COMBINED QED-QCD RESUMMATION FORMALISM

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arXiv:1805.11948 [hep-ph]

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LHC precision EW working group meeting

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Outline

- (Brief) Motivation and introduction
- 1- Combined QCD-QED resummation
- 2- Z production at *mixed NNLL QCD + NLL QED* accuracy
- Conclusions and outlook

Specific references:

1)- Collider effects: de Florian, Cieri, Ferrera and GS, **work in progress;** *partial results in PoS(EPS-HEP2017)398*

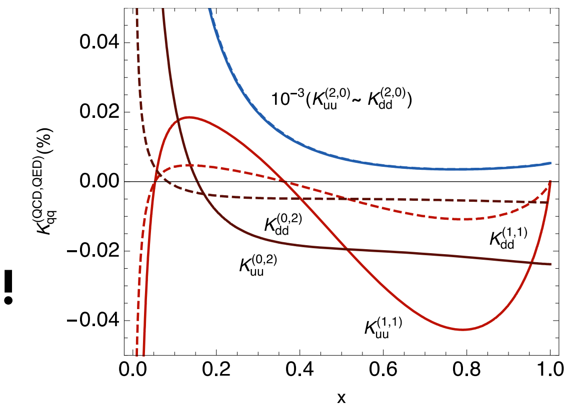
See next talk by Daniel!!!

2)- Mixed resummation: *Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]*

Motivation and introduction

Why we need QED corrections?

- Very precise experimental data is available!! We need to match the experimental precision!!
- **NNLO QCD** is the standard; but …
- **QED effects might compete with NNLO QCD (since $\alpha_S^2 \sim \alpha$) !!!**
- Inclusion of QED beyond LO leads to novel effects:
  - Quark-gluon interacting with leptons and photons
  - Charge separation
  - **Dependence on the photon content of the proton!**
  - Sizable contributions at high-energies (due to the running EM coupling)
  - QED radiation effects at low-energies (resummation needed)
- **Thus, QED corrections MUST be taken into account!**



Mixed corrections to AP kernels

Part 1: QCD-QED resummation

- Development of a formalism to deal with mixed QCD-QED computations, centering in the treatment of IR singularities
- Extension of the qt-subtraction/resummation framework to deal with simultaneous QCD-QED radiation

Mixed QCD-QED resummation

Introduction and qt-formalism

- **Case of study:** Drell-Yan (*W,Z,photon production at hadron colliders*)
- Soft photon radiation could provide non-negligible effects in the low q_T region → **Extend qt-resummation to deal with QCD-QED radiation!**
- Some formulae to introduce qt-resummation in QCD:
  - The singular (i.e. divergent) part has an universal structure:

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}\,S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\, f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

  - The **Sudakov factor** resums all the soft/collinear-emissions from the incoming legs; it is process independent
  - The **"hard-collinear"** coefficients **H** and **C** are related with the hard-virtual and collinear parts, and also contain the process dependence.

Catani et al, Nucl. Phys. B881 (2014) [arXiv:1311.1654]

Mixed QCD-QED resummation

- **More details about the resummation formula:**
  - The Sudakov factor contains the logarithmically enhanced contributions. It can be resumed to all orders within perturbation theory!

$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2)) \ \ln\frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$

$$A_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}$$

$$B_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)}$$

  - **A_c** and **B_c** depend on the leg responsible for the emission. *They are related to the splitting functions!*
  - Also, **C** and **H** are calculable within perturbation theory! **C** is process independent (**H** contains the virtuals, i.e. loops):

$$H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)}(x_1p_1, x_2p_2; \mathbf{\Omega}) \longrightarrow$$ Loop information (finite parts)

$$C_{q\,a}(z;\alpha_S) = \delta_{q\,a}\ \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{q\,a}^{(n)}(z) \longrightarrow$$ Radiation from incoming legs (transitions)

Catani et al, Nucl. Phys. B881 (2014) [arXiv:1311.1654]

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Path to QCD-QED resummation:**
- **Step I:** Transform all the QCD coefficients into the QED ones with the Abelianization algorithm (done!). Obtain QED resummation formula (done!).
  - *Subtlety I:* Charge separation effects due to up and down sectors.
  - *Subtlety II:* Photons and leptons must be included (closed loops), as well as the photon PDF ⇒ *Non trivial dependence!*

    SOLVED!
- **Step II:** Deal with QCD-QED radiation simultaneously. We need to Abelianizate all the coefficients, and perform the perturbative expansions with two couplings!
  - *Subtlety I:* Check of factorization formulae and its functional structure
  - *Subtlety II:* Compute *all* the coefficients, including the **mixed** ones!
  - *Subtlety III:* Applicable for **color-less neutral** final states…

Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - Originally, in the QCD formalism, the resumed component is given by

$$\frac{d\hat{\sigma}^{(\mathrm{res.})}_{a_1 a_2 \to F}}{dq_T^2}(q_T, M, \hat{s}; \mu_F) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(b\, q_T)\, \mathcal{W}^F_{a_1 a_2}(b, M, \hat{s}; \mu_F)$$

and we extend it by "exponentiating" photon/gluon radiation:

$$\mathcal{W}'^F_N(b, M; \mu_F) = \hat{\sigma}^{(0)}_F(M)\, \mathcal{H}'^F_N(\alpha_S, \alpha; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \times \exp\left\{\mathcal{G}'_N(\alpha_S, \alpha, L; M^2/\mu_R^2, M^2/Q^2)\right\}$$

Hard collinear part

Logarithmically-enhanced contributions

  - The hard-collinear part is expanded in a power series:

$$\mathcal{H}'^F_N(\alpha_S, \alpha) = \mathcal{H}^F_N(\alpha_S) + \frac{\alpha}{\pi}\mathcal{H}'^{F\,(1)}_N + \sum_{n=2}^{\infty}\left(\frac{\alpha}{\pi}\right)^n \mathcal{H}'^{F\,(n)}_N + \sum_{n,m=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \left(\frac{\alpha}{\pi}\right)^m \mathcal{H}'^{F\,(n,m)}_N$$

Pure QED part

Pure QCD

Mixed QCD-QED

Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - The Sudakov factor is also expanded:

$$\mathcal{G}'_N(\alpha_S,\alpha,L) = \mathcal{G}_N(\alpha_S,L) + L\, g'^{(1)}(\alpha L) + g'^{(2)}_N(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}_N(\alpha L)$$

$$+\, g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g'^{(n,m)}_N(\alpha_S L,\alpha L)$$

Pure QCD

Pure QED

(New) mixed QCD-QED!!

  - The *g-functions* for QED are:

$$\lambda = \tfrac{1}{\pi}\beta_0\,\alpha_S\,L$$

$$\lambda' = \tfrac{1}{\pi}\beta'_0\,\alpha\,L$$

Large log!!!

$$g'^{(1)}(\alpha L) = \frac{A'^{(1)}_q}{\beta'_0}\,\frac{\lambda' + \ln(1-\lambda')}{\lambda'}$$

$$g'^{(2)}_N(\alpha L) = \frac{\widetilde{B}'^{(1)}_{q,N}}{\beta'_0}\ln(1-\lambda') - \frac{A'^{(2)}_q}{\beta'^2_0}\left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda')\right)$$

$$+ \frac{A'^{(1)}_q\beta'_1}{\beta'^3_0}\left(\frac{1}{2}\ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'}\right)$$

Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - The new mixed first-order *g-function*:

$$\boxed{g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)}\beta_{0,1}}{\beta_0^2\beta_0'}\, h(\lambda,\lambda') + \frac{A_q'^{(1)}\beta_{0,1}'}{\beta_0'^2\beta_0}\, h(\lambda',\lambda)}$$

$$h(\lambda,\lambda') = -\frac{\lambda'}{\lambda-\lambda'}\ln(1-\lambda) + \ln(1-\lambda')\left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln\left(\frac{-\lambda'(1-\lambda)}{\lambda-\lambda'}\right)\right] - \mathrm{Li}_2\left(\frac{\lambda}{\lambda-\lambda'}\right) + \mathrm{Li}_2\left(\frac{\lambda(1-\lambda')}{\lambda-\lambda'}\right),$$

  - New **A, B** and **H** coefficients:

$$A_q'^{(1)} = e_q^2 \qquad A_q'^{(2)} = -\frac{5}{9}\, e_q^2\, N^{(2)}$$

$$\widetilde{B}_{q,N}'^{(1)} = B_q'^{(1)} + 2\gamma_{qq,N}'^{(1)}$$

$$B_q'^{(1)} = -\frac{3}{2}\, e_q^2$$

$$\gamma_{qq,N}'^{(1)} = e_q^2\left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1)\right)$$

$$\gamma_{q\gamma,N}'^{(1)} = \frac{3}{2}\, e_q^2\, \frac{N^2+N+2}{N(N+1)(N+2)}$$

$$\mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow q\bar{q},N} = \frac{e_q^2}{2}\left(\frac{2}{N(N+1)} - 8 + \pi^2\right)$$

$$\mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow \gamma q,N} = \mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow q\gamma,N} = \frac{3\,e_q^2}{(N+1)(N+2)}$$

$$\mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow \gamma\gamma,N} = \mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow qq,N} = \mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow \bar{q}\bar{q},N} = 0$$

Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Mixed QCD-QED coupling evolution

