# MSTW Global Fit Updates

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## Heavy Quark issues

Will discuss Charm $\sim 1.4\mathrm{GeV}$, bottom $\sim 4.75\mathrm{GeV}$ as heavy flavours.

Quick reminder.

Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Note that $n_f$ is effective number of light quarks. Can be $3, 4$ or $5$.

Does not sum $\alpha_S^n \ln^n Q^2/m_H^2$ terms in perturbative expansion. Usually achieved by definition of heavy flavour parton distributions and solution of evolution equations.

Additional problem FFNS known up to NLO (Laenen *et al*), but are not defined at NNLO – $\alpha_S^3 C_{2,Hg}^{FF,3}$ not fully known.

Recent progress by Blümlein *et al* reported at this meeting.

## Variable Flavour

High scales $Q^2 \gg m_H^2$ massless partons. Behave like up, down (strange always in this regime. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZM,n_f} \otimes f_j^{n_f}(Q^2).$$

Partons in different number regions related to each other perturbatively.

$$f_j^{n_f+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),$$

Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ (Buza *et al* $\mathcal{O}(\alpha_S^2)$, Blümlein *et al* $\mathcal{O}(\alpha_S^3)$) containing $\ln(Q^2/m_H^2)$ terms relate $f_i^{n_f}(Q^2)$ and $f_i^{n_f+1}(Q^2) \rightarrow$ correct evolution for both.

We use a **General-Mass Variable Flavour Number Scheme** (VFNS) taking one from the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

Particular definition. More on this later.

**Dependence on $m_c$ at NLO in 2008 fits.**

Vary $m_c$ in steps of $0.1\mathrm{GeV}$.

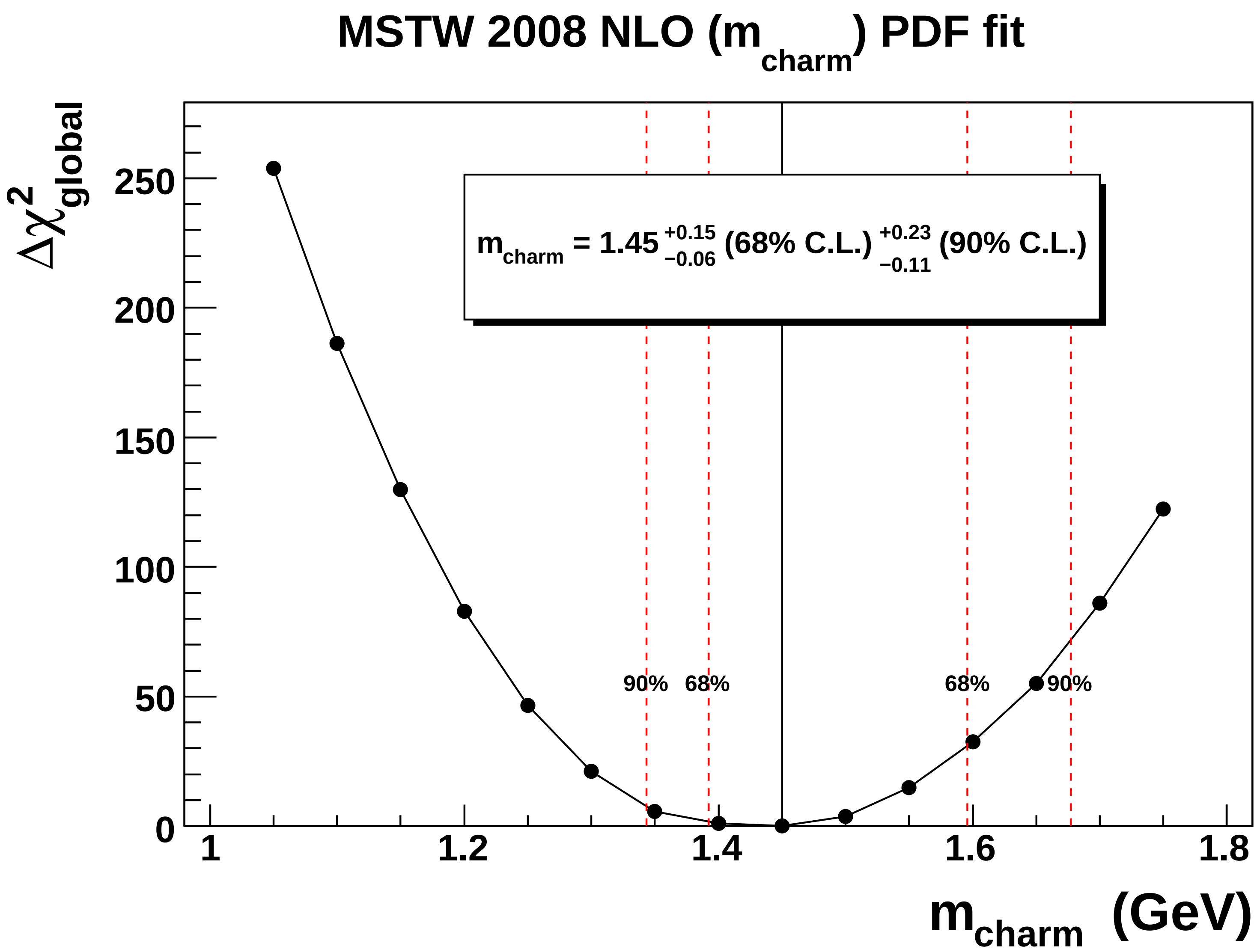
| $m_c$ (GeV) | $\chi^2_{global}$ 2699 pts | $\chi^2_{F_2^c}$ 83 pts | $\alpha_s(M_Z^2)$ |
|---|---|---|---|
| 1.1 | 2728 | 263 | 0.1182 |
| 1.2 | 2624 | 187 | 0.1187 |
| 1.3 | 2562 | 134 | 0.1194 |
| 1.4 | 2542 | 107 | 0.1201 |
| 1.45 | 2541 | 100 | 0.1204 |
| 1.5 | 2544 | 97 | 0.1208 |
| 1.6 | 2574 | 104 | 0.1215 |
| 1.7 | 2627 | 129 | 0.1222 |

Clear correlation between $m_c$ and $\alpha_S(M_Z^2)$.

For low $m_c$ overshoot low $Q^2$ medium $x$ data badly.

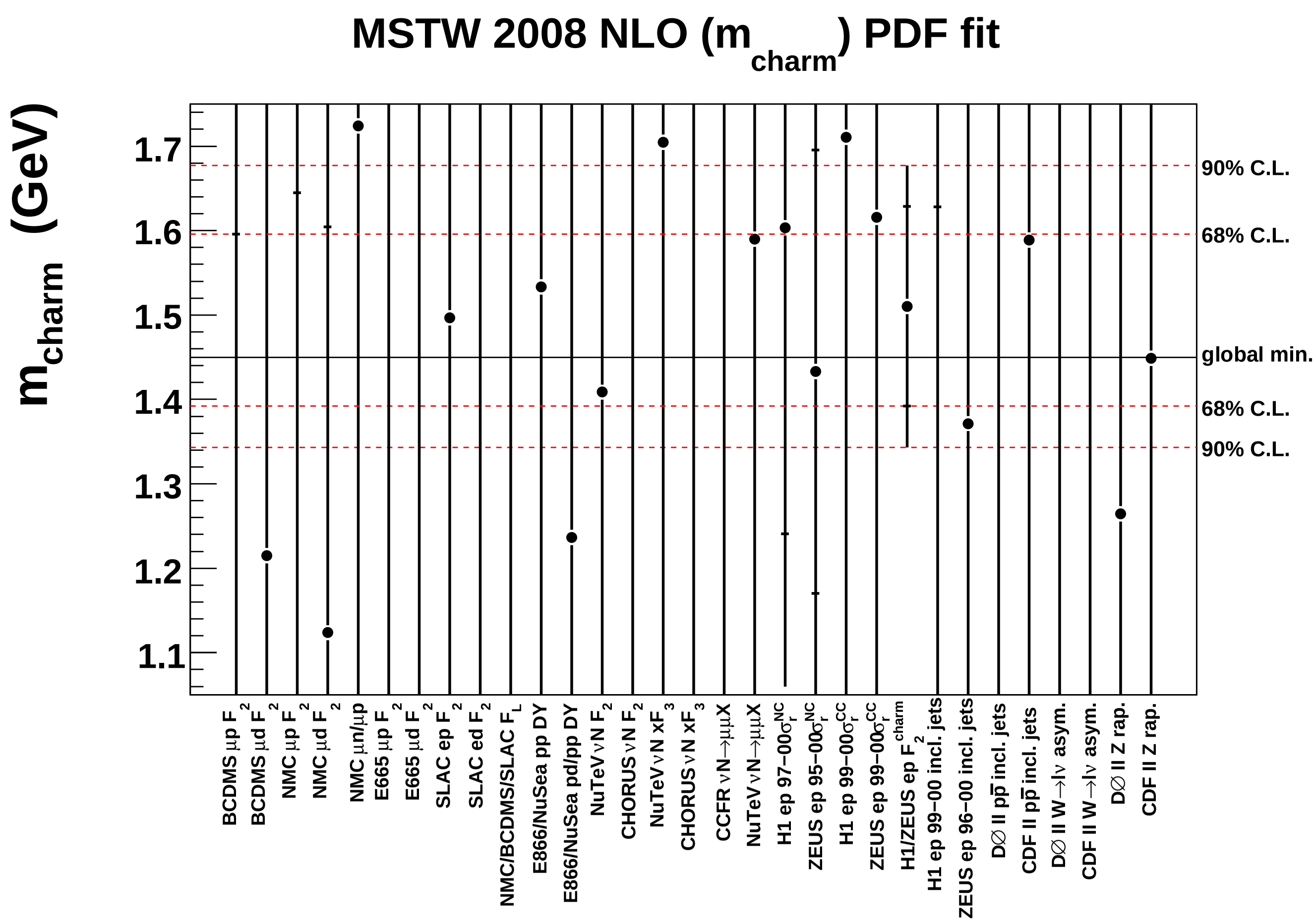
Preference for $m_c = 1.45\mathrm{GeV}$. Towards lower end of pole mass determinations. Uncertainty from fit $+0.15\mathrm{GeV}$ or $-0.06\mathrm{GeV}$.

Also illustrated as in the figure below.




Can look at more details

Preference of each data set.




$F_2^c(x, Q^2)$ data most discriminating. In HERA inclusive data changes in $m_c$ compensated for by change in gluon and $\alpha_S$.

NMC data prefer lower $m_c$ - quicker evolution near threshold. BCDMS mainly prefer correlated lower $\alpha_S$.

