Gabriela Barenboim presents

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PRESENTS

A NU HOPE
EPISODE I

STAR WARS: EPISODE I: PHANTOM MENACE / ATTACK OF THE CLONES / RETURN OF THE JEDI

Produced by George Lucas
Directed by George Lucas
Written by George Lucas
Music by John Williams

Revised Extended Version 10/03/08
Is The Whole Universe made of—
Electrons Protons Neutrons?

NO!
Electrons Protons Neutrons
are rareties!

For every one of them, the universe contains a billion neutrinos $\nu$!
Passing through each person on earth every second:
One hundred trillion neutrinos from the sun.

The sun shines because of nuclear fusion in its core.
This fusion produces—
• Energy, including visible light
• Neutrinos
• The atoms more complicated than hydrogen
Almost all neutrinos zipping through us do nothing at all.

Typically, a solar neutrino would have to zip through $10,000,000,000,000,000,000,000$ people before doing anything.

The probability that a particular solar neutrino will interact as it zips through one of us is $1 / 10,000,000,000,000,000,000,000,000$.
Are Neutrinos Important to Our Lives?

If there were no νs, the sun and stars would not shine.

• No energy from the sun to keep us warm.
• No atoms more complicated than hydrogen.
  No carbon. No oxygen. No water.
  No earth. No moon. No us.

No νs is very BAD news.
Summer Schools (if existed) were VERY short …..

$\beta$ decay was supposed to be a two body decay

$$n \rightarrow p^+ + e^-$$

$$E_e = \frac{m_n^2 + m_e^2 - m_p^2}{2m_n}$$
Studies of $\beta$ decay revealed a continuous energy spectrum.

Another anomaly was the fact that the nuclear recoil was not in the direction opposite to the momentum of the electron.

The emission of another particle was a probable explanation of this behaviour, but searches found no evidence of either mass or charge.
Fermi postulated a theory for $\beta$ decay in terms of spinors

$$H_{ew} = \frac{G_F}{\sqrt{2}} \bar{\Psi}_p \gamma_\mu \Psi_n \bar{\Psi}_e \gamma_\mu \Psi_\nu$$
Standard Model of Particle Physics

Gauge Theory based on the group:

\[ SU(3) \times SU(2) \times U(1) \]

\[ SU(3) \Rightarrow \text{Quantum Chromodynamics} \]

Strong Force (Quarks and Gluons)

\[ SU_L(2) \times U(1) \Rightarrow \text{ElectroWeak Interactions broken to } U_{EM}(1) \]

by HIGGS
\[ SU_L(2) \times U_Y(1) \Rightarrow U_{EM}(1) \]

**Force Carriers:** \( W^\pm, \, Z^0 \) and \( \gamma \) masses: 80, 91 and 0 GeV

**Quark, SU(2) doublets:**
\[
\begin{pmatrix}
  u \\ d
\end{pmatrix}_L, \quad \begin{pmatrix}
  c \\ s
\end{pmatrix}_L, \quad \begin{pmatrix}
  t \\ b
\end{pmatrix}_L
\]

- **Up-quark, SU(2) singlets:** \( u_R, c_R, t_R \)
- **Down-quark, SU(2) singlets:** \( d_R, s_R, b_R \)

**Lepton, SU(2) doublets:**
\[
\begin{pmatrix}
  \nu_e \\ e
\end{pmatrix}_L, \quad \begin{pmatrix}
  \nu_\mu \\ \mu
\end{pmatrix}_L, \quad \begin{pmatrix}
  \nu_\tau \\ \tau
\end{pmatrix}_L
\]

- **Neutrino, SU(2) singlets:** \(- - -\)
- **Charge lepton, SU(2) singlets:** \( e_R, \mu_R, \tau_R \)
Electron mass comes from a term of the form 

\[ \bar{L}\phi e_R \]

Absence of \( \nu_R \) forbids such a mass term (dim 4) for the Neutrino.

