Gauge Invariance and Spontaneous Symmetry Breaking

What happens when the transformation is a gauge invariance rather than a symmetry? Obviously there is no "breaking" — nothing to break! Except:

$$A_\mu \rightarrow \omega A_\mu + \xi_\mu \quad \phi \rightarrow e^{i\lambda} \phi$$

If $\lambda$ is a constant this doesn't remove any parts of $A$ — this is a true global symmetry and does what global symmetries do: puts particles in their multiplets, enforces charge conservation, etc.

Can this be broken? Sure! The usual potential that causes the field to get a VEV works the usual way. But something interesting happens —

Previously we used the Ward Identity to deduce the existence of a massless particle, the NGB.

$$\not{k}_n \langle 0 | T\{\bar{\psi}(k)\Theta_1 \ldots \Theta_n\} | 10 \rangle = \sum_{x} e^{i k_x} \langle 0 | \bar{\psi}(x) \Theta_1 \ldots \Theta_n | 10 \rangle$$

The same should be true here — but there is a loophole! We discovered that our description of gauge bosons includes fields that create and annihilate modes that are unphysical — they don't couple to physical, gauge invariant observables.

Could it be that the NGB is one of these? Not only "could be" but "always is"!
Easy way to see this: the current is modified \( J^u = i D^u \Phi^* \psi + c.c. \)

\[ D^u \Phi = (\partial^u + i e A^u) \Phi \]

If we write \( \Phi \sim u e^{i \theta} \)

\[ J^u = u^2 (\partial^u \theta - e A^u) \]

We still have to choose a gauge. I will choose Landau Gauge \( \partial^u A = 0 \). Why? \( \partial^u A \) is a (Lorentz) scalar part of \( A \) and can mix with other scalars. The remaining parts of \( A^u \) cannot mix in this way (they will be connected by Lorentz).

Now consider \( \partial^u J^u = 0 \to \Box \theta = 0 \), and \( \theta \), the phase in \( \Phi \) is our NGB.

But \( \theta \) doesn't couple to \( A^u \) (by gauge choice) and doesn't appear in Yang invariant operators \( \ast \) (Thus requires some effort to show if the radial, or "Higgs" modes, are included).

If \( \theta \) is unphysical, why not get rid of it? Perfectly possible: "Unitary Gauge". But this gauge breaks the charge symmetry, so not always a happy situation.

But shouldn't the gauge field give rise to massless poles? After all, the current is what the gauge field couples to.
No massless poles — the gauge field creates and annihilates massive spin 1 quanta.

"HIGGS MECHANISM"

Ex. Pseudo-Scalar Electrodynamics in $R_\xi$ gauge

Ignoring the radial mode, the Lagrangian may be written

$$e^2 L = \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + (\partial_\mu \theta + A_\mu)^2 M^2 + L_\xi$$

where

$$L_\xi = -\frac{1}{2\xi} (\partial_\mu A - M^2 \xi \theta)^2$$

fixes the gauge

$\xi \to 0$ Landau Gauge, $\xi \to \infty$ is Unitary Gauge

1) Expand the terms and show there is no mixing between $A$ and $\theta$; that is, $L = L_\theta + L_A$

Demonstrate that $\theta$ is a free massive particle.

What about $A$?

2) Compute the propagators for $\theta$ and $A$.

3) Verify the Ward Identity.
The Standard Model

All the requisite elements are in place—we need to
1) Specify the gauge degrees of freedom (and the associated global symmetries)
2) Specify the "matter" fields—all the non-gauge degrees of freedom
3) Any global symmetry constraints
4) All operators of dimension 4 ("relevant")

1) $SU(3)$ color
   $SU(2)$ "weak isospin" or "$SU(2)_L$
   $U(1)$ hypercharge

   $SU(3) \times SU(2) \times U(1)_Y$

2) We need to specify the matter field representations under these groups and Lorentz
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<th>SU(3)</th>
<th>SU(2)</th>
<th>U(1)</th>
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<td>( \frac{1}{2} )</td>
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We will NOT impose any additional symmetries.

Dimension \([ F_{\mu \nu} ] = 2 \) \quad \text{[J]} = 1

\([ \Psi_{\text{ferm}} ] = \frac{3}{2} \)

\([ H ] = 1 \)

1. Only gauge fields

\[ \sum_{\alpha} \left( -\frac{1}{4} F_{\mu \nu}^{\alpha} F^{\alpha}_{\mu \nu} \right) = -\frac{1}{2} \left( F, F \right) \]

Usual kinetic term — sum over each generator

\[ F_{\mu \nu}^{(\alpha)} = \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} F^{\beta (\alpha)} \]

\(( F, \tilde{F} \) but \(( F, \tilde{F} ) = \partial_{\mu} k^{\mu} \)

Total derivatives don't change the EOM →

No effect classically
Conventions:
\[ A_{\mu} = A^a_{\mu} T^a \quad g = e^{i \lambda^a T^a} \]

\[ A_{\mu} \rightarrow g A_{\mu} g^{-1} + g i \partial_{\mu} g^{-1} \]

\[ \phi \rightarrow g \phi \]

