Karlsruhe Institute of Technology

Double Higgs Production in the high- and low-energy limits

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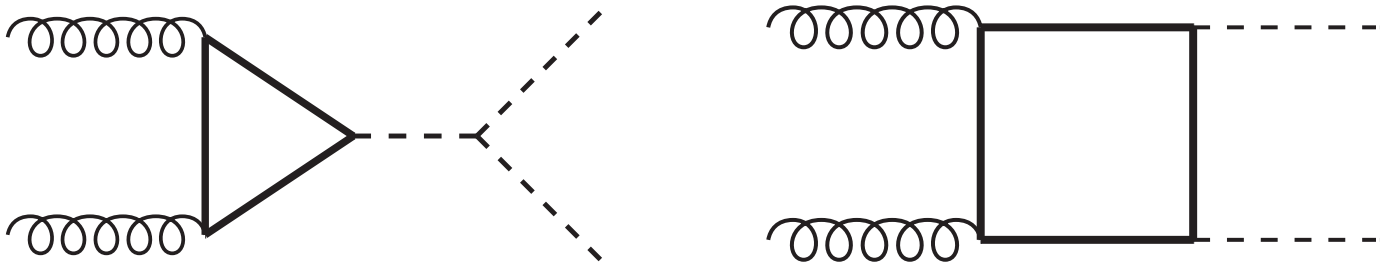
INSTITUTE FOR THEORETICAL PARTICLE PHYSICS (TTP)

Introduction

Dominant channel at a hadron collider: gluon fusion.



$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

Projectors: $\mathcal{F}_{tri} + \mathcal{F}_{box1} = P_{1\,\mu\nu}\mathcal{M}^{\mu\nu}$, $\mathcal{F}_{box2} = P_{2\,\mu\nu}\mathcal{M}^{\mu\nu}$.

Gives access to the Higgs self-coupling λ_{HHH} via $\mathcal{F}_{tri}$.

- experimentally challenging measurement (small cross-section)
- perhaps feasible with HL-LHC?

Theory Status

LO

- full result [Glover,van der Bij '88][Plehn,Spira,Zerwas '98]

NLO

- numerical result [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zicke '16]
  [Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher '18]
- large-m_t limit [Dawson,Dittmaier,Spira '98] [Grigo,Hoff,Melnikov,Steinhauser '13]
  [Degrassi,Giardine,Gröber '16]
- Padé approx. (large-m_t + threshold) [Gröber,Maier,Rauh '17]

NNLO

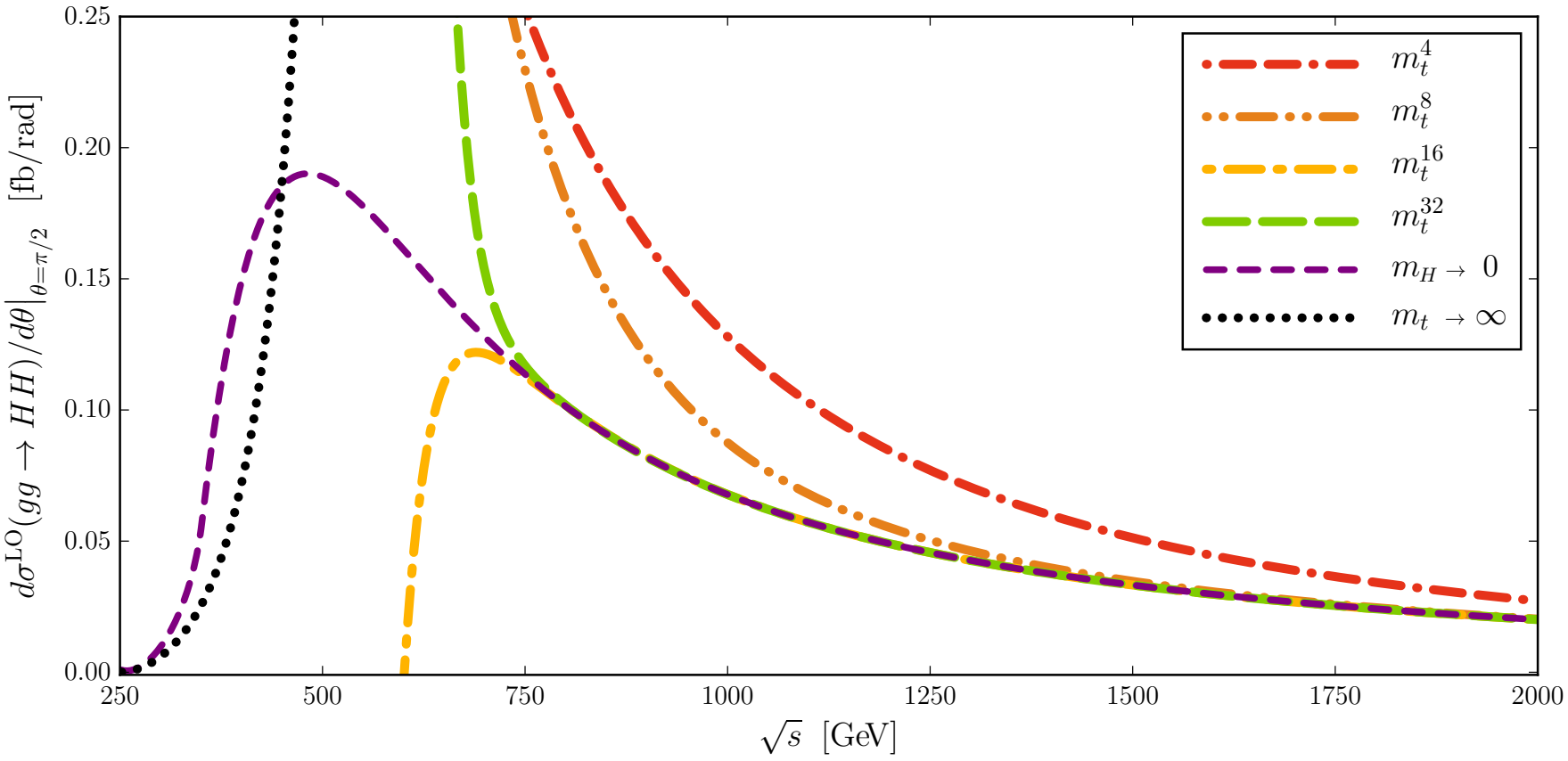
- large-m_t limit [de Florian,Mazzitelli '13] [Grigo,Melnikov,Steinhauser '14]
  [Grigo,Hoff,Steinhauser '15]
- finite-m_t estimate [Grazzini,Heinrich,Jones,Kallweit,Kerner,Lindert,Mazzitelli '18]

