QED and electroweak radiative corrections to polarized Bhabha scattering

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Outline

Motivation

Motivation:

- Development of the physical program for future high-energy e^+e^- colliders
- Having high-precision theoretical description of Bhabha scattering is of crucial importance
- Many of the future e^+e^- colliders foresee running with polarized beam(s)

QUESTIONS:

- What we have?
- What we need?
- What to do?
- How to do?

FUTURE e^+e^- COLLIDER PROJECTS

Linear Colliders

- ILC, CLIC
- ILC: technology is ready, to be built in Japan (?)

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$\mathcal{L} \approx 2 \cdot 10^{34}$ cm^{-2}s^{-1}

Stat. uncertainty $\sim 10^{-3}$

Beam polarization:
e^-beam: $P = 80 - 90\%$
e^+beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, TLEP
- CEPC
- muon collider (?)

E_{tot}

- 91; 160; 240; 350 GeV

$\mathcal{L} \approx 2 \cdot 10^{36}$ cm^{-2}s^{-1} (4 exp.)

Stat. uncertainty $< 10^{-3}$

Beam polarization: desirable

Super Charm-Tau Factory Project(s)

Budker Institute of Nuclear Physics in Novosibirsk and/or China

Colliding electron-positron beams with c.m.s. energies from 2 to 5 GeV with unprecedented high luminosity $10^{35} cm^{-2} c^{-1}$

The electron beam will be longitudinally polarized

The main goal of experiments at the Super Charm-Tau factory is to study the processes charmed mesons and tau leptons, using a data set that is 2 orders of magnitude more than the one collected by BESIII

Estimated experimental precision

Now:

Quantity	Theory error	Exp. error
M_W [MeV]	4	15
$\sin^2\theta^l_{eff}[10^{-5}]$	4.5	16
Γ_Z [MeV]	0.5	2.3
$R_b[10^{-5}]$	15	66

Quantity	ILC	FCC-ee	CEPC	Projected theory error
M_W [MeV]	3–4	1	3	1
$\sin^2\theta^l_{eff}[10^{-5}]$	1	0.6	2.3	1.5
Γ_Z [MeV]	0.8	0.1	0.5	0.2
$R_b[10^{-5}]$	14	6	17	5–10

The estimated error for the theoretical predictions of these quantities is given, under the assumption that $O(\alpha\alpha_s^2)$, fermionic $O(\alpha^2\alpha_s)$, fermionic $O(\alpha^3)$, and leading four-loop corrections entering through the ρ-parameter will become available.

Introduction to SANC

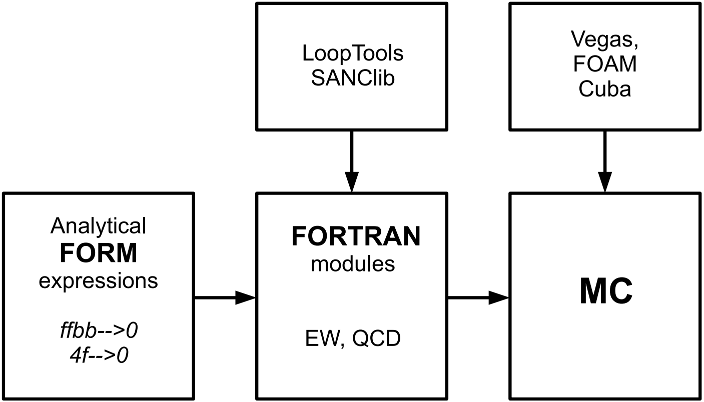
- The SANC system implements calculations of complete (real and virtual) NLO QCD and EW corrections for various processes at the partonic level

- All calculations are performed within the OMS (on-mass-shell) renormalization scheme in the R_ξ gauge which allows an explicit control of the gauge invariance by examining cancellation of the gauge parameters in the analytic expression of the matrix element

- Cross-sections of the processes at hadron level obtained by convoluting the partonic level cross-sections with PDFs