**Dependence on $m_c$ at NNLO in 2008 fits.**

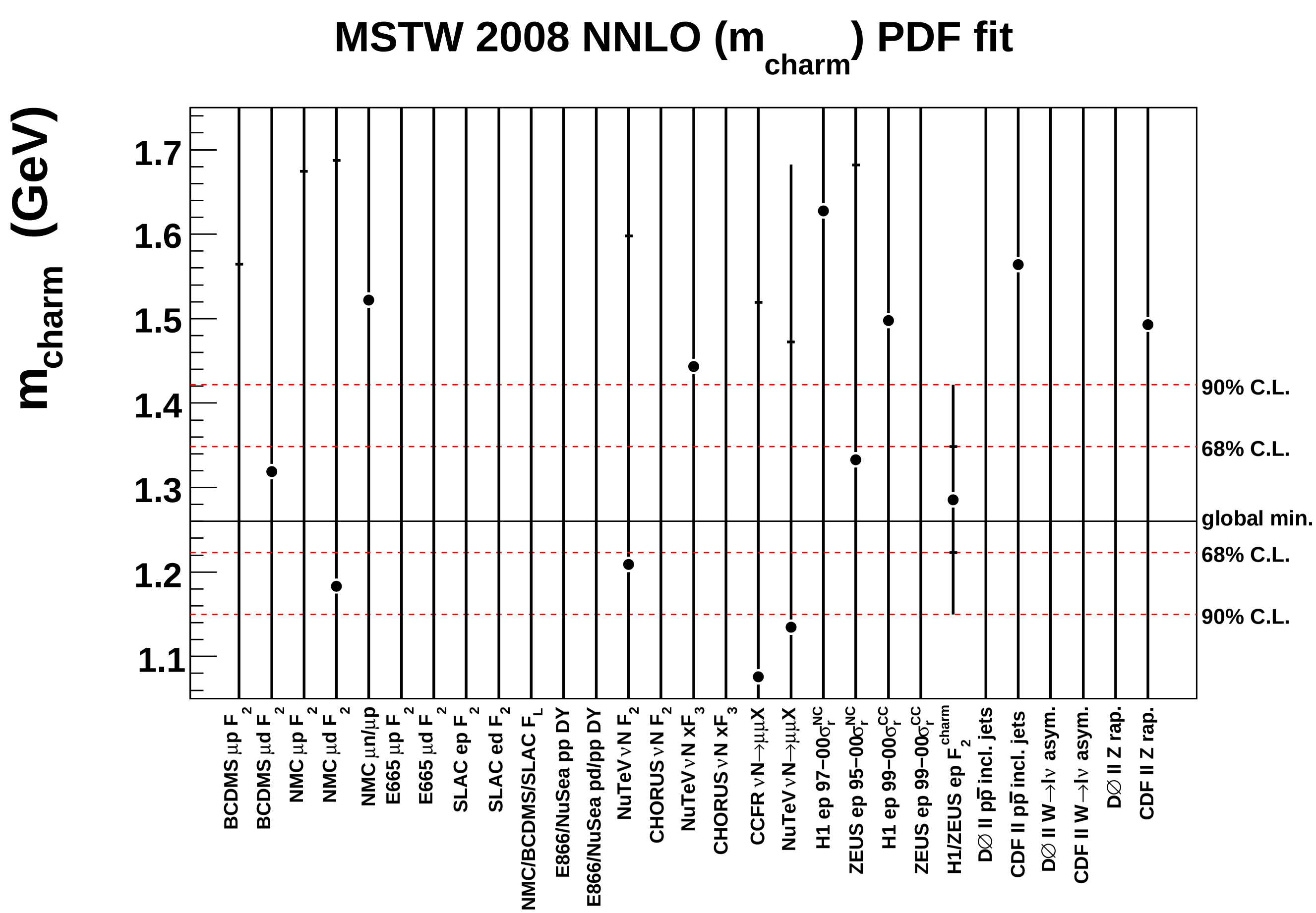
| $m_c$ (GeV) | $\chi^2_{global}$ 2615 pts | $\chi^2_{F_2^c}$ 83 pts | $\alpha_s(M_Z^2)$ |
|---|---|---|---|
| 1.1 | 2499 | 114 | 0.1158 |
| 1.2 | 2463 | 88 | 0.1162 |
| 1.28 | 2546 | 82 | 0.1165 |
| 1.3 | 2457 | 82 | 0.1166 |
| 1.4 | 2479 | 95 | 0.1170 |
| 1.5 | 2526 | 125 | 0.1175 |
| 1.6 | 2588 | 167 | 0.1179 |
| 1.7 | 2665 | 217 | 0.1184 |

Less correlation between $m_c$ and $\alpha_S(M_Z^2)$.

For high $m_c$ undershoot moderate $Q^2$ data badly.

Preference for low value of $m_c = 1.28\mathrm{GeV}$ with uncertainty of $+0.09\mathrm{GeV}$ and $-0.04\mathrm{GeV}$.

Preference of each data set.




NMC data already have quicker evolution near threshold at NNLO. BCDMS have less $\alpha_S$ dependence.

**Dependence on $m_b$ at NLO in 2008 fits.**

Vary $m_b$ in steps of $0.25\text{GeV}$.

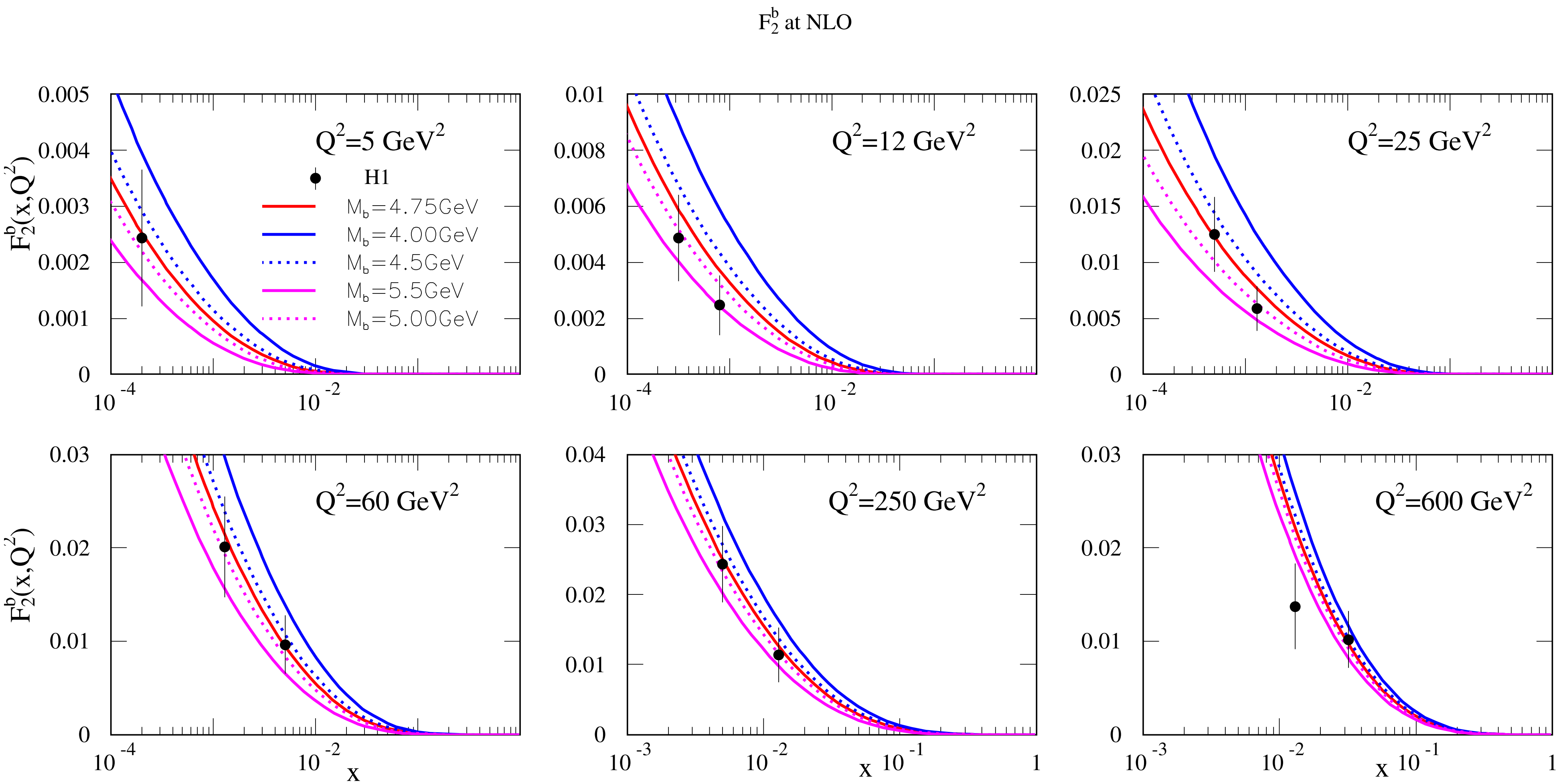
| $m_b$ (GeV) | $\chi^2_{global}$ 2699 pts | $\alpha_s(M_Z^2)$ |
|---|---|---|
| 4.00 | 2536 | 0.1201 |
| 4.25 | 2539 | 0.1201 |
| 4.50 | 2540 | 0.1201 |
| 4.75 | 2542 | 0.1201 |
| 5.00 | 2544 | 0.1200 |
| 5.25 | 2547 | 0.1201 |
| 5.50 | 2549 | 0.1200 |

Stays fairly flat all the way down to $m_b = 3\text{GeV}$.

For lower $m_b$ slightly better fit to HERA data, including $F_2^c(x, Q^2)$.

Similar at NNLO, but with about half the change in $\chi^2$.

# NLO comparisons to Beauty data (not in global fit) for varying $m_b$




Distinct preference for $m_b \approx 4.75 - 5\text{GeV}$.

Overall global fit, even including current beauty data, would prefer $m_b \approx 4.25 - 4.5\text{GeV}$.

## Considerations of the GM-VFNS - Our Definition

The GM-VFNS can be defined by demanding equivalence of the $n_f$ light flavour and $n_f+1$ light flavour descriptions at all orders – above transition point $n_f \to n_f+1$

$$F(x,Q^2) = C_k^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes f_j^{n_f+1}(Q^2)$$
$$\equiv C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2).$$

Hence, the VFNS coefficient functions satisfy

$$C_k^{FF,n_f}(Q^2/m_H^2) = C_j^{VF,n_f+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2),$$

which at $\mathcal{O}(\alpha_S)$ gives

$$C_{2,Hg}^{FF,n_f,(1)}(\frac{Q^2}{m_H^2}) = C_{2,HH}^{VF,n_f+1,(0)}(\frac{Q^2}{m_H^2}) \otimes P_{qg}^0 \ln(Q^2/m_H^2) + C_{2,Hg}^{VF,n_f+1,(1)}(\frac{Q^2}{m_H^2}),$$

The VFNS coefficient functions tend to the massless limits as $Q^2/m_H^2 \to \infty$.

However, $C_j^{VF}(Q^2/m_H^2)$ only uniquely defined in this limit.

Can swap $\mathcal{O}(m_H^2/Q^2)$ terms between $C_{2,HH}^{VF,0}(Q^2/m_H^2)$ and $C_{2,g}^{VF,1}(Q^2/m_H^2)$.

Various prescriptions (ACOT, TR, Chuvakin-Smith).

Some earlier versions violated threshold $W^2 > 4m_H^2$ in individual terms.

(TR-VFNS) highlighted freedom in choice and enforced kinematics in each term by making $(d\,F_2/d\,\ln Q^2)$ continuous at transition (in gluon sector). Complicated to extend.

(S)ACOT($\chi$) (Tung, *et al*) prescription says make simple choice

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) = \delta(z - Q^2/(Q^2 + 4m_H^2)).$$

$$\rightarrow F_2^{H,0}(x, Q^2) = (h + \bar{h})(x/x_{max}, Q^2), \qquad x_{max} = Q^2/(Q^2 + 4m_H^2)$$

$\rightarrow C^{ZM,0}_{2,HH}(z) = \delta(1 - z)$ for $Q^2/m_H^2 \rightarrow \infty$. Also $W^2 = Q^2(1 - x)/x \geq 4m_H^2$.

Have adopted this and obvious extensions to higher orders (and now simple modifications).

Still another difference.

ACOT type schemes have used e.g.

NLO $\frac{\alpha_S}{4\pi} C_{2,Hg}^{FF,n_f,(1)} \otimes g^{n_f} \rightarrow \frac{\alpha_S}{4\pi} (C_{2,HH}^{VF,n_f+1,(1)} \otimes (h + \bar{h}) + C_{2,Hg}^{VF,n_f+1,(1)} \otimes g^{n_f+1})$,

i.e., same order of $\alpha_S$ above and below.

But LO FFNS and evolution below and NLO definition and evolution above.

TR have used e.g.

LO $\frac{\alpha_S(Q^2)}{4\pi} C_{2,Hg}^{FF,n_f,(1)} (Q^2/m_H^2) \otimes g^{n_f}(Q^2) \rightarrow \frac{\alpha_S(M^2)}{4\pi} C_{2,Hg}^{FF,n_f,(1)}(1) \otimes g^{n_f}(M^2)$

$+ C_{2,HH}^{VF,n_f+1,(0)} (Q^2/m_H^2) \otimes (h + \bar{h})(Q^2)$,

i.e. freeze higher order $\alpha_S$ term when going upwards through $Q^2 = m_H^2$.