Then \[ [D\phi] \rightarrow g D\phi \quad Tr T^a T^b = \frac{1}{2} \delta_{ab} \]

and \[ D\phi = \partial_{\mu} \phi - i A_{\mu} \phi \]

\[ \mu_{\text{gauge}} = -\frac{1}{4} g^2 \left( F^{(i)}_{\mu\nu} F^{(i)}_{\mu\nu} \right) \]

\[ i = \text{SU}(3), \text{SU}(2), \text{U}(1) \]

The "gauge coupling"

\[ L_{\text{Higgs}} = \frac{1}{2} (D^\dagger \psi D \psi) - \frac{\lambda}{2} \left[ H^\dagger H - \frac{v^2}{2} \right]^2 \]

(NB more derivatives or higher powers are \( \delta > 4 \))

\[ L = \psi^+ \Gamma^{\mu} \overline{\psi} \nabla_\mu \psi \]

"\( L \)"

\( \psi \) is a large column of all the fermions.
Finally we may form operators of charm 4 involving 2 fermions and H. the gauge charges tell us there are only 3 types of terms.

\((QH)^c\), \((QH^cD^c)\), \((LH^c)^c\)

these are called “Yukawa” couplings.

That’s all! EXCEPT, each of fermions is replicated 3 times \(\rightarrow\) there are 3 families.
Gauge Couplings and Masses in EW.

\[ A^a = A^a_\mu T^a = A^a_\mu Q^a \] on Higgs doublet

\[ B = BY = B_{1/2} \]

Then \[ D^\mu = \partial^\mu - iA^\mu - iB^\mu \]

so \[ D^\mu H^+ D^\mu H = \left| \left( A^\mu + B^\mu \right) \left( 0 \right) \right|^2 \]

\[ = \frac{v^2}{2} \left( \frac{(A^a_\mu i A^a_\mu)}{4} \right) \left( (A^a_\mu + i A^a_\mu) \right) + \frac{v^2}{2} \left( \frac{(A^3_\mu^2 B^3_\mu)}{4} \right) \]

The gauge kinetic terms are

\[ -\frac{1}{4g^2} \left( F^2_{A1} + F^2_{A2} + F^2_{A3} \right) - \frac{1}{4g^2} F^2_B \]

Clearly \( A^3 \) and \( B \) mix while \( A^1 \) and \( A^2 \) do not.

\( A^1 \) and \( A^2 \) are degenerate fields with masses \( g v / \sqrt{2} \)

We combine them into a complex field

\[ W^\pm_\mu = \frac{1}{\sqrt{2}} \left( A^1_\mu + i A^2_\mu \right) \]

Defining \( T^\pm = T^1 \pm iT^2 \) (No \( V/A \) !)

The coupling to \( T^\pm \) currents is \( g \sqrt{2} \)
The neutral states are more interesting.

Clearly one boson stays massless: when
\[ A_3^\mu = B_3^\mu \]. This is the photon, and the
current it couples to is called Electromagnetic
charge. Since
\[ D_\mu = \partial_\mu - i (A_3^\mu T^3 + B_3^\mu Y) \]
\[ = \partial_\mu - i \left[ Y \mu Q + z_\mu (T_3 - \beta Q) \right] \]

when \( A_3^\mu = B_3^\mu \) this is \( Q \) so
\[ Q = T^3 + Y \]
and justifies calling \( W^+ \)
"charged."

What is the electromagnetic coupling?
\[ \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \rightarrow \]
\[ e = \frac{g g'}{\sqrt{g^2 + g'^2}} \]

What about the \( Z \) coupling?
\[ -\frac{1}{4} \left\{ \frac{1}{g^2} F_3^2 + \frac{1}{g'^2} F_{12}^2 \right\} = -\frac{1}{4} \left\{ \frac{1}{e^2} \frac{F_3^2}{Y} + \frac{1}{e^2} \frac{F_{12}^2}{z^2} \right\} \]

and \( A_3 = Y + Z(1 - \beta) \) \( B = Y - \beta Z \)
so eliminating the cross term \[ \beta = \frac{g'^2}{g^2 + g'^2} \]

and then \[ x = \sqrt{g^2 + g'^2} \]

and \[ m_Z^2 = \frac{g g'}{\sqrt{g^2 + g'^2}} \]

\[ \left\{ \sqrt{g^2 + g'^2} \right\} \]
It is conventional to define

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \quad \text{so} \quad \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

or

$$g = \frac{e}{\sin \theta_w}, \quad g' = \frac{e}{\sin \theta_w}$$

and

$$\frac{M_w}{M_z} = \cos \theta_w$$

Couplings:
- $Y$ to $Q = T^2 + Y$ strength $e$
- $W^\pm$ to $T^\pm$ strength $\frac{g}{\sqrt{2}}$
- $Z$ to $T^2$ strength $\sqrt{g^2 + g'^2}$
Global Symmetries

Notice we did NOT impose any global symmetries on the model. But nevertheless some symmetries seem to appear:

From our table we can assign

<table>
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<tr>
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The only terms that might violate these are the Yukawas — but they don’t!

