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1 Introduction

2 The forward–backward asymmetry

2.1 Conventions and leading-order predictions

The forward–backward asymmetry A_{FB} in di-lepton production at the LHC is defined as follows,

$$A_{\mathrm{FB}} = \frac{\mathrm{d}\sigma_{\mathrm{F}} - \mathrm{d}\sigma_{\mathrm{B}}}{\mathrm{d}\sigma_{\mathrm{F}} + \mathrm{d}\sigma_{\mathrm{B}}}. \tag{1}$$

Here $\mathrm{d}\sigma_{\mathrm{F}}(\mathrm{d}\sigma_{\mathrm{B}})$ denotes the cross section for forward (backward) events defined by $\cos\theta > 0 (< 0)$, where

$$\cos\theta = \frac{|p^3_{\ell^+\ell^-}|}{p^3_{\ell^+\ell^-}} \frac{2}{M_{\ell^+\ell^-}\sqrt{M^2_{\ell^+\ell^-} + p^2_{\mathrm{T},\ell^+\ell^-}}} \left(p^+_{\ell^-} p^-_{\ell^+} - p^-_{\ell^-} p^+_{\ell^+} \right) \tag{2}$$

is the cosine of the scattering angle in the Collins–Soper frame, with

$$p^{\pm} = \frac{1}{\sqrt{2}} \left(p^0 \pm p^3 \right), \tag{3}$$

where p^0 is the energy, p^3 the longitudinal component, and p_{T} the modulus of the transverse component of a four-momentum vector p^μ. The momentum of the $\ell^+\ell^-$ system is denoted $p^\mu_{\ell^+\ell^-} = p^\mu_{\ell^+} + p^\mu_{\ell^-}$, and $M^2_{\ell^+\ell^-} = p^2_{\ell^+\ell^-}$ is its squared invariant mass.

The parton-level differential cross section to charged-lepton-pair production via photon and Z-boson exchange in quark–antiquark annihilation ($\ell = e, \mu$)

$$q(p_q) + \bar{q}(p_{\bar{q}}) \to \gamma, \mathrm{Z} \to \ell^+(p_{\ell^+}) + \ell^-(p_{\ell^-})$$

can be written at LO as follows,

$$\mathrm{d}\hat{\sigma}^{(0)}_{q\bar{q}} = \mathrm{d}P_{2f} \, \frac{1}{12} \sum_{\mathrm{pol}} |\mathcal{M}^0_\gamma + \mathcal{M}^0_Z|^2(\hat{s}, \hat{t}) \;, \tag{4}$$

where $\sum_{\text{pol}}$ denotes the summation over the spin degrees of freedom of the initial and final state fermions and $\mathrm{d}P_{2f}$ is the two-particle phase-space element. The factor 1/12 results from averaging over the quark (spin and color) degrees of freedom. The matrix elements $\mathcal{M}_\gamma^0$ and $\mathcal{M}_Z^0$ describe the photon and Z-boson exchange processes, respectively, at lowest order in perturbation theory. In terms of the kinematical variables of the partonic system

$$\hat{s} = (p_q + p_{\bar{q}})^2, \quad \hat{t} = (p_q - p_{\ell^+})^2, \quad \hat{u} = (p_q - p_{\ell^-})^2, \tag{5}$$

the various contributions to the Born matrix elements squared for massless external fermions read

$$\begin{aligned}
\sum_{\text{pol}} |\mathcal{M}_\gamma^0|^2 &= 8\,(4\pi\alpha)^2\, Q_q^2\, Q_\ell^2\, \frac{(\hat{t}^2 + \hat{u}^2)}{\hat{s}^2}, \\
\sum_{\text{pol}} |\mathcal{M}_Z^0|^2 &= 8\,(4\pi\alpha)^2 \left[(v_q^2 + a_q^2)(v_\ell^2 + a_\ell^2)(\hat{t}^2 + \hat{u}^2) - 4 v_q a_q v_\ell a_\ell\,(\hat{t}^2 - \hat{u}^2)\right] \frac{|\chi(\hat{s})|^2}{\hat{s}^2}, \\
\sum_{\text{pol}} 2\mathrm{Re}(\mathcal{M}_Z^0 \mathcal{M}_\gamma^{0*}) &= 16\,(4\pi\alpha)^2\, Q_q\, Q_\ell\, a_q\, a_\ell \left[v_q v_\ell(\hat{t}^2 + \hat{u}^2) - a_q a_\ell(\hat{t}^2 - \hat{u}^2)\right] \frac{\mathrm{Re}[\chi(\hat{s})]}{\hat{s}^2},
\end{aligned} \tag{6}$$

with v_f and a_f parametrizing the $Zf\bar{f}$ $(f = \ell, q)$ couplings,

$$v_f = \frac{1}{2 s_\mathrm{W} c_\mathrm{W}}(I_f^3 - 2 s_\mathrm{W}^2 Q_f), \quad a_f = \frac{I_f^3}{2 s_\mathrm{W} c_\mathrm{W}}. \tag{7}$$

