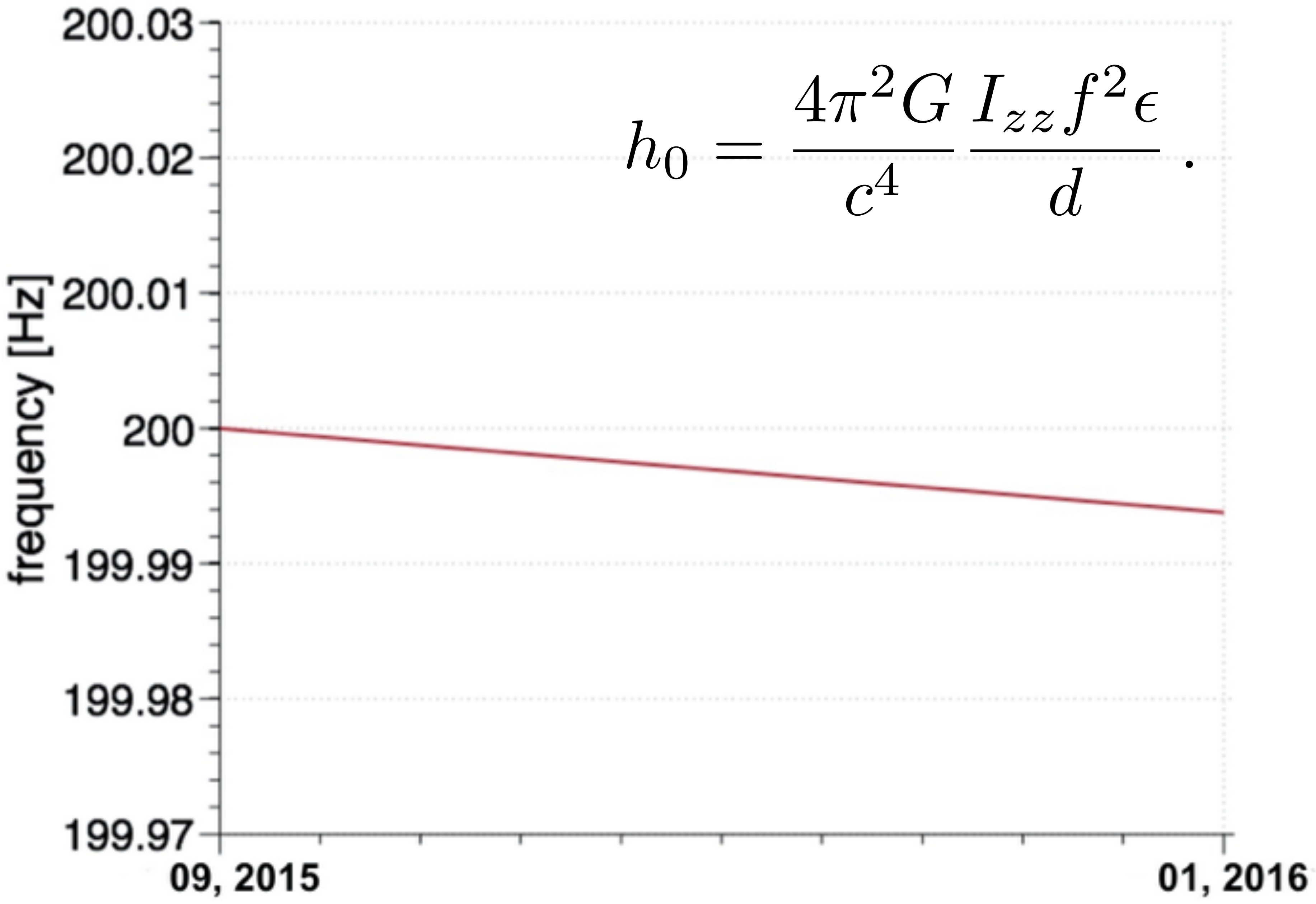
$$\varepsilon_x^{\mathrm{sd}} = \sqrt{\frac{5c^5}{32\pi^4 G}\frac{x|\dot{f}|}{I f^5}}\,.$$

Age upper limit of ellipticity

$$\tau_{\mathrm{c}} = \frac{1}{n-1}\frac{f}{|\dot{f}|}\,.$$

$$\varepsilon^{\mathrm{age}} = \frac{c^2}{4\pi^2 f^2}\sqrt{\frac{5c}{2(n-1)GI\tau_{\mathrm{c}}}}$$

