

ASIA-EUROPE-PACIFIC SCHOOL OF HEP
VIETNAM



GABRIELA BARENBOIM
PRESENTS

A NU HOPE EPISODE I

STAR WARS EPISODES I-III THE PHANTOM MENACE ATTACK OF THE CLONES REVENGE OF THE SITH
STORY BY EWAN MCGREGOR HAIDEN CHRISTENSEN NATALIE PORTMAN IAN MCDONALD
FRANK OZ ANTHONY DANIEL KINNO BAKER LAM NEESON JAZZ LUZZO FRANKLA ALBERT SMITH L. JACKSON CHRISTOPHER LEE
SCREENPLAY BY GEORGE LUCAS JONATHAN HALEY DIRECTED BY GEORGE LUCAS
PRODUCED BY RICK MCCALLUM MUSIC BY JOHN WILLIAMS
EDITED BY GEORGE LUCAS

STAR WARS EPISODES IV-VI A NEW HOPE THE EMPIRE STRIKES BACK RETURN OF THE JEDI
STARRING MARK HAMILL HARRISON FORD CARRIE FISHER ALEC GUINNESS
FRANK OZ ANTHONY DANIEL KINNO BAKER PETER MATHIEV DAVID PRINCE BILLY DEE WILLIAMS
SCREENPLAY BY GEORGE LUCAS LEIGH BRACKETT LAWRENCE KASDAN
DIRECTED BY GEORGE LUCAS IRVIN KERSHNER RICHARD MARQUAND
PRODUCED BY GARY KURTZ HOWARD KAZANJIAN MUSIC BY JOHN WILLIAMS
EDITED BY GEORGE LUCAS

Is The Whole Universe made of—
Electrons Protons Neutrons ?

NO!

Electrons Protons Neutrons
are rareties!

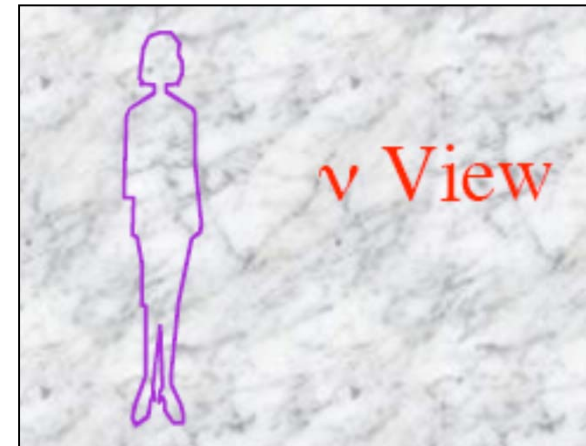
For every one of them, the universe contains a
billion neutrinos ν !

Passing through each person on earth every second:
One hundred trillion neutrinos from the sun.

The sun shines because of nuclear fusion in its core.

This fusion produces—

- Energy, including visible light
- Neutrinos
- The atoms more complicated than hydrogen



Almost all neutrinos zipping through us do nothing at all.

Typically, a solar neutrino would have to zip through 10,000,000,000,000,000,000 people before doing anything.

The probability that a particular solar neutrino will interact as it zips through one of us is $1 / 10,000,000,000,000,000,000$.

Are Neutrinos Important to Our Lives?

If there were no ν s, the sun and stars would not shine.

- No energy from the sun to keep us warm.
- No atoms more complicated than hydrogen.
No carbon. No oxygen. No water.
No earth. No moon. No us.

No ν s is very **BAD** news.

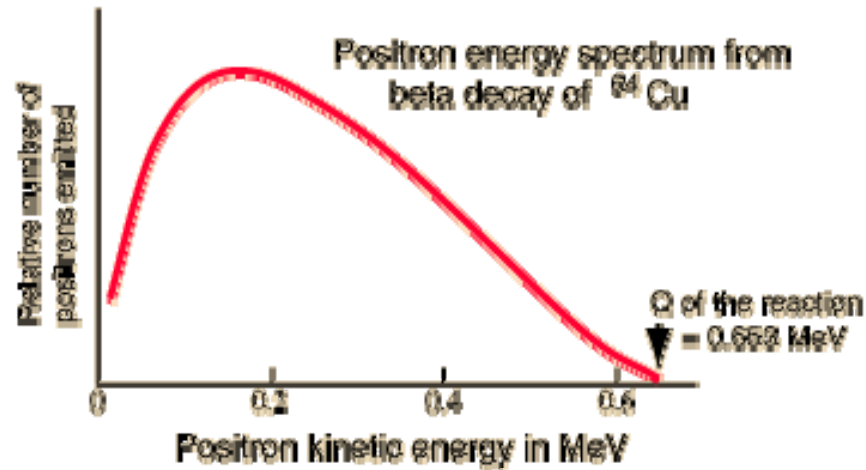
Summer Schools (if existed) were VERY short

β decay was supposed to be a two body decay



$$E_e = \frac{m_n^2 + m_e^2 - m_p^2}{2 m_n}$$

Studies of β decay revealed a continuous energy spectrum.



Another anomaly was the fact that the nuclear recoil was not in the direction opposite to the momentum of the electron.

The emission of another particle was a probable explanation of this behaviour, but searches found no evidence of either mass or charge.

...desperate remedy to save the law of conservation of energy...

Neutron Decay:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$



Fermi postulated a theory for β decay in terms of spinors

$$H_{ew} = \frac{G_F}{\sqrt{2}} \bar{\Psi}_p \gamma_\mu \Psi_n \bar{\Psi}_e \gamma^\mu \Psi_\nu$$

Standard Model of Particle Physics

Gauge Theory based on the group:

$$SU(3) \times SU(2) \times U(1)$$

$SU(3) \Rightarrow$ Quantum Chromodynamics

Strong Force (Quarks and Gluons)

$SU_L(2) \times U(1) \Rightarrow$ ElectroWeak Interactions broken to $U_{EM}(1)$

by HIGGS

$$\underline{SU_L(2) \times U_Y(1) \Rightarrow U_{EM}(1)}$$

Force Carriers: W^\pm , Z^0 and γ masses: 80, 91 and 0 GeV

quark, SU(2) doublets: $\begin{pmatrix} u \\ d \end{pmatrix}_L$, $\begin{pmatrix} c \\ s \end{pmatrix}_L$, $\begin{pmatrix} t \\ b \end{pmatrix}_L$

up-quark, SU(2) singlets: u_R, c_R, t_R

down-quark, SU(2) singlets: d_R, s_R, b_R

lepton, SU(2) doublets: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$, $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$, $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$

neutrino, SU(2) singlets: — — —

charge lepton, SU(2) singlets: e_R, μ_R, τ_R

Electron mass

comes from a term of the form

$$\bar{L}\phi e_R$$

Absence of ν_R

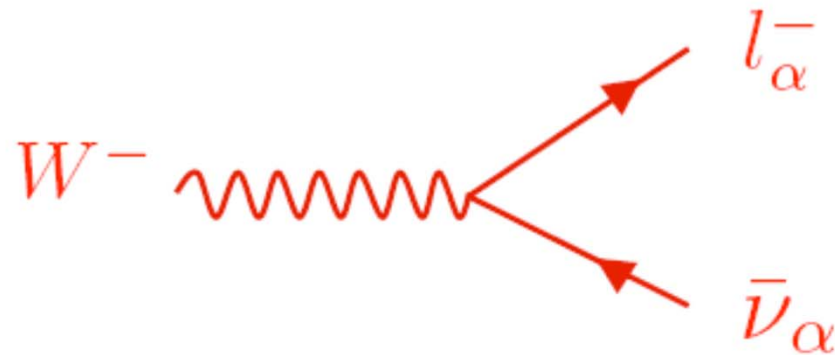
forbids such a mass term (dim 4)

for the Neutrino

Therefore in the SM neutrinos are massless
and hence travel at speed of light.

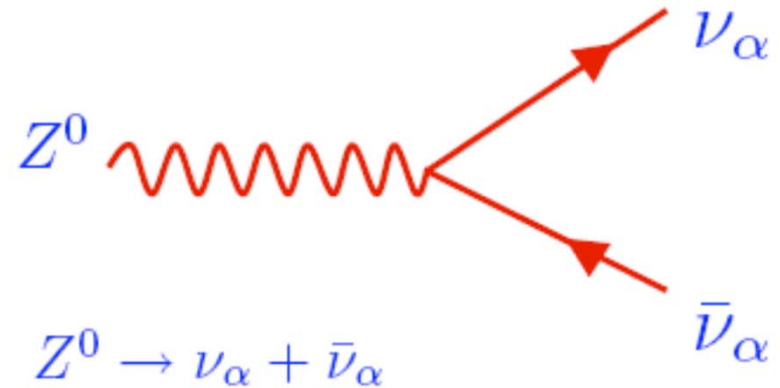
Interactions:

Charge Current (CC)

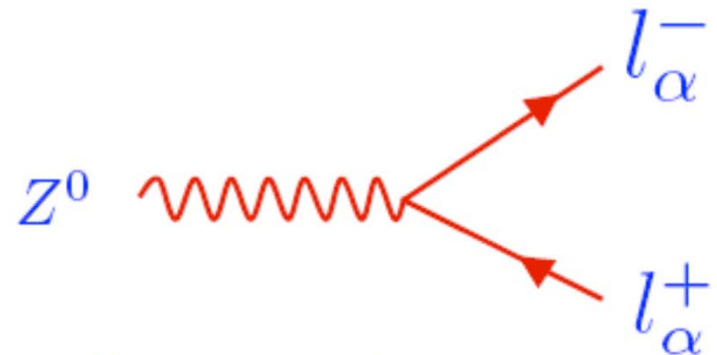


$$W^- \rightarrow l_{\alpha}^{-} + \bar{\nu}_{\alpha}$$

Neutral Current (NC)



$$Z^0 \rightarrow \nu_{\alpha} + \bar{\nu}_{\alpha}$$



$$Z^0 \rightarrow l_{\alpha}^{-} + l_{\alpha}^{+}$$

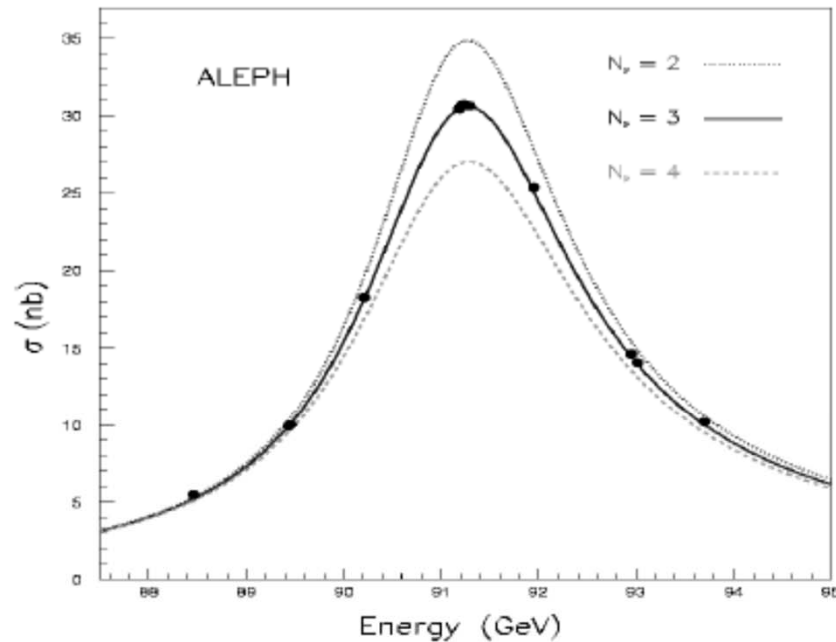
$$\Gamma(Z^0 \rightarrow f + \bar{f}) = K \frac{g_Z^2 M_Z}{48\pi} [|c_V^f|^2 + |c_A^f|^2]$$

$\alpha = e, \mu, \text{ or } \tau$

Invisible width of Z plus other data from LEP:

$$Z^0 \rightarrow \nu\bar{\nu}$$

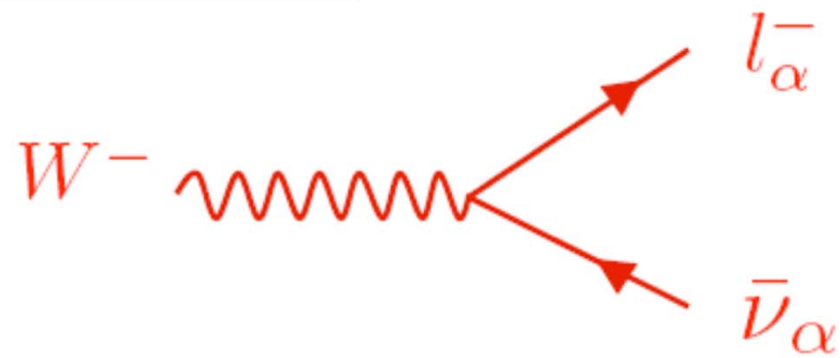
Implies $N_\nu = 2.99 \pm 0.01$



Three Active Neutrinos!!!