This difference in choice can be phenomenologically important.

In order to define our VFNS at NNLO, need $\mathcal{O}(\alpha_S^3)$ heavy flavour coefficient functions for $Q^2 \leq m_H^2$ and to be frozen for $Q^2 > m_H^2$. However, not calculated. Needs modelling. More later.

## Different type of Definition

Both the BMSN (Buza *et al*) and FONLL (Nason *et al*) applied the same type of reasoning in initially different contexts. In general terms (for structure functions)

$$F^{\rm GMVFNS}(x,Q^2) = F_2^{\rm FFNS}(x,Q^2) - F_2^{\rm asymp}(x,Q^2) + F_2^{\rm ZMVFNS}(x,Q^2)$$

where the second (subtraction) term is the asymptotic version of the first, i.e., all terms $\mathcal{O}(m_H^2/Q^2)$ omitted.

Differences in exactly how the second and third terms are defined in detail (e.g. Blümlein *et al* do not resum $\ln Q^2/m_H^2$ terms from PDF evolution in $F_2^{\rm ZMVFNS}$).

Question of whether one only uses above some transition point, else not exactly $F_2^{\rm FFNS}(x,Q^2)$ below $Q^2 = m_H^2$.

Realised from the beginning in FONLL approach that each term in the combination $(F_2^{\rm ZMVFNS} - F_2^{\rm asymp})$ can be modified by corrections which fall like $m_H^2/Q^2$.

In simplest application $\alpha_S$ order of $F^{\rm FFNS}(x,Q^2)$ at low $Q^2$ same as that of $F^{\rm ZMVFNS}(x,Q^2)$ as $Q^2 \to \infty$, like ACOT.

Modification in FONLL (Nason - CERN summer institute) can avoid this, but leads to extra (higher order) term as $Q^2 \to \infty$ – not exact cancellation in first two terms.

Ordering tricky problem. Would like any GMVFNS to reduce to exactly correct order FFNS at low $Q^2$ and exactly correct order ZMVFNS as $Q^2 \to \infty$. At present none do.

Return to particular TR version of the GMVFNS. Reason for violation of the above is frozen term $\alpha_S^n(m_H^2)\sum_i C_{2,i}^{\rm FFNS}(m_H^2) \otimes f_i(m_H^2)$ which still persists as $Q^2 \to \infty$ at order N$^{n-1}$LO.

Depends on size of PDFs at low scales, so rather small effect at large $Q^2$.

However, not strictly necessary. Frozen in original TR prescription from exact condition on derivative of $d\,F_2/d,\ln Q^2$. Could have instead

$$\left(\frac{m_H^2}{Q^2}\right)^a \alpha_S^n(m_H^2)\sum_i C_{2,i}^{\rm FF}(m_H^2)\otimes f_i(m_H^2) \text{ or } \left(\frac{m_H^2}{Q^2}\right)^a \alpha_S^n(Q^2)\sum_i C_{2,i}^{\rm FF}(Q^2)\otimes f_i(Q^2),$$

Any $a > 0$ provides both exactly correct asymptotic limits, though strictly should have $(m_H^2/Q^2)k\big(\ln(Q^2/m_H^2)\big)$ from factorization theorem.

Also have the freedom to modify the heavy quark coefficient function, by default

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) = \delta(z - x_{\rm max}).$$

Appears in convolutions for higher order subtraction terms, so do not want complicated $x$ dependence. Simple choice.

$$C^{VF,0}_{2,HH}(Q^2/m_H^2, z) \to (1 + b(m_H^2/Q^2)^c)\delta(z - x_{\rm max})),$$

where again $c$ really encompasses $(m_H^2/Q^2)$ with logarithmic corrections.

Can also modify argument of $\delta$-function, as in Intermediate Mass (IM) scheme of Nadolsky, Tung. Let argument of heavy quark contribution change like

$$\xi = x/x_{\rm max} \to x\big(1 + (x(1 + 4m_H^2/Q^2))^d 4m_H^2/Q^2\big),$$

so kinematic limit stays the same, but if $d > 0$ small $x$ less suppressed, or if $d < 0$ (must be $> -1$) small $x$ more suppressed.

Default $a, b, c, d$ all zero. Limit either by fit quality or *sensible* choices.

6 extreme variations tried.

GMVFNS1 – $b = -1, c = 1$.

GMVFNS2 – $b = -1, c = 0.5$.

GMVFNS1 – $a = 1$.

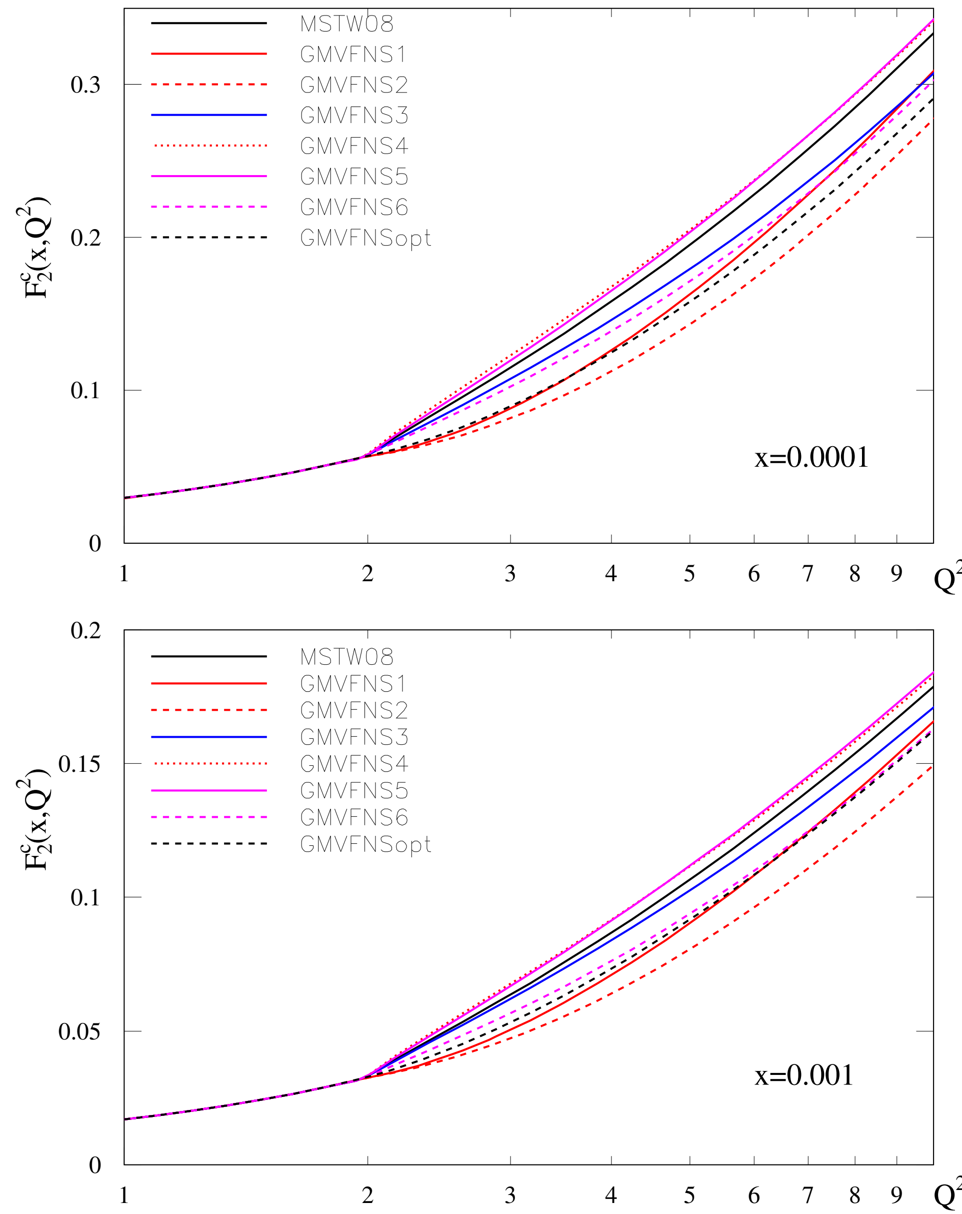
GMVFNS1 – $b = +0.3, c = 1$ – fit.

GMVFNS1 – $d = 0.1$ – fit.

GMVFNS1 – $d = -0.2$ – fit.

Variations in $F_2^c(x, Q^2)$ near the transition point at NLO due to different choices of GM-VFNS.

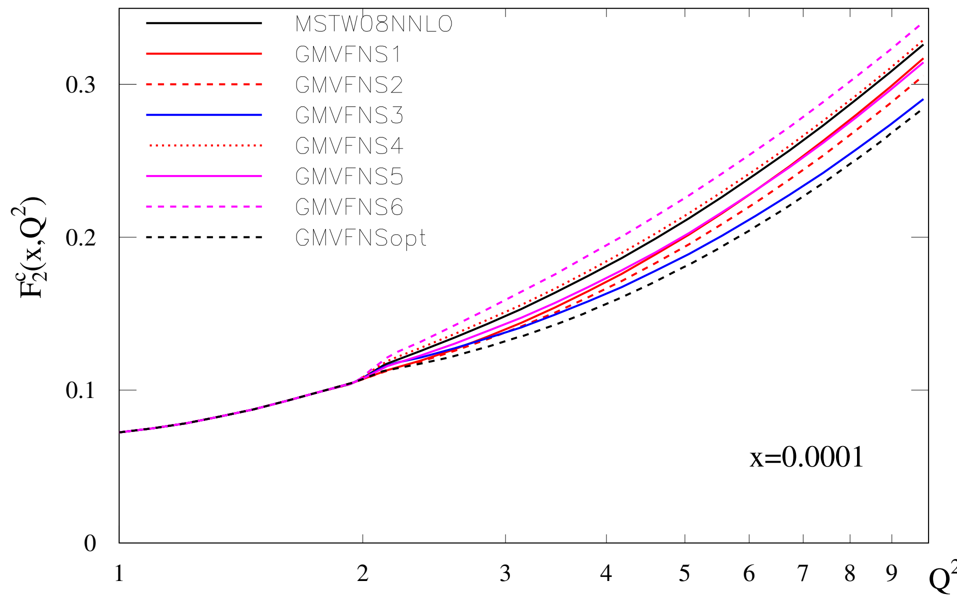
Optimal, $a = 1, b = -2/3, c = 1$, smooth behaviour.

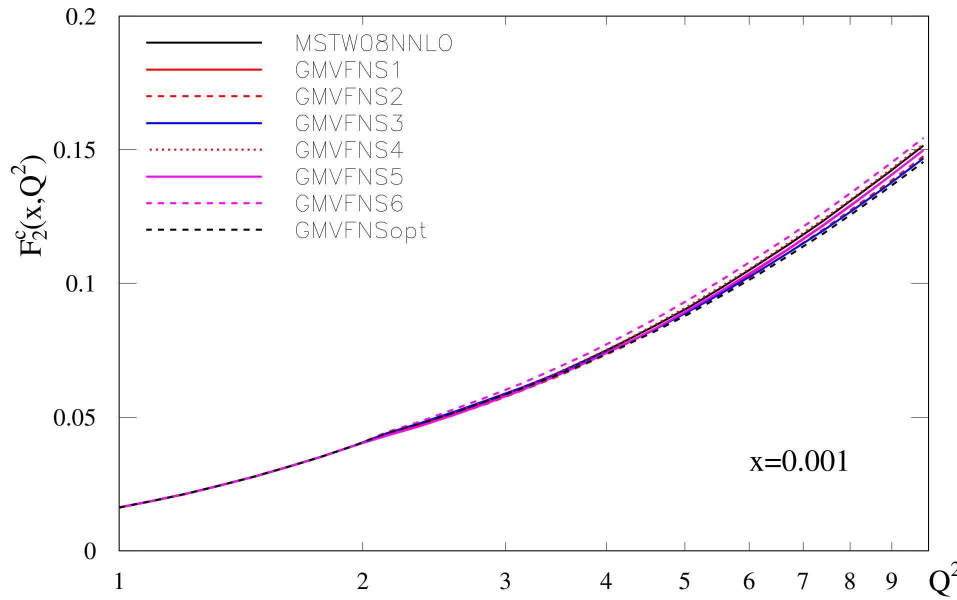


Variations in $F_2^c(x, Q^2)$ near the transition point due to different choices of GM-VFNS at NNLO.

