Study on PDF parametrisation uncertainties using Monte Carlo technique

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Outline

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Introduction

- PDFs are crucial inputs for studies at the LHC, therefore precise knowledge and understanding of them is essential.

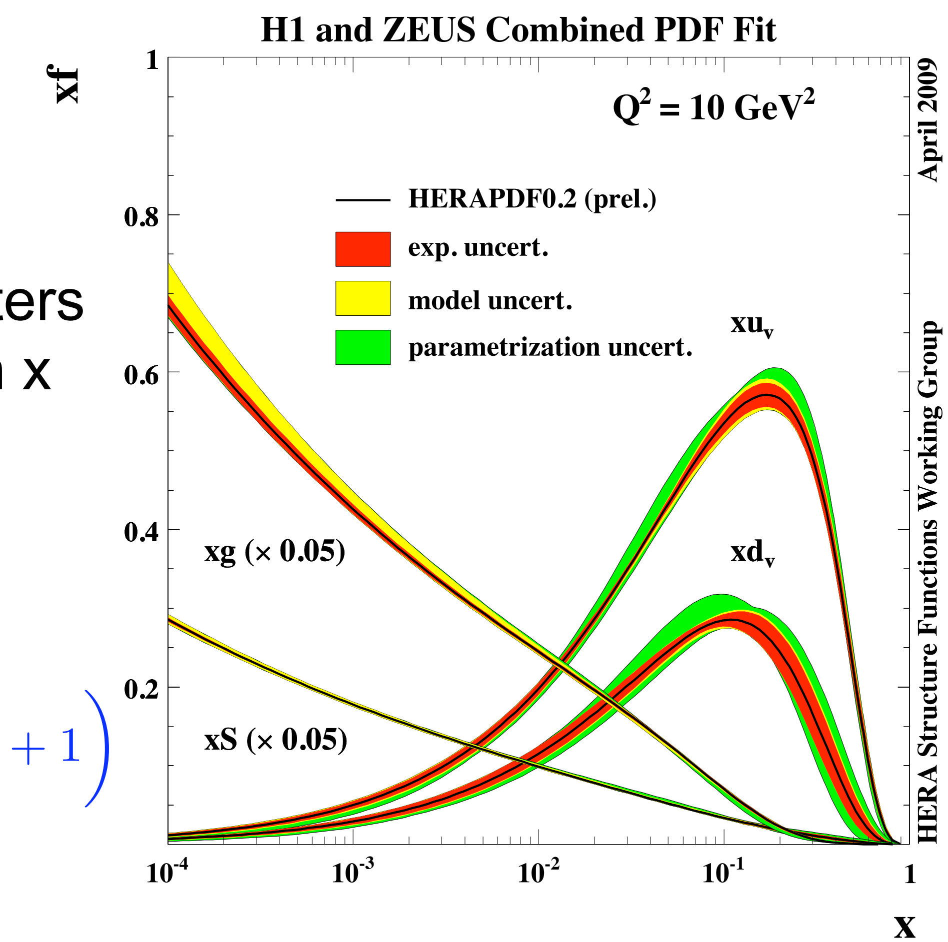
- PDFs are parametrised and extracted from fits, however the parametrisation uncertainty needs to be evaluated and studied.

  - Recent efforts by the H1 and ZEUS collaborations to estimate uncertainty on the HERAPDF parametrisation by scanning the parameter space affecting especially the high x region
  - Standard Parametrisation Form:

$$Ax^B(1-x)^C(1+Dx+..)$$

    - describes the shape of PDFs with few input parameters
    - difficult to study systematically both the low and high x regions
    - multiple similar solutions for $x>x_{min}$
      - equivalent solutions for D~0 and $Dx_{min}>>1$

$$Ax^B(1-x)^C(1+Dx) = \frac{A}{D}x^{B-1}(1-x)^C\left(\frac{1}{Dx}+1\right) = A'x^{B'}(1-x)^C\left(\frac{1}{Dx}+1\right)$$



  - Neural Network PDF group uses Neural Nets to study PDF param. biases

Chebyshev Polynomials

Another method, mathematically more robust to study parametrisation biases, is to use orthogonal polynomials to parametrise PDFs : Chebyshev Polynomials of Ist kind

- Orthogonally defined in the [-1,1] interval and given by the recurrence relation:

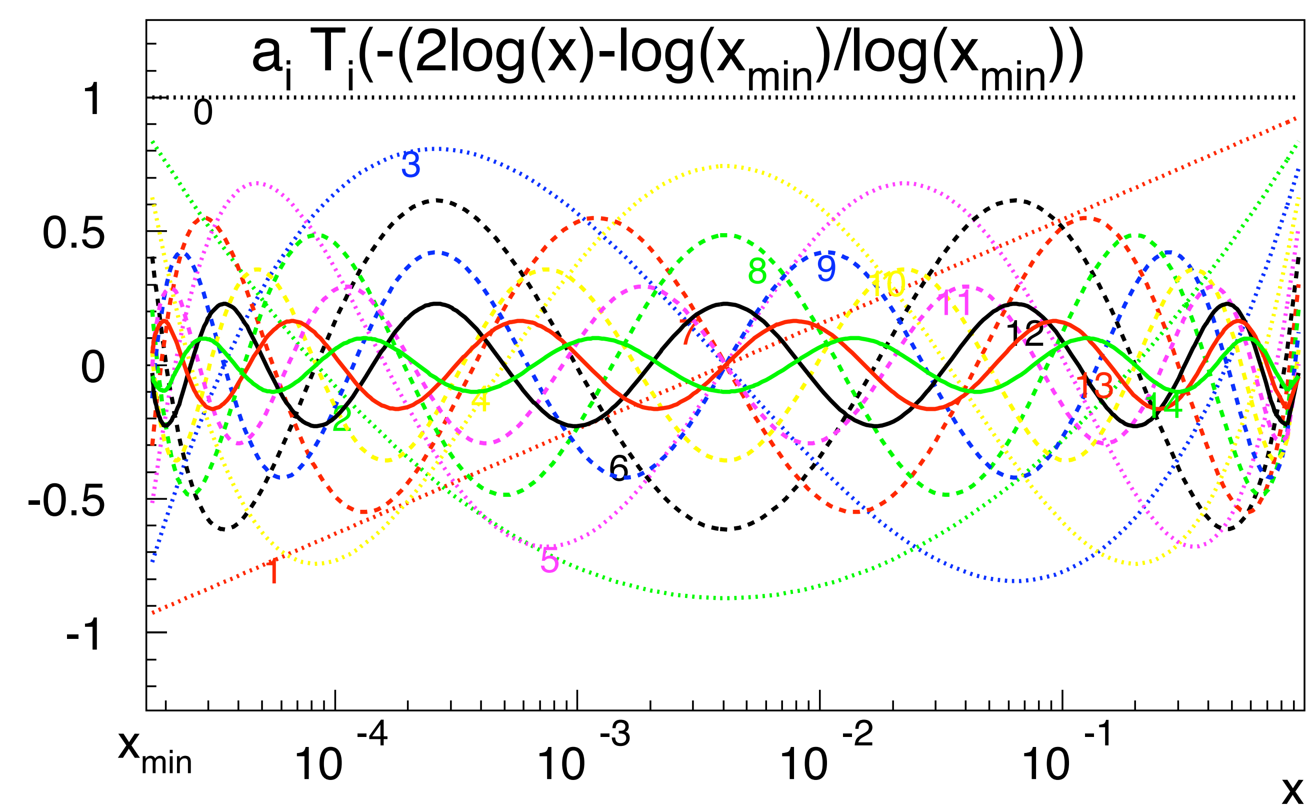
$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