This talk:

- **NLO high-energy limit** [Davies,Mishima,Steinhauser,Wellmann '18,'19]
- **NNLO large-m_t limit**

Leading Order

- High-energy limit: $s, t \gg m_t^2 > m_H^2$
- Large-m_t: $m_t \to \infty$



High-Energy Limit

Procedure:

Amplitude in terms of Feynman integrals: $I(m_H^2, m_t^2)$

$\downarrow$

Expand around $m_H^2 = 0$: $I(0, m_t^2) + m_H^2 I'(0, m_t^2) + \cdots$

$\downarrow$

IBP reduce Feynman integrals to master integrals: $J(0, m_t^2)$

$\downarrow$

Determine master integrals around $m_t^2 = 0$:
$J(0, m_t^2) = \sum_{m,n} C_{m,n} \left(m_t^2\right)^m \log\left(m_t^2\right)^n$

$\downarrow$

Amplitude for $\mathbf{s, t \gg m_t^2 > m_H^2}$

Software

Diagram generation	`qgraf`	[Nogueira '93]
Topology mapping	`q2e/exp`	[Harlander,Seidelsticker,Steinhauser '97]
Physics, projection	`TFORM 4.2`	[Ruijl,Ueda,Vermaseren '17]
$m_H^2 = 0$ expansion	`LiteRed`	[Lee '13]
IBP Reduction	`FIRE 5.2`	[Smirnov '14]
	(`LiteRed`)	[Lee '13]

Feynman Diagrams: $\mathbf{8}^{LO} + \mathbf{118}^{NLO}$

$\downarrow$

Feynman Integrals: **26K** (**+120K** (m_H^2 exp))

$\downarrow$

Masters Integrals: $\mathbf{10}^{LO} + \mathbf{161}^{NLO}$

IBP reduce topologies separately $\sim$ 3 weeks. Lots of memory required.

Compute masters: differential equations

Differentiate master integrals wrt $X \in \{s, t, m_t^2\}$. IBP reduce result:

$$\frac{\mathrm{d}}{\mathrm{d}X}\,\vec{J} = M(s, t, m_t^2, \epsilon) \cdot \vec{J}.$$

m_t^2 **equation:** substitute high-energy ansatz for each master integral,

$$J = \sum_i \sum_j \sum_k C_{ijk}(s, t)\, \epsilon^i\, (m_t^2)^j\, \log\left(m_t^2\right)^k.$$

