Status of HO_MCSANC

Andrey Sapronov on behalf of SANC, JINR

26 April 2018

One-loop effective form-factors

Z-exchange amplitude in the LQ basis has the following Born-like structure which allows to take the leading higher order corrections into account by means fo effective form-factors:

$$
\begin{aligned}
\mathcal{A}_Z^{OLA}(s,t) \;=\;& i\, e^2\, 4\, I_e^{(3)} I_f^{(3)} \frac{\chi_Z(s)}{s} \boldsymbol{\rho_{ef}}(s,t) \times \\
&\Big\{ \gamma_\mu (1+\gamma_5) \otimes \gamma_\mu (1+\gamma_5) \\
&-4|Q_e| s_W^2 \boldsymbol{\kappa_e}(s,t) \gamma_\mu \otimes \gamma_\mu (1+\gamma_5) \\
&-4|Q_f| s_W^2 \boldsymbol{\kappa_f}(s,t) \gamma_\mu (1+\gamma_5) \otimes \gamma_\mu \\
&+16|Q_e Q_f| s_W^4 \boldsymbol{\kappa_{e,f}}(s,t) \gamma_\mu \otimes \gamma_\mu \Big\}.
\end{aligned}
$$

One-loop effective form-factors

The effective ff's at the one-loop level are related to the $F_{LL}, F_{QL}, F_{LQ}, F_{QQ}$ formfactors:

$$
\begin{aligned}
\rho_{ef} &= 1 + F_{LL}(s,t) - s_W^2 \Delta r, \\
\kappa_e &= 1 + F_{QL}(s,t) - F_{LL}(s,t), \\
\kappa_f &= 1 + F_{LQ}(s,t) - F_{LL}(s,t), \\
\kappa_{ef} &= 1 + F_{QQ}(s,t) - F_{LL}(s,t).
\end{aligned}
$$

appearing from the one-loop amplitude parametrisation:

$$
\left[i\gamma_\mu \gamma_+ F_L^e(s) + i\gamma_\mu F_Q^e(s) \right] \otimes \left[i\gamma_\mu \gamma_+ F_L^t(s) + i\gamma_\mu F_Q^t(s) \right],
$$

due to 4 independent helicity amplitudes.

HO EW form-factors

The formfactors in turn, can be extended with higher order corrections to the ρ-parameter:

$$
\begin{aligned}
F_{LL}^{HO} &= F_{LL}^{1-loop} + \Delta\rho + \Delta\rho^2 \\
F_{QL}^{HO} &= F_{QL}^{1-loop} + (1 + c_W^2/s_W^2)\Delta\rho \\
F_{LQ}^{HO} &= F_{LQ}^{1-loop} + (1 + c_W^2/s_W^2)\Delta\rho \\
F_{QQ}^{HO} &= F_{QQ}^{1-loop} + (1 + 2c_W^2/s_W^2)\Delta\rho \\
F_{\gamma\gamma}^{HO} &= F_{\gamma\gamma}^{1-loop} + c_W^2/s_W^2\Delta\rho
\end{aligned}
$$

$\Delta\rho$ definition

The ρ-parameter is defined as the ratio of neutral and charged current effective coupling constants at zero momentum transfer:

$$\rho = \frac{G_{\mathrm{NC}}(0)}{G_{\mathrm{CC}}(0)} = \frac{1}{1 - \Delta\rho} \qquad (1)$$

Here $G_{\mathrm{CC}}(0) = G_\mu$ is the Fermi constant defined from the μ decay and $\Delta\rho$ is treated perturbatively:

$$\Delta\rho = \Delta\rho^{(1)} + \Delta\rho^{(2)} + ... \qquad (2)$$

$\Delta\rho$ definition

$\Delta\rho$ expansion to second order:

$$\rho = 1 + \Delta\rho + \Delta\rho^2 \tag{3}$$

The contribution to $\Delta\rho$ leading in $G_\mu m_t^2$ is explicitely given by:

$$\Delta\rho^{(1)}\big|^{G_\mu} = 3X_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2} \tag{4}$$

as in [S. Dittmaier et al., JHEP 1001 (2010) 060]