Therefore in the SM neutrinos are massless and hence travel at speed of light.
Interactions:

Charge Current (CC)

\[ W^- \rightarrow l^-_\alpha + \bar{\nu}_\alpha \]

Neutral Current (NC)

\[ Z^0 \rightarrow \nu_\alpha + \bar{\nu}_\alpha \]

\[ Z^0 \rightarrow l^-_\alpha + l^+_\alpha \]

\[ \Gamma(Z^0 \rightarrow f + \bar{f}) = K \frac{g^2 Z M_Z}{48 \pi} \left[ |c_V^f|^2 + |c_A^f|^2 \right] \]

\[ \alpha = e, \mu, \text{ or } \tau \]
Invisible width of $Z$ plus other data from LEP:

$$Z^0 \rightarrow \nu \bar{\nu}$$

Implies $N_\nu = 2.99 \pm 0.01$

Three Active Neutrinos!!!    Sterile Neutrinos don’t couple to $Z^0$
Note That

\[ W^- \rightarrow l^-_\alpha + \bar{\nu}_\alpha \]

Implies

\[ \nu_e, \nu_\mu, \nu_\tau \]

\[ e, \mu, \tau \]
Observed

small $L/E$  ($<< 1/\delta m^2$)
Not Observed

small $L/E$ \quad (<< 1/$\delta m^2$)
Observed

neutrino beam (not anti-neutrino beam)

Not Observed

large $E \gg m_\nu$
Standard Model

\[ W^- \rightarrow l^-_\alpha \bar{\nu}_\alpha \]
\[ Z^0 \rightarrow \nu_\alpha \bar{\nu}_\alpha \]

couplings conserve the Lepton Number \( L \) defined by—
\[ L(\nu) = L(l^-) = -L(\bar{\nu}) = -L(l^+) = 1. \]

Actually \( L_e, \ L_\mu, \) and \( L_\tau \) separately
Left Handed Nature of The Neutrino

Produce Left-Handed Neutrinos and Right-Handed Anti-Neutrinos

What about the RH neutrinos and LH anti-neutrino ????
There exist three fundamental and discrete transformations in nature:

- **Parity** \( \mathcal{P} \) \( \vec{x} \rightarrow -\vec{x} \)
- **Time reversal** \( \mathcal{T} \) \( t \rightarrow -t \)
- **Charge conjugation** \( \mathcal{C} \) \( q \rightarrow -q \)

\( \mathcal{P}, \mathcal{T} \) and \( \mathcal{C} \) are conserved in the classical theories of mechanics and electrodynamics!

\[ \mathcal{CPT} \leftrightarrow \text{Lorentz invariance} \oplus \text{unitarity}: \text{is an essential building block of field theory} \]

\[ \mathcal{CPT}: L \text{ particle} \leftrightarrow R \text{ antiparticle} \]

Neutrinos in the MSM are massless and exist only in two states: particle with negative helicity and antiparticle with positive one: **Weyl fermion**
\( \mathcal{P} : \) L particle \( \leftrightarrow \) R particle

Parity violation is nowhere more obvious than in the neutrino sector: the reflection of a left-handed neutrino in a mirror is nothing!
Summary of $\nu$’s in SM:

Three flavors of massless neutrinos

$$W^- \rightarrow l^-_\alpha + \bar{\nu}_\alpha$$
$$W^+ \rightarrow l^+_\alpha + \nu_\alpha$$

$\alpha = e, \mu, \text{ or } \tau$

Anti-neutrino, $\bar{\nu}_\alpha$, has $+$ve helicity, Right Handed

Neutrino, $\nu_\alpha$, has $-$ve helicity, Left Handed

$\nu_L$ and $\bar{\nu}_R$ are CPT conjugates

massless implies helicity $=$ chirality
Beyond the SM

What if Neutrino have a MASS?

speed is less than c therefore time can pass

and

Neutrinos can change character!!!

What are the stationary states?