- The list of processes implemented in the `MCSANC` integrator includes Drell-Yan processes (inclusive), associated Higgs and gauge boson production and single-top quark production in s- and t-channel (v1.01 – CPC 184 (2013) 2343), photon-induced contribution, EW corrections beyond NLO approximation to DY (v1.20 – JETP Lett. 103 (2016) 131)

SANC framework scheme



SANC FOR PROCESSES WITH POLARIZED BEAMS

- NLO EW corrections for polarized e^+e^- scattering:
  - Bhabha scattering (PRD 2018)
  - $e^+e^- \to ZH$ (arXiv:1812.10965)
  - $e^+e^- \to \mu^+\mu^-$(or $\tau^+\tau^-$) (preliminary)
  - $e^+e^- \to Z\gamma$ (preliminary)
  - $e^+e^- \to \gamma\gamma$ (preliminary)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

Bhabha: HA for Born and virtual contributions

At one-loop level we have 6 non-zero HAs (4 independent):

$$\mathcal{H}_{++++} = \mathcal{H}_{----} = -2e^2\,\frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) - \chi^t_Z\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right],$$

$$\mathcal{H}_{+-+-} = \mathcal{H}_{-+-+} = -e^2\,c_-\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) - \chi^s_Z\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right],$$

$$\mathcal{H}_{+--+} = -e^2\,c_+\Bigg(\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) + \chi^s_Z\left(\mathcal{F}^Z_{LL}(s,t,u) - 2\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right)\right]$$
$$+\frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) + \chi^t_Z\left(\mathcal{F}^Z_{LL}(t,s,u) - 2\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right)\right]\Bigg),$$

$$\mathcal{H}_{-++-} = -e^2\,c_+\Bigg(\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u)\right] + \frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u)\right]\Bigg),$$

where $\quad c_+ = 1+\cos\theta, \quad c_- = 1-\cos\theta$

$$\chi^s_Z = \frac{1}{4s^2_Wc^2_W}\,\frac{s}{s-M^2_Z+iM_Z\Gamma_Z}, \quad \chi^t_Z = \frac{1}{4s^2_Wc^2_W}\,\frac{t}{t-M^2_Z}, \quad \delta_e = v_e - a_e = 2s^2_W,$$

$$\mathcal{F}^{(\gamma,Z)}_{QQ}(a,b,c) = \mathcal{F}^{\gamma}_{QQ}(a,b,c) + \chi^a_Z\delta^2_e\mathcal{F}^Z_{QQ}(a,b,c)$$

We get the Born level HAs by replacing $\mathcal{F}^Z_{LL}\to 1$, $\mathcal{F}^Z_{QL}\to 1$, $\mathcal{F}^Z_{QQ}\to 1$ and $\mathcal{F}^{\gamma}_{QQ}\to 1$

SANC Monte Carlo generator for Bhabha

See the next talk by Renat Sadykov

We created a Monte Carlo generator of unweighted events for polarized Bhabha scattering $e^+e^- \to e^+e^-$ with complete one-loop EW corrections and with possibility to produce events in the standard Les Houches format.

This generator uses adaptive algorithm `mFOAM` (CPC 177:441-458,2007) which is part of the `ROOT` program

Setup for tuned comparison

We performed a tuned comparison of polarized Born and hard Bremsstrahlung by WHIZARD. The unpolarized soft and virtual parts were compared with the results of AÏtalc.

Initial parameters

$$\alpha^{-1}(0) = 137.03599976,$$

$$M_W = 80.451495 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.49977 \text{ GeV},$$

$$m_e = 0.5109990 \text{ MeV}, \quad m_\mu = 0.105658 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$

$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$

$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts

$$|\cos\theta| < 0.9,$$

$$E_\gamma > 1 \text{ GeV} \quad \text{(for comparison of hard Bremsstrahlung)}.$$

$e^+e^- \to e^+e^-$: WHIZARD VS SANC (HARD)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.62(1)	49.58(1)	48.74(1)	50.40(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.65(1)	49.56(1)	48.78(1)	50.44(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.14(1)	15.81(1)	13.54(1)	18.07(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.12(1)	15.79(1)	13.55(1)	18.11(2)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.693(1)	4.976(1)	3.912(1)	6.041(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.694(1)	4.975(1)	3.913(1)	6.043(1)