Here Q_f and $I_f^3 = \pm 1/2$ denote the charge and third component of the isospin quantum numbers of the fermion, respectively, and $s_\mathrm{W} \equiv \sin\theta_\mathrm{W}, c_\mathrm{W} \equiv \cos\theta_\mathrm{W}$ with θ_W being the weak mixing angle. At LO, the electromagnetic coupling α can be set to any appropriate value, such as the fine-structure constant $\alpha(0)$, the running coupling $\alpha(M_\mathrm{Z})$ at the Z pole, or the value α_{G_μ} derived from the Fermi constant G_μ ("G_μ scheme").

The Z resonance can be either parameterized by an s-dependent or a constant width (see Refs. [1, 2] for a discussion). We suggest to use the constant width approach,

$$\chi(\hat{s}) = \frac{\hat{s}}{\hat{s} - M_\mathrm{Z}^2 + \mathrm{i} M_\mathrm{Z} \Gamma_\mathrm{Z}}\,, \tag{8}$$

which identifies the complex mass squared $\mu_\mathrm{Z}^2 = M_\mathrm{Z}^2 - \mathrm{i} M_\mathrm{Z}\Gamma_\mathrm{Z}$ with the (gauge-invariant) location of the pole in the Z propagator. Note, however, that these "pole definitions" of mass and width of the W/Z bosons, $M_{\mathrm{W/Z}}$ and $\Gamma_{\mathrm{W/Z}}$, differ from the "on-shell (OS) definitions", as typically quoted by the LEP, Tevatron, and LHC collaborations, as follows

$$M_V = M_V^{\mathrm{OS}} / \sqrt{1 + (\Gamma_V^{\mathrm{OS}}/M_V^{\mathrm{OS}})^2}, \qquad \Gamma_V = \Gamma_V^{\mathrm{OS}} / \sqrt{1 + (\Gamma_V^{\mathrm{OS}}/M_V^{\mathrm{OS}})^2}, \qquad V = \mathrm{W}, \mathrm{Z}. \tag{9}$$

This difference formally matters at the two-loop level, but is numerically relevant in precision analyses, as M_V^{OS} differs from M_V by about 28 MeV and 34 MeV for W and Z bosons, respectively. The OS quantities M_V^{OS} and Γ_V^{OS} naturally appear in the parametrization of the V propagator by a running width.

2.2 Corrections at NLO and beyond

The NLO cross section receives QCD corrections of $\mathcal{O}(\alpha_s)$ and electroweak corrections of $\mathcal{O}(\alpha)$. The electroweak $\mathcal{O}(\alpha)$ corrections to neutral-current Drell–Yan processes naturally further decompose into QED and weak contributions, which individually build gauge-invariant subsets, and thus can be discussed separately. The observable pp cross section at NLO (with incoming proton momenta $P_{1,2}$) is obtained by convoluting the partonic cross section with the parton distribution functions $f_i(x_k, \mu_{\mathrm{F}}^2)$ ($p_1 = x_1 P_1$, $p_2 = x_2 P_2$),

$$
\begin{aligned}
\mathrm{d}\sigma^{\mathrm{NLO}}(P_1, P_2) &= \int_0^1 dx_1 dx_2 \sum_{i,j} f_i(x_1, \mu_{\mathrm{F}}^2)\, f_j(x_2, \mu_{\mathrm{F}}^2) \\
&\quad \times \Big[\mathrm{d}\hat{\sigma}_{ij}^{(0)}(p_1, p_2)\,(1 + \delta_{ij,\mathrm{weak}}) + \mathrm{d}\hat{\sigma}_{ij,\mathrm{QED}}^{(1)}(p_1, p_2, \mu_{\mathrm{F}}^2) + \mathrm{d}\hat{\sigma}_{ij,\mathrm{QCD}}^{(1)}(p_1, p_2, \mu_{\mathrm{F}}^2)\Big], \quad (10)
\end{aligned}
$$