Obtain a system of linear equations for coefficients $C_{ijk}(s, t)$. **Solve!**

... we require **Boundary Conditions**

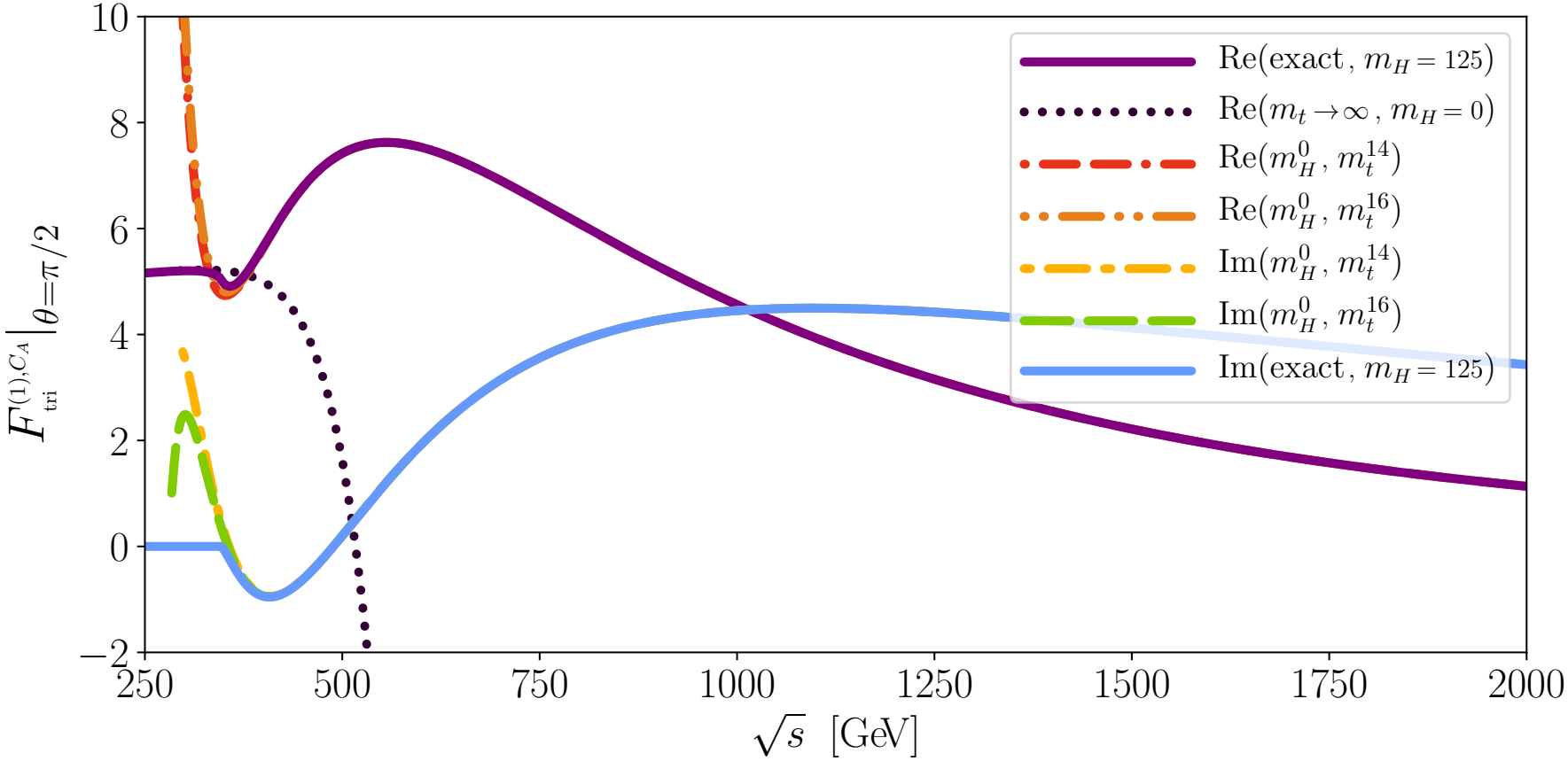
- determine leading powers in $m_t^2 \to$ fixes some $C_{ijk}(s, t)$

Here we determine the amplitude to m_t^{32} with `Mathematica` ... difficult!