Dominant RC

Instead of complete calculation of all perturbative orders, the dominant radiative corrections can be absorbed into the shift of the ρ parameter from it's lowest order value $\rho_{Born} = 1$:

$$\begin{aligned} \Delta\rho \;&=\; \Delta\rho_{X_t} + \Delta\rho_{\alpha\alpha_s} + \Delta\rho_{X_t\alpha_s^2} + \left(\Delta\rho_{X_t(zt)\alpha_s^2} + \Delta\rho_{X_t(alp)_f^2}\right) \\ &\quad + \Delta\rho_{X_t\alpha_s^3} + \Delta\rho_{X_t^2\alpha_s} + \Delta\rho_{X_t^2(bos)} + \Delta\rho_{X_t^3} \end{aligned} \tag{5}$$

For simplicity denote the corrections with indices:

notation	order of	DIZET v6.42	MCSANC
$\Delta\rho_1$	$O(\alpha\alpha_s)$	+	+
$\Delta\rho_2$	$O(\alpha^2)$	+	+
$\Delta\rho_3$	$O(\alpha_t\alpha_s^2)$	+	+
$\Delta\rho_4$	$O(\alpha_t^2\alpha_s)$	-	+
$\Delta\rho_5$	$O(\alpha_t\alpha_s^3)$	-	+

Treatment of dominant RC

There is some freedom in grouping:

- Direct sum:

$$\Delta\rho = \sum_{i=1}^{N} \Delta\rho_i$$

- Product:

$$\Delta\rho = \Delta\rho_0 \prod_{i=1}^{N} (1 + \Delta\rho_i/\Delta\rho_0) - \Delta\rho_0$$

Also a resummation can be applied (G.Degrassi et al., hep-ph/9507286):

$$\rho = 1 + \Delta\rho_0 + \sum_{i=1}^{N} \Delta\rho_i \rightarrow \frac{1}{1 - \Delta\rho_0^f}(1 + \Delta\rho_0^b + \sum_{i=1}^{N} \Delta\rho_i)$$

Effective formfactor comparison: 1-loop

The MCSANC 1-loop approximation agrees very well with DIZET (also for down):

up-quark

ρ	1.00649617	-4.19861512 $\cdot 10^{-3}$	DIZET v6.42
	1.00649616	-4.19861353 $\cdot 10^{-3}$	MCSANC v1.20
κ_q	1.04157647	1.39885391 $\cdot 10^{-2}$	DIZET v6.42
	1.04157647	1.39885391 $\cdot 10^{-2}$	MCSANC v1.20
κ_l	1.04114329	1.32567432 $\cdot 10^{-2}$	DIZET v6.42
	1.04114330	1.32567432 $\cdot 10^{-2}$	MCSANC v1.20
κ_{ql}	1.08271976	2.72452822 $\cdot 10^{-2}$	DIZET v6.42
	1.08271976	2.72452822 $\cdot 10^{-2}$	MCSANC v1.20

Effective formfactor comparison: HO corrections

The DIZET legacy resummation is difficult to reversely engineer. Here we are comparing the results for the first three components: $O(\alpha\alpha_s)$, $O(\alpha^2)$, $O(\alpha_t\alpha_s^2)$

ff values, up-quark				correction diff	
ff	`DIZET`	sum $\delta\rho_{1-3}$	prod $\delta\rho_{1-3}$	sum ε	prod ε
ρ	1.00540	1.00521	1.00526	-0.0351852	-0.0259259
κ_q	1.03670	1.03640	1.03654	-0.00817439	-0.00435967
κ_l	1.03622	1.03596	1.03611	-0.00717835	-0.003037
κ_{ql}	1.07424	1.07236	1.07266	-0.0253233	-0.0212823
down-quark					
ρ	1.00589	1.00567	1.00571	-0.0373514	-0.0305603
κ_q	1.03670	1.03640	1.03654	-0.00817439	-0.00435967
κ_l	1.03565	1.03545	1.03560	-0.0056101	-0.00140252
κ_{ql}	1.07365	1.07185	1.07215	-0.0244399	-0.0203666

$$\varepsilon = \frac{\rho_{\text{mcsanc}} - \rho_{\text{dizet}}}{\rho_{\text{dizet}} - 1}$$

HO Effective formfactor comparison, with resummation

ff values, up-quark				correction diff	
ff	`DIZET`	sum $\delta\rho_{1-3}$	prod $\delta\rho_{1-3}$	sum ε	prod ε
ρ	1.00540	1.00529	1.00541	-0.0203704	0.00185185
κ_q	1.03670	1.03664	1.03709	-0.00163488	0.0106267
κ_l	1.03622	1.03621	1.03665	-0.000276091	0.0118719
κ_{ql}	1.07424	1.07286	1.07374	-0.0185884	-0.00673491
down-quark					
ρ	1.00589	1.00574	1.00586	-0.0254669	-0.00509338
κ_q	1.03670	1.03664	1.03709	-0.00163488	0.0106267
κ_l	1.03565	1.03570	1.03614	0.00140252	0.0137447
κ_{ql}	1.07365	1.07235	1.07323	-0.0176511	-0.00570265

The product combination with resummation gives closest values to the DIZET results.