where the sum $\sum_{i,j}$ runs over all relevant parton configurations in the initial state and μ_{F} is the factorization scale, which is conveniently set to M_{Z} fopr physics near the Z pole. Here $\mathrm{d}\hat{\sigma}^{(0)}$ comprises the contribution of the ij parton configuration to the LO cross section, and $\delta_{ij,\mathrm{weak}}$ is the corresponding weak correction at NLO, which has the same kinematical dependence as the LO part. The NLO cross-section contributions induced by QED and QCD corrections are denoted as $\mathrm{d}\hat{\sigma}_{ij,\mathrm{QED}}^{(1)}$ and $\mathrm{d}\hat{\sigma}_{ij,\mathrm{QCD}}^{(1)}$, respectively, including both virtual and real emission contributions. Note that those parts explicitly depend on the factorization scale because of the PDF redefinition to absorb collinear initial-state singularities.

In detail, QED corrections comprise all contributions from photon exchange in loops between charged fermions or real photon emission off charged fermions, but not closed fermion loops in the photon propagator or loops with photons coupling to W bosons. Note that, in principle, there is a LO contribution from $\gamma\gamma$ collision as well, which should be counted as QED-like, but near the Z pole this part is negligibly small.

At LO, the above parametrization of the amplitude does not suffer from any problems with gauge invariance, but this statement deserves careful arguments. Gauge invariance dictates that the weak mixing angle θ_{w} and the gauge-boson masses M_{Z} and M_{W} are not independent parameters; in strict fixed-order calculations the relation $c_{\mathrm{w}} = M_{\mathrm{W}}/M_{\mathrm{Z}}$ is enforced. The above parametrization of the matrix element, however, goes beyond a pure lowest-order calculation, because the finite-width term in the propagator $\chi(\hat{s})$ results from a partial Dyson summation. This does not pose a problem with gauge invariance, since the W-boson mass M_{W} does not enter the LO prediction explicitly and the Z-boson mass only enters in the combination $\mu_{\mathrm{Z}}^2 = M_{\mathrm{Z}}^2 - \mathrm{i}M_{\mathrm{Z}}\Gamma_{\mathrm{Z}}$. The above parametrization, thus, effectively uses c_{w} and μ_{Z} as free parameters with a dependent W-boson mass $\mu_{\mathrm{W}} = c_{\mathrm{w}}\mu_{\mathrm{Z}}$, which does not enter the calculation directly.

The maintenance of gauge invariance becomes non-trivial after including electroweak corrections. Possible solutions are provided by the "complex-mass scheme" [3], the "pole scheme" [4], or by truncated expansions about resonances poles, where the validity of the latter "leading pole approximation" is restricted to window of the size of some Γ_V around the resonance at M_V. More details on the application of these schemes and explicit results can be found in Refs. [7, 8].

Beyond NLO, QCD corrections are known to fixed NNLO and beyond in terms of resummations of leading contributions [**?**]. Photonic (QED) and weak corrections to the full Drell–Yan process are only known to NLO [6, 7, **?**]. However, the dominant QED contributions, which are due to multi-photon final-state radiation, can be included in higher orders via parton showers [**?**] or QED structure functions [7, **?**]. Weak corrections beyond NLO are only known directly on resonance or in the high-energy region [**?**], where the latter is not relevant for Z-boson physics at the Z pole. Mixed QCD–EW corrections were worked out in the resonance region as well [8, 10], revealing that the dominant contributions can be accounted for by QCD precictions dressed by QED corrections based on parton showers or QED structure functions. In summary, a state-of-the-art cross-section contribution can be schematically written as

$$\sigma(P_1,P_2) = \int_0^1 dx_1 dx_2 \sum_{i,j} f_i(x_1,\mu_{\mathrm{F}}^2)\, f_j(x_2,\mu_{\mathrm{F}}^2) \int\!\!\int R_{\mathrm{QED}} \otimes R_{\mathrm{QCD}} \otimes \mathrm{d}\hat{\sigma}_{ij}^{(0)}(p_1,p_2)\,(1+\delta_{ij,\mathrm{weak}}), \quad (11)$$

where the convolution with the QED and QCD correction factors R_{QED} and R_{QCD} is indicated only in a sketchy way. In detail, those convolutions carefully have to retain full NLO accuracy while catching higher-order effects without double-counting or the creation of artifacts.

