

Status of HO_MCSANC

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One-loop effective form-factors

Z-exchange amplitude in the LQ basis has the following Born-like structure which allows to take the leading higher order corrections into account by means of effective form-factors:

$$\begin{aligned} \mathcal{A}_Z^{OLA}(s, t) = & i e^2 4 l_e^{(3)} l_f^{(3)} \frac{\chi_Z(s)}{s} \rho_{ef}(s, t) \times \\ & \left\{ \gamma_\mu (1 + \gamma_5) \otimes \gamma_\mu (1 + \gamma_5) \right. \\ & - 4 |Q_e| s_W^2 \kappa_e(s, t) \gamma_\mu \otimes \gamma_\mu (1 + \gamma_5) \\ & - 4 |Q_f| s_W^2 \kappa_f(s, t) \gamma_\mu (1 + \gamma_5) \otimes \gamma_\mu \\ & \left. + 16 |Q_e Q_f| s_W^4 \kappa_{e,f}(s, t) \gamma_\mu \otimes \gamma_\mu \right\}. \end{aligned}$$

One-loop effective form-factors

The effective ff's at the one-loop level are related to the $F_{LL}, F_{QL}, F_{LQ}, F_{QQ}$ formfactors:

$$\begin{aligned}\rho_{ef} &= 1 + F_{LL}(s, t) - s_W^2 \Delta r, \\ \kappa_e &= 1 + F_{QL}(s, t) - F_{LL}(s, t), \\ \kappa_f &= 1 + F_{LQ}(s, t) - F_{LL}(s, t), \\ \kappa_{ef} &= 1 + F_{QQ}(s, t) - F_{LL}(s, t).\end{aligned}$$

appearing from the one-loop amplitude parametrisation:

$$[i\gamma_\mu \gamma_+ F_L^e(s) + i\gamma_\mu F_Q^e(s)] \otimes [i\gamma_\mu \gamma_+ F_L^t(s) + i\gamma_\mu F_Q^t(s)],$$

due to 4 independent helicity amplitudes.

HO EW form-factors

The formfactors in turn, can be extended with higher order corrections to the ρ -parameter:

$$F_{LL}^{HO} = F_{LL}^{1-loop} + \Delta\rho + \Delta\rho^2$$

$$F_{QL}^{HO} = F_{QL}^{1-loop} + (1 + c_W^2/s_W^2)\Delta\rho$$

$$F_{LQ}^{HO} = F_{LQ}^{1-loop} + (1 + c_W^2/s_W^2)\Delta\rho$$

$$F_{QQ}^{HO} = F_{QQ}^{1-loop} + (1 + 2c_W^2/s_W^2)\Delta\rho$$

$$F_{\gamma\gamma}^{HO} = F_{\gamma\gamma}^{1-loop} + c_W^2/s_W^2\Delta\rho$$

$\Delta\rho$ definition

The ρ -parameter is defined as the ratio of neutral and charged current effective coupling constants at zero momentum transfer:

$$\rho = \frac{G_{\text{NC}}(0)}{G_{\text{CC}}(0)} = \frac{1}{1 - \Delta\rho} \quad (1)$$

Here $G_{\text{CC}}(0) = G_{\mu}$ is the Fermi constant defined from the μ decay and $\Delta\rho$ is treated perturbatively:

$$\Delta\rho = \Delta\rho^{(1)} + \Delta\rho^{(2)} + \dots \quad (2)$$

$\Delta\rho$ definition

$\Delta\rho$ expansion to second order:

$$\rho = 1 + \Delta\rho + \Delta\rho^2 \quad (3)$$

The contribution to $\Delta\rho$ leading in $G_\mu m_t^2$ is explicitly given by:

$$\Delta\rho^{(1)}|_{G_\mu} = 3X_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2} \quad (4)$$

as in [S. Dittmaier et al., JHEP 1001 (2010) 060]

Dominant RC

Instead of complete calculation of all perturbative orders, the dominant radiative corrections can be absorbed into the shift of the ρ parameter from it's lowest order value $\rho_{Born} = 1$:

$$\begin{aligned} \Delta\rho = & \Delta\rho_{X_t} + \Delta\rho_{\alpha\alpha_s} + \Delta\rho_{X_t\alpha_s^2} + \left(\Delta\rho_{X_t(zt)\alpha_s^2} + \Delta\rho_{X_t(alp)_f^2} \right) \quad (5) \\ & + \Delta\rho_{X_t\alpha_s^3} + \Delta\rho_{X_t^2\alpha_s} + \Delta\rho_{X_t^2(bos)} + \Delta\rho_{X_t^3} \end{aligned}$$