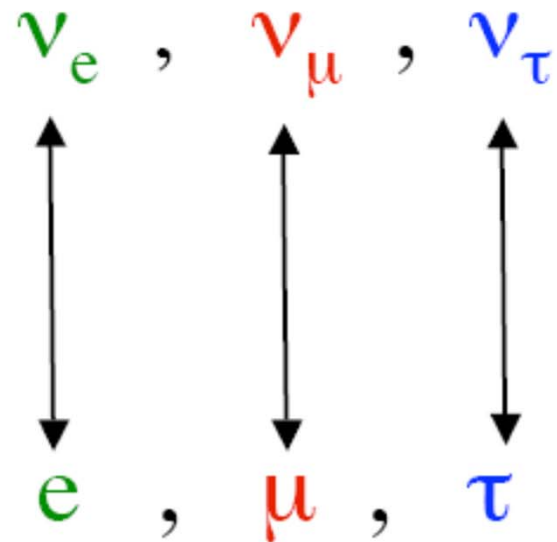
Sterile Neutrinos don't couple to Z^0

Note That

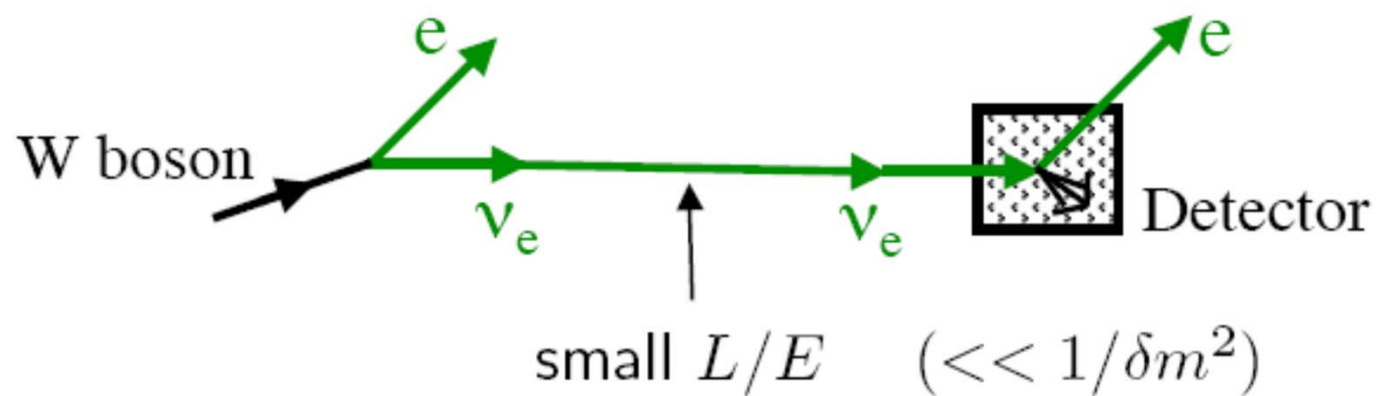


$$W^- \rightarrow l_\alpha^- + \bar{\nu}_\alpha$$

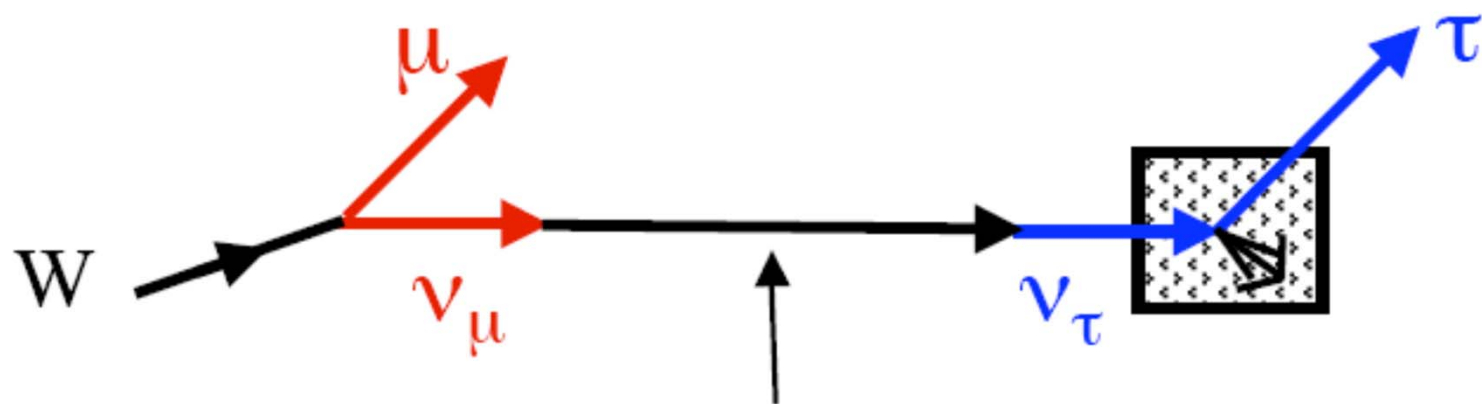
Implies



Observed

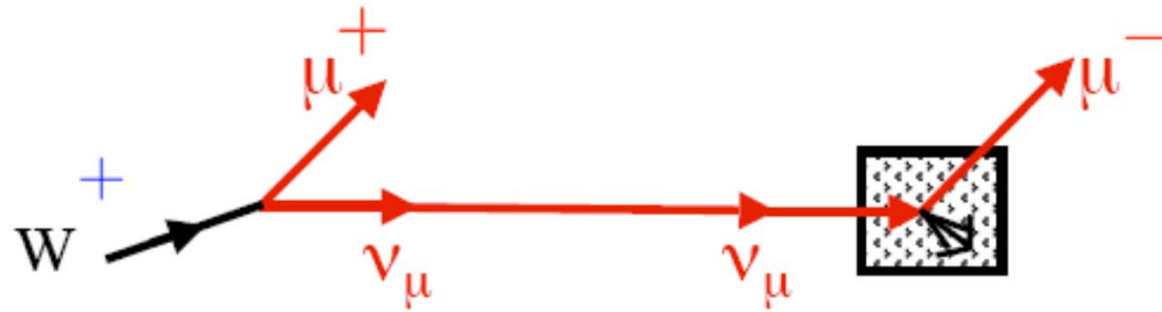


Not Observed



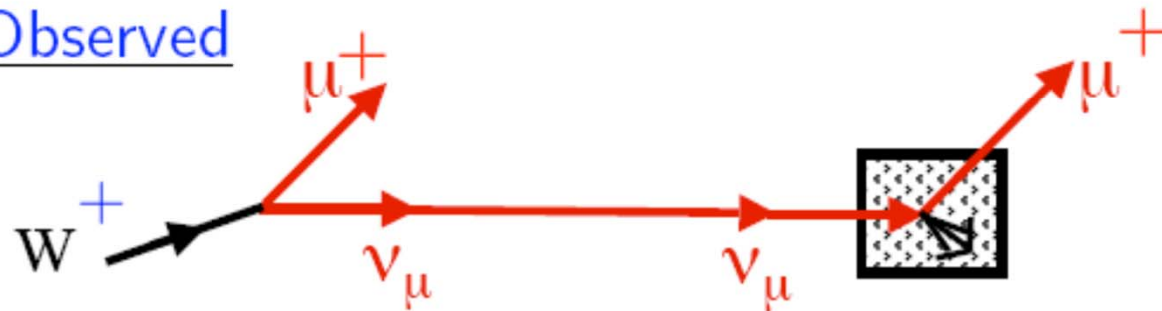
small L/E ($\ll 1/\delta m^2$)

Observed



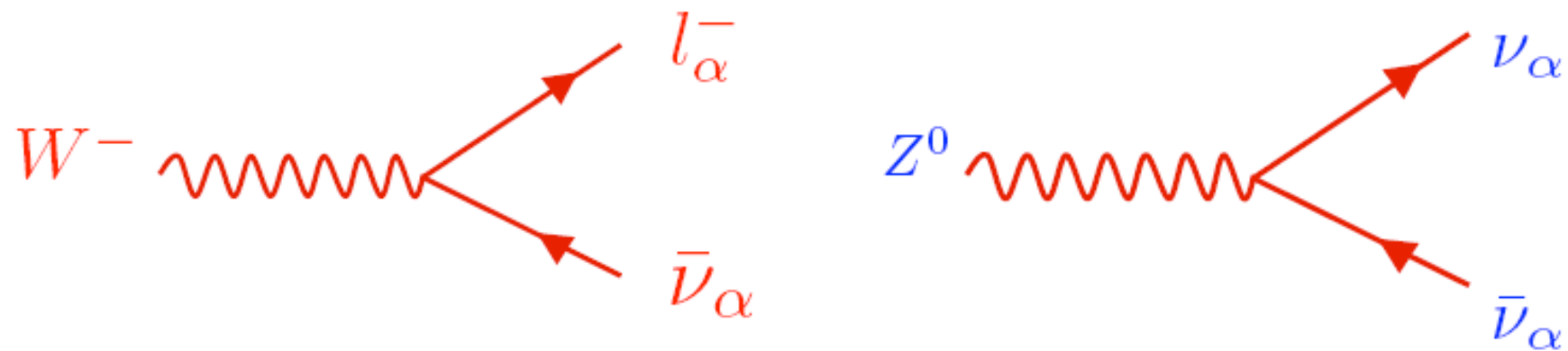
neutrino beam (not anti-neutrino beam)

Not Observed



large E ($\gg m_\nu$)

Standard Model

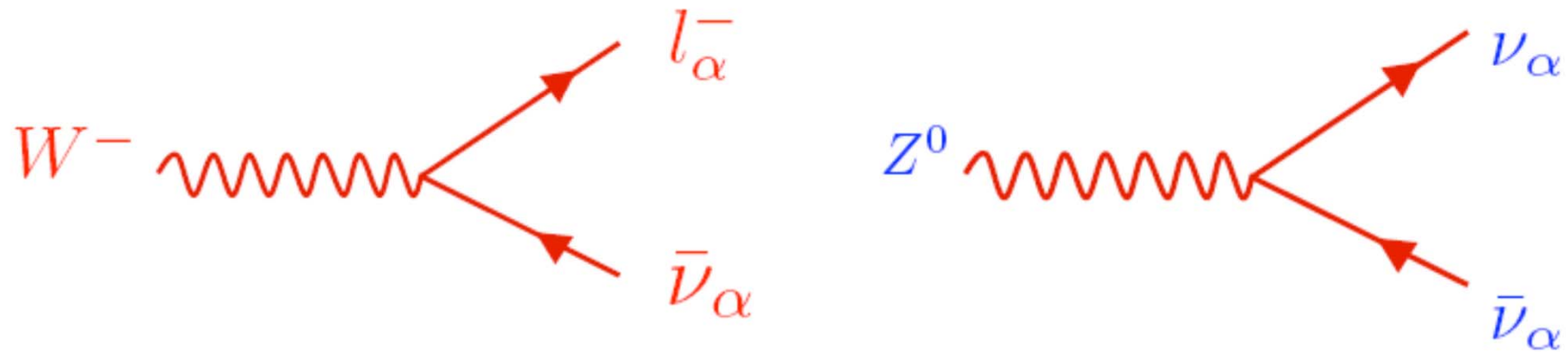


couplings conserve the **Lepton Number L**
defined by—

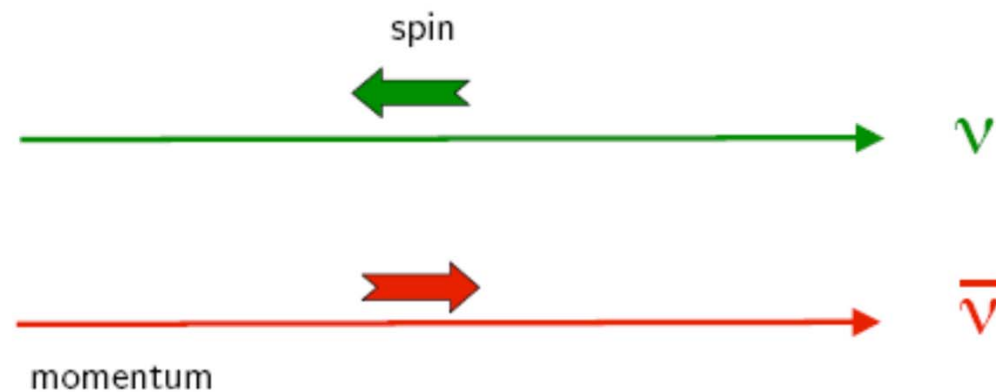
$$L(\nu) = L(l^-) = -L(\bar{\nu}) = -L(l^+) = 1.$$

Actually L_e , L_μ , and L_τ
separately

Left Handed Nature of The Neutrino



Produce Left-Handed Neutrinos
and Right-Handed Anti-Neutrinos



What about the RH neutrinos and LH anti-neutrino ????

There exist three fundamental and discrete transformations in nature:

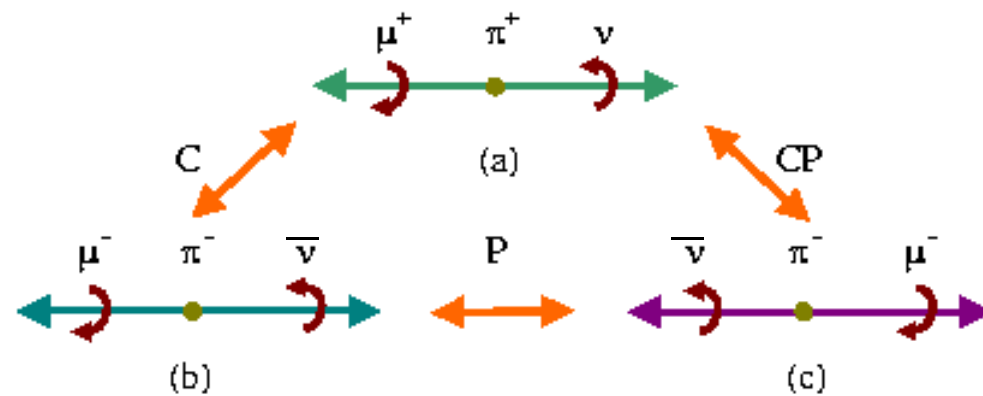
- Parity \mathcal{P} $\vec{x} \rightarrow -\vec{x}$
- Time reversal \mathcal{T} $t \rightarrow -t$
- Charge conjugation \mathcal{C} $q \rightarrow -q$

\mathcal{P} , \mathcal{T} and \mathcal{C} are conserved in the classical theories of mechanics and electrodynamics!

$CPT \leftrightarrow$ Lorentz invariance \oplus unitarity: is an essential building block of field theory

CPT : L particle \leftrightarrow R antiparticle

Neutrinos in the MSM are massless and exist only in two states: particle with negative helicity and antiparticle with positive one: **Weyl fermion**



\mathcal{P} : L particle \leftrightarrow R particle

Parity violation is nowhere more obvious than in the neutrino sector: the reflection of a left-handed neutrino in a mirror is nothing !

Summary of ν 's in SM:

Three flavors of massless neutrinos

$$W^- \rightarrow l_\alpha^- + \bar{\nu}_\alpha$$

$$W^+ \rightarrow l_\alpha^+ + \nu_\alpha$$

$$\alpha = e, \mu, \text{ or } \tau$$

Anti-neutrino, $\bar{\nu}_\alpha$, has +ve helicity, Right Handed

Neutrino, ν_α , has -ve helicity, Left Handed

ν_L and $\bar{\nu}_R$ are CPT conjugates

massless implies helicity = chirality

Beyond the SM

What if Neutrino have a MASS?

speed is less than c therefore time can pass

and

Neutrinos can change character!!!

What are the stationary states?

How are they related to the interaction states?

NEUTRINO OSCILLATIONS:

Two Flavors

flavor eigenstates \neq mass eigenstates

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

W's produce ν_μ and/or ν_τ 's

but ν_1 and ν_2 are the states

that change by a phase over time, mass eigenstates.

$$|\nu_j\rangle \rightarrow e^{-ip_j \cdot x} |\nu_j\rangle \quad p_j^2 = m_j^2$$

$\alpha, \beta \dots$ flavor index

$i, j \dots$ mass index

Production:

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

Propagation:

$$\cos\theta e^{-ip_1 \cdot x}|\nu_1\rangle + \sin\theta e^{-ip_2 \cdot x}|\nu_2\rangle$$

Detection:

$$|\nu_1\rangle = \cos\theta|\nu_\mu\rangle - \sin\theta|\nu_\tau\rangle$$

$$|\nu_2\rangle = \sin\theta|\nu_\mu\rangle + \cos\theta|\nu_\tau\rangle$$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos\theta(e^{-ip_1 \cdot x})(-\sin\theta) + \sin\theta(e^{-ip_2 \cdot x})\cos\theta|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos \theta (e^{-ip_1 \cdot x})(-\sin \theta) + \sin \theta (e^{-ip_2 \cdot x}) \cos \theta|^2$$

$$\text{Same } E, \text{ therefore } p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$$

$$e^{-ip_j \cdot x} = e^{-iEt} e^{-ip_j L} \approx e^{-i(Et - EL)} e^{-im_j^2 L / 2E}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta |e^{-im_2^2 L / 2E} - e^{-im_1^2 L / 2E}|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$\delta m^2 = m_2^2 - m_1^2 \text{ and } \frac{\delta m^2 L}{4E} \equiv \Delta \text{ kinematic phase:}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = |\cos \theta (e^{-ip_1 \cdot x}) (-\sin \theta) + \sin \theta (e^{-ip_2 \cdot x}) \cos \theta|^2$$

$$\text{Same } E, \text{ therefore } p_j = \sqrt{E^2 - m_j^2} \approx E - \frac{m_j^2}{2E}$$

$$e^{-ip_j \cdot x} = e^{-iEt} e^{-ip_j L} \approx e^{-i(Et - EL)} e^{-im_j^2 L / 2E}$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta |e^{-im_2^2 L / 2E} - e^{-im_1^2 L / 2E}|^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E} \frac{c^4}{hc}$$

Appearance:

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

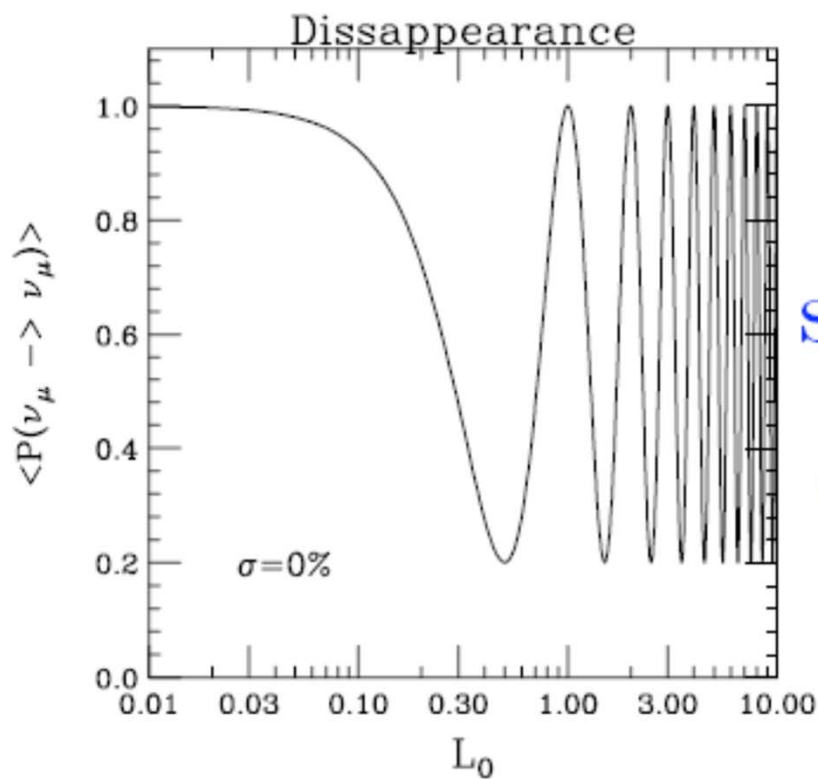
Disappearance:

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}$$

Oscillation Length $L_0 = 4\pi E / \delta m^2$

Fixed E_ν

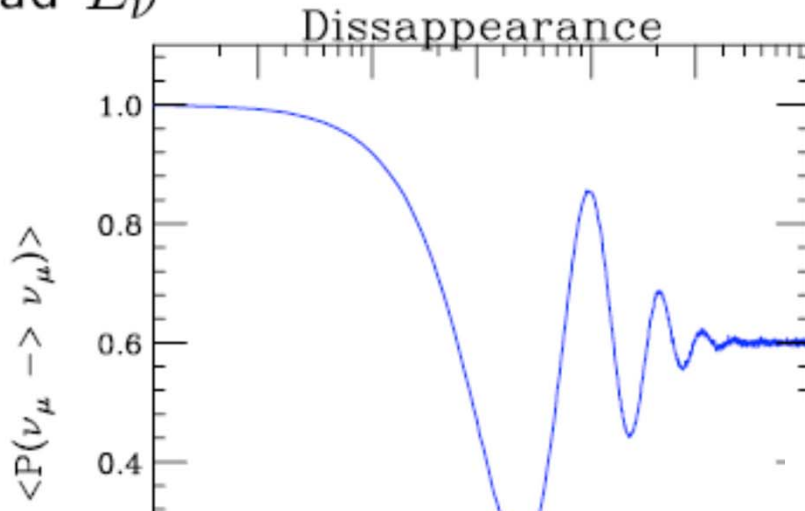


Amplitude of Oscillation

↑
 $\sin^2 2\theta$
 ↓

$$\langle P(\nu_\mu \rightarrow \nu_\mu) \rangle = 1 - \sin^2 2\theta \left\langle \sin^2 \frac{\delta m^2 L}{4E} \right\rangle$$