Mixed RGE equations

- **Coupled differential equations:** Crucial to recover non-trivial mixed terms in *g-functions*

$$\frac{d\ln\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S(\mu^2),\alpha(\mu^2)) = -\sum_{n=0}^{\infty}\beta_n\left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta_{n,m}\left(\frac{\alpha_S}{\pi}\right)^{n+1}\left(\frac{\alpha}{\pi}\right)^{m}$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2),\alpha_S(\mu^2)) = -\sum_{n=0}^{\infty}\beta'_n\left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta'_{n,m}\left(\frac{\alpha}{\pi}\right)^{n+1}\left(\frac{\alpha_S}{\pi}\right)^{m}$$

- **Mixed beta function coefficients:**

$$\beta_0 = \frac{1}{12}(11\,C_A - 2\,n_f)\,, \qquad \beta_{0,1} = -\frac{N_q^{(2)}}{8}\,,$$

$$\beta'_0 = -\frac{N^{(2)}}{3}\,, \qquad \beta'_1 = -\frac{N^{(4)}}{4}\,, \qquad \beta'_{0,1} = -\frac{C_F C_A N_q^{(2)}}{4},$$

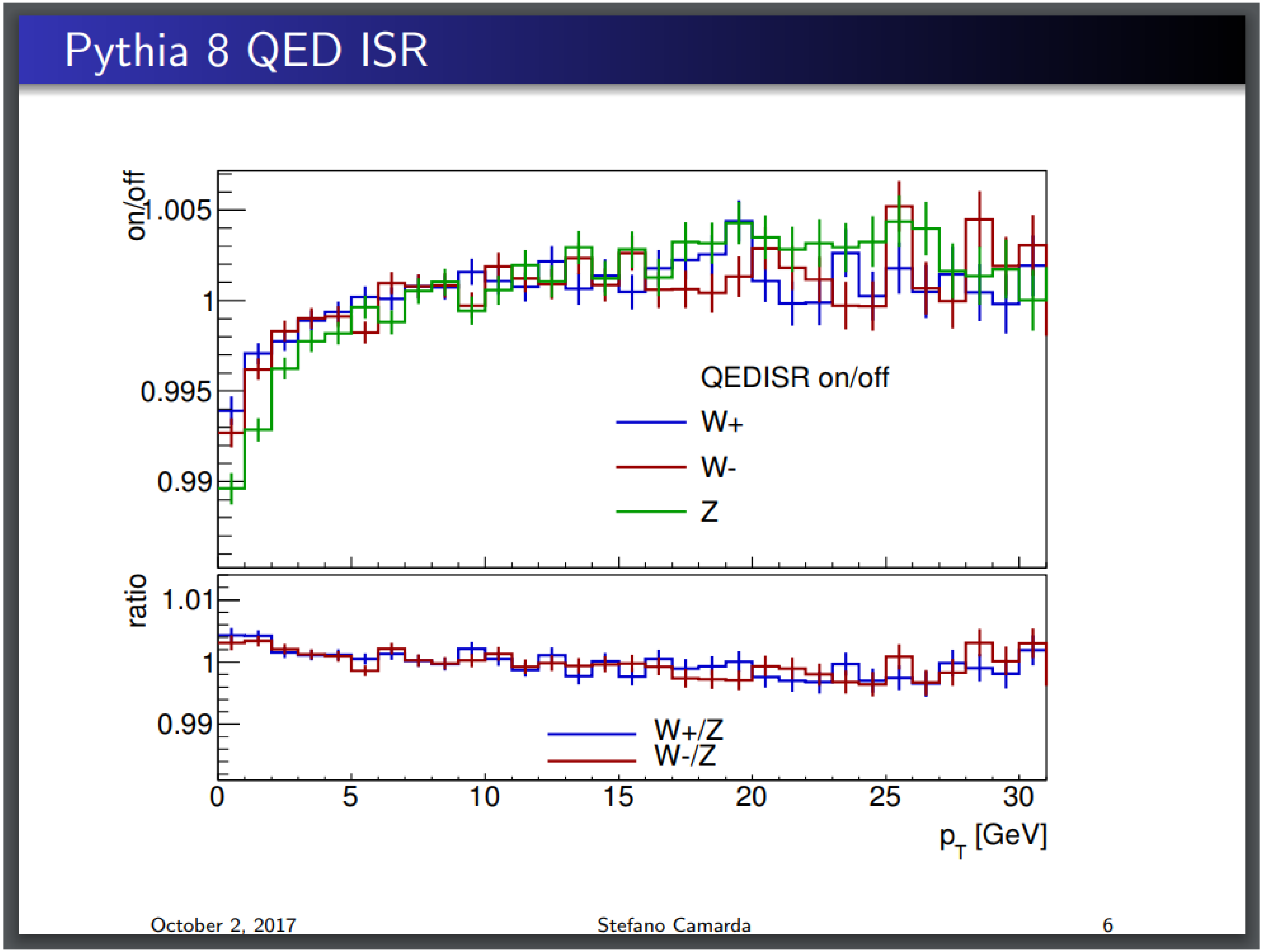
Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Part 3: Z production