Results: Form Factors

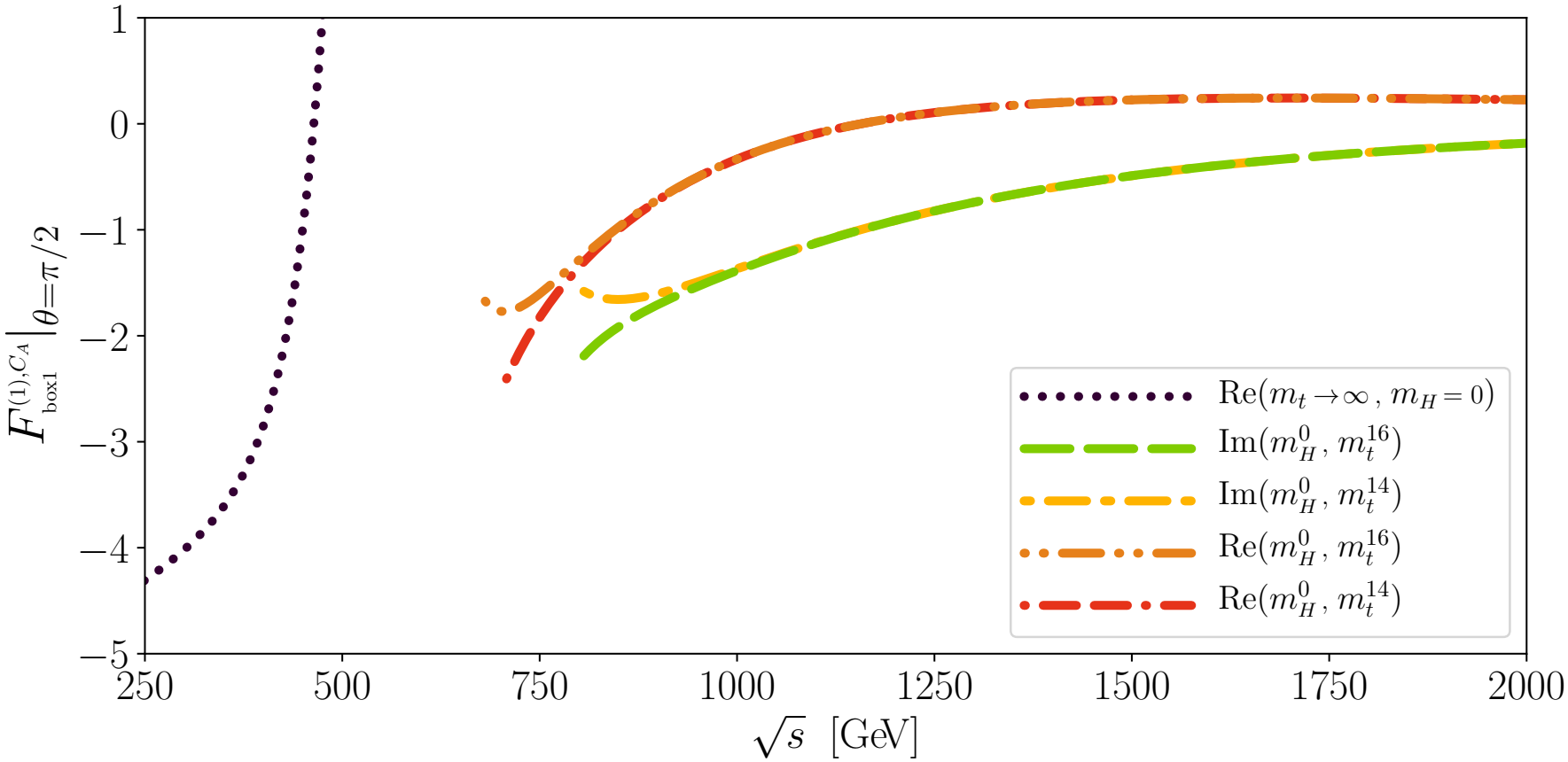
(Renorm. and IR subtraction: $\mathcal{F}_X^{(1)} = \mathcal{F}_X^{(1),IR-div.} - K_g^{(1)} \mathcal{F}_X^{(0)}$)

$\mathcal{F}_{tri}$ known exactly at NLO: $gg \to H$.