Spread E_ν



effectively incoherent
mass eigenstates

$$1 - \sin^2 2\theta \left(\frac{1}{2}\right) = \cos^4 \theta + \sin^4 \theta$$

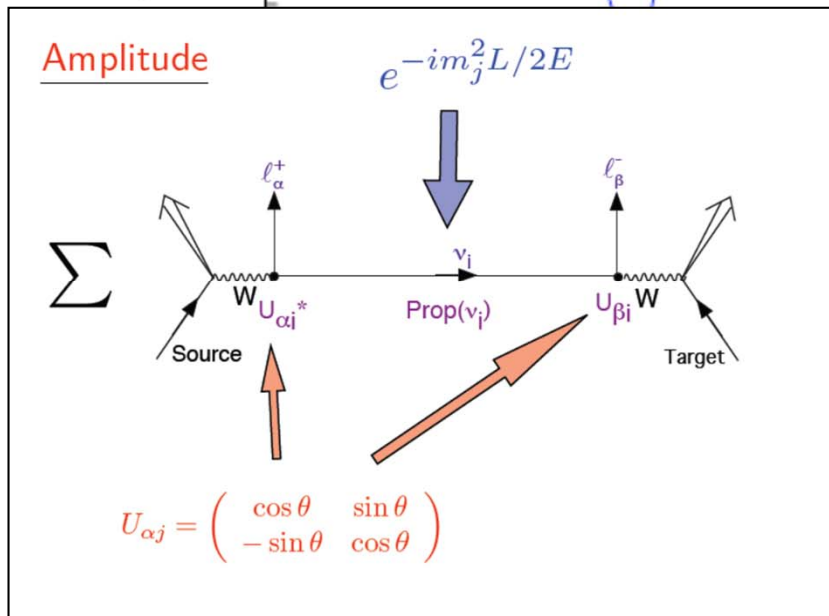
$W^+ \rightarrow \mu^+ + \nu_1$ probability $\cos^2 \theta$

$W^+ \rightarrow \mu^+ + \nu_2$ probability $\sin^2 \theta$

flavour fractions $|\nu_1\rangle$ and $|\nu_2\rangle$ during propagation remain unchanged

probability ν_1 contains ν_μ is $\cos^2 \theta$

probability ν_2 contains ν_μ is $\sin^2 \theta$



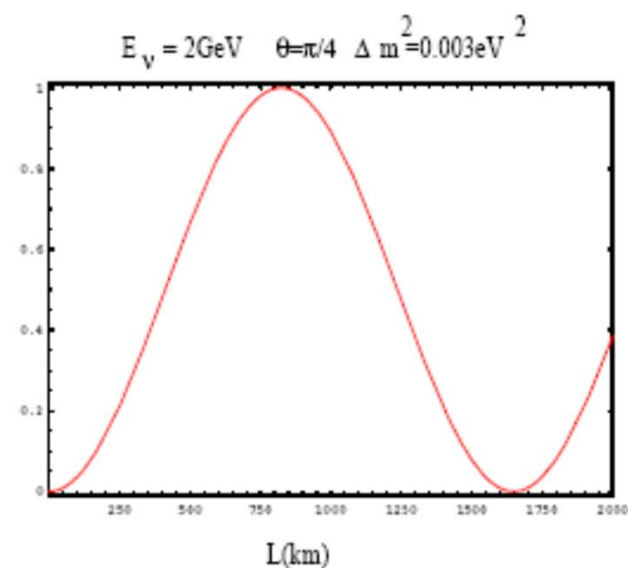
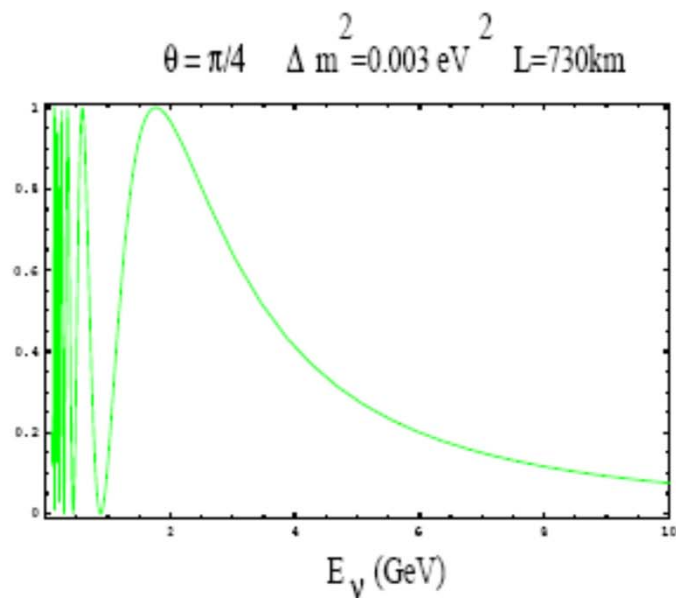
Using the unitarity of the mixing matrix: ($W_{\alpha\beta}^{jk} \equiv [V_{\alpha j} V_{\beta j}^* V_{\alpha k} V_{\beta k}]$)

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re}[W_{\alpha\beta}^{jk}] \sin^2 \left(\frac{\Delta m_{jk}^2 L}{4E_{\nu}} \right) \\ \pm 2 \sum_{k>j} \text{Im}[W_{\alpha\beta}^{jk}] \sin \left(\frac{\Delta m_{jk}^2 L}{2E_{\nu}} \right)$$

For 2 families: $V_{MNS} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E_{\nu}} \right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$



Oscillation probabilities show the expected **GIM** suppression of any flavour changing process: they vanish if the neutrinos are degenerate

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E_\nu} \right) \rightarrow \text{appearance}$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta} < 1 \rightarrow \text{disappearance}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4 E} \right)$$

$$P_{\alpha\alpha} = 1 - P_{\alpha\beta}$$

$$\left(1.27 \frac{\Delta m^2 (eV^2) L(km)}{E(GeV)} \right)$$

L/E becomes crucial !!!

Evidence for Flavor Change:

*** Atmospheric and Accelerator Neutrinos with $L/E = 500 \text{ km/GeV}$

*** Solar and Reactor Neutrinos with $L/E = 15 \text{ km/MeV}$

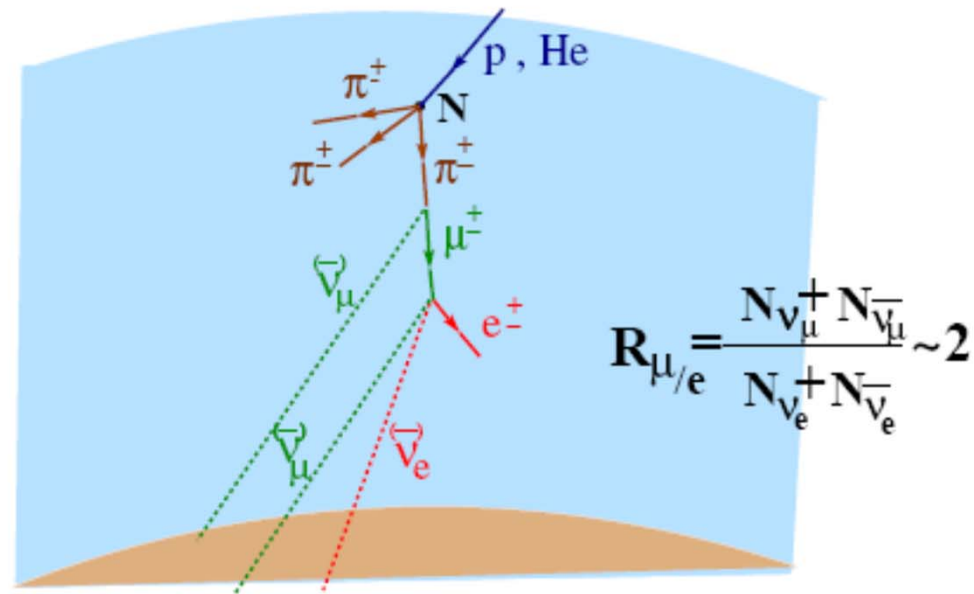
Neutrinos from Stopped muons $L/E = 2 \text{ m/MeV}$ (Unconfirmed)

Atmospheric neutrinos

- Atmospheric neutrinos are produced by the interaction of *cosmic rays* (p, He, \dots) with the Earth's atmosphere:

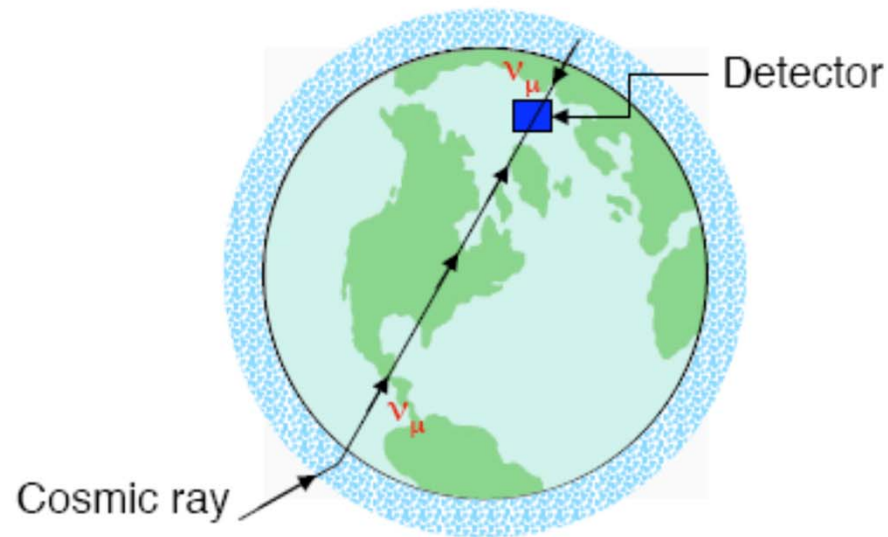
- 1 $A_{\text{cr}} + A_{\text{air}} \rightarrow \pi^{\pm}, K^{\pm}, K^0, \dots$
- 2 $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$,
- 3 $\mu^{\pm} \rightarrow e^{\pm} + \nu_e + \nu_{\mu}$;

- at the detector, some ν interacts and produces a **charged lepton**, which is observed.



A deficit was observed in the ratio μ/e events: **Soudan2, IMB, Kamiokande**

Atmospheric Neutrinos

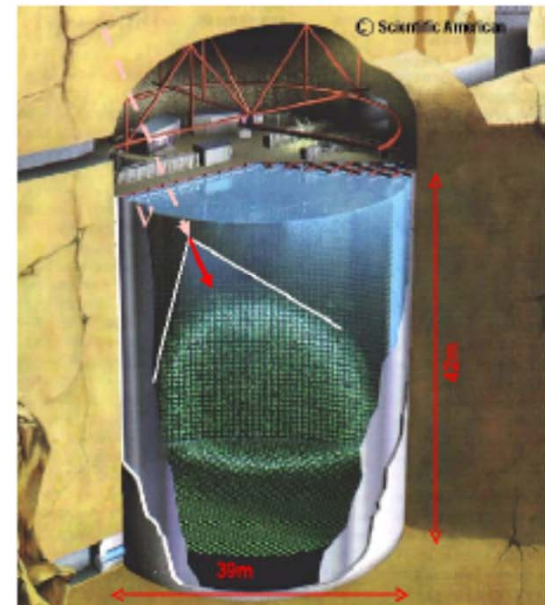


Isotropy of the ≥ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 1 .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

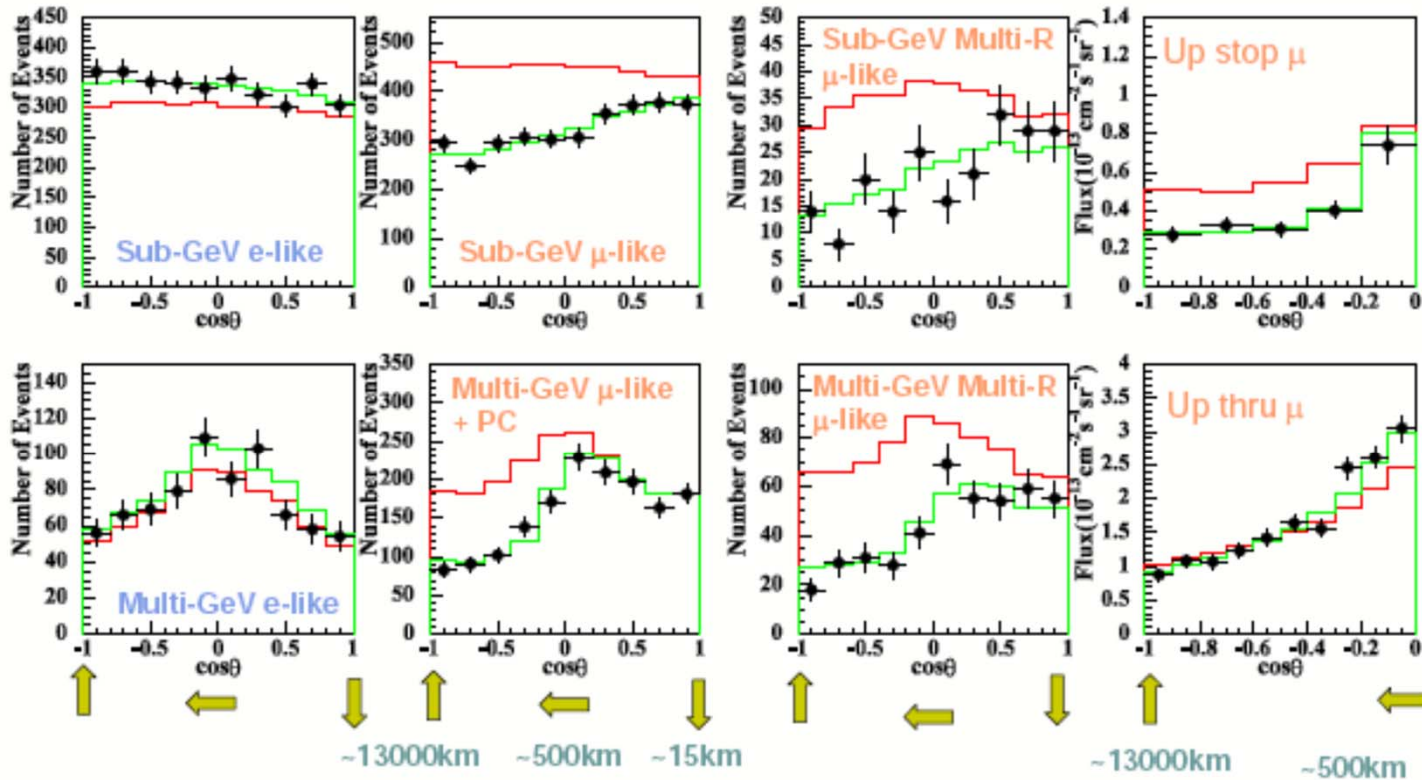
$$\frac{\phi_{\nu_\mu}(\text{Up})}{\phi_{\nu_\mu}(\text{Down})} = 0.54 \pm 0.04 .$$



Zenith angle distributions

$\nu_\mu \leftrightarrow \nu_\tau$
2-flavor oscillations

— Best fit
 $\sin^2 2\theta = 1.0, \Delta m^2 = 2.0 \times 10^{-3} \text{ eV}^2$
— Null oscillation



Half of the upward-going, long-distance-traveling ν_μ are disappearing.