- Effects of QED corrections in Z production
- **I)- Implementation of fixed order NLO**
- **II)- Proper treatment of photon radiation to reach up to NLL accuracy**
- **III)-Inclusion of mixed QCD-QED effects beyond the LL/LO**

Z production with *mixed NLL QED*

Motivation & some previous results

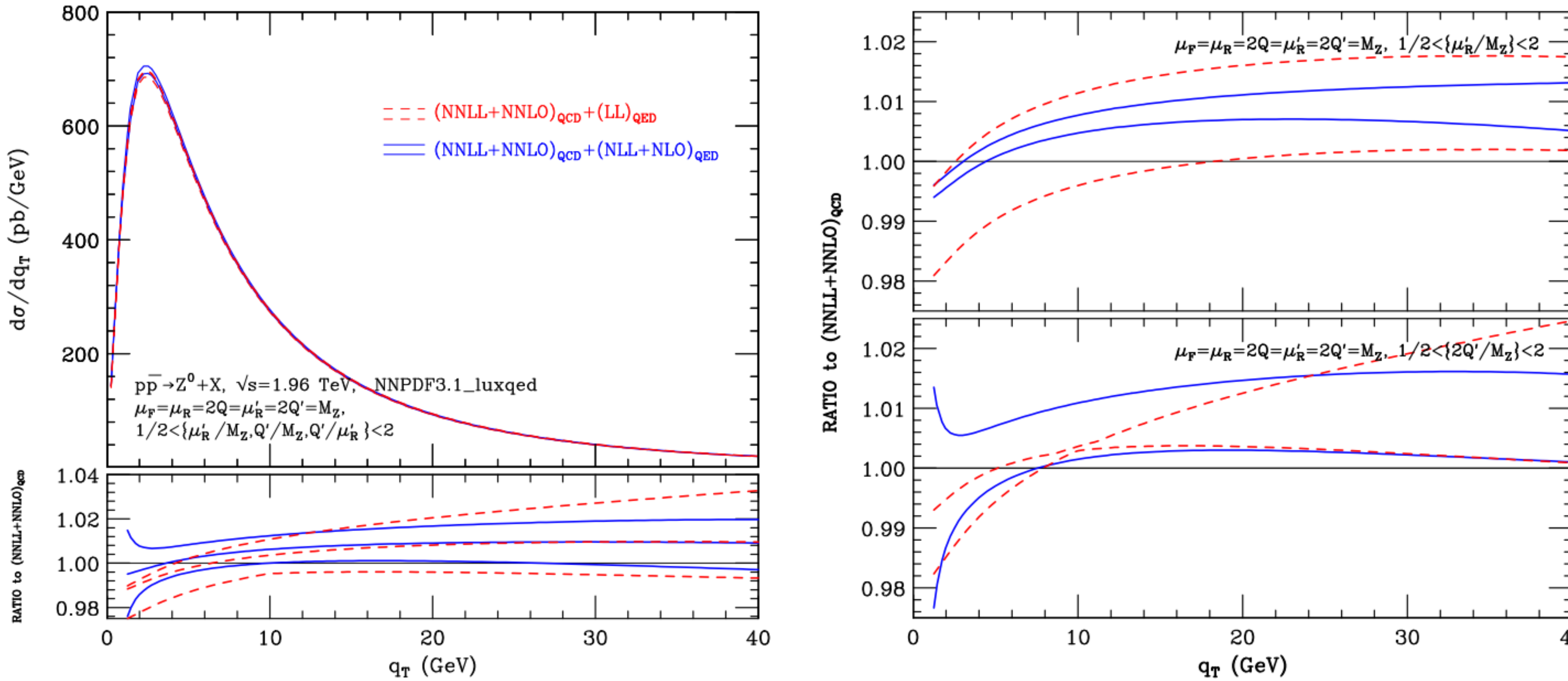


S. Camarda, "Studies of W/Z pT", Oct. 2017

Z production with *mixed NLL QED*

Some plots

- **Case of study:** Z production (implemented in DYqt)



  - Collider: Tevatron at 1.96 TeV
  - Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Z production with *mixed NLL QED*

