



# Using ZFITTER in CDF/Tevatron $A_{fb}$ template calculations

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LHC EWK precision subgroup meeting  
26 Apr 2018

# Outline

- Introduction
- Standard Model (SM) context adopted for ZFITTER calculations
- CDF Drell-Yan QCD calculations and ZFITTER
- PDF and mixing angle effects on  $A_{fb}$  at the Tevatron
- Tevatron Legacy  $\sin^2\theta_{eff}^{lept}$  measurement summary
  - ZFITTER based interpretation for  $M_W$
- Summary

# Introduction

- CDF 2011

- $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.2329 \pm 0.0008(\text{stat}) \pm 0.0010(\text{syst !??}): \langle A_4 \rangle$  method,  $2 \text{ fb}^{-1} ee$

- How to do better  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  extractions: ZFITTER deliberations begins

- ROKANC's  $ee \rightarrow Z \rightarrow q\bar{q}$  scattering amplitude looked like the promised land

- But is it suitable for Drell-Yan  $q\bar{q} \rightarrow \ell\ell$ ?

Yes, according to Tord Riemann

But CDF may be the first to try ... Oh boy ...

- ROKANC ( $\rho$ - $\kappa$ -NC) scattering amplitude:

- $A_q \propto 4 \rho_{\text{eq}} T_{3,e} T_{3,q} \{ \langle e|\gamma_\mu(1 + \gamma_5)|e\rangle \langle q|\gamma_\mu(1 + \gamma_5)|q\rangle$   
 $- 4|Q_e| \kappa_e \sin^2\theta_w \langle e|\gamma_\mu|e\rangle \langle q|\gamma_\mu(1 + \gamma_5)|q\rangle$   
 $- 4|Q_q| \kappa_q \sin^2\theta_w \langle e|\gamma_\mu(1 + \gamma_5)|e\rangle \langle q|\gamma_\mu|q\rangle$   
 $+ 16 |Q_e Q_q| \kappa_{\text{eq}} \sin^4\theta_w \langle e|\gamma_\mu|e\rangle \langle q|\gamma_\mu|q\rangle \}$

$$\leftarrow \sin^2\theta_{\text{eff}} = \kappa \sin^2\theta_w$$

- $\rho_{\text{eq}}$ ,  $\kappa_e$ ,  $\kappa_q$  and  $\kappa_{\text{eq}}$ : complex-valued form-factor functions

- External QED and QCD radiation turned off

- Include W/Z box contribution (only approximate factorization)

- Massless fermion approximation

- Functions of  $s$ ,  $t$ ,  $\sin^2\theta_w$ , and weak isospin

- Gauge invariant “pseudo-observables” (and hence so is  $\sin^2\theta_w$ )

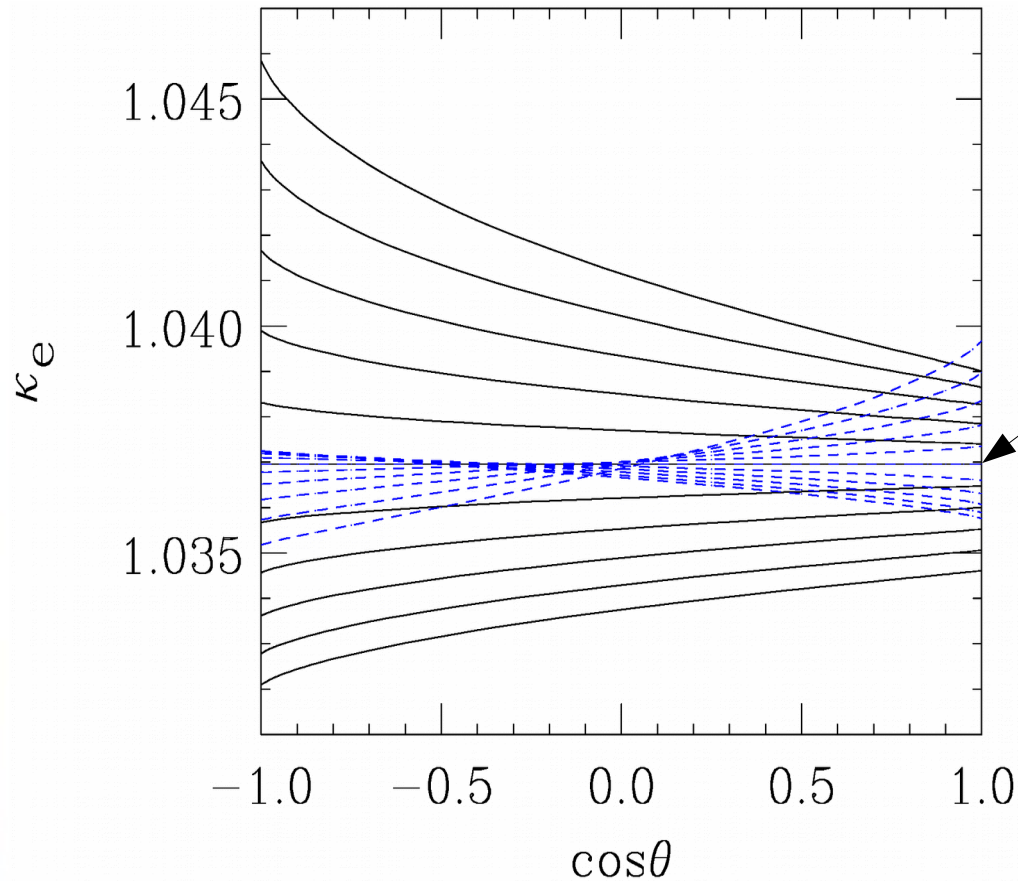
- $\sin^2\theta_{\text{eff}}$ : four effective mixing terms