Voluminous atmospheric neutrino data are well described by —

$$\nu_\mu \longrightarrow \nu_\tau$$

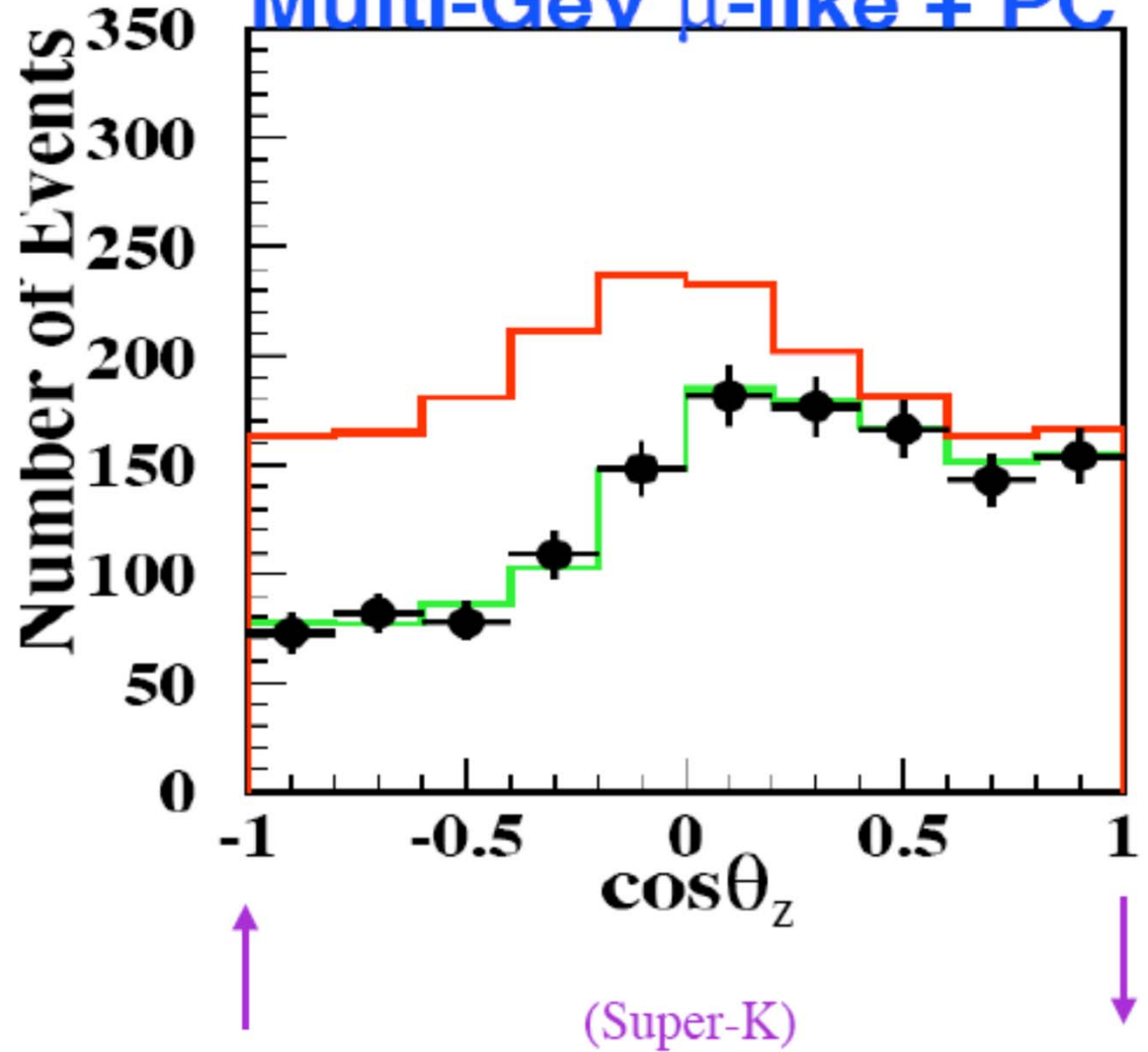
with —

$$\Delta m_{\text{atm}}^2 \cong 2.4 \cdot 10^{-3} \text{ eV}^2$$

and —

$$\sin^2 2\theta_{\text{atm}} \cong 1$$

Multi-GeV μ -like + PC



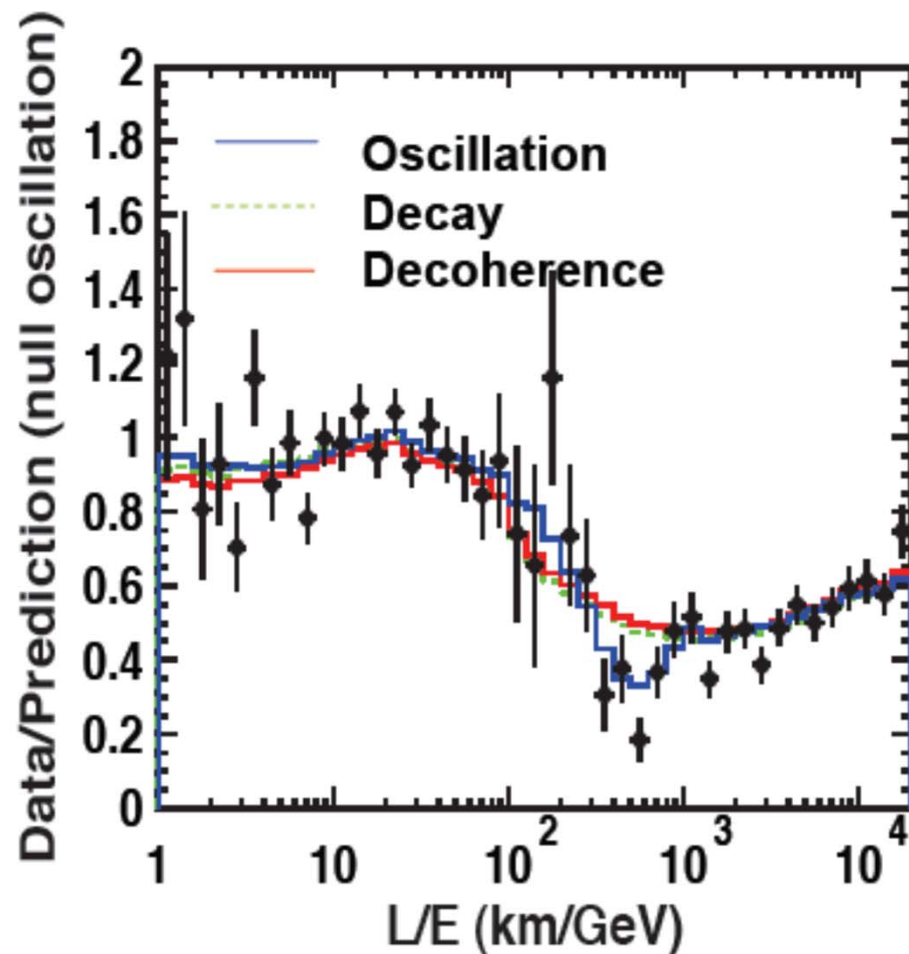
L/E Analysis

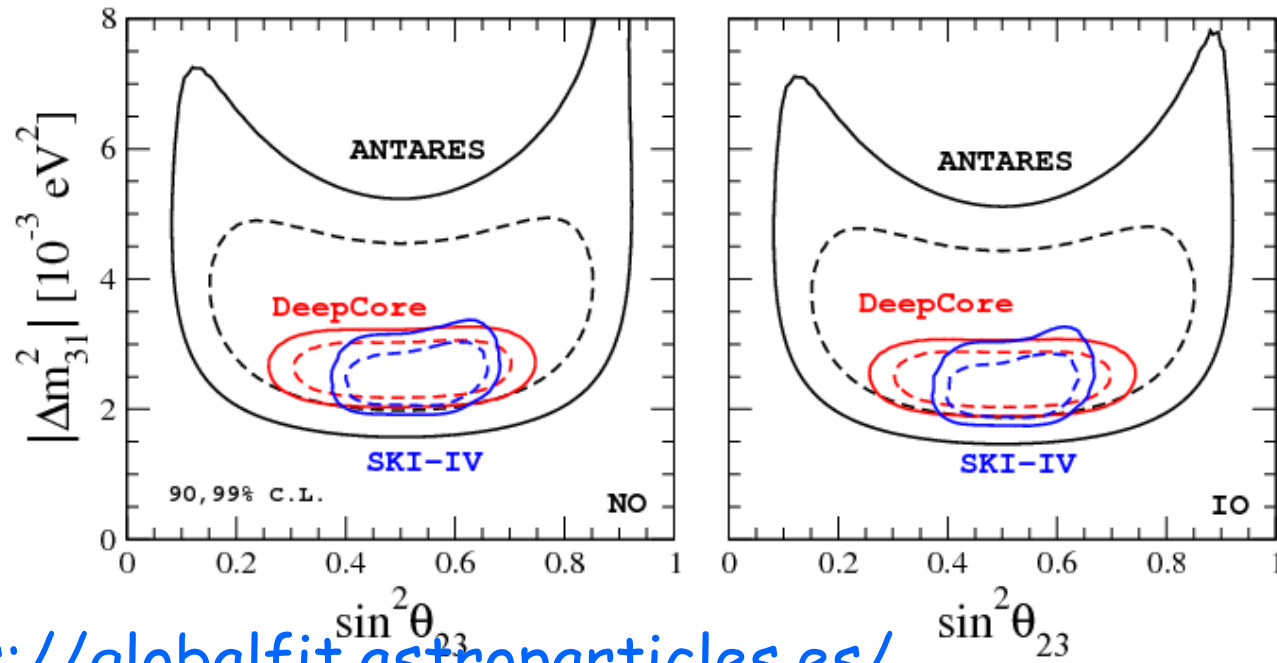
❖ Oscillation, decay and decoherence models tested

$$\chi^2_{\text{osc}} = 83.9/83$$

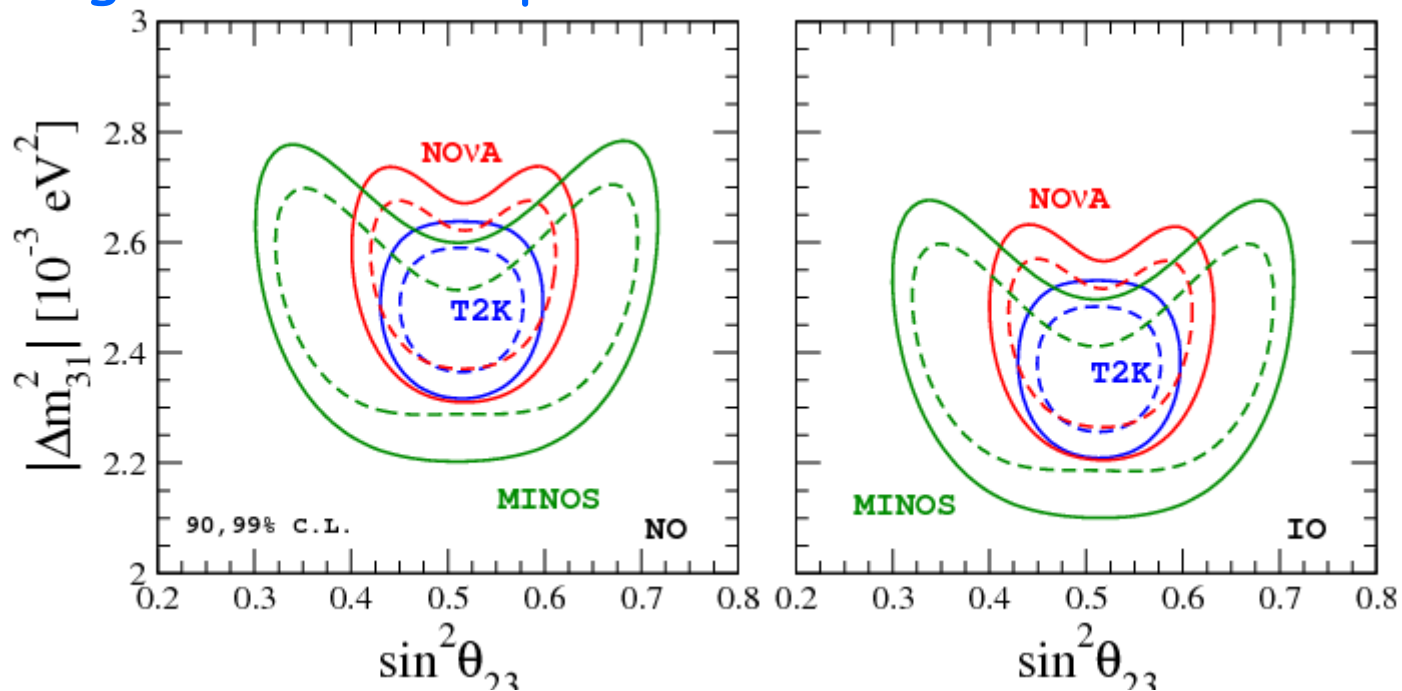
$$\chi^2_{\text{dcy}} = 107.1/83, \Delta\chi^2 = 23.2(4.8\sigma)$$

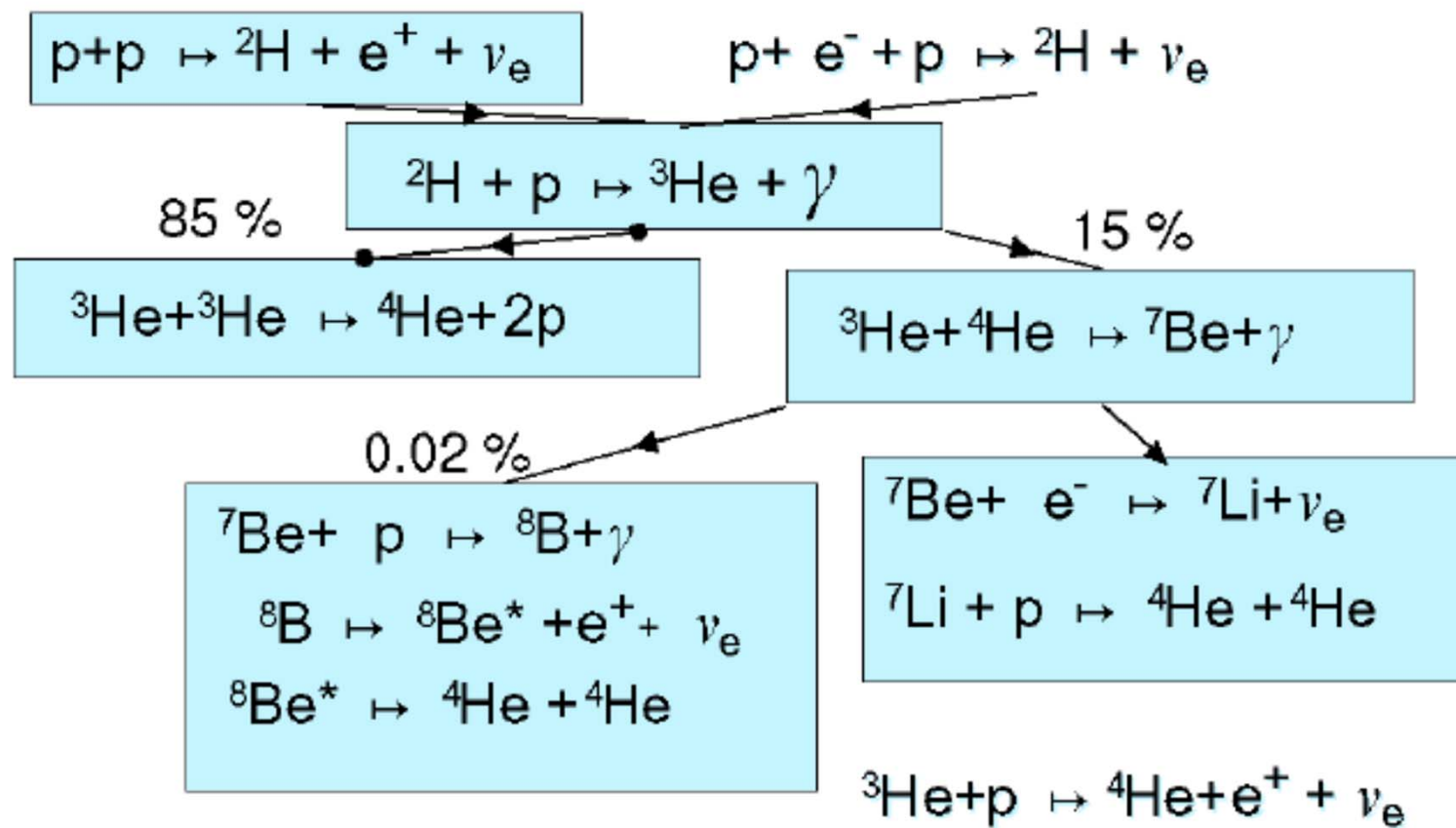
$$\chi^2_{\text{dec}} = 112.5/83, \Delta\chi^2 = 27.6(5.3\sigma)$$





<https://globalfit.astroparticles.es/>





Solar Spectrum:

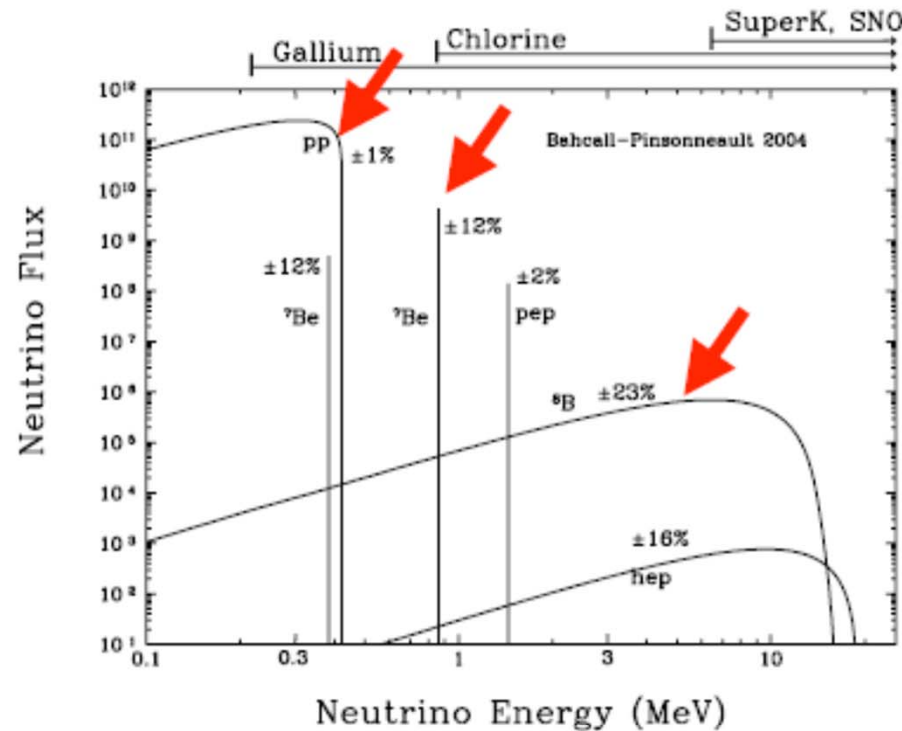
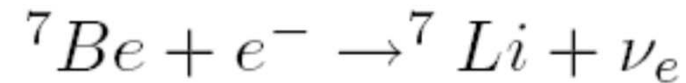