  - To approximate PDFs, change variable such that [log(x_{min}),0] interval is mapped to [-1,1] $x \rightarrow \frac{-(2\log x - \log x_{min})}{\log x_{min}}$
  - This allows to approximate PDF with few parameters:

$$xf(x) = \sum_{i=0}^{N-1} a_i T_i \left(\frac{-(2\log x - \log x_{min})}{\log x_{min}} \right)$$

- Momentum Sum Rule leads to simple finite integrals

$$T_0(x) = 1$$
$$T_1(x) = x$$
$$T_2(x) = 2x^2 - 1$$
$$T_3(x) = 4x^3 - 3x$$
$$T_4(x) = 8x^4 - 8x^2 + 1$$
$$T_5(x) = 16x^5 - 20x^3 + 5x$$
$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$
$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$$
$$T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$$
$$T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x.$$



Settings

The study is performed using:

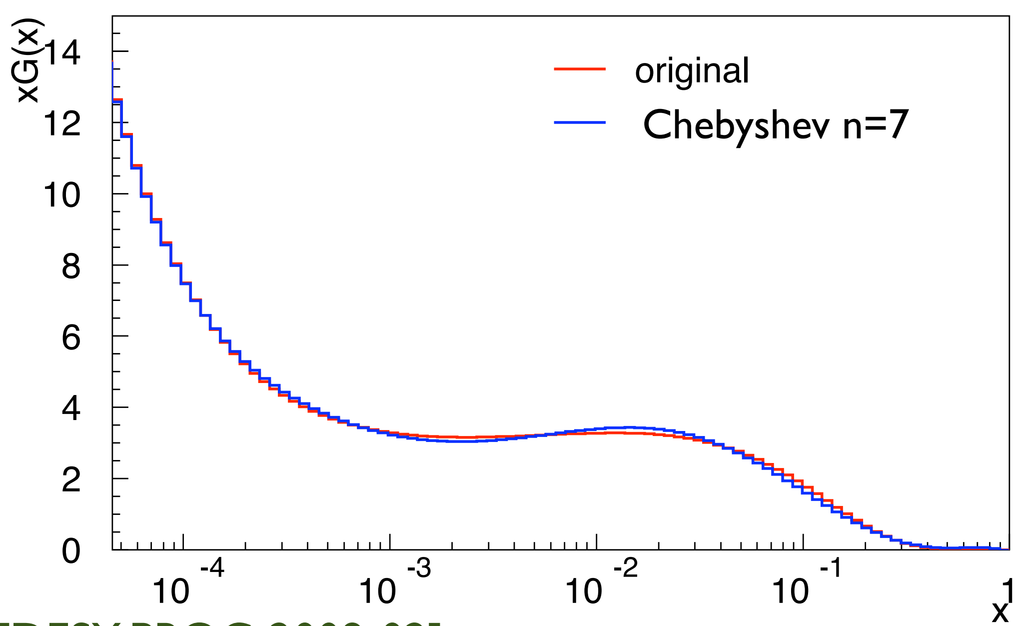
- Published H1-HERA I data of NC and CC $e^{\pm}p$ scattering cross sections
  - following EPJ C30,1 (2003)

- Fit program H1 QCDNUM implementation at NLO:
  - $\overline{MS}$ renormalisation scheme, DGLAP evolution at NLO, massless quarks (ZMVFNS)
  - starting scale Q_0^2= 4 GeV^2
- PDFs are parametrised a la ZEUS Parametrisation [EPJ C42,1(2005)hep-ph/0503274]
  - $\mathbf{xd_v(x), xu_v(x), x\Delta = x\bar{u}(x) - x\bar{d}(x)}$ with $x\Delta$ fixed

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + D_{u_v} x)$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$$

  - $\mathbf{xS(x) = 2x\left(\bar{u}(x) + \bar{d}(x) + \bar{s}(x) + \bar{c}(x)\right), xG(x)}$ using Chebyshev Polynomials up to the 15th order in series expansion
    - only for xG(x)
      - xS(x) with standard parametrisation
    - for xG(x) and xS(x)
    - Chebyshev Polynomials can reproduce the shape of the standard parametrisation



- Errors are estimated using Monte Carlo technique [DESY-PROC-2009-02]

Monte Carlo technique

- Method consists in preparing replicas of data sets allowing the central values of the cross sections to fluctuate within their systematic and statistical uncertainties taking into account all point to point correlations

    - Various assumptions can be considered for the error distributions: Gauss, Log-Normal ...

- Shift central values randomly within their uncorrelated errors assuming Gauss distributions of the errors:

$$\sigma_i = \sigma_i(1 + \delta_i^{uncorr} RAND_i)$$

- Shift central values with the same probability of the corresponding correlated systematic shift assuming Gauss distribution of the errors:

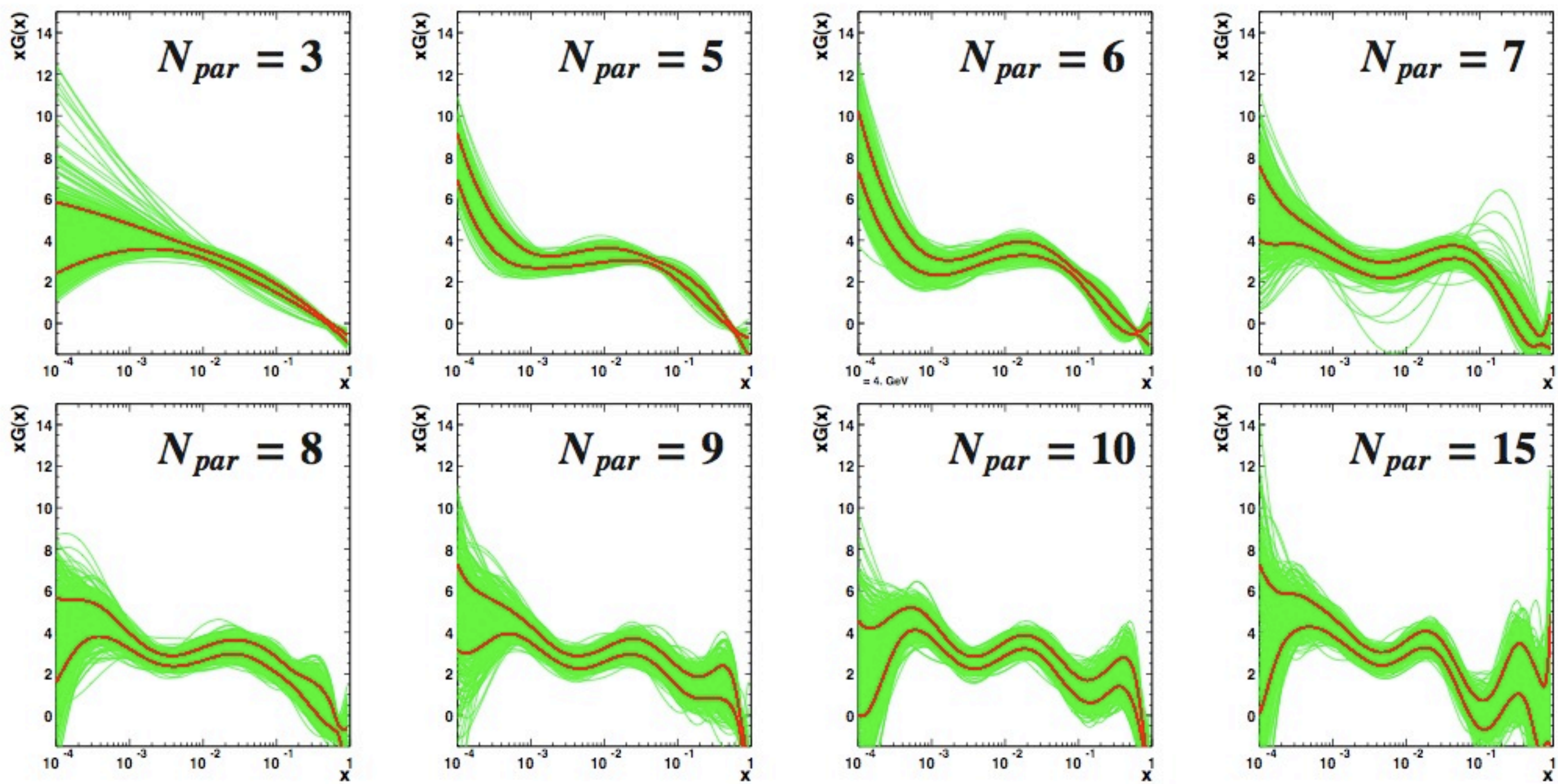
$$\sigma_i = \sigma_i(1 + \delta_i^{uncorr} RAND_i + \sum_j^{N_{sys}} \delta_{ij}^{corr} RAND_j)$$

- Preparation of the data is repeated for N times (N>100):
    - For each replicas NLO QCD fit is performed to extract the PDF set

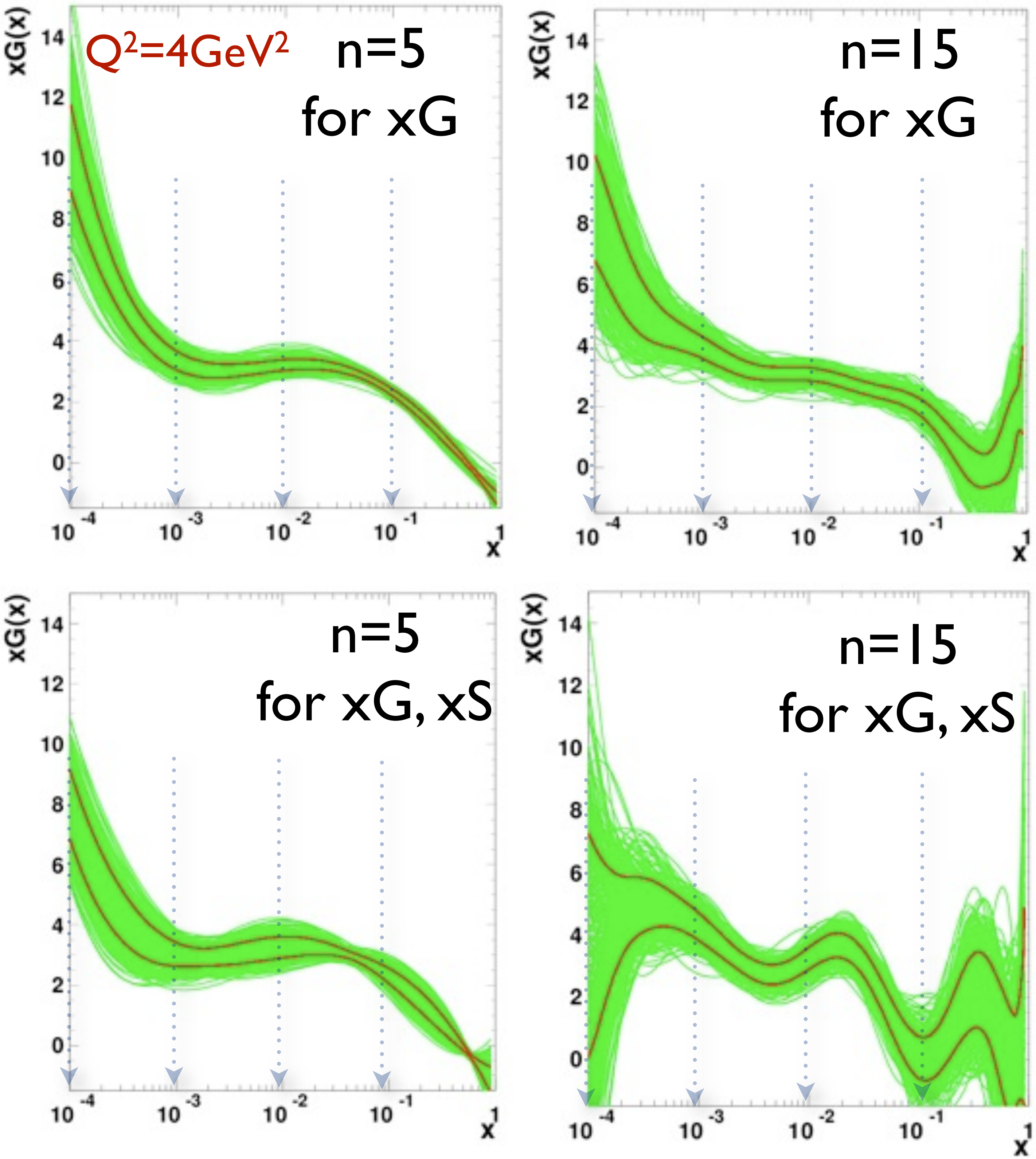
- Errors on the PDFs are estimated from the RMS of the spread of the N curves corresponding to the N individual extracted PDFs