Very much reduced, almost zero variation until very small $x$.

Shows that NNLO evolution effects most important in this regime.



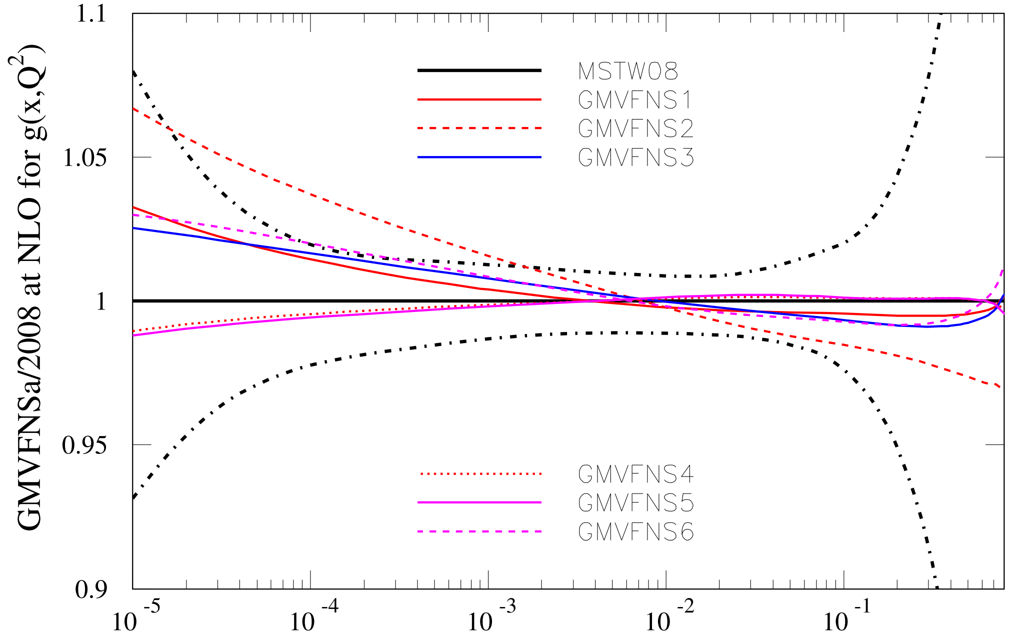
Variations in partons extracted from global fit due to different choices of GM-VFNS at NLO.

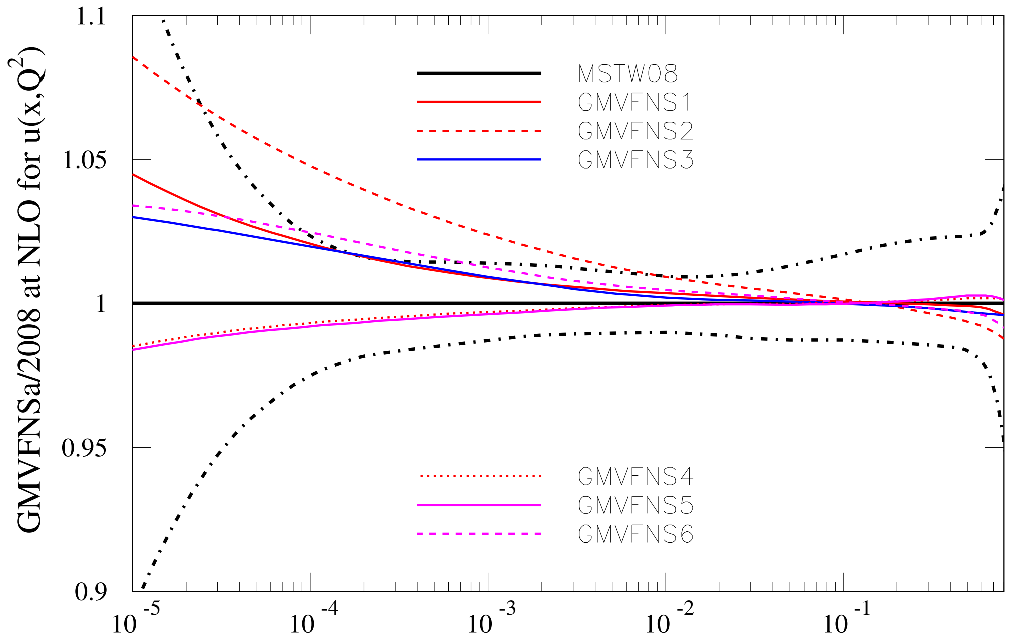
Initial $\chi^2$ can change by $250$.

Converges to at most about $15$ of original.

Better fit for GMVFNS1, GMVFNS3 and GMVFNS6.

Some changes in PDFs large compared to one-sigma *uncertainty*.



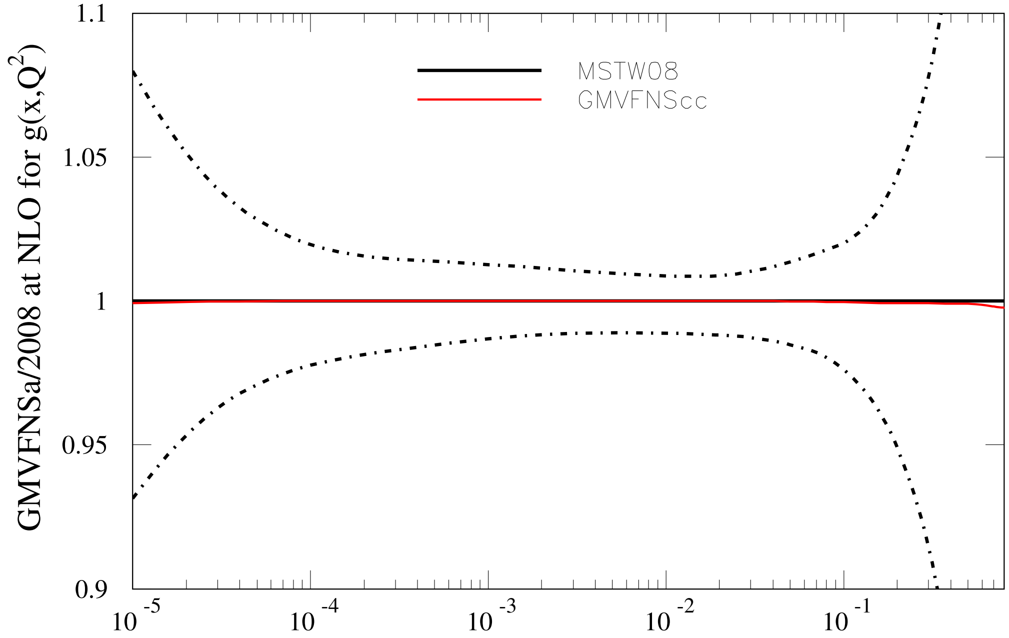
Also implement similar variations in GMVFNS for charged current processes.

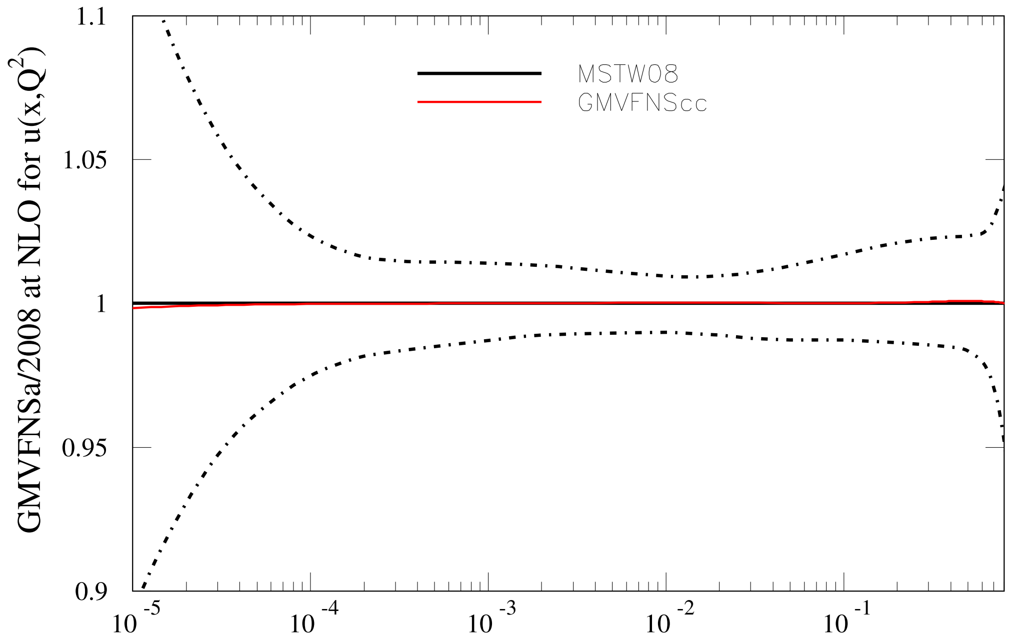
HERA data completely insensitive due to large $Q^2$.

Some effect on fixed target (anti)neutrino data. $\chi^2$ changes by at most $4$ and almost no change in this, or PDFs with refit.

Also make changes in cross-sections for dimuon data. In this case definition of separation into observable cross-section dependent on GMVFNS.

In practice $\chi^2$ changes by at most $1$ unit. Essentially no change in PDFs.



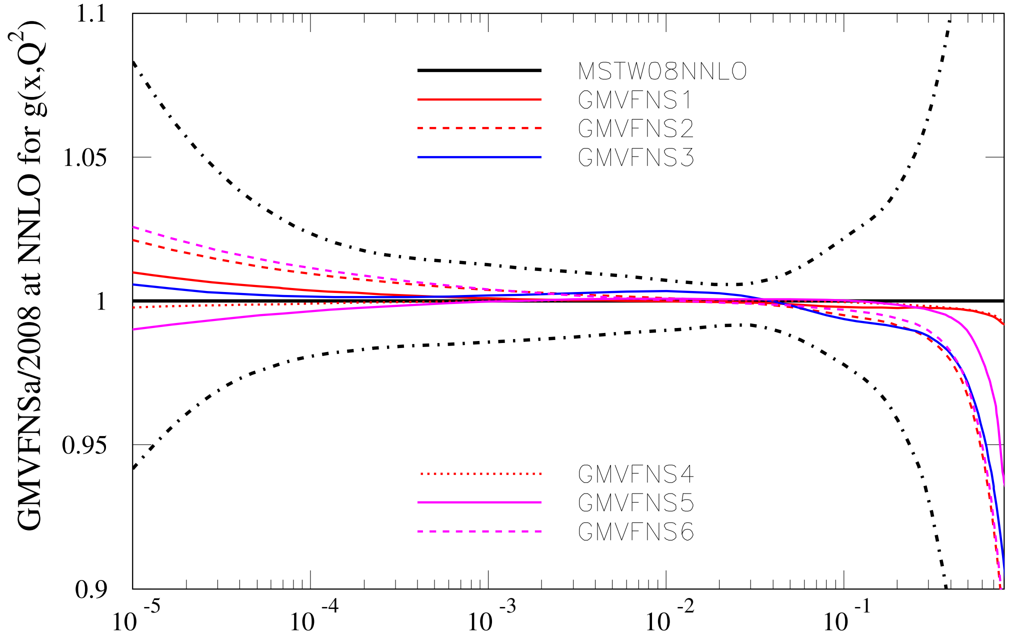
Variations in partons extracted from global fit due to different choices of GM-VFNS at NNLO.