- Overall coupling:  $\sqrt{2} G_F M_Z^2$  ( $G_F$  and  $\rho_{\text{eq}}$  must go together)

# SM context for ROKANC

- On-shell renormalization scheme is used
  - Particle masses are on shell:  $\sin^2\theta_W = 1 - M_W^2/M_Z^2$  to all orders
  - $M_Z$  is well measured:  $\sin^2\theta_W$  interpretation equivalent to  $M_W$  inference
- ZFITTER input parameters
  - $G_F$ ,  $\alpha_{em}$  ( $e^2/4\pi$ ), light particle masses
  - $M_Z = 91.1875 \pm 0.0021$  GeV,  $m_t = 173.2 \pm 0.9$  GeV
  - $\Delta\alpha_{em}^{(5)}(M_Z^2)[DALH5] = 0.0275 \pm 0.0001$ ,  $\alpha_s(M_Z^2) = 0.118$
- Fix  $m_H = 125$  GeV
  - Otherwise Tevatron  $A_{fb}$  fits give strange values for  $\sin^2\theta_W/m_H$
  - But now  $\sin^2\theta_W$  is not independent
  - To allow  $\sin^2\theta_W$  to vary independently, remove  $G_F$  constraint
- $\sin^2\theta_W$  as input parameter
  - $G_F$  calculated for the input  $\sin^2\theta_W$  ( $M_W$ ) with DIZET (IMOMS=3)
  - DIZET modified to use recalculated  $G_F$  in form factor calculations
  - This yields a family of physics models with SM-like couplings

# Example of s and t dependence of $\text{Re}(\kappa_e)$



$\theta$ :  $\angle$   $e^-$  and quark direction  
 Each curve: different  $M_{ee}$  (66-116 GeV)  
 u-type quark: black curves  
 d-type quark: blue curves

- $\kappa_e$ : lepton-Z vertex form factor for  $\sin^2\theta_w$
- W/Z box diagrams give small t dependence to form factors
- Form factor (FF) shape at  $s=M_Z^2$ :
  - All  $\kappa, \rho$  FF independent of  $\cos\theta$
  - $\kappa_e(\text{u-type}) = \kappa_e(\text{d-type})$
 one leptonic  $\sin^2\theta_{\text{eff}}$  at Z pole  
 $\sin^2\theta_{\text{eff}}^{\text{lept}} \equiv \kappa_e(M_Z^2) \sin^2\theta_w$   
 $\Rightarrow$  characterizes all FF's
- Implementation approximation
  - Average out t dependence with simple  $\gamma^*/Z$  angular model



- $\rho$  and  $\kappa$  form factor numbers

- $\sin^2\theta_w$  dependence: very small
- Quark type dependence: small
- $s = M_{ll}^2$  dependence: several percent
- Notation

- $\kappa_e$ : form factor to  $\sin^2\theta_w$  at lepton vertex
- $\kappa_q$ : form factor to  $\sin^2\theta_w$  at quark vertex