Results

- All Plots are shown for Gluon distribution at $Q^2=4GeV^2$
- MC replicas are shown in green lines (N>400)
- The uncertainty is estimated as the RMS of the spread and is shown in red
- It is interesting to observe the shape difference with the increased number of Chebyshev parameters:
  - for both Gluon and Sea parametrised by Chebyshev Polynomials



Results in more details

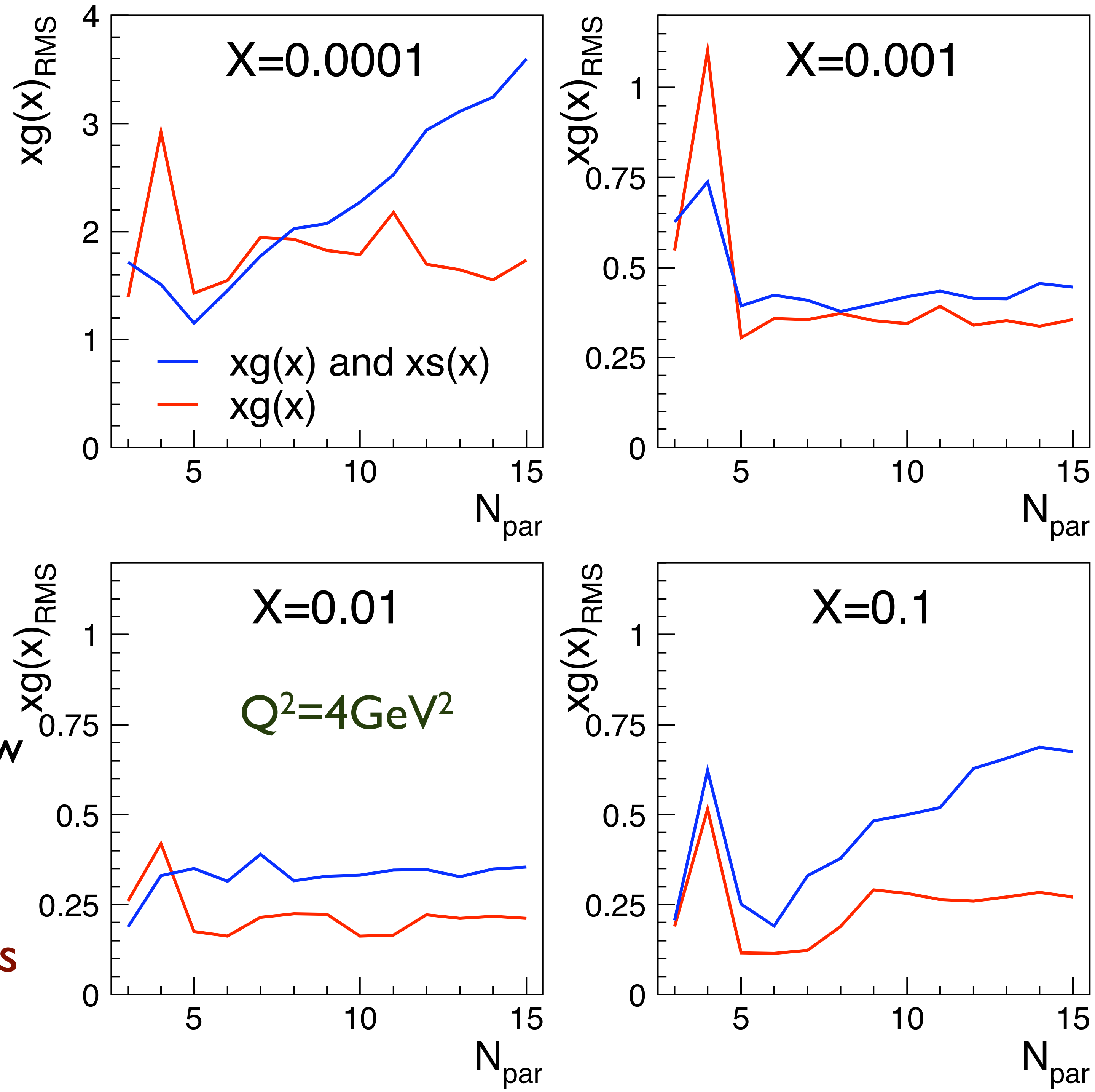


- All Plots are shown for gluon distribution at Q^2=4GeV^2
- MC replicas are shown in green lines (N>400)
- The uncertainty is estimated as the RMS of the spread and is shown in red
- Study in more details for fixed x points
  - x=0.0001
  - x=0.001
  - x=0.01
  - x=0.1

at the edges of sensitivity and for the bulk of precision.

RMS distributions at selected x

- We look at the RMS values for increasing number of Chebyshev parameters (3-15):
  - for Gluon only
  - for Sea and Gluon
- More fluctuations are observed for Sea and Gluon case than for Gluon only:
  - Gluon and Sea are strongly coupled by DGLAP evolution
  - Sea parametrisation is more rigid (standard param.) and it doesn't allow gluon to fluctuate that much
- RMS does not increase significantly with increasing number of parameters for x region 0.001-0.01, where most of data are.