Figure 1. The predicted solar neutrino energy spectrum. The figure shows the energy spectrum of solar neutrinos predicted by the BP04 solar model [22]. For continuum sources, the neutrino fluxes are given in number of neutrinos $\text{cm}^{-2} \text{s}^{-1} \text{MeV}^{-1}$ at the Earth's surface. For line sources, the units are number of neutrinos $\text{cm}^{-2} \text{s}^{-1}$. Total theoretical uncertainties taken from column 2 of table 1 are shown for each source. To avoid complication in the figure, we have omitted the difficult-to-detect CNO neutrino fluxes (see table 1).



$$\phi_{pp} = 5.94(1 \pm 0.01) \times 10^{10} \text{cm}^{-2} \text{sec}^{-1}$$



$$\phi_{{}^7\text{Be}} = 4.86(1 \pm 0.12) \times 10^9 \text{cm}^{-2} \text{sec}^{-1}$$

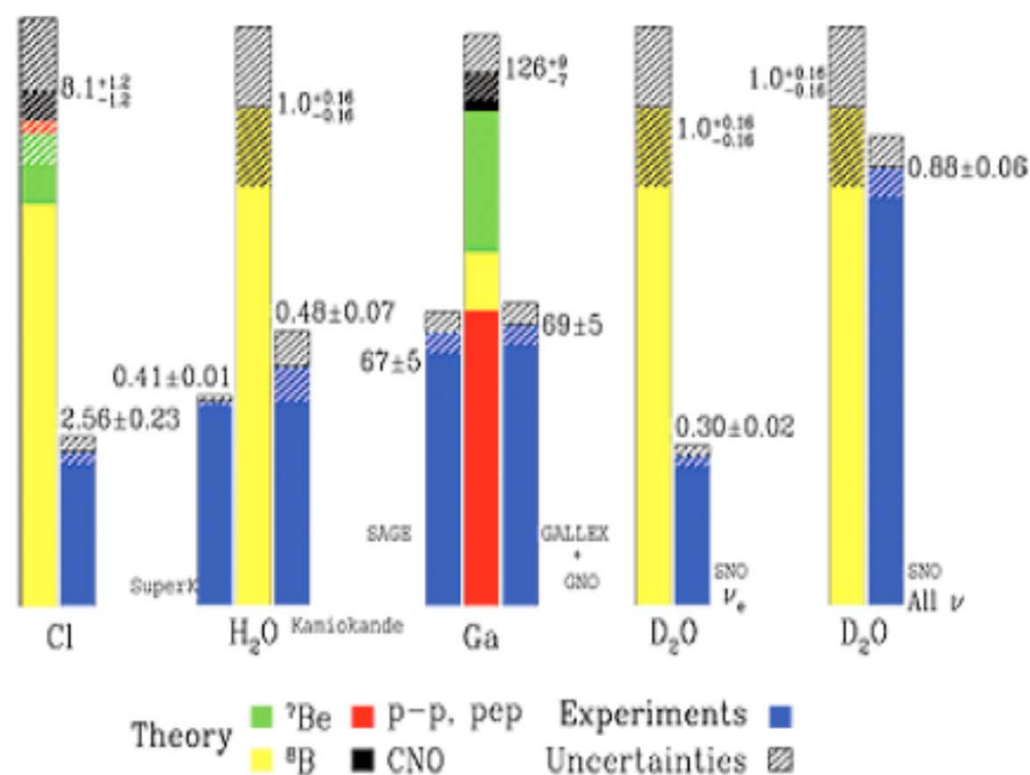


$$\phi_{{}^8\text{B}} = 5.82(1 \pm 0.23) \times 10^6 \text{cm}^{-2} \text{sec}^{-1}$$



Ray Davis & John Bahcall

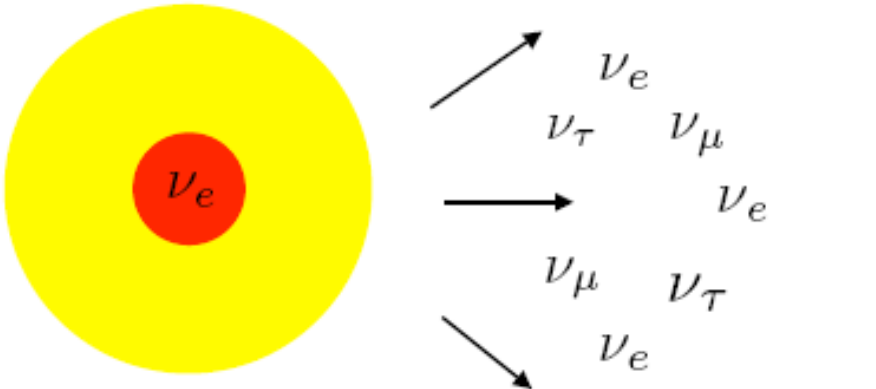
Total Rates: Standard Model vs. Experiment
Bahcall-Serenelli 2005 [BS05(OP)]



Theory v Exp.

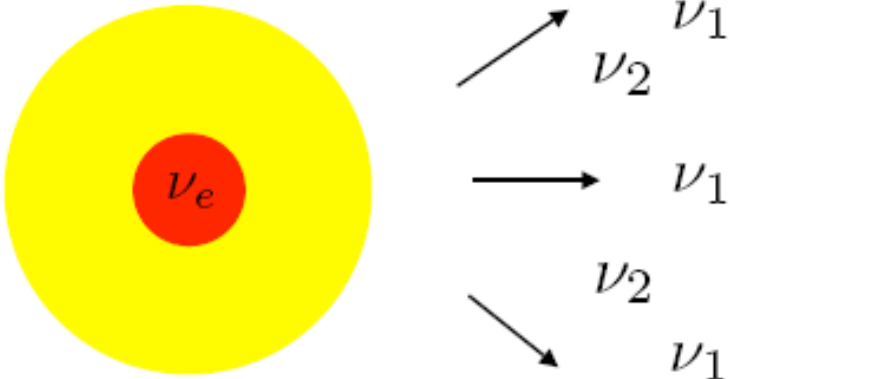
Neutrino Flavor Transitions!!!

Identical Solar Twins:



flavor eigenstates

??????



mass eigenstates

Kinematical Phase:

$$\delta m_{\odot}^2 = 8.0 \times 10^{-5} eV^2$$

$$\sin^2 \theta_{\odot} = 0.31$$

$$\Delta_{\odot} = \frac{\delta m_{\odot}^2 L}{4E} = 1.27 \frac{8 \times 10^{-5} eV^2 \cdot 1.5 \times 10^{11} m}{0.1-10 MeV}$$

$$\Delta_{\odot} \approx 10^{7 \pm 1}$$

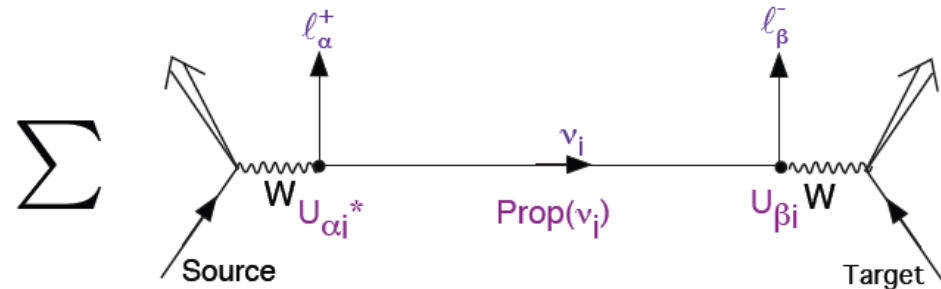
Effectively Incoherent !!!

Vacuum ν_e Survival Probability:

$$\langle P_{ee} \rangle = f_1 \cos^2 \theta_{\odot} + f_2 \sin^2 \theta_{\odot}$$

where f_1 and f_2 are the fraction of ν_1 and ν_2 at production.

In vacuum $f_1 = \cos^2 \theta_{\odot}$

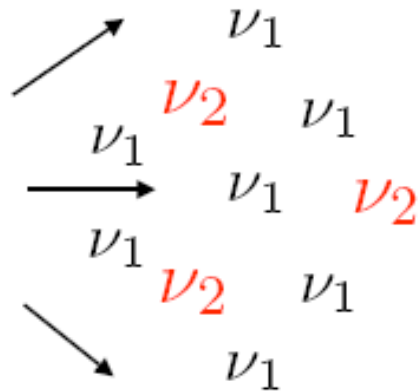
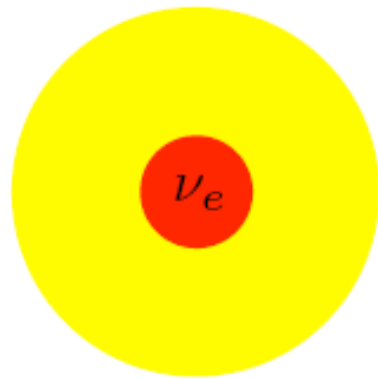


$$\langle P_{ee} \rangle = \cos^4 \theta_{\odot} + \sin^4 \theta_{\odot} = 1 - \frac{1}{2} \sin^2 2\theta_{\odot}$$

for pp and ${}^7\text{Be}$ this is approximately THE ANSWER.

$$f_1 \sim 69\% \text{ and } f_2 \sim 31\% \text{ and } \langle P_{ee} \rangle \approx 0.6$$

pp and ${}^7\text{Be}$



$$f_1 \sim 69\%$$

$$f_2 \sim 31\%$$

$$\langle P_{ee} \rangle \approx 0.6$$

$$f_3 = \sin^2 \theta_{13} < 4\%$$

What about 8B ?

SNO's CC/NC

CC: $\nu_e + d \rightarrow e^- + p + p$

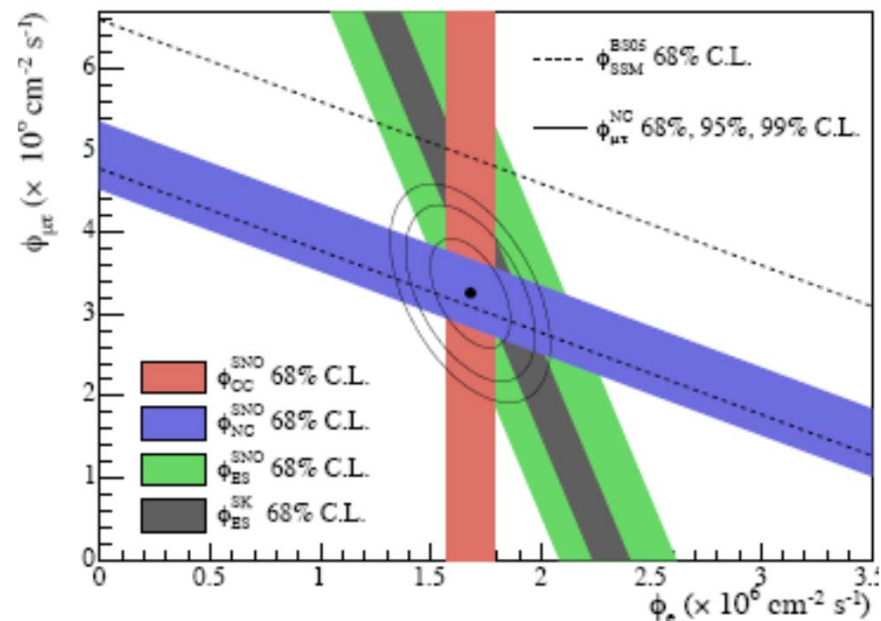
NC : $\nu_x + d \rightarrow \nu_x + p + n$

ES: $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$

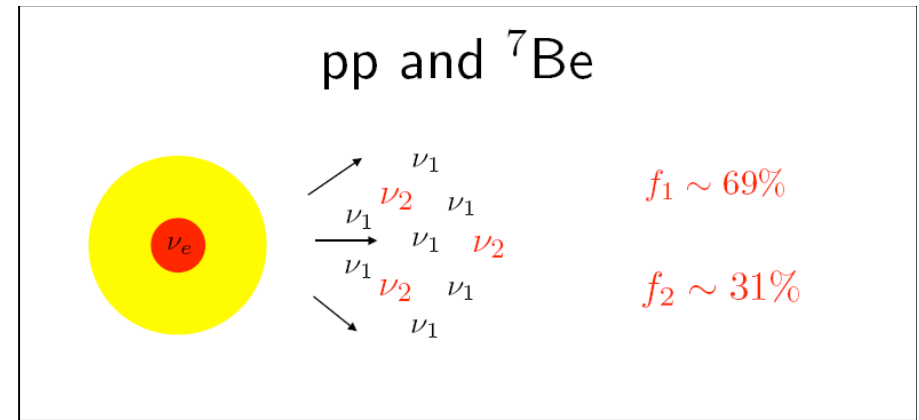
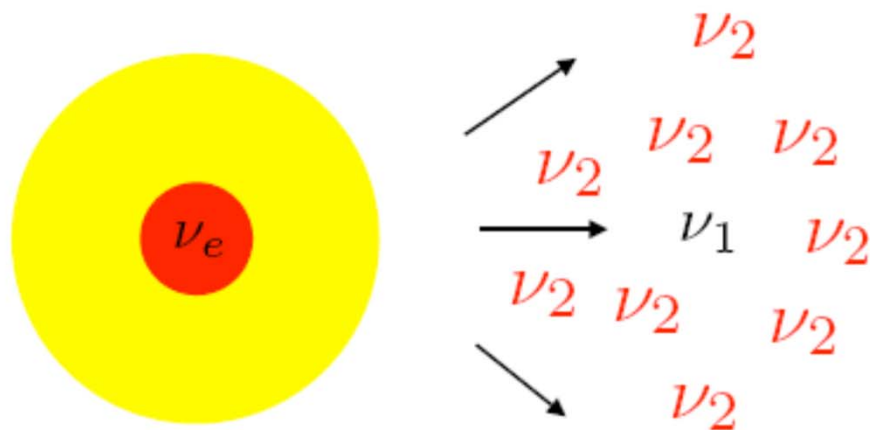
$$\frac{CC}{NC} = \langle P_{ee} \rangle = f_1 \cos^2 \theta_\odot + f_2 \sin^2 \theta_\odot$$

$$f_1 = \left(\frac{CC}{NC} - \sin^2 \theta_\odot \right) / \cos 2\theta_\odot$$

$$= (0.35 - 0.31) / 0.4 \approx 10 \%$$



8B



$f_1 \sim 69\%$

$f_2 \sim 31\%$

$f_2 \sim 90\%$

$f_1 \sim 10\%$

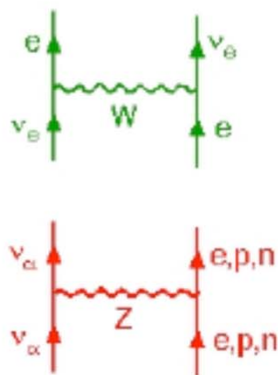
$$\langle P_{ee} \rangle = \sin^2 \theta + f_1 \cos 2\theta_{\odot} \approx \sin^2 \theta_{\odot} = 0.31$$

Wow!!! How did that happen???

energy dependence!!!