- Example values for  $\sin^2\theta_w = 0.22332$  and  $M_{ll} = M_Z$

	u-type quark	d-type quark	
$\rho$ :	(1.0054, -0.0042i)	(1.0059, -0.0034i)	
$\kappa_e$ :	(1.0369, 0.0135i)	(1.0369, 0.0135i)	← equal at $M_Z$
$\kappa_q$ :	(1.0364, 0.0128i)	(1.0358, 0.0120i)	← different from $\kappa_e$

- LEP1/SLD SM predictions [Phys. Rep. 427, 257 (2006): Eq. 8.5]

- $\sin^2\theta_w = 0.22331 \pm 0.00062$
- $\rho_l = 1.0051 \pm 0.0007$
- $\kappa_l = 1.0366 \pm 0.0025$



# Drell-Yan QCD calculations

- Standard fermion (f) couplings with  $\rho$  and  $\kappa$  form factors

$$- g_V^f = \sqrt{\rho_f} T_3^f (1 - 4|Q_f| \kappa_f \sin^2 \theta_W)$$

$$- g_A^f = \sqrt{\rho_f} T_3^f$$

- $\sin^4 \theta_W$  term of amplitude needs a small correction

$$- g_A/g_V \text{ amplitude: } \kappa_e \kappa_q$$

$$A_q \text{ amplitude: } \kappa_{eq}$$

$$- \text{Extra correction for } g_A/g_V \text{ amplitude: } \kappa_{eq} - \kappa_e \kappa_q$$

$$- \text{Fairly small: } |\kappa_{eq} - \kappa_e \kappa_q| \sim 0.01$$

- Photon propagator QED form factor for fermion-loop corrections utilized

$$- s \rightarrow s [ 1 - \Delta\alpha_{em}(s) ] \quad (\Delta\alpha_{em}(s) \rightarrow 0 \text{ as } s \rightarrow 0)$$



# W/Z box and FF extraction effects

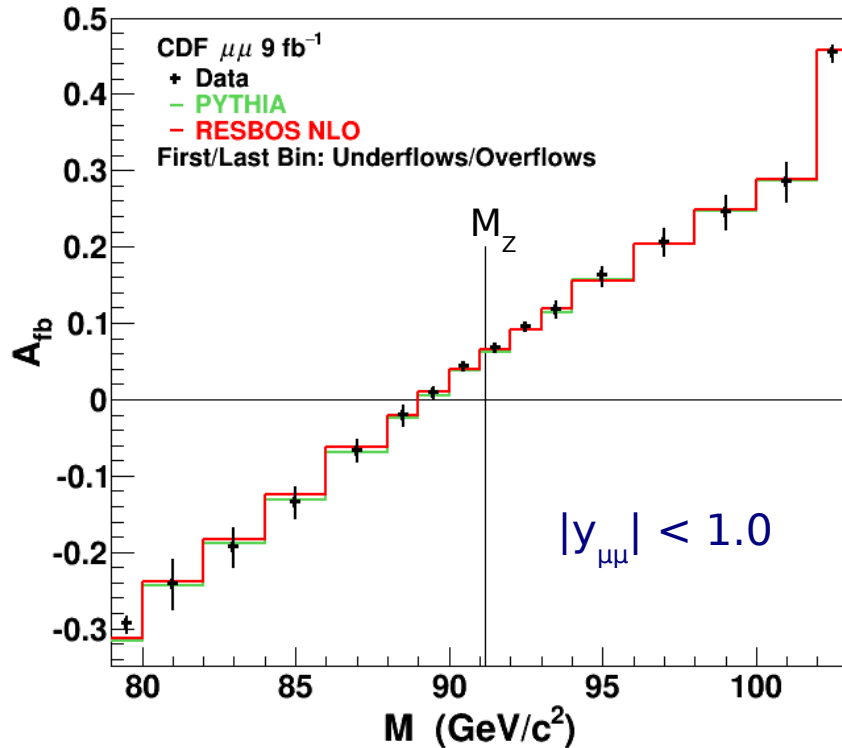
- Study W/Z box effect on  $A_4$  angular distribution coefficient
  - Drell-Yan angular distribution  $\propto 1 + \cos^2\theta + \dots + A_4 \cos\theta + \dots$   
(  $A_{fb} = 3/8 A_4$  )
  - Calculations of  $A_4$  over the region  $66 < M_{ll} < 116$  GeV:  $\langle A_4 \rangle$ 
    - ZGRad2 calculates box diagrams  
 $\langle A_4 \rangle$  with – without box diagrams  $\approx -0.0001$
    - Powheg-Box/Z with form factor implementation  
 $\langle A_4 \rangle$  with – without box diagrams  $\approx -0.0001$
  - Estimated effect of  $-0.0001$  shift of  $\langle A_4 \rangle$  on  $\sin^2\theta_W$ 
    - $\sin^2\theta_W \approx 0.2235 - 0.11 \cdot [ \langle A_4 \rangle - 0.12 ]$
    - $\sin^2\theta_W$  shift:  $\sim 0.00001$
- Relative effect of form factors on extraction of  $\sin^2\theta_{eff}^{lept}$ 
  - Extract  $\sin^2\theta_{eff}^{lept}$  using CDF ee and  $\mu\mu A_{fb}$  measurements
  - Relative shift:  $\sin^2\theta_{eff}^{lept}(\rho, \kappa) - \sin^2\theta_{eff}^{lept}(\rho=\kappa=1) = 0.00022 \pm 0.00004$ 
    - ↓
    - ↳ “one-size-fits-all” extraction
    - extract with CDF implementation of form factors
  - 11 MeV in terms of  $M_W$