Constraining the shape of PDFs

- Humpy shapes in x can be correlated with peaks in the hadronic state invariant mass W:

$$W \approx Q\sqrt{\frac{1-x}{x}}$$

  - Resonances are observed at low W but they disappear for W>10 GeV
    - [JLAB CLAS experiment]

- Idea: use "length" along the PDFs as an extra constraint [W. Giele]:

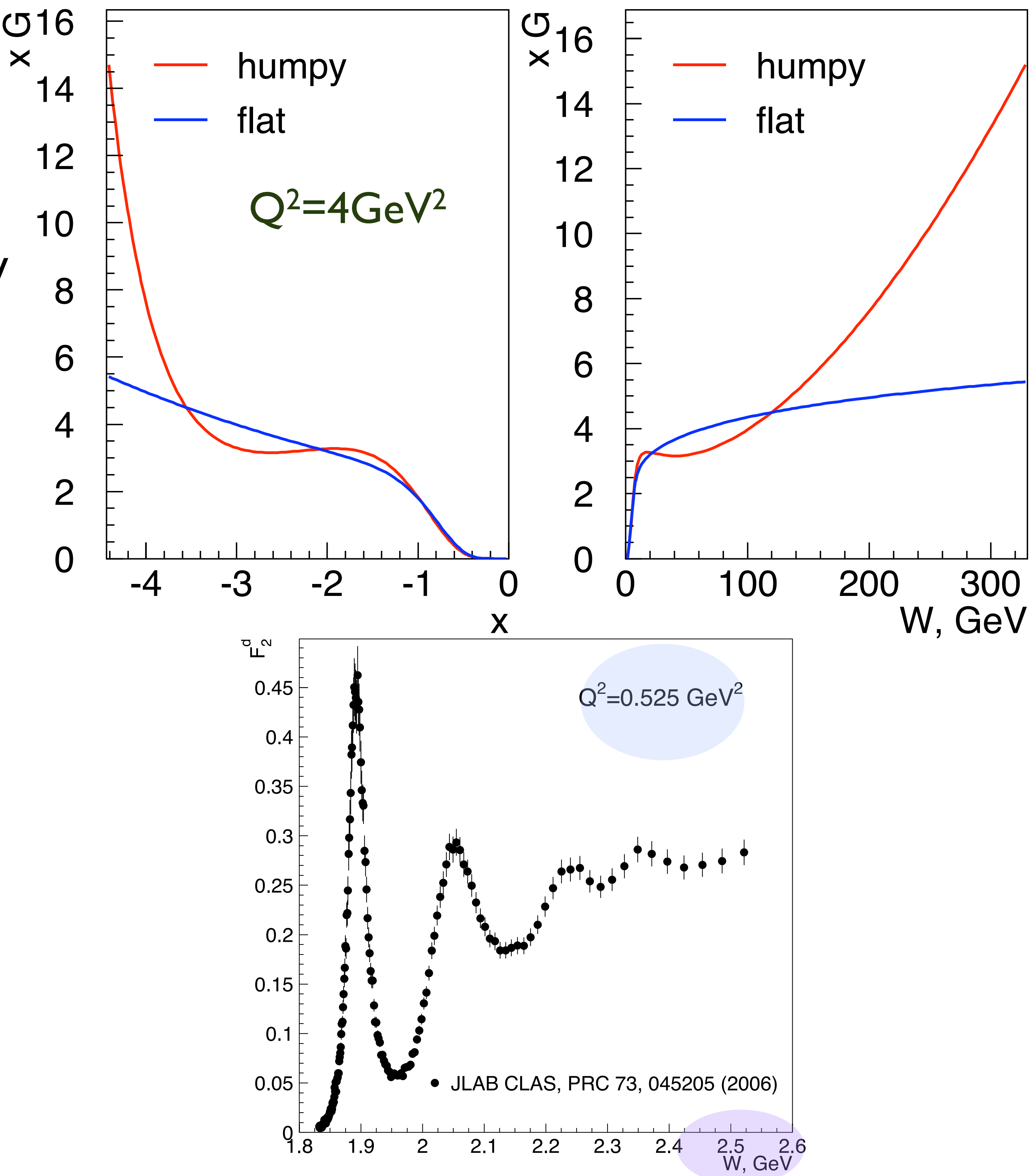
  - prefer solutions which are smoother in W
  - add length penalty to χ^2