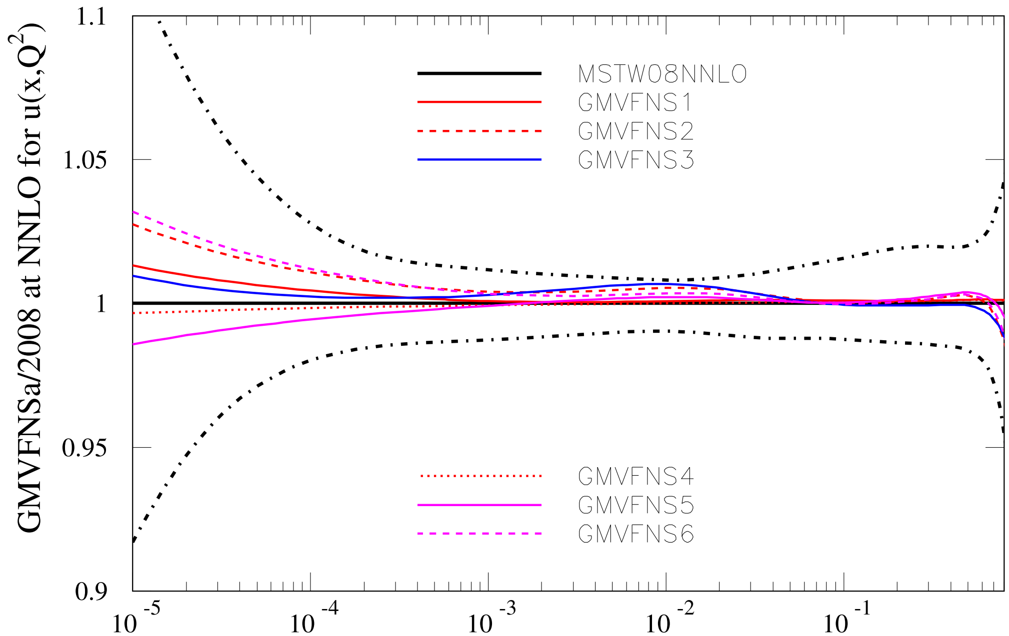
Initial changes in $\chi^2 < 20$.

Converge to about $10$. None a marked improvement.

At worst changes approach *uncertainty*.

Biggest variation in high-$x$ gluon, which has large uncertainty.



Model $\mathcal{O}(\alpha_S^3)$ at low $Q^2$ using known leading threshold logarithms (Laenen and Moch) and leading $\ln(1/x)$ term from $k_T$-dependent impact factors Catani, *et al*.

Include latter in form

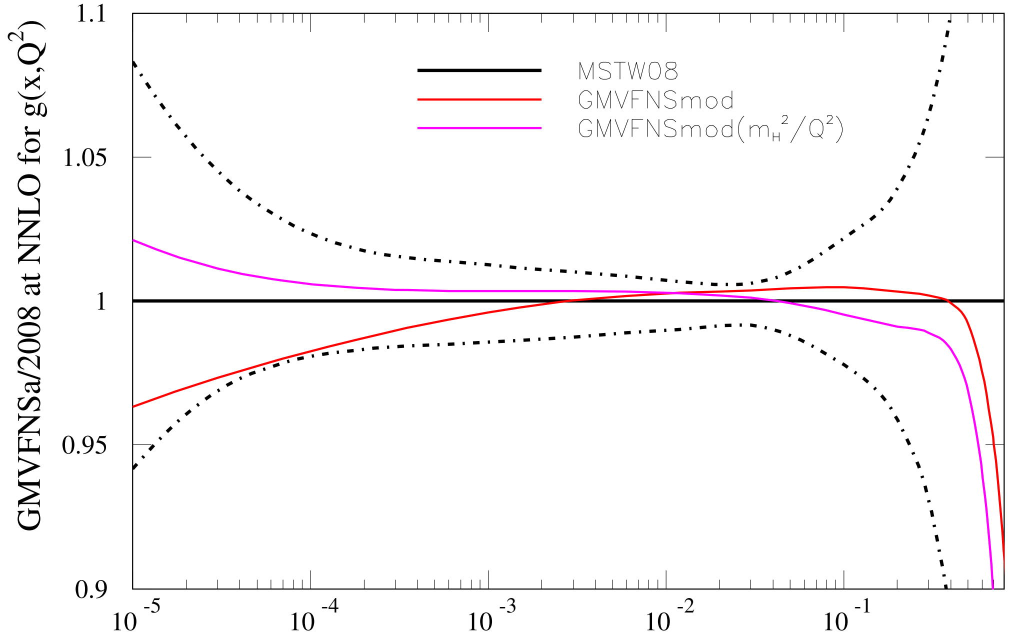
$$\propto (1 - z/x_{\max})^a (\ln(1/z) - b)/z,$$

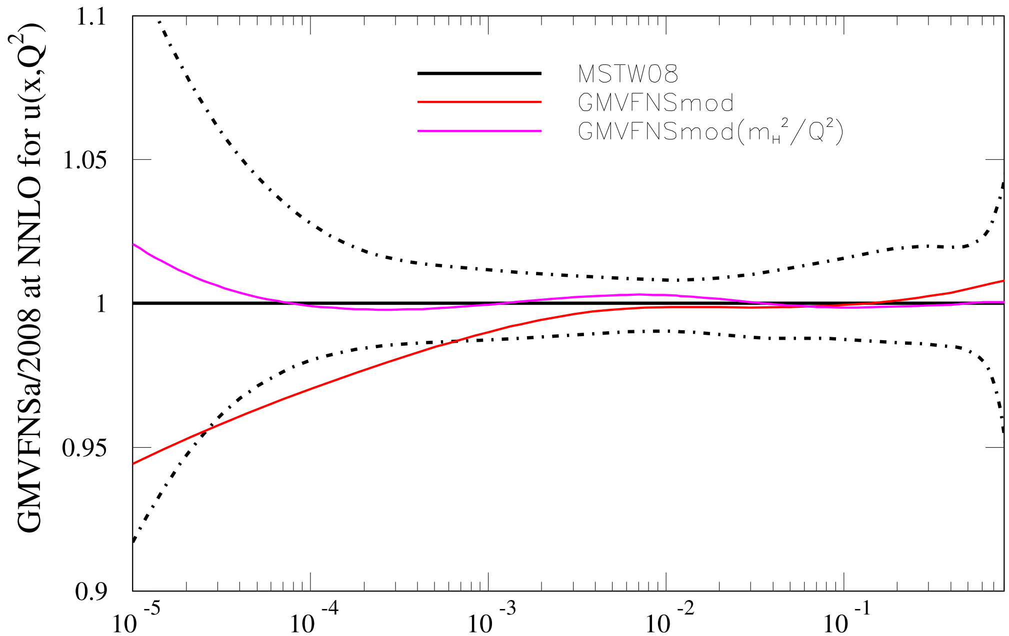
where default $a = 20, b = 4$.

Variations in $a$ make little difference. Maximum *sensible* variation of $b = 2$ leads to effect in PDFs shown.

Major effect at smallest $x$.

Moderated significantly if $\mathcal{O}(\alpha_S^3)$ falls away rather than frozen.



The values of the predicted cross-sections at NLO for $Z$ and a 120 GeV Higgs boson at the Tevatron and the LHC (latter for 14 TeV centre of mass energy).

| PDF set | $B_{l^+l^-} \cdot \sigma_Z$(nb) TeV | $\sigma_H$(pb)TeV | $B_{l^+l^-} \cdot \sigma_Z$(nb) LHC | $\sigma_H$(pb) LHC |
|---|---|---|---|---|
| MSTW08 | 0.2426 | 0.7462 | 2.001 | 40.69 |
| GMvar1 | 0.2433 | 0.7428 | 2.023 | 40.76 |
| GMvar2 | 0.2444 | 0.7383 | 2.061 | 41.29 |
| GMvar3 | 0.2429 | 0.7438 | 2.024 | 41.03 |
| GMvar4 | 0.2425 | 0.7457 | 1.993 | 40.60 |
| GMvar5 | 0.2423 | 0.7454 | 1.991 | 40.56 |
| GMvar6 | 0.2434 | 0.7431 | 2.032 | 41.00 |
| GMvarcc | 0.2427 | 0.7451 | 2.001 | 40.65 |

At most $1\%$ variation at Tevatron in $\sigma_Z$.

Up to $+3\%$ and $-0.5\%$ variation in $\sigma_Z$ at the LHC. About half as much in $\sigma_H$ due to higher average $x$ sampled.

Remember $8\%$ from ZMVFNS to GMVFNS in CTEQ6 ($6\%$ for completed NNLO GMVFNS in MRST06).

The values of the predicted cross-sections at NNLO. $\sigma_H$ calculated using Harlander, Kilgore code.

| PDF set | $B_{l^+l^-} \cdot \sigma_Z$(nb) TeV | $\sigma_H$(pb)TeV | $B_{l^+l^-} \cdot \sigma_Z$(nb) LHC | $\sigma_H$(pb) LHC |
|---|---|---|---|---|
| MSTW08 | 0.2507 | 0.9550 | 2.051 | 50.51 |
| GMvar1 | 0.2509 | 0.9505 | 2.054 | 50.39 |
| GMvar2 | 0.2514 | 0.9478 | 2.061 | 50.55 |
| GMvar3 | 0.2516 | 0.9539 | 2.062 | 50.88 |
| GMvar4 | 0.2507 | 0.9534 | 2.050 | 50.45 |
| GMvar5 | 0.2509 | 0.9519 | 2.046 | 50.37 |
| GMvar6 | 0.2509 | 0.9462 | 2.057 | 50.38 |
| GMvarmod | 0.2501 | 0.9511 | 2.022 | 50.03 |
| GMvarmod' | 0.2508 | 0.9482 | 2.052 | 50.57 |

Other than from model dependence maximum variations of order $0.5\%$ at LHC. High-$x$ gluon leads to $1\%$ on $\sigma_H$ at Tevatron.

Model uncertainties can be $> 1\%$ from region at very small $x$ and low $Q^2$. Can perhaps input more small-$x$ knowledge here. Effect far smaller when $\mathcal{O}(\alpha_S^3)$ term falls with $Q^2$.

## Observation on Relationship Between Gluon and $\alpha_S(M_Z^2)$

In study of $\alpha_S$ within global fit noticed that within full fit HERA cross-section data prefer large $\alpha_S(M_Z^2) \approx 0.125$ at NLO ($0.121$ at NNLO). Due to presence of other data?

Fitting only to these data using NLO find $\alpha_S(M_Z^2) = 0.127 \pm 0.005$ (using $\Delta\chi^2 = 1$) and $\chi^2 = 57$ lower than in global fit for $839$ points.

However, repeated fit removing second term from

$$xg(x, Q_0^2 = 1\mathrm{GeV}^2) = A_g(1-x)^{\eta_g}(1+\epsilon_g x^{0.5} + \gamma_g x)x^{\delta_g} + A_{g'}(1-x)^{\eta_{g'}}x^{\delta_{g'}}.$$

Obtain instead $\alpha_S(M_Z^2) = 0.110 \pm 0.002$ with $\chi^2$ now $17$ higher.

Use restricted parameterisation at $Q_0^2 = 1.5\mathrm{GeV}^2$. Now $\chi^2$ only $4$ higher than best fit. Data *happy* with positive input gluon - similar to other single power gluon fits? Obtain $\alpha_S(M_Z^2) = 0.117 \pm 0.0025$. (Find $\alpha_S(M_Z^2) = 0.132 \pm 0.006$ with free parameterisation.)

Both extracted $\alpha_S(M_Z^2)$ and its uncertainty (obtained from careful scan - higher value otherwise) sensitive to limited gluon parameterisation. Both become lower.

## Conclusions

Using our current default GM-VFNS MSTW have looked at the results of varying both the charm and bottom quark masses in the context of the MSTW2008 global fit. $m_c$ determined with good precision, but rather lower at NNLO than NLO. Global fit prefers (very) low $m_b$, but new direct $F_2^b(x, Q^2)$ data prefers $m_b \sim 4.75 - 5\text{GeV}$. Constraint certainly possible in future.

Discussed variations in definition of GMVFNS, and introduced options for exact reduction to correctly ordered high and low $Q^2/m_H^2$ limits.