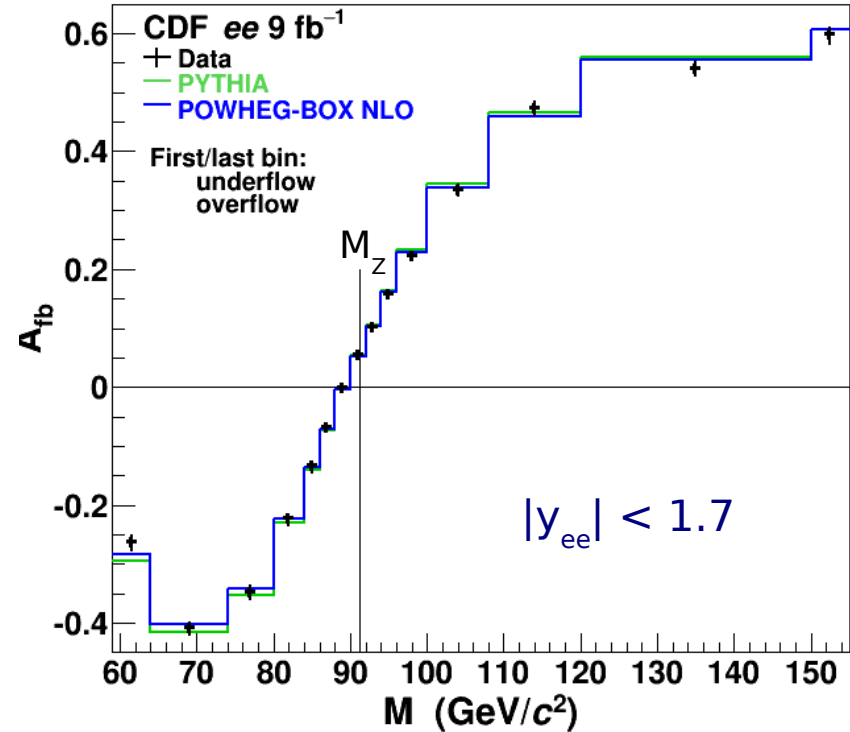


# $A_{fb}$ measurements

PRD 89, 072005 (2014)



PRD 93, 112016 (2016)