Results: Form Factors

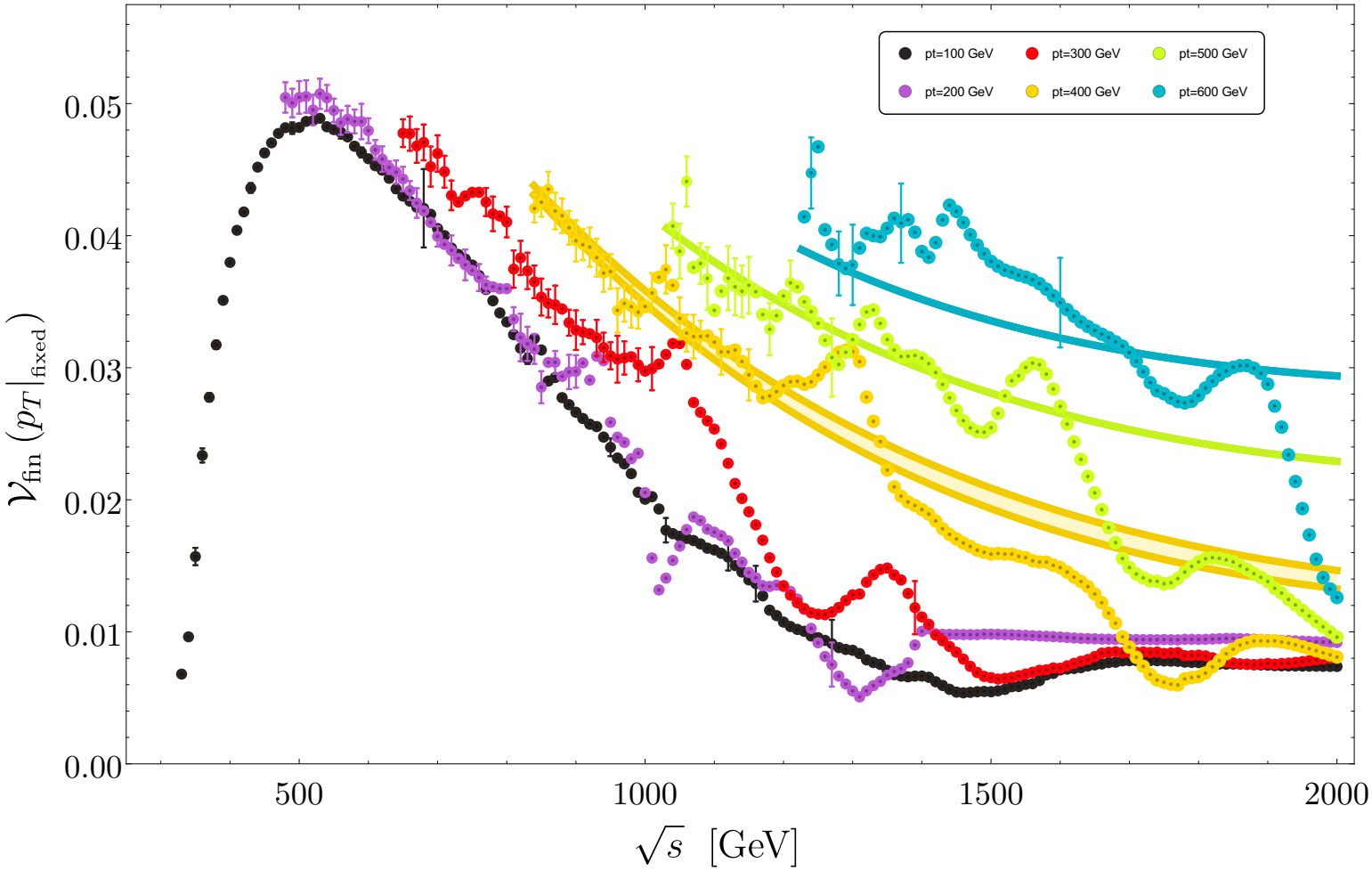
$\mathcal{F}_{box1}$, $\mathcal{F}_{box2}$: no exact result for comparison.



Results: V_{fin}

V_{fin}: IR finite (subtracted) virtual cross-section. Here, m_t^{30}, m_t^{32} terms.

- We can compare with `hhgrid` [Heinrich,Jones,Kerner,Luisoni,Vryonidou '17]



Padé Improved V_{fin}

Padé Approximant:

$$[n/m](m_t^2) = \frac{a_0 + a_1 m_t^2 + a_2 (m_t^2)^2 + \cdots + a_n (m_t^2)^n}{1 + b_1 m_t^2 + b_2 (m_t^2)^2 + \cdots + b_m (m_t^2)^m}$$