Examined limits of variation in definitions and looked at variations in PDFs and cross-sections. At NLO PDFs can vary significantly outside experimental uncertainties at small $x$ and cross-sections change by $3\%$. At NNLO PDFs usually (well) within uncertainties, and cross-sections rarely more than $1\%$ change. Full NNLO GMVFNS, has small amount of necessary modelling. Biggest effect at very small $x$.

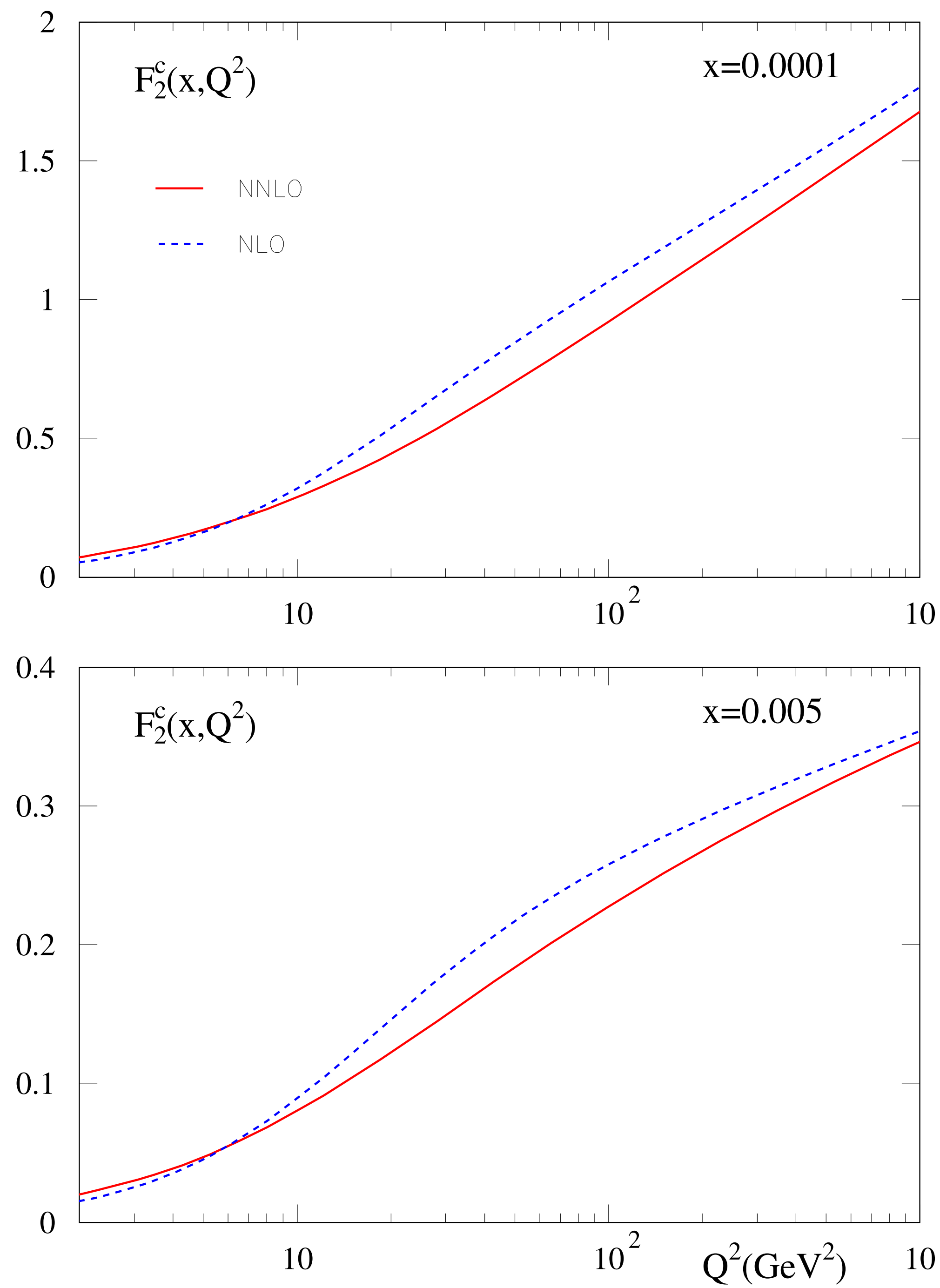
In general find that adding (or removing) parameters in procedure does not change PDFs nearly as much as uncertainty level. Hessian and Lagrange very similar despite latter having $\sim 1/3$ more parameters. Effect if new space opened up. Particular illustration with $\alpha_S$ and small-$x$ gluon.

NNLO consequences.

NNLO $F_2^c(x,Q^2)$ starts from higher value at low $Q^2$.

At high $Q^2$ dominated by $(c+\bar{c})(x,Q^2)$. This has started evolving from negative value at $Q^2=m_c^2$. Remains lower than at NLO for similar evolution.

General trend – $F_2^c(x,Q^2)$ flatter in $Q^2$ at NNLO than at NLO. Important effect on gluon distribution going from one to other.



## Parameterisation dependence reason for inflated $\Delta\chi^2 = 100$ Tolerance?

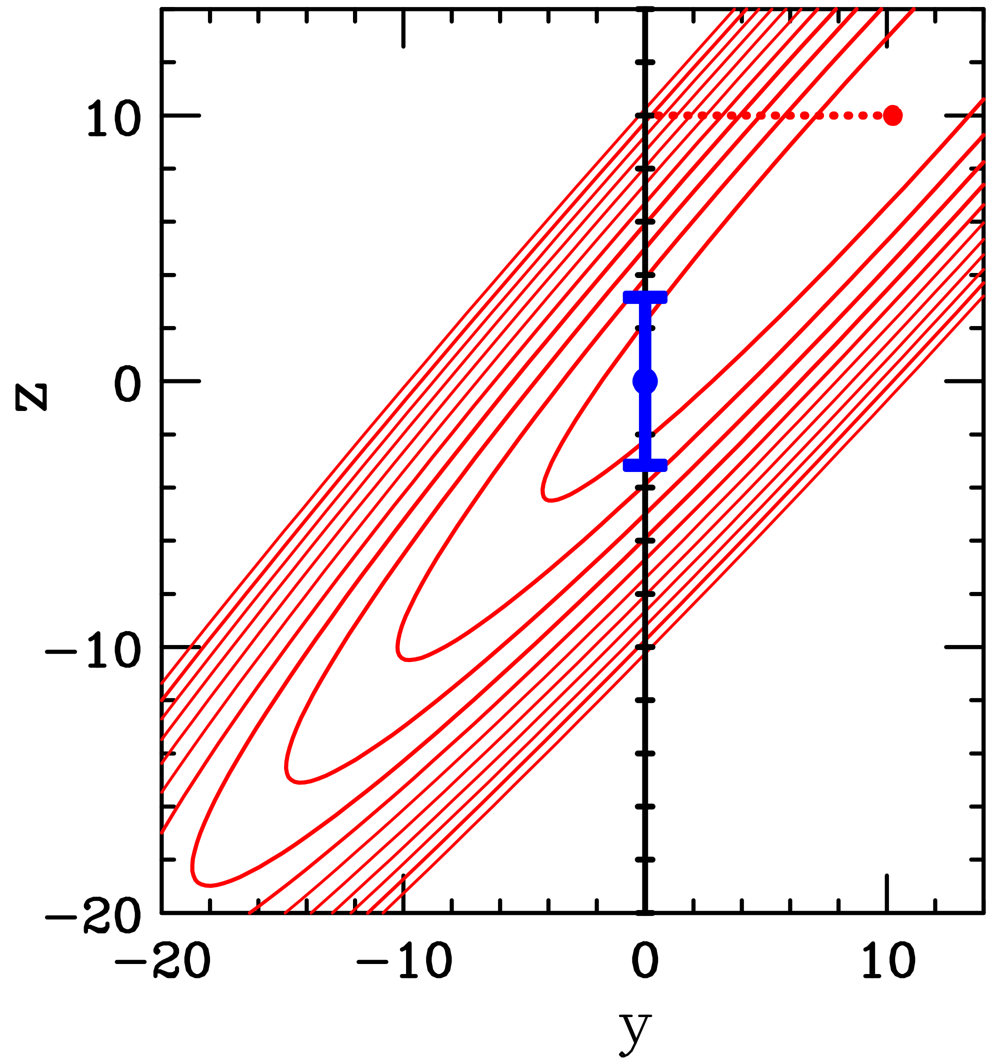
Proposal by Pumplin that this may be the case.

Simple model, fit depends on one parameter, min at $z = 0$ and $\chi^2 = z^2$.

Add second parameter $y$, could get $\chi^2$ profile as shown.

For long narrow ellipse can get shift in best fit $z$ such that value corresponds to $\chi^2$ in original model with magnitude much greater than improvement in best fit quality.

$\propto (1/R - R)^2$, where $R$ is ratio of minor to major axis.



## Why this doesn't apply to global fits

1. In MSTW/MRST and CTEQ global fits there are more free parameters in obtaining best fit minimum than in determining eigenvectors. Even if correct argument, doesn't directly apply since main effect – in change of minimum – already accounted for.

2. This very elliptical profile only occurs if two of the parameters are very correlated. This is in fact why we do not leave all our parameters free in eigenvectors. Along major axis very flat direction always suddenly turns up due to quartic and higher terms in $\chi^2$ distribution. Two parameters compensate almost exactly near minimum, then compensation suddenly breaks. Argument based on quadratic terms breaks down.

3. If $z$ and $y$ highly correlated a large change in $z$ is likely not a large change in a PDF distribution (explaining small improvement is $\chi^2$).

4. If a new parameter is introduced which is not highly correlated with one already there the $R$ is not small and change in old parameters in new best fit is commensurate with the improvement in $\chi^2$

Basic arguments seem to be validated by a variety of checks.

Parameterisation used in MSTW fits. Only those 20 in red appear in eigenvectors.

At input scale $Q_0^2 = 1$ GeV$^2$:

$$xu_v = A_u\, x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u\, x)$$

$$xd_v = A_d\, x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d\, x)$$

$$xS = A_S\, x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)$$

$$x\bar{d}-x\bar{u} = A_\Delta\, x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta\, x+\delta_\Delta\, x^2)$$

$$xg = A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g\, x)+A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}}$$

$$xs+x\bar{s} = A_+\, x^{\delta_S}\,(1-x)^{\eta_+}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)$$