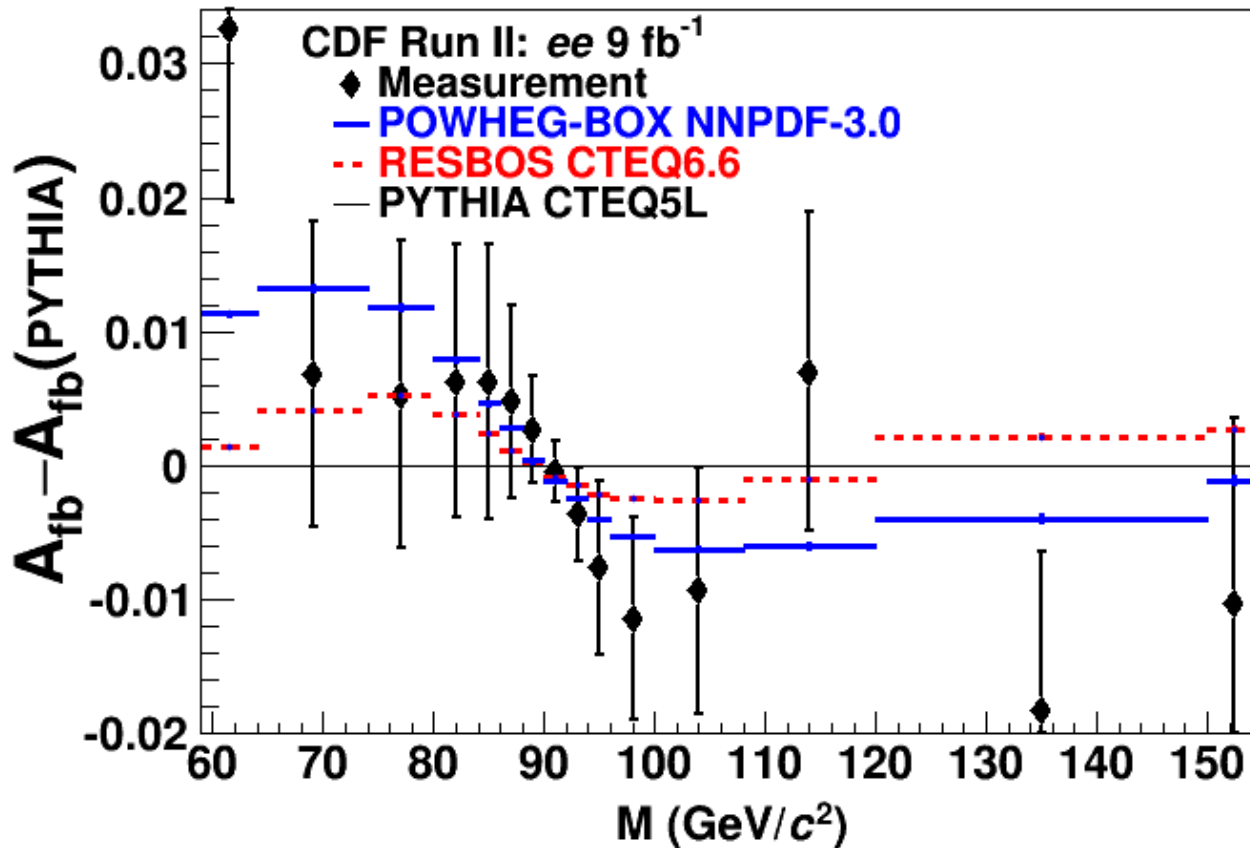
Measurements are fully corrected

RESBOS and POWHEG-BOX calculations include form factors  
PYTHIA is CDF standard



## PDF variations relative to CTEQ5L

- $A_{fb}$  offsets relative to PYTHIA with an older PDF
  - Calculations are best-fit
  - $\sin^2\theta_{eff}^{lept}$ : **PB** = 0.23248; **RB** = 0.23249; **PY\*** = 0.23207





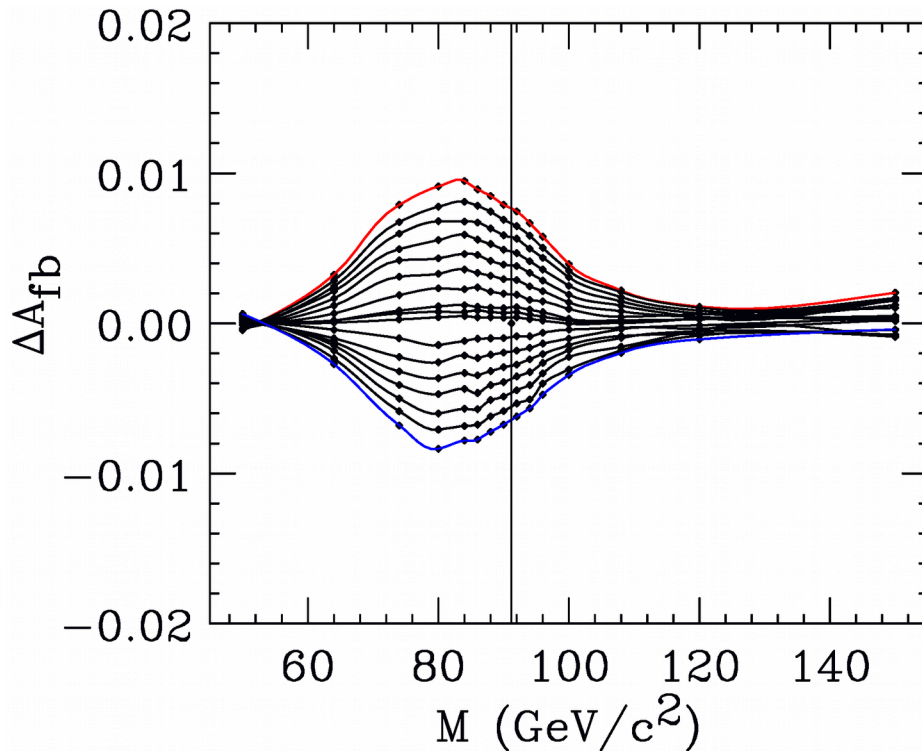
# $\sin^2\theta_W$ and PDF dependence study