2.3 Weak corrections, effective couplings, and improved Born approximation

The NLO differential partonic cross section including weak $\mathcal{O}(\alpha)$ corrections is of the following form [5, 6],

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{(1)}(p_1,p_2) &= \delta_{ij,\mathrm{weak}}\, \mathrm{d}\hat{\sigma}_{ij}^{(0)}(p_1,p_2) \\ &= \mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{self}}(\hat{s},\hat{t}) + \mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{vert}}(\hat{s},\hat{t}) + \mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{box}}(\hat{s},\hat{t}). \end{aligned} \quad (12)$$

The self-energy contributions $\mathrm{d}\hat{\sigma}_{\mathrm{weak}}^{ij,\mathrm{self}}$ are induced by the transverse parts of the $\gamma\gamma$, γZ, and ZZ self-energies in the s-channel. Note that in the "complete on-shell renormalization scheme", as formulated in [9] (or in its complex version [3]), the γZ and ZZ self-energies are renormalized in such a way that no resonant contribution to the one-loop corrected amplitude remains at the Z pole. Since the $\gamma\gamma$ self-energy contribution is not resonant at $\hat{s} = M_{\mathrm{Z}}^2$, there remains no resonant self-energy contribution at the Z pole in the on-shell renormalization scheme. Likewise, the contribution $\mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{box}}$, which comprises box diagrams with internal WW or ZZ pairs, is non-resonant.

This leaves the contribution $\mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{vert}}$ of the vertex corrections as the only source for weak corrections that are not suppressed on resonance. More precisely, only vertex corrections to the $\mathrm{Z}f\bar{f}$ vertices lead to resonant contributions, while $\gamma f\bar{f}$ vertex corrections remain non-resonant. For on-shell external fermions and Z bosons of virtuality q^2 the weak corrections to the $\mathrm{Z}f\bar{f}$ vertices can be described by (renormalized) formfactors $\hat{F}_{\mathrm{Z}ff,\mathrm{weak}}^{\sigma}(q^2)$ which effectively correct the vector and axial-vector couplings, v_f and a_f, introduced above. Note, however, that these formfactors are only gauge invariant for on-shell Z bosons, i.e. for $q^2 = M_{\mathrm{Z}}^2$. On the Z pole, the $\mathrm{Z}f\bar{f}$ vertex correction to the amplitude can, thus, be written as

$$\mathcal{M}_{ij,\mathrm{weak}}^{\mathrm{vert}} = \mathcal{M}_Z^0\Big|_{v_q\to\bar{g}_{\mathrm{V},q},\, a_q\to\bar{g}_{\mathrm{A},q}} + \mathcal{M}_Z^0\Big|_{v_\ell\to\bar{g}_{\mathrm{V},\ell},\, a_\ell\to\bar{g}_{\mathrm{A},\ell}} \quad (13)$$

with the corrected ("effective") vector and axial-vector couplings

$$\begin{aligned} \bar{g}_{\mathrm{V},f} &= v_f\left(1+\hat{F}_{\mathrm{Z}ff,\mathrm{weak}}^{\mathrm{V}}(M_{\mathrm{Z}}^2)\right), \\ \bar{g}_{\mathrm{A},f} &= a_f\left(1+\hat{F}_{\mathrm{Z}ff,\mathrm{weak}}^{\mathrm{A}}(M_{\mathrm{Z}}^2)\right). \end{aligned} \quad (14)$$

Explicit results on the formfactors can, e.g., be found in Refs. [7, ?].[1] At NLO, the weak corrections near the Z resonance can, thus, be included by the following modification of the LO cross section,

$$\mathrm{d}\hat{\sigma}_{ij}^{(0)} \;\to\; \mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{IBA}} \equiv \mathrm{d}\hat{\sigma}_{ij}^{(0)}\Big|_{v_f\to\bar{g}_{\mathrm{V},f},\, a_f\to\bar{g}_{\mathrm{A},f}}, \tag{15}$$

which means that all (quark and lepton) LO couplings v_f and a_f to the Z boson are replaced by the effective couplings $\bar{g}_{\mathrm{V},f}$ and $\bar{g}_{\mathrm{A},f}$, respectively. Note that the values of the effective couplings depend on the chosen input scheme for the electromagnetic coupling constant α. We recommend to take the "G_μ scheme" where $\alpha = \alpha_{G_\mu}$, which absorbs universal corrections from the running of α from $\alpha(0)$ to $\alpha(M_\mathrm{Z})$ as well as some leading corrections from the ρ parameter into the coupling factors (see, e.g., Ref. [7] for details).