How are they related to the interaction states?
NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates \neq mass eigenstates

\[
\begin{pmatrix}
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\]

W’s produce \( \nu_\mu \) and/or \( \nu_\tau \)’s

but \( \nu_1 \) and \( \nu_2 \) are the states

that change by a phase over time, mass eigenstates.

\[
|\nu_j\rangle \rightarrow e^{-ip_j \cdot x} |\nu_j\rangle \quad p_j^2 = m_j^2
\]

\( \alpha, \beta \ldots \) flavor index \quad \( i, j \ldots \) mass index
Production:
\[ |\nu_\mu\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle \]

Propagation:
\[ \cos \theta e^{-i p_1 \cdot x} |\nu_1\rangle + \sin \theta e^{-i p_2 \cdot x} |\nu_2\rangle \]

Detection:
\[ |\nu_1\rangle = \cos \theta |\nu_\mu\rangle - \sin \theta |\nu_\tau\rangle \]
\[ |\nu_2\rangle = \sin \theta |\nu_\mu\rangle + \cos \theta |\nu_\tau\rangle \]

\[
\begin{pmatrix}
\nu_\mu \\
\nu_\tau
\end{pmatrix} = 
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\]

\[ P(\nu_\mu \rightarrow \nu_\tau) = | \cos \theta (e^{-i p_1 \cdot x})(-\sin \theta) + \sin \theta (e^{-i p_2 \cdot x}) \cos \theta |^2 \]
\[ P(\nu_\mu \rightarrow \nu_\tau) = | \cos \theta (e^{-ip_1 \cdot x})(- \sin \theta) + \sin \theta (e^{-ip_2 \cdot x}) \cos \theta |^2 \]

Same E, therefore \( p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E} \)

\[ e^{-ip_j \cdot x} = e^{-iEt} e^{-ip_j L} \approx e^{-i(ET - EL)} e^{-im_j^2 L/2E} \]

\[ P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta |e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E}|^2 \]

\[ P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E} \]

\( \delta m^2 = m_2^2 - m_1^2 \) and \( \frac{\delta m^2 L}{4E} \equiv \Delta \) kinematic phase:
\[ P(\nu_\mu \rightarrow \nu_\tau) = | \cos \theta (e^{-ip_1 \cdot x}) (- \sin \theta) + \sin \theta (e^{-ip_2 \cdot x}) \cos \theta |^2 \]

Same E, therefore \( p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E} \)

\[ e^{-ip_j \cdot x} = e^{-iEt} e^{-ip_j L} \approx e^{-i(Et - EL)} e^{-im_j^2 L/2E} \]

\[ P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta |e^{-im_2^2 L/2E} - e^{-im_1^2 L/2E}|^2 \]

\[ P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E} \frac{c^4}{\hbar c} \]
Appearance:

\[ P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E} \]

Disappearance:

\[ P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E} \]
\[ P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E} \]

Oscillation Length
\[ L_0 = \frac{4\pi E}{\delta m^2} \]

Fixed \( E_\nu \)

Dissappearance
\[ \langle P(\nu_\mu \rightarrow \nu_\mu) \rangle \]

Amplitude of Oscillation
\[ \uparrow \sin^2 2\theta \]
\[ \downarrow \]

\( \sigma = 0\% \)

\( L_0 \)
\[ \langle P(\nu_\mu \rightarrow \nu_\mu) \rangle = 1 - \sin^2 2\theta \left( \frac{\sin^2 \delta m^2 L}{4E} \right) \]

Spread \( E_\nu \)

\[ 1 - \sin^2 2\theta \left( \frac{1}{2} \right) = \cos^4 \theta + \sin^4 \theta \]

\( W^+ \rightarrow \mu^+ + \nu_1 \) probability \( \cos^2 \theta \)

\( W^+ \rightarrow \mu^+ + \nu_2 \) probability \( \sin^2 \theta \)

flavour fractions \( |\nu_1\rangle \) and \( |\nu_2\rangle \) during propagation remain unchanged

probability \( \nu_1 \) contains \( \nu_\mu \) is \( \cos^2 \theta \)

probability \( \nu_2 \) contains \( \nu_\mu \) is \( \sin^2 \theta \)

effectively incoherent mass eigenstates
Using the unitarity of the mixing matrix: 

\[ W_{\alpha\beta}^{jk} \equiv [V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k}] \]