$$\Delta A_{fb} = A_{fb}(\text{Test}) - A_{fb}(\text{Reference})$$

Ref: POWHEG-BOX/Z + NNPDF3 261000

$\sin^2\theta_W = 0.2244$

Test  $\sin^2\theta_W$ : 0.2220 to 0.2262

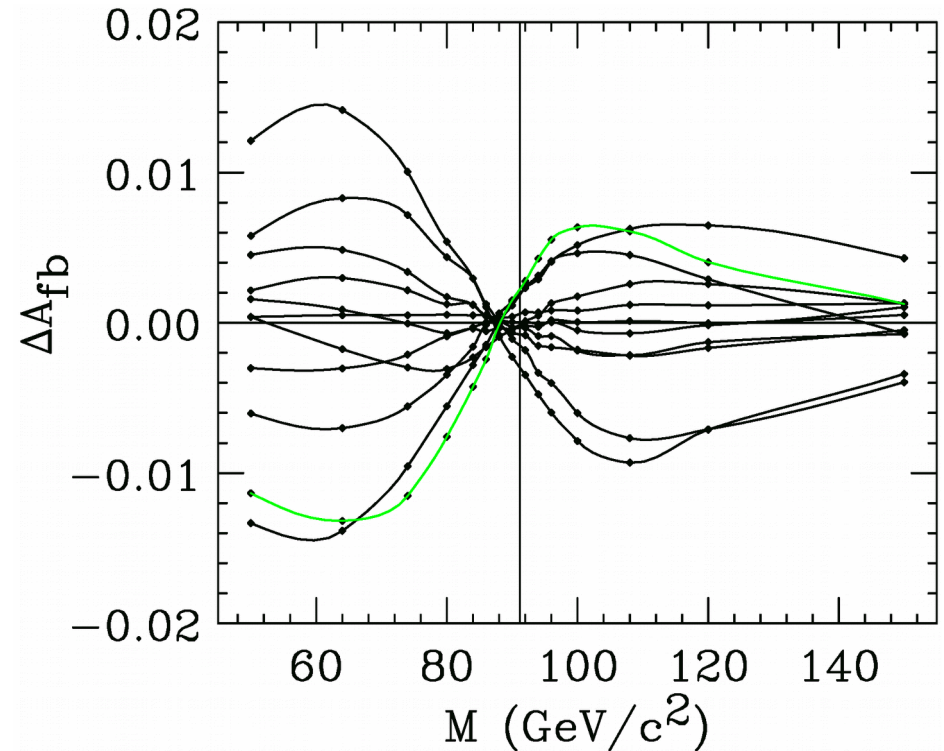


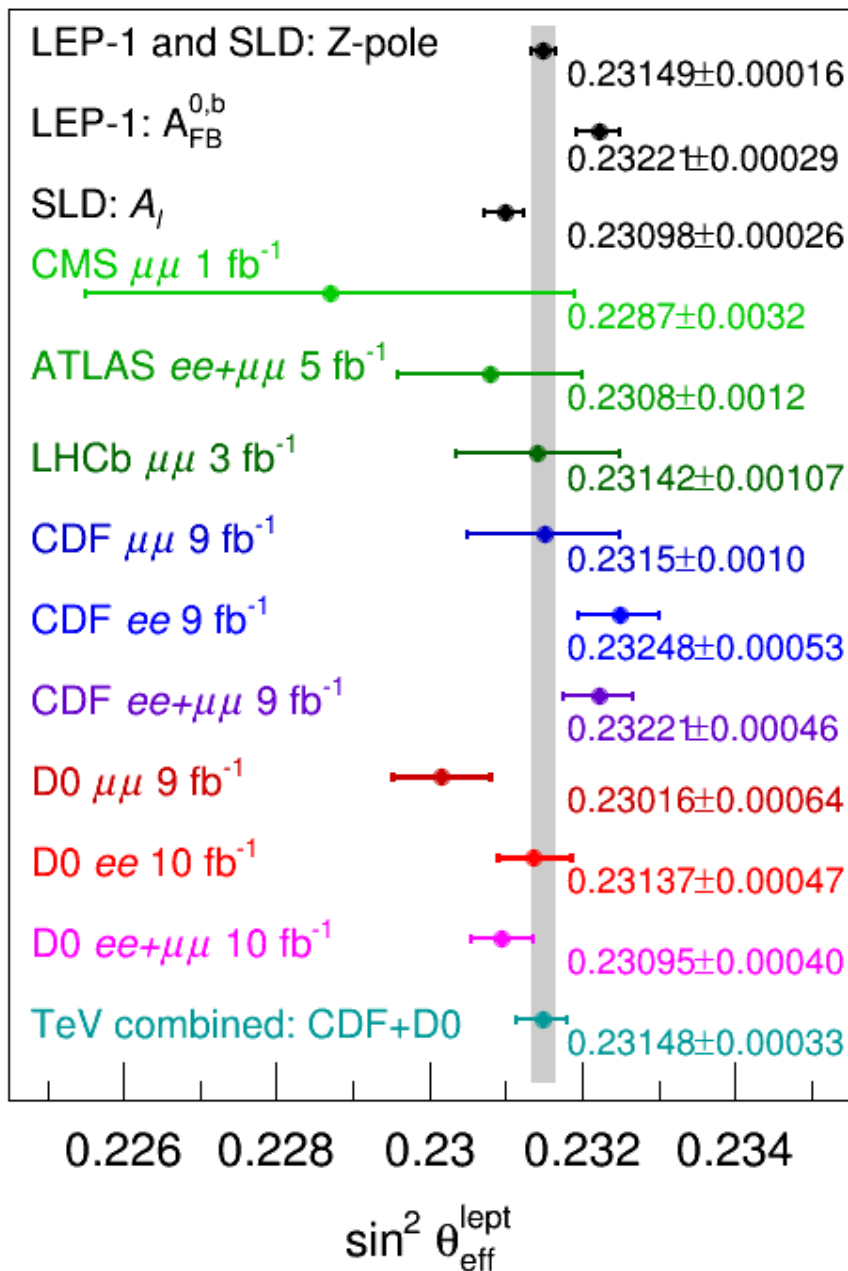
Ref: POWHEG-BOX/Z + NNPDF3 261000

$\sin^2\theta_W = 0.2244$

Test: various NNPDFs with  $\sin^2\theta_W = 0.2244$ ;

PYTHIA+CTEQ5L





Phys. Rep. 427, 257 (2006)

Phys. Rep. 532, 119 (2013)

Phys. Rev. D84, 11202 (2011)

J. High Energy Phys. 09 (2015) 049

J. High Energy Phys. 11 (2015) 190

Phys. Rev. D89, 072005 (2014)

Phys. Rev. D93, 112016 (2016)

Accepted by PRD:  
arXiv:1801.06283 [hep-ex]



# $\sin^2\theta_w / M_w$ interpretations



- ZFITTER with SM context of slide 4 provides the interpretation

$$- \sin^2\theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_e(M_Z^2, \sin^2\theta_w)] \sin^2\theta_w$$