$$xs-x\bar{s} = A_-\, x^{\delta_-}(1-x)^{\eta_-}(1-x/x_0)$$

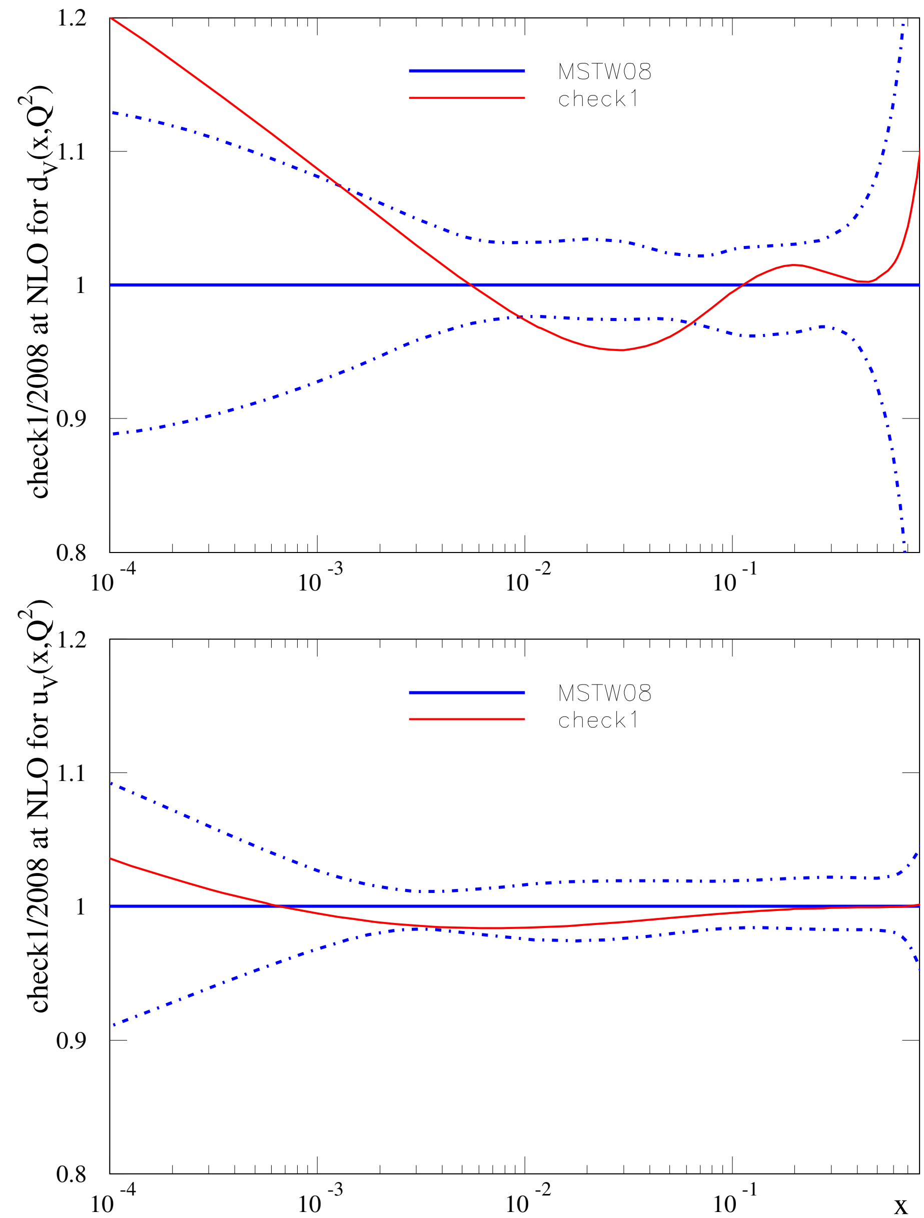
Of others only $A_v, A_d, A_g$ and $x_0$ fixed by sum rules and $\delta_{s\_}$ fixed due to total correlation.

Try checking by setting all the parameters not in eigenvectors equal to zero, or if this is not sensible a round value. Fit quality awful. Need these parameters in global fit.

Move each by about $10\%$ and refit. Like having $y$ away from best fit. Refit $8$ worse. Some parameters left free move by $2-3$ times quoted uncertainty.

Main change in PDFs in valence quarks shown. Worst change $1.8$ bigger than uncertainty in $d_V$. Size of change in PDF not well correlated to relative size of change in parameters.

Trace to eigenvectors $14$ and $18$ in direction such that $\Delta\chi^2 \sim 10$ for uncertainty. – change in PDF twice that expected from change in $\chi^2$ for global fit.



Try removing parameter which is not highly correlated, i.e. one in eigenvector definition.

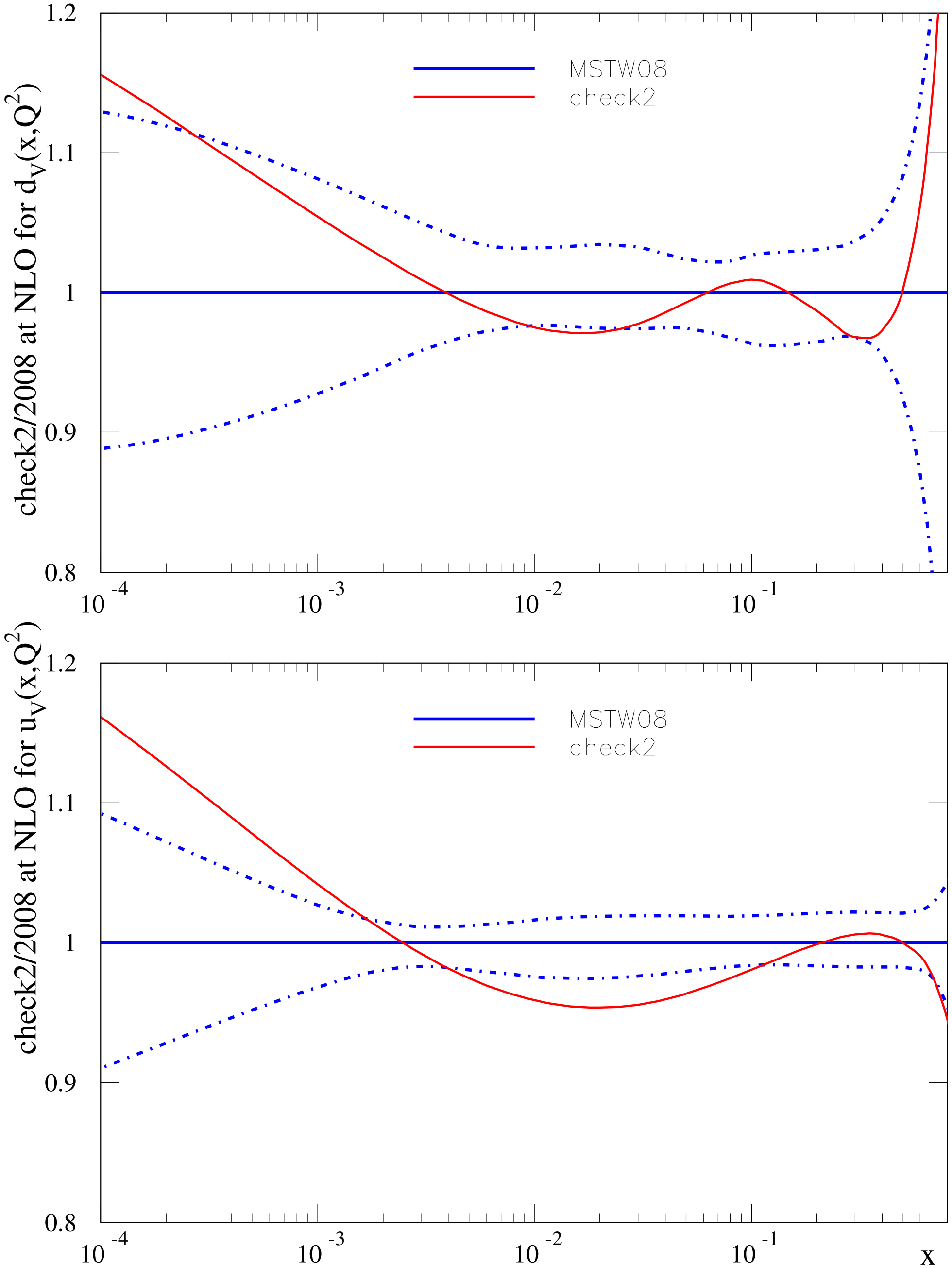
Set $\epsilon x^{0.5}$ term in $u_V$ to zero. Usual magnitude is about twice the uncertainty.

Refit with usual eigenvector parameters free. $\chi^2$ is $30$ worse.

Biggest change in PDFs shown. At most variation about $1.8$ uncertainty, in $u_V$.

Again relevant eigenvectors suggest uncertainty corresponds to $\Delta\chi^2 \sim 10$.

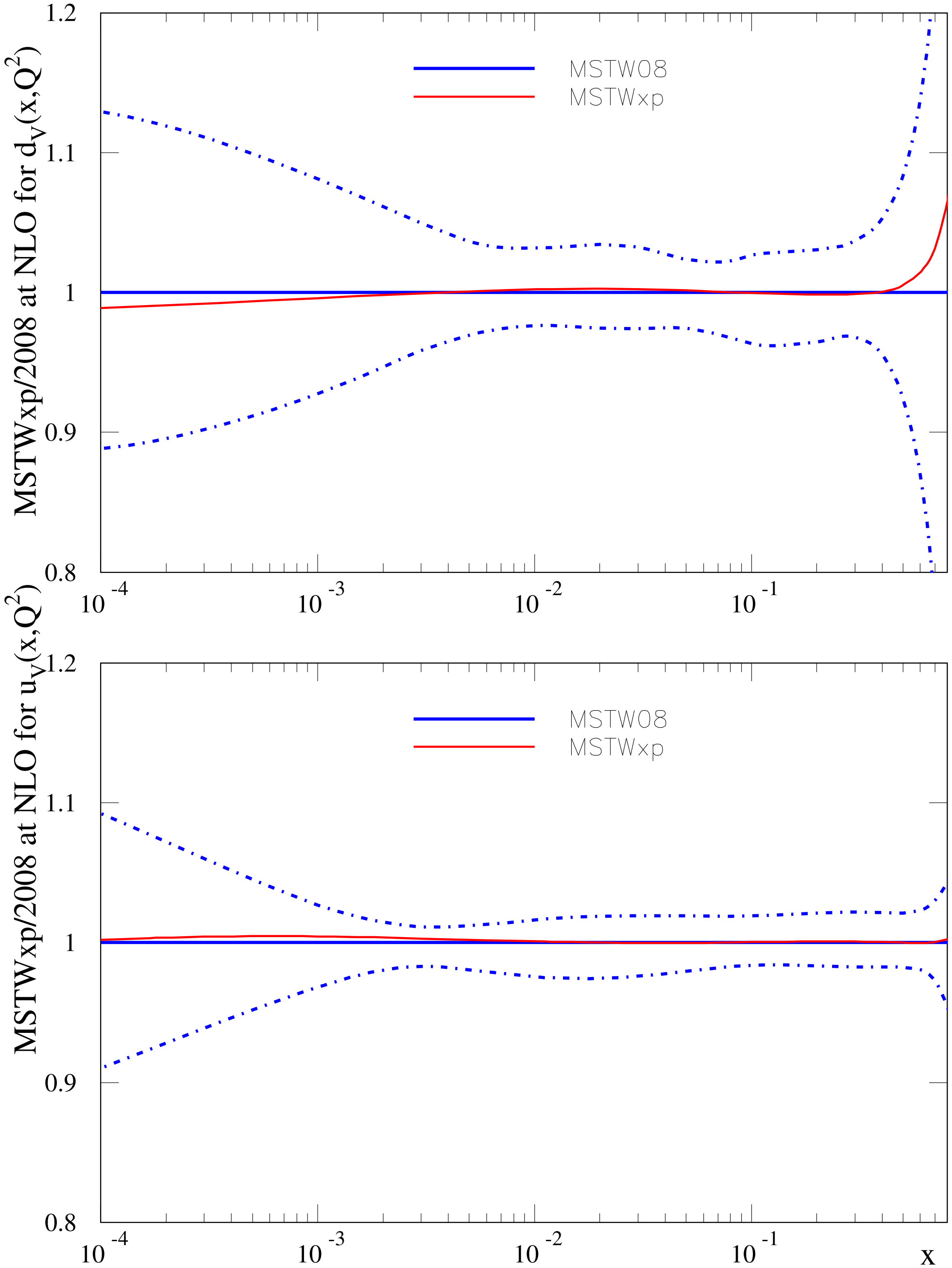
This time change in PDF pretty much what deterioration in fit quality suggests.



Also tried adding $x^2$ terms to polynomial in two valence parameterisations.

Fit quality improved by $2$ units.

Change in partons negligible.