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re}[W_{\alpha\beta}^{jk}] \sin^2 \left( \frac{\Delta m_{jk}^2 L}{4E_\nu} \right) \]

\[ \pm 2 \sum_{k>j} \text{Im}[W_{\alpha\beta}^{jk}] \sin \left( \frac{\Delta m_{jk}^2 L}{2E_\nu} \right) \]

For 2 families: 

\[ V_{MNS} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]

\[ P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left( \frac{\Delta m_{jk}^2 L}{4E_\nu} \right) \rightarrow \text{appearance} \]

\[ P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance} \]
Oscillation probabilities show the expected GIM suppression of any flavour changing process: they vanish if the neutrinos are degenerate
Probability for Neutrino Oscillation in Vacuum

\[ P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 = \]

\[ P_{\alpha\beta} = \sin^2 2\theta \; \sin^2 \left( \frac{\Delta m^2 L}{4E\nu} \right) \rightarrow \text{appearance} \]

\[ P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance} \]
Probability for Neutrino Oscillation in Vacuum

\[ P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 = \]

\[ P_{\alpha\beta} = \sin^2 2\theta \quad \text{appearance} \]

\[ P_{\alpha\alpha} = 1 - P_{\alpha\beta} \quad \text{disappearance} \]

\[ \left( \frac{\Delta m^2 L}{4E} \right) \left( \frac{1.27 \Delta m^2 (eV^2) L(km)}{E(GeV)} \right) \]

L/E becomes crucial !!!
Evidence for Flavor Change:

- Atmospheric and Accelerator Neutrinos with $L/E = 500 \text{ km}/\text{GeV}$

- Solar and Reactor Neutrinos with $L/E = 15 \text{ km}/\text{MeV}$

Neutrinos from Stopped muons $L/E = 2\text{ m}/\text{MeV}$ (Unconfirmed)
Atmospheric neutrinos

- Atmospheric neutrinos are produced by the interaction of cosmic rays ($p, \text{He}, \ldots$) with the Earth’s atmosphere:

1. $A_{\text{cr}} + A_{\text{air}} \rightarrow \pi^\pm, K^\pm, K^0, \ldots$
2. $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$,
3. $\mu^\pm \rightarrow e^\pm + \nu_e + \nu_\mu$;

- at the detector, some $\nu$ interacts and produces a charged lepton, which is observed.

A deficit was observed in the ratio $\mu/e$ events: Soudan2, IMB, Kamiokande
Atmospheric Neutrinos

Isotropy of the $\geq 2 \text{ GeV}$ cosmic rays + Gauss’ Law + No $\nu_\mu$ disappearance

$$\Rightarrow \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1.$$ 

But Super-Kamiokande finds for $E_\nu > 1.3 \text{ GeV}$

$$\frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 0.54 \pm 0.04.$$
Zenith angle distributions

$\nu_\mu \leftrightarrow \nu_\tau$

2-flavor oscillations

Best fit

$\sin^2 2\theta = 1.0$, $\Delta m^2 = 2.0 \times 10^{-3} \text{ eV}^2$

Null oscillation

---

Sub-GeV e-like

Sub-GeV $\mu$-like

Multi-GeV Multi-R $\mu$-like

Up stop $\mu$

Multi-GeV e-like

Multi-GeV $\mu$-like + PC

Multi-GeV Multi-R $\mu$-like

Up thru $\mu$

---

$\approx 13000 \text{ km}$

$\approx 500 \text{ km}$

$\approx 15 \text{ km}$
Half of the upward-going, long-distance-traveling $\nu_\mu$ are disappearing.