“Lepton Number” and “Baryon Number”

These are called “accidental symmetries” — we can’t violate them even if we want too! The constraints of gauge invariance and dim 4, along with our particle content, enforce this. HOWEVER it’s only true at dim 4 or less.

EX: Construct operators that violate L and B (especially those of low dimension)
There is a subtlety—although these transformations give rise to conserved currents at the classical level, they are actually violated by EW quantum effects.

Firstly, even at 1-loop, there is an effect called (unfortunately) the anomaly:

\[ J^u_B \left\{ \begin{align*}
\mathcal{E}_\mu J^u_B &= \frac{g^2}{32\pi^2} N_{1/2} (F, \vec{F}) \\
\mathcal{E}_\mu J^u_L &= 0
\end{align*} \right. \]

where \( F_{\mu\nu} \) includes the SU(2) gauge fields

BJL not conserved! Why doesn't the proton decay?

1) \((F, \vec{F})\) is a total derivative and shouldn't matter. This isn't quite true, but almost—

the effects are strictly non-perturbative

\[ -\frac{g_1^2}{8\pi^2} e^{-\frac{\sqrt{2}g_1}{g^2}} \]

This is unobservably small for the weak interaction but is significant for QCD

\[ \rightarrow \text{ should not ignore } (\tilde{G}, \hat{G}) \text{ operator} \]

\[ \therefore \text{ New coupling } \Theta(G, \hat{G}) \quad \text{Expt. } \tilde{\Theta} < 10^{-9} \quad \text{"Strong CP Problem"} \]
2) Actually $\Delta B=3$ so proton is stable anyway.

3) NB B-L conserved "non-anomalous"

But anyway, these accidental symmetries are important — if they were not accidental we would expect operators of $\Delta m \leq 4$ to receive contributions from short distances and would no longer have an explanation of the smallness of $B$ and $L$ violation.

$\Delta m \leq 4$ operators are the ones sensitive to short distance physics.
Inadequacies of the SM

1) Doesn't include gravity
   \[ \text{Gravity is described by a dimensionful parameter, } M_p \sim 10^{19} \text{ GeV. It is possible to postpone any issue to a high scale } \] (see later)

2) Doesn't have massive neutrinos

do last → ii) Can always add degrees of freedom

\[
\begin{array}{cccc}
\text{N}^c & 1 & 1 & 1 \\
\end{array}
\]

ii. Totally neutral. A "singlet neutrino" or often a "sterile neutrino"

Lagrangian? \( (LH)N^c \) New Yukawa

\[
\text{And: } M N^c N^c
\]

Only possible since \( N^c \) is a singlet.

This operator violates the global lepton symmetry (but that's OK)

Ex: If \( M \) very large, we may integrate it out. What does this do?
1) No new light degrees of freedom.
   → Neutrinos are automatically massless unless from higher-d operators
   (\"success\"?)

Ex: What is the lowest dim operator for 1) mass?

3) Doesn't have a dark matter candidate.

   Strong observational evidence (Galactic Rotation Curves, CMB, BAL, ILS, Grav Lensing) implies the existence of non-SM objects behaving today like non-relativistic material.

Remarkably (but not uniquely) new degrees of freedom near the TeV scale fit the bill.

4) Dark Energy
   No idea how dark energy fits in

5) Strong CP problem

   the operator $\frac{G^2}{32\pi^2}$ gives rise to a new coupling constant ($\overline{\Theta} = \Theta + \arg \det M$) that violates CP
If we were to set the Yukawas to zero we would have a large global symmetry. We might also ignore the weak gauge couplings.

Example: 

$Q^3$ families $\rightarrow SU(3)$

$U^3 D^3 \rightarrow$ families $\rightarrow SU(0)$

These couplings break the symmetry in very specific ways (GIM).

The higher dimension (dim 6) operators break the symmetries in different ways, giving rise to flavor effects that don't arise in the SM (except possibly at high loop order).

$\rightarrow$ The scale of these new ops are strongly constrained.
Multi TeV for essentially all of them
Multi 10 TeV for many (most?)
Multi 1000 TeV for some!

7) Hierarchy -

Recall: the physics at high scales contributes to the coefficients of the operators in our effective field theory (called the "SM")

These operator coefficients give us (imprecise) information about high scales

Ex: Absence of proton decay tells us the dim 6 operator has a coefficient less than \( \frac{1}{n^2} \leq \left( \frac{1}{10^{16} \text{GeV}} \right)^2 \)

๐. New physics respects symmetry up to very high scale.

Similar conclusions apply the flavor higher dimension operators

But this leads to a puzzle - there are operators in the SM of dim < 4 - these have coefficients that depend sensitively on the short distance theory
There are 2 such operators:

1. The identity. Normally we adjust the vacuum energy to remove this. But in cosmology (or gravity more generally) this operator has consequences and can be measured

\[ C_2 \sim \Lambda^4 \rho_c \sim \pi \times 10^{-11} \text{eV}^4 \]

\[ \sim (1 \text{ TeV})^4 \times \pi \times 10^{-59} \]

2. \( H^+ H \) (175 GeV)^2