For simplicity denote the corrections with indices:

notation	order of	DIZET v6.42	MCSANC
$\Delta\rho_1$	$O(\alpha\alpha_s)$	+	+
$\Delta\rho_2$	$O(\alpha^2)$	+	+
$\Delta\rho_3$	$O(\alpha_t\alpha_s^2)$	+	+
$\Delta\rho_4$	$O(\alpha_t^2\alpha_s)$	-	+
$\Delta\rho_5$	$O(\alpha_t\alpha_s^3)$	-	+

Treatment of dominant RC

There is some freedom in grouping:

- Direct sum:

$$\Delta\rho = \sum_{i=1}^N \Delta\rho_i$$

- Product:

$$\Delta\rho = \Delta\rho_0 \prod_{i=1}^N (1 + \Delta\rho_i/\Delta\rho_0) - \Delta\rho_0$$

Also a resummation can be applied (G.Degrassi et al., hep-ph/9507286):

$$\rho = 1 + \Delta\rho_0 + \sum_{i=1}^N \Delta\rho_i \rightarrow \frac{1}{1 - \Delta\rho_0^f} (1 + \Delta\rho_0^b + \sum_{i=1}^N \Delta\rho_i)$$

Effective formfactor comparison: 1-loop

The MCSANC 1-loop approximation agrees very well with DIZET (also for down):

up-quark

ρ	1.00649617	$-4.19861512 \cdot 10^{-3}$	DIZET v6.42
	1.00649616	$-4.19861353 \cdot 10^{-3}$	MCSANC v1.20
κ_q	1.04157647	$1.39885391 \cdot 10^{-2}$	DIZET v6.42
	1.04157647	$1.39885391 \cdot 10^{-2}$	MCSANC v1.20
κ_l	1.04114329	$1.32567432 \cdot 10^{-2}$	DIZET v6.42
	1.04114330	$1.32567432 \cdot 10^{-2}$	MCSANC v1.20
κ_{ql}	1.08271976	$2.72452822 \cdot 10^{-2}$	DIZET v6.42
	1.08271976	$2.72452822 \cdot 10^{-2}$	MCSANC v1.20

Effective formfactor comparison: HO corrections

The DIZET legacy resummation is difficult to reversely engineer. Here we are comparing the results for the first three components: $O(\alpha\alpha_s)$, $O(\alpha^2)$, $O(\alpha_t\alpha_s^2)$

ff values, up-quark				correction diff	
ff	DIZET	sum $\delta\rho_{1-3}$	prod $\delta\rho_{1-3}$	sum ε	prod ε
ρ	1.00540	1.00521	1.00526	-0.0351852	-0.0259259
κ_q	1.03670	1.03640	1.03654	-0.00817439	-0.00435967
κ_l	1.03622	1.03596	1.03611	-0.00717835	-0.003037
κ_{ql}	1.07424	1.07236	1.07266	-0.0253233	-0.0212823
down-quark					
ρ	1.00589	1.00567	1.00571	-0.0373514	-0.0305603
κ_q	1.03670	1.03640	1.03654	-0.00817439	-0.00435967
κ_l	1.03565	1.03545	1.03560	-0.0056101	-0.00140252
κ_{ql}	1.07365	1.07185	1.07215	-0.0244399	-0.0203666

$$\varepsilon = \frac{\rho_{\text{mcsanc}} - \rho_{\text{dizet}}}{\rho_{\text{dizet}} - 1}$$

HO Effective formfactor comparison, with resummation

ff values, up-quark				correction diff	
ff	DIZET	sum $\delta\rho_{1-3}$	prod $\delta\rho_{1-3}$	sum ε	prod ε
ρ	1.00540	1.00529	1.00541	-0.0203704	0.00185185
κ_q	1.03670	1.03664	1.03709	-0.00163488	0.0106267
κ_l	1.03622	1.03621	1.03665	-0.000276091	0.0118719
κ_{ql}	1.07424	1.07286	1.07374	-0.0185884	-0.00673491
down-quark					
ρ	1.00589	1.00574	1.00586	-0.0254669	-0.00509338
κ_q	1.03670	1.03664	1.03709	-0.00163488	0.0106267
κ_l	1.03565	1.03570	1.03614	0.00140252	0.0137447
κ_{ql}	1.07365	1.07235	1.07323	-0.0176511	-0.00570265

The product combination with resummation gives closest values to the DIZET results.