Voluminous atmospheric neutrino data are well described by —

$$\nu_\mu \rightarrow \nu_\tau$$

with —

$$\Delta m_{atm}^2 \approx 2.4 \times 10^{-3} \text{ eV}^2$$

and —

$$\sin^2 2\theta_{atm} \approx 1$$
L/E Analysis

- Oscillation, decay and decoherence models tested

\[
\begin{align*}
\chi^2_{\text{osc}} &= 83.9/83 \\
\chi^2_{\text{decy}} &= 107.1/83, \Delta\chi^2 = 23.2(4.8\sigma) \\
\chi^2_{\text{dec}} &= 112.5/83, \Delta\chi^2 = 27.6(5.3\sigma)
\end{align*}
\]
\[ p + p \rightarrow ^2H + e^+ + \nu_e \]
\[ p + e^- + p \rightarrow ^2H + \nu_e \]
\[ ^2H + p \rightarrow ^3He + \gamma \]
\[ ^3He + ^3He \rightarrow ^4He + 2p \]
\[ ^3He + ^4He \rightarrow ^7Be + \gamma \]
\[ ^7Be + p \rightarrow ^8B + \gamma \]
\[ ^8B \rightarrow ^8Be^* + e^+ + \nu_e \]
\[ ^8Be^* \rightarrow ^4He + ^4He \]
\[ ^7Be + e^- \rightarrow ^7Li + \nu_e \]
\[ ^7Li + p \rightarrow ^4He + ^4He \]
\[ ^3He + p \rightarrow ^4He + e^+ + \nu_e \]
Solar Spectrum:

\[ p + p \rightarrow ^2 H + e^+ + \nu_e \]
\[ \phi_{pp} = 5.94(1 \pm 0.01) \times 10^{10} \text{ cm}^{-2} \text{ sec}^{-1} \]

\[ ^7 Be + e^- \rightarrow ^7 Li + \nu_e \]
\[ \phi_{^7 Be} = 4.86(1 \pm 0.12) \times 10^9 \text{ cm}^{-2} \text{ sec}^{-1} \]

\[ ^7 Be + p \rightarrow ^8 B \rightarrow ^8 Be^* + e^- + \nu_e \]
\[ \phi_{^8 B} = 5.82(1 \pm 0.23) \times 10^6 \text{ cm}^{-2} \text{ sec}^{-1} \]

**Figure 1.** The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model [22]. For continuum sources, the neutrino fluxes are given in number of neutrinos cm\(^{-2}\) s\(^{-1}\) MeV\(^{-1}\) at the Earth's surface. For line sources, the units are number of neutrinos cm\(^{-2}\) s\(^{-1}\). Total theoretical uncertainties taken from column 2 of table 1 are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes (see table 1).
Ray Davis & John Bahcall

Neutrino Flavor Transitions!!!
Identical Solar Twins:

Flavor eigenstates:

Mass eigenstates:
Kinematical Phase:

\[ \delta m^2_\odot = 8.0 \times 10^{-5} \text{eV}^2 \]
\[ \sin^2 \theta_\odot = 0.31 \]

\[ \Delta_\odot = \frac{\delta m^2_\odot L}{4E} = 1.27 \frac{8 \times 10^{-5} \text{eV}^2 \cdot 1.5 \times 10^{11} \text{m}}{0.1-10 \text{MeV}} \]

\[ \Delta_\odot \approx 10^7 \pm 1 \]

Effectively Incoherent !!!
Vacuum $\nu_e$ Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot$$

where $f_1$ and $f_2$ are the fraction of $\nu_1$ and $\nu_2$ at production.

In vacuum $f_1 = c_0 \sum W U_{\alpha i}^* \nu_i \text{Prop}(\nu_i) U_{\beta i} W$

$$\langle P_{ee} \rangle = \cos^4 \theta_\odot + \sin^4 \theta_\odot = 1 - \frac{1}{2} \sin^2 2\theta_\odot$$

for pp and $^7\text{Be}$ this is approximately THE ANSWER.