Some plots

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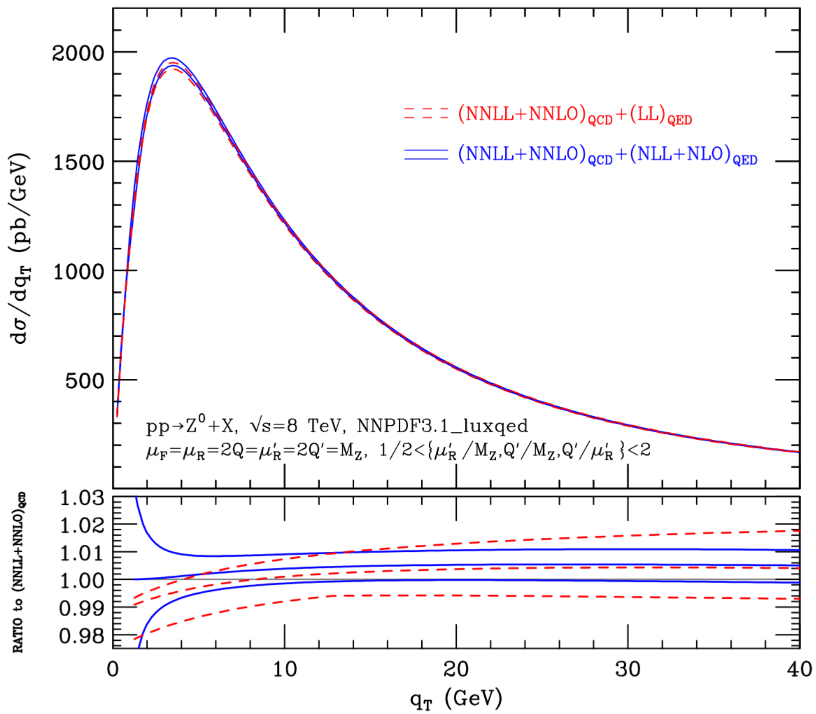

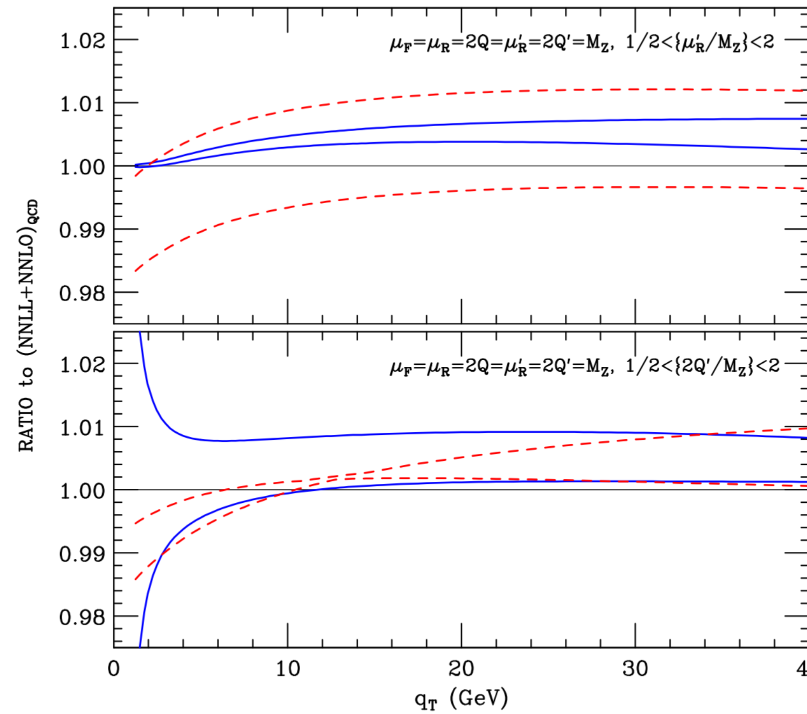

  - Collider: LHC at 8 TeV
  - Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Z production with *mixed NLL QED*