Inclusion of the full list of HO corrections available in the MCSANC with resummation:

	up-quark		down-quark		
ρ	1.00540	$-4.16376 \cdot 10^{-3}$	1.00589	$-3.40169 \cdot 10^{-3}$	DIZET v6.42
	1.00538	$-4.16678 \cdot 10^{-3}$	1.00583	$-3.40462 \cdot 10^{-3}$	sum $\delta\rho_{1-5}$
	1.00549	$-4.16678 \cdot 10^{-3}$	1.00594	$-3.40462 \cdot 10^{-3}$	prod $\delta\rho_{1-5}$
κ_q	1.03670	$1.35206 \cdot 10^{-2}$	1.03670	$1.35206 \cdot 10^{-2}$	DIZET v6.42
	1.03696	$1.36787 \cdot 10^{-2}$	1.03696	$1.36787 \cdot 10^{-2}$	sum $\delta\rho_{1-5}$
	1.03736	$1.36787 \cdot 10^{-2}$	1.03736	$1.36787 \cdot 10^{-2}$	prod $\delta\rho_{1-5}$
κ_l	1.03622	$1.27903 \cdot 10^{-2}$	1.03565	$1.19328 \cdot 10^{-2}$	DIZET v6.42
	1.03653	$1.29483 \cdot 10^{-2}$	1.03602	$1.20909 \cdot 10^{-2}$	sum $\delta\rho_{1-5}$
	1.03693	$1.29483 \cdot 10^{-2}$	1.03642	$1.20909 \cdot 10^{-2}$	prod $\delta\rho_{1-5}$
κ_{ql}	1.07424	$2.63109 \cdot 10^{-2}$	1.07365	$2.54534 \cdot 10^{-2}$	DIZET v6.42
	1.07349	$2.66271 \cdot 10^{-2}$	1.07298	$2.57696 \cdot 10^{-2}$	sum $\delta\rho_{1-5}$
	1.07429	$2.66271 \cdot 10^{-2}$	1.07378	$2.57696 \cdot 10^{-2}$	prod $\delta\rho_{1-5}$

Summary

- A set of HO leading corrections are introduced to the MCSANC framework.
- Multiple options for combining together with resummation are allowed
- The best comparison with DIZET corresponds to product-like combination with resummation.
- The difference between DIZET and MCSANC can be attributed to theoretical uncertainties, since there is no defined rule for the combination of the corrections.
- The HO corrections are implemented in the MCSANC integrator for pp processes. e^+e^- processes to follow.

BACKUP SLIDES

References: $\Delta\rho$ corrections

$O(\alpha\alpha_s)$ (complete): B. Kniehl, D. Bardin, ZF manual: https://arxiv.org/pdf/hep-ph/9908433.pdf

$O(\alpha^2)$ (fermionic, leading+subleading): G. Degrassi, P. Gambino, and A. Vicini, Phys. Lett. B383 (1996) 219–226. G. Degrassi, P. Gambino, and A. Sirlin, Phys. Lett. B394 (1997) 188–194. G. Degrassi and P. Gambino, hep-ph/9905472.

$O(\alpha_t\alpha_s^2)$: (leading) L. Avdeev et al., hep-ph/946362 K. Chetyrkin et al., hep-ph/9504413

$O(\alpha_t^2\alpha_s)$: (leading) J.J. van der Bij et al., hep-ph/0011373

$O(\alpha_t\alpha_s^3)$: (leading) Y. Schoeder et al., hep-ph/0504055

Δr

The one-loop expression for Δr Sirlin's parameter is given by

$$\Delta r = \frac{g^2}{16\pi^2}\left\{-\frac{2}{3} - \Pi^{\mathrm{fer,F}}_{\gamma\gamma}(0) + \frac{c_W^2}{s_W^2}\Delta\rho^F + \frac{1}{s_W^2}\right.$$
$$\left.\left[\Delta\rho^F_W + \frac{11}{2} - \frac{5}{8}c_W^2(1+c_W^2) + \frac{9}{4}\frac{c_W^2}{s_W^2}\ln c_W^2\right]\right\},$$