$f_1 \sim 69\%$ and $f_2 \sim 31\%$ and $\langle P_{ee} \rangle \approx 0.6$
pp and $^7$Be

\[ \langle P_{ee} \rangle \approx 0.6 \]

\[ f_1 \sim 69\% \]
\[ f_2 \sim 31\% \]

\[ f_3 = \sin^2 \theta_{13} < 4\% \]
What about $^8 B$?

**SNO's CC/NC**

**CC**: $\nu_e + d \rightarrow e^- + p + p$

**NC**: $\nu_x + d \rightarrow \nu_x + p + n$

**ES**: $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$

$$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot$$

$$f_1 = \left( \frac{CC}{NC} - \sin^2 \theta_\odot \right) / \cos 2\theta_\odot$$

$$= (0.35 - 0.31) / 0.4 \approx 10$$
\[ \langle P_{ee} \rangle = \sin^2 \theta + f_1 \cos 2\theta_\odot \approx \sin^2 \theta_\odot = 0.31 \]

Wow!!! How did that happen???

energy dependence!!!
MSW

Coherent Forward Scattering:

Wolfenstein '78

MATTER EFFECTS
CHANGE THE NEUTRINO MASSES AND MIXINGS

Neutrino Spectrum in Sun

Mikheyev + Smirnov Resonance WIN '85
Neutrino Evolution:

\[-i \frac{\partial}{\partial t} \nu = H \nu\]

in the mass eigenstate basis

\[\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} \sqrt{p^2 + m_1^2} & 0 \\ 0 & \sqrt{p^2 + m_2^2} \end{pmatrix}\]

\[E = \sqrt{p^2 + m^2}\]

\[H = (p + \frac{m_1^2 + m_2^2}{4p})I + \frac{1}{4E} \begin{pmatrix} -\delta m^2 & 0 \\ 0 & \delta m^2 \end{pmatrix}\]

\[\delta m^2 = m_2^2 - m_1^2 > 0\]
in the flavor basis

$$\nu \rightarrow U\nu \text{ and } H \rightarrow UHU^\dagger$$

where $\nu = \begin{pmatrix} \nu_e \\ \nu_\sigma \end{pmatrix}$ and $U = \begin{pmatrix} \cos \theta_\odot & \sin \theta_\odot \\ -\sin \theta_\odot & \cos \theta_\odot \end{pmatrix}$

and therefore in flavor basis

$$H = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}$$

i.e. $\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}_{\text{mass}} \Rightarrow \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}_{\text{flavor}}$

$0 < \theta_\odot < \frac{\pi}{2}$
Coherent Forward Scattering:

$$\pm \sqrt{2} G_F N_e \delta_{ee}$$

$N_e$ is number density of electrons

$+(-)$ for neutrinos (anti-neutrinos)

Wolfenstein '78

Same for all active flavors, therefore overall phases

\[
\begin{pmatrix}
+ \sqrt{2} G_F N_e & 0 \\
0 & 0
\end{pmatrix} \rightarrow \frac{G_F N_e}{\sqrt{2}} I_2 + \frac{1}{2} \begin{pmatrix}
+ \sqrt{2} G_F N_e & 0 \\
0 & - \sqrt{2} G_F N_e
\end{pmatrix}
\]
Including Matter Effects in the Flavor Basis:

\[
H_{\text{flavor}} = \frac{1}{4E_\nu} \begin{pmatrix}
-\delta m^2 \cos 2\theta_\odot + 2\sqrt{2}G_FN_eE_\nu & \delta m^2 \sin 2\theta_\odot \\
\delta m^2 \sin 2\theta_\odot & \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_FN_eE_\nu
\end{pmatrix}
\]

Diagonalize by identifying with

\[
H_{\text{flavor}} = \frac{1}{4E_\nu} \begin{pmatrix}
-\delta m_N^2 \cos 2\theta_\odot^N & \delta m_N^2 \sin 2\theta_\odot^N \\
\delta m_N^2 \sin 2\theta_\odot^N & \delta m_N^2 \cos 2\theta_\odot^N
\end{pmatrix}
\]