All MCSANC corrections for ρ

Inclusion of the full list of HO corrections available in the MCSANC with resummation:

	up-quark		down-quark		
ρ	1.00540	$-4.16376 \cdot 10^{-3}$	1.00589	$-3.40169 \cdot 10^{-3}$	DIZET v6.42
	1.00538	$-4.16678 \cdot 10^{-3}$	1.00583	$-3.40462 \cdot 10^{-3}$	sum $\delta\rho_{1-5}$
	1.00549	$-4.16678 \cdot 10^{-3}$	1.00594	$-3.40462 \cdot 10^{-3}$	prod $\delta\rho_{1-5}$
κ_q	1.03670	$1.35206 \cdot 10^{-2}$	1.03670	$1.35206 \cdot 10^{-2}$	DIZET v6.42
	1.03696	$1.36787 \cdot 10^{-2}$	1.03696	$1.36787 \cdot 10^{-2}$	sum $\delta\rho_{1-5}$
	1.03736	$1.36787 \cdot 10^{-2}$	1.03736	$1.36787 \cdot 10^{-2}$	prod $\delta\rho_{1-5}$
κ_l	1.03622	$1.27903 \cdot 10^{-2}$	1.03565	$1.19328 \cdot 10^{-2}$	DIZET v6.42
	1.03653	$1.29483 \cdot 10^{-2}$	1.03602	$1.20909 \cdot 10^{-2}$	sum $\delta\rho_{1-5}$
	1.03693	$1.29483 \cdot 10^{-2}$	1.03642	$1.20909 \cdot 10^{-2}$	prod $\delta\rho_{1-5}$
κ_{ql}	1.07424	$2.63109 \cdot 10^{-2}$	1.07365	$2.54534 \cdot 10^{-2}$	DIZET v6.42
	1.07349	$2.66271 \cdot 10^{-2}$	1.07298	$2.57696 \cdot 10^{-2}$	sum $\delta\rho_{1-5}$
	1.07429	$2.66271 \cdot 10^{-2}$	1.07378	$2.57696 \cdot 10^{-2}$	prod $\delta\rho_{1-5}$

Summary

- A set of HO leading corrections are introduced to the MCSANC framework.
- Multiple options for combining together with resummation are allowed
- The best comparison with DIZET corresponds to product-like combination with resummation.
- The difference between DIZET and MCSANC can be attributed to theoretical uncertainties, since there is no defined rule for the combination of the corrections.
- The HO corrections are implemented in the MCSANC integrator for pp processes. e^+e^- processes to follow.

BACKUP SLIDES

References: $\Delta\rho$ corrections

$O(\alpha\alpha_s)$ (complete): B. Kniehl, D. Bardin, ZF manual:
<https://arxiv.org/pdf/hep-ph/9908433.pdf>

$O(\alpha^2)$ (fermionic, leading+subleading): G. Degrassi, P. Gambino, and A. Vicini, Phys. Lett. B383 (1996) 219–226. G. Degrassi, P. Gambino, and A. Sirlin, Phys. Lett. B394 (1997) 188–194. G. Degrassi and P. Gambino, hep-ph/9905472.

$O(\alpha_t\alpha_s^2)$: (leading) L. Avdeev et al., hep-ph/946362 K. Chetyrkin et al., hep-ph/9504413

$O(\alpha_t^2\alpha_s)$: (leading) J.J. van der Bij et al., hep-ph/0011373

$O(\alpha_t\alpha_s^3)$: (leading) Y. Schoeder et al., hep-ph/0504055

The one-loop expression for Δr Sirlin's parameter is given by

$$\Delta r = \frac{g^2}{16\pi^2} \left\{ -\frac{2}{3} - \Pi_{\gamma\gamma}^{\text{fer},F}(0) + \frac{c_W^2}{s_W^2} \Delta\rho^F + \frac{1}{s_W^2} \left[\Delta\rho_W^F + \frac{11}{2} - \frac{5}{8} c_W^2 (1 + c_W^2) + \frac{9}{4} \frac{c_W^2}{s_W^2} \ln c_W^2 \right] \right\},$$