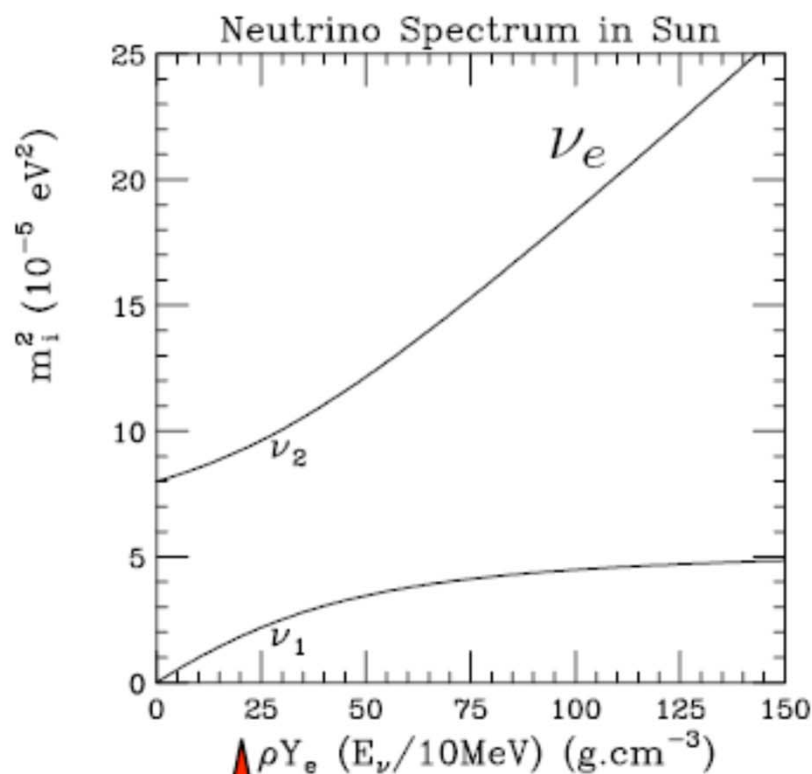
MSW

Coherent Forward Scattering:



Wolfenstein '78

MATTER EFFECTS
CHANGE THE NEUTRINO
MASSES AND MIXINGS



Mikheyev + Smirnov Resonance WIN '85

Neutrino Evolution:

$$-i \frac{\partial}{\partial t} \nu = H \nu$$

in the mass eigenstate basis

$$\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ and } H = \begin{pmatrix} \sqrt{p^2 + m_1^2} & 0 \\ 0 & \sqrt{p^2 + m_2^2} \end{pmatrix}$$

$E = \sqrt{p^2 + m^2}$

$$H = \left(p + \frac{m_1^2 + m_2^2}{4p} \right) I + \frac{1}{4E} \begin{pmatrix} -\delta m^2 & 0 \\ 0 & \delta m^2 \end{pmatrix}$$

$$\delta m^2 = m_2^2 - m_1^2 > 0$$

in the flavor basis

$$\nu \rightarrow U\nu \text{ and } H \rightarrow UHU^\dagger$$

$$\text{where } \nu = \begin{pmatrix} \nu_e \\ \nu_\sigma \end{pmatrix} \text{ and } U = \begin{pmatrix} \cos \theta_\odot & \sin \theta_\odot \\ -\sin \theta_\odot & \cos \theta_\odot \end{pmatrix}$$

and therefore in flavor basis

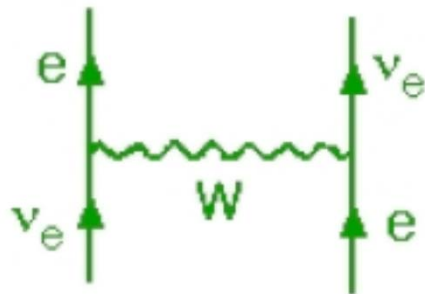
$$0 < \theta_\odot < \frac{\pi}{2}$$

$$H = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}$$

$$\text{i.e. } \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}_{mass} \Rightarrow \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{pmatrix}_{flavor}$$

Coherent Forward Scattering:

dimensions $[G_F N_e] = M^{-2} L^{-3} = M$

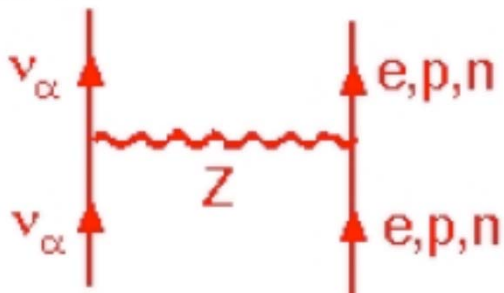


$$\pm \sqrt{2} G_F N_e \delta_{ee}$$

N_e is number density of electrons

+(-) for neutrinos (anti-neutrinos)

Wolfenstein '78



Same for all active flavors,
therefore overall phases

$$\begin{pmatrix} +\sqrt{2}G_F N_e & 0 \\ 0 & 0 \end{pmatrix} \rightarrow \frac{G_F N_e}{\sqrt{2}} I_2 + \frac{1}{2} \begin{pmatrix} +\sqrt{2}G_F N_e & 0 \\ 0 & -\sqrt{2}G_F N_e \end{pmatrix}$$

Including Matter Effects in the Flavor Basis:

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m^2 \cos 2\theta_\odot + 2\sqrt{2}G_F N_e E_\nu & \delta m^2 \sin 2\theta_\odot \\ \delta m^2 \sin 2\theta_\odot & \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu \end{pmatrix}$$

Diagonalize by identifying with

$$H_{flavor} = \frac{1}{4E_\nu} \begin{pmatrix} -\delta m_N^2 \cos 2\theta_\odot^N & \delta m_N^2 \sin 2\theta_\odot^N \\ \delta m_N^2 \sin 2\theta_\odot^N & \delta m_N^2 \cos 2\theta_\odot^N \end{pmatrix}$$

Masses and Mixings in MATTER: δm_N^2 and θ_\odot^N

$$\begin{aligned} \delta m_N^2 \cos 2\theta_\odot^N &= \delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu \\ \delta m_N^2 \sin 2\theta_\odot^N &= \delta m^2 \sin 2\theta_\odot \end{aligned}$$

Notice:

- (1) Possible zero when $\delta m^2 \cos 2\theta_\odot = 2\sqrt{2}G_F N_e E_\nu$
- (2) the invariance of the product $\delta m^2 \sin 2\theta_\odot$

ν_e disappearance in Looooong Block of Lead:

$$1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta_{\odot}^N \sin^2 \Delta_N$$

$$\Delta_N = \frac{\delta m_N^2 L}{4E}$$

same form as vacuum

The Solution:

$$\delta m_N^2 = \sqrt{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu)^2 + (\delta m^2 \sin 2\theta_\odot)^2}$$

$$\sin^2 \theta_\odot^N = \frac{1}{2} \left(1 - \frac{(\delta m^2 \cos 2\theta_\odot - 2\sqrt{2}G_F N_e E_\nu)}{\delta m_N^2} \right) \quad \theta_\odot^N > \theta_\odot$$

Quasi-Vacuum: $2\sqrt{2}G_F N_e E_\nu \ll \delta m^2 \cos 2\theta_\odot$

pp and ${}^7\text{Be}$

$$\delta m_N^2 = \delta m^2 \text{ and } \theta_\odot^N = \theta_\odot$$

Resonance (Mikheyev + Smirnov '85): $2\sqrt{2}G_F N_e E_\nu = \delta m^2 \cos 2\theta_\odot$

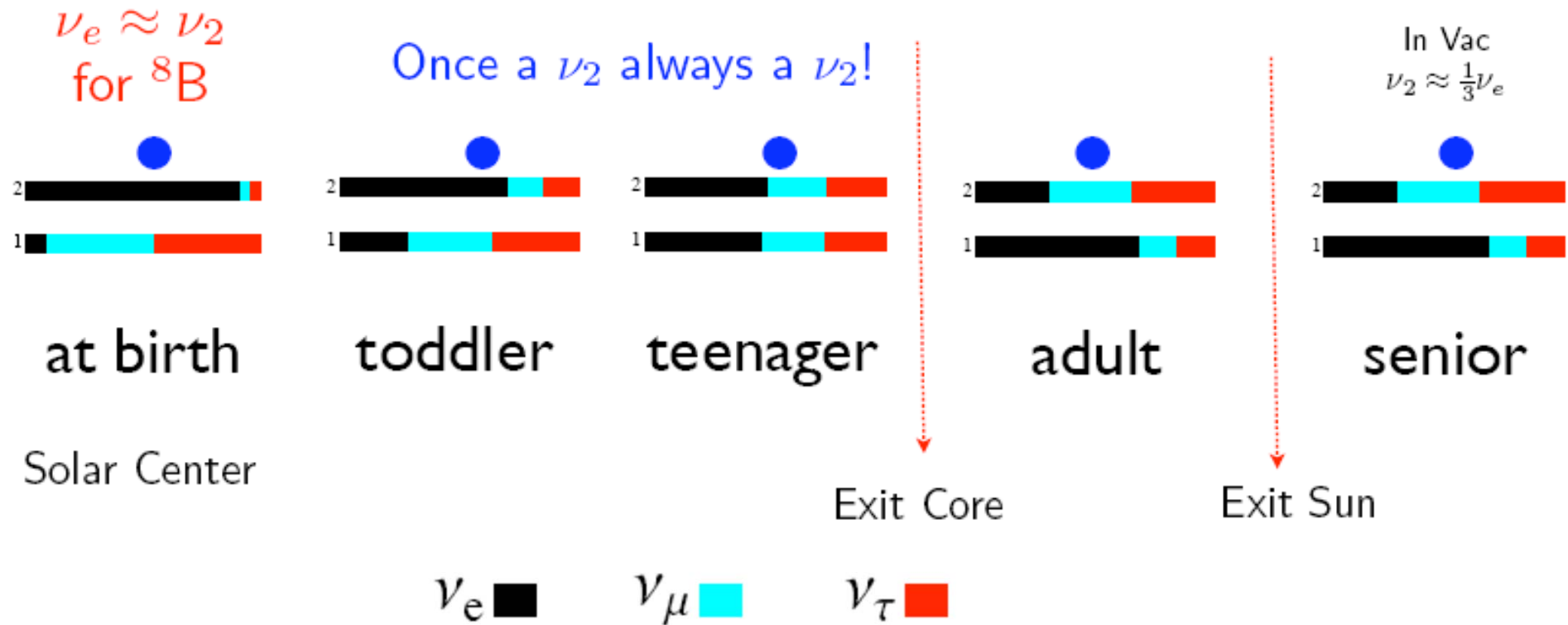
$$\delta m_N^2 = \delta m^2 \sin 2\theta_\odot \text{ and } \theta_\odot^N = \pi/4$$

Matter Dominated: $2\sqrt{2}G_F N_e E_\nu \gg \delta m^2 \cos 2\theta_\odot$

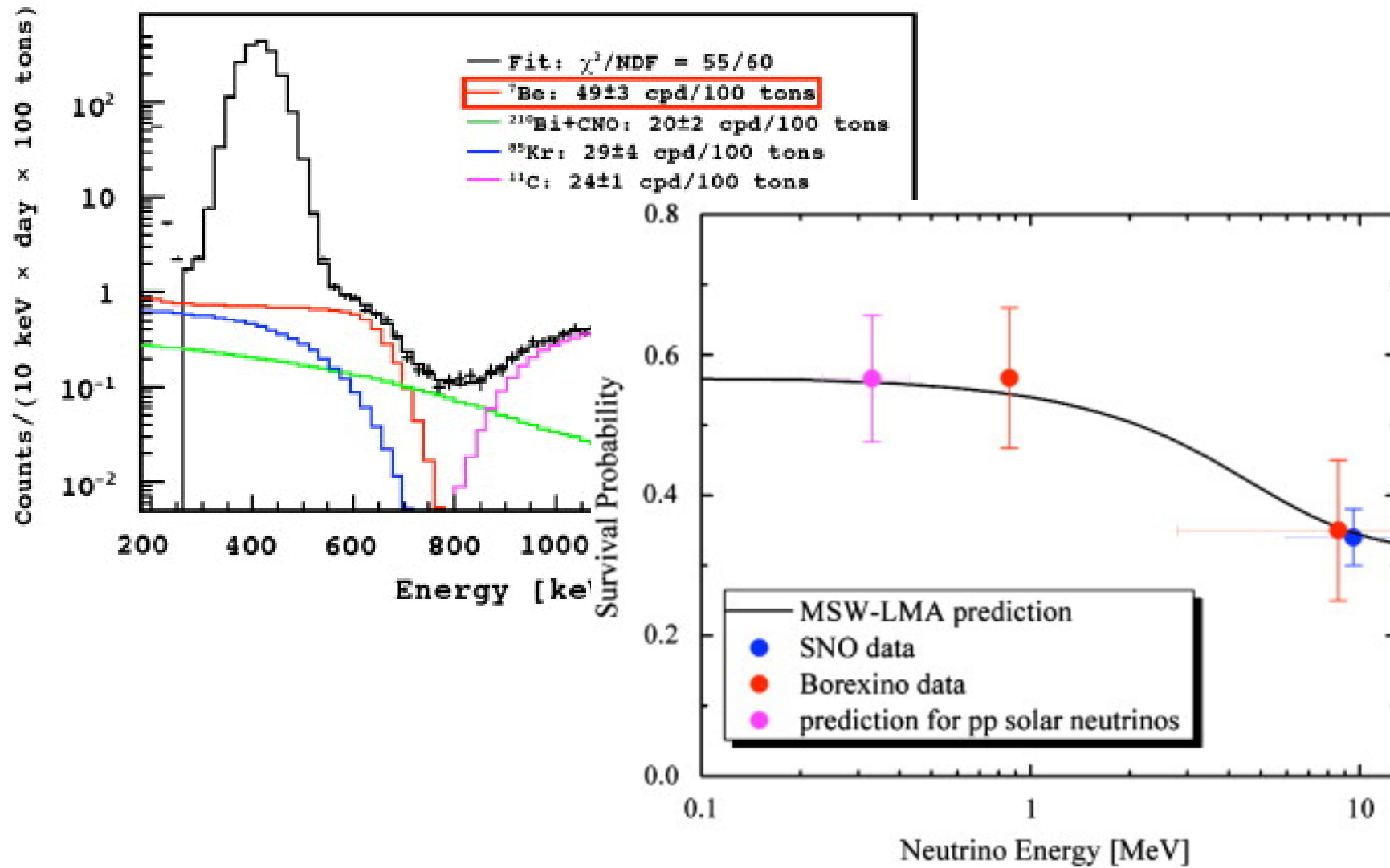
$$\delta m_N^2 \rightarrow 2\sqrt{2}G_F N_e E_\nu \text{ and } \theta_\odot^N \rightarrow \pi/2$$

${}^8\text{B}$

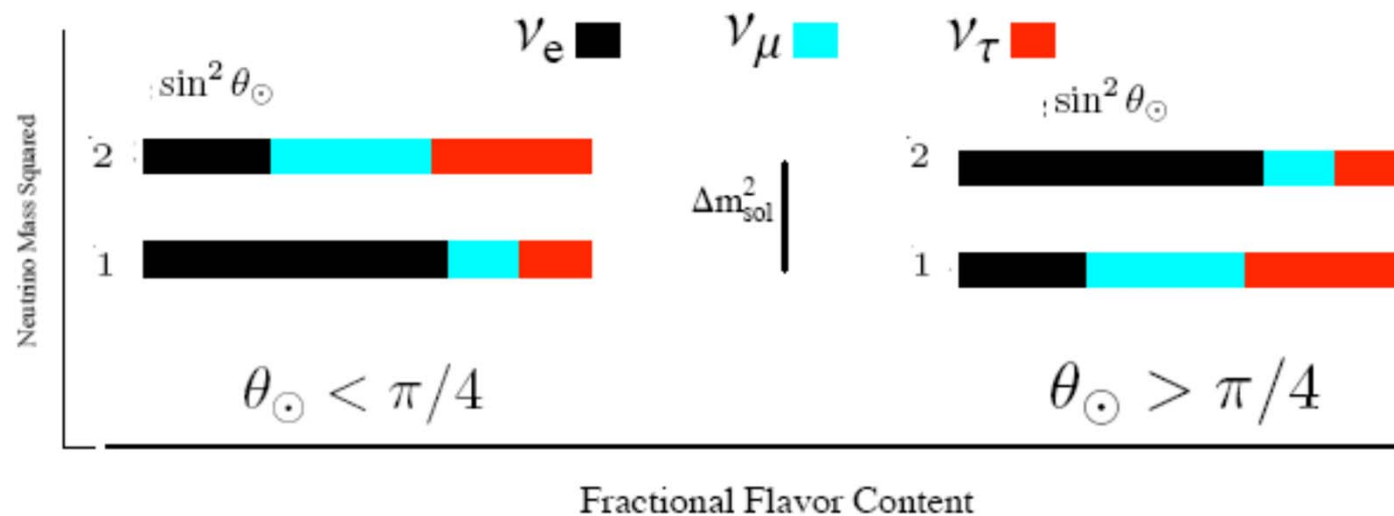
Life of a Boron-8 Solar Neutrino:



Borexino results (2012)



Solar Pair Mass Hierarchy:



Who cares ?
SNO does !!!

for neutrino in matter
 $\theta_\odot^N > \theta_\odot$

$$\langle P_{ee} \rangle = \cos^2 \theta_\odot^N \cos^2 \theta_\odot + \sin^2 \theta_\odot^N \sin^2 \theta_\odot = \frac{1}{2} + \frac{1}{2} \cos 2\theta_\odot^N \cos 2\theta_\odot$$

$$\text{if } \theta_\odot < \pi/4 \\ \langle P_{ee} \rangle \geq \sin^2 \theta_\odot$$

$$\text{if } \theta_\odot > \pi/4 \\ \langle P_{ee} \rangle \geq \frac{1}{2}(1 + \cos^2 2\theta_\odot) \geq \frac{1}{2}$$

$$\text{SNO: } \langle P_{ee} \rangle_{\text{day}} = 0.347 \pm 0.038$$

Solar Hierarchy
Determined !!!