$$L = \int_{W_{min}}^{W_{max}} \sqrt{1+\left(\frac{dxf(W)}{dW}\right)^2} dW$$

$$\Delta\chi^2 = P\cdot(L - L_{min})$$

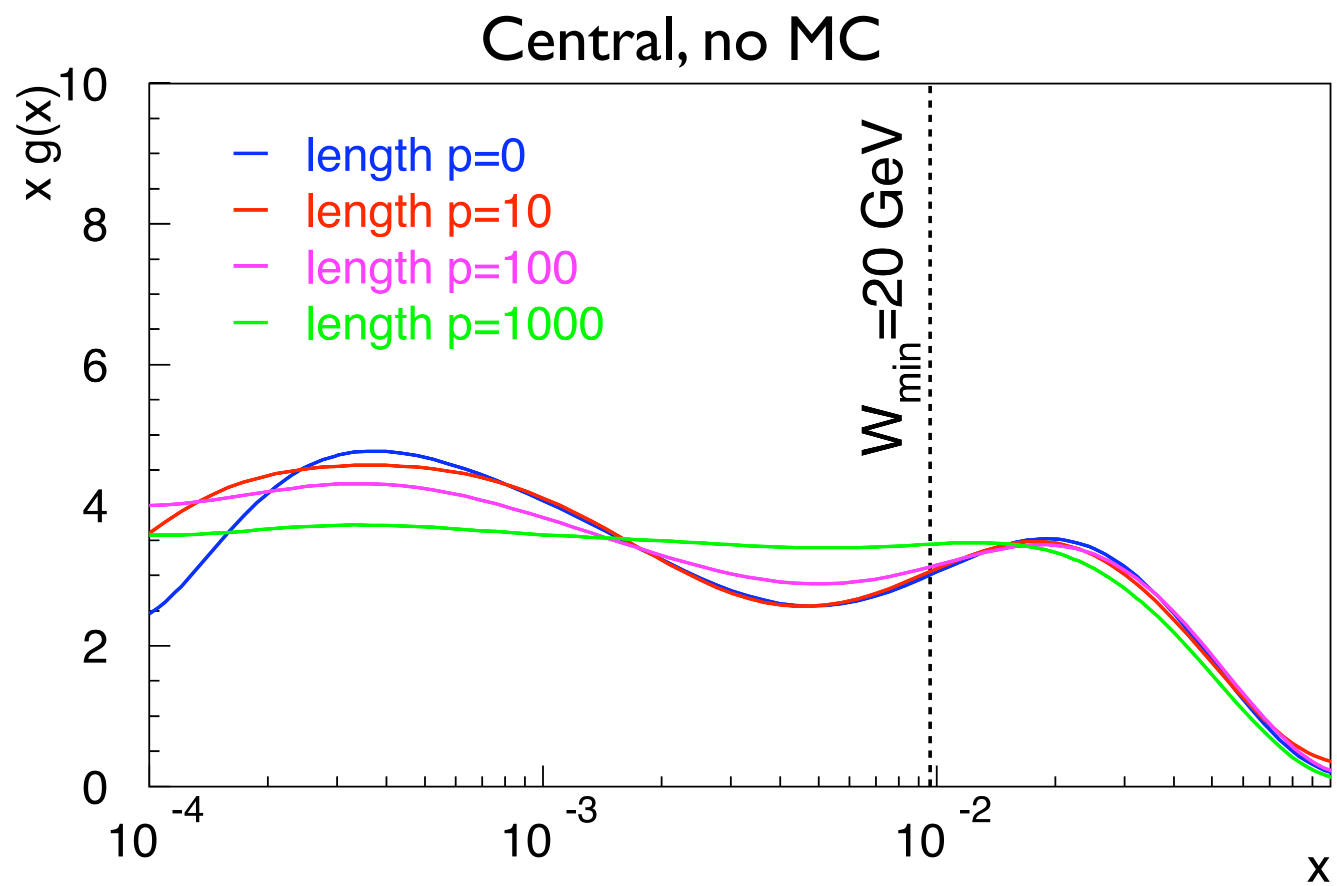
$$L_{min} = W_{max} - W_{min}$$

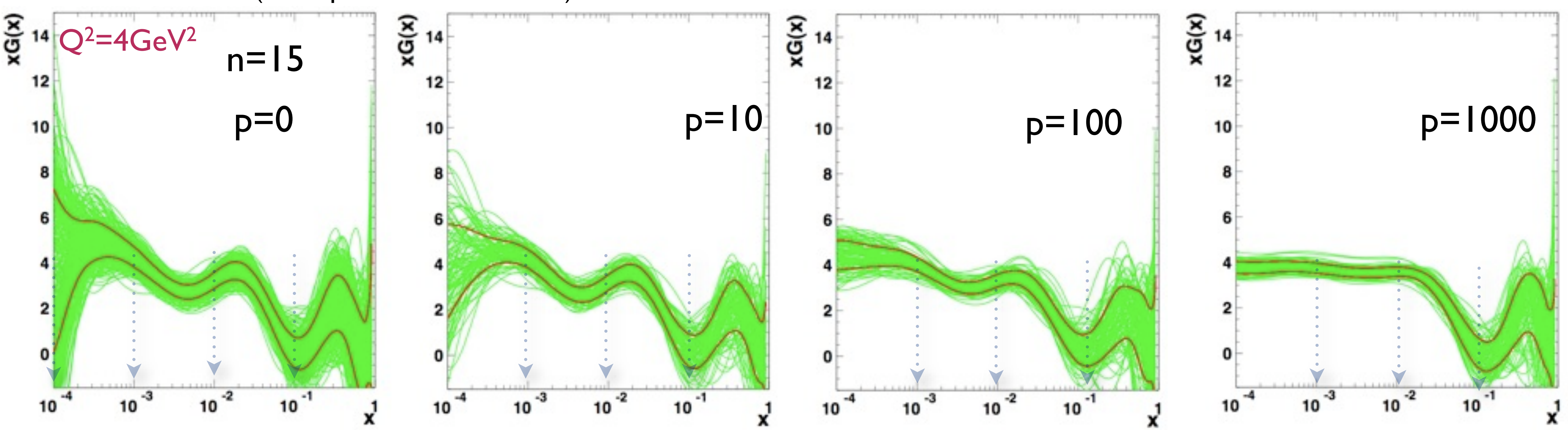
    - Wmax=320 GeV



Effect of the length constraint

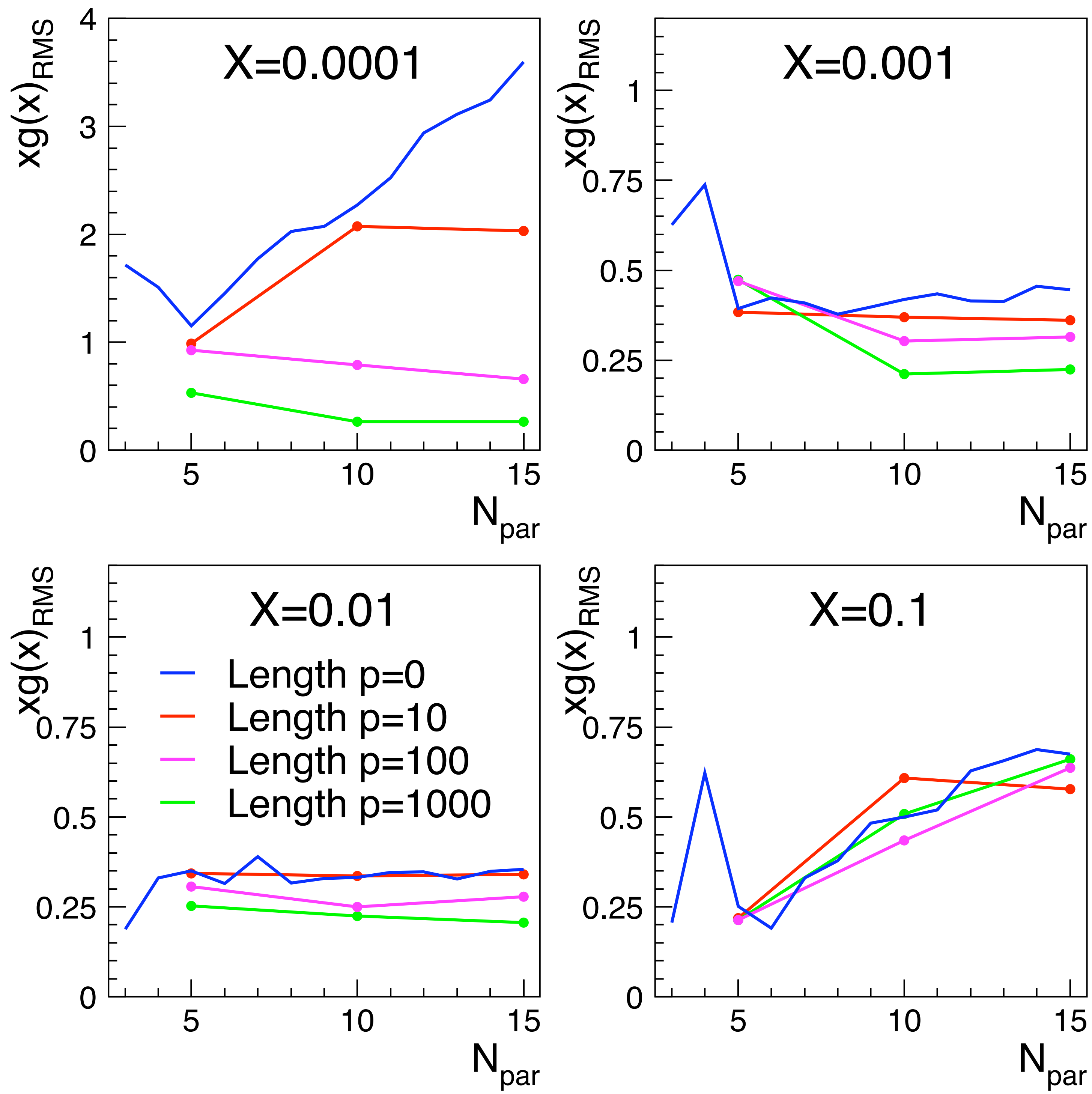
- Run the MC replicas by applying the length constraint for the following cases:
  - Penalty p=10, 100, 1000
    - the length penalty is only added at the starting scale $Q^2 = 4\ GeV^2$
  - choose W_{min} = 20 GeV
    - concentrate on the low x region
- The Constraint is efficient (χ^2/ndf)
  - (15x15) p=0 479 / 588
  - (15x15) p=10 481 / 588
  - (15x15) p=100 486 / 588
  - (15x15) p=1000 514 / 588
  - (stand.param 551 / 611)