Spindown



$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} f^2 \epsilon}{d} \, .$$

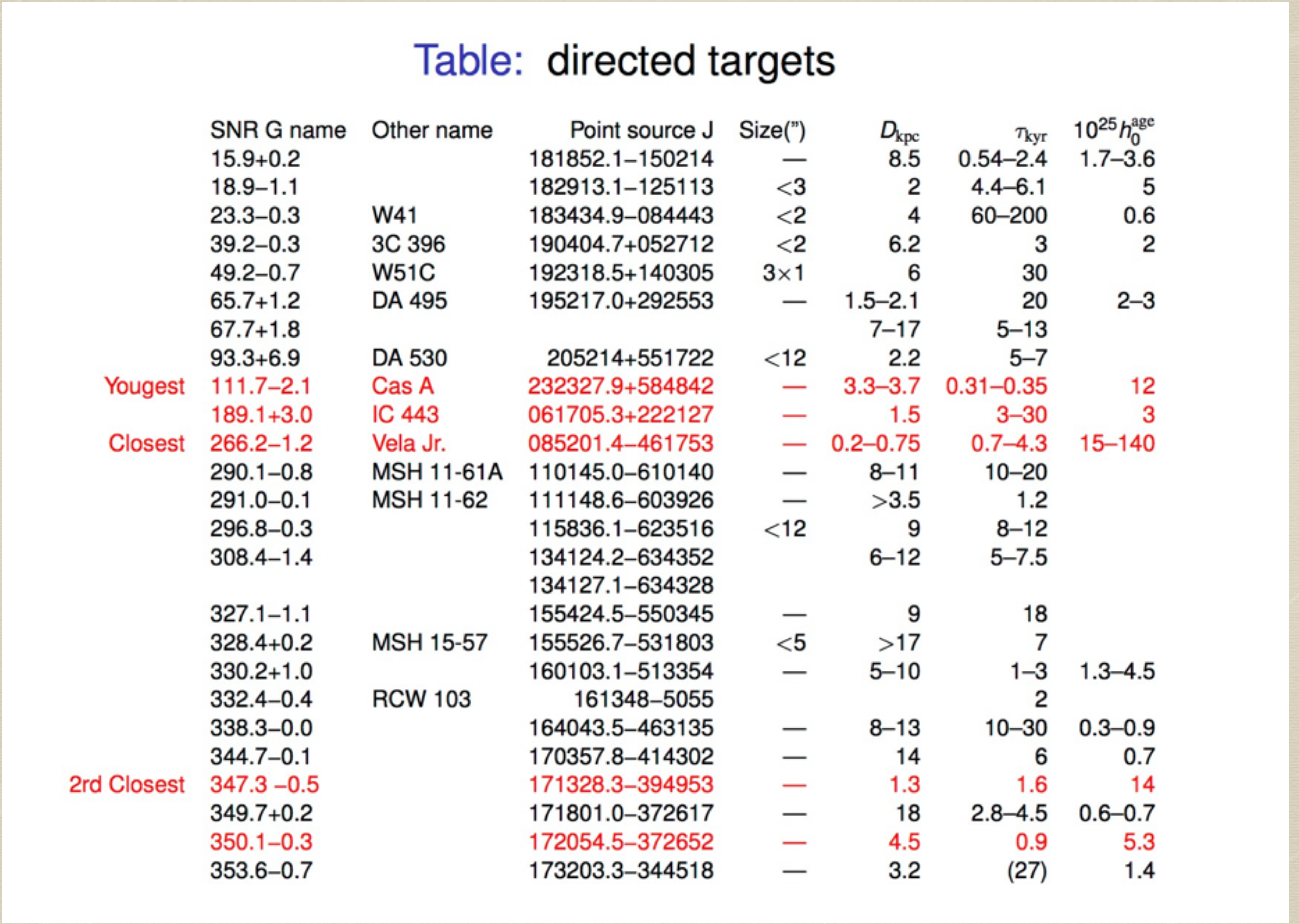


Table: directed targets

	SNR G name	Other name	Point source J	Size(")	D_{kpc}	τ_{kyr}	$10^{25} h_0^{\mathrm{age}}$
	15.9+0.2		181852.1–150214	—	8.5	0.54–2.4	1.7–3.6
	18.9–1.1		182913.1–125113	<3	2	4.4–6.1	5
	23.3–0.3	W41	183434.9–084443	<2	4	60–200	0.6
	39.2–0.3	3C 396	190404.7+052712	<2	6.2	3	2
	49.2–0.7	W51C	192318.5+140305	3×1	6	30	
	65.7+1.2	DA 495	195217.0+292553	—	1.5–2.1	20	2–3
	67.7+1.8				7–17	5–13	
	93.3+6.9	DA 530	205214+551722	<12	2.2	5–7	
Yougest	111.7–2.1	Cas A	232327.9+584842	—	3.3–3.7	0.31–0.35	12
	189.1+3.0	IC 443	061705.3+222127	—	1.5	3–30	3
Closest	266.2–1.2	Vela Jr.	085201.4–461753	—	0.2–0.75	0.7–4.3	15–140
	290.1–0.8	MSH 11-61A	110145.0–610140	—	8–11	10–20	
	291.0–0.1	MSH 11-62	111148.6–603926	—	>3.5	1.2	
	296.8–0.3		115836.1–623516	<12	9	8–12	
	308.4–1.4		134124.2–634352 134127.1–634328		6–12	5–7.5	
	327.1–1.1		155424.5–550345	—	9	18	
	328.4+0.2	MSH 15-57	155526.7–531803	<5	>17	7	
	330.2+1.0		160103.1–513354	—	5–10	1–3	1.3–4.5
	332.4–0.4	RCW 103	161348–5055			2	
	338.3–0.0		164043.5–463135	—	8–13	10–30	0.3–0.9
	344.7–0.1		170357.8–414302	—	14	6	0.7
2rd Closest	347.3 –0.5		171328.3–394953	—	1.3	1.6	14
	349.7+0.2		171801.0–372617	—	18	2.8–4.5	0.6–0.7
	350.1–0.3		172054.5–372652	—	4.5	0.9	5.3
	353.6–0.7		173203.3–344518	—	3.2	(27)	1.4