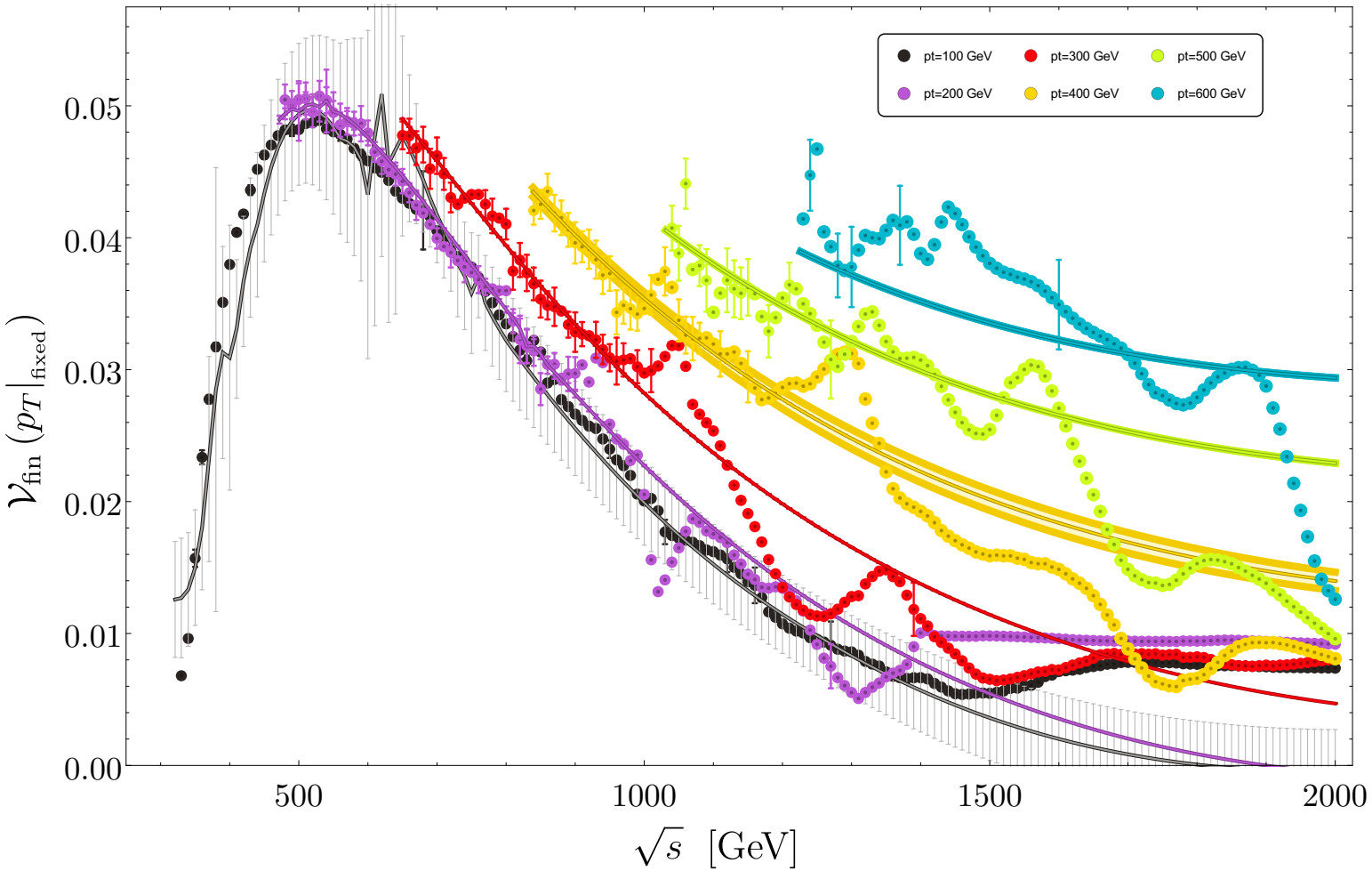
- use high-energy expansion to fix coefficients a_i, b_i.
- evaluate for $m_t = 173$ GeV

Compute Padé approximants with $n + m = 16$, $|n - m| \leq 2$:

- $[8/8], [7/8], [8/7], [7/9], [9/7]$

Take mean value, stdev for error estimate.

Results: Padé Improved V_{fin}



In progress: use high-energy input to improve `hhgrid`.

Large-m_t Limit

Previous NNLO calculation:

- leading [de Florian,Mazzitelli '13][Grigo,Melnikov,Steinhauser '14]
- terms to 3rd order $(1/m_t^4)$ for diff. XS [Grigo,Hoff,Steinhauser '15]

Here:

- terms to 5th order $(1/m_t^8)$ for form factors

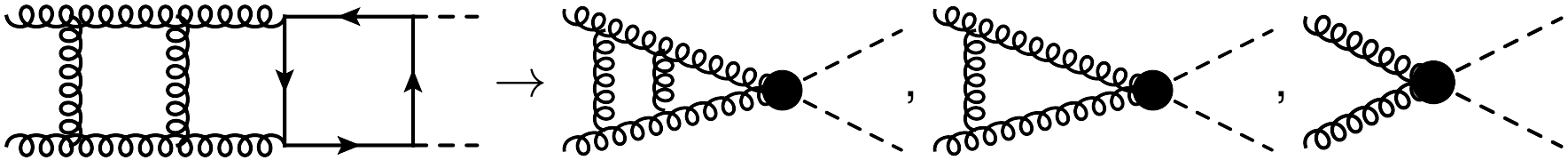
Diagram generation	`qgraf`	[Nogueira '93]
Asymp. exp. code	`q2e/exp`	[Harlander,Seidelsticker,Steinhauser '97]
Expansion	`TFORM 4.2`	[Ruijl,Ueda,Vermaseren '17]
Massive vacuum gr.	`MATAD`	[Steinhauser '00]

Asymptotic Expansion

Expand diagrams for $m_t \gg q_1, q_2, q_3$:

- Yields expansion in powers of $\{q_3 \cdot q_3,\ q_1 \cdot q_2,\ q_1 \cdot q_3,\ q_2 \cdot q_3\}/m_t^2$.
- Diagrams factorize into products of lower-loop massless graphs and hard sub-graphs.
- Expansion of these hard sub-graphs yield massive vacuum graphs (can be treated by `MATAD`).