$e^+e^- \to e^+e^-$: AI̊TALC vs SANC $\sqrt{s} = 500 GeV$

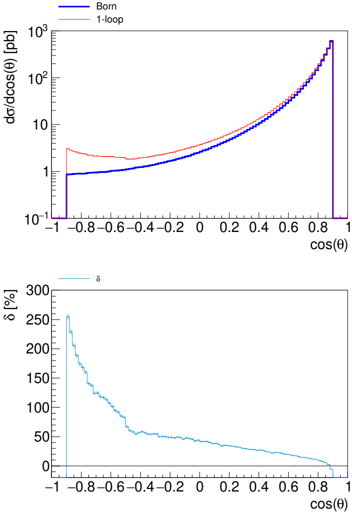
$\cos\theta$	$\sigma^{\text{Born}}_{e^+e^-}$, pb	$\sigma^{\text{Born+virt+soft}}_{e^+e^-}$, pb
-0.9	$2.16999 \cdot 10^{-1}$ $2.16999 \cdot 10^{-1}$	$1.93445 \cdot 10^{-1}$ $1.93445 \cdot 10^{-1}$
-0.5	$2.61360 \cdot 10^{-1}$ $2.61360 \cdot 10^{-1}$	$2.38707 \cdot 10^{-1}$ $2.38707 \cdot 10^{-1}$
0	$5.98142 \cdot 10^{-1}$ $5.98142 \cdot 10^{-1}$	$5.46677 \cdot 10^{-1}$ $5.46677 \cdot 10^{-1}$
$+0.5$	$4.21273 \cdot 10^{0}$ $4.21273 \cdot 10^{0}$	$3.81301 \cdot 10^{0}$ $3.81301 \cdot 10^{0}$
$+0.9$	$1.89160 \cdot 10^{2}$ $1.89160 \cdot 10^{2}$	$1.72928 \cdot 10^{2}$ $1.72928 \cdot 10^{2}$
$+0.99$	$2.06556 \cdot 10^{4}$ $2.06555 \cdot 10^{4}$	$1.90607 \cdot 10^{4}$ $1.90607 \cdot 10^{4}$
$+0.999$	$2.08236 \cdot 10^{6}$ $2.08236 \cdot 10^{6}$	$1.91624 \cdot 10^{6}$ $1.91624 \cdot 10^{6}$
$+0.9999$	$2.08429 \cdot 10^{8}$ $2.08429 \cdot 10^{8}$	$1.91402 \cdot 10^{8}$ $1.91402 \cdot 10^{8}$

Born vs 1-loop (SANC)

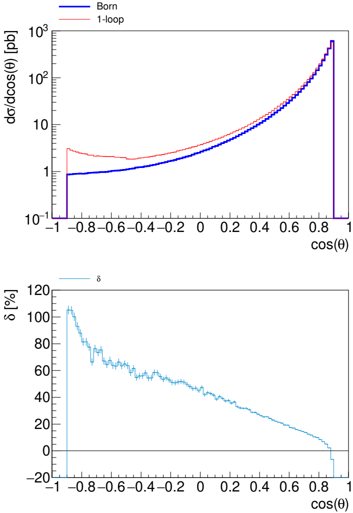
P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.6763(1)	57.7738(1)	56.2725(4)	59.2753(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	61.731(6)	62.587(6)	61.878(6)	63.287(7)
δ, %	8.92(1)	8.33(1)	9.96(1)	6.77(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.3789(1)	15.0305(1)	12.7061(1)	17.3550(2)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	15.465(2)	15.870(2)	13.861(1)	17.884(2)
δ, %	7.56(1)	5.59(1)	9.09(1)	3.05(1)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.67921(1)	3.90568(1)	3.03577(3)	4.77562(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	3.8637(4)	3.9445(4)	3.2332(3)	4.6542(7)
δ, %	5.02(1)	0.99(1)	6.50(1)	-2.54(1)