Some plots

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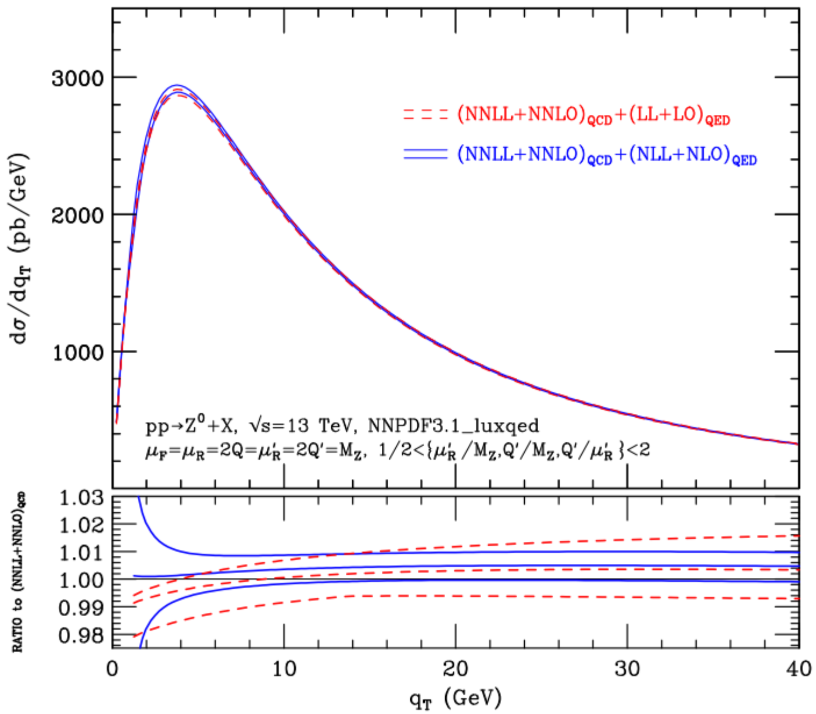
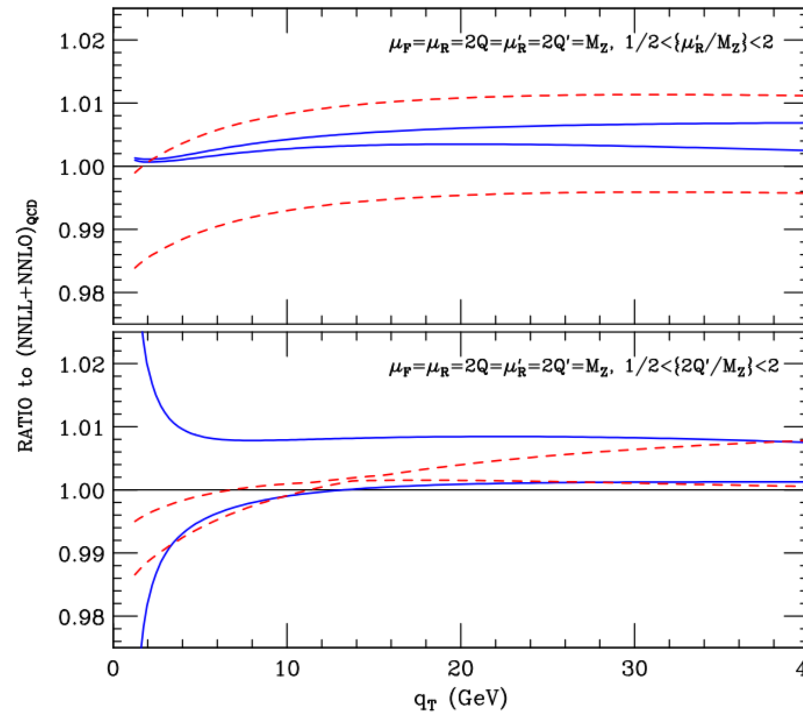




  - Collider: LHC at 13 TeV
  - Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

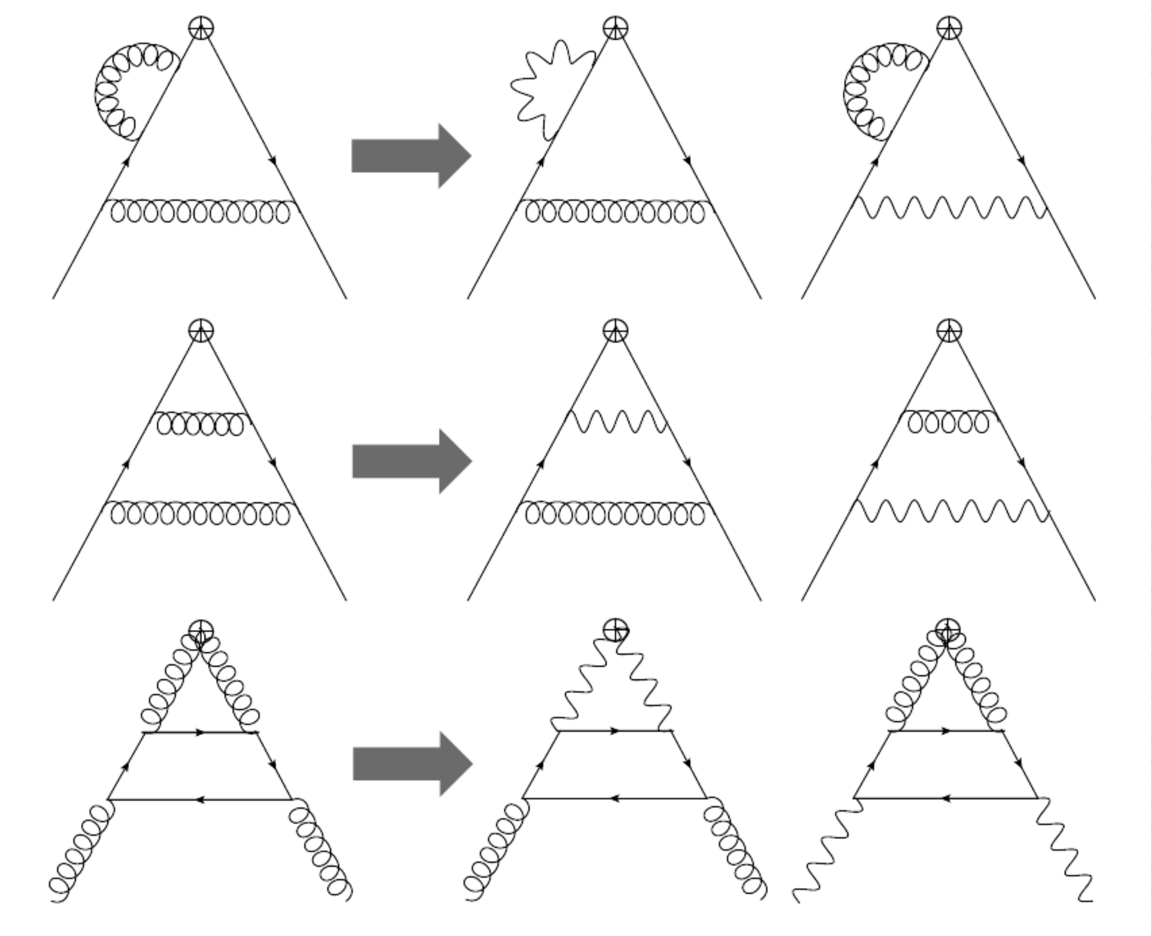
Cieri, Ferrera and GS, arXiv:1805.11948 [hep-ph]