Effect of the length constraint

- Even soft constraint against extra minima reduces error at low X
  - p=10 (red line)
- Tighter constraints limit the uncertainty better than the data.
- For x=0.1 constraint does not do much
- For the bulk of the data constraint does not do much for RMS but it becomes more constant vs Npar.



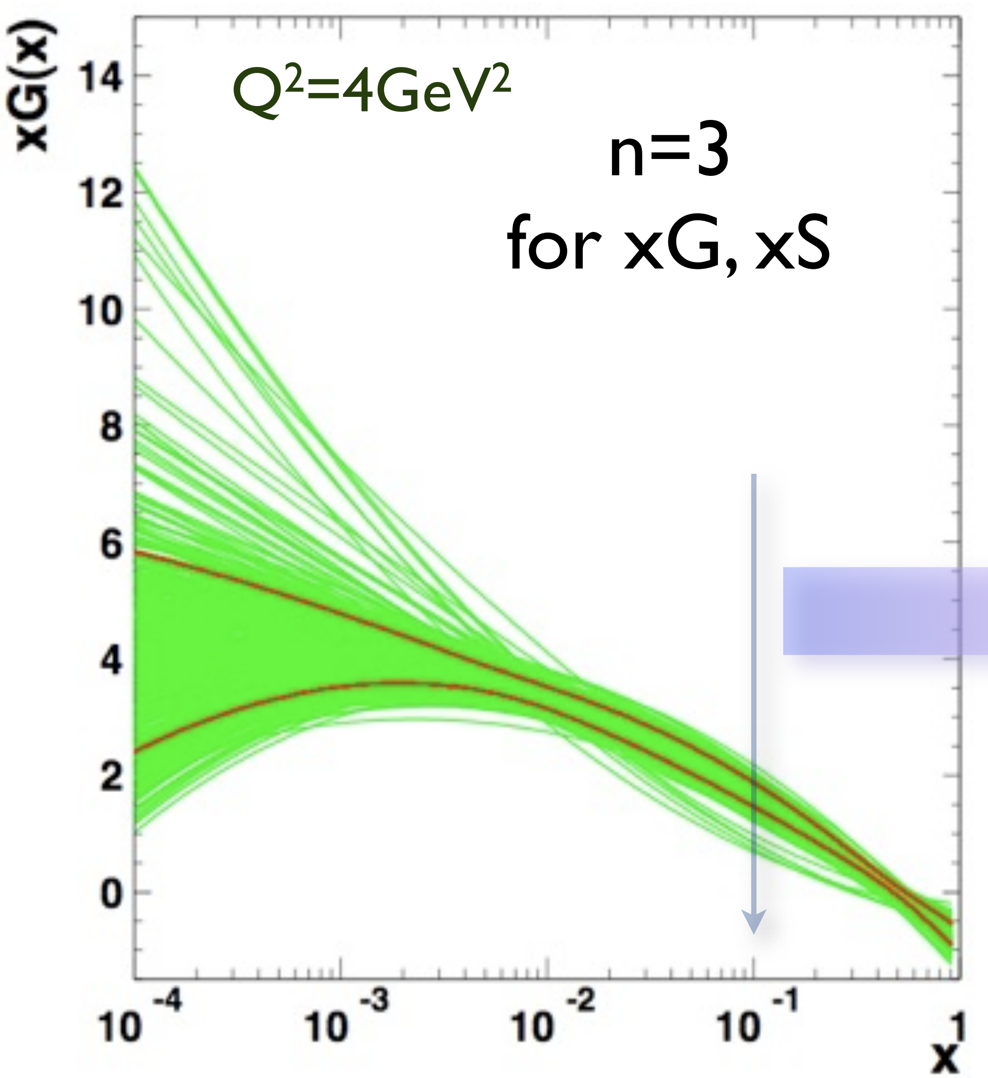
Summary

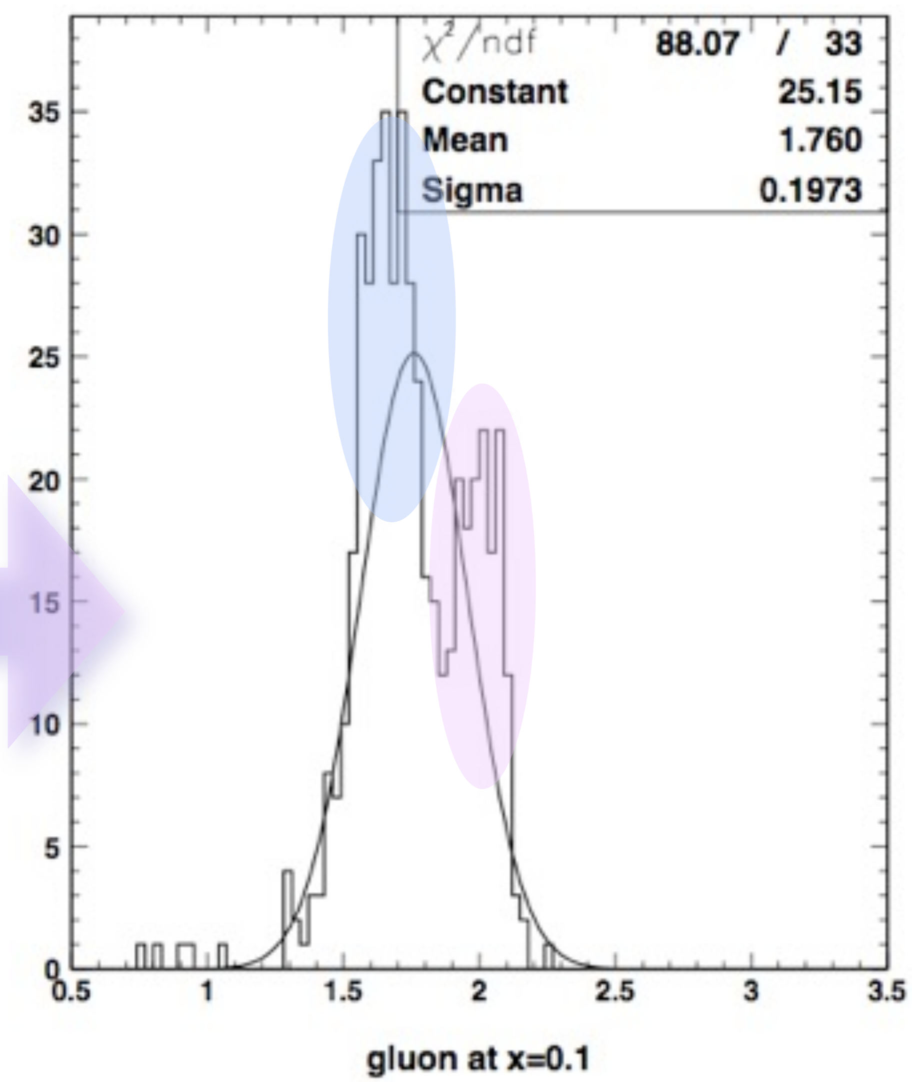
- Presented a new study on the PDF parametrisation using Chebyshev polynomial with the emphasis on the low x region for gluon and sea quarks:
  - Parametrisation with Chebyshev Polynomials offers more flexibility for the PDFs than the standard parametrisation form
  - Observe larger variations of the Gluon uncertainty at the edges of data sensitivity

- Presented a method to constrain PDFs using simple, physically motivated penalty term against extra minima/maxima vs W.