Masses and Mixings in MATTER: \( \delta m_N^2 \) and \( \theta_\odot^N \)

\[
\begin{align*}
\delta m_N^2 \cos 2\theta_\odot^N &= \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_FN_eE_\nu \\
\delta m_N^2 \sin 2\theta_\odot^N &= \delta m^2 \sin 2\theta_\odot
\end{align*}
\]

Notice:
(1) Possible zero when \( \delta m^2 \cos 2\theta_\odot = 2\sqrt{2}G_FN_eE_\nu \)
(2) the invariance of the product \( \delta m^2 \sin 2\theta_\odot \)
\( \nu_e \) disappearance in Loooon Block of Lead:

\[
1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta^N_\odot \sin^2 \Delta_N
\]

\[
\Delta_N = \frac{\delta m_N^2 L}{4E}
\]

same form as vacuum
The Solution:

\[
\delta m^2_N = \sqrt{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_FN_eE_\nu)^2 + (\delta m^2 \sin 2\theta_\odot)^2}
\]

\[
\sin^2 \theta^N_\odot = \frac{1}{2} \left( 1 - \frac{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_FN_eE_\nu)}{\delta m^2_N} \right)
\]

\(\theta^N_\odot > \theta_\odot\)

Quasi-Vacuum: \(2\sqrt{2}G_FN_eE_\nu \ll \delta m^2 \cos 2\theta_\odot\)

\(\delta m^2_N = \delta m^2\) and \(\theta^N_\odot = \theta_\odot\)

Resonance (Mikheyev + Smirnov '85): \(2\sqrt{2}G_FN_eE_\nu = \delta m^2 \cos 2\theta_\odot\)

\(\delta m^2_N = \delta m^2 \sin 2\theta_\odot\) and \(\theta^N_\odot = \pi/4\)

Matter Dominated: \(2\sqrt{2}G_FN_eE_\nu \gg \delta m^2 \cos 2\theta_\odot\)

\(\delta m^2_N \to 2\sqrt{2}G_FN_eE_\nu\) and \(\theta^N_\odot \to \pi/2\)
Life of a Boron-8 Solar Neutrino:

$\nu_e \approx \nu_2$

for $^8B$

at birth

Solar Center

Once a $\nu_2$ always a $\nu_2$!

Solar Center

In Vac

$\nu_2 \approx \frac{1}{3} \nu_e$

Exit Sun

Exit Core
Borexino results (2012)

- Fit: $\chi^2/\text{NDF} = 55/60$
- $^7\text{Be}$: $49\pm3$ cpd/100 tons
- $^{210}\text{Bi} + \text{CNO}$: $20\pm2$ cpd/100 tons
- $^{85}\text{Kr}$: $29\pm4$ cpd/100 tons
- $^{11}\text{C}$: $24\pm1$ cpd/100 tons

![Graphs showing energy and survival probability distributions with data points and error bars.]

- MSW-LMA prediction
- SNO data
- Borexino data
- Prediction for pp solar neutrinos

Energy [keV] vs Counts/(10 keV x day x 100 tons)

Survival Probability vs Neutrino Energy [MeV]
Solar Pair Mass Hierarchy:

\[ \sin^2 \theta^N \]

\[ \Delta m_{\text{sol}}^2 \]

\[ \theta_{\odot} < \frac{\pi}{4} \quad \theta_{\odot} > \frac{\pi}{4} \]

Fractional Flavor Content

Who cares? SNO does!!!

for neutrino in matter \( \theta^N_{\odot} > \theta_{\odot} \)

\[
\langle P_{ee} \rangle = \cos^2 \theta^N_{\odot} \cos^2 \theta_{\odot} + \sin^2 \theta^N_{\odot} \sin^2 \theta_{\odot} = \frac{1}{2} + \frac{1}{2} \cos 2\theta^N_{\odot} \cos 2\theta_{\odot}
\]

if \( \theta_{\odot} < \frac{\pi}{4} \)

\[
\langle P_{ee} \rangle \geq \sin^2 \theta_{\odot}
\]

if \( \theta_{\odot} > \frac{\pi}{4} \)

\[
\langle P_{ee} \rangle \geq \frac{1}{2}(1 + \cos^2 2\theta_{\odot}) \geq \frac{1}{2}
\]