Distributions in $\cos\theta$

$\sqrt{s} = 250$ GeV

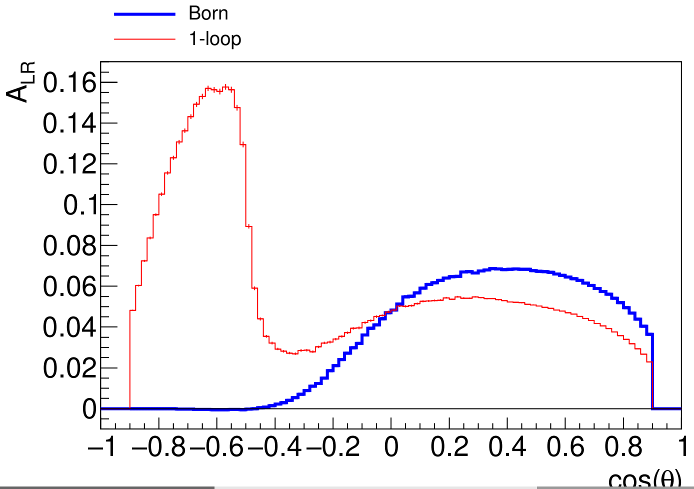


$\sqrt{s} = 500$ GeV



LEFT-RIGHT ASYMMETRY ($E_{CM} = 250$ GEV)

$$A_{LR} \equiv \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$



LEADING AND NEXT-TO-LEADING LOG APPROXIMATIONS

The QED leading (LO) logarithmic corrections

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^n \frac{s}{m_e^2}$$

were relevant for LEP measurements of Bhabha, $e^+e^- \to \mu^+\mu^-$ etc. for $n \leq 3$ since $\ln(M_Z^2/m_e^2) \approx 24$

NLO contributions

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^{n-1} \frac{s}{m_e^2}$$

with $n = 3$ are required for future colliders

In the collinear approximation we can get them within the NLO QED structure function formalism

QED NLO master formula

The NLO Bhabha cross section reads

$$
\begin{aligned}
d\sigma &= \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\text{str}}_{ae}(z_1)\mathcal{D}^{\text{str}}_{b\bar{e}}(z_2) \\
&\times \left[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) \right] \\
&\times \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\text{frg}}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\text{frg}}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right) + \mathcal{O}(\alpha^n L^{n-2})
\end{aligned}
$$

$\alpha^2 L^1$ terms are completely reproduced [A.A., E.Scherbakova, JETP Lett. 2006; PLB 2008]

$\alpha^3 L^2$ terms are required at the Z peak and for e^+e^- annihilation channels at higher energies

QED NLO evolution

$$\frac{d\mathcal{D}_{ee}(x,\mu_f,m_e)}{d\ln\mu_f^2} = \sum_{a=e,\gamma,\bar{e}} \int_z^1 \frac{dz}{z} P_{ea}(z,\bar{\alpha}(\mu_f))\mathcal{D}_{ae}\left(\frac{x}{z},\mu_f,m_e\right)$$

with perturbative splitting functions

$$P_{ea}(z,\bar{\alpha}(\mu_f)) = \frac{\bar{\alpha}(\mu_f)}{2\pi}P_{ea}^{(0)}(z) + \left(\frac{\bar{\alpha}(\mu_f)}{2\pi}\right)^2 P_{ea}^{(1)}(z) + \mathcal{O}(\alpha^3)$$

and initial conditions

$$\mathcal{D}_{ee}^{\rm ini}(x,\mu_0,m_e) = \delta(1-x) + \frac{\bar{\alpha}(\mu_0)}{2\pi}d_1(x,\mu_0,m_e) + \mathrm{O}(\alpha^2)$$

$$d_1(x,\mu_0,m_e) = \left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_0^2}{m_e^2} - 2\ln(1-x) - 1\right)\right]_+$$

$$\mathcal{D}_{\gamma e}^{\rm ini}(x,\mu_0,m_e) = \frac{\bar{\alpha}(\mu_0)}{2\pi}P_{\gamma e}^{(0)}(x) + \mathrm{O}(\alpha^2)$$