Conclusions and perspectives

- Combined QCD-QED corrections studied and computed!
- **Mixed** resummation effects
  - Physical example: Z production
  - Combined NNLL QCD + NLL QED accuracy (properly matched to the fixed order, with non-trivial mixing effects)
  - **Reduction of ambiguities due to EM coupling choice! (O(3%) at LL QED reduced by factor 2 at NLL QED)**
  - **In general, H.O. QED effects are under control (stable predictions and small contribution)**
- **Outlook:**
  - **Extension to W production!!**
  - **Implementation in DYRes (Z->ll decay) and inclusion of QED effects from final state leptons.**

Thanks for the attention!!!

BACKUP SLIDES



$P_{qq}^{(2,0)} \rightarrow P_{qq}^{(1,1)}$

Non-observable gluon leads to non-equivalent diagrams contributing to the same kernel

$P_{gg}^{(2,0)} \rightarrow P_{g\gamma}^{(1,1)} \oplus P_{\gamma g}^{(1,1)}$

Replacement of external gluons leads to different kernels (no need of factor 2)

Abelianization algorithm: graphical explanation

The *Abelianization* is an algorithm defined to extract QCD-QED corrections from QCD ones. **However,** the structure of **mixed corrections is not trivial** (involves expanding in two different couplings, potential crossed terms might appear…)

Splittings and DGLAP within QCD-QED

Extended DGLAP equations (easiest ones)

- New optimized DGLAP equations! They become completely diagonal at some perturbative orders (due to vanishing kernels).

Roth& Weinzierl '04

$$\frac{dq_{v_i}}{dt} = P^-_{q_i} \otimes q_{v_i} + \sum_{j=1}^{n_F} \Delta P^S_{q_i q_j} \otimes q_{v_j} + \Delta P^S_{q_i l} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right) ,$$

$$\frac{dl_{v_i}}{dt} = P^-_{l} \otimes l_{v_i} + \sum_{j=1}^{n_F} \Delta P^S_{l q_j} \otimes q_{v_j} + \Delta P^S_{ll} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right) ,$$

Valence PDFs

$$\frac{d\{\Delta_{uc}, \Delta_{ct}\}}{dt} = P^+_u \otimes \{\Delta_{uc}, \Delta_{ct}\} ,$$

$$\frac{d\{\Delta_{ds}, \Delta_{sb}\}}{dt} = P^+_d \otimes \{\Delta_{ds}, \Delta_{sb}\} ,$$

$$\frac{d\Delta^l_{\{2,3\}}}{dt} = P^+_l \otimes \Delta^l_{\{2,3\}} ,$$

Diagonal equations

There are some remaining equations to describe the full coupled system, but they are more complicated…

- These equations are usually solved with Mellin transformations. The coupled differential system is reduced to an algebraic one for the Mellin momenta.