SNO: \( \langle P_{ee} \rangle_{\text{day}} = 0.347 \pm 0.038 \)

Solar Hierarchy Determined!!!
Day/Night Asymmetry:

\[
\sin^2 \theta_\odot \rightarrow \sin^2 \theta_\oplus = \sin^2 \theta_\odot + \frac{1}{2} \sin^2 2\theta_\odot \left( \frac{A_{\oplus}}{\delta m^2_\odot} \right) \text{ in the earth.}
\]

\[A = 2(D-N)/(D+N)\] expected to be few %

Spectral Distortion:

A characteristic of matter effects is that the Fraction of \( \nu_2 \) is energy dependent.

Smaller at smaller E.

Implies an increase in \( P_{ee} \) near threshold.
Summary:

The low energy pp and $^7$Be Solar Neutrinos exit the sun as two thirds $\nu_1$ and one third $\nu_2$ due to (quasi-) vacuum oscillations.

$$f_1 = 65 \pm 2\%, \quad f_2 = 35 \pm 2\% \text{ with } P_{ee} \approx 0.56$$

The high energy $^8$B Solar Neutrinos exit the sun as "PURE" $\nu_2$ mass eigenstates due to matter effects.

$$f_2 = 91 \pm 2\% \text{ and } f_1 =$$
Testing solar neutrino oscillations with reactors

\[ 1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta_\odot \sin^2 \Delta \]

\[ \Delta = \frac{\delta m^2 L}{4E} \]

\[ 10^5 \text{ eV}^2 \]

\[ 10^5 \text{m} = 100 \text{ km} \]

\[ 1 \text{ MeV} \]
expected no-oscillation neutrino event rate at KamLAND
Summary:

The low energy pp and $^7$Be Solar Neutrinos exit the sun as two thirds $\nu_1$ and one third $\nu_2$.

$$f_1 = 65 \pm 2\%, \ f_2 =$$

The high energy $^8$B Sol "PURE" $\nu_2$ mass eigenstates:

$$f_2 = 91 \pm 2\% \text{ and } j$$

![Diagram showing $\Delta m^2_{sol}$ and $\sin^2 \theta$]
Excess: Neutrino vs Anti-neutrino
With 3 different $\Delta m^2$ 4 light neutrinos needed!

4th $\nu$: cannot be active – must be sterile. Mixing matrix: 6 $\theta_{ij}$, 3 Dirac-type $\mathcal{CP}$ phases. But: simplifications occur – only two possible type of schemes: 2+2 and 3+1
On March 2011 ... ArXiv 1101.2755

New reactor antineutrino spectra have been provided for $^{235}$U, $^{239}$Pu, $^{241}$Pu, and $^{238}$U, increasing the mean flux by about 3 percent.

This reevaluation applies to all reactor neutrino experiments.

It means that for experiments at reactor-detector distances < 100 m, the ratio of observed event rate to predicted rate shifts.

$0.976 \pm 0.024 \rightarrow 0.943 \pm 0.023$
The Gallium Anomaly

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

\[ \sin^2(2\theta) \approx 0.50 \quad \Delta m^2 \approx 2 \text{ eV}^2 \]

Signals at SBL are at the 2-4\sigma level
All pointing in the same direction

\[ \sigma^{(\text{Cr})} = 58.1 \times 10^{-46} \text{ cm}^2 \left( 1 \pm 0.028 \right)_{1\sigma} \quad \Rightarrow \quad R_{\text{Ga}} = 0.76_{-0.08}^{+0.09} \]


\[ \sigma^{(\text{Cr})} = 63.9 \times 10^{-46} \text{ cm}^2 \left( 1 \pm 0.106 \right)_{1\sigma} \quad \Rightarrow \quad R_{\text{Ga}} = 0.76_{-0.08}^{+0.09} \]