Iterative solution

The pure photonic non-singlet part of the electron structure function

$$\mathcal{D}_{ee}^{(\gamma)}(x,\mu_f,m_e) = \delta(1-x) + \frac{\alpha}{2\pi}d_1(x,\mu_0,m_e) + \frac{\alpha}{2\pi}L_f P_{ee}^{(0)}(x)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{1}{2}L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(0)}(x) + L_f P_{ee}^{(0)} \otimes d_1(x,\mu_0,m_e) + L_f P_{ee}^{(1,\gamma)}(x)\right)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^3 \left(\frac{1}{6}L_f^3 P_{ee}^{(0)} \otimes P_{ee}^{(0)} \otimes P_{ee}^{(0)}(x) + L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(1,\gamma)}(x)\right.$$
$$\left. + L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(0)} \otimes d_1(x,\mu_0,m_e)\right) + \mathcal{O}(\alpha^3 L^1)$$

The large logarithm $L_f \equiv \ln\frac{\mu_f^2}{m_e^2}$ with factorization scale $\mu_f^2 \sim s$ or $\sim -t$

NLO contributions of e^+e^- pairs are recovered in the same way

All required convolution integrals are listed in [A.A. hep-ph/0304063]

Example of convolution (I)

$$P_{ee}^{(0)}(x) = \left[\frac{1+x^2}{1-x}\right]_+$$

$$P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \delta(1-x)\left(\frac{3}{8} - 3\zeta(2) + 6\zeta(3)\right)$$
$$+\frac{1+x^2}{1-x}\left(-2\ln x\ln(1-x) + \ln^2 x + 2\mathrm{Li}_2\,(1-x)\right)$$
$$-\frac{1}{2}(1+x)\ln^2 x + 2\ln x + 3 - 2x$$

Plus prescription

$$\int_{x_{\min}}^{1} dx\,[V(x)]_+\,W(x) = \int_0^1 dx V(x)\Big[W(x)\Theta(x - x_{\min}) - W(1)\Big]$$

Convolution

$$A \otimes B(x) = \int_0^1 dz \int_0^1 dz'\delta(x - zz')A(z)B(z')$$

Example of convolution (II)

$$
\begin{aligned}
P_{ee}^{(0)} \otimes P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \Bigg[\frac{1+x^2}{1-x} \bigg(& -4\mathrm{S}_{12}(x) + 4\mathrm{Li}_2(1-x)(\ln(1-x) - \ln(x)) \\
& -4\ln(x)\ln^2(1-x) + 4\ln^2(x)\ln(1-x) - \frac{2}{3}\ln^3(x) + 3\mathrm{Li}_2(1-x) \\
& -3\ln(x)\ln(1-x) + \frac{3}{2}\ln^2(x) + 4\zeta(2)\ln(x) + 6\zeta(3) - 3\zeta(2) + \frac{3}{8} \bigg) \\
& +4(1+x)\mathrm{S}_{12}(x) + \frac{1+x}{2}\ln^3(x) - 2(1+x)\ln^2(x)\ln(1-x) \\
& +(6x-2)\mathrm{Li}_2(1-x) + (6-2x)\ln(x)\ln(1-x) + \left(\frac{11}{4}x - \frac{9}{4}\right)\ln^2(x) \\
& +(6-4x)\ln(1-x) + (5x-3)\ln(x) + 2x - \frac{1}{2} \Bigg]_+
\end{aligned}
$$

OUTLOOK

- Having high theoretical precision for the normalization processes $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, and $e^+e^- \to 2\gamma$ with *polarized* beams is crucial for future e^+e^- colliders
- There are several two-loop QED results, but most of them are w/o polarization yet
- Two-loop EW RC are in progress, polarization to be foreseen from the beginning
- The SANC computer system is being upgraded
- New Monte Carlo codes are required
- $\mathrm{O}(\alpha^3 L^2)$ collinear radiator factors are derived. First, they will be implemented in the ZFITTER code.