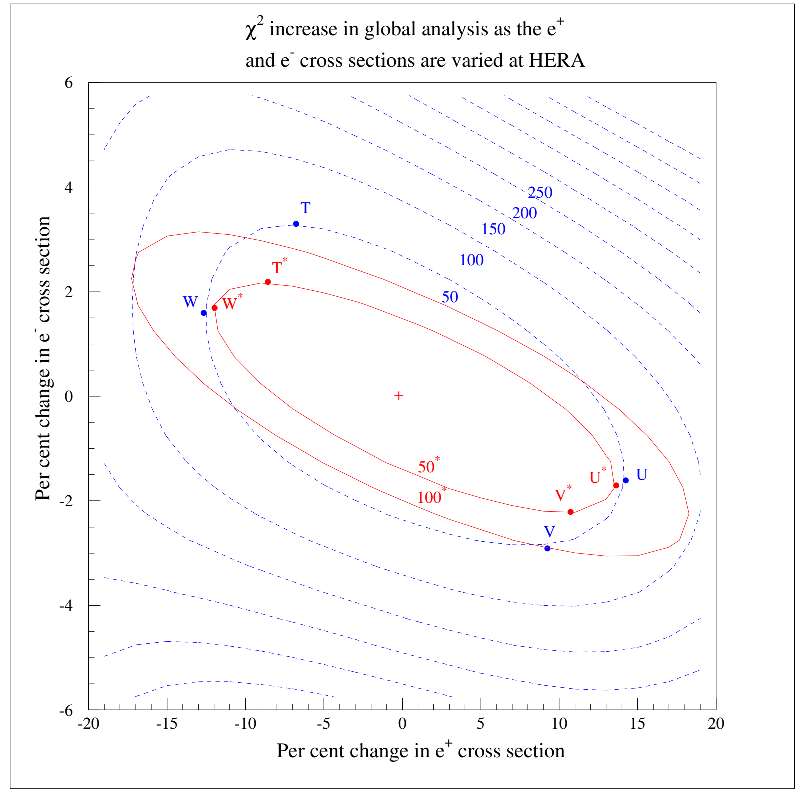
Recall study in first MRST uncertainties paper comparing the Hessian approach with 15 parameters and Lagrange multiplier with 22 parameters and same $\Delta\chi^2 = 50$ for both.

Plot shown for Lagrange multiplier method for charged current HERA structure functions at $x = 0.5$ (Red curve – fixed $\alpha_S$).

Uncertainty using Hessian approach was $2\%$ for $F_2^{CC}(e^-p)$ and $10\%$ for $F_2^{CC}(e^+p)$.

Excellent agreement between two for $F_2^{CC}(e^-p)$.

Factor of up to $50\%$ too low for $F_2^{CC}(e^+p)$.



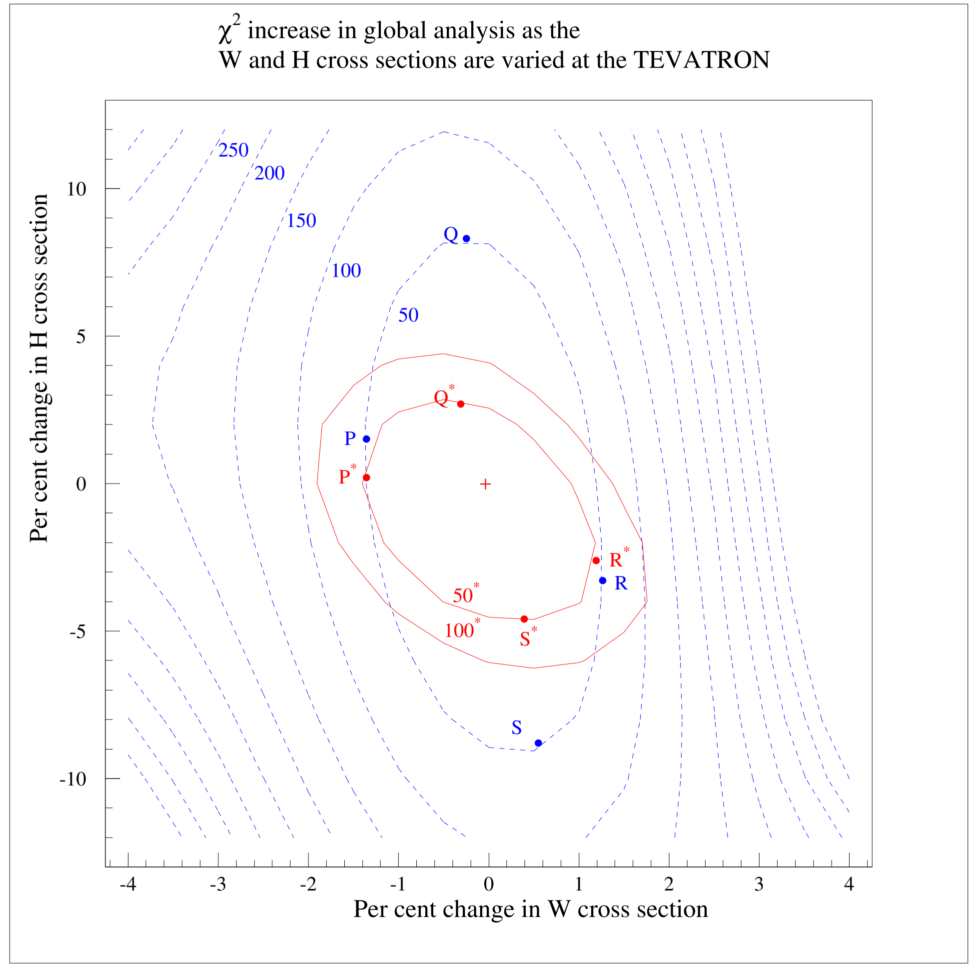

However, used non-optimum choice of parameters in eigenvectors for $d_V(x, Q^2)$ in MRST2001. Correction of this lead to automatic increase in uncertainty of about $50\%$ at $x = 0.5$, with no new free parameters.

Lagrange multiplier method for $W$ and $120\text{GeV}$ Higgs at the Tevatron.

Uncertainty using Hessian approach is $1.2\%$ for $W$ and $3\%$ for Higgs.

Slightly smaller in latter case. Using $3$ parameters lead to narrowing uncertainty in gluon at $x \approx 0.2$ – affects Tevatron uncertainty.

Extra parameter in eigenvectors for gluon increases uncertainty by about the amount expected.



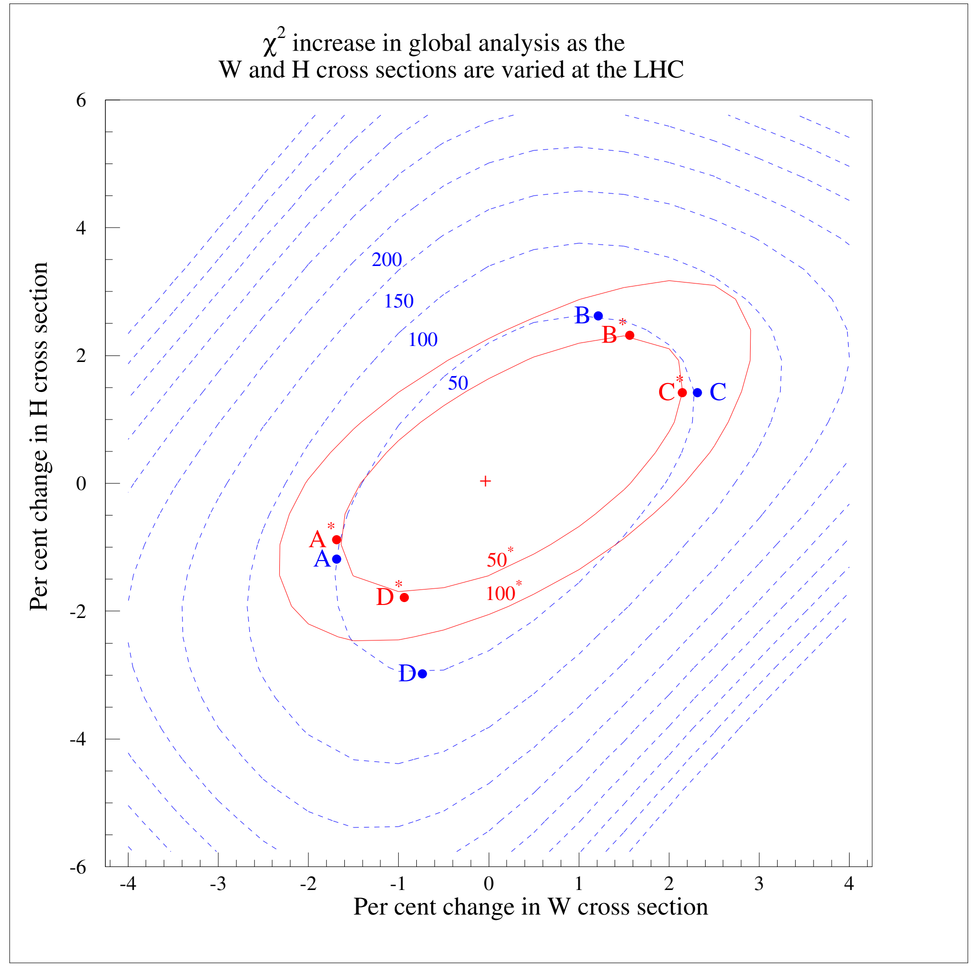
Lagrange multiplier method for $W$ and $120\mathrm{GeV}$ Higgs at the LHC.

Uncertainty using Hessian approach about $10\%$ smaller.

Also looked at uncertainties on moments of $u$–$d$ using Hessian and Lagrange multiplier approaches. Very similar and latter could be slightly smaller.

In all cases introduction of extra parameters in the Lagrange multiplier method led to at most a moderate increase in uncertainty.

If this was clearly more than $10\%$ the limitations in parameters were addressed and the problem solved.



## Not looking for $\Delta\chi^2 = 100$ anyway

```
# iEigen x68plusMin x68minusMax
1 4.53655 -3.76623 (ZEUS ep 95-00 #sigma_{r}^{NC}, H1 ep 97-00 #sigma_{r}^{NC})
2 3.38422 -3.79217 (NMC #mud F_{2}, NuTeV #nuN#rightarrow#mu#muX)
3 1.46292 -2.17007 (NuTeV #nuN#rightarrow#mu#muX, CCFR #nuN#rightarrow#mu#muX)
4 3.45159 -2.31949 (NMC #mun/#mup, E866/NuSea pd/pp DY)
5 1.49487 -2.12523 (NuTeV #nuN#rightarrow#mu#muX, NuTeV #nuN xF_{3})
6 5.2242 -3.21227 (H1 ep 97-00 #sigma_{r}^{NC}, NuTeV #nuN#rightarrow#mu#muX)
7 2.03521 -2.78497 (D#oslash II W#rightarrowl#nu asym., BCDMS #mud F_{2})
8 5.20184 -1.84172 (NuTeV #nuN F_{2}, BCDMS #mup F_{2})
9 3.89046 -3.63201 (H1 ep 97-00 #sigma_{r}^{NC}, ZEUS ep 95-00 #sigma_{r}^{NC})
10 2.99034 -2.67972 (D#oslash II W#rightarrowl#nu asym., SLAC ed F_{2})
11 3.74202 -6.58278 (H1 ep 97-00 #sigma_{r}^{NC}, ZEUS ep 95-00 #sigma_{r}^{NC})
12 5.18993 -3.20527 (SLAC ep F_{2}, BCDMS #mup F_{2})
13 3.32487 -1.57418 (E866/NuSea pp DY, NuTeV #nuN xF_{3})
14 4.21973 -3.62346 (NMC #mud F_{2}, D#oslash II W#rightarrowl#nu asym.)
15 2.63335 -4.3632 (H1 ep 97-00 #sigma_{r}^{NC}, NuTeV #nuN F_{2})
16 2.32169 -0.925389 (CCFR #nuN#rightarrow#mu#muX, E866/NuSea pd/pp DY)
17 2.31104 -1.51795 (NuTeV #nuN#rightarrow#mu#muX, CCFR #nuN#rightarrow#mu#muX)
18 2.88709 -1.42061 (D#oslash II W#rightarrowl#nu asym., E866/NuSea pd/pp DY)
19 4.20991 -4.1461 (H1 ep 97-00 #sigma_{r}^{NC}, CDF II p#bar{p} incl. jets )
20 3.70876 -2.47281 (NuTeV #nuN#rightarrow#mu#muX, NuTeV #nuN#rightarrow#mu#muX)
```

Majority of eigenvectors correspond to $\sqrt{\Delta\chi^2} \sim 2 - 3$.

More types of data and weaker cuts than CTEQ. Even more discrepancy?

Comparison of full uncertainty and that from no normalization uncertainties (except in best fit).

Normalization uncertainty $\sim 1 - 1.5\%$, for all partons.

Difficult to account for in tolerance for eigenvectors – some very sensitive (size of quarks) others insensitive ($\bar{u} - \bar{d}$ determined from ratios).

Use of normalisation uncertainties increases uncertainties on partons significantly.

Not applied by CTEQ. Part of the reason for large tolerance?