Eg,



Each ● is a large sum of massive vacuum (**tensor**) integrals. We wish to expand to 5th order $(1/m_t^8)$.

- If treated by `MATAD`, **large bottleneck** (esp. at 3 loops).

Projection

Avoid computing three-loop vacuum tensor integrals by projecting each diagram onto a basis

$$B = \sum_{L=0}^{L_{max}} \sum_{k+l+m+n}^{=L} C_{k,l,m,n} \left(q_3 \cdot q_3\right)^k \left(q_1 \cdot q_2\right)^l \left(q_1 \cdot q_3\right)^m \left(q_2 \cdot q_3\right)^n,$$

using projectors $P_{k,l,m,n} B = C_{k,l,m,n}$.

$P_{k,l,m,n}$ defined in terms of derivative operators $\Box_{a,b} = \frac{\partial}{\partial q_{a\,\mu}} \frac{\partial}{\partial q_b^\mu}$. Eg:

$$P_{0,0,0,2} = \frac{1}{2d^2 + 2d - 4} \Box_{2,3} \Box_{2,3} - \frac{1}{2d^3 + 2d^2 - 4d} \Box_{2,2} \Box_{3,3}.$$

For $1/m_t^{\{0,2,4,6,8\}}$ expansion there are $\{15, 38, 88, 174, 324\}$ derivative combinations to evaluate, for each diagram.

After derivatives, only scalar vacuum integrals remain – easy for `MATAD`.

Computation

Procedure:

- Sum diagrams with same colour factor: 9 “super-diagrams”
- Apply 5 structures of form-factor projectors: 45 projected objects

45 hard tasks $\left\{ \begin{array}{c} 1/m_t^2 \text{ expansion, to 5th order.} \\ \downarrow \\ \text{Store intermediate result.} \end{array} \right.$

Total stored: 324GB (compressed with `gzip` – crucial).

29K easy tasks $\left\{ \begin{array}{c} \text{Compute a derivative. Load } \textbf{only} \text{ necessary terms.} \\ \downarrow \\ \text{Scalar vacuum integrals for } \texttt{MATAD}. \end{array} \right.$

Combine all results together.

Total time $\sim$ 4.5 yr ($\sim$ 1 month).

Essential Optimizations

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Terms have the form:

```
+ Den(l1,mt) * Den(l1+q1,mt) * ...  * ( many terms )
```

where `( many terms )` do not take part in the expansion.

Expansion of the denominators takes place over many `FORM` modules. Repeated sorting of huge expressions to disk is a **large bottleneck**.

"Hide" these terms with the construction

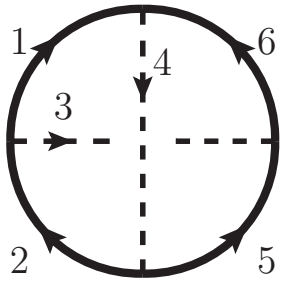
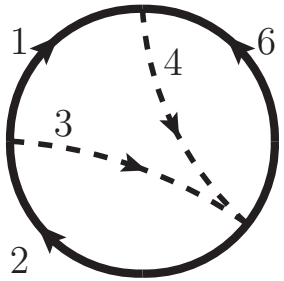
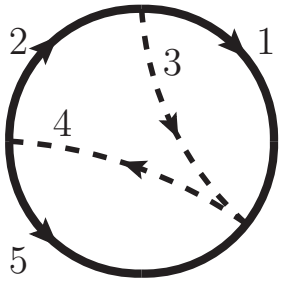
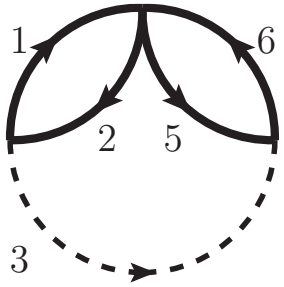
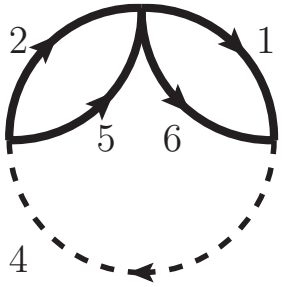
```
Bracket Den;
.sort
Collect f;
ArgToExtraSymbol f;
```

and reinstate them after expansion is complete.

Larger memory requirement, but smaller expressions.