Search range

$$\ddot{f} = n\dot{f}^2/f \qquad\qquad \tau_c = \frac{1}{n-1}\frac{f}{|\dot{f}|}$$

The conservative search ranges are then

$$\begin{cases} |\dot{f}| < & f/\tau_c \\ |\ddot{f}| < & 5f/\tau_c^2\,, \end{cases}$$

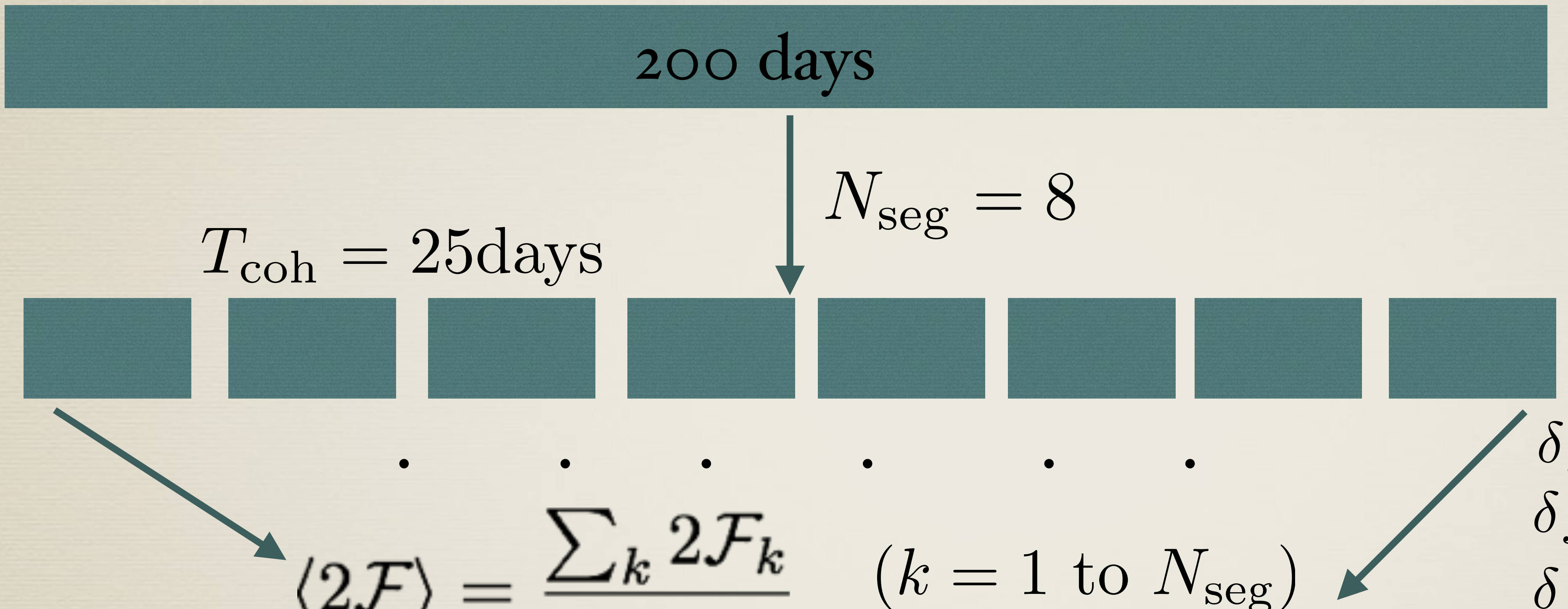
Note that these maximum ranges correspond to different n values, namely 2 and 5. This is physically inconsistent for any single source but it ensures the broadest prior range over the search values now, allowing for deviations from the constant braking index model in the past evolution of the star. Pure GW emission n=5, pure electromagnetic emission n=3

Semi-coherent method

PRD **58** 063001 (1998)
PRL **103** 181102 (2010)

For directed searches: one sky position (α, δ)

$$T_{\rm obs} = 200{\rm days}$$



200 days

$$N_{\rm seg} = 8$$

$$T_{\rm coh} = 25{\rm days}$$

$$\langle 2\mathcal{F} \rangle = \frac{\sum_k 2\mathcal{F}_k}{N} \qquad (k = 1 \text{ to } N_{\rm seg})$$

$$\begin{aligned} &\delta f \propto 1/T_{\rm coh}, && \\ &\delta \dot{f} \propto 1/T_{\rm coh}^2, && \gamma^{(1)} \\ &\delta \ddot{f} \propto 1/T_{\rm coh}^3, && \gamma^{(2)} \end{aligned}$$

$$N_{\rm temp} \propto N_{\rm seg} T_{\rm coh}^6$$

Still Expensive!