In the IBA, the Z width is either considered to be an input parameter as well, set to the measured value, or calculated in terms of the effective couplings to include the NLO weak corrections and dressed by further QED and QCD corrections.

The IBA for the weak corrections can be dressed with QED and QCD corrections rather easily. To this end, it is only necessary to replace the LO cross section $\mathrm{d}\hat{\sigma}_{ij}^{(0)}$ in a combined QCD×QED prediction by $\mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{IBA}}$, which depends on the same kinematical variables as its LO counterpart. The schematic cross-section prediction (11), thus, turns into

$$\sigma^{\mathrm{IBA}}(P_1,P_2) = \int_0^1 dx_1 dx_2 \sum_{i,j} f_i(x_1,\mu_\mathrm{F}^2)\, f_j(x_2,\mu_\mathrm{F}^2) \int\!\!\int R_\mathrm{QED}\otimes R_\mathrm{QCD}\otimes \mathrm{d}\hat{\sigma}_{ij,\mathrm{weak}}^{\mathrm{IBA}}(p_1,p_2). \tag{16}$$

Note that this IBA description far away from the Z pole becomes insufficient for two reasons: The effective couplings are not static, but are functions of $\hat{s}$, and the non-resonant weak corrections (e.g. from photon exchange or box graphs) are no longer negligible, but increase strongly with the energy and hence contribute sizeably at high invariant masses of the lepton pair. Moreover, we recall that effective couplings simply based on off-shell formfactors would not be gauge invariant (and thus not useful in phenomenology). The validity of the IBA should, thus, be carefully validated, i.e. the dependence of its approximative quality on the size of the neighbourhood of the Z resonance should be carefully investigated.

In summary, the IBA can be characterized by employing the pole approximation for the weak corrections, while taking QCD and QED corrections with the full off-shell kinematics.

2.4 Effective weak mixing angle

The effective weak mixing angle for a generic fermion f, quantified by $\bar{s}^2_{\mathrm{eff},f}$, is related to the ratio of vector to axial-vector effective couplings as follows,

$$\bar{s}^2_{\mathrm{eff},f} = \frac{1}{4|Q_f|}\left(1 - \frac{\mathrm{Re}\bar{g}_{\mathrm{V},f}}{\mathrm{Re}\bar{g}_{\mathrm{A},f}}\right). \tag{17}$$

While the absolute size of the effective couplings depends rather sensitively on the fermion flavour f, the value of $\bar{s}^2_{\mathrm{eff},f}$ is quite robust against the change of the defining flavour. In fact, at NLO the

[1] In Ref. [7], the formfactors actually are given in the chirality basis as $F^{\pm}_{Zff,\mathrm{weak}}$, which translate into the $v - a\gamma_5$ basis according to $F^\mathrm{V} = [(v-a)F^+ + (v+a)F^-]/(2v)$ and $F^\mathrm{A} = [(v+a)F^- - (v-a)F^+]/(2a)$. Following the conventions of Ref. [9], the sign of s_w in Ref. [7] differs from the one of this work, but this difference drops out in $F^\pm$, which depends only on s^2_w.

difference in $\bar{s}^2_{\mathrm{eff},f}$ taken between different flavours $f \neq \mathrm{b},\mathrm{t}$, where the masses of f and its weak isospin partner are negligible, is due to one-loop diagrams with $\mathrm{Z}f\bar{f}$ or $\mathrm{W}f\bar{f}'$ couplings only, but not dependent on graphs with non-Abelian γWW/ZWW interactions or Higgs-boson exchange. For $f = \mathrm{b}$, there are additional contributions involving the top-quark mass m_t, which also arise from the vertex correction with ZWW interaction. Explicitly the difference to the leptonic effective weak mixing angle reads at NLO:

$$\begin{aligned}
\left[\bar{s}^2_{\mathrm{eff},f} - \bar{s}^2_{\mathrm{eff},\ell}\right]_{\mathrm{NLO}} &= \frac{\alpha s^2_\mathrm{W}|Q_{f'}|}{8\pi}\Bigg[11 - 4s^2_\mathrm{W} + 4(2-s^2_\mathrm{W})^2\left(\frac{\pi^2}{6} - \mathrm{Re}\left\{\mathrm{Li}_2\left(1+c_\mathrm{W}^{-2}\right)\right\}\right) \\
&\qquad + 4(5-2s^2_\mathrm{W})\ln(c_\mathrm{W})\Bigg] \\
&\quad + \frac{\alpha s^2_\mathrm{W}}{16\pi c^2_\mathrm{W}}\left[3|Q_{f'}| - 4(1-Q_f^2)s^2_\mathrm{W}\right]\left(11 - \frac{4\pi^2}{3}\right), \qquad f \neq \mathrm{b}, \qquad (18)\\
\left[\bar{s}^2_{\mathrm{eff},\mathrm{b}} - \bar{s}^2_{\mathrm{eff},\ell}\right]_{\mathrm{NLO}} &= \left[\bar{s}^2_{\mathrm{eff},\mathrm{d}} - \bar{s}^2_{\mathrm{eff},\ell}\right]_{\mathrm{NLO}} + (\text{some longer expression for } m_\mathrm{t}\text{-dependence}), \qquad (19)
\end{aligned}$$

where f' is the weak-isospin partner to fermion f, and $Q_{f,f'}$ the respective electric charges in units of the elementary charge e. For illustration we recite here some rough numbers on those differences:

	ν_ℓ	u	d	b
$(\bar{s}^2_{\mathrm{eff},f} - \bar{s}^2_{\mathrm{eff},\ell})/10^{-4}$	$-3.6...$	$-1.0...$	$-2.2...$	...

(20)

Note that LEP and SLC have measured $\bar{s}^2_{\mathrm{eff},\ell}$ to an accuracy of 2.9×10^{-4} and 2.6×10^{-4}, respectively, i.e. the flavour differences in $\bar{s}^2_{\mathrm{eff},\mathrm{b}}$ are of the typical order of existing measurements.

2.5 Issues in extracting $\bar{s}^2_{\mathrm{eff},\ell}$ from A_FB

- How well do the available higher-order calculations agree?

  Perform a tuned comparison of predictions for $A_{FB}(M_{ll})$ at NLO EW, NLO QCD, NLO+PS accuracy.

- How well does the IBA describe the weak corrections to $A_\mathrm{FB}(M_{\ell\ell})$ in the resonance region where $\bar{s}^2_{\mathrm{eff},\ell}$ is extracted?

- What is the actual fit procedure in a "template fit" of the IBA to data?

  - Take the leptonic $\bar{s}^2_{\mathrm{eff},\ell}$ as free parameter in the IBA, but the differences to $\bar{s}^2_{\mathrm{eff},f}$ with $f \neq \ell$ from theory?
  - How are the normalizations of the effective couplings $\bar{g}_{\mathrm{V},f}$ and $\bar{g}_{\mathrm{A},f}$, which depend on f, treated in the fit? Use three global normalization parameters for the partonic channels $q\bar{q} \to \ell^+\ell^-$ with $q = \mathrm{u},\mathrm{d},\mathrm{b}$?
  - How are imaginary parts in $\bar{g}_{\mathrm{V},f}$, $\bar{g}_{\mathrm{A},f}$ treated in the fit? Treat $\bar{g}_{\mathrm{V},f}$, $\bar{g}_{\mathrm{A},f}$ as complex quantities in the above formulas (some should be changed!).
  - Is the Z width Γ_Z, expressed in terms of $\bar{g}_{\mathrm{V},f}$, $\bar{g}_{\mathrm{A},f}$ and treated as a fit parameter, or is it taken from the LEP measurement? In the former case, this would have an impact on the previous question of normalization.

- Is the IBA introducing a systematic bias in fit results for $\bar{s}^2_{\mathrm{eff},f}$?

  Fit IBA to SM state-of-the-art predictions and compare fit result on $\bar{s}^2_{\mathrm{eff},f}$ with known SM prediction for given input. Extract systematic shifts in $\bar{s}^2_{\mathrm{eff},f}$ induced by the IBA and correct fit result by it (including some conservative error).

- How are closure tests of the EW SM done including higher-order corrections?

  Take predictions of A_{FB} at different levels of sophistication and perform template fits for different values of $\bar{s}^2_{\mathrm{eff},\ell}$ to determine the impact of higher-order corrections in terms of shifts in $\bar{s}^2_{\mathrm{eff},\ell}$.

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