  - The data are stable vs parameterisation change in the bulk region x=0.001-0.01.
  - Minimal constraint improve precision for smallest x.

Double minima issue

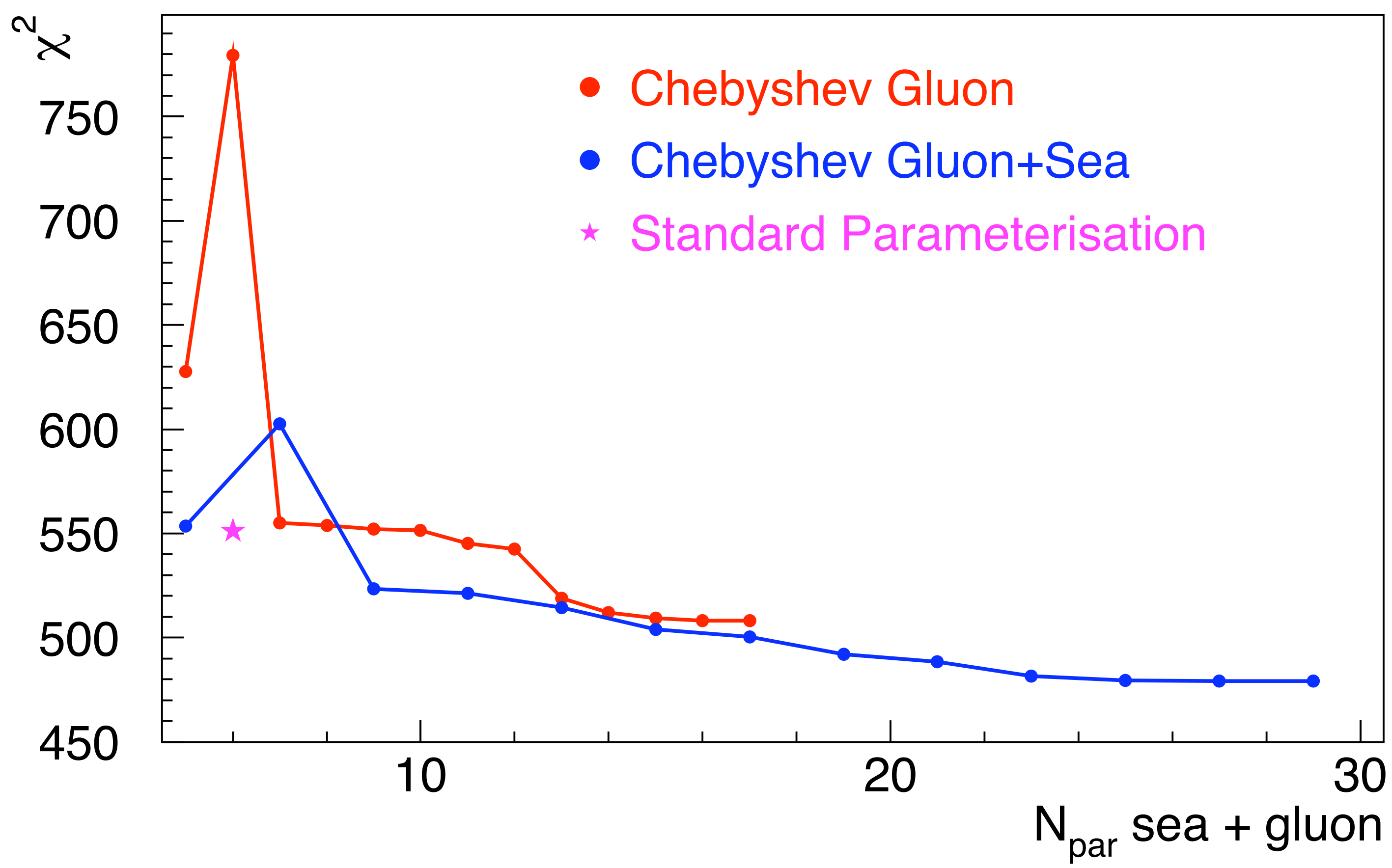
- A feature is observed for low number of Chebyshev parameters.
- Two solutions are preferred by the minimisation procedure for n=3 Chebyshev parameters, clearly observed at x=0.1
  - both gluon and sea distributions are parametrised by Chebyshev polynomials





First General Remarks

- Using Chebyshev Polynomials offers more flexibility for the PDFs than the standard parametrisation form
- Larger variations of the xG(x) uncertainty at the edges of sensitivity are observed
- Slow increase of the uncertainty for the region where the data is very precise



- Problem of 2 solutions goes away starting from n=5
- Is it physical to allow for oscillations vs x in PDFs ?