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Splittings and DGLAP within QCD-QED

Extending sum rules and new effects

- **Extended sum rules** *(impose physical constraints in AP kernels)*
  - *Fermion number conservation* $\Rightarrow$ $\int_0^1 dx\, P_f^- = 0$
  - *Momentum conservation* $\Rightarrow$ $0 = \frac{dP}{dt} = \int_0^1 dx\, x \left(\frac{dg}{dt} + \frac{d\gamma}{dt} + \sum_f \left(\frac{df}{dt} + \frac{d\bar{f}}{dt} \right) \right)$

- Some general remarks:
  - **Charge separation** effects introduced by QED
  - Non-trivial **quark-lepton mixing** (although simplified in the optimized basis)
  - **Explicit formulae** involving AP kernels can be obtained by replacing the **evolution equations**
  - Sum rules allow to **fix the behaviour** of AP kernels in the end-point (**x=1**)
  - *Also, they are useful for checking the consistency of the results.*

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

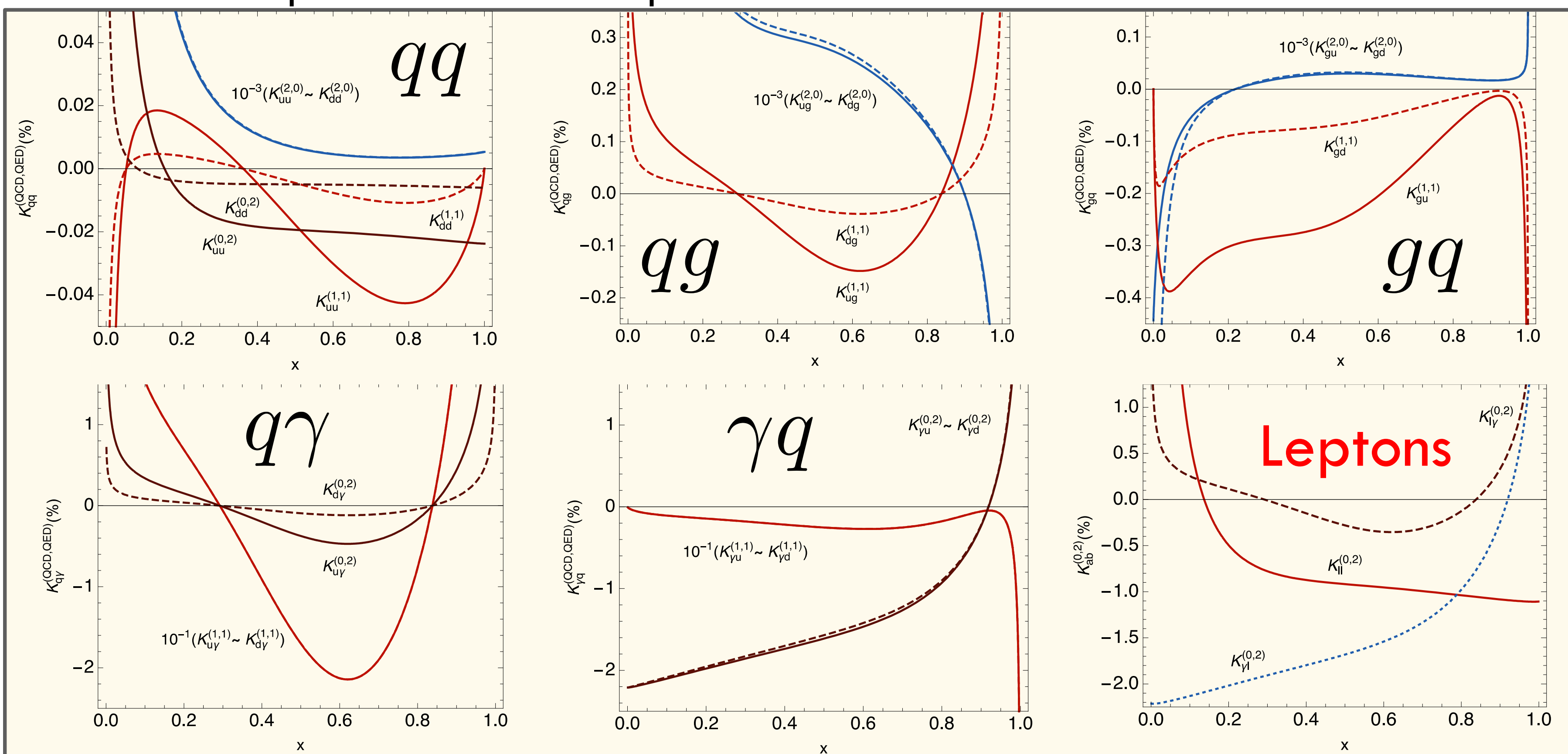
Splittings and DGLAP within QCD-QED

Splittings: quantifying QED effects

- We define a ratio to quantify the effect of H.O. QED corrections

$$K_{ab}^{(i,j)} = a_{\rm S}^i\, a^j\, \frac{P_{ab}^{(i,j)}(x)}{P_{ab}^{\rm LO}(x)} \qquad \text{with} \qquad P_{ab}^{\rm LO} = a_{\rm S}\, P_{ab}^{(1,0)} + a\, P_{ab}^{(0,1)}$$

- Some plots to show the impact in the evolution kernels:



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