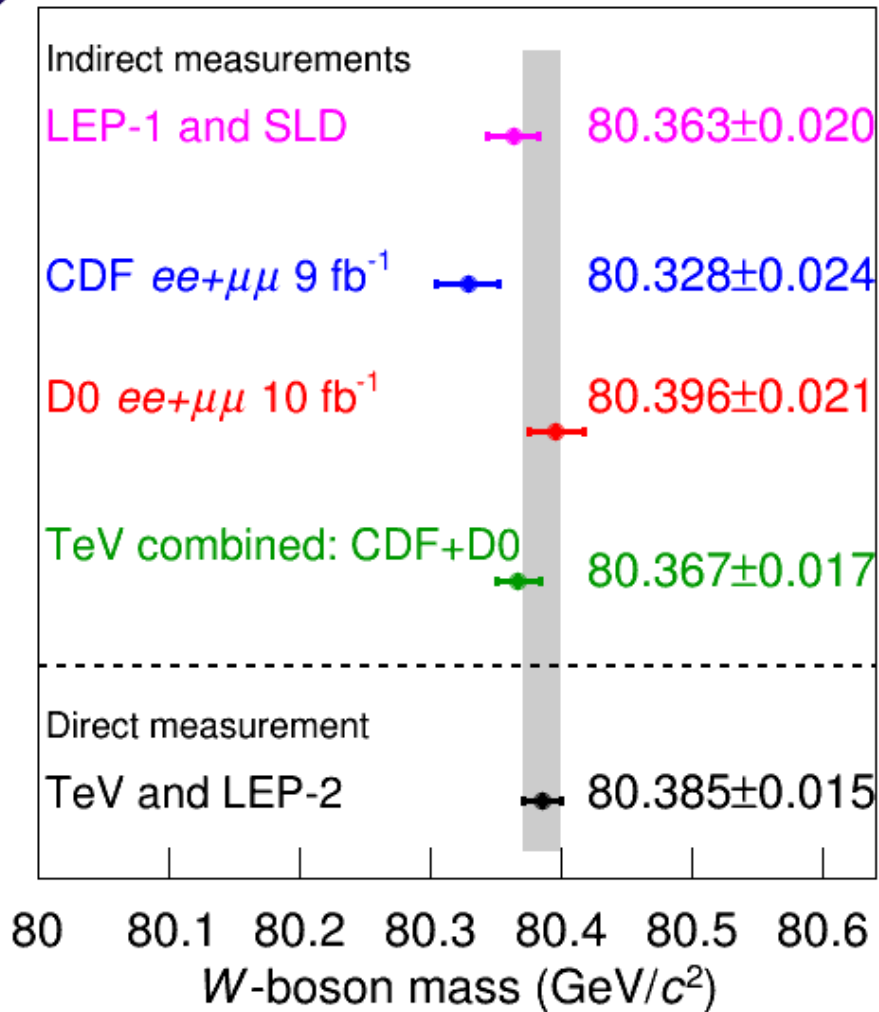
$$\hookrightarrow \approx 1.037$$

– Input uncertainty considered

- Form factors most sensitive to top-quark mass  $173.2 \pm 0.9 \text{ GeV}/c^2$
- $m_t$  uncertainty to  $\sin^2\theta_w$ : 0.00008

- Tevatron Legacy inferences

	$\sin^2\theta_w$	$M_w$
– CDF only:	$0.22400 \pm 0.00041 \pm 0.00019$	$80.328 \pm 0.021 \pm 0.010 \text{ GeV}/c^2$
– D0 only:	$0.22269 \pm 0.00034 \pm 0.00021$	$80.396 \pm 0.017 \pm 0.011 \text{ GeV}/c^2$
– Combination:	$0.22324 \pm 0.00026 \pm 0.00019$	$80.367 \pm 0.014 \pm 0.010 \text{ GeV}/c^2$
	(stat) (syst)	(stat) (syst)



Phys. Rep. 427, 257 (2006)  
Phys. Rep. 532, 119 (2013)

Phys. Rev. D89, 072005 (2014)  
Phys. Rev. D93, 112016 (2016)

Phys. Rev. D88, 05218 (2013)



# Summary



- CDF timeline
  - 2011:  $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.2329 \pm 0.0008 \pm 0.0010$ ;  $\langle A_4 \rangle$ ,  $2 \text{ fb}^{-1} \text{ ee}$
  - 2012: Add ZFITTER technology to CDF QCD Drell-Yan calculations
    - Goal: Improve  $\sin^2\theta_{\text{eff}}^{\text{lept}}$  extraction model relative to one-size-fits-all
    - POWHEG-BOX/Z, RESBOS and QCD tree level calculations
  - 2013:  $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.2328 \pm 0.0009 \pm 0.0003$ ;  $\langle A_4 \rangle$ ,  $2 \text{ fb}^{-1} \text{ ee}$
  - 2014:  $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.2315 \pm 0.0009 \pm 0.0004$ ;  $\mu\mu$ ,  $9 \text{ fb}^{-1}$
  - 2016:  $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23221 \pm 0.00043 \pm 0.00018$ ;  $\mu\mu + \text{ee}$ ,  $9 \text{ fb}^{-1}$
- Tevatron Legacy combination: 2018
  - $\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23148 \pm 0.00027(\text{stat}) \pm 0.00019(\text{syst})$
  - $M_W$  interpretation:  $80.367 \pm 0.014(\text{stat}) \pm 0.010(\text{syst})$
  - Calls for improving the extraction model and understanding
- ZFITTER: may be nice if development continued ... (?)
  - 6.43 precision region:  $50 < s < 150 \text{ GeV}$  ... Problem?