Day/Night Asymmetry:

$$\sin^2 \theta_{\odot} \rightarrow \sin^2 \theta_{\oplus} = \sin^2 \theta_{\odot} + \frac{1}{2} \sin^2 2\theta_{\odot} \left(\frac{A_{\oplus}}{\delta m_{\odot}^2} \right) \text{ in the earth.}$$

$A=2(D-N)/(D+N)$ expected to be few %

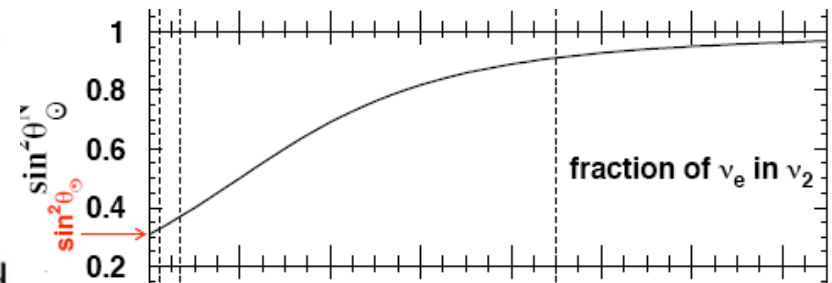
	Amplitude fit		separate D, N:
	Δm	Δm	$(D-N)/((D+N)/2)$
SK-I	$-2.0 \pm 1.8 \pm 1.0\%$	$-1.9 \pm 1.7 \pm 1.0\%$	$-2.1 \pm 2.0 \pm 1.3\%$
SK-II	$-4.4 \pm 3.8 \pm 1.0\%$	$-4.4 \pm 3.6 \pm 1.0\%$	$-5.5 \pm 4.2 \pm 3.7\%$
SK-III	$-4.2 \pm 2.7 \pm 0.7\%$	$-3.8 \pm 2.6 \pm 0.7\%$	$-5.9 \pm 3.2 \pm 1.3\%$
SK-IV	$-3.6 \pm 1.6 \pm 0.6\%$	$-3.3 \pm 1.5 \pm 0.6\%$	$-4.9 \pm 1.8 \pm 1.4\%$
comb	$-3.3 \pm 1.0 \pm 0.5\%$	$-3.1 \pm 1.0 \pm 0.5\%$	$-4.1 \pm 1.2 \pm 0.8\%$
non-zero signif.	3.0σ	2.8σ	2.8σ

Spectral Distortion:

A characteristic of matter effects is that the Fraction of ν_2 is energy dependent .

Smaller at smaller E.

Implies an increase in P_{ee} near threshold.

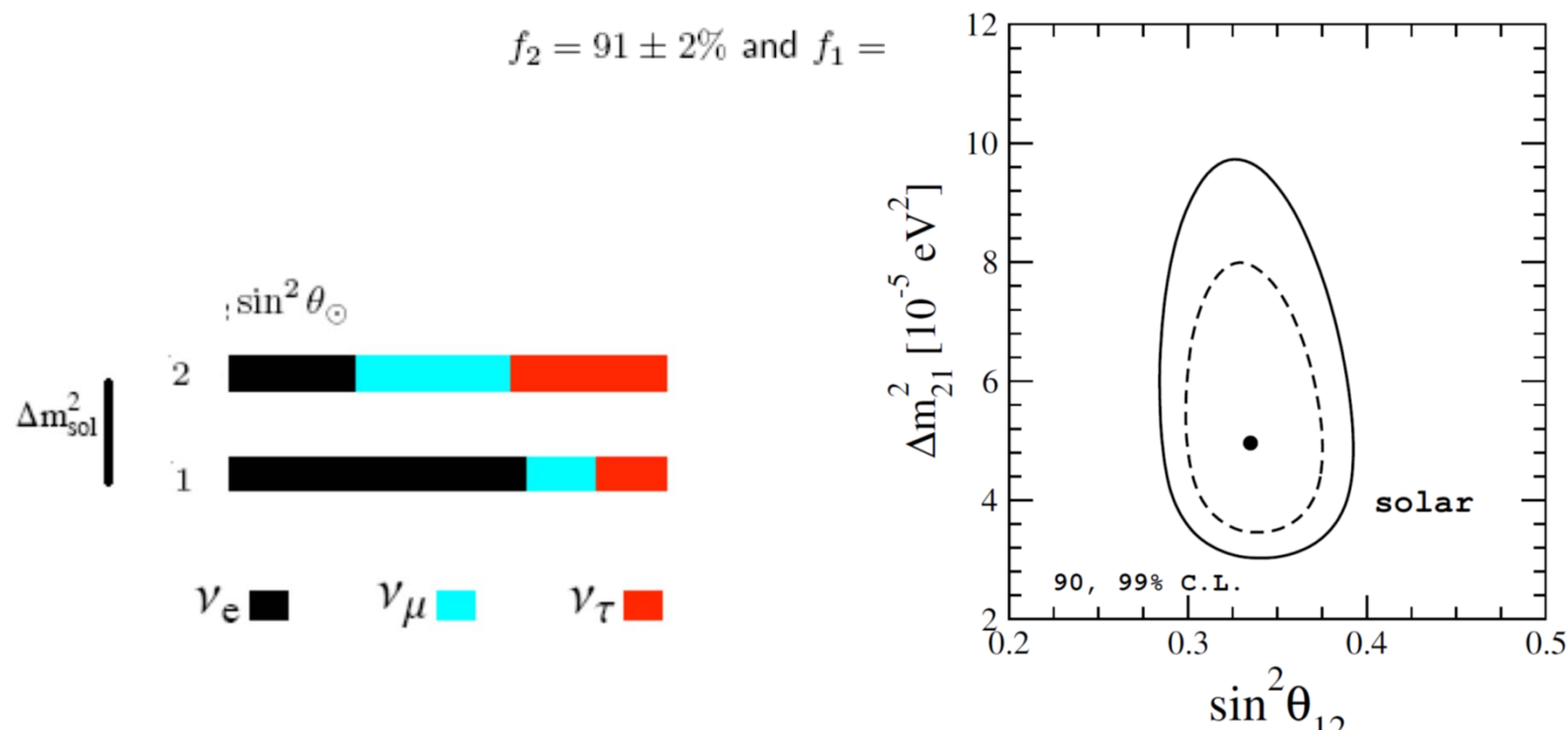


Summary:

The low energy pp and ${}^7\text{Be}$ Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2 due to (quasi-) vacuum oscillations.

$$f_1 = 65 \pm 2\%, f_2 = 35 \mp 2\% \text{ with } P_{ee} \approx 0.56$$

The high energy ${}^8\text{B}$ Solar Neutrinos exit the sun as "PURE" ν_2 mass eigenstates due to matter effects.



Testing solar neutrino oscillations with reactors

$$1 - P(\nu_e \rightarrow \nu_e) = \sin^2 2\theta_{\odot} \sin^2 \Delta$$

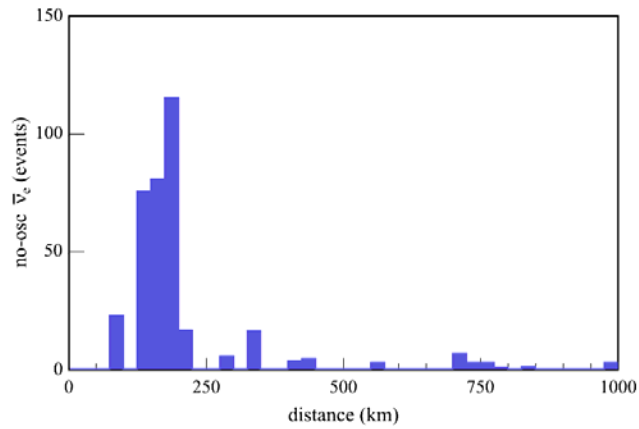
$$\Delta = \frac{\delta m^2 L}{4E}$$

10^{-5} eV^2

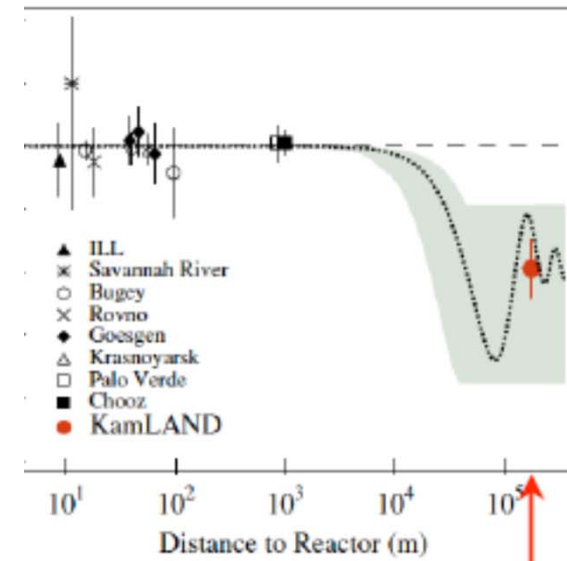
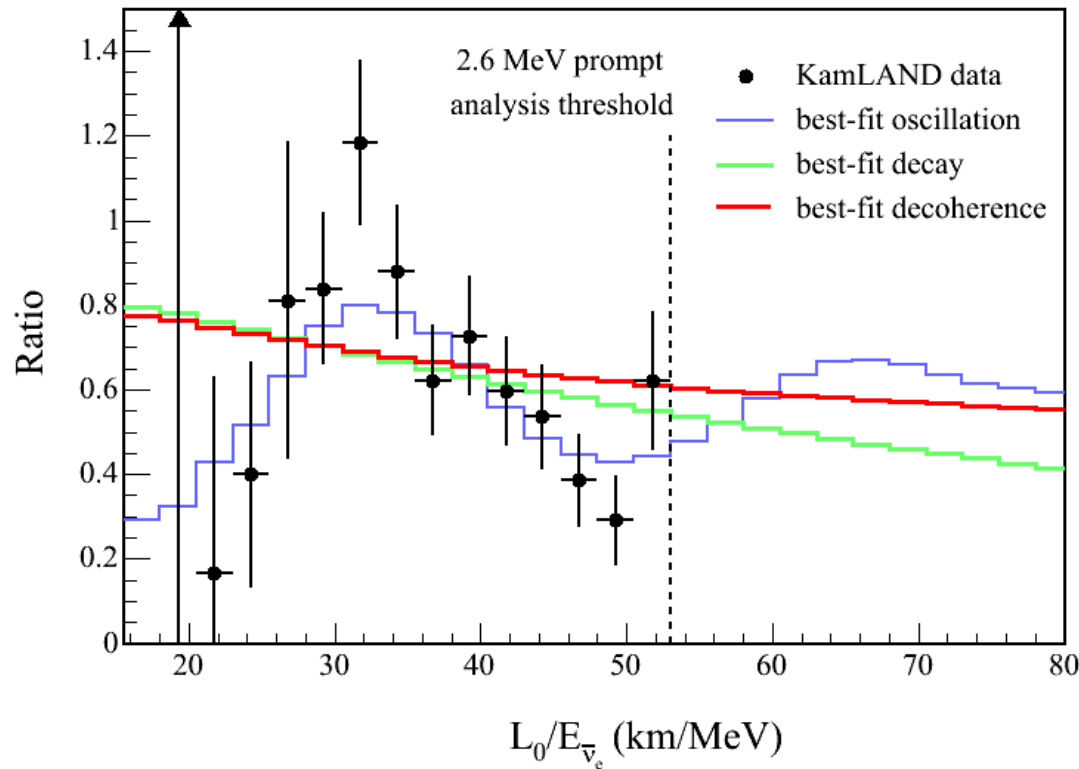
$10^5 \text{ m} = 100 \text{ km}$

1 MeV

expected no-oscillation neutrino event rate at KamLAND



180 km



180 km

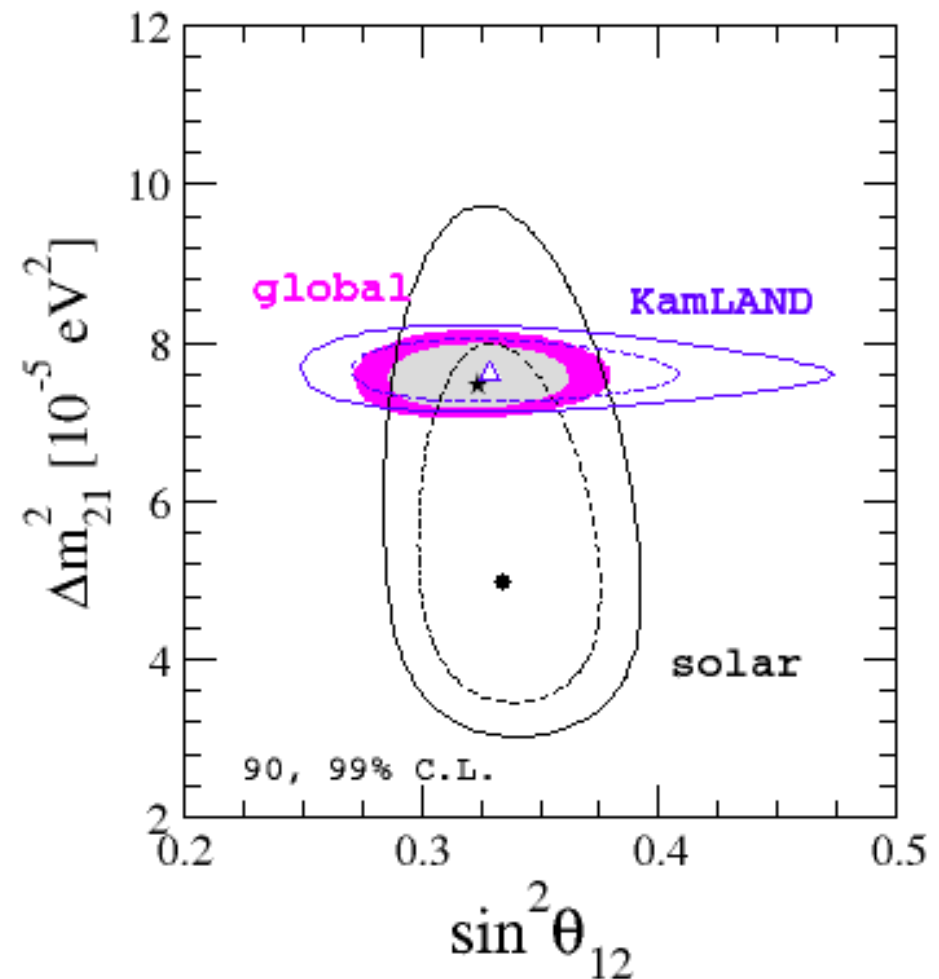
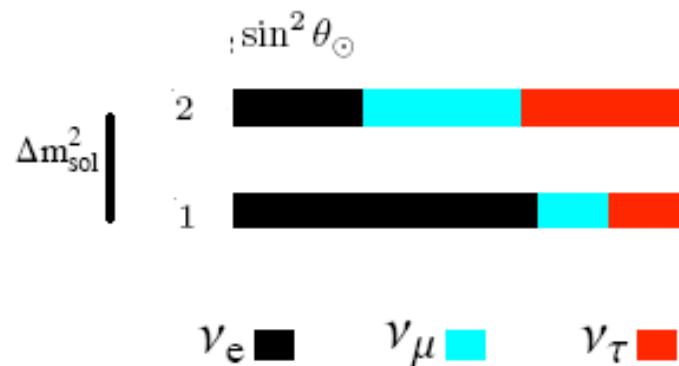
Summary:

The low energy pp and ${}^7\text{Be}$ Solar Neutrinos exit the sun as two thirds ν_1 and one third ν_2