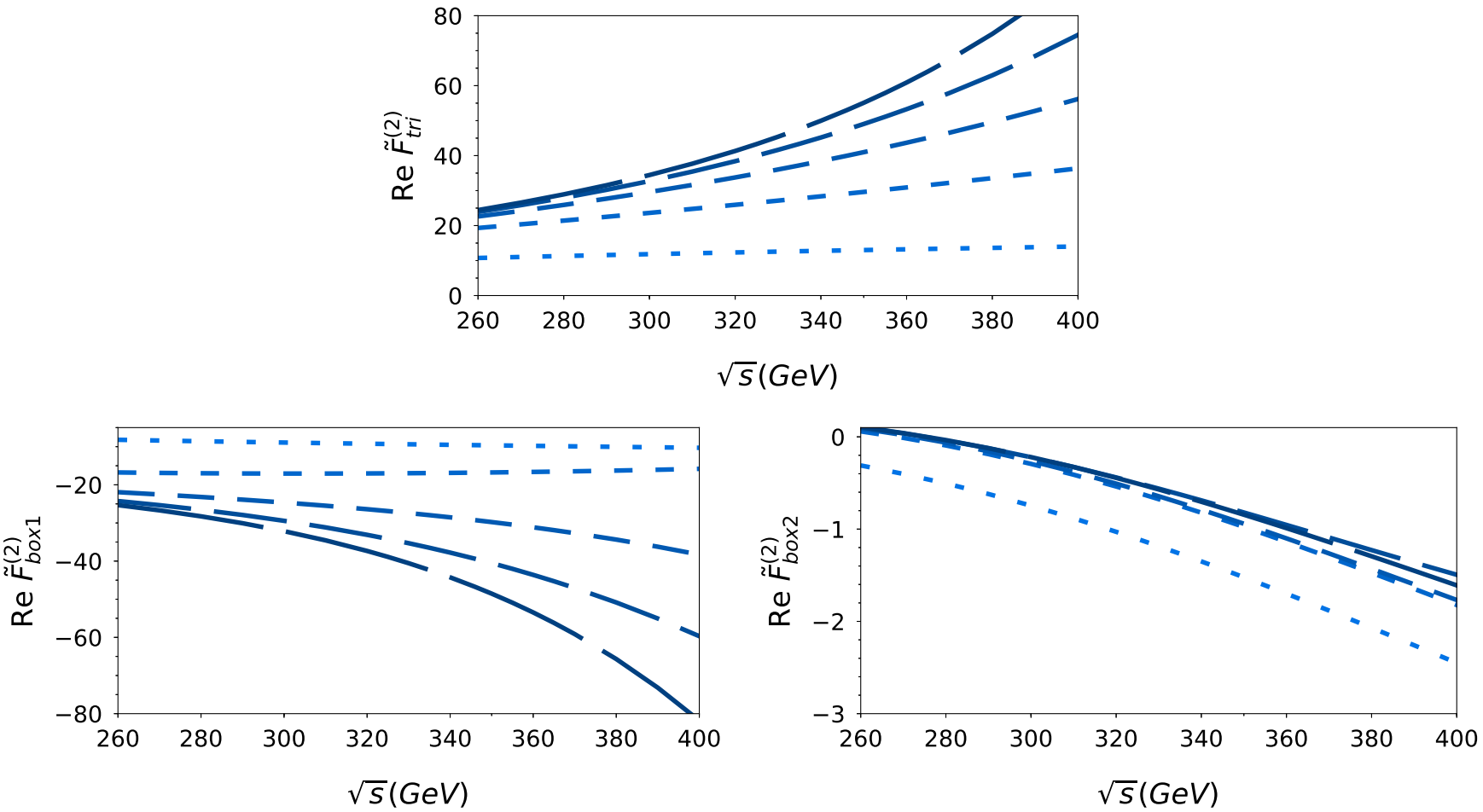
Essential Optimizations

Vacuum diagrams are highly symmetric:

Top-level Topology	Graph 1	Graph 2	Relabelling
			$p1 \to -p2$ $p2 \to -p1$ $p5 \to -p6$ $p6 \to -p5$ $p4 \to -p4$
			$p1 \to p5$ $p5 \to p1$ $p2 \to -p2$ $p6 \to -p6$ $p3 \to -p4$ $p4 \to -p3$

Apply symmetries **before and after** large-m_t expansion.

Results



In progress: NNLO Padé approx in combination with threshold

- c.f. NLO large-m_t + threshold approx [Gröber,Maier,Rauh '18]

Conclusions

NLO:

- high-energy expansion gives information in previously-unknown region of phase space
- combine with other inputs to improve approximation of NLO virtual corrections

NNLO:

- additional terms in large-m_t expansion computed
- combine with NNLO threshold expansion to approximate NNLO virtual corrections