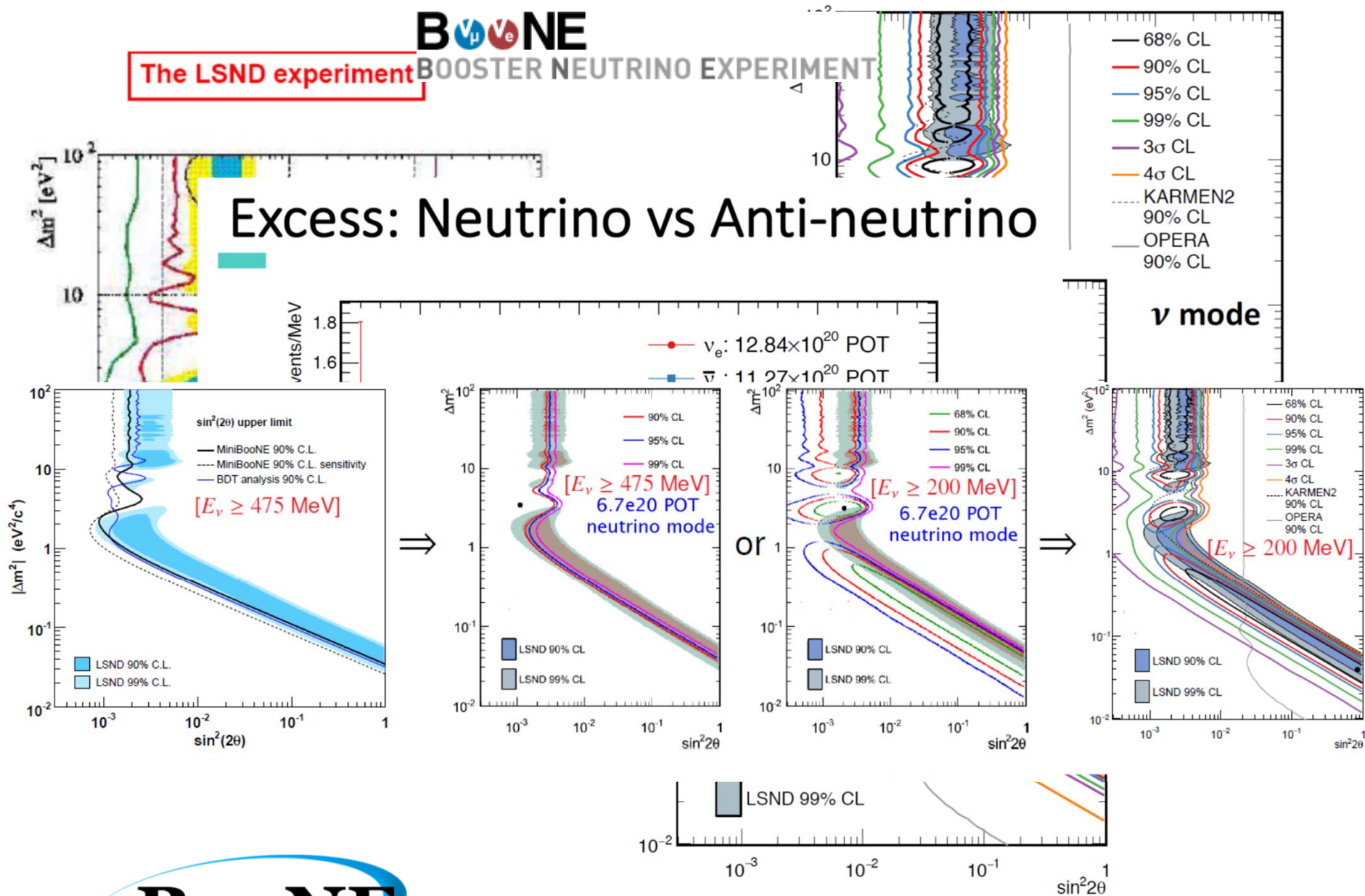
$$f_1 = 65 \pm 2\%, f_2 =$$

The high energy ${}^8\text{B}$ Sol
"PURE" ν_2 mass eigen:

$$f_2 = 91 \pm 2\% \text{ and } j$$

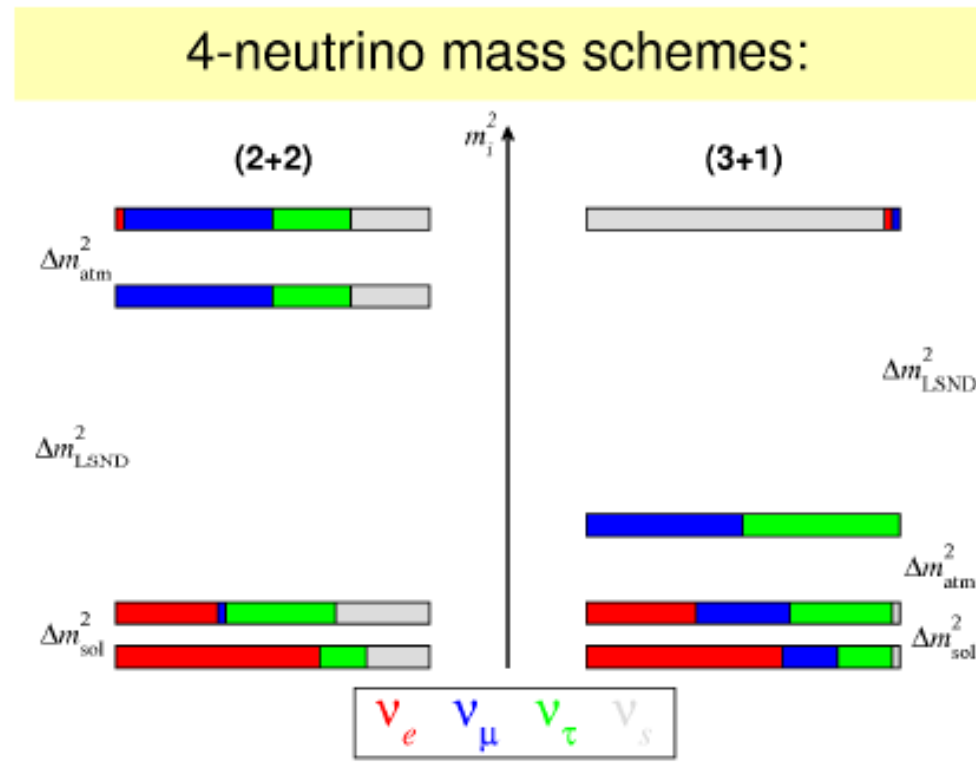


Excess: Neutrino vs Anti-neutrino



With 3 different Δm^2 4 light neutrinos needed!

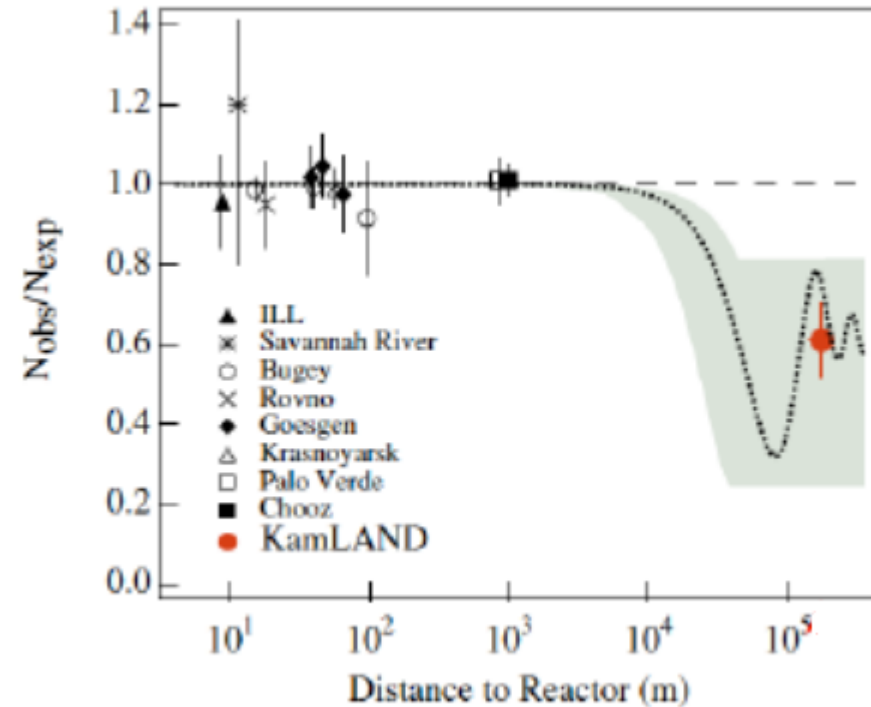
4th ν : cannot be active – must be sterile. Mixing matrix: 6 θ_{ij} , 3 Dirac-type \mathcal{CP} phases. But: simplifications occur – only two possible type of schemes: 2+2 and 3+1



On March 2011 ArXiv 1101.2755

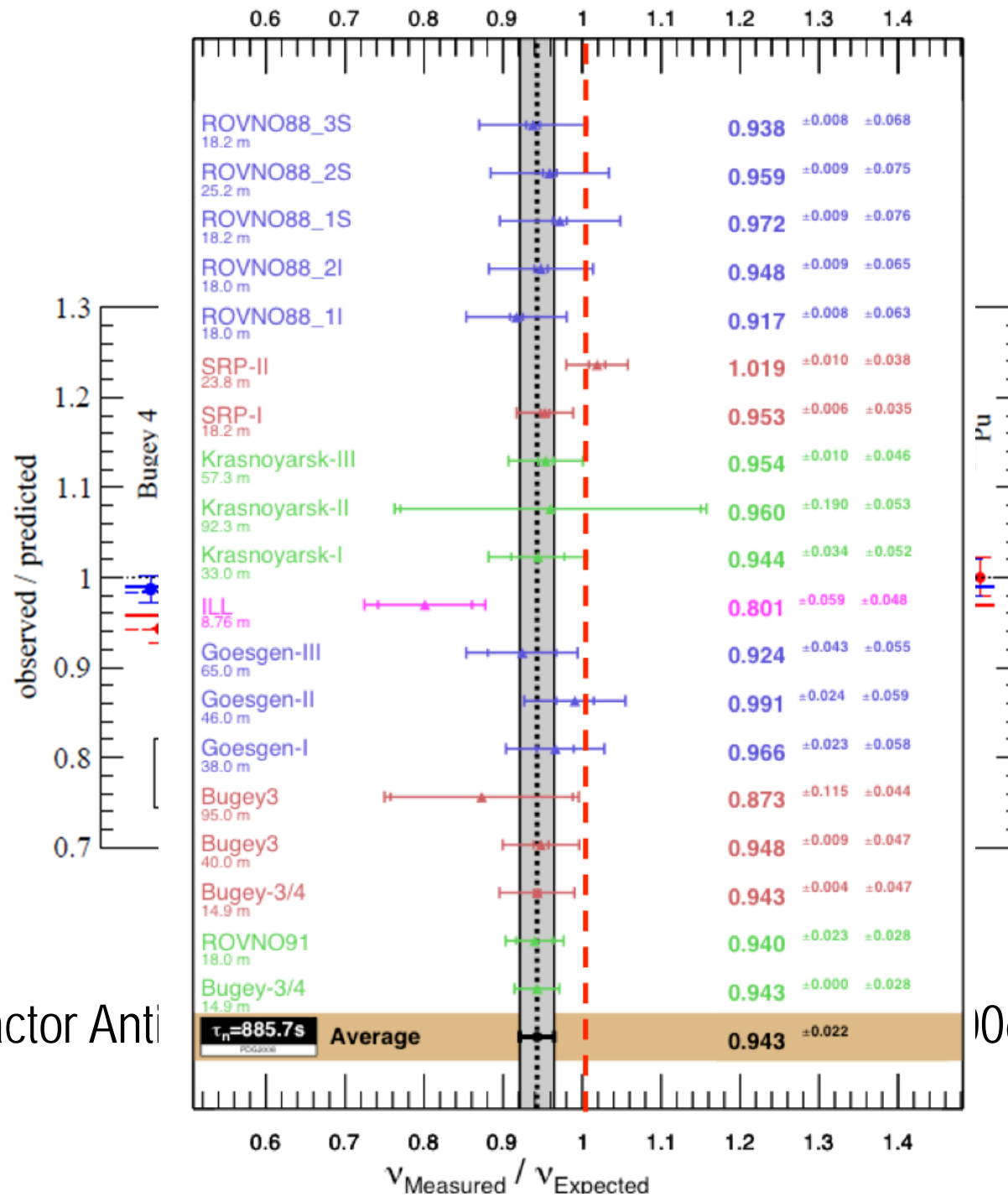
New reactor antineutrino spectra have been measured using ^{238}U , increasing the mean flux by

This reevaluation applies to all reactor



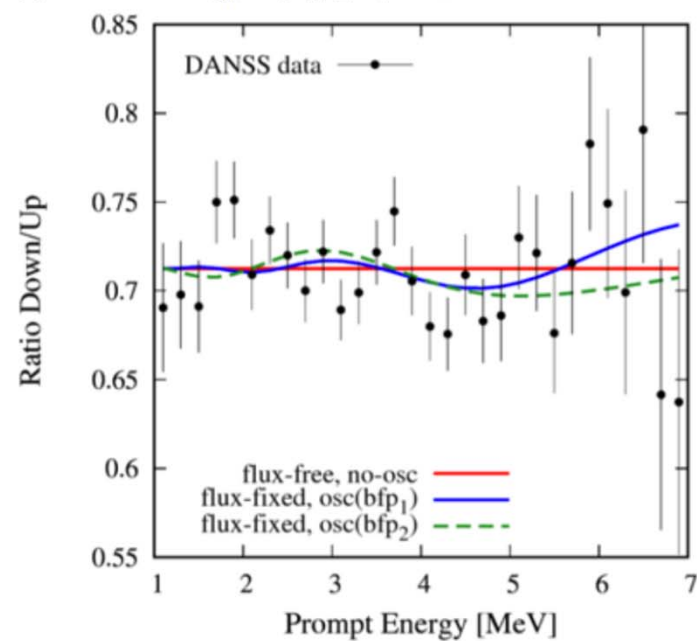
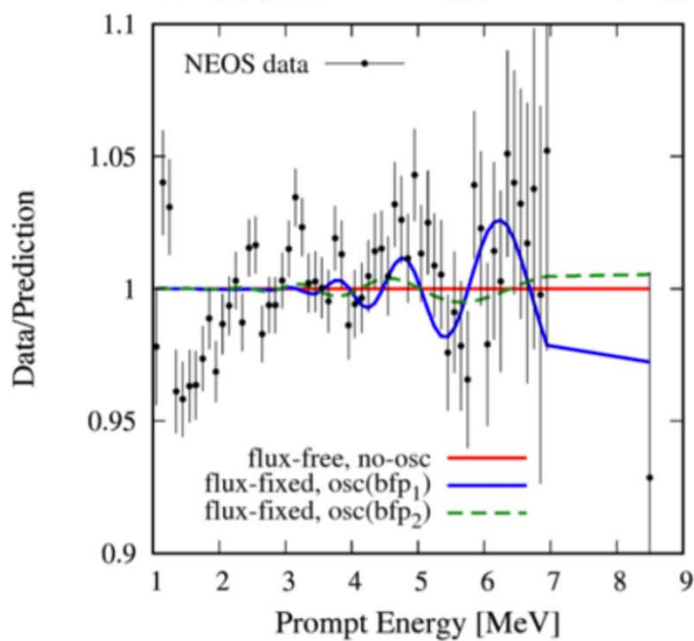
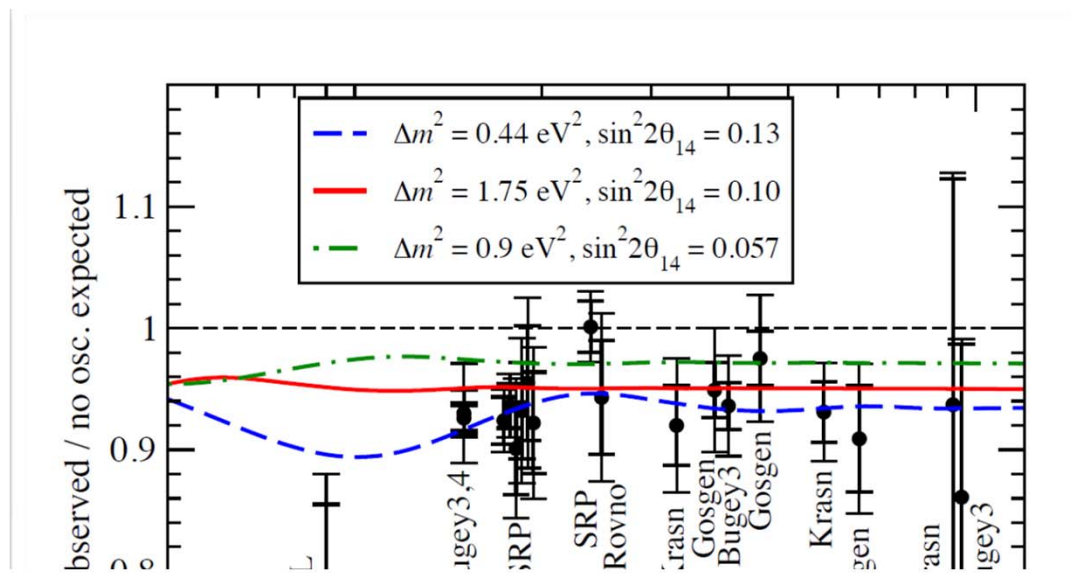
It means that for experiments at reactor-detector distances < 100 m the ratio of observed event rate to predicted rate shifts

$$0.976 \pm 0.024 \rightarrow 0.943 \pm 0.023$$



“The Reactor Anti

06, 2011



The Gallium Anomaly

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

$$\sin^2(2\theta) \approx 0.50 \quad \Delta m^2 \approx 2 \text{ eV}^2$$

Signals at SBL are at the 2-4 σ level
All pointing in the same direction

Experiment	Type	Channel	Significance
LSND	DAR	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ CC	3.8 σ
MiniBooNE	SBL accelerator	$\nu_\mu \rightarrow \nu_e$ CC	3.4 σ
MiniBooNE	SBL accelerator	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ CC	2.8 σ
GALLEX/SAGE	Source - e capture	ν_e disappearance	2.8 σ
Reactors	Beta-decay	$\bar{\nu}_e$ disappearance	3.0 σ

K. N. Abazajian et al. "Light Sterile Neutrinos: A Whitepaper", arXiv:1204.5379 [hep-ph], (2012)

(e⁻)

10491]

$$\sigma(\bar{\nu}_\mu \text{Cr}) = 58.1 \times 10^{-44} \text{ cm}^2 (1 \pm 0.028)_{1\sigma} \implies R_{\text{Ga}} = 0.86 \pm 0.05$$

[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

Haxton: [Hata, Haxton, PLB 353 (1995) 422, nucl-th/9503017; Haxton, PLB 431 (1998) 110, nucl-th/9804011]

$$\sigma(^{51}\text{Cr}) = 63.9 \times 10^{-46} \text{ cm}^2 (1 \pm 0.106)_{1\sigma} \implies R_{\text{Ga}} = 0.76^{+0.09}_{-0.08}$$

[SAGE, PRC 59 (1999) 2246, hep-ph/9803418]